

In this document, we aim to analysis the function

$$f(x) = e^{-2tx^2} - \cos^{4t}(x)$$

Property 1: $f(x) \geq 0$

Because e^{-x} is mono decrease, we only need to prove

$$2tx^2 < -4t \ln(\cos(x)).$$

let $g(x) = 1 - \cos^2(x)e^{x^2}$. $g(0) = 0$, $g(\frac{\pi}{2}) = 1$

$$g'(x) = 2e^{x^2} \cos^2(x)(x - \sin(x)) > 0$$

so $g(x) > 0$. thereby the origin formula is true.

Property 2: $f(x) \leq -(2tx^2 + 4t \ln(\cos x))e^{-2tx^2}$

Because e^{-x} is convex function,

$$e^{-a} - e^{-b} \leq (b - a)e^{-a}$$

where $a < b$.