Notations

Let

$$A_{i+j} := \sum_{\mu=1}^{N-j} P_i(\mu,t)P_i(\mu+j,t)$$
$$B_{i+j} := \sum_{\mu=1}^{N-j} I_i(\mu,\mu+j,t)$$

Estimate the difference of P

$$P_i(\mu,t) - P_i(\mu+1,t) = \frac{4}{n}\sum_{k=1}^{N-1} \sin\left[\frac{k\pi}{2n}\right]\sin\left[\frac{k\pi\mu}{n}\right]\cos\left[\left(i-\frac{1}{2}\right)\frac{\pi k}{n}\right]\cos^{2t}\left[\frac{\pi k}{2n}\right]$$

$$\sin\left[\frac{k\pi}{2n}\right]\sin\left[\frac{k\pi\mu}{n}\right]\cos\left[\left(i-\frac{1}{2}\right)\frac{\pi k}{n}\right]$$
$$= \frac{1}{2}\left\{\cos\left[\frac{k\pi(-1+i+\mu)}{n}\right] + \cos\left[\frac{k\pi(i-\mu)}{n}\right] - \cos\left[\frac{k\pi(\mu-i+1)}{n}\right] - \cos\left[\frac{k\pi(i+\mu)}{n}\right]\right\}$$

From the estimation of $\mathcal{P}$, we have

$\sum \cos(a\beta k)\cos^{2t}\left(\frac{1}{2}\beta k\right) \sim \frac{1}{2\sqrt{2\pi t}} e^{-\frac{a^2}{2t}} +$