Recursion and Dynamic Programming

People often joke that in order to understand recursion, you must first understand recursion

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A - Almost Union Find

Technique

Well...guess what.
Problem applying straightforward UFDS: the "move" operation.

Suggestions

- We can easily move elements from one DS to another if they are leaves
- Things get messy if we need to move an internal node
- Note: if an element of the set is a leaf, it will continue to be a leaf no matter how many merge operations we invoke.

A - Almost Union Find

Solution

Initialization: Make the set $S[i]$ be a child of $S[n+i]$

Move (p, q): Find roots: `x = find(p); y = find(q)`
Then, set `root[p] = y`.
Note that p, q will always be leaves.

Union: No change needed.

Note

Declare a FT of 64 bit integers (perhaps 32 bit unsigned could have been enough).

B - 10 kinds of people

Technique

Seems like flood fill / find connected components. A BFS/DFS will probably result in TLE.
Use Union Find.

Solution

- Each cell of the matrix belongs to one and only one set.
- Merge connected sets.
- Find roots of queries' elements. If the root is the same, solution may be `decimal` or `binary`. Otherwise is `neither`

C - Guess the Data Structure

Technique

Simulation

Solution

- Create a stack, a queue, a pqueue.
- Insertion: operate insertions on all containers.
- Extraction: extract from all containers, compare result with input.

D - Fenwick Tree

Technique

Fenwick Tree - Range Sum, Point Update

Solution

- See last week's slides
- Use 64 bit integers
- Heavy I/O, use `stdio.h`

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Introduction

Notes

- No Kattis contest today (yeah, I know you love me)
- Small amount of theory today (You're loving me even more)
- We will solve some random problems together
- I'm planning to publish some notes on Dynamic Programming on `https://www.fralotito.me`

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You need to have an account on

- AtCoder `https://atcoder.jp/`
- Kattis `https://open.kattis.com/`

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Divide et whaaat?

Let's Google it

"A divide-and-conquer algorithm works by recursively breaking down a problem into two or more sub-problems of the same or related type, until these become simple enough to be solved directly. The solutions to the sub-problems are then combined to give a solution to the original problem."

Some examples

- Binary search
- Euclidean Algorithm (GCD)
- Merge sort / Quick sort
- Fast Fourier Transform

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Note: You can click on them and get a tutorial

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Fibonacci

Problem definition

The Fibonacci numbers are the numbers in the following integer sequence:
$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ..$
In mathematical terms, the sequence F_n of Fibonacci numbers is defined by the recurrence relation:

$$F(n) = \begin{cases} 0 & \text{if } n = 0. \\ 1 & \text{if } n = 1. \\ F(n-1) + F(n-2) & \text{otherwise.} \end{cases} \tag{1}$$

Print the n-th Fibonacci Number.

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Print the n-th Fibonacci Number.

Solution

Well.. there are lots of them :)

Fibonacci

Solution

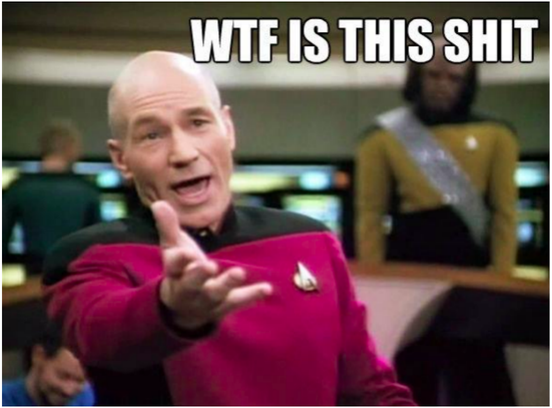
- Use recursion: exponential

Fibonacci

Solution

- Use recursion: exponential

After one day of computation..



Fibonacci

Let's make Fibonacci great again

- Add memoization (DP): $O(n) time, space$

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Fibonacci

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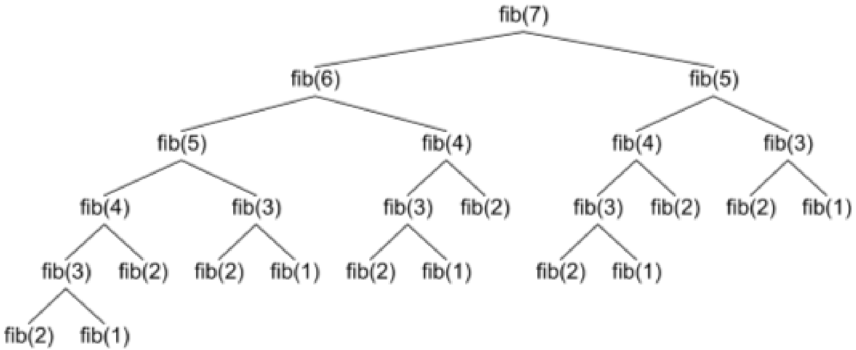
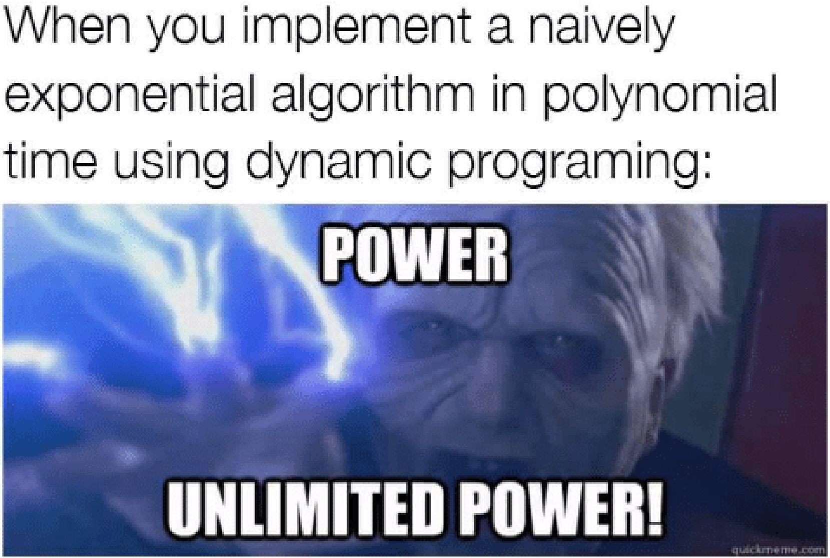
- Add memoization (DP): $O(n) time, space$
- Bottom-up (DP): $O(n) time, space$
- Bottom-up + space optimization (DP): $O(n) time, O(1) space$
- Closed formula: $O(1) time, space$

$$f(n) = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2} \right)^{n+1} - \left(\frac{1-\sqrt{5}}{2} \right)^{n+1} \right]$$

Why does DP work?




When you implement a naively exponential algorithm in polynomial time using dynamic programing:
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Dynamic Programming

Idea

The idea is very simple, if you have solved a problem with the given input, then save the result for future reference, so as to avoid solving the same problem again.. shortly 'Remember your Past'

Dynamic Programming

Top-down vs Bottom up

- If all subproblems must be solved at least once, a bottom-up dynamic-programming algorithm usually outperforms a top-down memoized algorithm by a constant factor.
    - No overhead for recursion and less overhead for maintaining table.
    - There are some problems for which the regular pattern of table accesses in the dynamic-programming algorithm can be exploited to reduce the time or space requirements even further.
- If some subproblems in the subproblem space need not be solved at all, the memoized solution has the advantage of solving only those subproblems that are definitely required.

Educational DP contest: problems A to G, **NO E**