

Estimate the difference of \mathbf{P}

$$\begin{aligned} P_i(\mu, t) - P_i(\mu + 1, t) &= \frac{4}{n} \sum_{k=1}^{N-1} \sin\left[\frac{k\pi}{2n}\right] \sin\left[\frac{k\pi\mu}{n}\right] \cos\left[\left(i - \frac{1}{2}\right) \frac{\pi k}{n}\right] \cos^{2t}\left[\frac{\pi k}{2n}\right] \\ &\quad \sin\left[\frac{k\pi}{2n}\right] \sin\left[\frac{k\pi\mu}{n}\right] \cos\left[\left(i - \frac{1}{2}\right) \frac{\pi k}{n}\right] \\ &= \frac{1}{2} \left\{ \cos\left[\frac{k\pi(-1+i+\mu)}{n}\right] + \cos\left[\frac{k\pi(i-\mu)}{n}\right] - \cos\left[\frac{k\pi(\mu-i+1)}{n}\right] - \cos\left[\frac{k\pi(i+\mu)}{n}\right] \right\} \end{aligned}$$

From the estimation of \mathcal{P} , we have

$$\sum \cos(a\beta k) \cos^{2t}\left(\frac{1}{2}\beta k\right) \sim \frac{1}{2\sqrt{2\pi t}} e^{-\frac{a^2}{2t}} +$$