

# Constant-delay Enumeration for Lorem Ipsum

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## 6 — Abstract —

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12 2012 ACM Subject Classification Theory of computation → Database theory

13 Keywords and phrases Streams, query evaluation, enumeration algorithms.

## 14 1 Introduction

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## 22 2 Preliminaries

23 **Sets and intervals.** Given a set  $A$ , we denote by  $2^A$  the *powerset* of  $A$ . We denote by  $\mathbb{N}$   
24 the natural numbers. Given  $n, m \in \mathbb{N}$  with  $n \leq m$ , we denote by  $[n]$  the set  $\{1, \dots, n\}$  and by  
25  $[..m]$  the interval  $\{n, n+1, \dots, m\}$  over  $\mathbb{N}$ .

26 **Events and streams.** We fix a set  $\mathbf{T}$  of *event types*, a set  $\mathbf{A}$  of *attributes names*, and a  
27 set  $\mathbf{D}$  of *data values* (e.g., integers, floats, strings). An *event*  $e$  is a partial mapping  $e : \mathbf{A} \rightarrow \mathbf{D}$   
28 that maps attributes names in  $\mathbf{A}$  to data values in  $\mathbf{D}$ . We denote  $\text{att}(e)$  the domain  
29 of  $e$ , called the attributes of  $e$ , and we assume that  $\text{att}(e)$  is finite. We denote by  $e(A)$  the  
30 data value of the attribute  $A \in \mathbf{A}$  whenever  $A \in \text{att}(e)$ . Further, each event  $e$  has a type in  $\mathbf{T}$   
31 denoted by  $\text{type}(e)$ . We write  $\mathbf{E}$  to denote the set of all events over event types  $\mathbf{T}$ , attributes  
32 names  $\mathbf{A}$ , and data values  $\mathbf{D}$ . A *stream* is an (arbitrary long) sequence  $\bar{S} = e_1 e_2 \dots e_n$  of  
33 events where  $|S| = n$  is the length of the stream.

34 **Complex events.** Fix a finite set  $\mathbf{X}$  of variables and assume that  $\mathbf{T} \subseteq \mathbf{X}$ , where  $\mathbf{T}$  is the  
35 set of event types as defined earlier, this is to say that all event types are a variable. Let  $\bar{S}$   
36 be a stream of length  $n$ . A complex event of  $\bar{S}$  is a triple  $(i, j, \mu)$  where  $i, j \in [n]$ ,  $i \leq j$ , and  
37  $\mu : \mathbf{X} \rightarrow 2^{[i..j]}$  is a function with finite domain. Intuitively,  $i$  and  $j$  marks the beginning  
38 and end of the interval where the complex event happens, and  $\mu$  stores the events in the  
39 interval  $[i..j]$  that fired the complex event. In the following, we will usually use  $C$  to denote  
40 a complex event  $(i, j, \mu)$  of  $\bar{S}$  and omit  $\bar{S}$  if the stream is clear from the context. We will  
41 use  $\text{interval}(C)$ ,  $\text{start}(C)$ , and  $\text{end}(C)$  to denote the interval  $[i..j]$ , the start  $i$ , and the end  $j$

$$\begin{aligned}
\llbracket R \rrbracket(\bar{S}) &= \{ (i, i, \mu) \mid \text{type}(e_i) = R \wedge \mu(R) = \{i\} \wedge \forall X \neq R. \mu(X) = \emptyset \} \\
\llbracket \varphi \text{ AS } X \rrbracket(\bar{S}) &= \{ C \mid \exists C' \in \llbracket \varphi \rrbracket(\bar{S}). \text{interval}(C) = \text{interval}(C') \wedge C(X) = \bigcup_Y C'(Y) \\
&\quad \wedge \forall Z \neq X. C(Z) = C'(Z) \} \\
\llbracket \varphi \text{ FILTER } X[P] \rrbracket(\bar{S}) &= \{ C \mid C \in \llbracket \varphi \rrbracket(\bar{S}) \wedge C(X) \models P \} \\
\llbracket \varphi_1 \text{ OR } \varphi_2 \rrbracket(\bar{S}) &= \llbracket \varphi_1 \rrbracket(\bar{S}) \cup \llbracket \varphi_2 \rrbracket(\bar{S}) \\
\llbracket \varphi_1 \text{ AND } \varphi_2 \rrbracket(\bar{S}) &= \llbracket \varphi_1 \rrbracket(\bar{S}) \cap \llbracket \varphi_2 \rrbracket(\bar{S}) \\
\llbracket \varphi_1 ; \varphi_2 \rrbracket(\bar{S}) &= \{ C_1 \cup C_2 \mid C_1 \in \llbracket \varphi_1 \rrbracket(\bar{S}) \wedge C_2 \in \llbracket \varphi_2 \rrbracket(\bar{S}) \wedge \text{end}(C_1) < \text{start}(C_2) \} \\
\llbracket \varphi_1 : \varphi_2 \rrbracket(\bar{S}) &= \{ C_1 \cup C_2 \mid C_1 \in \llbracket \varphi_1 \rrbracket(\bar{S}) \wedge C_2 \in \llbracket \varphi_2 \rrbracket(\bar{S}) \wedge \text{end}(C_1) + 1 = \text{start}(C_2) \} \\
\llbracket \varphi^+ \rrbracket(\bar{S}) &= \llbracket \varphi \rrbracket(\bar{S}) \cup \llbracket \varphi ; \varphi^+ \rrbracket(\bar{S}) \\
\llbracket \varphi^\oplus \rrbracket(\bar{S}) &= \llbracket \varphi \rrbracket(\bar{S}) \cup \llbracket \varphi : \varphi^\oplus \rrbracket(\bar{S}) \\
\llbracket \pi_L(\varphi) \rrbracket(\bar{S}) &= \{ \pi_L(C) \mid C \in \llbracket \varphi \rrbracket(\bar{S}) \}
\end{aligned}$$

Figure 1: The semantics of CEL formulas defined over a stream  $\bar{S} = e_1 e_2 \dots e_n$  where each  $e_i$  is an event.

42 of  $C$ , respectively. Further, by some abuse of notation we will also use  $C(X)$  for  $X \in \mathbf{X}$  to  
43 denote the set  $\mu(X)$  of  $C$ .

44 The following operations on complex events will be useful throughout the paper. We  
45 define the union of complex events  $C_1$  and  $C_2$ , denoted by  $C_1 \cup C_2$ , as the complex event  
46  $C'$  such that  $\text{start}(C') = \min\{\text{start}(C_1), \text{start}(C_2)\}$ ,  $\text{end}(C') = \max\{\text{end}(C_1), \text{end}(C_2)\}$ , and  
47  $C'(X) = C_1(X) \cup C_2(X)$  for every  $X \in \mathbf{X}$ . Further, we define the *projection over L* of a  
48 complex event  $C$ , denoted by  $\pi_L(C)$ , as the complex event  $C'$  such that  $\text{interval}(C') =$   
49  $\text{interval}(C)$  and  $C'(X) = C(X)$  whenever  $X \in L$ , and  $C'(X) = \emptyset$ , otherwise. Finally, we  
50 denote by  $(i, j, \mu_\emptyset)$  the complex event with trivial mapping  $\mu_\emptyset$  such that  $\mu_\emptyset(X) = \emptyset$  for  
51 every  $X \in \mathbf{X}$ .

52 **Predicate of events.** A *predicate* is a possibly infinite set  $\mathbf{P}$  of events. We say that an event  
53  $e$  satisfies predicate  $P$ , denoted  $e \models P$ , if, and only if,  $e \in P$ . We generalize this notation from  
54 events to a set of events  $E$  such that  $E \models P$  if, and only if,  $e \models P$  for every  $e \in E$ . We assume  
55 a fixed set of predicates  $\mathbf{P}$ . Further, we assume that there is a basic set of predicates  $P_{basic}$   
56  $\subseteq \mathbf{P}$  and  $\mathbf{P}$  is the closure of  $P_{basic}$  under intersection and negation (i.e.,  $P_1 \cap P_2 \in \mathbf{P}$  and  
57  $\mathbf{E} P \in \mathbf{P}$  for every  $P, P_1, P_2 \in \mathbf{P}$ ) where  $\mathbf{E}$  is a predicate in  $\mathbf{P}$ , that we usually denote by true.

58 **Complex event logic.** In this work, we use the Complex Event Logic (CEL) introduced in  
59 [21] and implemented in CORE [11] as our basic query language for CER. The syntax of a  
60 CEL formula  $\varphi$  is given by the grammar:

$$61 \quad \varphi := R \mid \varphi \text{ AS } X \mid \varphi \text{ FILTER } X[P] \mid \varphi \text{ OR } \varphi \mid \varphi \text{ AND } \varphi \mid \varphi ; \varphi \mid \varphi : \varphi \mid \varphi^+ \mid \varphi^\oplus \mid \pi_L(\varphi)$$

62 where  $R \in \mathbf{T}$  is an event type,  $X \in \mathbf{X}$  is a variable,  $P \in \mathbf{P}$  is a predicate, and  $L \subseteq \mathbf{X}$  is a set  
63 of variables. We define the semantics of a CEL formula  $\varphi$  over a stream  $\bar{S}$ , recursively, as a  
64 set of complex events over  $\bar{S}$ . In Figure 1, we define the semantics of each CEL operator like  
65 in [11, 21].

### 66 3 Main results

67 In this section we introduce an extension to the semantics of CEL, namely we introduce a  
68 new operator using [allen interval algebra] *overlap*. We then extend the formal computational

69 model for evaluating CEL formulas and prove its correctness. We start by recalling the  
 70 notion of a CEA to later extend the proof.

71 **Fernando:** this intro is just to get the point across, not polished

**Complex Event Automata.** A *Complex Event Automata* (CEA) is a tuple  $\mathcal{A} = (Q, \mathbf{P}, \mathbf{X}, \Delta, q_0, F)$  where  $Q$  is a finite set of states,  $\mathbf{P}$  is the set of predicates,  $\mathbf{X}$  is a finite set of variables,  $\Delta \subseteq Q \times \mathbf{P} \times 2^{\mathbf{X}} \times Q$  is a finite relation (called the transition relation),  $q_0 \in Q$  is the initial state, and  $F$  is the set of final states. A run  $\rho$  of  $\mathcal{A}$  over the stream  $\bar{S} = e_1 e_2 \dots e_n$  from position  $i$  to  $j$  is a sequence:

$$\rho := p_i \xrightarrow{P_i, L_i} p_{i+1} \xrightarrow{P_{i+1}, L_{i+1}} p_{i+2} \xrightarrow{P_{i+2}, L_{i+2}} \dots \xrightarrow{P_j, L_j} p_{j+1}$$

where  $p_i = q_0$ ,  $(p_k, P_k, L_k, p_{k+1}) \in \Delta$ , and  $e_k \models P_k$  for all  $k \in [i..j]$ . We say that the run is accepting if  $p_{j+1} \in F$ . A run  $\rho$  from positions  $i$  to  $j$  like above defines the complex event  $C_\rho = (i, j, \mu_\rho)$  such that  $\mu_\rho(X) = k \in [i..j] \mid X \in L_k$  for every  $X \in \mathbf{X}$ . Note that the starting and ending positions  $i, j$  of the run define the interval of the complex event, and the labels  $L_k \subseteq \mathbf{X}$  define the mapping  $\mu_\rho$  of  $C_\rho$ . We define the set of all complex events of  $\mathcal{A}$  over  $\bar{S}$  as:

$$\llbracket \mathcal{A} \rrbracket(\bar{S}) = \{C_\rho \mid \rho \text{ is an accepting run of } \mathcal{A} \text{ over } \bar{S}\}$$

We present then the overlap operator for CEL as with the following definition:

$$\begin{aligned} \llbracket \varphi_1 :o \varphi_2 \rrbracket(\bar{S}) &= \{C_1 \cup C_2 \mid C_1 \in \llbracket \varphi_1 \rrbracket(\bar{S}) \wedge C_2 \in \llbracket \varphi_2 \rrbracket(\bar{S}) \\ &\quad \wedge \text{start}(C_1) \leq \text{start}(C_2) \leq \text{end}(C_1) \leq \text{end}(C_2)\} \end{aligned}$$

72 We also know from [11,22] the following theorem:

73 ► **Theorem 1** (CEA and CEL equivalence). *For every CEL formula  $\varphi$  there exists a CEA  $\mathcal{A}_\varphi$   
 74 such that  $\llbracket \varphi \rrbracket(\bar{S}) = \llbracket \mathcal{A}_\varphi \rrbracket(\bar{S})$  for every stream  $\bar{S}$*

75 To maintain the correctness of it true, we extend the induction proof [11,22] by adding an  
 76 extra case for this operator.

77 Lets assume then that there exists an automaton that satisfies the previous property for  $\varphi_1$  and  
 78  $\varphi_2$ , therefore we know there exists  $\mathcal{A}_{\varphi_1} = (Q_1, \mathbf{P}_1, \mathbf{X}_1, q_0, F_1)$  and  $\mathcal{A}_{\varphi_2} = (Q_2, \mathbf{P}_2, \mathbf{X}_2, p_0, F_2)$   
 79 Then the construction for the operator will be  $\mathcal{A}_{:o}$  be a CEA such that  $\mathcal{A}_{:o} = ()$

## 80 4 Conclusions

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**86 References**

- 87 1 Guillaume Bagan, Arnaud Durand, and Etienne Grandjean. On acyclic conjunctive queries  
88 and constant delay enumeration. In *CSL*, pages 208–222, 2007.
- 89 2 Marco Bucchi, Alejandro Grez, Andrés Quintana, Cristian Riveros, and Stijn Vansumeren.  
90 CORE: a complex event recognition engine. *VLDB*, 15(9):1951–1964, 2022.
- 91 3 Arnaud Durand and Etienne Grandjean. First-order queries on structures of bounded degree  
92 are computable with constant delay. *ACM Trans. Comput. Log.*, 8(4):21, 2007.

**93 A Proofs from Section 2**

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**99 B Proofs of Section 3****100 B.1 Proof of Lemma ??**

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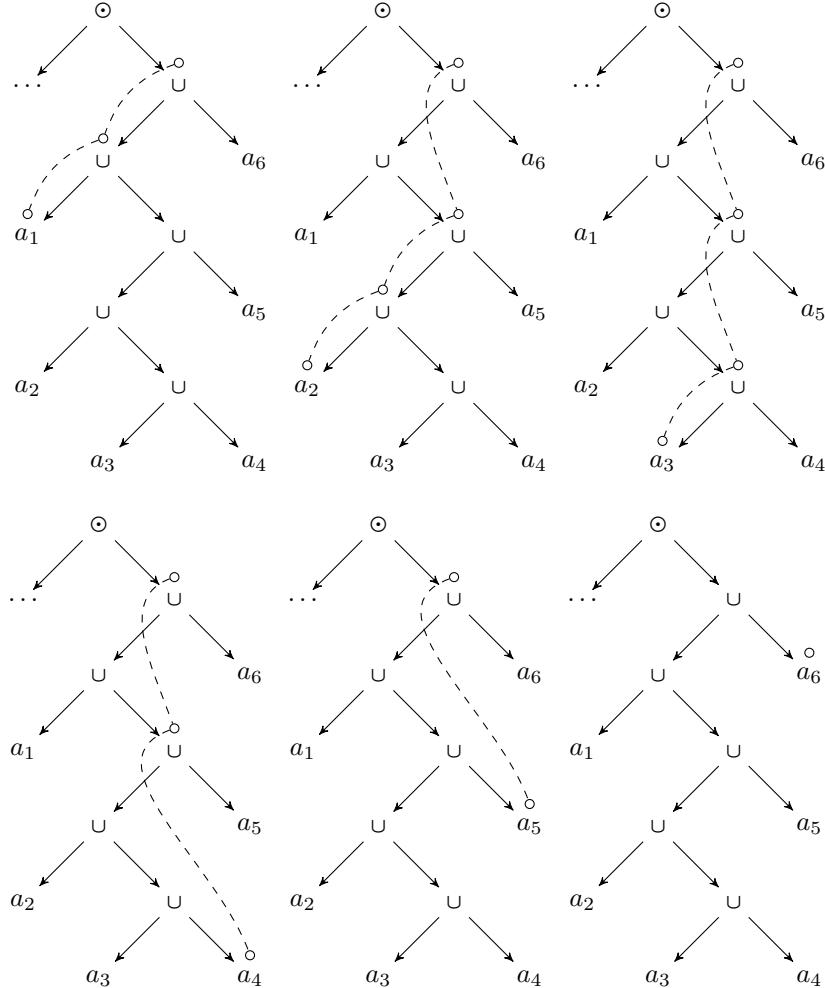
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**131 B.2 Proof of Theorem 1**

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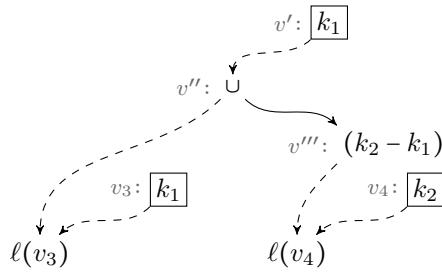


**Figure 1** An example iteration of `trav` and `move`. The sequences of nodes joined by dashed lines represent a stack  $St$ , where the first one was obtained after calling `trav` over the topmost union node, and the following five are obtained by repeated applications of `move( $St$ )`.

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**Figure 2** Gadget used in Theorem 1.

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### 162 B.3 Proof of Proposition ??

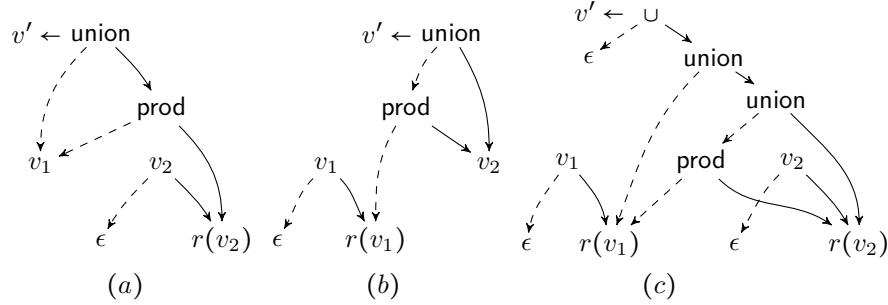
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168 ▷ **Claim 2.** Fix  $k \in \mathbb{N}$ . Let  $\mathcal{C}_k$  be the class of all duplicate-free and  $k$ -bounded  $D$  that satisfy  
 169 the  $\epsilon$  condition. Then one can solve the problem **Enum**[ $\mathcal{C}_k$ ] with output-linear delay and  
 170 without preprocessing (i.e. constant preprocessing time).

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■ **Figure 3** Gadgets for product as defined for an  $\mathcal{D}$  with the  $\epsilon$ -node.

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