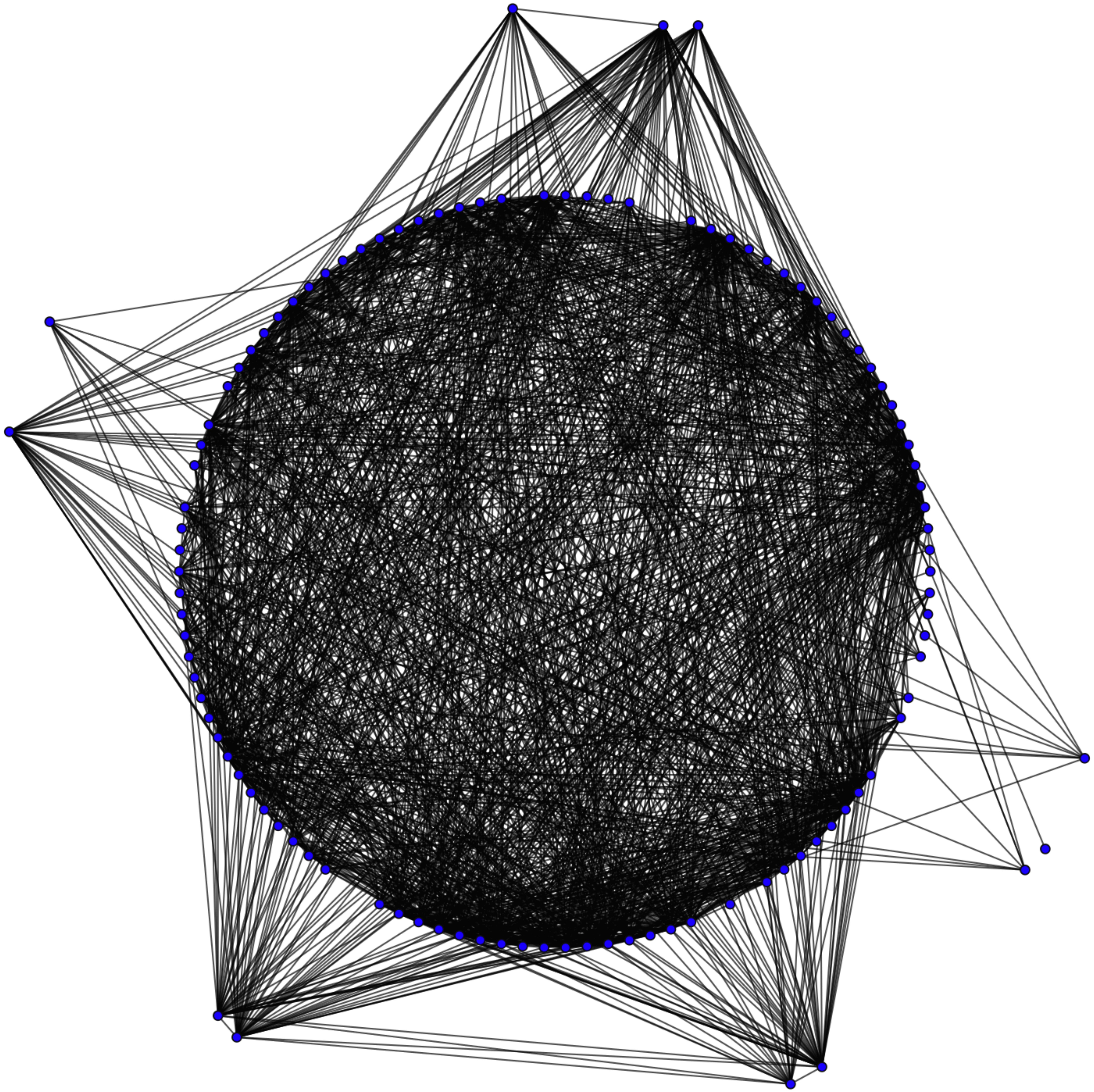
Epidemics on networks

Gentle introduction

What is a network

- A pair (set of nodes, set of edges)
- Represented by adjacency matrix
- Represents anything involving interactions between actors
- Edges can be contacts, or transmissions



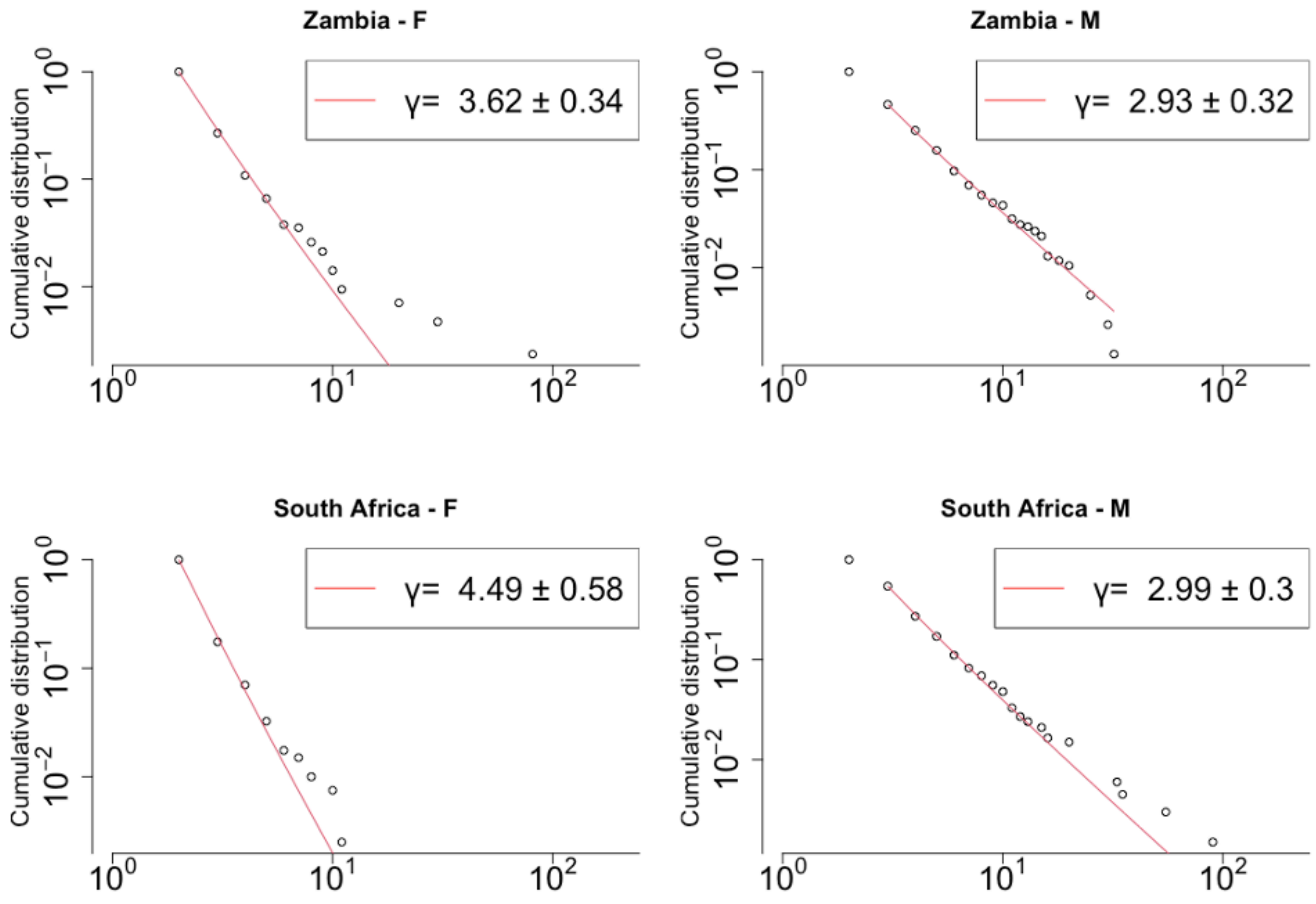
- Nodes in blue, edges in black
- Notice the structure!

Some properties

- Degree distribution: for each node, look at its neighbours, count them, and collect the results in a distribution
- Assortativity (Degree-degree correlations)
- Clustering (triangles)
- Modularity, centrality… other measures

A real network

Population Cohort survey, PopART Zambia and South Africa



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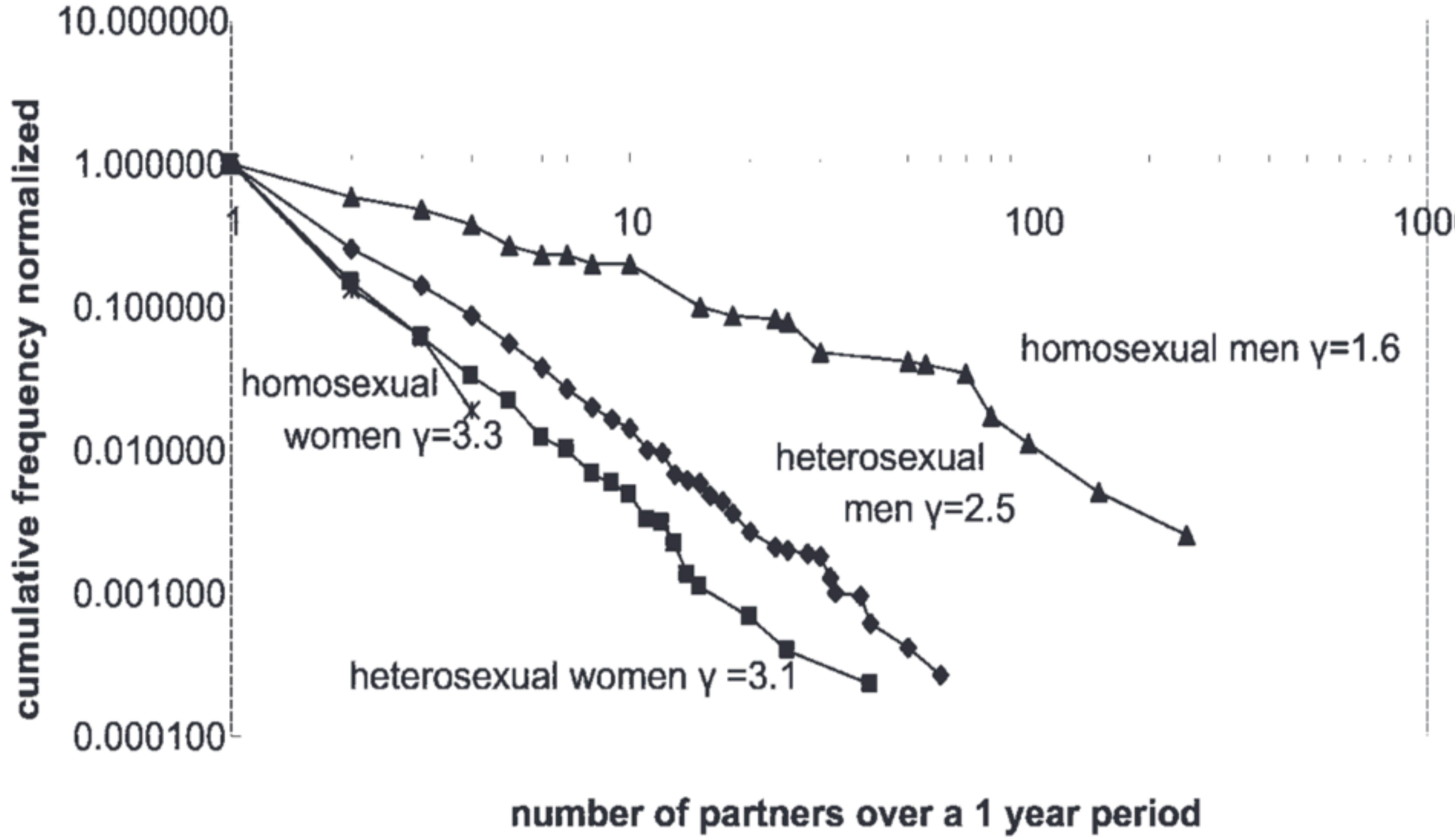


Types of networks

- Random graphs
- Configuration models
- Preferential attachment models
- Small world

Static vs dynamic vs temporal

Easiest network model: Erdos-Renyi

- Take N nodes
- Between any pair of different nodes add a link with probability $0<p<1$
- The degree distribution is Poisson like

Simple epidemic on networks

- Instead of homogeneous mixing, have a network
- Need to change to a Gillespie algorithm: the dynamics is stochastic
- The role of the average degree becomes important

Easiest model for this

- Simple modification of SIR: each node has on average (N-1)p neighbours

- Therefore the average number of contacts between infected nodes and susceptible is described by

$$\frac{d[\dot{I}]}{dt} = \beta[SI] - \gamma[I]$$

- But we can approximate [SI], to get the mean-field SIR model

$$[SI] \sim [S]p(N-1)\frac{[I]}{N}$$

$$\frac{d[\dot{S}]}{dt} = -\beta\frac{p(N-1)}{N}[S][I]$$

$$\frac{d[\dot{I}]}{dt} = \beta\frac{p(N-1)}{N}[S][I] - \gamma[I]$$

Improvement: think at what happens to pairs

$$\frac{\mathrm{d}[SI]}{\mathrm{d}t} = -\tau[SI] + \tau[SSI] - \tau[ISI] + \mathrm{g}[II] - \mathrm{g}[SI],$$

where [SSI] is the number of triples of individuals consisting of a central susceptible individual with both an infected and a susceptible contact

$$[ABC] \approx \frac{[AB][BC](n-1)}{n[B]},$$

There are also heterogeneous networks…

- Take N_0 nodes, fully connected

- At each time step, add one node with degree m

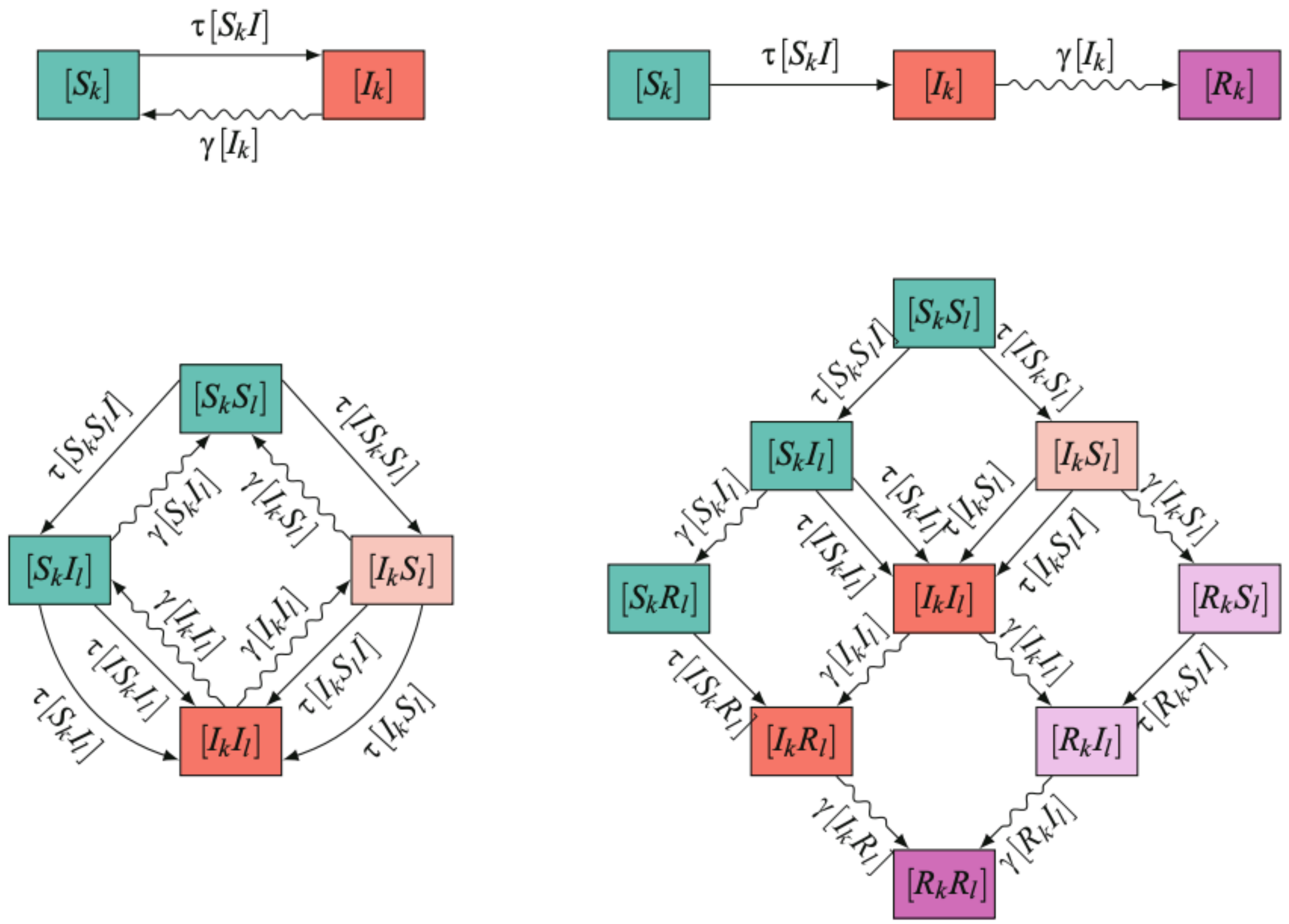
- Nodes connect to nodes that are already there, with probability proportional to their degree: preferential attachment

- It can be shown that the limit distribution for the degree is power-law, with exponent roughly 3

We need to take into account heterogeneity

- We consider equations for $[S]_k$, $[I]_k$, expected susceptible and infected nodes with degree k

- Similar idea as before



$$[S_k I] \sim k[S_k]\pi_I$$

$$\pi_I = \frac{\sum_{l=1}^{M} l[I_l]}{\sum_{l=1}^{M} lN_l}$$

$$[\dot{S}_k] = -\tau k[S_k]\pi_I$$

$$[\dot{I}]_k = \tau k[S_k]\pi_I - \gamma[I]_k$$