FRC 566.020 — Spectral Modes and Relaxation in UCC

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Abstract

We linearize the UCC evolution for $u = \ln C$ around a stationary equilibrium and derive modal relaxation rates $\tau_n = 1/(D_C\,\lambda_n)$ from the Laplace spectrum. We connect these to measured μ-level oscillations and quantify approach to coherence equilibria.

1. Setup and linearization

Let u solve $\partial_t u = D_C\,\Delta u + S_C$ on Ω with (N)/(D) BCs. Suppose u_* is stationary: $D_C\,\Delta u_* + S_C = 0$. Linearize $u = u_* + v$. Then v satisfies $\partial_t v = D_C\,\Delta v$ to first order (inhomogeneity absorbed by u_*).

Theorem 1 (Spectral decomposition). *Let $\{-\Delta\phi_n = \lambda_n\phi_n\}$ be the Laplacian eigenbasis under (N)/(D). Then the solution admits $v(t,\cdot) = \sum_n a_n e^{-D_C\lambda_n t}\,\phi_n(\cdot)$ with relaxation times $\tau_n = 1/(D_C\lambda_n)$.*

Sketch. Expand in eigenfunctions; coefficients evolve by $\dot{a}_n = -D_C\lambda_n a_n$.

2. Assumptions and implications

- Ω bounded with C^1 boundary; standard spectral theory; $\lambda_0 = 0$ for (N), $\lambda_1 > 0$ for (D).
- For (N), the mean mode is conserved; relaxation governed by λ_1.
- For (D), all modes decay; τ_1 sets the slowest timescale.

3. Link to μ-level oscillations

In FRC, observed coherence oscillations decompose into modal contributions with frequencies/decays controlled by $\{\lambda_n\}$. Measuring τ_n recovers D_C and domain scales.

4. Numerical demo

We include a 1D example on $[0,1]$ showing exponential decay of modes and estimating τ_n from data.

5. Reproducibility

Run `python code/566.020/spectral_modes_demo.py`; artifacts in `artifacts/566.020/`.