Research Track 2
DIBRIS
Università degli Studi di Genova

FIRST ASSIGNMENT

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31-05-2022

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1. Introduction

The **First Assignment** of the course **Research Track 2** is about the statistical analysis of two algorithms related to robotic solutions.

The objective of the implementations is to drive autonomously a mobile robot inside a given environment, avoiding obstacles and grasping and placing behind itself target tokens. This shall be done following a counter clockwise path in the map.

The environment of the simulation is the following:

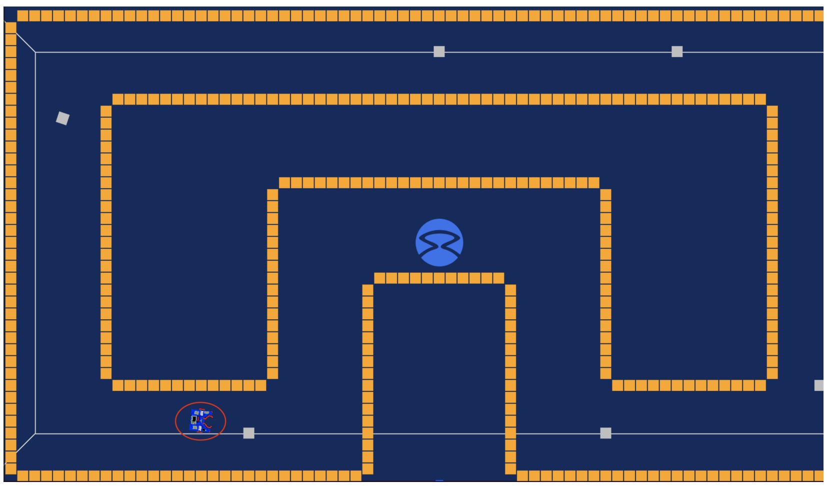

Figure 1.1: Simulation Environment

where the golden tokens are the obstacles that must be avoided and the silver tokens are the targets that must be grasped and placed behind. In Figure 1.1 the obstacles (golden tokens) on the right are missing even if they are present. The reason of their absence is that the simulation does not allow to reduce the dimension in order to see the whole map.

The mobile robot driving around the map is the one circled in red.

The two algorithms taken into account are introduced below:

- **Algorithm 1**: is a previously developed solution made by myself for the Research Track 1 course done in the first semester.
- **Algorithm 2**: is an algorithm developed by Professor Recchiuto to let students perform statistical analysis comparing it with their algorithms.

2. Statistical Analysis

2.1 Formulation of Hypothesis

In this statistical analysis, in fact, there are two algorithms. Let's call them A and B for a moment.

The **Null Hypothesis** (H_0) is a way to compare A and B stating that the two methods are equally good. If the Null Hypothesis is accepted it means that A is equally good compared to B. If the Null Hypothesis is rejected it means that the **Alternative Hypothesis** (H_a) is accepted and H_0 rejected. Accepting H_a means that one of the two methods is superior to the other one.

First of all there is the need of defining an hypothesis that must be tested, chosen before the sample is drawn.

As a performance evaluator, it was decided to consider the average time required to complete one lap of the circuit.

To compare the algorithms a Null Hypothesis H_0 and an Alternative Hypothesis H_a were selected. The two are the following:

H_0: The robot complete a lap of the circuit in about the same time, on average, using Algorithm 1 and Algorithm 2, so $\mu_1 = \mu_2$.

H_a: The robot does not complete a lap of the circuit in about the same time, on average, using Algorithm 1 and Algorithm 2, so $\mu_1 \neq \mu_2$.

Another important concept is the **level of significance**. It is a percentage that must be considered when testing an hypothesis. The choice of a certain significance level implies that the hypothesis that has been tested will be rejected when the sampling result has less than the decided probability of occurring if the hypothesis is true.

To verify if the Null Hypothesis should be rejected a 5% level of significance was chosen.

2.2 Test Choice

Once that the hypothesis has been stated and the level of significance has been defined, it is time to take samples. Samples are taken by performing experiments.

In this case, both algorithms have been tested one at a time in the map. The position of the silver tokens placed inside the environment changes every time the same algorithm is launched, whereas the displacement of the golden tokens does not vary. Even if the scenario keeps changing at each run, the two algorithms are compared in couples using the same map. In this way, when Algorithm 1 is launched inside Map 1, also Algorithm 2 is launched in Map 1. Once both are tested, the environment changes in Map 2 and so on.

This allows to have the possibility of comparing the two solutions using the same environment and make the test fair. For this reason it can be stated that there is a correlation between the observations.

Since there are two samples in which the observations of one sample can be paired with the observations of the other due to the fact that they are taken in the same scenario, a **Paired T-Test** is the most appropriate test to choose.

As a rule of thumb, it must be considered a sample size of at least 30 units since the data is assumed to come from a normally distributed population.

A **two-tailed test** is adopted to compare the two approaches. In the context of hypothesis testing, a two tailed-test rejects the null hypothesis if the sample mean is significantly higher or lower than the hypothesis value of the mean of the control group.

In other words, both tails of the distribution must be considered. If the test static value is either in the lower tail or in the upper tail, the null hypothesis is rejected. If the static is within the two reference lines, then i fail to reject the null hypothesis.

3. Paired T-Test

3.1 Test Description

It is now time to perform a Paired T-test to see whether the hypothesis H_0 stated in the previous chapter shall be rejected or accepted.

The procedure to follow is reported below:

- Calculate the **difference** $d_i = y_i - x_i$ between the two observations on each pair, making sure to distinguish between positive and negative differences.
- Compute the **mean** difference, $\overline{d}$.
- Calculate the **standard deviation** of the differences, s_d.
- Use the standard deviation to compute the **standard error of the mean difference**, $SE(\overline{d}) = \frac{s_d}{\sqrt{n}}$
- Calculate the **t-statistic**, which is given by $T = \frac{\overline{d}}{SE(\overline{d})}$. Under the null hypothesis, this statistics follows a t-distribution with $n-1$ Degrees of Freedom.
- Compare the value found at the previous step with the value found in a t-distribution table with $n-1$ Degrees of Freedom where n is the sample size.

3.2 Result

All the steps reported in the previous subsection were performed to obtain a statistical analysis for Algorithm 1 and Algorithm 2.

	TIME SAMPLES [s]																													
Sampling #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Algorithm 1	204	210	216	217	210	207	213	197	187	186	200	237	219	210	207	215	199	220	220	182	222	220	241	212	200	207	199	224	209	210
Algorithm 2	203	206	202	206	223	190	201	196	181	185	208	202	207	207	210	202	207	215	189	216	211	206	232	178	224	215	202	205	223	226
Difference	1	4	14	11	-13	17	12	1	6	1	-8	35	12	3	-3	13	-8	5	31	-34	11	14	9	34	-24	-8	-3	19	-14	-16
Mean	4,067																													
Std. Deviation	15,887																													
Std. Error	2,901																													
T	1,402																													
T [from tables]	2,045																													
t-Test Result	**Ho (null hypothesis) accepted**																													

Figure 3.1: Paired T-Test Result

In the Figure 3.1 can be seen step by step what was done and the result achieved. At the top of the graph, in the two rows "Algorithm 1" and "Algorithm 2" the values in seconds needed to complete one lap of the circuit are reported. The difference between a sample couple is d_i and the mean value of the differences is $\overline{d}$. The standard deviation is s_d, the standard error is $SE(\overline{d})$.

The "**T** [from tables]" value was obtained by looking online at t-distribution tables keeping into account a sample size of 29 DoF and a level of significance of 0.05 (5%) using a two-tailed test.

As it is possible to see in the graph, $T < T\ \ [from\ tables]$ so the **Null Hypothesis is accepted**. This means that, on average, Algorithm 1 and Algorithm 2 make the robot complete a lap of the circuit in about the same time.