**Comparison of Three Optimization Scenarios:**

**Safety and Total Value: A 0-1 Bi-Objective Knapsack Problem**

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This report compares the results obtained from three different optimization scenarios, each with varying weights assigned to profit and trust. In addition, we apply Pareto frontier analysis using the ϵ-constrained method to explore the trade-offs between maximizing profit and trust.

The three scenarios are:

* **Scenario 1**: Original weights from the given problem.
* **Scenario 2**: Increased weights for both profit and trust.
* **Scenario 3**: Decreased weights for both profit and trust.

The Pareto frontier analysis using the ϵ-constrained method provides further insight into how the balance between trust and profit influences the outcomes.

**Results for Each Scenario**

**Scenario 1: Original Weights**  
In the first scenario, the model was solved using the original weights, yielding the following results:

* **Safety (S)**: 133
* **Total Value (T)**: 154
* **Selected Investments**: 1, 3, 5, 7, 9, 14

**Scenario 2: Increased Weights**  
In the second scenario, where the weights for both profit and trust were increased, the results were:

* **Safety (S)**: 286 (Increased from Scenario 1)
* **Total Value (T)**: 145 (Decreased from Scenario 1)
* **Selected Investments**: 3, 7, 8, 14

This scenario demonstrates that increasing the weights results in a significant increase in safety but a decrease in total value. The higher safety reflects the increased emphasis on trust, while the decrease in total value indicates a trade-off with profit.

**Scenario 3: Decreased Weights**  
In the third scenario, where the weights for both profit and trust were decreased, the results were:

* **Safety (S)**: 66 (Decreased from Scenario 1 and Scenario 2)
* **Total Value (T)**: 76 (Decreased from Scenario 1 and Scenario 2)
* **Selected Investments**: 1, 3, 5, 7, 9, 14

Decreasing the weights resulted in a significant reduction in both safety and total value, suggesting that a lower emphasis on trust and profit leads to a less favorable solution.

**Pareto Frontier Analysis with the ϵ-Constrained Method**

To further explore the trade-off between profit and trust, we applied the ϵ-constrained method. This method maximizes profit while imposing incremental constraints on trust, as outlined below:

**Step 1: Maximizing Profit with No Trust Constraints**  
The first step involved solving the problem by maximizing profit alone, yielding the following results:

* **Profit (P)**: 159
* **Trust (S)**: 40
* **Selected Investments**: 3, 7, 8, 14

This represents the solution that focuses exclusively on profit maximization.

**Step 2: Maximizing Profit with a Trust Constraint (S + 1)**  
In the second step, a trust constraint was imposed, ensuring that trust is at least 41 (S ≥ 41). The results were:

* **Profit (P)**: 154
* **Trust (S)**: 105
* **Selected Investments**: 1, 3, 5, 7, 9, 14

This iteration demonstrates a trade-off, where profit is slightly reduced, but trust is significantly increased compared to the previous step.

**Step 3: Iterating the Trust Constraint**  
The solution was iterated by adjusting the trust constraint after each step, until no feasible solutions remained. This process illustrates how different trade-offs between profit and trust evolve as the constraints are modified.

**Comparison of Results for All Scenarios**

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| |  |  |  | | --- | --- | --- | | **Scenario** | **Safety (S)** | **Total Value (T)** | | **Scenario 1** | 133 | 154 | | **Scenario 2** | 286 (Increased) | 145 (Decreased) | | **Scenario 3** | 66 (Decreased) | 76 (Decreased) | |  |  |

**Observations**

* **Scenario 1**: The original weights provide a balanced solution between safety and total value, making this scenario a useful baseline.
* **Scenario 2**: Increasing the weights results in a notable increase in safety but a reduction in total value, demonstrating the trade-off where prioritizing trust leads to less profit.
* **Scenario 3**: Decreasing the weights leads to a decrease in both safety and total value, highlighting that a less balanced approach may result in suboptimal outcomes.

The comparison of the three scenarios, along with the Pareto frontier analysis, shows that the balance between trust and profit is crucial in determining the optimal outcome. Scenario 2, which increases the weights for both profit and trust, yields the highest safety value but at the cost of a lower total value. Conversely, Scenario 3, which decreases the weights, results in the lowest safety and total value, indicating that a more balanced approach is necessary to achieve better results in both objectives.