

## Exercise 02 – September 25, 2024

1. Calculate the Trimean for a dataset below

Data set:

10, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 50

**Solution:**

**Step 1:** Sort the Data

The data is already sorted in ascending order:

10, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 50

**Step 2:** Find the Median

Since there are 15 values (an odd number), the median is the 8<sup>th</sup> value in the sorted list:

Median = 30

**Step 3:** Find Q1 and Q3

Q1 (1<sup>st</sup> Quartile): This is the median of the lower half of the data (excluding the median itself):

Lower half = 10, 12, 15, 18, 21, 24, 27

The median of this lower half (7 values) is the 4<sup>th</sup> value:

Q1 = 18

Q3 (3<sup>rd</sup> Quartile): This is the median of the upper half of the data (excluding the median itself):

Upper half = 33, 36, 39, 42, 45, 48, 50

The median of this upper half (7 values) is the 4<sup>th</sup> value:

Q3 = 42

**Step 4:** Calculate the Trimean

The formula for the trimean is:

$$\text{Trimean} = \frac{Q1 + 2 \times \text{Median} + Q3}{4}$$

Substitute the values:

$$\text{Trimean} = \frac{18 + 2 \times 30 + 42}{4} = \frac{18 + 60 + 42}{4} = \frac{120}{4} = 30$$

### Conclusion

The trimean of this dataset is 30. This measure incorporates the median (with twice the weight) and the quartiles, providing a balanced estimate of central tendency that is robust to outliers.

2. Geometric Mean

Suppose the population of a city changes over four years with the following annual growth rates:

Year 1: +5%

Year 2: +10%

Year 3: -3%

Year 4: +6%

Calculate the geometric mean of the growth rates to find the average population growth rate over these 4 years

**Solution:**

**Step 1:** Convert percentages to growth factors

Convert each percentage into a decimal growth factor by adding 1:

$$\text{Year 1: } 1 + 0.05 = 1.05$$

$$\text{Year 2: } 1 + 0.10 = 1.10$$

$$\text{Year 3: } 1 - 0.03 = 0.97$$

$$\text{Year 4: } 1 + 0.06 = 1.06$$

**Step 2:** Calculate the product of the growth factors

$$1.05 \times 1.10 \times 0.97 \times 1.06 = 1.181841$$

**Step 3:** Calculate the geometric mean

Since there are 4 years, the geometric mean is the 4th root of the product:

$$GM = \sqrt[4]{1.181841} \approx 1.0426$$

**Step 4:** Convert back to percentage

To express this as a percentage growth rate, subtract 1 and multiply by 100:

$$1.0426 - 1 = 0.0426 \text{ or } 4.26\%$$

**Conclusion:**

The average annual population growth rate over these 4 years is approximately 4.26% per year, even though the individual rates varied each year. The geometric mean smooths out the effect of the fluctuations and gives a meaningful long-term average growth rate.

3. Trimmed Mean

Consider the following dataset of 10 values representing exam scores:

65, 70, 72, 75, 80, 85, 90, 92, 95, 100

Calculate the 10% trimmed mean

**Solution:**

**Step 1:** Sort the data (if not already sorted)

In this case, the data is already sorted in ascending order:

65, 70, 72, 75, 80, 85, 90, 92, 95, 100

**Step 2:** Trim 10% from both ends

Since we have 10 values, trimming 10% means removing 10% of the values from each end.

10% of 10 = 1 value from the top and 1 value from the bottom.

So, remove the smallest value (65) and the largest value (100).

**Step 3:** Calculate the mean of the remaining values

The remaining values are:

70, 72, 75, 80, 85, 90, 92, 95

Now, calculate the mean of these 8 values:

Sum =  $70 + 72 + 75 + 80 + 85 + 90 + 92 + 95 = 659$

$$\text{Mean} = \frac{659}{8} = 82.375$$

**Conclusion:**

The 10% trimmed mean of the dataset is 82.375.

By trimming the lowest and highest values (65 and 100), we remove the possible influence of any outliers, providing a more robust average value.