FPT (US Treasury bill replication), DYT (capital-efficient leveraged yield trading) and a Novel AMM

December 26, 2023

1 Abstract

In this white paper, we analyze the capital efficiency of YT tokens, as we introduce a novel decomposition of PT+YT (into FPT and DYT) that is compatible to Pendle's PT-SY framework, to allow for a more capital efficient way of fixed income investing or speculating in yields. Specifically, FPT replicates a cash flow similar to US Treasury bills, and it will capture the stable (and constant) cash flow part of yield generating asset, resembling a true "fixed income asset" that has a constant income stream daily, and a fixed income at expiration; DYT, on the other hand, will capture the more variable (and more volatile, made easier for speculation) cash flow of a yield generating asset, borrowing some resemblance from interest rate swaps (IRS), making it a vehicle better fit for betting on yield movement. We then discuss the well-definedness and feasibility of such idea as well as a few implementation considerations, and its broad applications.

2 Table of Content

3 Introduction

As was introduced by Pendle, YT (yield token) is the yield component of a yield bearing asset. The users can obtain the exposure to yield movement by holding a long position in YT. YT allows the user to long the yield change, and get a profit when yield goes up after purchase, or record a loss when yield goes down after purchase. Two yield strategies introduced by Pendle, one being holding (long yield strategy) and another being speculating (active yield trading) strategy all rely on the nature of YT that captures the yield component of the underlying asset.

It is worth noting that YT enables user to trade yield with (implicit) leverage, which is another main contribution from PT-YT separation. PT is very capital intensive, because all the yield or return received on PT requires an upfront capital of roughly same amount of total face value (although less by a few percentage points); YT is hence more capital efficient by allowing users to gain exposure by only purchasing the yield component, which is also only a relatively small chunk of market value of SY. If we borrow the terminology from traditional finance, PT implements the function of a Zero-Coupon-Bond (ZCB) while YT implements a composite cash flow of variable coupons. Putting them side by side gives us products similar to strip bonds.

Being capital efficient as YT is, there is still inefficiencies in its active yield trading aspect. Imagine the yield is very stale (or fluctuates very little), and the "moving" part of YT is very little compared to the current yield level, we still need to put in a certain amount of capital (which is an increasing function of current yield) to gain exposure to the small fluctuations. Indeed, we face the similar problem in traditional finance, when interest rate measures do not usually move around as much, traders need (and more importantly, are willing to pay some haircut) to gain access to an investment vehicle that tracks changes of yield, and reducing the capital upfront relating to the current level of yield. A typical example of such products is futures, where traders are allowed to put aside margins to leverage up, and capture volatility with less capital. A more aggressive solution (and hence higher counterparty risk, and with more constrained use) is interest rate swap (IRS), where one leg receives floating rate, and another leg receives a fixed rate such that no upfront capital is needed! Of course IRS leads to significant counter-party risk than the first, but both of then hint us a solution where we can likewise mitigate the problem of stale yields and further improve capital efficiency.

In the following sections we design a novel separation scheme by introducing FPT (fixed yield PT) and DYT (dynamic-YT). We then explain how the separation leads to higher leverage and more efficient capital usage. Additionally, we obtain a byproduct, which represents cash flow of fixed coupon, and face value payment in the end, which also has a wide usage in finance. With FPT and DYT defined, we design a liquidity pool consist of FPT and SY, allowing users to explicitly gain leverage through trading with AMM. We then present a few good properties of this market, and in the end show potential applications of this new separation scheme.

4 DYT and FPT

In this section we define the DYT and FPT products, as a decomposition of YT+PT. Before that we start with a numerical example to show what DYT intuitively is:

Assume we have a bond, expiring in 2 periods, and at end of each period, a coupon is paid. The amount of coupon paid is determined by the interest rate immediately before end of each period.

For the simplicity of argument, we define end of two periods as t_0 and t_1 (time now is 0). The interest rate paid by end of each period is i_0 and i_1 respectively. Suppose the current yield (as discounting rate) in the market is r, and face value is just 1. Immediately we know the "fair market value" of this yield stream is the following, assuming i_0 and i_1 are known:

$$v = e^{-rt_0} i_0 + e^{-rt_1} i_1$$

Without loss of generality, we assume the market has a different expectation of i_1, denoting as $i_1' < i_1$, and has the same expectation of i_0. Then the current market price of the yield stream is

$$v' = e^{-rt_0} i_0 + e^{-rt_1} i_1'$$

An active speculator with perfect prediction of market (sometimes also called an oracle) arrives at the market, observes the price $v' < v$, she would then purchase the yield stream v', in expectation of an appreciation when i_1 is priced in. The trade takes a capital upfront of v', and earns a profit of $e^{-rt_1}(i_1 - i_1')$. The return is hence

$$\frac{e^{-rt_1}(i_1 - i_1')}{e^{-rt_0} i_0 + e^{-rt_1} i_1'} = \frac{i_1 - i_1'}{e^{-r(t_0 - t_1)} i_0 + i_1 1}$$

noticing that $e^{-r(t_0 - t_1)}$ is close to 1. Hence the lower $i_1 - i_1'$ is in relates to scale of i_0 and i_1, the lower the return is. Then a natural idea would be to increase the ratio between $i_1 - i_1'$ (i.e. interest rate volatility) and $i_0 + i_1$ (i.e. interest rate level).

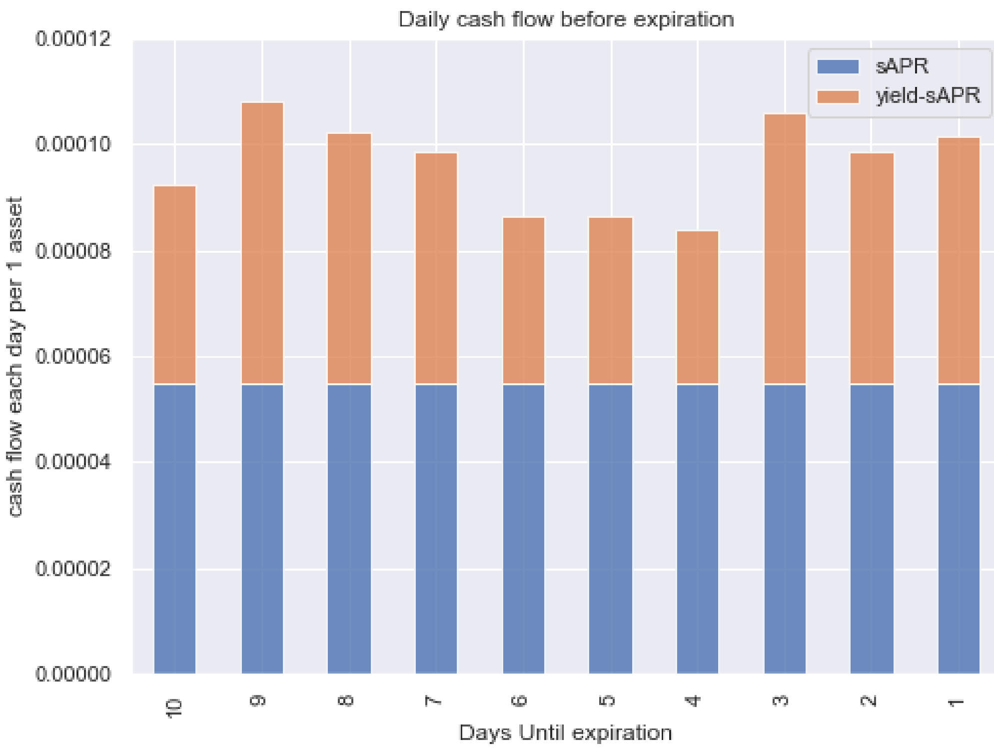
Plugging in real numbers, let $r = 0$ for simplicity, $i_0 = 10\%$, $i_1 = 11\%$, $t_0 = 1$, $t_1 = 2$, $i_1' = 10\%$, it is easy to see $v = 0.21$, $v' = 0.2$, and profit is 0.01. The speculator will have return of 5%. Now suppose we know that before t_1, it is almost impossible for rate to fall under 8%, we can further separate the cash flow of the bond into two parts: one part that pays a fixed cash flow of 8% (stable), and another part that pays a variable cash flow (fluctuating) of 2% by end of first term and 2% by end of second term (or maybe 3%, nobody knows). The type of investors who want a fixed income stream and are risk averse could invest in stable product, and the type of speculators mentioned above could invest in fluctuating product instead. Redoing the example above, it is easy to see, without any friction, her return is $0.01/0.04 = 25\%$, magnified by the separation and made possible by the fluctuating product. This example leads to the introduction of **FPT** and **DYT**, which we define below:

Stable APR (sAPR) We first need to define an APR which we make sure YT will not yield lower than. This could be a theoretical threshold that exists by definition, or could be a practical threshold that is calculated using historical data and forward outlooks. It is worth noting that we can have a higher stable APR when YT gets closer to expiration, and the higher stable APR is, the more capital efficiency we can achieve

Fixed profit token (FPT) we define FPT as a fixed coupon income stream, with coupon rate equal to **sAPR**, along with a payment of 1 underlying asset upon expiration (in other word, PT). Coupons are paid at same time as YT

Dynamic yield token (DYT) we define DYT as the fluctuating part of YT, a variable yield income stream, with yield equal to yield of YT deducting **sAPR**. Yield are paid at same time as YT

Below is a demonstration of how the daily cash flow looks like (except last day, for the scale of plotting). Here we assume an **sAPR** of 0.02% out of a randomized yield. We also assume an expiration of 10 days, for the ease of plotting. A back of envelop calculation would tell us how much leverage we achieve in this scenario: by allowing DYT only betting on the variable cash flow (orange portion), suppose the discounting rate is 4% in the market, the fair value is 0.00096 for the aggregated cash flow, but only 0.00042 for the variable portion of the cash flow, giving us more than 2x leverage to bet on yield.



5 FPT-SY Market

5.1 Definitions

Having defined FPT and DYT we define a market that allows FPT to be obtained through AMM. Here we mostly follow the PT-SY pool that is present in Pendle, as we observe a lot of similarity between the two. We use same naming convention as Pendle's V2_AMM whitepaper, and we define a few additional terms (overriding V2_AMM whitepaper definition if terminology conflicts):

FPT (fixed profit token) price at time t is $price_s(t)$, which is the price of PFT in terms of asset

asset price in FPT at time t, $price_a(t)$ is simply $\frac{1}{price_s(t)}$, which is the price of the asset in FPT

yield payment time the timestamps t_i (in normalized time, ie expiry=0) when YT holder is deposited yield.

internal rate of return (IRR) is the annual average yield rate r from now (t in normalised time) until the expiry that the market is implying, by trading **FPT**. By discounting cash flow, we can have the relationship between FPT price $price_s(t)$ and r

$$price_s(t) = \sum_{\forall i,\, t_i \le t} e^{r(t_i - t)} \cdot \text{sAPR}/365 + e^{-rt}$$

here the summation means the aggregated present value of all future cash flow ($t_i \le t$) discounted by r. r is a variable that is solvable in the first equation (given $price_s(t)$ is known to the market, and sAPR, t_i all fixed; the financial meaning of r is the IRR (internal rate of return) of FPT.

implied interest rate is the annualized interest rate directly linked with IRR r. Specifically the implied rate is e^r. Note here the interest rate is a number above 1, made consistent with the set up by Pendle

fixed profit token AMM is an AMM that allows users to buy or sell FPT against the asset. Buying **FPT** will push price $price_s(t)$ up (exchange more FPT against asset) and r down

5.2 Notional AMM

Following the defintion of Notional AMM in V2_AMM whitepaper, here we define the Notional AMM in similar fashion. The asset price in FPT $price_a(t)$ is

$$price_a(t) = \frac{\ln\left(\frac{p_t}{1-p_t}\right)}{\text{rateScalar}(\lceil t \rceil)} + \text{rateAnchor}(t)$$

where $\lceil t \rceil$ means to ceiling t to the time-to-expiry of last yield distribution.

Similar to AMM for PT, we need to define precomputation every time when a new trade comes, with t elapsing. Between any previous trading timestamp t' and current trading timestamp t, there are two mechanisms we need to consider. For simplicity, we assume that t' and t are within a day (24 hour). We will further extend this argument to if t' and t are further away:

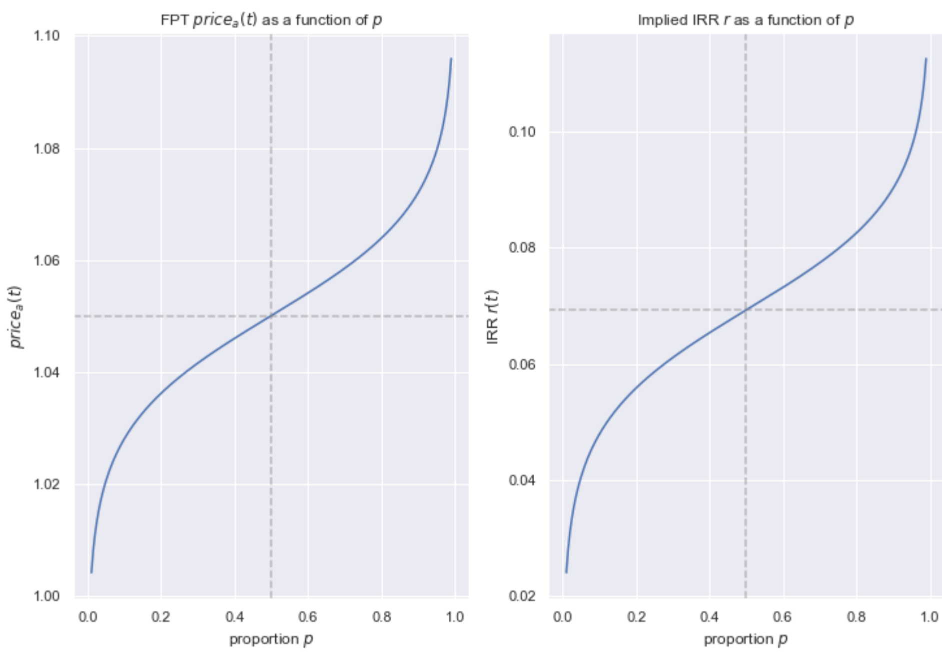
yield distribution: if there is an yield distribution event for FPT, on the moment of distribution, the market value of FPT will reduce by exact amount of yield distributed. Then $\lceil t \rceil$ will also update, leading us to recalculate rateScalar and also rateAnchor

continuous trading (non-yield distribution): otherwise, if there is no yield distribution during this period, the change of price will be determined by trading pressure; during this period, we keep t unchanged in the rate scalar calculation, by flooring t to last yield distribution time. If there is no trade happening between yield distributions. rateScalar does not change, hence we do not need to recalculate rateAnchor (as none of the terms in the MM equation has changed)

If t' and t covers more than one yield distribution, the basic principles listed above still hold, but in essence we will need to subtract more than one yield from price, and then we only need to calculate the rateScalar and rateAnchor once for t. There are more details to this calculation, if t' and t covers more than one period. We will cover more of this discussion in the Implementation Details section.

Based on the DCF formula, after each trade, we can calculate r from most recent price. We demostrate properties of the market with the following market making curves. Left graph: how

$price_a(t)$ changes with p, with $t = 0.5$, expiration in 1 year, and sAPR = 2%, rateAnchor = 1.05 (this implies a $price_a(0.5)$ of 1.05 at $p = 0.5$) and $rateScalar = 100$. Right graph: how equivalent IRR r changes with different p, at the same market configuration as the left graph:



5.3 Discussions

Below we provide some discussions and explanations into the model set up above.

5.3.1 Connections and differences with principal-asset Notional AMM

Connections: it is easy to see that the FPT-SY market is designed largely borrowing the concept of PT-SY market, where the impact of a liquidity shock (characterized by a trade that moves the needle of $p(t)$) will move the market in a non-linear manner, with impact staying small if $p(t)$ is relatively small, and impact growing rapidly as $p(t)$ becomes significant. The elasticity of the market is (almost) controlled by a single parameter scalarRoot which makes it very easy and intuitive to initialize.

Differences: the main differences come from the differences between PT and FPT. The process of PT is a continuous process, where price of PT changes smoothly over time until it reaches a certain point which is 1 asset at expiry; price of FPT changes smoothly (when there is no coupon paid), and will gap (when coupon is paid), and over time converges to a certain point which is 1 asset at expiry, with all yields distributed.

The similarities make it possible that we can have a similar set up, but the differences requires the additional update steps which are not present in PT-SY market. We will elaborate more in the following section.

5.3.2 Well-definedness of AMM formula

If just thinking of FPT as a fixed coupon payment, we will have the discounted cash flow (DCF) pricing method outlined above:

$$price_s(t) = \sum_{\forall i,\, t_i \le t} e^{-r(t-t_i)} \cdot \text{sAPR}/365 + e^{-rt}$$

When there is a coupon payment, we study $price_s(t)$ immediately preceding and following coupon payment:

$$\Delta price_s(t) = \sum_{\forall i,\, t_i \le t-\Delta t} e^{r(t_i-(t-\Delta t))} \cdot \text{sAPR}/365 - \left(\sum_{\forall i,\, t_i \le t-\Delta t} e^{r(t_i-t)} \cdot \text{sAPR}/365 + \text{sAPR}/365 \right)$$

Having argued that the terms in parenthesis is close to 0 above, we have that around coupon payment, $\Delta price_s(t) \to -\text{sAPR}/365$

The arguments above explained why we design the additional steps before initializing a Notional AMM pool of FPT-SY, that differ from PT-SY pool.

5.3.3 Buying/Selling DYT

Similar to the relationship between PT and YT, we have a new relationship

$$P(\text{FPT}) + P(\text{DYT}) = P(\text{underlying})$$

Buying/Selling FPT if an user wants to buy FPT, she can just utilize FPT-SY pool to buy FPT.

Buying DYT if an user wants to buy DYT, she can achieve that using flash swaps, similar to how Pendle handles flash swaps:

1. Buyer sends SY into swap contract
2. Contract withdraws more SY from the pool
3. Mint DYT and FPT from all of the SY
4. Send the DYT to the buyer
5. FPT is sold for SY in FPT-SY pool, and the aggregated amount is returned from step 2

Selling DYT similarly,

1. Seller sends DYT into swap contract
2. Contract borrows an equivalent amount of FPT from the pool
3. FPT and DYT are used to redeem SY
4. Send the SY to the seller
5. A portion of SY is sold for FPT in FPT-SY pool, and amount is returned from step 2

Selling short DYT something new we propose, is the ability to sell short DYT to bet on the opposite of rate movement. Speficially here we will need to borrow underlying asset, and use borrowed asset to buy FPT, and that will generate an effect of shorting DYT.

5.3.4 Stylized facts about FPT-YT market

We present a few stylized facts about FPT-YT market that are implied by the set up:

Information flow: The market is made more complete now with FPT and DYT, and we are able to gauge market consensus from the market price and IRR of FPT (and DYT). When the market is more risk averse, we will see a shrinking demand for DYT, and an expanding demand for FPT, that in turn will lower the IRR of FPT, and pushing market to self-correct (or mean-revert).

Funding/Margin: The first law of finance, there is no free lunch. The active speculator we mentioned in the example above gained exposure, but at what cost? There are two costs at play, one cost is the cost incurred whenever a trade is finalized, including swap fee paid to liquidity providers etc.; the other form of cost is implicit, through the market impact of the swap trade. The market will make sure that IRR of FPT is always higher than sAPR (otherwise there will be risk-free arbitrage opportunities), and given FPT is risk-free asset with known cash flow, the rate differential IRR-sAPR represents a market-consensus view of the cost of providing leverage to DYT.

5.3.5 Upper bound of FPT price

Borrowing from the notion of par bond in US Treasury market, we can derive a theoretical upper bond for price of FPT. Specifically, we recall the definition and condition of a par bond:

Par bond is a bond that currently trades at its face value. Par bond has a coupon rate that iequals to the market interest rate

With the definition, immediately we can get the following result, that the price of FPT will be at par (ie, 1) if and only if sAPR (coupon rate) equals that of IRR. Otherwise if sAPR is lower than IRR, FPT will trade a a discount below par, and vice versa. Note that sAPR is a number we choose to be lower than the prevailing IRR (or even low enough so that IRR will likely not reach), we will almost certain to have price of FPT lower than 1; this will also serve as a bound we set for FPT AMM.

5.4 Implementation Details

5.4.1 FPT market empirical constraints

We know that FPT is a stable income stream, and its yield is very representative of market perception of a "discounting rate". In the mean time, because our market is an AMM, there could be trades that cause abnormal price movements of FPT. When FPT prices are pushed down too much, its yield will go up. As is the property of a healthy market, we will allow yield to fluctuate (or be reasonably higher than the market yield we deem sensible), hence we will not set an upper bound too tight of how FPT yield will be, instead, we set the yield to be high enough (say 10%) as more of a sanity check.

The lower bound is more intuitive: we constrain that the lower bound of yield should not be less than 0, i.e. the price of FPT should be below 1, as explained above.

5.4.2 $price_s(t)$ and r computation

The following pricing formula links $price_s(t)$ and r

$$price_s(t) = \sum_{\forall i,\, t_i \le t} e^{r(t_i - t)} \cdot \text{sAPR}/365 + e^{-rt}$$

To reduce the formula above, noticing that yields are distributed at around same time of a day, we can rewrite $t - t_i$ as $i/365$, assume that FPT expires in n days, we can write $price_s(t)$ immediately after a yield distribution as following

$$price_s(t) = \sum_{i=1..n} e^{-ri/365}\text{sAPR}/365 + e^{-rt} = \frac{\text{sAPR}}{365} e^{-r/365} \left(\frac{1 - e^{-rn/365}}{1 - e^{-r/365}} \right) + e^{-rn/365}$$

Hence given r, it is easy to calculate out $price_s(t)$ exactly:

- calculate $d = e^{-r/365}$
- calculate $d^n = e^{-rn/365}$
- plug in the formula

To calculate r given $price_s(t)$ is less intuitive, because of the transcendental nature of discounted cash flow. The common solution is to solve for r numerically. In fact this is a very standard and fundamental operation. We can use binary search which should yield good results in a few iterations. Alternatively, we can use Newton methods, given we know r of previous trading time, and the new r should not be too far from it. That should lead to convergence in fewer steps.

5.4.3 Precomputation after multi-day inactivity

We have mentioned in our previous section that we allow price between yield distributions to be driven by trading activity, and at yield distribution, we deduct price by the amount of distributed yield. That leads to a potential problem with multiple days without any trading. We propose our way to make it more computationally viable.

Inactivity of pro-longed periods It is easy to show the problem with a FPT with 3 yield distribution remaining at time t'. Suppose there is no trading over the whole time periods until the last yield distribution. When that happens, notice that $price_s(t')$ itself is calculated via discounted cash flow, and the new $price_s(t)$ is also calculated via discounted cash flow, so instead of having to "deduct" or "adjust" $price_s(t')$ with yield distributed, we can just recalculate $price_s(t)$ using the new DCF calculation immediately after the last yield distribution before t (with same IRR, because no trading is equivalent to no change on the market-consensus of yield rate). That will always allow us to have a positive price, as well as to keep IRR unchanged over long period of inactivity.

6 Applications

Here we present three most common applications of our new market

Fixed-coupon bond Similar to how US treasury bounds are designed, we can use FPT to achieve a "bond" product with fixed coupon payments (at **sAPR**) and a payment equal to face value at expiration. This is a more well-behaved and well-studied product than yield generating asset itself, which bears an uncertain yield. We also expect great demand in products like this, given how important US treasury (or in general, fixed coupon bonds) are in traditional finance.

Active yield trading/hedging To trade/hedge out variability of yields, instead of having to buy YT, now one can just buy DYT, which requires less intense capital, but achieves same level of variability. This can be used by traders to actively speculate on yield change, or by hedgers who are exposed to yield change and want to hedge.

7 References

- https://medium.com/pendle/yt-capital-efficiency-bc447366d38b
- https://docs.pendle.finance/ProtocolMechanics/YieldTokenization/YT
- https://docs.pendle.finance/ProtocolMechanics/AMM#flash-swaps
- https://github.com/pendle-finance/pendle-v2-resources/blob/main/whitepapers/V2_AMM.pdf