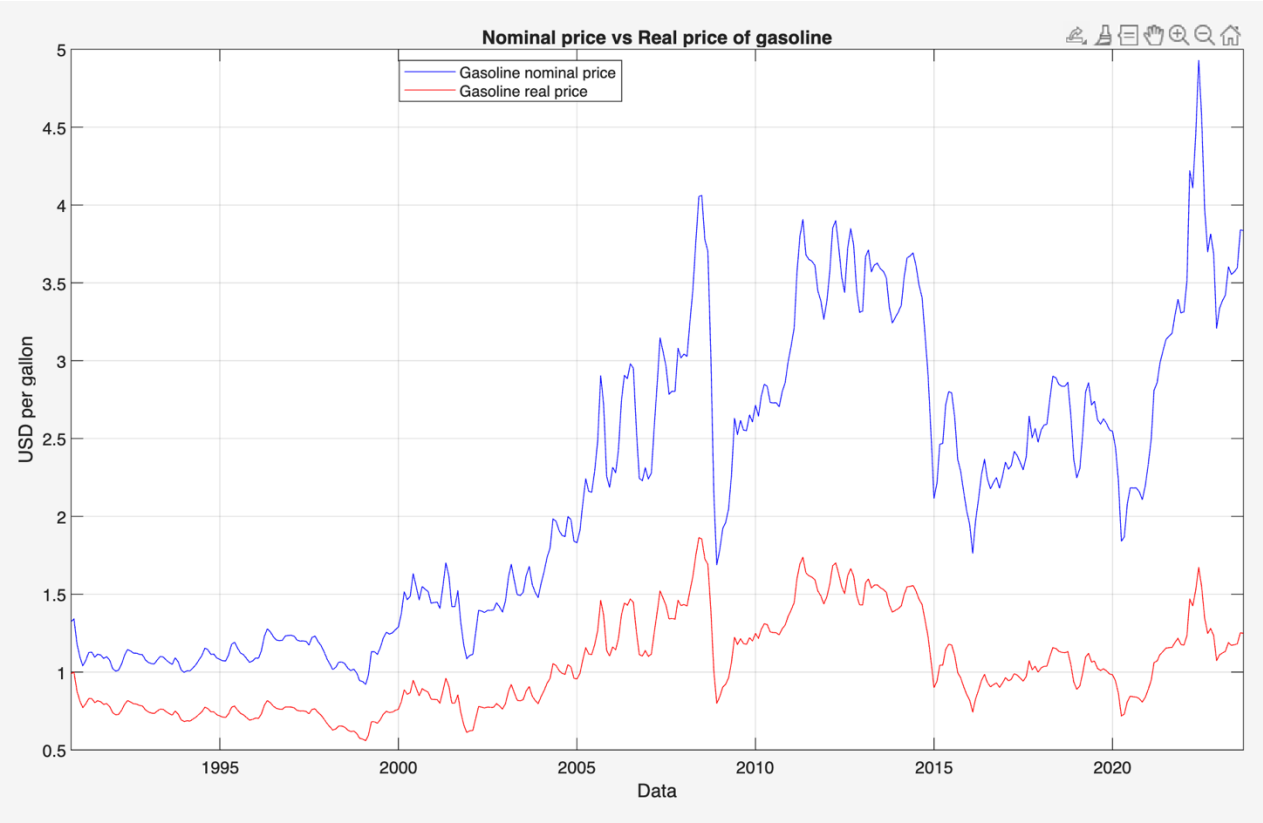
FRANCESCO SEBASTIANO MEMMOLA

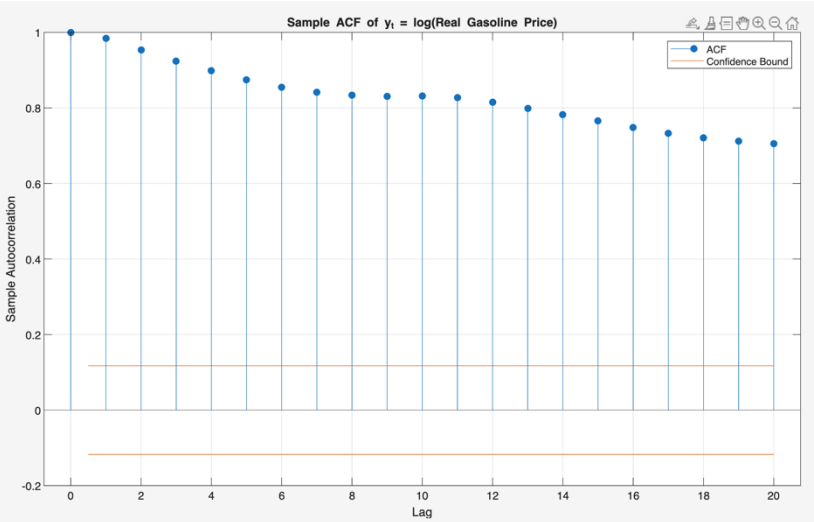
50893A

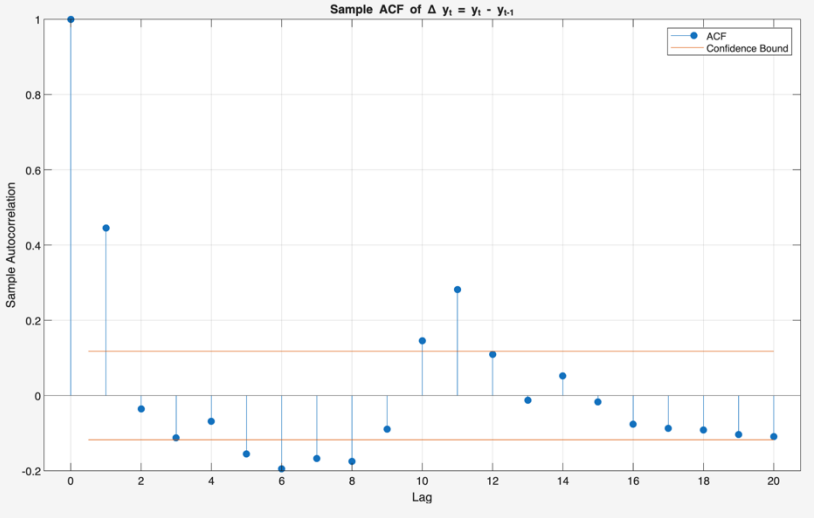
Real price gasoline



observation_date	GASREGW	CPIAUCSL	Real_Gasoline
01-Nov-1990	1.324	133.7	0.99028
01-Dec-1990	1.341	134.2	0.99925
01-Jan-1991	1.18	134.7	0.87602
01-Feb-1991	1.0942	134.8	0.81176
01-Mar-1991	1.04	134.8	0.77151
01-Apr-1991	1.0762	135.1	0.7966
01-May-1991	1.1258	135.6	0.8302
01-Jun-1991	1.1282	136	0.8296

Sample ACF of log(Real Gasoline Price) and its First Difference





The ACF of y_t = *log(Real Gasoline Price$_t$)* exhibits very slow decay, with autocorrelations remaining high and positive over many lags. This pattern indicates strong persistence and suggests that y_t is non-stationary, behaving like a random walk.

In contrast, the ACF of the first difference $\Delta y_t = y_t - y_{t-1}$ drops sharply after lag 1 and fluctuates around zero, indicating that the differenced series is approximately stationary.

These results confirm that the log of real gasoline prices follows a process close to an integrated order one series, *I(1)* .

AR(1) Estimation for y_t and Δy_t

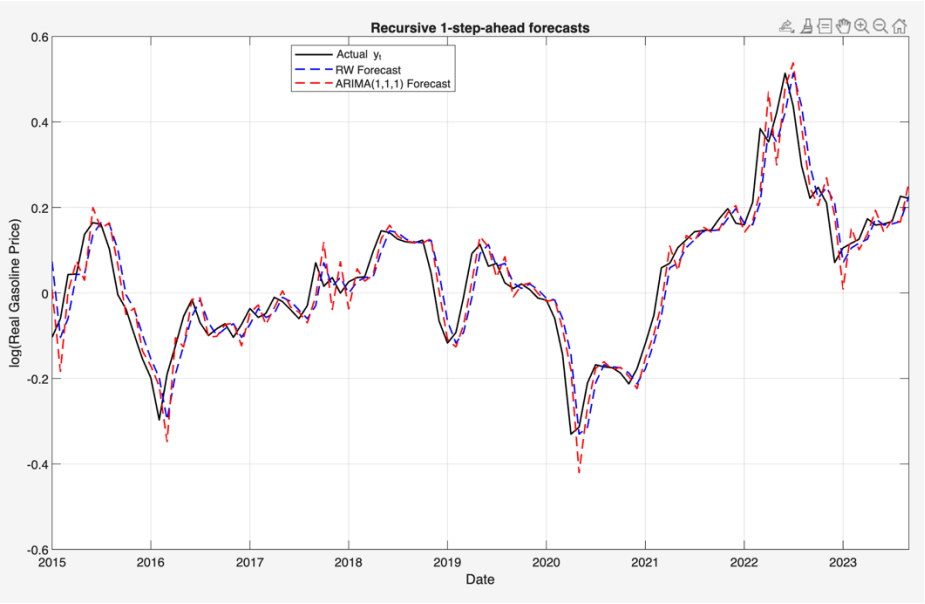
```
AR(1) on levels (y_t):
  phi (coefficient on y_{t-1}) = 0.9845

AR(1) on first differences (Δy_t):
  rho (coefficient on Δy_{t-1}) = 0.4547
```

The AR(1) coefficient for y_t (0.9845) is close to 1, indicating near–unit-root behavior.

After differencing, the coefficient (0.4547) confirms stationarity with moderate short-term correlation.

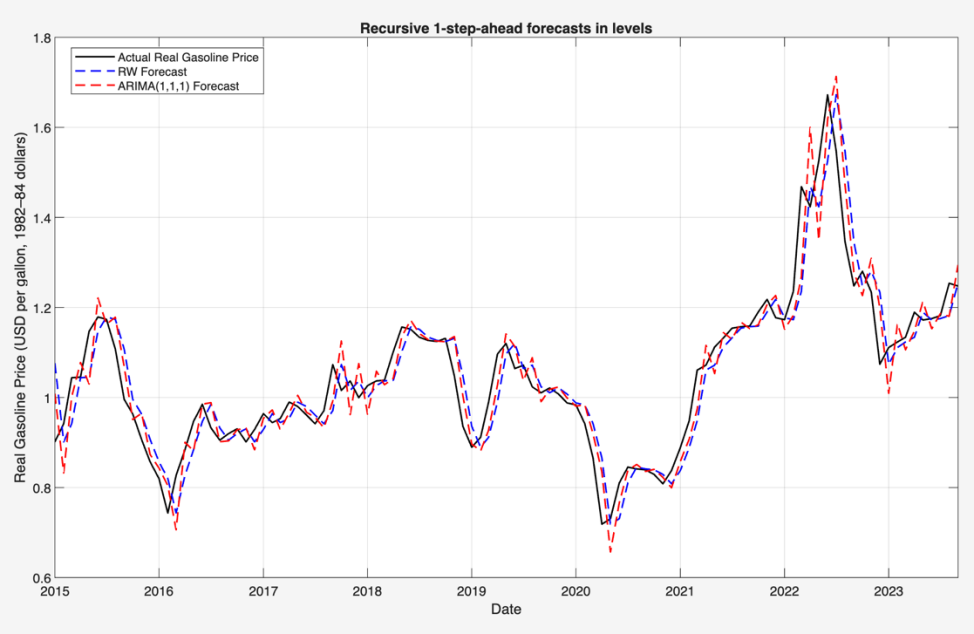
Recursive 1-Step-Ahead Forecasts for log(Real Gasoline Price)



All models produce very similar 1-step-ahead forecasts, closely tracking the actual log real gasoline price. The **Random Walk** serves as a strong benchmark, while ARIMA models only marginally improve short-term dynamics. This confirms that y_t is highly persistent and behaves almost like a **random walk**.

Date	Actual_y	Forecast_RW	Forecast_ARIMA_110	Forecast_ARIMA_011	Forecast_ARIMA_111
01-Jan-2015	-0.10393	0.073458	0.013086	0.014999	0.0083982
01-Feb-2015	-0.060053	-0.10393	-0.18817	-0.1703	-0.18323
01-Mar-2015	0.043064	-0.060053	-0.040447	-0.00048496	0.0014545
01-Apr-2015	0.044211	0.043064	0.089881	0.066376	0.074929
01-May-2015	0.13709	0.044211	0.044825	0.031514	0.029456
01-Jun-2015	0.16453	0.13709	0.17899	0.19552	0.19965
01-Jul-2015	0.16005	0.16453	0.17711	0.14757	0.15069
01-Aug-2015	0.1021	0.16005	0.15837	0.16686	0.16415

Transforming Forecasts to Real Prices (Levels)



The forecasts in levels track the realized real gasoline price very closely.
All models (Random Walk and ARIMA) are able to follow both medium-run movements and large spikes, so the transformation back to levels preserves forecast accuracy in economically meaningful units.

Date	Actual_y	Forecast_RW	Actual_Level __ Actual_Level	Forecast_RW_Level Forecast_RW_Level	Forecast_ARIMA_110_Level Forecast_ARIMA_110_Level	Forecast_ARIMA_011_Level Forecast_ARIMA_011_Level	Forecast_ARIMA_111_Level Forecast_ARIMA_111_Level
01-Jan-2015	-0.10393	0.073458					
01-Feb-2015	-0.060053	-0.10393	0.90129	1.0762	1.0132	1.0151	1.0084
01-Mar-2015	0.043064	-0.060053	0.94171	0.90129	0.82848	0.84341	0.83258
01-Apr-2015	0.044211	0.043064	1.044	0.94171	0.96036	0.99952	1.0015
01-May-2015	0.13709	0.044211	1.0452	1.044	1.094	1.0686	1.0778
01-Jun-2015	0.16453	0.13709	1.1469	1.0452	1.0458	1.032	1.0299
01-Jul-2015	0.16005	0.16453	1.1788	1.1469	1.196	1.2159	1.221
01-Aug-2015	0.1021	0.16005	1.1736	1.1788	1.1938	1.159	1.1626
			1.1075	1.1736	1.1716	1.1816	1.1784

Mean Squared Forecast Error (MSFE) Evaluation

When forecasts are transformed back to real price levels, the ranking remains similar.
Again, **ARIMA(0,1,1)** yields the smallest MSFE, but the differences among models are **minor**.

This means that more complex models provide **only modest gains** compared to the Random Walk, which captures most of the variation due to the strong autocorrelation in the series.

The Mean Squared Forecast Errors (MSFE) indicate that all ARIMA specifications perform slightly better than the Random Walk, both in log and level forecasts.

The ARIMA(0,1,1) model provides the lowest MSFE, but the improvement is marginal, confirming that the real gasoline price behaves almost like a **random walk**, and simple models already achieve high predictive accuracy.

```
MSFE (log of real gasoline price):
 Random Walk:     0.003603
 ARIMA(1,1,0):    0.003300
 ARIMA(0,1,1):    0.003215
 ARIMA(1,1,1):    0.003272

MSFE (real gasoline price levels):
 Random Walk:     0.004207
 ARIMA(1,1,0):    0.003945
 ARIMA(0,1,1):    0.003814
 ARIMA(1,1,1):    0.003895
```