

# *Lecture 2*

## **Forwards, Futures, Arbitrage**

Presented by: Yan  
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# Introduction

- 1 Forward Contract**
- 2 Arbitrage**
- 3 Forward Price**
- 4 Future contract**

# 01 Forward Contract

## The definition of forward contract.

- The forward contract on a stock with maturity  $T$  and strike  $K$  is a legally binding sale contract of one stock at time  $T$  for a price of  $K$

## Obligations of the Trading Parties.

- **Buyer (Long Position):** The buyer is obligated to pay  $K$  dollars for purchasing the stock at maturity  $T$  regardless of the market price at that time.
- **Seller (Short Position):** The seller is obligated to sell the stock at maturity  $T$  for a price  $K$  regardless of the market price at that time.

## Uses of Forward Contracts.

- Risk Management
- Arbitrage
- Speculation
- Asset Allocation
- Price Discovery

# 01 Forward Contract

## Characteristics of Forward Contracts.

- ✓ **Non-standardization:** Forward contracts are tailored to the specific needs of the parties involved, including the asset being traded, the quantity, price, and delivery time.
- ✓ **Flexibility:** Due to the customizable contract terms, forward contracts offer a high degree of flexibility.
- ✓ **Over-the-counter (OTC) trading:** Forward contracts are not traded on public exchanges but are negotiated privately.
- ✓ **Liquidity:** They can be bought or sold.
- ✓ **No margin system:** Unlike futures contracts, forward contracts typically do not have margin requirements, which increases counterparty risk.

# 01 Forward Contract

## Example.



**Seller**

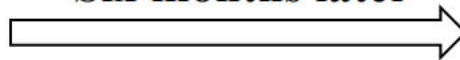
+\$2000

-\$1000

**10,000kg Wheat**



**Six months later**



**Price K: \$ 2.1**

Present market price: \$2.0

Future market price: \$1.9

Future market price: \$2.2



**Buyer**

-\$2000

+\$1000



# 01 Forward Contract

## The definition of forward price.

- The *forward price*  $F$  of a stock with maturity  $T$  is the strike that is agreed upon at time zero satisfying the followings:
  - ✓ 1. Holder of the forward contract will buy the stock at time  $T$  for a price  $F$  regardless of the actual price of the stock at that time.
  - ✓ 2. The seller of the forward contract will sell the stock at time  $T$  for a price  $F$  regardless of the actual price of the stock at that time.
  - ✓ 3. At time zero, **no up-front payment** is made by either party.
  - ✓ 4. The contract is **liquidly traded**, i.e., there are investors willing to take the long and the short position of this contract.

# 01 Forward Contract

## The definition of price of forward contract.

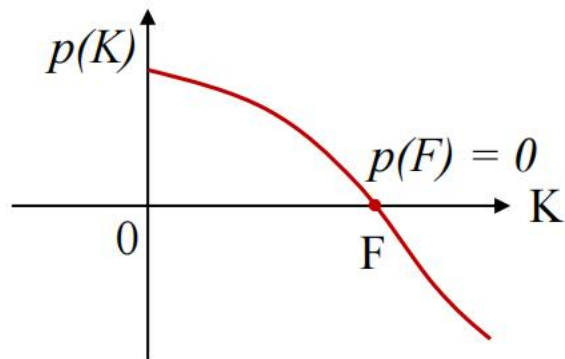
- Consider a forward contract with a general strike  $K$ . The price  $p(K)$  of this contract is paid at time zero. If  $p(K) > 0$ , the buyer pays this amount to the seller, and if  $p(K) < 0$ , the seller pays this amount  $-p(K)$  to the buyer. Then, the contract is settled at maturity, namely, at time  $T$ , the buyer pays  $K$  dollars to the seller who gives one share of the stock to the buyer.

## The determination of the forward price.

- Let  $p(K)$  be the price of the price of a forward contract with strike  $K$ . Then, the forward price  $F$  is the unique solution of  $p(F) = 0$ .

- **Proof:**

- $p(0) > 0$ .
- $\lim_{K \rightarrow \infty} p(K) < 0$ .



## 02 Arbitrage

### The definition of price of arbitrage.

- An arbitrage is a linear combination of assets which has non-zero future cash-flows that is not zero all the time and has a price less than or equal to zero.
- The core of this definition is that arbitrage offers a risk-free profit opportunity.

### The characteristics of the arbitrage.

- Non-zero future cash flows: Arbitrage opportunities allow investors to obtain positive cash flows in the future, meaning they can gain additional returns from current investments.
- Price less than or equal to zero: Another feature of arbitrage opportunities is that the cost of constructing such a portfolio is zero or negative.



## 02 Arbitrage

### Example.



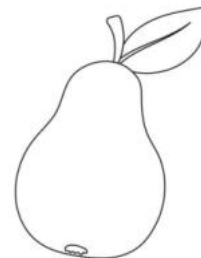
Apple: \$1.0

Pear: \$3.0



Apple: \$2.0

Pear: \$2.5



Apple: \$3.0

Pear: \$2.0

## 02 Arbitrage

### The definition of Arbitrage Pricing Rule.

- All products must be priced in such a way that the resulting market is free of arbitrage. In particular, any price that leads to an arbitrage is not allowed.
- The core of this definition is to ensure the consistency and integrity of the market pricing system. By following the Arbitrage Pricing Rule, market participants can be confident that the transactions they engage in are based on fair and transparent prices, which reflect all available information and do not contain hidden risks or unfair advantages. In this way, the market can allocate resources more effectively and provide an equal competitive environment for all participants.
- In complex markets there could be more than one price system that has no arbitrage. Uniqueness of the prices is related to the completeness of the financial market which is introduced later.

## 02 Arbitrage

### The definition of Law of One Price.

- In the absence of trade frictions, manipulations and tariffs, contracts possibly different locations must have the same price, if they produce identical cash-flows when expressed in a common currency.

### Significance of Law of One Price.

- It emphasizes the importance of price consistency in financial markets.
- It is the foundation for the pricing of financial derivatives and risk management, as it ensures fairness in pricing and the efficiency of the market.

### Factors that cause the law of one price not to hold.

- |  |  |
|--|--|
| <input type="checkbox"/> Trading Costs                       | <input type="checkbox"/> Risk Preferences                  |
| <input type="checkbox"/> Market Segmentation                 | <input type="checkbox"/> Market Expectations               |
| <input type="checkbox"/> Currency Exchange Rate Fluctuations | <input type="checkbox"/> Policy and Regulatory Environment |
| <input type="checkbox"/> Tax Policies                        | <input type="checkbox"/> Market Manipulation               |
| <input type="checkbox"/> Information Asymmetry               |  |
| <input type="checkbox"/> Liquidity Differences               |  |

## 03 Forward Price

### Forward contract pricing.

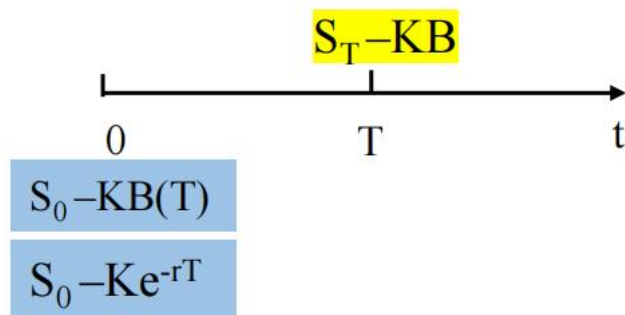
- Let  $p(K)$  be the price of the price of a forward contract with maturity  $T$  and strike  $K$ . Then,

$$p(K) = S_0 - KB(T) = 0$$

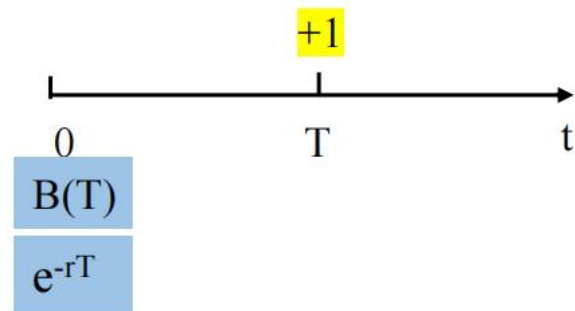
where  $S_0$  is current price of the stock and  $B(T)$  is the price of a government bond with maturity  $T$  and face value one. Consequently, the forward price  $F$  at time  $T$  of a stock is given by

$$F = \frac{S_0}{B(T)}$$

$$F = S_0 e^{rT}$$



Forward Contract



Government Zero-coupon bond



## 03 Forward Price

### Short sales

- **Definition:** investors sell securities they do not actually own with the expectation that the price will fall in the future, allowing them to buy back and return the securities at a lower price and profit from the price difference.
- **Steps:**
  - ✓ **Borrowing Securities:** Investors borrow securities from a brokerage firm or other holders.
  - ✓ **Selling Securities:** The borrowed securities are sold, and investors receive the cash from the sale.
  - ✓ **Buying Back Securities:** At some future date, investors must purchase an equal number of securities at the market price.
  - ✓ **Returning Securities:** The purchased securities are returned to the original holders.

## The definition of future contract.

- A futures contract is a standardized agreement established by a futures exchange, stipulating the delivery of a specified quantity and quality of goods at a certain time and place in the future.

## The characteristics of future contract.

- Contract standardization.
- Centralized trading.
- Two-way trading and hedging mechanism.
- Margin trading (Leverage mechanism).
- Daily mark-to-market settlement system.

## Uses of Forward Contracts.

- Risk Management
- Arbitrage
- Investment and Speculation
- Asset Allocation
- Price Discovery

## 04 Future Contract

### Comparison of Forward Contracts and Futures Contracts.

Forward Contract	Futures Contract
Over-the-Counter (OTC) Trading	Exchange Trading
Contract Agreement Content (a paper contract)	Standardized (invisible contract)
Negotiation on whether and how much margin to pay	Payment of margin 5%-10%
Used for hedging and physical delivery	Used for risk management and speculation
No trading system, negotiated individually	Trading system is in place
Transactions and prices are not transparent	Transactions and prices are public and transparent
One-to-one	Many-to-many
No clearinghouse	Clearinghouse
Mainly physical delivery	Mainly cash settlement
Credit risk is high	Credit risk is zero
Liquidity is poor, contracts are not easily transferable	Liquidity is strong, contracts are easily transferable

*Thanks!*

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