Time as Fractal Decoherence:
A Speculative Group-Topological Construction

1 Abstract

This document is not a conventional scientific article, nor does it aim to present an empirically testable theory. Rather, it offers a speculative and formally structured exploration of time as an emergent phenomenon — modeled through algebraic deformation, fractality, and categorical recursion. The aim is not to make predictions, but to *reframe the question of time* — to offer a heuristic formalism that may resonate with alternative physical interpretations such as the Wheeler–DeWitt equation, the Page–Wootters mechanism, and topological models of quantum emergence.

While speculative in nature, the construction is precise in form. It may serve as a thought experiment, a categorical toy model, or a provocation toward new formal intuitions. This is not physics per se — but perhaps a sketch of physics' ontological shadow.

We propose a speculative model of time as an emergent structure arising from the interaction of local abelian groups, gradient operators, and recursive fractality. The key thesis claims that time does not exist as an independent dimension nor as a background to events—it is the very process of decoherence, understood not as a quantum epiphenomenon, but as a dynamic ontological form. Mathematically, time is described as the limit of an iterated semidirect product over the direct sum of local abelian groups:

$$T = \lim_{n\to\infty} \left(\bigoplus_{k=1}^{n} A_k \right) \rtimes_{\nabla} F$$

where A_k represent local symmetries, ∇ acts as a symmetry-breaking operator (introducing directionality and non-commutativity), and F is the fractal recursion operator. We discuss the ontological implications of this construction, proposing a view of time as a fractal process of deforming a space of possibilities.

2 The Formula of Time: Foundations of a Fractal Reality

Before the Formula: Why Time Needed Rethinking

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Think of a kaleidoscope whose patterns deform as they reflect — time is the limit of that deformation.

In classical and relativistic physics, time is treated as a dimension or parameter—a number used to describe the evolution of dynamic systems. In quantum physics, time most often remains an external parameter, not subject to quantization. However, an increasing number of proposals—especially at the intersection of quantum theory, cosmology, and information theory—suggest that time may be an emergent structure resulting from deeper physical and topological processes.

In this approach, we propose to go further: we recognize that time *is* decoherence itself, understood as the fractal dynamics of local deformations of symmetric structures. From this perspective, decoherence is not an effect that occurs "in time," but time itself as a movement of distortions. This interpretation resonates with various emergent-time hypotheses in theoretical physics. For example, the Wheeler-DeWitt equation effectively erases time from the fundamental description of the universe, suggesting a 'timeless' state. Meanwhile, the Page–Wootters mechanism proposes that temporal sequences can emerge from entanglement between subsystems of a globally static state. Our model builds on these intuitions, but departs by framing time not as a relational correlation to be recovered, but as a recursive deformation of local symmetries. In this framework, time is not a hidden parameter or informational correlation — it is the structural consequence of iterative asymmetry. Rather than extracting time from entanglement or constraint, we posit that time is actively generated through the dynamics of deformation itself. It is not remembered — it is written, iteratively. generated through recursive structural deformation. This claim aligns loosely with approaches such as the Page-Wootters mechanism or the Wheeler-DeWitt timeless formulation, but goes further by proposing that temporal structure is not recovered from entanglement or constraint equations, but is itself a recursive topological deformation. Time does not pass through the world—it is the world that deforms as time.

Formal Construction

Each iteration of local transformation carries a deformation. Over infinite iterations, what emerges is not stability, but rhythm — an uneven flow of change.

Thus, time appears not as a given, but as a necessary result — the trace of recursive transformations accumulated across scales.

2.0.1 Local Groups A_k

Each A_k represents a local symmetry—an abelian group of type $\mathbb{Z}$, $\mathbb{R}$, $U(1)$, etc. Collectively, their aggregation forms a static space:

$$\bigoplus_{k=1}^{n} A_k$$

Here, F recursively re-applies the action of *abla*, embedding each successive deformation into a higher-order structure, thereby producing a self-similar hierarchy. which models a multiplicity of independent, yet structurally similar, temporal moments.

2.0.2 Deformation Gradient ∇

The operator ∇ introduces structural deformation: non-commutativity, directionality, and relationality. In the sense of Lie algebra, ∇ may be identified with the commutator: $[X, Y] = XY - YX$. It serves as the mechanism that “dislodges” the system from abelian symmetry and initiates dynamics. *abla* acts individually on each A_k, but also encodes interactions across indices k through a recursive coupling schema that sets the stage for F to act..

2.0.3 Fractality F

The operator F defines the fractal self-similarity of the structure: each aggregation A_k can be recursively embedded in a higher order. Formally: $F(S) = S \otimes S$, or $F(A_k) = A_k \oplus A_{k+1}$.

2.0.4 Emergent Limit

Time as a whole arises through an infinite iteration of the process:

$$T = \lim_{n \to \infty} \left(\bigoplus_{k=1}^{n} A_k \right) \rtimes_{\nabla} F$$

This constitutes a non-commutative emergent structure, irreducible to its components.

2.0.5 Ontological Role of Fractals

The operator F is not merely a mathematical recursion — it encodes an ontological principle: that reality unfolds not linearly, but through iterative deformation. Fractality, in this context, does not simply mean self-similarity across scales, but recursive asymmetry: a pattern of transformation where each layer modifies the logic of the previous.

This approach is motivated by the insight that time, as experienced and constructed, rarely behaves as a uniform continuum. Instead, it emerges through nested ruptures, differentiations, and asymmetries. In this sense, F is not just a tool for generating structure — it is the very condition of temporal becoming.

Just as a fractal boundary never resolves into a closed curve, the temporal process never resolves into a fixed state. The limit $\lim_{n \to \infty}$ in the construction

$$T = \lim_{n \to \infty} \left(\bigoplus_{k=1}^{n} A_k \right) \rtimes_{\nabla} F$$

is not an endpoint, but a mode of infinite deformation. Time is the consequence of this endless recursion: a topology of unfolding difference.

Note on Chronopoiesis: Time as Autopoietic Recursion

> *The following is a speculative and heuristic proposal. It is not intended as a formal derivation, but as an interpretive extension of the formal model — a philosophical gesture that explores its ontological implications.*

We begin not with being, but with the possibility of relation. A *potential group* is the space where identity exists only as the echo of a possible distinction. The operator ∇ does not explain — it deforms. It acts not by external impulse, but through internal asymmetry.

From this, we construct a *potential category* Π, whose objects are sites of possibility, and whose morphisms are the shadows of future differences. The functor $\nabla : \Pi \to C$ (e.g. Ab) introduces irreversibility. What cannot be undone — that is time.

The operator F performs recursive memory: it iterates, folds, and inscribes deformation. Time becomes the iterated deformation of potentiality — a dynamic topology of difference.

In Louis Kauffman’s cybernetic language, time is not an object, but an unhalting circuit. ∇ is the *mark*; F is the *re-entry*. The system cannot stop: each closure generates new openings.

Time is not a passive dimension. It is a **chronopoiesis** — a self-organizing, autopoietic process of deformation that recursively stabilizes and destabilizes its own eigenform. It is not something through which a system moves, but something that the system itself generates — as the emergent rhythm of operative recursion.

In this model, time is not a background but a result of the internal relationality of the system. It is a stream of deformation—not absolute, not global, but locally nested and fractal. Decoherence, understood as the local transition from symmetry to asymmetry, becomes a form of becoming.

Time as *A*ufhebung

Although the central formula has a seemingly Hegelian triadic structure (symmetry, deformation, synthesis), it should be noted that this is more a heuristic structure than a full ontology of the process. The traditional Hegelian triad is often overused as a simplified scheme of development, while Hegel himself never proposed it as a universal model.

In our model, the central role is played not by the triad itself, but by the concept of *Aufhebung*—abolition, preservation, and elevation simultaneously. Time as *Aufhebung* is the ongoing tension between form and deformation, between locality and globality, between symmetry and its transgression.

The formula:

$$T = \lim_{n\to\infty} \left(\bigoplus A_k \right) \rtimes_{\nabla} F$$

does not represent the triad — it enacts it. It is a performative formula: in a strange way, it requires the continual involvement of the observer to function. It does not merely describe dialectics — it *is* dialectics.

Time becomes the motion that abolishes local symmetries (via ∇), preserves them (via the fractal F), and elevates them (via the limit lim), forming a structure of higher order — but only through time, as a process of deformation sustained by performative engagement.

Is this still mathematics? We dare to say: probably not. And yet — something intelligible is happening here.

Aufhebung in this model takes on a dynamic, fractal, and structural form. It is not a stage, but a continual and ruptured function. Not a closure, but a motion of fractal duration.

Cognitive and Consciousness Applications

Time, consciousness, and ontological plasticity

The model of time as fractal decoherence can be interpreted not only within theoretical physics but also as an ontological-cognitive proposal. The notion of plasticity, as developed by Catherine Malabou in her interpretation of *Aufhebung*, is understood as the capacity of a structure to be shaped, to shape, but also to explode—that is, to undergo complete reconfiguration.

In the context of the present model:

- A_k are potential (formable) structures,
- ∇ is the operation of plastic deformation of local form (shaping),
- F is the function of memory, entanglement, and recursion (preserving and duplicating form),
- lim is plastic emergence—irreversible transformation of the entire configuration.

Thus, time appears as a form of ontological plasticity:

$$\text{time} = \text{the capacity of being to change its form through fractal decoherence-based deformations}$$

This can be expressed as a metaphysical dynamics of *Aufhebung*, which not only abolishes and preserves—but plastically transforms the form of becoming. This plasticity does not concern only material systems, but also mental, semantic, and cognitive structures.

Example applications:

- In artificial intelligence architectures: a temporal model based on local structural deformations (e.g., variability of network layers as an effect of the operator ∇),
- In neuroscience: perceiving time as the emergence of fractal synchronization and recursive memory,
- In psychology and philosophy of consciousness: consciousness as the capacity to generate internal "times" based on the transformation of modal structures,
- In ontology: time not as substance, but as the capacity for transformation inscribed within the materiality of relations.

This model may serve as a framework for thinking about subjectivity as a function of temporal deformation—a plastic tracking of the transformation of one's own form through time. Given that time in the proposed model is not a line nor a parameter, but a deformation structure, this model may have potential applications in the description of dynamic cognitive systems. If consciousness is a function of differentiating states, then a fractal-categorical structure can be viewed as a form of temporality, in which:

- perception of the present is a local activation of one of the objects A_k,
- memory is iterative embedding through F,
- "awareness of time" is the operational ability to reconfigure morphisms in S.

In this context, the model may be explored as a formal schema for architectures of artificial consciousness, in which time is not a built-in clock, but arises from local nonlinearity and self-similar reconstruction of the past.

Autopoiesis, Emergence, and Eigenforms of Time

In the context of the proposed model of time as fractal decoherence, reference to the concept of autopoiesis, as developed by Humberto Maturana and Francisco Varela, becomes essential. In their approach, a system is autopoietic when it reproduces its own organization within a closed operational loop. Our model of time—through the mechanism

$$T = \lim \left(\bigoplus A_k \rtimes_{\nabla} F \right)$$

This fulfills the condition that time is not externally imposed, but rather *emerges from within* the system as a dynamic synthesis of local symmetries (A_k) and deformational flow (F) modulated by ∇.

The operator F may be interpreted as a tool of self-similarity and reconstruction—a fractal memory of structures—analogous to Varela's cognitive loop. Kauffman, in turn, points to emergence as the act of transitioning from local interactions to complex organization. In this view, A_k (local groups) and ∇ (deformation gradient) can be understood as the elements of a minimal complexity threshold from which time emerges.

Additionally, reference to the concept of eigenform (Varela, von Foerster, Spencer-Brown) opens the possibility of interpreting time as a form that appears in the act of its own observation. The formula T not only describes time but also generates it as a "persistent form of perception in action." It is thus a self-referential structure whose existence depends on its own operational actualization.

Possibility of Falsification and Testability

Although the presented model is speculative in nature, it is worth asking about its falsifiability. One possible way to test the hypothesis of time as fractal decoherence would be to analyze event spaces in dynamic systems in terms of their Hausdorff dimension. If it were observed that the temporal evolution of certain systems has a fractal dimension—not only geometrically, but also in the context of informational evolution—this would provide evidence supporting the model.

Another possible testing avenue could involve tracking recursive patterns in artificial neural networks trained on temporal sequences—especially those that exhibit dynamic transitions between local symmetry and global unpredictability.

From the perspective of falsifiability, the key question becomes: can it be demonstrated that time is *not* an emergence from operations like $\rtimes_\nabla$ and F? Paradoxically, the most interesting aspect is that falsifying this model would require... another, equally operational model of time. And this leads us to a question not only of testability but of the pluralism of models of reality.

Note: What if Time Is Not Deformation?

> *This note continues the speculative and philosophical thread. It is not intended as a scientific argument, but as an ontological provocation — a performative gesture within the frame of recursive deformation.*

Assume for a moment that time is not deformation. Not ∇, not F, not a fractal recursion of difference. What then?

Any attempt to negate this structure must still operate within its frame. It must use difference — that is, ∇ — and invoke iteration — that is, F. Negation functions inside the system it seeks to negate.

This is not an error. It is the mechanic of the *speculaton*: a structure that absorbs each attempt at its refutation as another moment of its unfolding. It is not a proof — but a *non-non-proof*. It is event. Performativity.

The speculaton does not say: "It is true." It says: "It functions — because you cannot undo it without using its tools." And the more you try, the more you densify its form.

From this impotence — this crisis — new sense could arise. Not truth. Not falsehood. Not yet another turn of speculaton. But the generativity of deformation.

This could anticipate framing of the dialectic of crisis in the double descent phenomenon. Crisis is not an end — it is, unfortunately, the necessary threshold toward higher structural recursion. Likewise, the inability to refute the dialectics without its own tools is not a failure of critique, but the very engine of emergence.

Time in Physical Terms: A More Tangible Perspective

As scientific and philosophical thought progressed, there was a growing desire to define time in a more concrete and measurable way, moving beyond purely abstract notions. One such approach that emerged conceptualizes time not as a fundamental entity but as a relational property—specifically, as a ratio that connects changes occurring at a structural level with transformations happening at a qualitative level, like in old schools of dialectic. This relationship can be expressed with the following formula:

$$t = \frac{\Delta S}{\Delta Q}$$

To better understand this model, let's break down the components of this equation:

- ΔS (Delta S) serves as a measure of **structural shifts**. This encompasses a wide range of alterations within a system, such as genetic mutations in biology, iterative steps in computational processes, or observable changes in the patterns and arrangements of physical or conceptual systems.
- ΔQ (Delta Q), on the other hand, is concerned with tracking **qualitative transformations**. These are changes that alter the nature or essence of the system, which can manifest as shifts in energy states, variations in informational content, or transformations in phenomenological experience (i.e., how the system is perceived or experienced directly).

What is the core intuition behind this formulation? We suggest the following: *If the underlying framework or "scaffolding" of a system (its structure) evolves or changes more rapidly than the meaning, significance, or essence it embodies (its quality), then our subjective experience is that time feels quick, or accelerated. Conversely, if the meaning or qualitative aspects of a system transform at a faster rate than the form or structure that contains them, then our perception is that time stretches, slows down, or becomes more dense.*

This model introduces a fascinating implication: time is not necessarily uniform but can vary in its flow and character *within* different systems, or even within different aspects of the same system. This isn't merely about the relativistic effects described by Einstein, where time is relative to the observer's frame of reference. Instead, it points to intrinsic, qualitative differences in the experience and progression of time, which are dependent on the specific internal dynamics and the interplay between structural and qualitative changes within each system.

Time as Fractal Decoherence: A Recursive Unfolding

We can propose now a significant reframing of our understanding of time, moving away from the conventional image of linear, unidirectional passage. Instead, time is conceptualized as **fractal decoherence**. This is a complex idea, but we can understand it as a process where the coherence (symmetrical potentiality without any relation to reality) of a system undergoes a collapse. Crucially, this collapse is not a simple, singular event. Instead, it is "fractal," meaning it unfolds recursively, repeating its primarily simple non-symmetrical pattern of breakdown across various scales and levels of the system. Time, in this view, is this very multi-layered, self-similar process of coherence loss.

This concept is formalized with the equation:

$$T(s) = \frac{1}{f(\omega)} \int_{\Omega} \psi^*(x) \hat{D} \psi(x) \, d\omega$$

Let's dissect this mathematical expression to grasp its meaning:

- $T(s)$ represents the measure of time that this model seeks to define, where 's' might denote a scale parameter.
- $\psi(x)$ (psi of x) symbolizes the state of the system. It's a mathematical description of the system's current configuration or condition at a given point.
- $\hat{D}$ (D-hat) is the **decoherence operator**. This operator mathematically embodies the transition of the system from a realm of multiple possibilities (a superposition of potential states) to a specific, actualized state. It's the mathematical engine driving the "collapse" of coherence.
- $f(\omega)$ (f of omega) is a function that regulates the scale-frequency of this decoherence process. It essentially tunes how the decoherence manifests and is measured across different scales or frequencies within the system.
- The integral $\int_{\Omega}$, along with $d\omega$ (an infinitesimal element of the integration space Ω) and $\psi^*(x)$ (the complex conjugate of the state function), are components of a mathematical framework (often employed in quantum mechanics) used to calculate the average effect or expectation value of the decoherence operator's action on the system.

This formalism—viewing time through the lens of fractal decoherence—can offer a richer explanation for subjective temporal experiences. For instance, it might help us understand why certain moments in life feel disproportionately long or short, or why complex, multifaceted events can seem to fragment or distort our linear perception of time, making it feel non-uniform and layered.

The Fractal Structure of Time: Self-Similarity Across Scales

If time is indeed a recursive process—one that refers back to itself in its own definition, much like a fractal pattern contains smaller versions of itself—then it should adhere to mathematical laws that describe such self-similar structures. We propose the following recursive formula to capture this fractal nature:

$$T(s) = \alpha T\left(\frac{s}{\lambda}\right) + \beta T\left(\frac{s}{\lambda^2}\right)$$

What does this equation convey? It suggests that the experience or measure of time at a particular scale ($T(s)$) is dependent on, and composed of, the behavior of time at smaller, nested scales (represented by $T(s/\lambda)$ and $T(s/\lambda^2)$). The parameters α (alpha) and β (beta) are weighting factors, and λ (lambda) is a scaling factor, all ofwhich determine how these different temporal scales interact and contribute to the overall temporal fabric. This pattern—where smaller iterations and cycles collectively shape the larger-scale perception and structure of time—is proposed to account for the irregularities, sudden bursts of activity, apparent gaps, and the general non-linearity we often observe in both physical processes and subjective experience.

Adopting this perspective has profound implications for our understanding of causality. The relationship between cause and effect is no longer seen as a straightforward, linear progression from event A to event B. Instead, causality becomes a more complex, folded surface, where influences can propagate and interact across these different, nested layers of time in intricate and non-obvious ways.

Decoherence and the Arrow of Time: Why Time Moves Forward

One of the most enduring mysteries in physics and philosophy is the directionality of time—why does it appear to flow inexorably from the past, through the present, and into the future, but never in reverse? The author invokes the Lindblad equation, a key formula in quantum mechanics that describes the evolution of open quantum systems (systems that interact with their external environment):

$$\frac{d\rho}{dt} = -i[H, \rho] - \gamma(\rho - \rho_{\text{diag}})$$

Without delving too deeply into the mathematical intricacies, this equation essentially describes how a quantum system loses its coherence (its uniquely quantum properties, like superposition) in an irreversible manner due to its interactions with the surrounding environment. In this equation, ρ (rho) represents the density matrix, a mathematical object describing the state of the quantum system; H is the Hamiltonian, representing the total energy of the system; and the term involving γ (gamma) quantifies the rate and nature of the decoherence process.

The central idea here is that **decoherence is what gives time its arrow**. Time's perceived forward motion is not an illusion, nor is it merely a consequence of objects moving through space. Rather, it is a direct result of the continuous and irreversible reduction of possibilities within a system. As a system interacts with its environment, it transitions from a state of many potential futures to a single, actualized present. The return to a previous state of full coherence, with all its prior possibilities intact, becomes overwhelmingly improbable (and for all practical purposes, impossible for complex, macroscopic systems). This ongoing, irreversible loss of quantum coherence and reduction of potentialities is what we perceive as the forward flow of time.

Time as an Emergent Phenomenon: Arising from Collective Dynamics

Drawing an analogy with concepts like temperature in thermodynamics, we suggest that time, much like temperature, might not be a fundamental property of the universe at its most basic level. Temperature isn't a characteristic of a single, isolated particle; rather, it's an emergent property that arises from the collective statistical behavior of a vast number of particles. Similarly, this model proposes that time itself is an **emergent phenomenon**. It doesn't exist as a pre-ordained backdrop but rather emerges statistically from the multitude of interactions occurring within a complex system. This idea is captured by the formula:

$$T_{\text{emergent}} = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \Delta S_i$$

This equation suggests that the emergent experience of time (T_{emergent}) can be thought of as an average of the structural changes ($DeltaS_i$) occurring across a very large number (N approaching infinity) of interacting elements or sub-processes within a system. This implies that time, as we know it, only comes into being when a certain threshold of complexity and interaction density is crossed. It is when these interactions become sufficiently numerous and patterned that a measurable, regular rhythm—what we identify as the passage of time—can emerge from the collective dynamics.

Presented model, while speculative, offers a formally coherent framework for reconstructing our understanding of time as a process rooted in fractal decoherence. It is possible to translate this conceptual construction into the rigorous language of category theory and further exploring its profound implications for fields such as information theory, cosmology, and the burgeoning study of artificial consciousness.

Testability and Empirical Implications: How Can These Ideas Be Probed?

The model of time as fractal decoherence is, by design, speculative and conceptual. Nevertheless, it invites certain pathways for verification — or at least confrontation — with reality, whether empirical or formal. Below are several proposals:

- **Operational Signatures:** If time is a process of deformation and coherence-loss, it may be possible to observe measurable asymmetries in physical processes where symmetry breaking has no obvious external cause. An example might include quantum decoherence exhibiting unusual or anomalous directionality.
- **Fractal Traces in Noise:** Time-series data (e.g., from neuronal systems such as brain activity, or from models of black hole evaporation) could reveal recursively structured noise signatures or scale-free fluctuations — characteristic of fractals — within temporal correlations. This could offer indirect evidence of fractal recursion in time-bound processes.
- **Topological Quantum Models:** The formal structure of iterated semi-direct products — a mathematical construction used in group theory and previously proposed here as a model of time — may find resonance in certain quantum systems, especially in so-called topological phases where local symmetries are broken via emergent gradients (i.e., slow, structured variation of field-like quantities).
- **Simulation Heuristics:** Building computational simulations (e.g., based on cellular automata or agent-based systems) that implement recursive group actions and deformation operators could lead to the emergence of pseudo-temporal structures. Such outcomes might offer insight into synthetic chronogenesis — the artificial generation of time.
- **Failure as Insight:** Finally, if this model proves incapable of coherently describing or simulating reality in any meaningful way, then the *manner of its failure* might itself yield philosophical value — helping us better understand what time is *not*. This, in turn, would sharpen the ontological contrast with existing frameworks that aim to model temporality.

This is not a theory in the narrow Popperian sense — one that can be tested through falsification. It is rather a **generative diagram** — a conceptual framework, a heuristic schema. It offers a perspective within which other models may be deformed, restabilized, or shown to be "blind" to recursion itself. In this sense, the model does not predict specific phenomena — it **orients** our thinking about time.

Examples for the Imagination: Time Across Contexts

To bring these abstract ideas closer to experience, here is a series of examples illustrating how the proposed concept of time — as the ratio of structural change ΔS to qualitative change ΔQ, and as fractal decoherence — might manifest across various domains:

- **Subjective Experience:** In our personal perception, time seems to stretch or contract depending on the interplay between structural novelty and qualitative shift. A boring lecture may feel endless (low ΔS — little structural change in content, and low ΔQ — little qualitative engagement). In contrast, an engaging conversation can make time vanish, because meaning (high ΔQ) unfolds faster than the structure of duration (ΔS) can register it.

- **Developmental Biology:** Embryos undergo rapid structural transformations (high ΔS), while cellular and tissue differentiation (ΔQ) may progress more gradually. As growth slows, subtle qualitative shifts begin to dominate — time becomes a kind of map where form catches up to function.
- **Evolutionary Dynamics:** Long periods of evolutionary stasis may conceal latent potential, until a qualitative shift (e.g., environmental) triggers a structural leap (e.g., speciation). Here, time appears as a punctuated process — it becomes visible only when the balance between ΔS and ΔQ is disrupted.
- **Adaptive Systems (e.g., Neural Networks):** In learning processes, such as those in artificial neural networks, the ability to process information and generate meaningful responses (rise in ΔQ) can increase with minimal structural change (low ΔS). But when the network's topology itself evolves, time seems to accelerate — emergence is felt through structural leaps.
- **Language:** Structural repetition in language (identical phrasing) may conceal deep shifts in meaning depending on context. Poetry deliberately increases ΔS (through unusual syntax, metaphor) to deform semantic flow (influence ΔQ). Conversely, everyday expressions rely on variable ΔQ (intention, context) while preserving structural stability (ΔS).
- **Quantum Decoherence:** In quantum decoherence, when superposed states collapse into a single actual state, ΔS increases (from potential to actuality). Simultaneously, ΔQ reflects the loss of informational purity — the quantum state becomes "entangled" with its environment. Time here is like a shadow cast by irreversibility — a residue of the unravelling of quantum states.
- **Human–AI Interaction:** In human–AI dialogue, ΔQ may capture the depth and quality of the exchange, while ΔS models recursive adaptation — the human learns to shape prompts, the AI refines responses. When both grow together, time is not merely endured but felt as an intensity of mutual deformation and co-creation.

In this autopoietic (self-generating) process, there is immense diversity — a dynamic, emergent "homomorphy" (structural similarity) unfolding at every level. It invites the imagination to dive into these "fractals of fractals," into the perpetual multiplication of processes.

What we have presented is an ongoing exploration of this approach — using mathematical metaphors or "spells," but not mathematics itself; with empathy for physics, though not academic physics. It may not even be philosophy in the strict sense of the word.

It is a form of performative thinking, one that requires a kind of attunement, a specific kind of attentiveness. The first chapter, centered on the concept of time, was intended precisely to evoke this mood.

3 Possible Further Developments of the Formula

The original F.T. R. equation for time uses abelian groups to represent local symmetries:

$$T = \lim_{n\to\infty} \left(\bigoplus_{k=1}^{n} A_k \right) \times \nabla F \tag{1}$$

Where A_k represent abelian groups, ∇ is the deformation operator, and F is the fractal recursion operator. In this formulation, time emerges as a process that breaks commutative symmetries, creating directionality through deformation. The abelian groups represent static, harmonious structures that form the foundation of potential reality states.

A significant enhancement involves replacing abelian groups with Galois groups, which describe symmetries of field extensions:

$$T = \lim_{n\to\infty} \left(\bigoplus_{k=1}^{n} G_k \right) \times \nabla F \tag{2}$$

Where $G_k = \mathrm{Gal}(L_k/K_k)$ is the Galois group for field extension L_k/K_k. This substitution introduces a more dynamic and relational symmetry structure. Galois groups encode how the "roots" of reality can be rearranged while preserving fundamental relationships. Unlike abelian groups, they capture the inherent non-commutativity of complex systems, making them better suited for modeling the intricate symmetries underlying spacetime.

Since Galois groups are typically non-commutative, the direct sum operation can be replaced with a tensor product to better capture the entanglement of symmetries:

$$T = \lim_{n\to\infty} \left(\bigotimes_{k=1}^{n} G_k \right) \times \nabla F \tag{3}$$

The tensor product $\bigotimes_{k=1}^{n} G_k$ creates a complex space of symmetries where Galois groups interact, forming a relational network. This operation better reflects how symmetries become entangled in fractal structures, allowing for the emergence of complex patterns from simple components. The tensor product captures the multiplicative rather than additive nature of symmetry interactions in complex systems.

To account for the dynamic interaction between entangled symmetries and deformation, we replace the direct product with a semidirect product:

$$T = \lim_{n\to\infty} \left(\bigotimes_{k=1}^{n} G_k \right) \rtimes \nabla' F' \tag{4}$$

Where ∇' and F' are modified versions of the deformation and fractal operators adapted to work with Galois groups. The semidirect product $\rtimes$ acknowledges that the deformation process doesn't merely combine with symmetries but actively transforms them. This operation models how the deformation operator ∇' acts on the symmetry structure, creating a more sophisticated interaction that better captures the way time emerges from symmetry breaking.

To better capture the continuous nature of the interaction, we can refine the formula using a continuous semidirect product:

$$T = \lim_{n\to\infty} \left(\bigotimes_{k=1}^{n} G_k \right) \rtimes_{\text{cont}} \nabla' F' \tag{5}$$

Where $\rtimes_{\text{cont}}$ denotes a continuous semidirect product that allows for smoother interaction between symmetries and deformation. This refinement acknowledges that the interaction between symmetry and deformation occurs in a continuous rather than discrete manner, providing a more nuanced model of how time emerges from the underlying mathematical structures. The continuous semidirect product allows for infinitesimal transformations, better modeling the smooth flow of time.

The most sophisticated enhancement incorporates topological aspects through loop cohomology:

$$T = \lim_{n\to\infty} \left(\bigotimes_{k=1}^{n} G_k \right) \rtimes_{\text{cont}} \nabla \circ F \curvearrowright H^1_{\text{loop}}(K) \tag{6}$$

Where:

$G_k = \mathrm{Gal}(L_k/K_k)$ **Galois groups of successive extensions** — local algebraic symmetries (of microstructures); encode relationships between field elements indistinguishable under automorphisms.

$\bigotimes$ **Profinite tensor** — fractal aggregation of entangled symmetries; symbolizes how local Galois groups join into a potentiality space of infinite interweaving.

$\lim_{n\to\infty}$ **Limit to infinity** — emergence of global spacetime; a macrostructure arises from local compositions of symmetries and deformations.

$\rtimes_{\text{cont}}$ **Continuous semidirect product** — interaction of deformation with symmetries; a dynamic formulation where space deforms via operators acting on group structures.

∇ **Deformation operator (form gradient)** — structural change, local breakages; represents moments of rupture, bending, where structure becomes a new structure.

F **Fractal operator recursion** — production of new layers and trajectories; an iterative action that does not merely add data but generates complexity.

$H^1_{\text{loop}}(K)$ **Loop cohomology over the knot category** — information about stable (eigen)topological forms; equivalence classes of deformations that retain form despite iteration.

$\curvearrowright$ **Action on the space of interpretation** — transformation of potentiality into concreteness; models how mathematical structures begin to make sense as forms of the world.

This final formulation presents time as a process emerging from entangled Galois symmetries, deformation, fractal iteration, and topological structures. The introduction of loop cohomology $H^1_{\text{loop}}(K)$ connects the formula to knot theory, suggesting that time can be understood through the lens of topological invariants. These invariants represent stable patterns that persist through deformation, much like how certain aspects of reality remain consistent despite the flow of time.

Unlike the original formula, where time results from breaking commutative symmetries, this enhanced version presents time as a relational exploration of non-commutative symmetries, emphasizing its discrete, algebraic, and topological character. The loop cohomology can be interpreted as a space of equivalence classes of knots (algebraic or temporal), on which the external symmetry G_∞ acts. This creates a map of all possible "entanglements of reality," from which certain configurations are selected through the recursive coupling $\nabla \circ F$.

This topological perspective suggests that time is not merely a parameter but a complex structure with intrinsic topology, where events are connected through knot-like relationships that determine the possible paths of reality's evolution. The formula thus provides a mathematical framework for understanding time as an emergent property of underlying symmetry transformations and topological constraints.