

In this document, we aim to analysis the function

$$f(x) = e^{-2tx^2} - \cos^{4t}(x)$$

**Property 1:**  $f(x) \geq 0$

Because  $e^{-x}$  is mono decrease, we only need to prove

$$2tx^2 < -4t \ln(\cos(x)).$$

let  $g(x) = 1 - \cos^2(x)e^{x^2}$ .  $g(0) = 0$ ,  $g(\frac{\pi}{2}) = 1$

$$g'(x) = 2e^{x^2} \cos^2(x)(x - \sin(x)) > 0$$

so  $g(x) > 0$ . thereby the origin formula is true.

**Property 2:**  $f(x) \leq (2tx^2 + 4t \ln(\cos x))e^{-2tx^2}$

Because  $e^{-x}$  is convex function,

$$e^{-a} - e^{-b} \leq (a - b)e^{-a}$$

where  $a < b$ .