Lecture 1
Introduction, Time Value of Money

H. Mete Soner

Introduction to Financial Mathematics

October 12, 2024

Instructors

List of classes are given in the syllabus.
Lecture notes will be provided at every lecture.

Instructors :

- Mete Soner, hmsoner@gmail.com
- Yan Zhang, yanzhang2@link.cuhk.edu.cn.

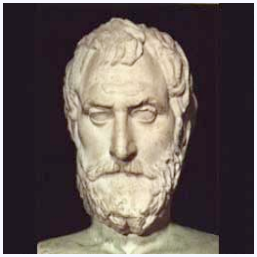
Context

- Financial industry is the largest in the world.
- It impacts everybody through retirement plans, mortgages.
- It is essential in the development of all other technologies : all companies are financed through equities. Pharmaceutical or tech research need investments to conduct research.
- All governments rely on the government bonds to finance the necessary services such as education, health.
- Quantitative methods provide liquidity and allows for innovation.
- Risk management and assessment is the central task.
- Speculation on the financial market is inevitable but constitutes a small portion of the market.

In social sciences, no mathematical model can be exact. At best they approximate the reality to a high degree of accuracy. In most cases, the level of accuracy varies with the particular application and also it is extremely time and context dependent. So one has to always remember this discrepancy.

In his report A regulatory response to the financial crisis of May 2009, Lord Turner states that

"More fundamentally, however, it is important to realize that the assumption that past distribution patterns carry robust inferences for the probability of future patterns is methodologically insecure. It involves applying to the world of social and economics relationships to a technique drawn from the world of physics, . . .it is unclear whether this analogy is valid when applied to economic and social relationships, or whether instead, we need to recognize that we are not dealing with mathematically modelable risk, but with inherent 'Knightian uncertainty'. "



- Thales lived in Milet from 624BC to 546 BC.
- It is believed that he has correctly guessed the sun eclipse of 585BC.
- Several theorems in geometry are attributed to him and he was the first of the seven wise men of Ancient Greece.

According to Aristotle

Following is from Politics, Book 1 (part XI) by Aristotle. He tells this story in explaining the importance of monopoly. But it is also the first account of a forward contract.

"According to the story, he knew by his skill in the stars while it was yet winter that there would be a great harvest of olives in the coming year; so having little money, he gave deposits for the use of all olive-presses in Chios and Miletus, which he hired at a low price. When the harvest-time came, and many wanted all at once and of a sudden, he let them out at any rate he pleased.

Thus he showed the world that philosophers can easily be rich if they like, but their ambition is another sort."

According to Aristotle

Following is from Politics, Book 1 (part XI) by Aristotle. He tells this story in explaining the importance of monopoly. But it is also the first account of a forward contract.

"According to the story, he knew by his skill in the stars while it was yet winter that there would be a great harvest of olives in the coming year ; so having little money, he gave deposits for the use of all olive-presses in Chios and Miletus, which he hired at a low price. When the harvest-time came, and many wanted all at once and of a sudden, he let them out at any rate he pleased.

Thus he showed the world that philosophers can easily be rich if they like, but their ambition is another sort."

- All questions and tasks in quantitative finance are decisions under uncertainty.
- There is no sure-decision.
- One needs to reduce risk by proper modeling through theory and statistical learning.
- Techniques developed for financial markets are also useful in other contexts as well.

Some examples

1 Month Treasury Rate

3.90% for Dec 21 2022

Overview | Interactive Chart

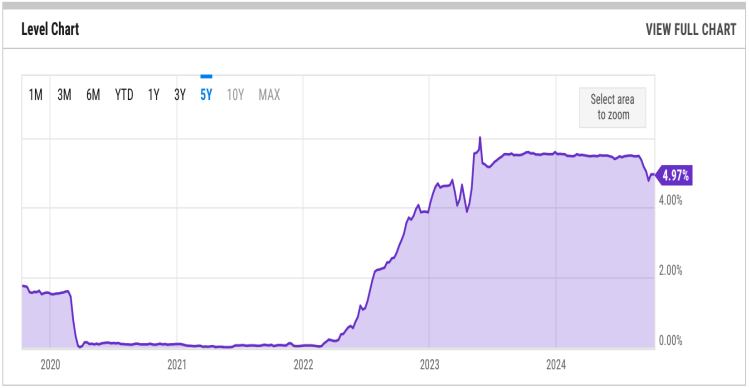
Level Chart — VIEW FULL CHART

1 Month Treasury Rate (I:1MTCMR)

4.97% for Oct 11 2024

Overview | Interactive Chart

Level Chart — VIEW FULL CHART



Market Summary > GameStop Corp.

99.65 USD

Following

+75.52 (310.65%) ↑ past 5 years

Jan 21, 11:54 AM EST • Disclaimer

1D | 5D | 1M | 6M | YTD | 1Y | 5Y | Max

GameStop Corp (GME)

20.81 ↓ -0.10 (-0.48%) USD | NYSE | Oct 11, 16:00 **20.86** ↑ +0.05 (+0.24%) After-Hours: 18:08

Quote | Performance | Key Stats | Financials | Estimates | News | Events | Y-Rating | Valuation | Multichart | Fundar

Price Chart — PRICE | RETURNS | VIEW FULL CHART

Market Summary > Bitcoin

16,467.70 USD

+ Follow

+1,157.72 (7.56%) ↑ past 5 years

Nov 29, 10:40 PM UTC · Disclaimer

1D 5D 1M 6M YTD 1Y 5Y Max

Market Summary > Bitcoin

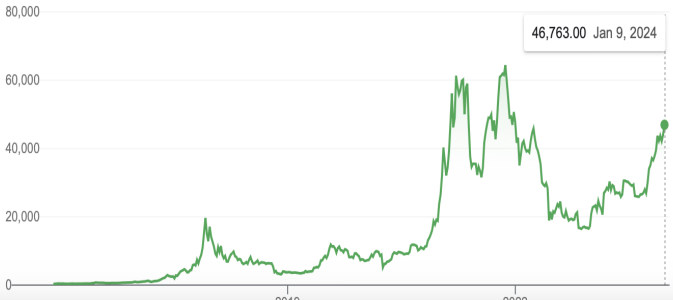
46,763.00 USD

+ Follow

+46,436.00 (14,200.61%) ↑ all time

Jan 9, 1:20 PM UTC · Disclaimer

1D | 5D | 1M | 6M | YTD | 1Y | 5Y | Max




4,763.54
Following
+4,596.52 (2,752.08%) all time
Jan 8, 4:51 PM EST • Disclaimer
1D
5D
1M
6M
YTD
1Y
5Y
Max
292.88 Mar 10, 1989
5,000
4,000
3,000
2,000
1,000
0
1994
2004
2014
2024

SP500-2024-now

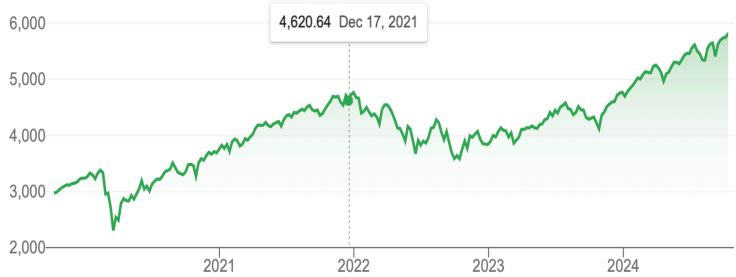
Market Summary > S&P 500

5,815.03

+2,844.76 (95.77%) ↑ past 5 years

Oct 11, 4:50 PM EDT • Disclaimer

1D | 5D | 1M | 6M | YTD | 1Y | 5Y | Max



Financial Instruments

A government bond or sovereign bond is a contract issued by a national government, that promises to pay periodic interest payments called coupon payments and to repay the face value on the maturity date.

The aim of a government bond is to support government spending.

Government bonds are usually denominated in the country's own currency. So it carries the exchange rate risk.

A sovereign debt crisis is when a government is close to default on its debt. Most recent example was Greece in 2007 although the Greek bonds are in Euros. In 1998, Russia delayed her public and private debt. This year, Puerto Rico.

Government bonds have three important characteristics :

- *Maturity* T is the length of the bond. It can vary from few weeks to 30/40 years.
- *Face Value* F is the amount of the debt that will paid in full at maturity. Banks can divide them into other amounts as well. So we assume any F value is possible. The convention is to quote prices for $F = 100$.
- *Coupon rate* C is percentage of the face value that will paid annually or semi-annually. Typical values of C are zero for short maturities and close to the inflation rate for longer maturities.

- Treasury bills have the shortest maturities, with durations only up to a year. The Treasury offers T-bills with maturities of four, eight, 13, 26 and 52 weeks and pay no coupon payments.
- Treasury notes are offered for 2, 3, 5, 7 and 10-year terms and pay semiannual coupon payments. The 10-year T-note is the most widely tracked government debt instrument in finance, and its yield is often used as a benchmark for other interest rates, such as mortgage rates.
- Treasury bonds like T-notes, pay semiannual coupon payments, but are issued in terms of 30 years.

Depending on T, C and the market view on the future, the bonds have an initial price. In the US, the initial prices are determined by public auctions. This price will then fluctuate into the future.

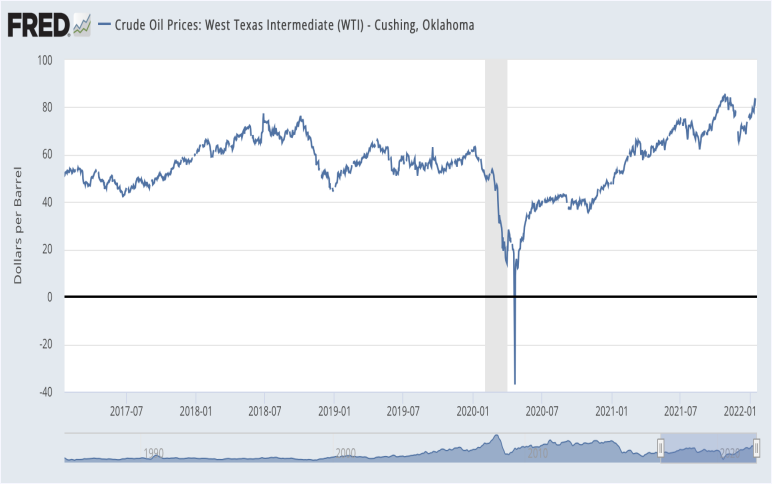
This a simple pricing exercise showing how many of the financial instruments are inter-related. We start with the definitions.

Definition

The forward contract on a stock with maturity T and strike K is a binding contract to buy one stock at time T for a price of K.

Each forward contract has a buyer and a seller. We will say that the buyer has the long position and the seller the short position. It is important to note that the holder of this contract has the legal obligation to buy the stock at time T for a price K regardless of the actual price of the stock at that time. Also, the seller of the forward contract has the obligation to sell the stock at time T for a price K regardless of the actual price of the stock at that time.

Price on 1-21-2022 was 87.69 (Brent Oil per barrel).



These are the most important financial instruments after the stocks and bonds. They are very similar to forward contracts, but they give to the holder of the contract the option either to buy or to sell as opposed to the forward contracts which require the sale is made. Therefore, the owner of the options are sure not to lose money at maturity. On the other hand, they have to pay an initial fee to obtain these contracts. Hence, their net position after maturity can be both positive or negative.

Options are written on a particular stock which is called the *underlying*.

A call option with maturity T and strike K gives its holder the option (not the obligation) to buy the stock at time T for a price K (regardless the price of the stock at time T). The future random pay-off of this option is

$$(S_T - K)^+,$$

where $S_T$is the future random value of the stock and $(a)^+ := \max\{a, 0\}$ for a real number a.

Note that the pay-off of a forward contact is $S_T - K$. However, since the owners of a call option are not required to exercise the option, they will choose their option of no-action when the future stock price S_T is less than the agreed strike price of K. This is the reason why the payoff is the positive part of $S_T - K$.

A put option with maturity T and strike K gives its holder the option (not the obligation) to sell the stock at time T for a price K (regardless the price of the stock at time T). The future random pay-off of this option is

$$(K - S_T)^+ .$$

Note that the pay-off of the short position of a forward contact is $K - S_T$. However, since the owners of a put option are not required to exercise the option, they will choose their option of no-action when the future stock price S_T is greater than the agreed strike price of K. This is the reason why the payoff is the positive part of $K - S_T$.

A *straddle* is formally known as *going long volatility*. Its construction involves in buying both a call option and a put option on the same underlying. These options are bought at the same strike price and expire at the same time. The owner of a straddle makes a profit if the underlying price moves a long way from the strike price, either above or below. Thus, investors may take a long straddle position if they think the market is more volatile than option prices suggest, but does not know in which direction it is going to move. This position is a limited risk, since the most a purchaser may lose is the cost of both options. At the same time, there is unlimited profit potential.

The payoff of a straddle is given by $(S_T - K)^+ + (K - S_T)^+$, i.e.

$$\text{pay-off} = \begin{cases} K - S_T & \text{if} \quad S_T \leq K, \\ S_T - K & \text{if} \quad S_T \geq K. \end{cases}$$

There are other options like this. See the butterly options in the lecture notes.