数理逻辑基础 作业 1

习题 1. 列出以下复合命题的真值表. (其中支命题 p, q, r, s 视为问题变元.)

(a) $(\neg p \wedge q) \to (\neg q \wedge r)$

(b) $(p \to q) \to (p \to r)$

(c) $\neg(p \vee (q \wedge r)) \leftrightarrow ((p \vee q) \wedge (p \vee r))$

解: (a)

$(\neg$	p	$\wedge$	$q)$	$\to$	$(\neg$	q	$\wedge$	$r)$
1	0	0	0	1	1	0	0	0
1	0	0	0	1	1	0	1	1
1	0	1	1	0	0	1	0	0
1	0	1	1	0	0	1	0	1
0	1	0	0	1	1	0	0	0
0	1	0	0	1	1	0	1	1
0	1	0	1	1	0	1	0	0
0	1	0	1	1	0	1	0	1

(b)

$(p$	$\to$	$q)$	$\to$	$(p$	$\to$	$r)$
0	1	0	1	0	1	0
0	1	0	1	0	1	1
0	1	1	1	0	1	0
0	1	1	1	0	1	1
1	0	0	1	1	0	0
1	0	0	1	1	1	1
1	1	1	0	1	0	0
1	1	1	1	1	1	1

(c)

$\neg$	$(p$	$\vee$	$(q$	$\wedge$	$r))$	$\leftrightarrow$	$((p$	$\vee$	$q)$	$\wedge$	$(p$	$\vee$	$r))$
1	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	1	0	0	0	0	0	0	1	1
1	0	0	1	0	0	0	0	1	1	0	0	0	0
0	0	1	1	1	1	0	0	1	1	1	0	1	1
0	1	1	0	0	0	0	1	1	0	1	1	1	0
0	1	1	0	0	1	0	1	1	0	1	1	1	1
0	1	1	1	0	0	0	1	1	1	1	1	1	0
0	1	1	1	1	1	0	1	1	1	1	1	1	1

习题 2. 写出由 $X_2 = \{x_1, x_2\}$ 生成的公式集 $L(X_2)$ 的三个层次: L_0, L_1 和 L_2.

解:

$$L_0 = X_2 = \{x_1, x_2\} \tag{2.1}$$

$$L_1 = \{\neg x_1, \neg x_2, x_1 \to x_1, x_1 \to x_2, x_2 \to x_1, x_2 \to x_2\} \tag{2.2}$$

$$
\begin{aligned}
L_2 = \{ & \neg(\neg x_1), \neg(\neg x_2), \\
& \neg(x_1 \to x_1), \neg(x_1 \to x_2), \neg(x_2 \to x_1), \neg(x_2 \to x_2), \\
& x_1 \to (\neg x_1), x_1 \to (\neg x_2), x_2 \to (\neg x_1), x_2 \to (\neg x_2), \\
& (\neg x_1) \to x_1, (\neg x_1) \to x_2, (\neg x_2) \to x_1, (\neg x_2) \to x_2, \\
& x_1 \to (x_1 \to x_1), x_1 \to (x_1 \to x_2), x_1 \to (x_2 \to x_1), x_1 \to (x_2 \to x_2), \\
& x_2 \to (x_1 \to x_1), x_2 \to (x_1 \to x_2), x_2 \to (x_2 \to x_1), x_2 \to (x_2 \to x_2), \\
& (x_1 \to x_1) \to x_1, (x_1 \to x_2) \to x_1, (x_2 \to x_1) \to x_1, (x_2 \to x_2) \to x_1, \\
& (x_1 \to x_1) \to x_2, (x_1 \to x_2) \to x_2, (x_2 \to x_1) \to x_2, (x_2 \to x_2) \to x_2 \}
\end{aligned}
\tag{2.3}
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