INSTITUTO POLITECNICO NACIONAL

ESCUELA SUPERIOR DE COMPUTO

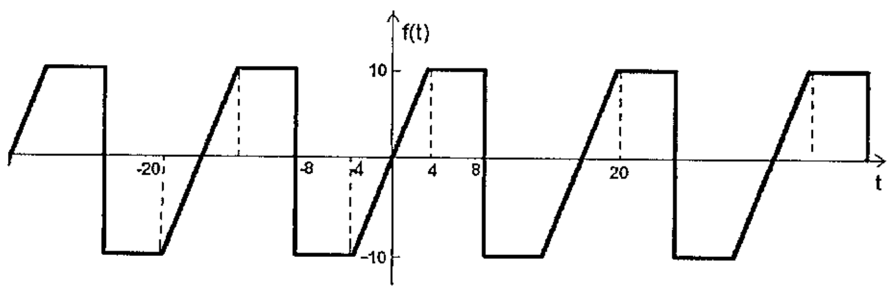
Teoría de Comunicaciones y Señales

1er. Exámen departamental

NOMBRE:____ TIPO: B

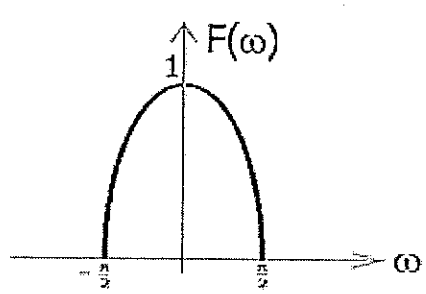
GRUPO:______________________

Problema 1. Encuentre la Serie Trigonométrica de Fourier de x(t)



Problema 2. A partir de la serie obtenida en el problema anterior, encuentra la Serie Exponencial de Fourier.

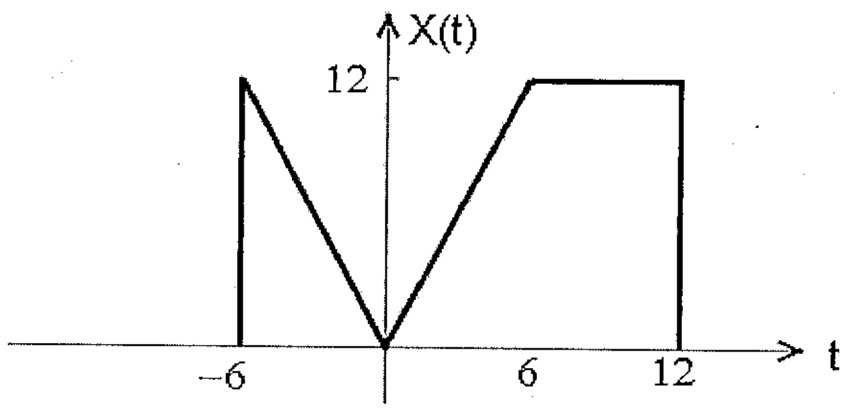
Problema 3. Encuentre la transformada inversa de Fourier de la siguiente función:



Problema 4. Usando propiedades, y a partir de la tabla de transformadas, complete la transformada de Fourier que se indica:

$$2\delta\left[\frac{t}{5}-10\right]\cdot sen(31t)+e^{j4t}\cdot(t-1)\cdot u(t)\leftrightarrow ?$$

Problema 5. Usando Propiedades encuentra la transformada de x(t)



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TIPO B

Problema 1

Encontrar la S.T.F de f(t)

$$f(t) = \begin{cases} -10 & -8 < t \le -4 \\ \frac{5}{2}t & -4 < t \le 4 \\ 10 & 4 < t \le 8 \\ f(t+16) & \text{otro caso} \end{cases}$$

$$T = 16 \therefore \omega_0 = \frac{2\pi}{16} = \frac{\pi}{8}$$

como f(t) es Impar ∴ $a_0 = a_n = 0$ &

$$b_n = \frac{4}{T}\int_0^{\frac{T}{2}} f(t)\cdot \operatorname{Sen} n\omega_0 t\, dt$$

$$b_n = \frac{4}{16}\int_0^4 \frac{5}{2}t \cdot \operatorname{Sen}\frac{n\pi}{8}t\, dt + \frac{4}{16}\int_4^8 10 \operatorname{Sen}\frac{n\pi}{8}t\, dt$$

$$b_n = \frac{5}{8}\int_0^4 t \operatorname{Sen}\frac{n\pi}{8}t\, dt + \frac{5}{2}\int_4^8 \operatorname{Sen}\frac{n\pi}{8}t\, dt$$

$u = t \quad dv = \operatorname{sen}\frac{n\pi}{8}t\, dt$

$du = dt \quad v = -\frac{8}{n\pi}\operatorname{Cos}\frac{n\pi}{8}t$

$$b_n = \frac{5}{8}\left[\frac{-8t}{n\pi}\cos\frac{n\pi}{8}t\Big|_0^4 + \frac{8}{n\pi}\int_0^4 \cos\frac{n\pi}{8}t\, dt\right] - \left(\frac{5}{2}\right)\left(\frac{8}{n\pi}\right)\operatorname{Cos}\frac{n\pi}{8}t\Big|_4^8$$

$$b_n = \frac{5}{8}\left[\frac{-32}{n\pi}\cos\frac{n\pi}{2} + \emptyset + \frac{64}{n^2\pi^2}\operatorname{Sen}\frac{n\pi}{8}t\right] - \frac{20}{n\pi}\left[\cos n\pi - \cos\frac{n\pi}{2}\right]$$

$$b_n = \cancel{\frac{-20}{n\pi}\cos\frac{n\pi}{2}} + \frac{40}{n^2\pi^2}\left[\operatorname{sen}\frac{n\pi}{2} - \varphi\right] - \frac{20}{n\pi}\cos n\pi + \cancel{\frac{20}{n\pi}\cos\frac{n\pi}{2}}$$

$$b_n = \frac{40}{n^2\pi^2}\operatorname{sen}\frac{n\pi}{2} - \frac{20}{n\pi}\cos n\pi$$

Finalmente

$$f(t) = \sum_{n=1}^{\infty}\left[\frac{40}{n^2\pi^2}\operatorname{Sen}\frac{n\pi}{2} - \frac{20}{n\pi}\cos n\pi\right]\cdot \operatorname{Sen}\frac{n\pi}{8}t$$

2.0 pts

Problema 2

A partir de la serie

$$f(t)=\sum_{n=-\infty}^{\infty}\left[\frac{40}{n^2\pi^2}\operatorname{Sen}\frac{n\pi}{2}-\frac{20}{n\pi}\cos n\pi\right]\operatorname{Sen}\frac{n\pi}{8}t$$

Encontremos la Serie exponenc

Si $C_n=\frac{1}{2}(a_n-jb_n)$

como $a_n=0$ ∴

$$C_n=\frac{-j}{2}\left[\frac{40}{n^2\pi^2}\operatorname{Sen}\frac{n\pi}{2}-\frac{20}{n\pi}\cos n\pi\right]$$

$$C_n=j\left[\frac{-20}{n^2\pi^2}\operatorname{Sen}\frac{n\pi}{2}+\frac{10}{n\pi}\cos n\pi\right]$$

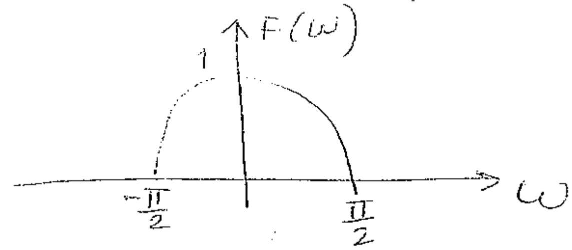
$$C_n=j\left[\frac{10}{n\pi}\cos n\pi-\frac{20}{n^2\pi^2}\operatorname{Sen}\frac{n\pi}{2}\right]$$

Finalmente:

$$f(t)=\sum_{n=-\infty}^{\infty}j\left[\frac{10}{n\pi}\cos n\pi-\frac{20}{n^2\pi^2}\operatorname{Sen}\frac{n\pi}{2}\right]e^{-j\frac{n\pi}{8}t}$$

2.0 pts

Problema 3



$$F(\omega)=\begin{cases}\cos\omega & -\frac{\pi}{2}<\omega\le\frac{\pi}{2}\\ 0 & \text{otro caso}\end{cases}$$

$$\mathcal{F}^{-1}\{F(\omega)\}=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(\omega)e^{j\omega t}\,d\omega$$

$$=\frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\omega\, e^{j\omega t}\,d\omega \quad \ldots \quad 1$$

$$\left\{\begin{matrix} u=\cos\omega & dv=e^{j\omega t}d\omega \\ du=-\operatorname{sen}\omega\, d\omega & v=\frac{1}{jt}e^{j\omega t}\end{matrix}\right\}$$

$$=\frac{1}{2\pi}\left[\frac{\cos\omega}{jt}e^{j\omega t}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}+\frac{1}{jt}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\operatorname{sen}\omega\, e^{j\omega t}\ldots\right.$$

$$u=\operatorname{sen}\omega \quad dv=e^{j\omega t}d\omega$$

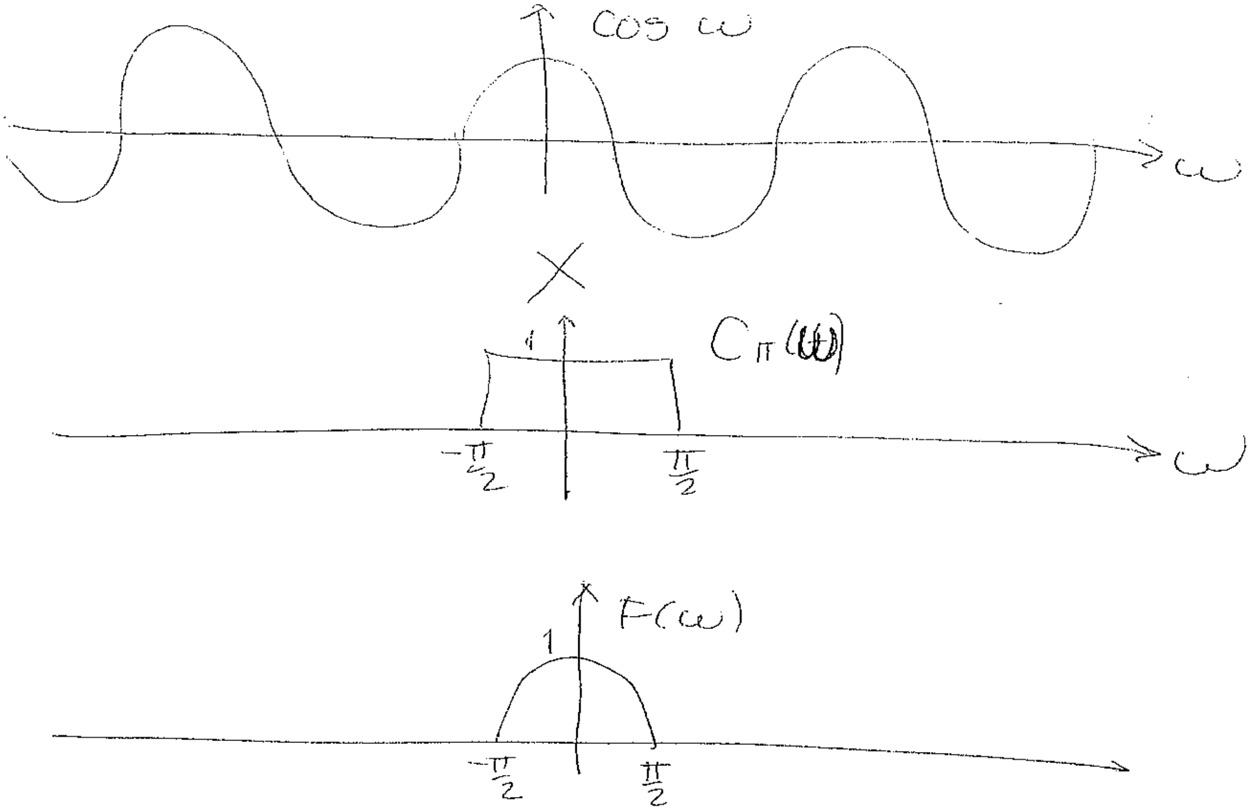
$$du=\cos\omega\, d\omega \quad v=\frac{1}{jt}e^{j\omega t}$$

$$=\frac{1}{2\pi}\left[\frac{\cancel{\cos\frac{\pi}{2}}}{jt_0}e^{j\frac{\pi}{2}t}-\frac{\cancel{\cos(-\frac{\pi}{2})}}{jt_0}e^{-j\frac{\pi}{2}t}\right.$$

$$+\frac{1}{jt}\left(\frac{1}{jt}\operatorname{sen}\omega\, e^{j\omega t}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}-\frac{1}{jt}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\omega\, e\ldots\right.$$

Problema 3. Otra forma

Ya que $F(\omega)$ es una función compuesta



$$\mathcal{F}^{-1}\{F(\omega)\} = \mathcal{F}^{-1}\{\cos\omega \cdot C_\pi(\omega)\}$$

De tablas

$$A C_d(t) \longleftrightarrow A d \, Sa \frac{\omega d}{2}$$

$$A d \, Sa \frac{td}{2} \longleftrightarrow 2\pi A C_d(-\omega)$$

$$\frac{\cancel{A} d}{2\pi \cancel{A}} Sa \frac{td}{2} \longleftrightarrow C_d(\omega)$$

$$\frac{d}{2\pi} Sa \frac{td}{2} \longleftrightarrow C_d(\omega)$$

$$\frac{\pi}{2\pi} Sa \frac{\pi}{2} t \longleftrightarrow C_\pi(\omega)$$

Por la Prop. de Modulación Dual

si $f(t) \longleftrightarrow F(\omega)$

$$\left[f(t+t_0) + f(t-t_0)\right] \longleftrightarrow F(\omega) \cos t_0 \omega$$

$$\frac{1}{2}\left[\frac{1}{2} Sa \frac{\pi}{2}(t+1) + \frac{1}{2} Sa \frac{\pi}{2}(t-1)\right] \longleftrightarrow C_\pi(\omega) \cdot \cos \omega$$

$$\frac{1}{4}\left[Sa \frac{\pi}{2}(t+1) + Sa \frac{\pi}{2}(t-1)\right] \longleftrightarrow$$

20 pts

... continuación ↓

$$\mathcal{F}^{-1}\{F(\omega)\} = \frac{1}{2\pi j t}\left[\frac{1}{jt}\operatorname{sen}\frac{\pi}{2}e^{j\frac{\pi}{2}t} - \frac{1}{jt}\operatorname{sen}\left(-\frac{\pi}{2}\right)e^{-j\frac{\pi}{2}t}\right]$$

$$\frac{1}{2\pi j^2 t^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\omega\, e^{j\omega t}\, d\omega$$

$$-\frac{1}{2\pi t^2}\left[e^{j\frac{\pi}{2}t}\right] + \frac{1}{2\pi t^2}\left(-e^{-j\frac{\pi}{2}t}\right)$$

$$+\frac{1}{2\pi t^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\omega\, e^{j\omega t}\, d\omega$$

$$-\frac{1}{2\pi t^2}e^{j\frac{\pi}{2}t} - \frac{1}{2\pi t^2}e^{-j\frac{\pi}{2}t}$$

$$+\frac{1}{2\pi t^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\omega\, e^{j\omega t}\, d\omega$$

$$-\frac{1}{\cancel{2}\pi t^2}\cdot\cancel{2}\cos\frac{\pi}{2}t + \frac{1}{2\pi t^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\omega\, e^{j\omega t}\, d\omega$$

De (1)

$$\frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\omega\, e^{j\omega t}\, d\omega = -\frac{1}{\pi t^2}\cos\frac{\pi}{2}t + \frac{1}{2\pi t^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\omega\, e^{j\omega t}\, d\omega$$

$$\left(\frac{1}{2\pi} - \frac{1}{2\pi t^2}\right)\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\omega\, e^{j\omega t}\, d\omega = -\frac{1}{\pi t^2}\cos\frac{\pi}{2}t$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\omega\, e^{j\omega t}\, d\omega = \left(\frac{2\pi t^2}{t^2-1}\right)\left(-\frac{1}{\pi t^2}\cos\frac{\pi}{2}t\right)$$

$$= \frac{-2\cos\frac{\pi}{2}t}{t^2-1}$$

así de (1)

$$\mathcal{F}^{-1}\{F(\omega)\} = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos\omega\, e^{j\omega t}\, d\omega$$

$$\frac{1}{2\pi}\left[\frac{-2\cos\frac{\pi}{2}t}{t^2-1}\right] = -\frac{\cos\frac{\pi}{2}t}{\pi(t^2-1)}$$

$$f(t) = -\frac{\cos\frac{\pi}{2}t}{\pi(t^2-1)}$$

Demostracion (Continuacion Problema 5)

$$\frac{1}{4}\,Sa\,\frac{\pi}{2}(t+1) + \frac{1}{4}\,Sa\,\frac{\pi}{2}(t-1)$$

$$= \frac{1}{4}\,\frac{\operatorname{sen}\frac{\pi}{2}(t+1)}{\frac{\pi}{2}(t+1)} + \frac{1}{4}\,\frac{\operatorname{sen}\frac{\pi}{2}(t-1)}{\frac{\pi}{2}(t-1)}$$

$$= \frac{1}{2\pi}\,\frac{\operatorname{sen}\frac{\pi}{2}(t+1)}{(t+1)} + \frac{1}{2\pi}\cdot\frac{\operatorname{sen}\frac{\pi}{2}(t-1)}{t-1}$$

$$= \frac{1}{2\pi}\cdot\frac{e^{j\frac{\pi}{2}(t+1)} - e^{-j\frac{\pi}{2}(t+1)}}{(t+1)2i} + \frac{1}{2\pi}\cdot\frac{e^{j\frac{\pi}{2}(t-1)} - e^{-j\frac{\pi}{2}(t-1)}}{(t-1)2i}$$

$$= \frac{1}{4\pi i}\cdot\frac{e^{j\frac{\pi}{2}t}\cdot\underbrace{e^{j\frac{\pi}{2}}}_{j} - e^{-j\frac{\pi}{2}t}\cdot\underbrace{e^{-j\frac{\pi}{2}}}_{-j}}{t+1} + \frac{j}{4\pi i}\cdot\frac{e^{j\frac{\pi}{2}t}\cdot e^{-j\frac{\pi}{2}} - e^{-j\frac{\pi}{2}t}\cdot e^{j\frac{\pi}{2}}}{t-1}$$

$$= \frac{j e^{j\frac{\pi}{2}} + j e^{-j\frac{\pi}{2}t}}{4\pi i\,(t+1)} + \frac{-j e^{j\frac{\pi}{2}t} - j e^{-j\frac{\pi}{2}t}}{4\pi i\,(t-1)}$$

$$= \frac{(t-1)\left(2i\cos\frac{\pi}{2}t\right) + (t+1)\left(-2j\cos\frac{\pi}{2}t\right)}{4\pi i\,(t+1)(t-1)}$$

$$= \frac{\cancel{2it\cos\frac{\pi}{2}t} - 2i\cos\frac{\pi}{2}t \cancel{- 2it\cos\frac{\pi}{2}t} - 2i\cos\frac{\pi}{2}t}{4\pi i\,(t^2-1)}$$

$$= \frac{-\cancel{4i}\cos\frac{\pi}{2}t}{\cancel{4}\pi\cancel{i}\,(t^2-1)}$$

$$= \frac{-\cos\frac{\pi}{2}t}{\pi\,(t^2-1)}$$

Problema 4

$$\underbrace{2\,\delta\left(\tfrac{t}{5}-10\right)\,\text{Sen } 31t}_{\text{I}} + \underbrace{e^{j4t}(t-1)u(t)}_{\text{II}} \longleftrightarrow ?$$

I

Si $\delta(t) \longleftrightarrow 1$

$$\delta(t-10) \longleftrightarrow e^{-j10\omega}$$

$a = \frac{1}{5}$

$$\delta\left(\tfrac{t}{5}-10\right) \longleftrightarrow 5\,e^{-j10\cdot 5\omega}$$

$$2\cdot\delta\left(\tfrac{t}{5}-10\right) \longleftrightarrow (2)5\,e^{-j50\omega}$$

$$2\delta\left(\tfrac{t}{5}-10\right)\text{Sen } 31t \longleftrightarrow \frac{j}{2}\left[10\,e^{-j50(\omega+31)} - 10\,e^{-j50(\omega-31)}\right]$$

$$2\delta\left(\tfrac{t}{5}-10\right)\text{Sen } 31t \longleftrightarrow 5i\left[e^{-j50(\omega+31)} - e^{-j50(\omega-31)}\right]$$

II Si $u(t) \longleftrightarrow \pi\delta(\omega) + \frac{1}{i\omega}$

$$u(t+1) \longleftrightarrow \left(\pi\delta(\omega) + \frac{1}{i\omega}\right)e^{j\omega}$$

$$-jt\,u(t+1) \longleftrightarrow \frac{d}{d\omega}\left[\left(\pi\delta(\omega) + \frac{1}{j\omega}\right)e^{j\omega}\right]$$

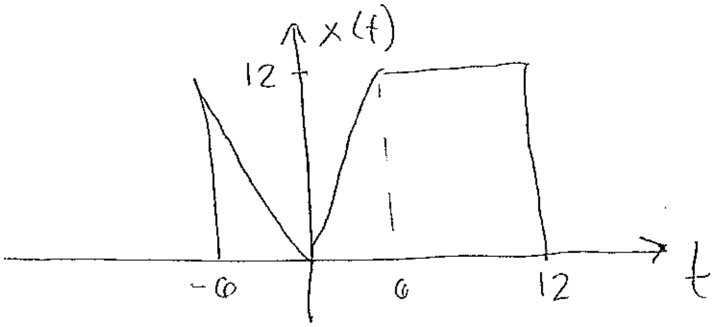
$$(t-1)\,u(t) \longleftrightarrow j\frac{d}{d\omega}\left[\left(\pi\delta(\omega) + \frac{1}{i\omega}\right)e^{j\omega}\right]e^{-i\omega}$$

$$e^{j4t}(t-1)u(t) \longleftrightarrow j\,\frac{d}{d\omega}\left[\pi\delta(\omega-4) + \frac{1}{i(\omega-4)}e^{i(\omega-4)}\right]e^{-i(\omega-4)}$$

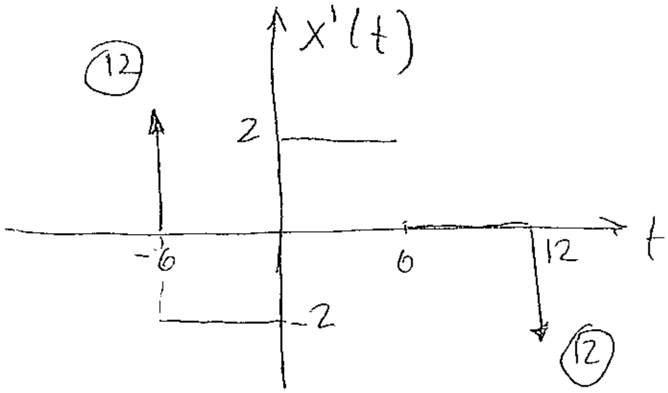
2,0 pts

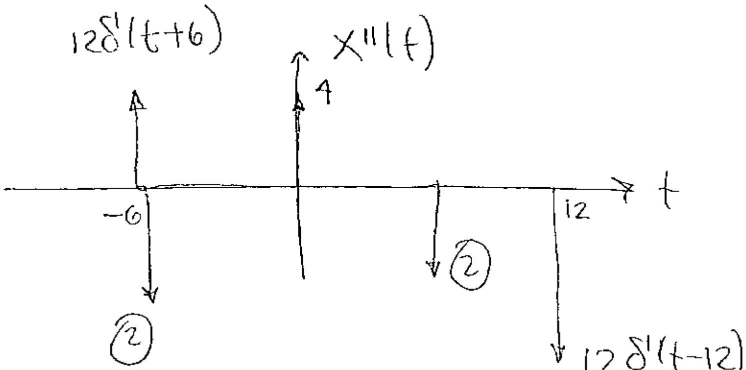
Problema 5

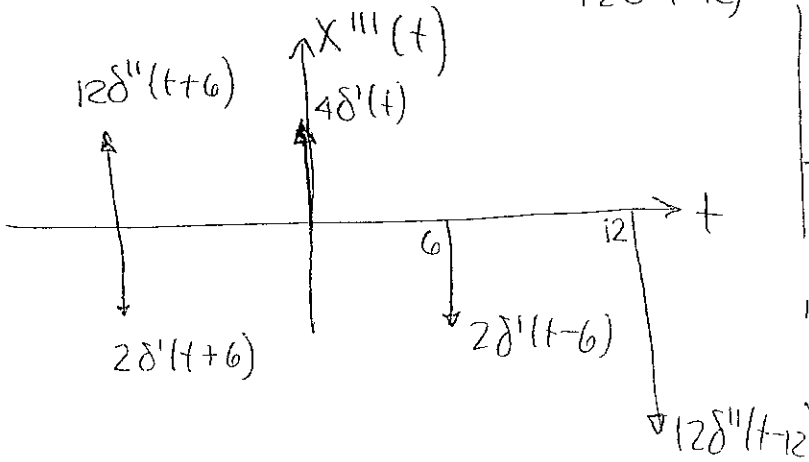
Encontremos la transformada de $x(t)$ por Propiedades



(1)







(2)

$$x'''(t) = 12\delta''(t+6) - 12\delta''(t-6) - 2\delta'(t+6) - 2\delta'(t-6) + 4\delta'(t)$$

(3)

$$\mathcal{F}\{x'''(t)\} = 12\mathcal{F}\{\delta''(t+6)\} - 12\mathcal{F}\{\delta''(t-6)\} - 2\mathcal{F}\{\delta'(t+6)\} - 2\mathcal{F}\{\delta'(t-6)\} + 4\mathcal{F}\{\delta'(t)\}$$

Si $\delta(t) \longleftrightarrow 1$

$\delta(t\pm 6) \longleftrightarrow e^{\pm i6\omega}$

$\delta'(t\pm 6) \longleftrightarrow j\omega e^{\pm i6\omega}$

$\delta''(t\pm 6) \longleftrightarrow (j\omega)^2 e^{\pm i6\omega}$

$$\mathcal{F}\{x'''(t)\} = 12(j\omega)^2 e^{j6\omega} - 12(j\omega)^2 e^{-j6\omega} - 2j\omega e^{j6\omega} - 2j\omega e^{-i6\omega} + 4j\omega$$

$$\mathcal{F}\{x'''(t)\} = 12(j\omega)^2 \left[\frac{e^{j6\omega} - e^{-i6\omega}}{2i}\right] - 2j\omega\left[\frac{e^{j6\omega} + e^{-i6\omega}}{2}\right](2) + 4i\omega$$

$$\mathcal{F}\{x'''(t)\} = -24i\omega^2 \operatorname{sen} 6\omega - 4i\omega\cos 6\omega + 4i\omega$$

(4) Si $f(t) \longleftrightarrow F(\omega)$

$\therefore f'''(t) \longleftrightarrow (j\omega)^3 F(\omega)$

Asi $(j\omega)^3 F(\omega) = -24i\omega^2 \operatorname{Sen} 6\omega - 4i\omega\cos 6\omega + 4i\omega$

$$F(\omega) = \frac{24}{\omega}\operatorname{sen} 6\omega + \frac{4}{\omega^2}\cos 6\omega - \frac{4}{\omega^2}$$

$F(\omega) = 144 Sa\, 6\omega - 72 Sa\, 3$