

Lecture 1

Time Value of Money

Presented by: Yan
12-13/10/2024

Introduction

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Coursework Explained

01 The Course Schedule

Specific topics are:

- Lecture 1, October 12 : Time value of money, bonds, interest rates.
- Lecture 2, October 19 : Internal rate of return, Futures, Arbitrage.
- Lecture 3, October 26 : Binomial model, one and two step cases.
[Homework assignment.](#)
- Lecture 4, November 2 : Multi-step model.
[Homework assignment.](#)
- Lecture 5, November 9 : Introduction to Probability and python based pricing computations. **Note the time change.**
- **November 16 : Midterm.**
- Lecture 6, November 23 : Dynamic Programming.
- Lecture 7, November 30 : Random Walk: properties and simulations.
- Lecture 8, December 7 : Black & Scholes formula.
[Homework assignment.](#)
- Lecture 9, December 14: Black & Scholes price and volatility.
[Homework assignment.](#)
- Lecture 10, December 21 : Mean-Variance Problem and CAPM.

Lecture 11 : Proposal

Lecture 12 : Proposal

01 The Course Schedule

Grading :

- **Exam:** 70% (35% Middle, 35% Final)
- **Homework:** 20% (5% each homework (total of 4))
- **Class Participation:** 10% (Attendance, Time, Attitude, Interaction, etc.)

02 Review of class content

Time Value of Money

THREE

- **Core Concepts:** Time Value of Money (TVM), Cash Flow (CF), Interest Rates
- **Financial Instruments:** Bonds, Forward Contract, Options

TWO

- **Important Rate:** Interest Rate, Internal Rate of Return (IRR)
- **Important Calculation Methods:** Discounting, IRR

ONE

- **Analytical Tool:** Yield Curve

02 Review of class content

2.1 Core Concepts

- **Time Value of Money (TVM)**

A given amount of cash today is worth more than the same amount at a future date, considering its potential to generate earnings.

- **Cash Flow (CF)**

A sequence of future payments. (*times* and *sizes* of the payments)

- **Interest Rates**

- The amount charged when one takes a loan or the amount earned when one makes a cash investment in a bank or in a bond. It is expressed as a percentage of the principal amount. (*maturity, face value*)

02 Review of class content

2.2 Financial Instruments

- **Bonds**

- Definition: A *government bond or sovereign bond* is a contract issued by a national government, that promises to pay periodic payments called coupon payments and to repay the face value on the maturity date.
- Characteristics: Maturity T ; Face value F ; Coupon Rate C .

- **Forward Contract**

A forward contract is a financial agreement where two parties commit to exchanging an asset at a specified price on a future date.

- **Options**

Options are derivative financial instruments that provide the holder with the right, but not the obligation, to buy (call option) or sell (put option) an underlying asset at a predetermined price within a specific time frame or at expiration.

02 Review of class content

2.3 Interest Rate

- **The amount of interest**
 - Maturity: length of the loan
 - Face value: the amount of the loan
- **Categories:** (F dollars, T years, r annual interest rate)
 - Simple Annual : rTF
 - Compounded Multiple Times: $\left[(1+rT)^n - 1\right]F$
 - Continuously Compounded: $j \left[e^{rT} - 1\right]F$

02 Review of class content

2.4 Yield Curve

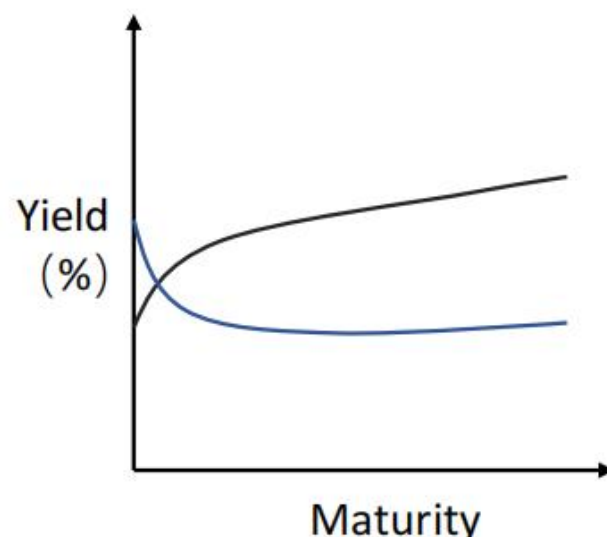
- **Definition**

- The yield: The value $r(t)$ represents the annualized continuously compounded interest rate for maturity t .
- The yield curve is a graphical representation of the relationship between the yields of bonds and their maturities.

- **Calculation:**

$$p(t) = e^{-tr(t)} F$$

$$r(t) = - \frac{\ln(p(t)/F)}{t}$$



02 Review of class content

2.5 Discounting

- **Discounting**

Discounting is the process of determining the present value of a future cash flow by applying a discount rate, which reflects the time value of money and the risk associated with the cash flow.

- **Present Value**

Present Value is the concept of calculating the worth of a future sum of money or a series of future cash flows in today's terms.

- **Discount factor**

The price $e^{-tr(t)}$ of the zero-coupon bond with face value zero is called the discount factor.

- **Calculation**

$$PV(\mathbf{c}) := \sum_{i=1}^n c(t_i) e^{-t_i r(t_i)}.$$

02 Review of class content

2.6 Internal Rate of Return

- **Definition**

The internal rate of return (IRR) is the yield of a cash flow that makes the present value of the cash flow equal to its cost or purchase price. For bonds, the IRR is often referred to as the yield-to-maturity (YTM).

- **Calculation**

$$\boxed{p(\mathbf{c})} = \sum_{i=1}^n c(t_i) e^{-r t_i} = \boxed{PV(\mathbf{c})}$$

- **Characteristics (Theorem 1.7)**

Suppose that $c(t_i) > 0$ for every i and $p(c) > 0$.

- Uniquely defined
- Inversely related to the price
- Strictly positive $p(c) > \sum_{i=1}^n c(t_i)$

1. Consider a bond with face value ¥100,000, semi-annual coupon of 3.5% and maturity 5 years.
 - a. If the price is ¥100,000, find the internal rate of return.
 - b. If the price is ¥101,586.07, find the internal rate of return.
 - c. If the price is ¥98,939.72, find the internal rate of return.

The definition of the internal rate of return (IRR).

The calculation of the internal rate of return (IRR).

Ensure that the cash flows are correctly identified.

2. Consider a bond with face value F , semi-annual coupon of $c\%$ and maturity T years. Decide on the following statement
- a. If the price is exactly equal to F , then internal rate of return is approximately equal to c . Correct? Not correct?
 - b. If the price is strictly more than F , then internal rate of return is approximately less than c . Correct? Not correct?
 - c. If the price is strictly more than F , then internal rate of return is approximately greater than c . Correct? Not correct?
 - d. If the price is strictly less than F , then internal rate of return is approximately greater than c . Correct? Not correct?
 - e. If the price is strictly less than F , then internal rate of return is approximately less than c . Correct? Not correct?

The relationship between bond price and its face value.

The relationship between the internal rate of return and coupon rate of bonds.

Thanks!

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