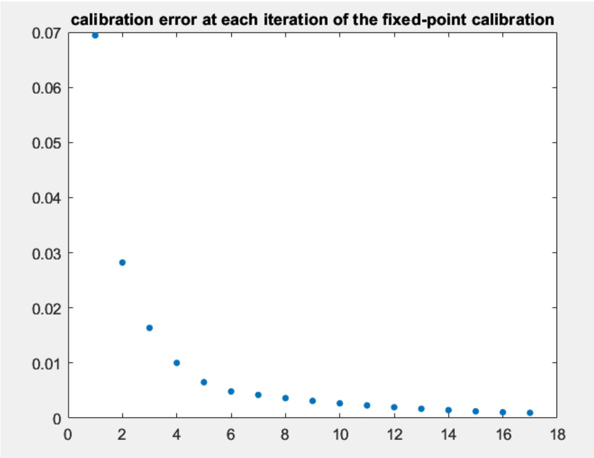
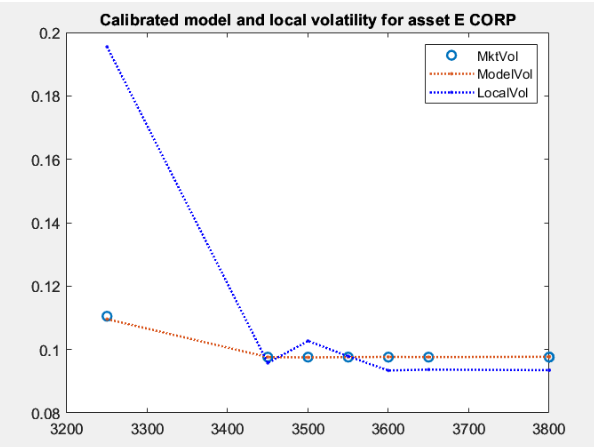
REPORT LVM GROUP 12

BIANCINI–BOTTINELLI–IPPOLITO

1.1 Calibration

Given the market data contained in the ECORP section of the file MarketData_12.xlsx, we calibrate the model with a threshold of 0.001and we obtain the calibrated local volatility matrix V, the model implied volatility at the normalized strikes and the vector of calibration error at each iteration. We then proceed to plot such data as follow:





The first plot clearly shows a monotone descending slope in the behaviour of the calibration error likely with exponential decay.

The second one shows how well our calibrated model implied volatility (in orange with blue circles at the normalized strikes) stacks against the local volatility market with the first column of the market strikes on the x-axis.

Given the two graphs the result is in line with the expected outcome and thus the calibration is deemed successful.

1.2 Pricing of call options

Using the calibrated model just found we can price call options with either with Dupire equation (Punto1_2a.m, Punto1_2b.m) or by Monte Carlo simulation (Punto1_2aMC.m, Punto1_2bMC.m).

With Dupire:

Price 454.3703 and implied volatility 0.3270 with strike 0.9*S(0)

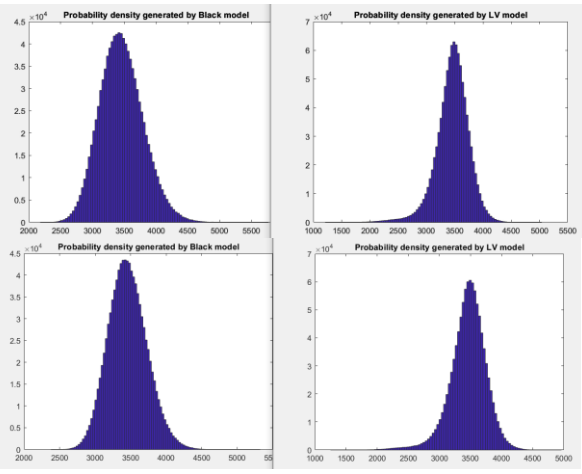
Price 160.2404 and implied volatility 0.3270 with strike 1.1*S(0)

With Monte Carlo simulation:

Price 294.8152 and implied volatility 0.1308 with strike 0.9*S(0) (Black MC price 300.4472 and volatility 0.1392)

Price 5.1283 and implied volatility 0.1029 with strike 1.1*S(0) (Black MC price 7.4287 and volatility 0.1111)

We notice Monte Carlo simulation is in line with the Black model, but obviously does not coincide, as demonstrated by the comparison of the probability densities that also explains the difference in value:



The difference between the price with Dupire pricing and Monte carlo simulation is due to their different nature: the first one is a deterministic computation while the second one is a stochastic simulation.

1.3 Pricing of forwards

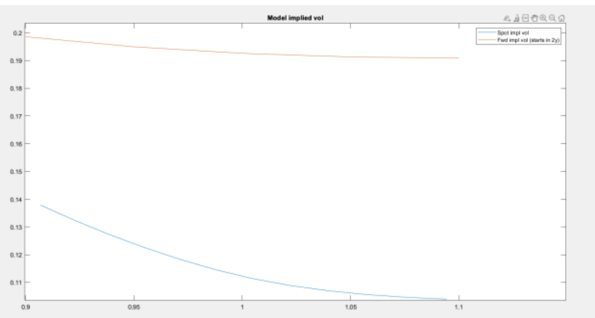
We proceed to price two forward starting option with start date 2 and expiry 2.5.

Strike 0.9: Price 325.0997 Model implied volatility 0.1993

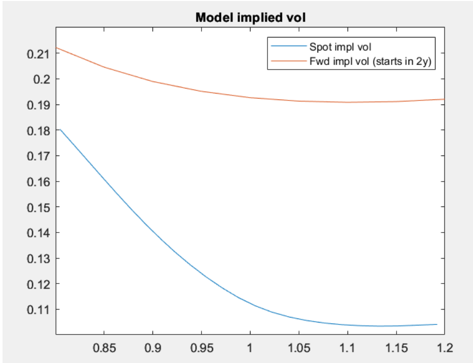
Strike 1.1: Price 46.7048 Model implied volatility 0.1910

1.4 Skew of the spot and forward smile

The slope of the spot/forward implied volatility as a function of strike K, as expected, follows a smile pattern:

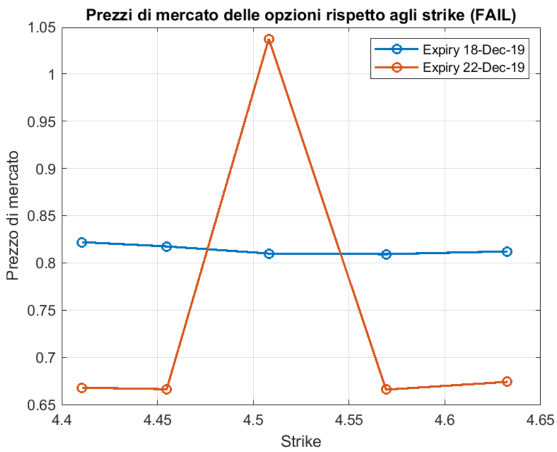


The feature is even more evident by using a bigger interval of K as in the file Punto1_4b.m:



2. Analysis of the Asset FAIL

For the “FAIL” asset, we attempted to calibrate the local volatility model, but the process failed. The main reason for the failure is the non-convexity of the price function with respect to the strike prices. In particular, we calculate the market prices of options using Black’s formula and observe that the relation $K_{i,j} \mapsto C_0(T_i, K_{i,j})$ does not maintain the convexity required for a correct calibration of the local volatility model.



This violation of convexity indicates that the local volatility model is not suitable for the “FAIL” asset, which may have market characteristics that are incompatible with the fundamental assumptions of this type of model. Therefore, the calibration was rejected, and it is suggested to use an alternative model to analyze this asset.

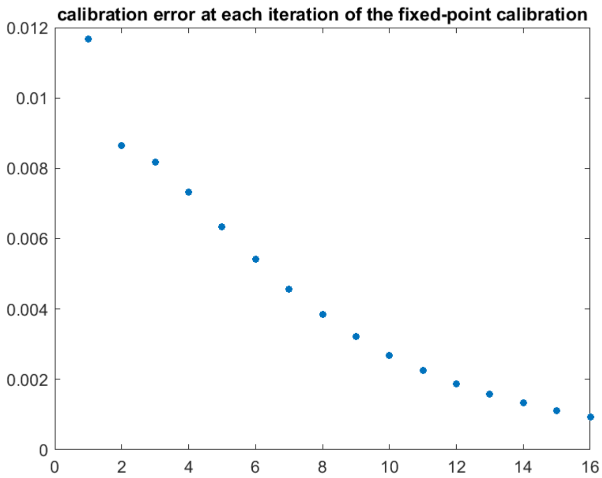
3. Asset EUR/USD analysis

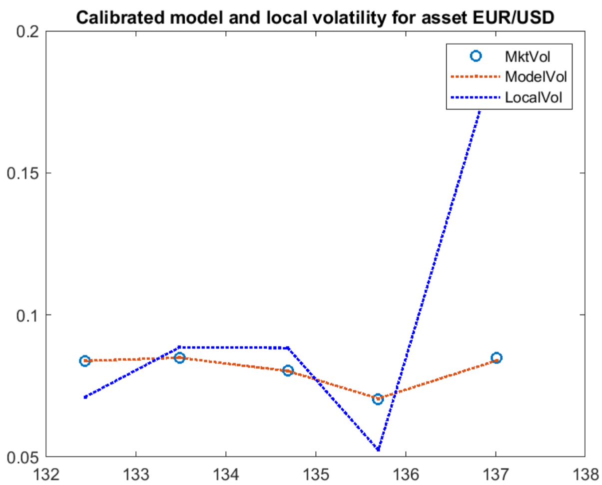
3.1 Local Volatility Model Calibration

We analyzed the EUR/USD exchange rate using the provided market data. In particular, the market strike prices were derived from the quoted deltas for the options. The forwards were calculated using the formula:

$$F_0(T_i) = Y(0)\frac{Df_0(T_i)}{Dd_0(T_i)}$$

Once obtained the necessary parameters, we calibrate the local volatility model with a maximum calibration error of 10 bps (0.001).





The calibration result showed a decreasing maximum error, indicating a good fit between the market prices and those predicted by the local volatility model. The maximum error for the different strikes varied from lower values (e.g., 0.0009) for lower strikes to higher values (e.g., 0.0117) for higher strikes, suggesting a successful calibration.

3.2 Montecarlo price of the exotic derivative with the Local Volatility Model

We considered an exotic derivative with expiration T_5 and a payoff:

$$\Phi(T_5) = \begin{cases} 2 & \text{if } Y(T_5) < K_{5,2} \\ (Y(T_5) - K_{5,2})^+ & \text{otherwise} \end{cases}$$

where $K_{5,2}$ is the strike corresponding to the 25-Delta. Using a Monte Carlo simulation, we calculate the price of this derivative.

The estimated price using the local volatility model was 7.3339, with a 95% confidence interval ranging from 7.2952 to 7.3726. This model taking in account different volatilities appears to adequately capture the behaviour of the exotic derivative.

3.3 Price of the exotic derivative with Black dynamics

For comparison, we calculate the same exotic derivative price using a constant volatility Black-Scholes model, with $\sigma=\sigma_{5,2}^{mkt}$,namely the 25-Delta market volatility at expiry T_5.

The price calculated using Black dynamics was 3.6600, with a 95% confidence interval of 3.6264 to 3.6935.

The average payoff in this model is 7.221830±0.1 while in the previous one was 9.033634±0.1, additionally, the Black-Scholes model resulted in a much lower percentage of times the derivative had a positive payoff (49.71%), compared to the local volatility model(75.77%), which clarifies the reason why the estimate of the price are so far apart.