Exercise 05 – October 23-24, 2024

1. The following data set represents the scores of 5 students in a quiz:
   Scores: 70,85,78,90,88

   Find the standard deviation from those data.

   Solution:

   1. **Mean (average):**

$$\text{Mean} = \frac{70 + 85 + 78 + 90 + 88}{5} = 82.2$$

   2. **Deviation of each score from the mean:**
      - $70 - 82.2 = -12.2$
      - $85 - 82.2 = 2.8$
      - $78 - 82.2 = -4.2$
      - $90 - 82.2 = 7.8$
      - $88 - 82.2 = 5.8$

   3. **Squared deviations:**
      - $(-12.2)^2 = 148.84$
      - $(2.8)^2 = 7.84$
      - $(-4.2)^2 = 17.64$
      - $(7.8)^2 = 60.84$
      - $(5.8)^2 = 33.64$

   4. **Variance** (average of squared deviations):

$$\text{Variance} = \frac{148.84 + 7.84 + 17.64 + 60.84 + 33.64}{5} = 53.76$$

   5. **Standard deviation** (square root of the variance):

$$\text{Standard deviation} = \sqrt{53.76} \approx 7.33$$

   So, the standard deviation of the quiz scores is approximately 7.33.

2. Suppose a survey indicates that 30% of people prefer coffee over tea. If you randomly select 100 people, what is the probability that fewer than 25 people prefer coffee? Use z-table

Solution:

1. $n = 100$, $p = 0.30$, and $q = 0.70$.
2. Check conditions:
   - $n \cdot p = 100 \cdot 0.30 = 30$,
   - $n \cdot (1 - p) = 100 \cdot 0.70 = 70$.

   Both are greater than 5, so the normal approximation can be used.
3. Calculate the mean and standard deviation:
   - $\mu = 100 \cdot 0.30 = 30$,
   - $\sigma = \sqrt{100 \cdot 0.30 \cdot 0.70} = \sqrt{21} \approx 4.58$.
4. Apply the continuity correction:
   - You want $P(X < 25)$, so with the continuity correction, calculate $P(X \leq 24.5)$.
5. Standardize:

$$Z = \frac{24.5 - 30}{4.58} = \frac{-5.5}{4.58} \approx -1.20$$

6. Find the z-score in the z-table:
   - For $Z = -1.20$, the z-table gives $P(Z \leq -1.20) \approx 0.1151$.

The probability that fewer than 25 people prefer coffee is approximately 0.1151 (or 11.51%).

3. You are conducting an experiment with 100 trials (n = 100), and the probability of success in each trial is p = 0.4. You want to find the probability that at least 45 successes will occur.

Solution:

1. **Check the conditions:**
   - $n \cdot p = 100 \cdot 0.4 = 40$,
   - $n \cdot (1 - p) = 100 \cdot 0.6 = 60$. Both conditions are satisfied.
2. **Calculate the mean and standard deviation:**
   - $\mu = 100 \cdot 0.4 = 40$,
   - $\sigma = \sqrt{100 \cdot 0.4 \cdot 0.6} = \sqrt{24} \approx 4.9$.

3. **Apply continuity correction**: We want $P(X \geq 45)$. Using the continuity correction, we calculate $P(X \geq 44.5)$.
4. **Convert to a z-score**:

$$Z = \frac{44.5 - 40}{4.9} = \frac{4.5}{4.9} \approx 0.92$$

5. **Find the probability using the z-table**: From the z-table, $P(Z \leq 0.92) \approx 0.8212$.

   Since we are looking for $P(X \geq 45)$, we calculate $P(Z \geq 0.92)$:

$$P(Z \geq 0.92) = 1 - 0.8212 = 0.1788$$

The probability of having at least 45 successes is approximately **0.1788**, or 17.88%