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Lecture 11. Graph SLAM II

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1 Chi squared error

$$\chi^2 = \sum \|\cdot\|^2_{\Sigma_i} + \|\cdot\|^2_{\Sigma_k} = \|A\delta - b\|^2_2$$

If χ^2 has converged ($\delta = 0$), then

$$\chi^2\big|_{\delta=0} = b^T b$$

1.1 Information matrix ($A^T A = \Lambda$)

SAM is a MAP estimator of $x_{0:t_i}$

$$\arg\max_\theta P(\mathcal{X}, \mathcal{M}, \mathcal{Z}, \mathcal{U}) \xrightarrow{(-\log\,,\ \text{linearisation})} \arg\min_\delta \|A\delta - b\|^2_2$$

In fact, all these factors express a distribution as well.

$$\begin{aligned}\|A\delta - b\|^2_2 = (A\delta - b)^T(A\delta - b) = \delta^T A^T A\delta - \delta^T A^T b - b^T A\delta + b^T b &\overset{\text{if } b = A\mu}{=} \\ \delta^T A^T A\delta - 2\delta^T A^T A\mu + \mu^T A^T A\mu &= \\ (\delta - \mu)^T \underbrace{A^T A}_{\Lambda}(\delta - \mu)&\end{aligned}$$

1.2 Normal equation

$$\begin{aligned} A\delta &= b \Rightarrow (\delta = A^{-1} \cdot b) \\ A^T A\delta &= A^T b \\ \delta &= (A^T A)^{-1} A^T b. \qquad \text{complexity of this: } \mathcal{O}(n^3)\end{aligned}$$

A is space $\to$ exploit by SOTA Linear algebra.

1.3 Cholesky factorisation

$\Lambda = A^T A = L \cdot L^T = R^T R$, where R is an upper-triangular matrix (can be a lower-triang.)

$$\begin{aligned} A^T A\delta &= A^T b \\ &\Downarrow \text{(cholesky)} \\ R^T R\delta &= A^T b \qquad \text{(Squared root method)}\end{aligned}$$

$$\left[\begin{aligned} R^T \cdot y &= A^T b \\ R \cdot \delta &= y \end{aligned}\right.$$

Equations above solved efficiency by back-substitution

Example:

$$\begin{bmatrix} 1 & 0 & 0 \\ 5 & 7 & 0 \\ 6 & -7 & 3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 5 \end{bmatrix}$$

Solving:

1. $y_1 = 2$
2. $5 \cdot 2 + 7y_2 = 5 \Rightarrow y_2 = -\frac{5}{7}$
3. $6y_1 - 7y_2 + 3y_3 = 5$
   $6 \cdot 2 - 7(-\frac{5}{7}) + 3y_3 \Rightarrow y_3 = \dfrac{5 - 12 - 5}{3} = 4$

Cholesky factorisation regiven to solve 2 systems by back-substitution

Other efficient factorizations include:

- QR factorization
- Schur complement (to marginalize landmarks)
- Exploiting sparsity of A, so calling sparse methods
- Ordering of nodes factorizations resulting in more zero elements.

2 Pose SLAM

Only poses are estimated.

Example: 2D

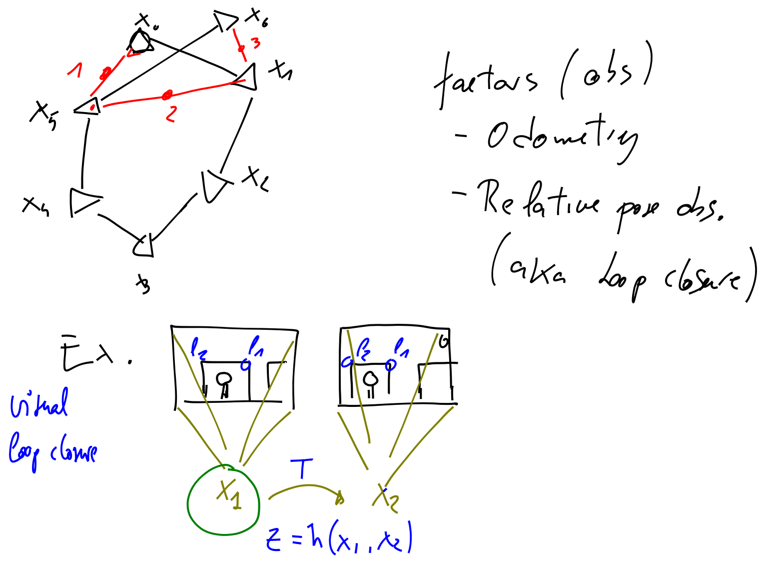

Figure 1: Poses are observed from different poses, for instance, from x_5 we observe the initial pose x_0. In the bottom part there is an example for visual loop closure, relating a pair of poses from visual information.

2.1 Observations in 2D poses

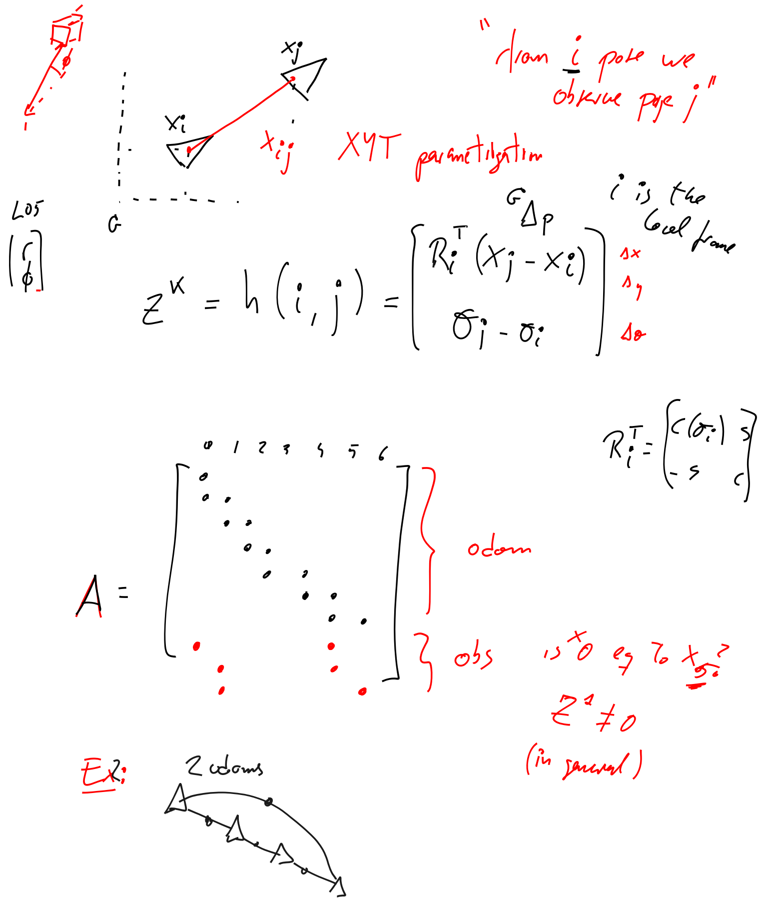

Figure 2: Observations in the adjacency matrix indicate relation between poses.

$$2DPoseJacobian \qquad H_k^j = \begin{bmatrix} R_i^T & 0 \\ 0 & 1 \end{bmatrix} \qquad H_k^i = \begin{bmatrix} -R_i^T & \begin{matrix} -s\Delta x + c\Delta x \\ -c\Delta x - s\Delta x \end{matrix} \\ 0 & -1 \end{bmatrix}$$