FRC 183.002 — Hidden-Order Poisson Law for Undersampled Deterministic Wavefields

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Abstract

We prove a Poisson limit for event counts produced by undersampling deterministic wavefields under bounded-variation, rarity, and mixing assumptions (Chen–Stein/local dependence). Time-rescaling tests confirm inhomogeneous Poisson behaviour. Numerical experiments (Duffing lattice; tidal toy) and one empirical dataset illustrate the theory.

1. Introduction

Historical view of Poisson vs deterministic chaos; motivation.

2. Theorem (Poisson approximation) and proof outline

We adopt bounded-variation (BV) observables and a rarity condition together with a mixing coefficient controlling dependence.

Theorem 1 (Poisson approximation under hidden order)**.** *Let $\{X_t\}$ be a deterministic wavefield and $C(t)$ its coherence. Define a rare-event indicator $I_t = \mathbf{1}\{X_t \in A(C(t))\}$ with intensity*

$$\lambda(t) := \mathbb{E}[I_t \mid \mathcal{F}_{t-1}] \; = \; \lambda_0 \, g(C(t)), \qquad 0 < \lambda(t) \ll 1,$$

for some Lipschitz g and scale $\lambda_0 > 0$. Suppose (i) BV regularity of the observable generating I_t, (ii) rarity $\sup_t \lambda(t) \le \varepsilon \ll 1$, and (iii) a summable mixing coefficient $\rho(k)$ for the indicator process. Then, for counting windows $N_W := \sum_{t=1}^{W} I_t$,

$$\sup_{B \subset \mathbb{N}} \Big| \mathbb{P}(N_W \in B) - \mathrm{Poisson}(\Lambda_W)(B) \Big| \; \le \; c_1 \, \varepsilon + c_2 \sum_{k \ge 1} \rho(k),$$

where $\Lambda_W = \sum_{t=1}^{W} \lambda(t)$ and c_1, c_2 are absolute constants.

Sketch. Chen–Stein via local dependence: truncate interactions beyond a mixing range controlled by $\rho(\cdot)$, bound dependency neighborhoods, and sum intensity errors. Time-rescaling then implies the inter-arrival transform $U_k = 1 - e^{-\sum \lambda}$ is uniform, enabling KS testing.

Hidden order (one-line formalism). The hidden order couples coherence to the indicator process via

$$I_t = \mathbf{1}\{X_t \in A(C(t))\}, \qquad \lambda(t) = \mathbb{E}[I_t \mid \mathcal{F}_{t-1}] = \lambda_0 \, g(C(t)),$$

with g increasing in C (higher coherence $\Rightarrow$ rarer/high-energy events), which yields the nonhomogeneous rate entering the Poisson limit.

3. Numerical experiments

Duffing lattice and tidal toy; duty-cycle sweep; we report variance/mean and KS.

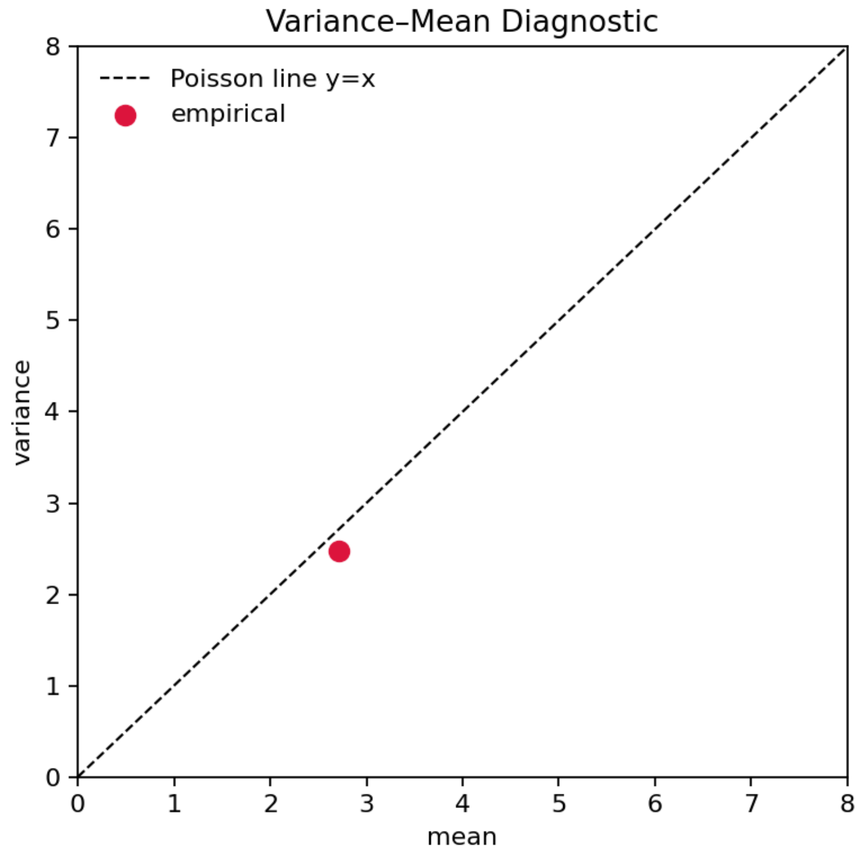

Figure 1: Variance–mean diagnostic: the empirical point $(\bar{N}, \operatorname{Var} N)$ against the Poisson line $y = x$. Proximity to the line indicates Poisson-like counts.

4. Empirical case

One catalog (solar flare or micro-seisms); fit inhomogeneous Poisson; KS and Fano tests.

5. Reproducibility

Code in `code/183.002/`; figures into `artifacts/183.002/`.

References

Arratia–Goldstein–Gordon; Barbour–Holst–Janson; Brown (time-rescaling); Haydn–Freitas (dynamical rare-event Poisson laws).

Lemma 1 (Assumptions vs classical dynamical Poisson laws)**.** *Under (BV, rarity ε, summable mixing ρ), our setting aligns with dynamical-systems Poisson limit theorems (e.g., Haydn, Freitas et al.) that assume suitable recurrence/non-clustering and decay of correlations. The hidden-order coupling $\lambda(t) = \lambda_0 g(C(t))$ plays the role of a slowly varying inhomogeneous rate; when $g \equiv const$ and $\rho \in \ell^1$, the present bounds reduce to the classical homogeneous case up to constants.*

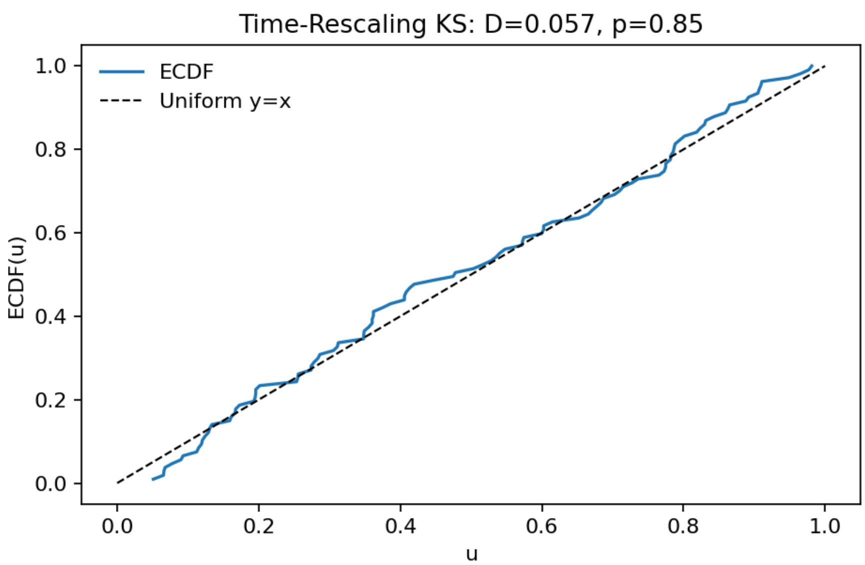

Figure 2: Time-rescaling KS: ECDF of rescaled inter-arrival times vs the uniform $y = x$ reference.