INSTITUTO POLITECNICO NACIONAL

ESCUELA SUPERIOR DE COMPUTO

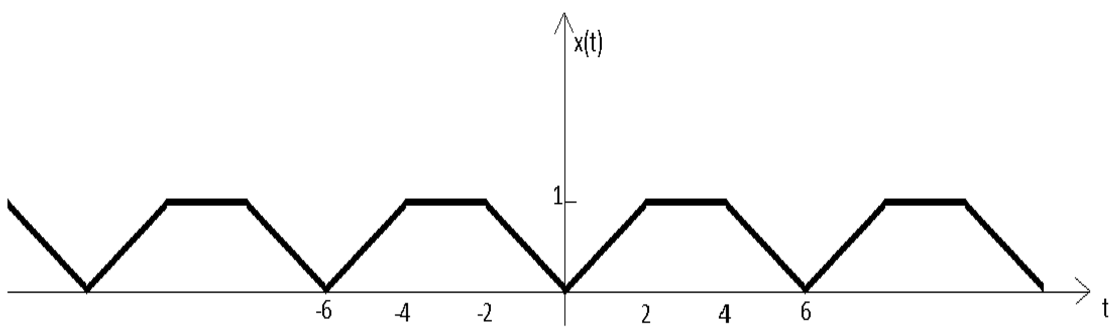
Teoría de Comunicaciones y Señales

1er. Exámen departamental

NOMBRE:____ **TIPO: A**

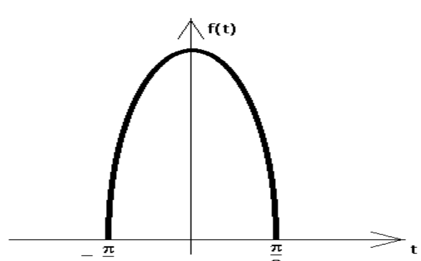
GRUPO:____________________________

Problema 1 ***(valor 2.0 ptos).*** Encuentre la Serie Trigonométrica de Fourier de x(t)



Problema 2 ***(valor 1.5 ptos).*** A partir de la serie obtenida en el problema anterior, encuentra la Serie Exponencial de Fourier.

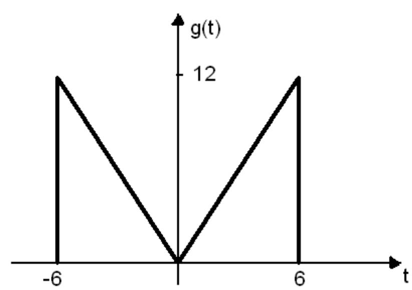
Problema 3 ***(valor 2.0 ptos).*** Encuentre la transformada de Fourier de la siguiente función



Problema 4 ***(valor 2.5 ptos)*****.** Usando propiedades, encuentre la transformada de Fourier de:

$$-6\delta\left[5t-10\right]\cdot\cos 15t \quad + \quad \frac{1}{3-jt}\cdot t^{2} \quad + \quad e^{j4t}(t-1) \quad \leftrightarrow \quad ?$$

Problema 5 ***(valor 2.0 ptos)*****.** Usando Propiedades de la transformada de Fourier encuentre la transformada de g(t)



1er Departamental TIPO A

Problema 1

$$x(t) = \begin{cases} \frac{1}{2}t & 0 < t \leq 2 \\ 1 & 2 < t \leq 4 \\ -\frac{1}{2}(t-6) & 4 < t \leq 6 \\ x(t+6) & \text{otro caso} \end{cases}$$

$T = 6 \therefore \omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}$

como $x(t)$ es par

$\therefore b_n = 0$

&

$$a_n = \frac{4}{T}\int_0^{\frac{T}{2}} x(t) \cos n\omega_0 t \, dt \quad ; \quad a_0 = \frac{2}{T}\int_0^{\frac{T}{2}} x(t) dt$$

$$a_n = \frac{4}{6}\int_0^{3} x(t) \cos \frac{n\pi}{3} t \, dt$$

$$a_n = \frac{2}{3}\int_0^{2} \frac{1}{2} t \cdot \cos \frac{n\pi}{3} t \, dt + \frac{2}{3}\int_2^{3} \cos \frac{n\pi}{3} t \, dt$$

$u = t \quad dv = \cos \frac{n\pi}{3} t \, dt$

$du = dt \quad v = \frac{3}{n\pi} \operatorname{sen} \frac{n\pi}{3} t$

$$a_n = \frac{1}{3}\left\{ \frac{3t}{n\pi} \operatorname{sen} \frac{n\pi}{3} t \Big|_0^2 - \frac{3}{n\pi}\int_0^2 \operatorname{sen} \frac{n\pi}{3} t \, dt \right\} + \left(\frac{2}{\cancel{3}}\right)\left(\frac{\cancel{3}}{n\pi}\right) \operatorname{sen} \frac{n\pi}{3} t \Big|_2^3$$

$$a_n = \frac{1}{3}\left\{ \frac{6}{n\pi} \operatorname{sen} \frac{2}{3} n\pi + \frac{9}{n^2\pi^2} \cos \frac{n\pi}{3} t \Big|_0^2 \right. + \frac{2}{n\pi}\left(\underbrace{\operatorname{sen} n\pi}_{0} - \operatorname{sen} \frac{2}{3} n\pi \right)$$

$$a_n = \cancel{\frac{2}{n\pi} \operatorname{sen} \frac{2}{3} n\pi} + \frac{3}{n^2\pi^2}\left[\cos \frac{2}{3} n\pi - 1\right] - \cancel{\frac{2}{n\pi} \operatorname{sen} \frac{2}{3} n\pi} \quad \forall \, n \neq 0$$

$$a_n = \frac{3}{n^2\pi^2}\left[\cos \frac{2}{3} n\pi - 1\right]$$

$$a_0 = \frac{2}{6}\int_0^{3} x(t) dt$$

$$a_0 = \frac{1}{3}\int_0^{2} \frac{1}{2} t \, dt + \frac{1}{3}\int_2^{3} dt$$

$$a_0 = \frac{1}{6}\left[\frac{t^2}{2}\right]_0^2 + \frac{1}{3} t \Big|_2^3$$

$$a_0 = \frac{1}{12}\left[4 - 0\right] + \frac{1}{3}\left[3 - 2\right]$$

$$a_0 = \frac{1}{3} + \frac{1}{3}$$

$$a_0 = \frac{2}{3}$$

Finalmente:

$$x(t) = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{3}{n^2\pi^2}\left(\cos \frac{2}{3} n\pi - 1\right) \cdot \cos \frac{n\pi}{3} t$$

Problema 2

Si $x(t) = \frac{2}{3} + \sum_{n=1}^{\infty} \frac{3}{n^2\pi^2}\left(\cos \frac{2}{3}n\pi - 1\right) \cdot \cos \frac{n\pi}{3} t$

entonces su serie exponencial está dada por:

$C_n = \frac{1}{2}(a_n - i b_n)$, con $b_n = 0$

así $C_n = \frac{1}{2} \cdot \frac{3}{n^2\pi^2}\left(\cos\left(\frac{2}{3}n\pi\right) - 1\right) \quad \forall\, n \neq 0$

$C_0 = a_0 = \frac{2}{3}$

Finalmente:

$$x(t) = \frac{2}{3} + \sum_{n=-\infty}^{\infty} \frac{3}{2n^2\pi^2}\left(\cos \frac{2}{3}n\pi - 1\right) \cdot e^{\frac{in\pi}{3}t}$$

Problema 3

Usando Propiedades y dado que $f(t)$ es una función compuesta y definida por:

$$f(t) = \cos t \cdot C_\pi(t)$$

entonces la pareja que buscamos completar es:

$$\cos t \cdot C_\pi(t) \longleftrightarrow ?$$

Asi de tablas

$$A C_d(t) \longleftrightarrow Ad\, Sa \frac{\omega d}{2}$$

$$C_\pi(t) \longleftrightarrow \pi\, Sa \frac{\pi}{2}\omega$$

$$C_\pi(t) \cdot \cos t \longleftrightarrow \frac{1}{2}\left[\pi\, Sa \frac{\pi}{2}(\omega+1) + \pi\, Sa \frac{\pi}{2}(\omega-1)\right]$$

$$C_\pi(t) \cdot \cos t \longleftrightarrow \frac{\pi}{2}\left[Sa \frac{\pi}{2}(\omega+1) + Sa \frac{\pi}{2}(\omega-1)\right]$$

Problema 3 por definición

$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt \qquad \text{Si } f(t) = \begin{cases} \cos t & -\frac{\pi}{2} < t < \frac{\pi}{2} \\ \phi & \text{otro caso} \end{cases}$$

$$\mathcal{F}\{f(t)\} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t \cdot e^{-j\omega t}\, dt = \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (e^{jt} + e^{-jt})\, e^{-j\omega t}\, dt$$

$$\mathcal{F}\{f(t)\} = F(\omega) = \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[e^{-j(\omega-1)t} \cdot e^{-j(\omega+1)t} \right] dt$$

$$= \frac{1}{2}\left\{ -\frac{1}{j(\omega-1)} e^{-j(\omega-1)t}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{1}{j(\omega+1)} e^{-j(\omega+1)t}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \right\}$$

$$= -\frac{1}{2i}\left\{ \frac{e^{-j(\omega-1)\frac{\pi}{2}} - e^{+j(\omega-1)\frac{\pi}{2}}}{\omega-1} + \frac{e^{-j(\omega+1)\frac{\pi}{2}} - e^{j(\omega+1)\frac{\pi}{2}}}{\omega+1} \right\}$$

$$= -\frac{1}{2i}\left\{ \frac{e^{-j\frac{\pi}{2}\omega} \cdot \underbrace{\cancel{e^{j\frac{\pi}{2}}}}_{j} - e^{j\frac{\pi}{2}\omega} \cdot \underbrace{\cancel{e^{-j\frac{\pi}{2}}}}_{-j}}{\omega-1} + \frac{e^{-j\frac{\pi}{2}\omega} \cdot \underbrace{\cancel{e^{-j\frac{\pi}{2}}}}_{-j} - e^{j\frac{\pi}{2}\omega} \cdot \underbrace{\cancel{e^{j\frac{\pi}{2}}}}_{j}}{\omega+1} \right\}$$

$$= -\frac{1}{2i}\left\{ \frac{j e^{-j\frac{\pi}{2}\omega} + j e^{j\frac{\pi}{2}\omega}}{\omega-1} + \frac{-j e^{-j\frac{\pi}{2}\omega} - j e^{j\frac{\pi}{2}\omega}}{\omega+1} \right\}$$

$$= -\frac{1}{2i}\left\{ \frac{(\omega+1)(j e^{-j\frac{\pi}{2}\omega} + j e^{j\frac{\pi}{2}\omega}) + (\omega-1)(-j e^{-j\frac{\pi}{2}\omega} - j e^{j\frac{\pi}{2}\omega})}{(\omega-1)(\omega+1)} \right\}$$

$$= -\frac{1}{2i}\left\{ \frac{j(\omega+1)(2\cos\frac{\pi}{2}\omega) + j(\omega-1)(2\cos\frac{\pi}{2}\omega)}{\omega^2-1} \right\}$$

$$= -\frac{1}{2i}\left\{ \frac{\cancel{2i\omega\cos\frac{\pi}{2}\omega} + 2i\cos\frac{\pi}{2}\omega - \cancel{2i\omega\cos\frac{\pi}{2}\omega} + 2i\cos\frac{\pi}{2}\omega}{\omega^2-1} \right\}$$

$$= -\frac{1}{2i}\left\{ \frac{4i\cos\frac{\pi}{2}\omega}{\omega^2-1} \right\} = -\frac{2\cos\frac{\pi}{2}\omega}{\omega^2-1}$$

$$\mathcal{F}\{f(t)\} = F(\omega) = \frac{2\cos\frac{\pi}{2}\omega}{1-\omega^2}$$

Problema 4

$$\underbrace{-6\,\delta(5t-10)\cdot \cos 15t}_{\mathrm{I}} + \underbrace{\frac{1}{3-jt}t^2}_{\mathrm{II}} + \underbrace{e^{j4t}(t-1)}_{\mathrm{III}} \longleftrightarrow ?$$

SOLUCION:

I Si $\delta(t) \longleftrightarrow 1$

$$\delta(t-10) \longleftrightarrow e^{-j10\omega}$$

$$\delta(5t-10) \longleftrightarrow \frac{1}{|5|}e^{-j\frac{10}{5}\omega}$$

$$\delta(5t-10)\cdot \cos 15t \longleftarrow \frac{1}{10}\left[e^{-j2(\omega+15)} + e^{-j2(\omega-15)}\right]$$

$$-6\,\delta(5t-10)\cdot \cos 15t \longleftrightarrow -\frac{3}{5}\left(e^{-j2(\omega+15)} + e^{-j2(\omega-15)}\right]$$

II Si $e^{-at}u(t) \longleftrightarrow \frac{1}{a+j\omega}$

$$\frac{1}{a+jt} \longleftrightarrow 2\pi e^{-a(-\omega)}u(-\omega)$$

$$\frac{1}{3+jt} \longleftrightarrow 2\pi e^{3\omega}u(-\omega)$$

$$\frac{1}{3-jt} \longleftrightarrow 2\pi e^{-3\omega}u(\omega)$$

$$\frac{(-jt)^2}{3-jt} \longleftrightarrow 2\pi\frac{d^2}{d^2\omega}\left[e^{-3\omega}u(\omega)\right]$$

$$\frac{t^2}{3-jt} \longleftrightarrow -2\pi\frac{d^2}{d^2\omega}\left[e^{-3\omega}u(\omega)\right]$$

III Si $\delta(t) \longleftrightarrow 1$

$$1 \longleftrightarrow 2\pi\delta(-\omega)$$

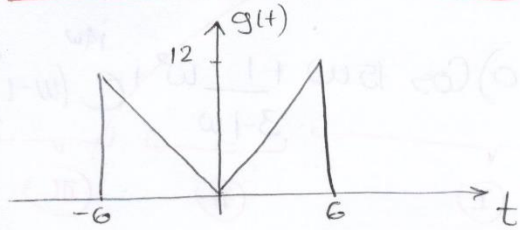
$$-jt \longleftrightarrow 2\pi\frac{d}{d\omega}\delta(\omega)$$

$$t-1 \longleftrightarrow 2\pi j\frac{d}{d\omega}\delta(\omega)\cdot e^{-j\omega}$$

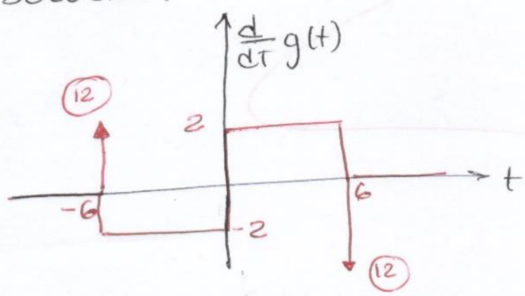
$$e^{j4t}\cdot(t-1) \longleftrightarrow 2\pi j\frac{d}{d\omega}\delta(\omega+4)e^{-j(\omega+4)}$$

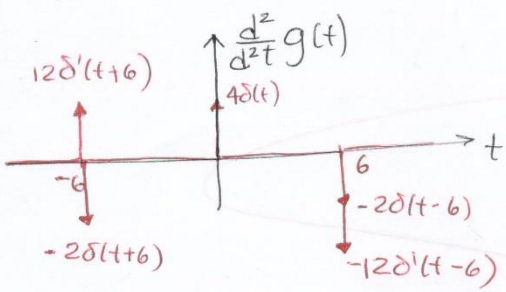
$$e^{j4t}\cdot(t-1) \longleftrightarrow j2\pi e^{-j(\omega-4)}\frac{d}{d\omega}\delta(\omega-4)$$

Problema 5 (2.0 pts)



SOLUCION:





$$\frac{d^2}{d^2t} g(t) = -2\delta(t+6) + 12\delta'(t+6) + 4\delta(t) - 2\delta(t-6) - 12\delta'(t-6)$$

$$\mathcal{F}\left\{\frac{d^2}{d^2t} g(t)\right\} = -2\mathcal{F}\{\delta(t+6)\} + 12\mathcal{F}\{\delta'(t+6)\} + 4\mathcal{F}\{\delta(t)\} - 2\mathcal{F}\{\delta(t-6)\} - 12\mathcal{F}\{\delta'(t-6)\}$$

$$\mathcal{F}\left\{\frac{d^2}{d^2t} g(t)\right\} = -2e^{i6\omega} + 12 j\omega e^{i6\omega} + 4 - 2e^{-i6\omega} - 12 j\omega e^{-i6\omega}$$

$$= -2(e^{i6\omega} + e^{-i6\omega}) + 12 j\omega (e^{i6\omega} - e^{-i6\omega}) + 4$$

$$= -4\cos 6\omega - 24\omega \operatorname{Sen} 6\omega + 4$$

$$= 4(1 - \cos 6\omega) - 24\omega \operatorname{Sen} 6\omega$$

$$= 8 \cdot \operatorname{sen}^2 3\omega - 24\omega \operatorname{Sen} 6\omega$$

Así

$$\frac{d^2}{d^2t} g(t) \longleftrightarrow 8\operatorname{Sen}^2 3\omega - 24\omega \operatorname{Sen} 6\omega$$

De la Prop. de diferenciación en t

Si $g(t) \longleftrightarrow G(\omega)$

$$\frac{d^2}{d^2t} g(t) \longleftrightarrow (j\omega)^2 G(\omega)$$

Finalmente

$$(j\omega)^2 G(\omega) = 8\operatorname{sen}^2 3\omega - 24\omega \operatorname{Sen} 6\omega$$

$$G(\omega) = \frac{-1}{\omega^2}\left[8\operatorname{sen}^2 3\omega - 24\omega \operatorname{Sen} 6\omega\right]$$

$$G(\omega) = \frac{24}{\omega}\operatorname{Sen} 6\omega - \frac{8}{\omega^2}\operatorname{sen}^2 3\omega$$

$$G(\omega) = 144\, Sa\, 6\omega - 8(9)\frac{\operatorname{sen} 3\omega}{(3)\omega} \cdot \frac{\operatorname{sen} 3\omega}{(3)\omega}$$

$$G(\omega) = 144\, Sa\, 6\omega - 72\, Sa^2\, 3\omega$$