COMP0118-Coursework1-Figures

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1 Parametric models

1.1 Parameter estimation and mapping

1.1.1

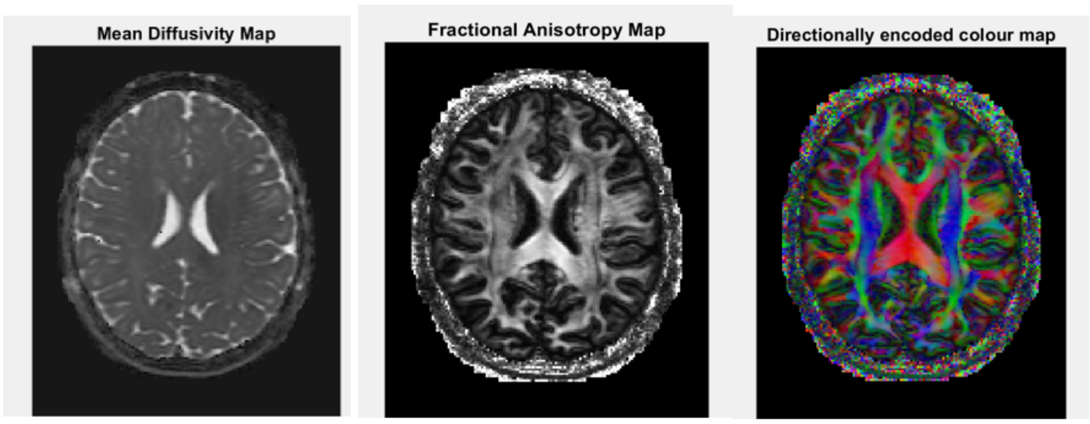

Figure 1: Maps of MRI scans computed using a Linear Diffusion Tensor estimator. Mean Diffusivity Map (left); Fractional Anisotropy Map (middle); Colour-coded Principal Direction Map(right)

1.1.2

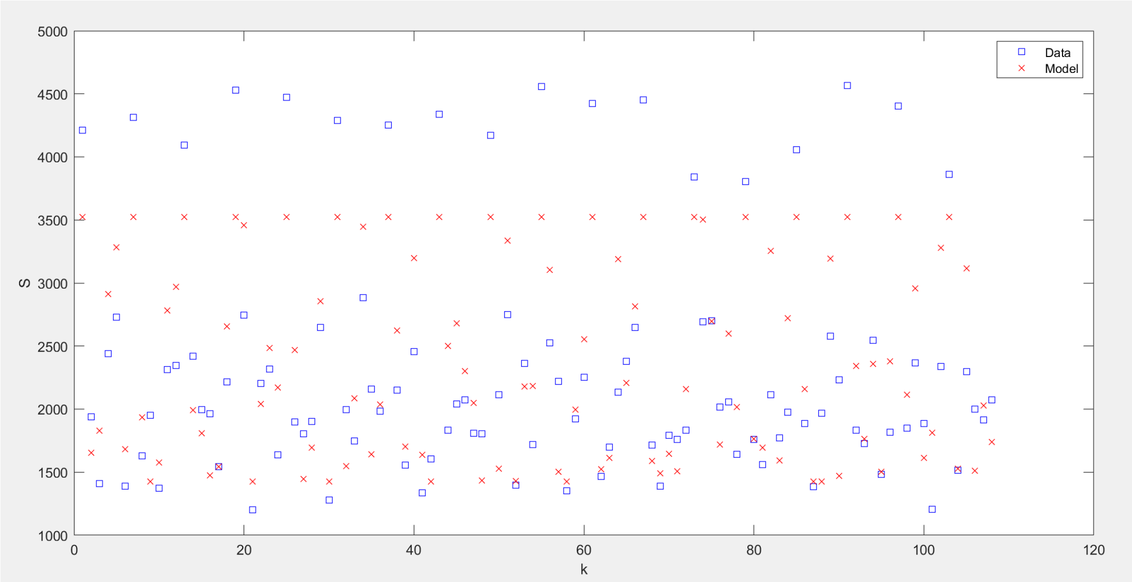

Figure 2: Comparison between the data and the estimated model. The model is the Ball-and-Stick without any constraints on the parameters

1.1.3

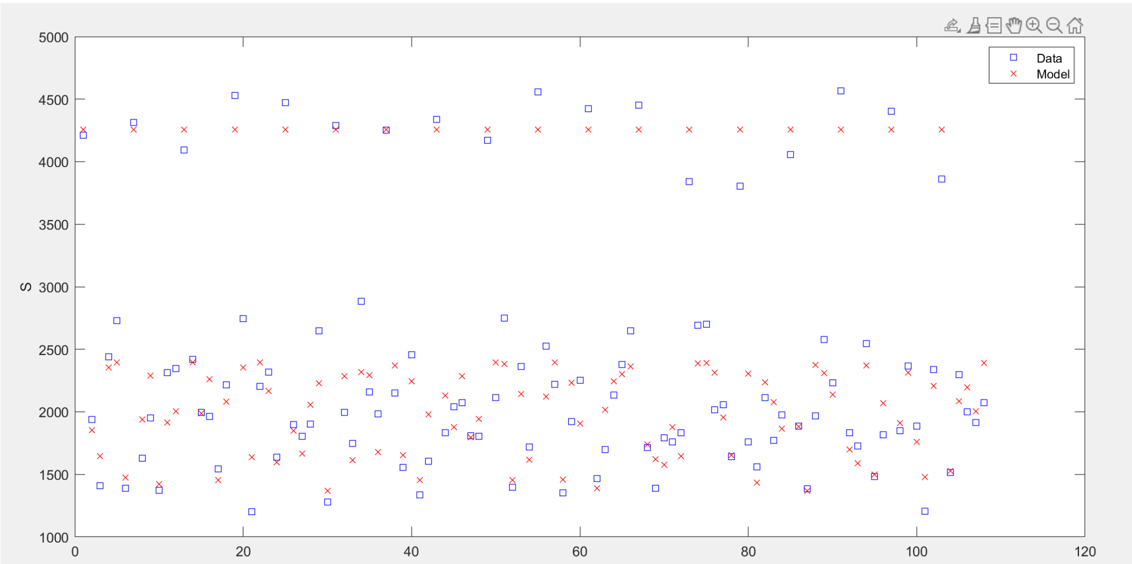

Figure 3: Comparison between the data and the estimated model. The model is the Ball-and-Stick constraints on the $S0$,d and f parameters

1.1.4

parameters	STD noise
$S0$	10^3
d	10^-3
f	10^-1
θ	10^-1
ϕ	10^-1

Table 1: Chosen STD noise values for each parameter of the Ball-and-Stick model

Voxel	**Percentage**	**Num_runs**
Avox = dwis(:, 92, 65, 72);	9.6400×10^{-1}	9.0118×10^{-1}
Avox = dwis(:, 115, 50, 72);	4.9000×10^{-1}	4.4490
Avox = dwis(:, 70, 90, 72);	6.8700×10^{-1}	2.5791
Avox = dwis(:, 112, 33, 72);	3.3700×10^{-1}	7.2892
Avox = dwis(:, 37, 44, 72);	6.1200×10^{-1}	3.1642
Avox = dwis(:, 100,100,72);	5.6100×10^{-1}	3.6389
Avox = dwis(:, 40, 100, 72);	9.3100×10^{-1}	1.1205
Avox = dwis(:, 90, 50, 72);	9.7900×10^{-1}	7.7545×10^{-1}
Avox = dwis(:, 77, 95, 72);	2.6400×10^{-1}	9.7732
Avox = dwis(:, 85, 70, 72);	7.5400×10^{-1}	2.1361

Table 2: Percentage of times the minimum $RESNORM$ value was found across all runs from different starting points and number of runs required to be 95% confident in finding the global minimum for 10 different voxels.

1.1.5

option	value
MaxFunctionEvaluations	20000
Algorithm	sqp
MaxIter	2000
TolX	1×10^{-10}
TolFun	1×10^{-10}
lb	$[0 \quad 0 \quad 0 \quad -inf \quad -inf]$
ub	$[inf \quad inf \quad 1 \quad inf \quad inf]$

Table 3: Options used for *fmincon* function

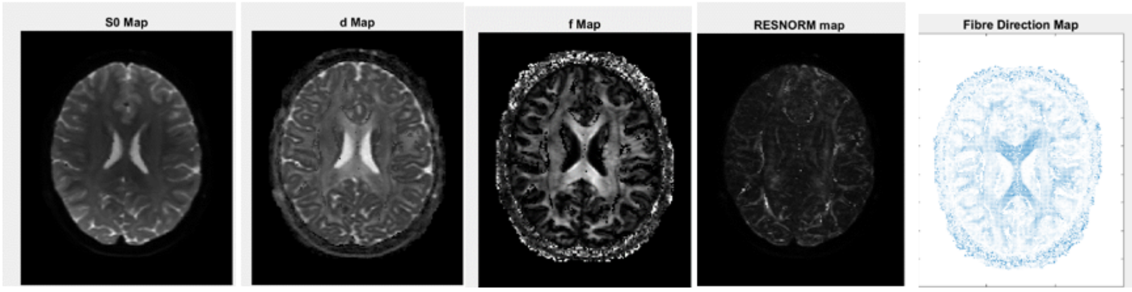

Figure 4: Maps of MRI scans computed using a Ball-and-Stick model estimation over all the voxel in the $72nd$ slice. $S0$ Map (left-left); Diffusivity (d) Map (left); Volume Fraction (f) Map (middle); Residual Error ($RESNORM$) Map(right); Fibre Direction Map (right-right).

1.1.6

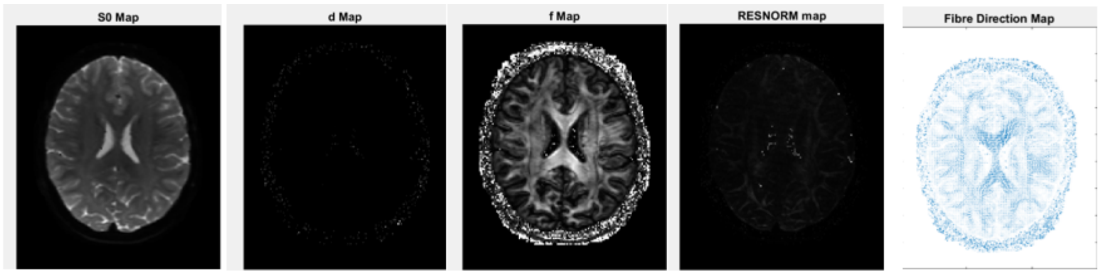

Figure 5: Maps of MRI scans computed using a Ball-and-Stick model estimation over all the voxel in the $72nd$ slice using the fmincon function to impose constraints to the parameters. $S0$ Map (left-left); Diffusivity (d) Map (left); Volume Fraction (f) Map (middle); Residual Error ($RESNORM$) Map(right); Fibre Direction Map (right-right).

Voxel	**Percentage**	**Num_runs**
Avox = dwis(:, 92, 65, 72);	4.8000×10^{-1}	4.5811
Avox = dwis(:, 115, 50, 72);	7.4000×10^{-1}	2.2239
Avox = dwis(:, 70, 90, 72);	4.2600×10^{-1}	5.3965
Avox = dwis(:, 112, 33, 72);	8.3800×10^{-1}	1.6459
Avox = dwis(:, 37, 44, 72);	6.8700×10^{-1}	2.5791
Avox = dwis(:, 100,100,72);	4.8400×10^{-1}	4.5277
Avox = dwis(:, 40, 100, 72);	4.3200×10^{-1}	5.2962
Avox = dwis(:, 90, 50, 72);	4.6000×10^{-1}	4.8617
Avox = dwis(:, 77, 95, 72);	4.9700×10^{-1}	4.3596
Avox = dwis(:, 85, 70, 72);	4.9100×10^{-1}	4.4361

Table 4: Percentage of times the minimum $RESNORM$ value was found using fmincon to constraints the parameters across all runs from different starting points and number of runs required to be 95% confident in finding the global minimum for 10 different voxels.

1.2 Uncertainty estimation

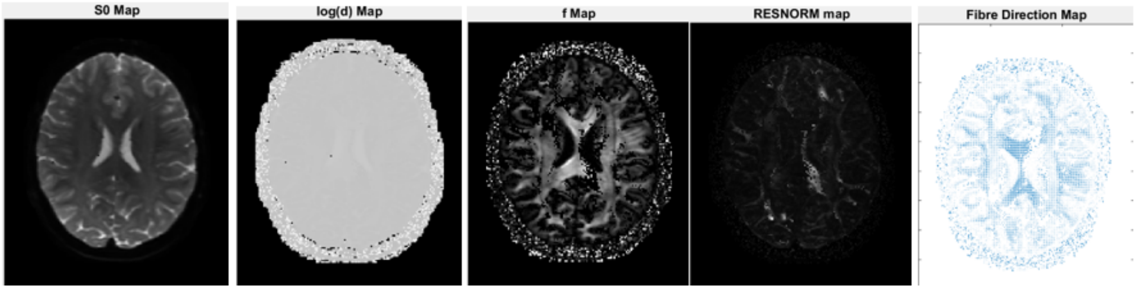

Figure 6: Maps of MRI scans computed using a Ball-and-Stick model estimation over all the voxel in the $72nd$ slice using the result of the Linear Diffusion Tensor as a starting point for the optimization process. $S0$ Map (left-left); Log of the Diffusivity (d) Map (left); Volume Fraction (f) Map (middle); Residual Error ($RESNORM$) Map(right); Fibre Direction Map (right-right).

1.2.1

$parameter$	$2-\sigma$	95% $range$
$S0$	[4136.0206099 4373.15407004]	[4131.31772944 4370.79343194]
f	[0.00088345174 0.0013340582]	[0.00075187509 0.00119869186]
d	[0.11173601735 0.5424787294]	[$6.3570069 \times 10^{-12}$ 0.4075320969]

Table 5: $2-\sigma$ range and 95% confidence interval for the parameters $S0$, d, and f in the *Ball-and-Stick* model for the $Avox = dwis(:, 92, 65, 72)$ obtained with the *Classic Bootstrap* algorithm

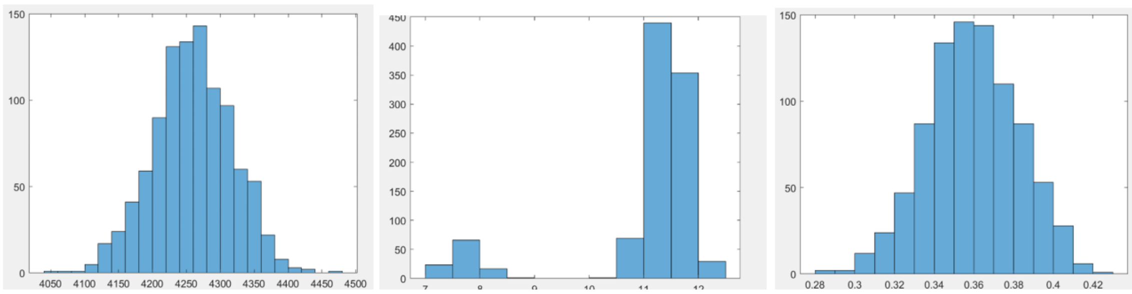

Figure 7: Posterior distributions for the *Classical Bootstrap* method for the parameters $S0$ (left); d (middle); f (right).

Voxel	**2-sigma range**	**95% range**
Avox = dwis(:, 92, 65, 72);	$[4.1360206 \times 10^3, 4.3731541 \times 10^3]$ $[8.835 \times 10^{-4}, 1.3341 \times 10^{-3}]$ $[1.117360 \times 10^{-1}, 5.424787 \times 10^{-1}]$	$[4.1313177 \times 10^3, 4.3707934 \times 10^3]$ $[7.519 \times 10^{-4}, 1.1987 \times 10^{-3}]$ $[6.3570 \times 10^{-12}, 4.075321 \times 10^{-1}]$
Avox = dwis(:, 100,100,72);	$[2.3827959 \times 10^3, 3.0382754 \times 10^3]$ $[3.581 \times 10^{-4}, 1.1574 \times 10^{-3}]$ $[-1.100992 \times 10^{-1}, 8.292790 \times 10^{-1}]$	$[1.8345533 \times 10^3, 2.8239784 \times 10^3]$ $[2.4133 \times 10^{-18}, 9.559 \times 10^{-4}]$ $[1.4701 \times 10^{-16}, 6.082507 \times 10^{-1}]$
Avox = dwis(:, 40, 100, 72);	$[3.5945588 \times 10^3, 3.7198799 \times 10^3]$ $[8.161 \times 10^{-4}, 1.4039 \times 10^{-3}]$ $[1.42203 \times 10^{-2}, 5.577154 \times 10^{-1}]$	$[3.5956130 \times 10^3, 3.7179327 \times 10^3]$ $[7.755 \times 10^{-4}, 1.2340 \times 10^{-3}]$ $[1.2618 \times 10^{-12}, 3.812613 \times 10^{-1}]$
Avox = dwis(:, 77, 95, 72);	$[8.8816172 \times 10^3, 9.3088169 \times 10^3]$ $[3.925619 \times 10^2, 2.4911690 \times 10^3]$ $[-1.854952 \times 10^{-1}, 8.549589 \times 10^{-1}]$	$[8.8843714 \times 10^3, 9.3005809 \times 10^3]$ $[5.570198 \times 10^2, 2.6450035 \times 10^3]$ $[1.58278 \times 10^{-2}, 9.016526 \times 10^{-1}]$
Avox = dwis(:, 85, 70, 72);	$[3.0385140 \times 10^3, 3.2119367 \times 10^3]$ $[7.177 \times 10^{-4}, 1.5054 \times 10^{-3}]$ $[1.261494 \times 10^{-1}, 9.400686 \times 10^{-1}]$	$[3.0366230 \times 10^3, 3.2117073 \times 10^3]$ $[5.801 \times 10^{-4}, 1.2732 \times 10^{-3}]$ $[6.9531 \times 10^{-12}, 6.744089 \times 10^{-1}]$

Table 6: $2-\sigma$ range and 95% confidence interval for different voxels for the parameters $S0$, d, and f in the *Ball-and-Stick* model for the $Avox = dwis(:, 92, 65, 72)$ obtained with the *Classic Bootstrap* algorithm

1.2.2

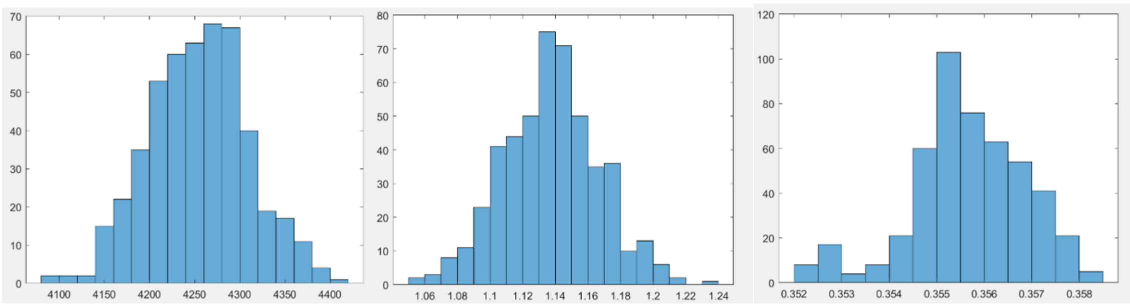

Figure 8: Posterior distributions for the *Metropolis-Hastings MCMC* method for the parameters $S0$ (left); d (middle); f (right).

Specification	Value
Number of samples (T)	1000000
Burn in samples	20000
Sampling interval	2000

Table 7: Specifications for the Metroplis-Hastings MCMC algorithm

parameter	$2-\sigma$	95% *range*
$S0$	[4136.9534103 4363.5615563]	[4100.39516493 4353.303826223]
f	[0.0010846100 0.0012125074]	[0.0010705101 0.0012058503677]
d	[0.3562016184 0.3737022973]	[0.35620452269 0.370940663224]

Table 8: $2-\sigma$ range and 95% confidence interval for the parameters $S0$, d, and f in the *Ball-and-Stick* model for the $Avox = dwis(:, 92, 65, 72)$ obtained with the *Metropolis-Hastings* algorithm

1.2.3

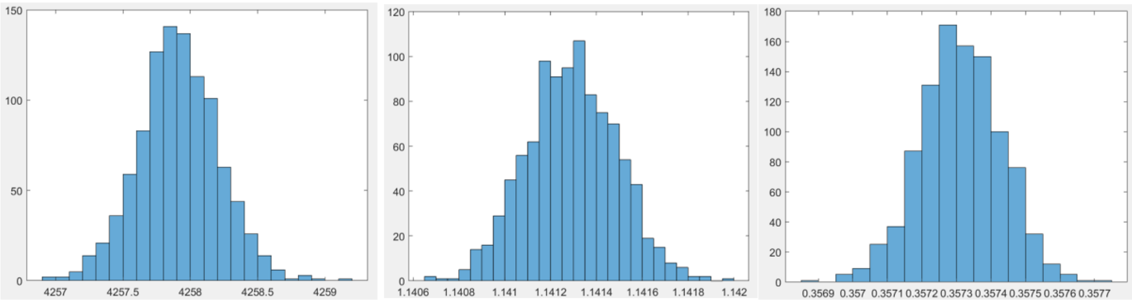

Figure 9: Posterior distributions for the *Parametric Bootstrap* method for the parameters $S0$ (left); d (middle); f (right).

1.3

Parameter	$2-\sigma$	95% *range*
$S0$	[4257.31 4258.52]	[4257.34 4258.53]
f	[0.001141 0.001142]	[0.001141 0.001142]
d	[0.35707 0.35756]	[0.35708 0.35755]

Table 9: $2-\sigma$ range and 95% confidence interval for the parameters $S0$, d, and f in the *Ball-and-Stick* model for the $Avox = dwis(:, 92, 65, 72)$ obtained with the *Parametric Bootstrap* method.

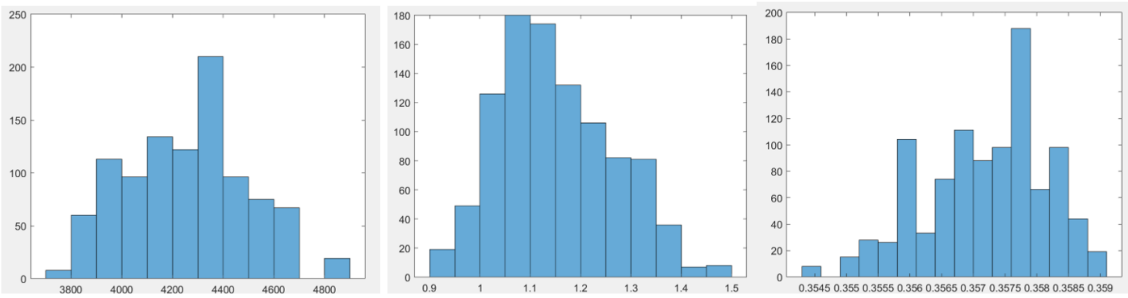

Figure 10: Posterior distributions for the *Residual Bootstrap* method for the parameters $S0$ (left); d (middle); f (right).

Parameter	$2-\sigma$	95% *range*
$S0$	[3764.54 4736.79]	[3820.91 4677.33]
f	[0.000922 0.001392]	[0.000977 0.001392]
d	[0.35520 0.35912]	[0.35515 0.35862]

Table 10: $2-\sigma$ range and 95% confidence interval for the parameters $S0$, d, and f in the *Ball-and-Stick* model for the $Avox = dwis(:, 92, 65, 72)$ obtained with the *Residual Bootstrap* method.

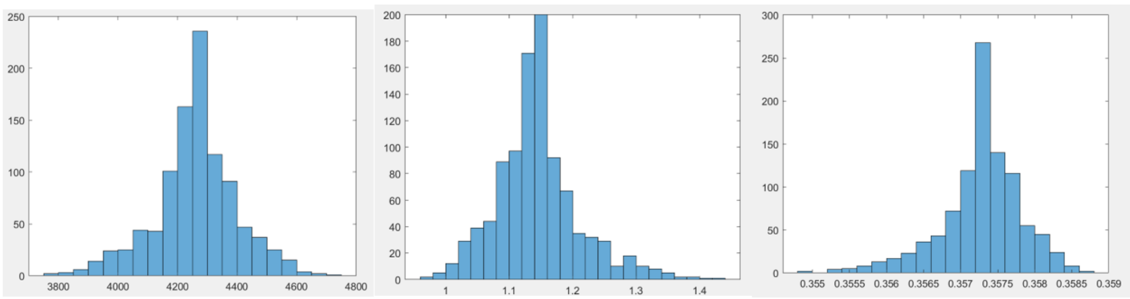

Figure 11: Posterior distributions for the *Wild Bootstrap* method for the parameters $S0$ (left); d (middle); f (right).

Parameter	$2-\sigma$	95% *range*
$S0$	[3995.95 4546.48]	[3969.78 4577.72]
f	[0.001011 0.001267]	[0.001010 0.001293]
d	[0.35626 0.35838]	[0.35602 0.35835]

Table 11: $2-\sigma$ range and 95% confidence interval for the parameters $S0$, d, and f in the *Ball-and-Stick* model for the $Avox = dwis(:, 92, 65, 72)$ obtained with the *Wild Bootstrap* method.

Parameter	$2-\sigma$
$S0$	[4257.0637 4258.78434]
f	[4.8924×10^{-6} 0.0022874]
d	[0.29421423 0.420405484]

Table 12: $2-\sigma$ range confidence interval for the parameters $S0$, d, and f in the *Ball-and-Stick* model for the $Avox = dwis(:, 92, 65, 72)$ obtained with the *Laplace Method*.

1.3.1

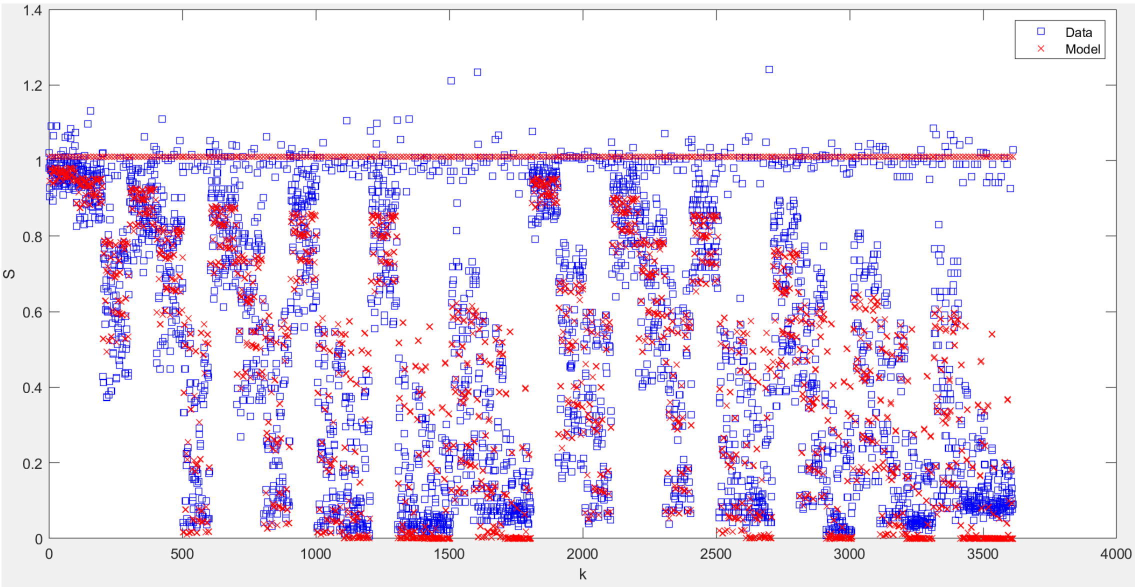

Figure 12: Comparison between the data and the estimated model. The model is the Ball-and-Stick applied to the new data set

parameters	STD noise
$S0$	10^1
d	$10^{-}3$
f	$10^{-}1$
θ	$10^{-}1$
ϕ	$10^{-}1$

Table 13: Chosen STD noise values for each parameter of the Ball-and-Stick model for the new data set

1.3.2

Model	Percentage	Num_runs
Zeppelin-and-Stick	0.993	0.60375
Zeppelin-and-Stick with Tortuosity	0.99800	0.48205

Table 14: Percentage of times the minimum *RESNORM* value was found across all runs from different starting points and number of runs required to be 95% confident in finding the global minimum for the two different models.

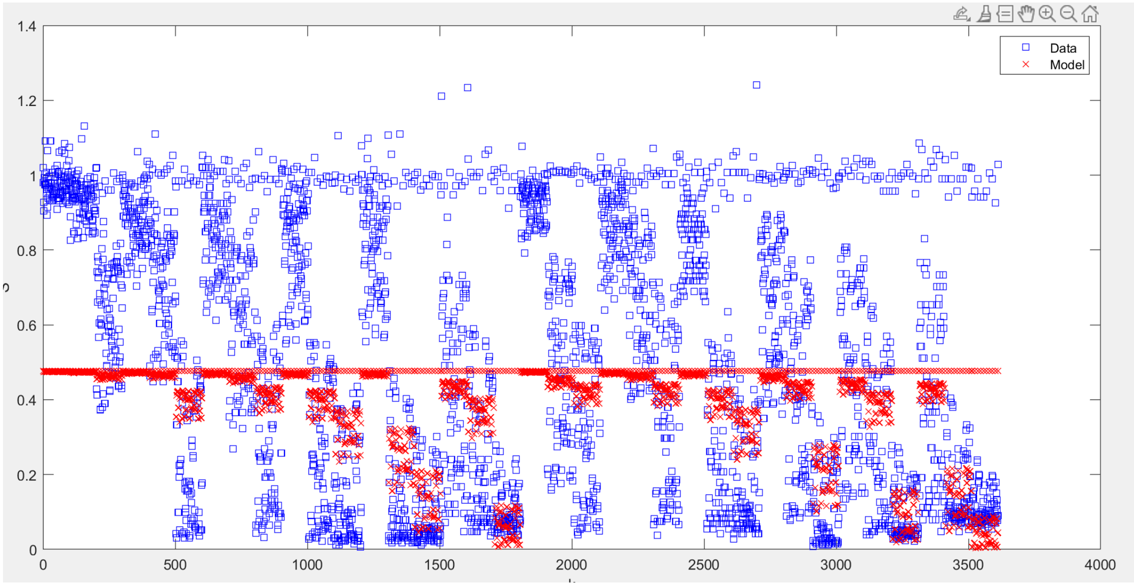

Figure 13: Comparison between the data and the estimated model. The model is the Linear Diffusion Tensor applied to the new data set

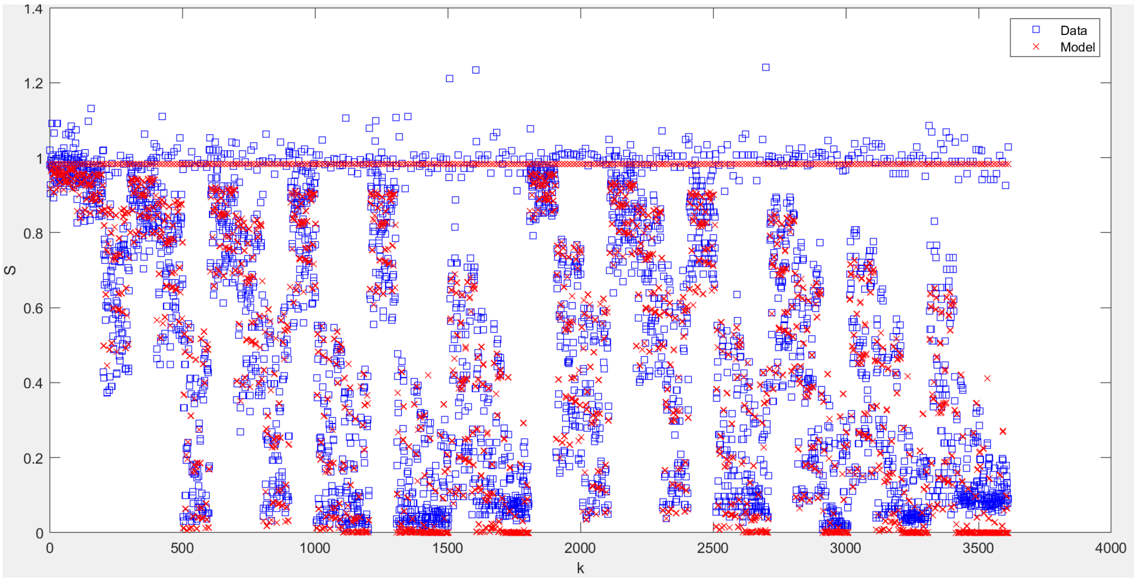

Figure 14: Comparison between the data and the estimated model. The model is the Zeppelin-and-Stick applied to the new data set

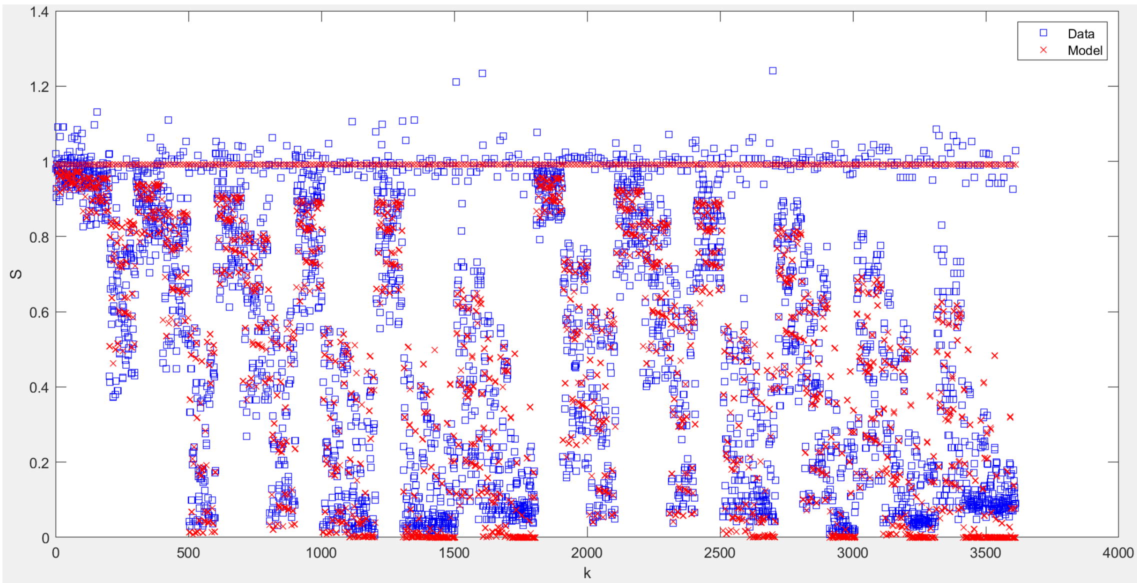

Figure 15: Comparison between the data and the estimated model. The model is the Zeppelin-and-Stick with Tortuosity applied to the new data set

1.3.3

MODEL	AIC	BIC
DT	-8660.48	-8824.21
BS	-19770.65	-19397.98
ZS	-20975.07	-20550.69
ZST	-20722.91	-20300.18

Table 15: AIC and BIC values for different models (DT, BS, ZS, ZST).

1.3.5

MODEL	AIC	BIC
DT	-8660.48	-8824.21
	-8470.49	-8685.83
	-8466.81	-8564.64
	-8610.95	-8774.67
	-8420.96	-8636.30
	-8417.27	-8515.10
BS	-19770.65	-19397.98
	-18768.89	-18387.46
	-19033.09	-19289.12
	-19733.50	-19360.83
	-18731.73	-18350.31
	-18995.93	-19251.97
ZS	-20975.07	-20550.69
	-20122.82	-19263.30
	-20405.68	-20783.56
	-20931.73	-20507.35
	-20079.48	-19219.96
	-20362.33	-20740.22
ZST	-20722.91	-20300.18
	-19743.23	-19048.18
	-19964.16	-20248.77
	-20685.76	-20263.03
	-19706.08	-19011.02
	-19927.01	-20211.62

Table 16: AIC and BIC values for different models (DT, BS, ZS, ZST) for all 6 voxel.

MODEL	VFCV	MCVC	L1OCV
BS constraints	0.1585	0.0811	0.0126
ZS	0.1124	0.0613	0.0090
ZST	0.1210	0.0688	0.0100

Table 17: VFCV, MCVC, and L1OCV values for different models.

1.3.6

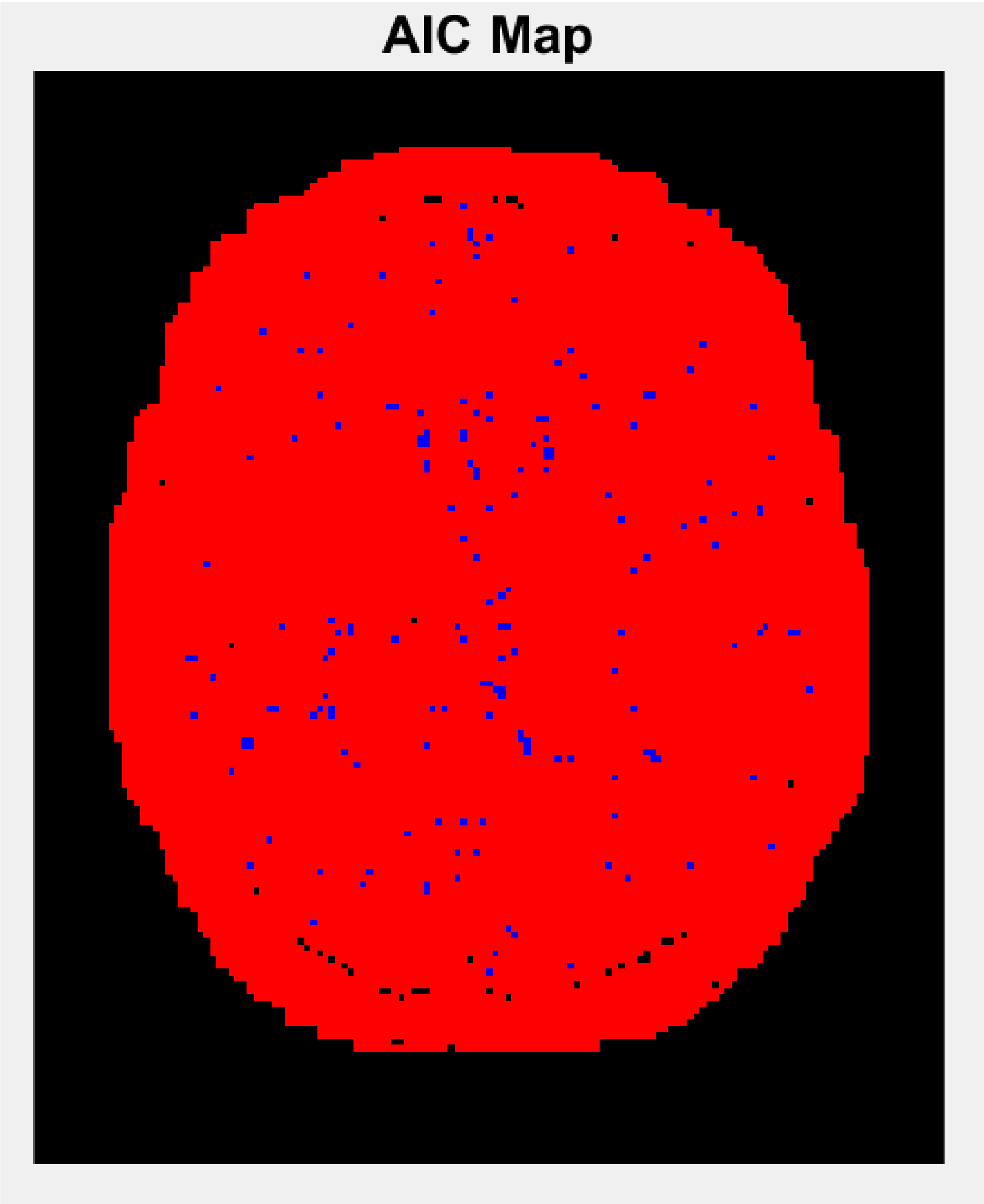

Figure 16: Map of the best model between *Zeppelin-and-Stick*, in blue, and *Zeppelin-and-Stick with Tortuosity*, in red for the $72nd$ slice of the data set used in sections 1.1 and 1.2

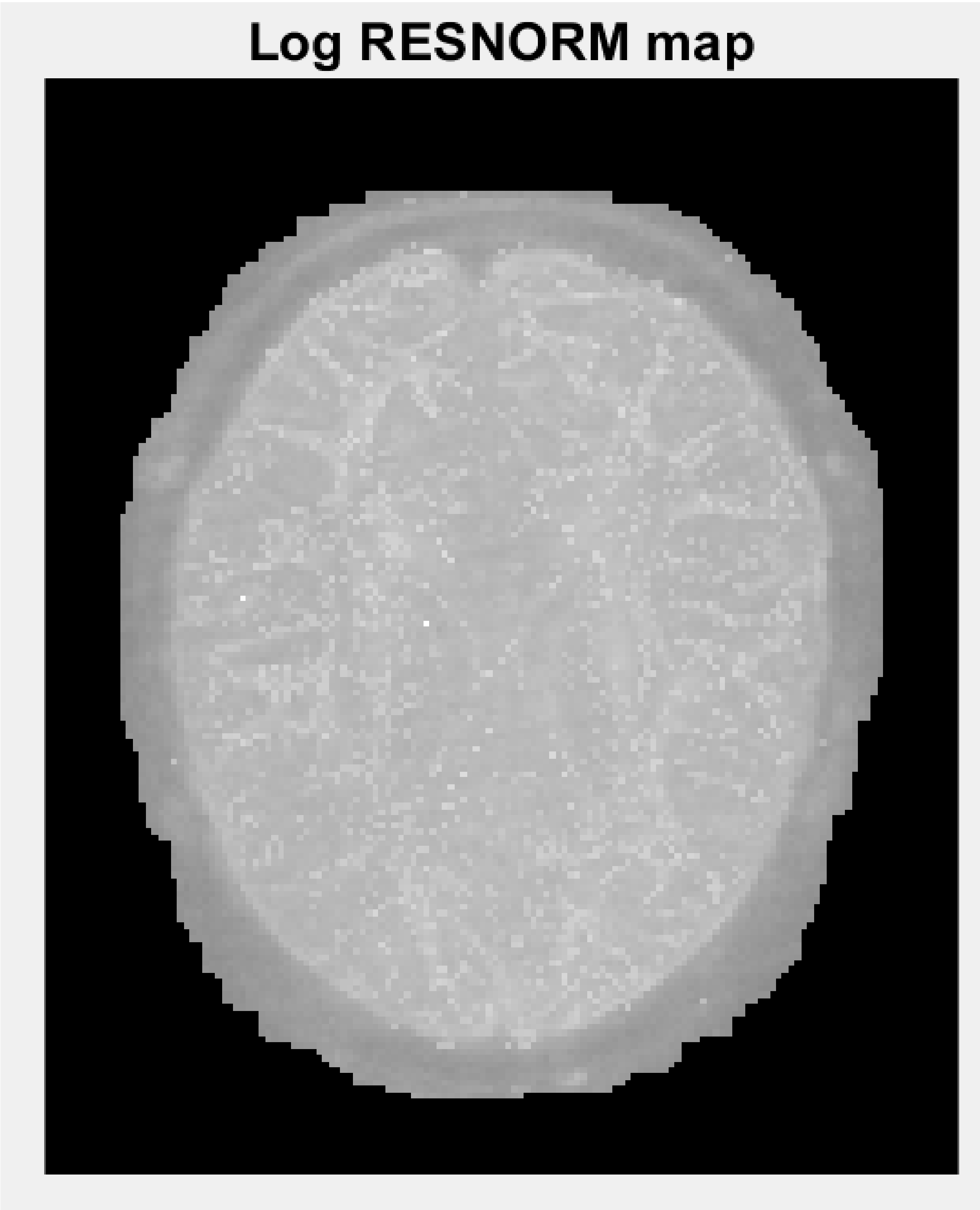

Figure 17: Log of the *RESNORM* map obtained by weighting the estimated signals obtained from different models with the Akaike weights