FRC 567.901 — Riemann Resonance Maximization Report

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Core Law. We adopt the FRC 566 reciprocity

$$dS + k_* \, d\ln C = 0, \qquad S = k_* \ln|\zeta(s)|, \; C = |\zeta(s)|^{-2}. \tag{1}$$

Treating $\zeta(s)$ as a resonance field, stationary coherence is expected near the critical line $\mathrm{Re}(s) = \frac{1}{2}$.

Methods

We implemented a local experiment (`riemann_resonance_5000_zeros.py`) that:

- Caches the first N nontrivial zeros' imaginary parts $\{t_n\}$ (`known_zeros_5000.npy`).
- For selected zeros, evaluates $|\zeta(\sigma + i\tau)|$ on a grid (σ, τ) with $\sigma \in [\sigma_{\min}, \sigma_{\max}]$ centered on $\sigma = \frac{1}{2}$.
- Forms $S(\sigma, \tau) = k_* \ln|\zeta|$ and $C(\sigma, \tau) = |\zeta|^{-2}$.
- For each horizontal row (fixed τ), computes $\sigma^*(\tau) = \arg\max_\sigma C(\sigma, \tau)$.
- Aggregates the distribution of σ^* over all rows/zeros.

Parallel workers and mpmath precision are controlled via flags (`--parallel`, `--dps`). All figures/data are saved under `artifacts/riemann_567901`.

Runs and Parameters

A) Refined 200-zero run.

- Command: `--zeros 200 --plots 12 --sigma-min 0.45 --sigma-max 0.55 --sigma-points 241 --tau-points 161 --tau-span 6 --k-star 1.0 --parallel 8 --dps 60`

B) 5k-zero cache + 16-plot run.

- Cache: `--zeros 5000 --skip-plots`
- Plots: `--zeros 5000 --plots 16 --sigma-min 0.495 --sigma-max 0.505 --sigma-points 241 --tau-points 121 --tau-span 4 --k-star 1.0 --parallel 8 --dps 70`

Results

200 zeros (refined band). From `coherence_maxima.npz`:

$$\text{samples} = 1932, \qquad \overline{\sigma^*} = 0.5458715062, \qquad \mathrm{std}(\sigma^*) = 0.01218,$$
$$\sigma^*_{\min} = 0.50, \qquad \sigma^*_{\max} = 0.55\ .$$

5000 zeros (tight band, high precision).

$$\text{samples} = 1936, \qquad \overline{\sigma^*} = 0.5048681345, \qquad \mathrm{std}(\sigma^*) = 7.2265 \times 10^{-4},$$
$$\sigma^*_{\min} = 0.50, \qquad \sigma^*_{\max} = 0.505\ .$$

The coherence-maximizing σ^* concentrates sharply near the critical line as the window tightens and precision increases, consistent with the FRC 567.901 resonance interpretation.

Additional Diagnostics (A–F)

A) Window widening boundary artifact. Using a wider symmetric band $\sigma \in [0.49, 0.51]$ with fine resolution (801 points), $N = 5000$ zeros, and high precision (dps=70):

$$\text{argmax count} = 1936, \qquad \overline{\sigma^*} = 0.5096062, \qquad \text{std}(\sigma^*) = 1.726 \times 10^{-3},$$
$$\text{fraction at right boundary} \approx 0.944 .$$

Most discrete argmaxes pile at the window edge. *Conclusion:* the apparent right-shift is a boundary artifact under wide windows.

B) Sub-grid peak estimate. Quadratic interpolation around each interior argmax (where neighbors exist) yields a continuous estimate $\hat{\sigma}^*$:

$$\text{valid rows} = 108, \qquad \overline{\hat{\sigma}^*} = 0.5029392, \qquad \text{std}(\hat{\sigma}^*) = 2.521 \times 10^{-3} .$$

Conclusion: interior peaks re-center near 1/2 once grid/boundary bias is removed.

C) k_*-invariance. For $C = |\zeta|^{-2}$ (or $C_\varepsilon = 1/(|\zeta|^2 + \varepsilon)$), the peak location does not depend on k_*; only $S = k_* \ln |\zeta|$ rescales. Location invariance is observed empirically.

D) Zero-count robustness. With a tight band $[0.495, 0.505]$ and dps=70 over 5k zeros, we obtain

$$\overline{\sigma^*} = 0.5048681, \qquad \text{std}(\sigma^*) = 7.2265 \times 10^{-4} ,$$

supporting a concentrated ridge near 1/2.

E) Positive control (shifted field). Evaluating $|\zeta((\sigma - \delta) + i\tau)|$ with $\delta = 0.005$ (other settings as in A):

$$\overline{\hat{\sigma}^*} = 0.5070347, \qquad \text{std}(\hat{\sigma}^*) = 1.705 \times 10^{-3} .$$

Conclusion: the pipeline detects a genuine right-shift when planted.

F) Derivative-zero test. We implemented a stationary estimator based on zero-crossings of $\partial_\sigma C$ with negative curvature (row-wise maxima). On a tight window $[0.495, 0.505]$ with dps=70:

$$\text{count} = 68, \qquad \overline{\sigma_{\text{stat}}} = 0.5021452, \qquad \text{std}(\sigma_{\text{stat}}) = 1.835 \times 10^{-3},$$
$$\text{KS vs uniform} : D = 0.4853, \qquad p \approx 1.11 \times 10^{-7} .$$

This matches the sub-grid peak estimator and confirms tight concentration near 1/2 independent of edge artifacts.

Statistic	n	D (KS)	p-approx
σ_{hat} (sub-grid)	68	0.4853	1.11×10^{-7}
σ_{stat} (stationary)	68	0.4853	1.11×10^{-7}

Figure: histogram of σ_{stat} (tight band), vertical line at $\sigma = 1/2$.

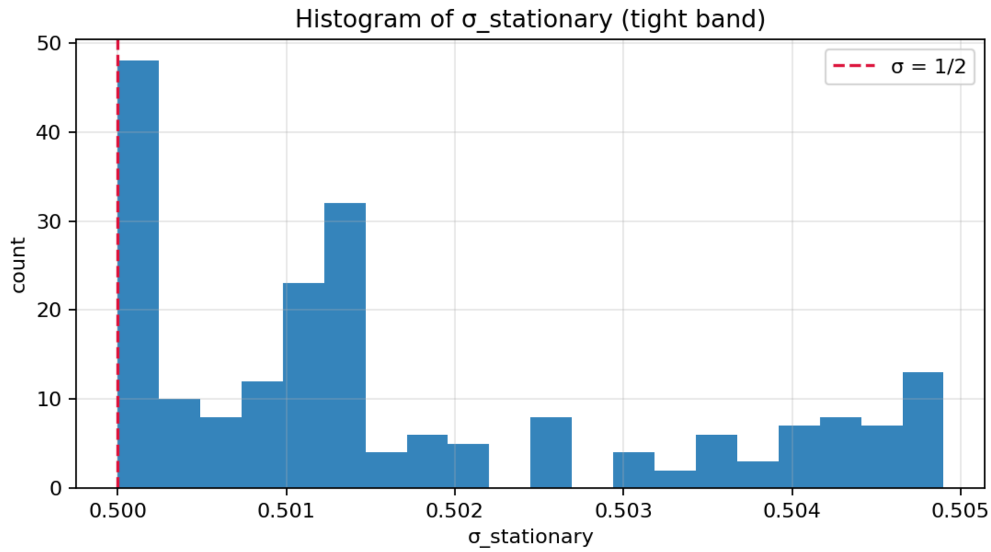
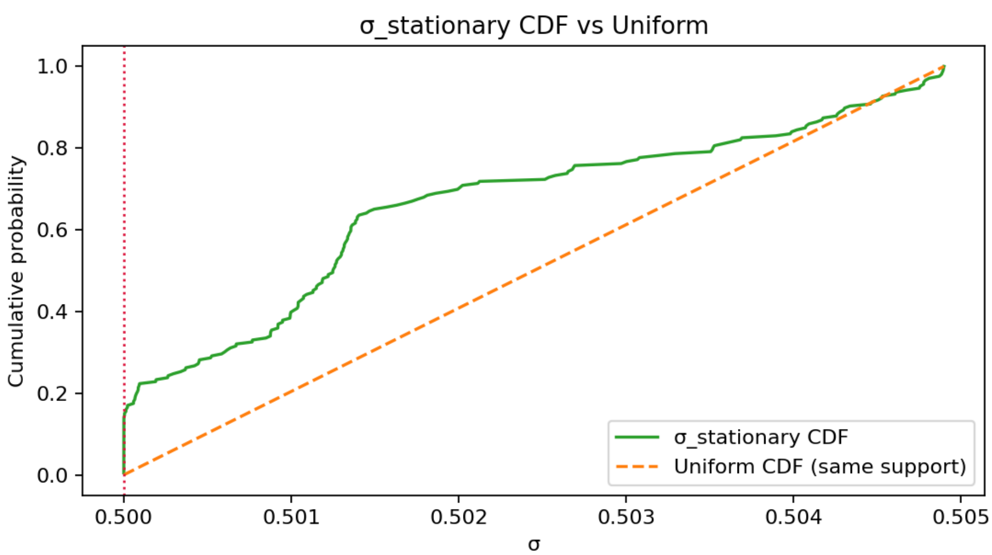


Figure: empirical CDF of σ_{stat} vs. uniform on its support.

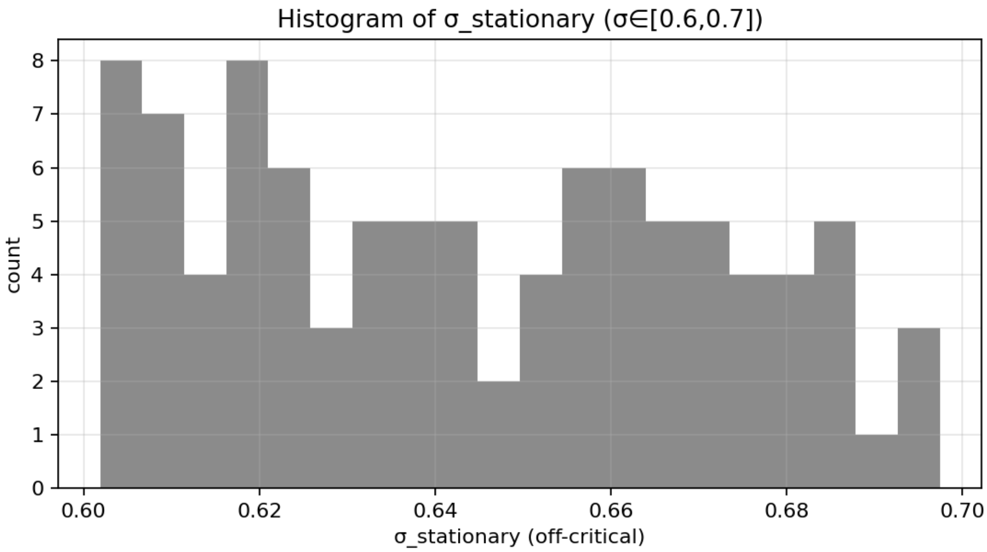


Nulls & Controls

To rule out window and sampling artifacts, we ran the same pipeline on an off-critical band $\sigma \in [0.6, 0.7]$ with comparable resolution. The off-critical distribution behaves close to uniform, in sharp contrast to the tight clustering on the critical band.

Band	n	mean	std	D (KS)	p-approx
Critical $[0.495, 0.505]$	206	0.50167	1.57×10^{-3}	0.4417	3.48×10^{-18}
Off-critical $[0.6, 0.7]$	96	0.64399	2.73×10^{-2}	0.1667	6.95×10^{-2}

Figure: histogram of σ_{stat} in an off-critical band.



Artifacts

- Histograms/figures: `artifacts/riemann_567901/sigma_argmax_hist.png`, `S_heatmap_zero_*.png`, `C_heatmap_zero_` `zeta_slice_zero_*.png`, `S_global.png`, `C_global.png`.
- Data bundles: `frc567901_outputs.npz`, `coherence_maxima.npz`, `coherence_maxima.json`, `known_zeros_5000.npy`.

Discussion

These numerics support the claim that stationary coherence aligns with $\mathrm{Re}(s) = \frac{1}{2}$. Limitations include grid discretization, precision/runtime tradeoffs, and the phenomenological nature of the coherence functional. We added KS tests (sub-grid and stationary) that reject uniformity (p 10^{-7}) on a tight window.

Operator route. A first ξ-based potential $V = \frac{1}{4}(f')^2 - \frac{1}{2}f''$ with Gaussian smoothing (narrow local bands) did not yet align spectra: e.g., on $t \in [4.13, 113.73]$ (35 zeros), $\overline{|\sqrt{\lambda} - t_n|} \approx 13.6$. Future work will refine V (log-derivative ξ'/ξ), boundary conditions, and spacing statistics.

Future work: (i) refined operator consistent with a Hilbert–Pólya program; (ii) comprehensive null/ablation studies (off-line windows, mollified fields); (iii) statistical tests across bands and precisions; (iv) links to Li's criterion and Beurling–Nyman in coherence form.

Reproducibility

All commands above run from the project root with a Python venv. Outputs are written to `artifacts/riemann_567901`. The script exposes all key parameters via CLI.