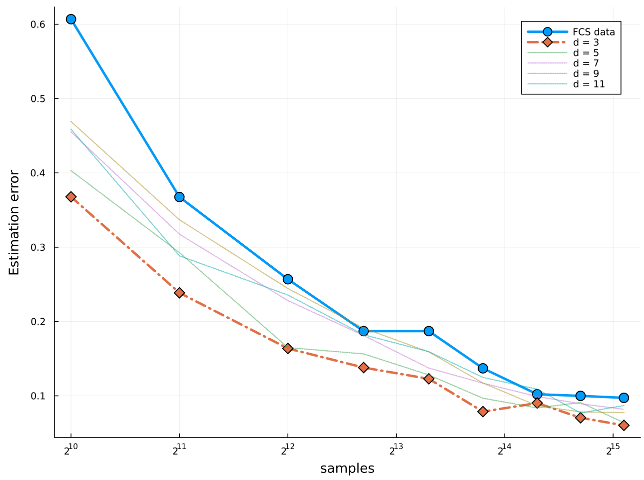
Dear Editors,

We are delighted to present our manuscript titled **"Adaptive-depth randomized measurement for fermionic observables"** for your esteemed consideration for publication in the "Quantum Science and Technology".

Background Estimating properties of fermionic systems is an important problem in quantum information theory and quantum computing since it captures the intricacies of electron correlations and dynamics. The fermionic classical shadow (FCS) algorithm is introduced to efficiently predict many properties, such as the ground state energy, in a fermionic system. However, studies suggest that it requires a random measurement circuit with $\mathcal{O}(n^2)$ size quantum circuit, which exceeds the capabilities of current quantum devices. An intriguing open question remains: *Is there an efficient shallow-depth FCS algorithm for specific sets of fermionic observables?*

Main results We propose an adaptive-depth fermionic classical shadow (ADFCS) protocol that reduces the depth requirements of FCS while preserving the measurement efficiency. Through theoretical analysis and numerical fitting, we demonstrate that the required depth for approximating a fermionic observable H can be upper bounded by $\mathcal{O}\left(\max\left\{\frac{d_{\text{int}}(H)^2}{\log n}, d_{\text{int}}(H)\right\}\right)$, where $d_{\text{int}}(H)$ denotes the interaction distance of H, as detailed in the manuscript. Numerical results demonstrate that the ADFCS algorithm achieves comparable estimation accuracy while requiring shallower circuit depths for fermionic observables with small interaction distances. The following figure illustrates this comparison by presenting the estimation error for the expectation values of the Kitaev chain Hamiltonian using the ADFCS and FCS algorithms.

Significance Our work offers the first insight into the capabilities of the FCS protocol using shallow-depth random matchgate circuits. We introduce an ADFCS measurement protocol that correlates with the interaction distance of the fermionic observable H, offering a novel approach to estimating fermionic properties. The algorithm can be applied to a broad range of quantum simulation approaches like VQE. This advancement is poised to significantly impact the learning fermionic Hamiltonians, paving the way for more efficient quantum simulations.



Recognizing the substantial impact our research could deliver in the field of quantum information and computation, we believe that our work is of broad interest to "Quantum Science and Technology" readers.

Thank you very much for your time and consideration.

Yours sincerely,
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