Tarea #2 Procesos numéricos

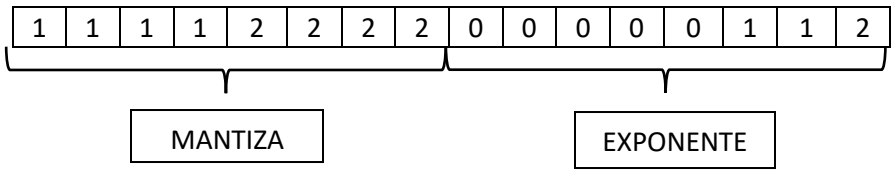
Fraider Rentería Usuga
Dairon Alberto Zapata David
Juan Pablo Garcia
Sebastián Madrid Taborda

2. Cambie cada uno de los siguientes números a base 10.

- $235_6 \rightarrow 2*6^2 + 3*6^1 + 5*6^0 = 95_{10}$
- $65_9 \rightarrow 6*9^1 + 5*9^0 = 59_{10}$
- $11112222220000 \rightarrow 1*3^{13} + 1*3^{12} + 1*3^{11} + 1*3^{10} + 2*3^9 + 2*3^8 + 2*3^7 + 2*3^6 + 2*3^5 + 2*3^4 + 0*0^3 + 0*0^2 + 0*0^1 + 0*0^0 = 2420928_{10}$
- $555ABC_{13} \rightarrow 5*13^5 + 5*13^4 + 5*13^3 + A*13^2 + B*13^1 + C*13^0 = 2012100_{10}$
- $A1B2C3D4E5F6_{16} \rightarrow A*16^{11} + 1*16^{10} + B*16^9 + 2*16^8 + C*16^7 + 3*16^6 + D*16^5 + 4*16^4 + E*16^3 + 5*16^2 + F*16^1 + 6*16^0 = 177789162812934_{10}$

6. Considere una máquina de 16 bits que trabaja en base tres. Los bits los tienen distribuidos por partes iguales para mantisa y exponente (ignore los signos). ¿Cómo se almacena cada uno de los siguientes números?

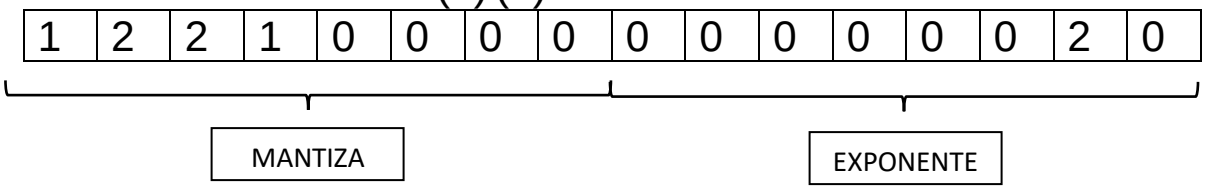
- 11112222220000_3



- $222222.111111113 \rightarrow 2730.333339691162_{10}$

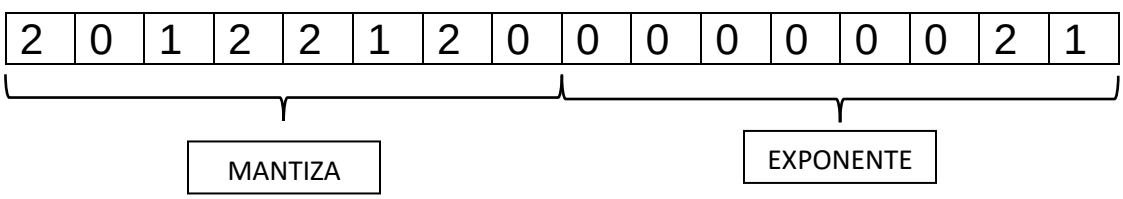
- **468** $\rightarrow 122100_3 \rightarrow 0.122100 \times 3^{20}$

468 | 3
(0) 156 | 3
(0) 52 | 3
(1) 17 | 3
(2) 5 | 3
(2) (1)



- **123A11**$\rightarrow 1616_{10} \rightarrow 2012212_3 \rightarrow 0.2012212 \times 3^{21}$

1616 | 3
(2) 538 | 3
(1) 179 | 3
(2) 59 | 3
(2) 19 | 3
(1) 6 | 3
(0) (2)



- 22354.217 → 1260.276 → 1201200.02 → 0.12012000×3^{21}

1260 | 3 — 0.216 x 3 = 0.828

(0) 420 | 3 — 0.828 x 3 =2.484

(0) 140 | 3

(2) 46 | 3

(1) 15 | 3

(0) 5 | 3

(2) (1)

0.12012000×3^{21}

1	2	0	1	2	0	0	0	0	0	0	0	0	2	1

- $11000111000.0001111_2 \rightarrow 1592.1171875_{10} \rightarrow 2011222.01_3 \rightarrow 0.20112220 \times 3^{22}$

1593 | 3 — 0.1171875 x 3 =0.3515625

(2) 530 | 3 — 0.3515625 x 3 =1.0546875

(2) 176 | 3

(2) 58 | 3

(1) 19 | 3

(1) 6 | 3

(0) (2)

0.20112220×3^{22}

2	0	1	1	2	2	2	0	0	0	0	0	0	0	2	1

- $222222.111111113_4 \longrightarrow 2730.333339691162_{10} \longrightarrow 10202010_3 \longrightarrow 0.10202010 \times 3^{22}$

2730 | 3

(0) 910 | 3

(1) 303 | 3

(0) 101 | 3

(2) 33 | 3

(0) 11 | 3

(2) 3 | 3

(0) (1)

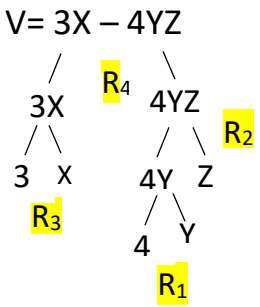
0.10202010×3^{22}

1	0	2	0	2	0	1	0	0	0	0	0	0	0	2	2

- Ejercicio número 13, hallar máximo Error Absoluto

EJERCICIO #1

$V = 3X - 4YZ$



$X = 23.45 \pm 0.43 \times 10^{-4}$
$Y = 75.35 \pm 0.23 \times 10^{-5}$
$Z = 31.135 \pm 0.78 \times 10^{-5}$

$E_{4Y} = YE_4 + 4E_Y + r1$

$E_{4Y.Z} = ZE_{4Y} + 4YE_Z + R_2$
$= Z(YE_4 + 4E_Y + R_1) + 4YE_Z + r_2$
$= ZYE_4 + 4ZE_Y + ZR_1 + 4YE_Z + R_2$

$E_{3X} = XE_3 + 3E_X + R_3$

$EV = E_{3X} - E_{4YZ}$
$EV = XE_3 + 3E_X + R_3 - ZYE_4 - 4ZE_Y - ZR_1 - 4YE_Z + R_2$
$EV = XE_3 + 3E_X - ZYE_4 - 4ZE_Y - 4YE_Z + R_2 + R_3 + ZR_1$

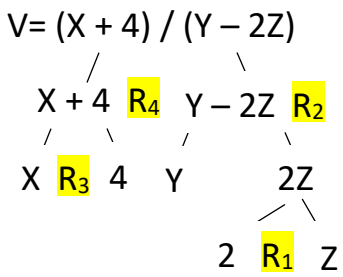
$$|V| = |3E_X - 4ZE_Y - 4YE_Z + R_2 + R_3 + ZR_1|$$
$$\leq |3E_X| + |4ZE_Y| + |4YE_Z| + |R_2| + |R_3| + |ZR_1|$$
$$\leq 3|E_X| + |4Z||E_Y| + |4Y||E_Z| + |R_2| + |R_3| + |Z||R_1|$$
$$\leq 3E_X + 4ZE_Y + 4YE_Z + 0.5 \times 10^{-d} + 0.5 \times 10^{-d} + (Z \times 0.5 \times 10^{-d})$$
$$\leq 3E_X + 4ZE_Y + 4YE_Z + [\langle 1 + 1 + (Z \times 1)\rangle \times 0.5 \times 10^{-d}]$$
$$\leq 3E_X + 4ZE_Y + 4YE_Z + [\langle 1 + 1 + (Z \times 1)\rangle \times 0.5 \times 10^{-d}]$$

$$\leq \mathbf{0.2766362} \times 10^{-2} + \mathbf{1.70678} \times 10^{-d+1}$$

$R1 \leq 0.5 \times 10^{-d}$
$R2 \leq 0.5 \times 10^{-d}$
$R3 \leq 0.5 \times 10^{-d}$

EJERCICIO #2

$$X = 23.45 \pm 0.43 \times 10^{-4}$$
$$Y = 75.35 \pm 0.23 \times 10^{-5}$$
$$Z = 31.135 \pm 0.78 \times 10^{-5}$$



$E_{2Z} = ZE_2 + 2E_Z + R_1$

$E_{Y-2Z} = E_Y - E_{2Z} + R_2$
$= E_Y - ZE_2 + 2E_Z + R_1 + R_2$

$E_{X+4} = E_X + E_4 + R_3$

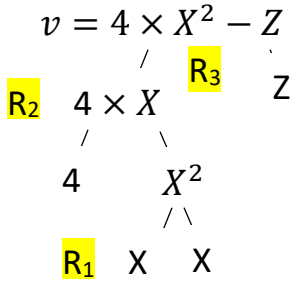
$R1 \leq 0.5 \times 10^{-d}$
$R2 \leq 0.5 \times 10^{-d}$
$R3 \leq 0.5 \times 10^{-d}$
$R4 \leq 0.5 \times 10^{-d}$

$$E_V = \frac{1}{Y-2Z} E_{X+4} - \frac{X+4}{(Y-2Z)^2} E_{Y-2Z} + R_4$$
$$= \frac{1}{Y-2Z}(E_X + E_4 + R_3) - \frac{X+4}{(Y-2Z)^2}(E_Y - ZE_2 + 2E_Z + R_1 + R_2) + R_4$$
$$= \frac{1}{Y-2Z} E_X - \frac{X+4}{(Y-2Z)^2} E_Y - \frac{X+4}{(Y-2Z)^2} 2E_z + \left(\frac{R3}{Y-2Z} - \frac{X+4}{(Y-2Z)^2}(R1) - \frac{X+4}{(Y-2Z)^2}(R2) + R4\right)$$
$$|V| = \left|\frac{1}{Y-2Z} EX - \frac{X+4}{(Y-2Z)^2} EY - \frac{X+4}{(Y-2Z)^2} 2Ez + \left(\frac{R3}{Y-2Z} - \frac{X+4}{(Y-2Z)^2}(R1) - \frac{X+4}{(Y-2Z)^2}(R2) + R4\right)\right|$$
$$\leq \left|\frac{1}{Y-2Z} EX\right| + \left|\frac{X+4}{(Y-2Z)^2} EY\right| + \left|\frac{X+4}{(Y-2Z)^2} 2Ez\right| + \left|\left(\frac{R3}{Y-2Z} - \frac{X+4}{(Y-2Z)^2}(R1) - \frac{X+4}{(Y-2Z)^2}(R2) + R4\right)\right|$$
$$\leq \left|\frac{1}{Y-2Z}\right| |EX| + \left|\frac{X+4}{(Y-2Z)^2}\right| |EY| + \left|\frac{X+4}{(Y-2Z)^2}\right| |2Ez| + \left(\left|\frac{R3}{Y-2Z}\right| - \left|\frac{X+4}{(Y-2Z)^2}\right| (|R1|) + \left|\frac{X+4}{(Y-2Z)^2}\right| (|R2|) + |R4|\right)$$
$$\leq \frac{1}{Y-2Z} EX + \frac{X+4}{(Y-2Z)^2} EY + \frac{X+4}{(Y-2Z)^2} 2Ez + \left[\left(\frac{R3}{Y-2Z} + \frac{X+4}{(Y-2Z)^2}(R1) + \frac{X+4}{(Y-2Z)^2}(R2) + R4\right).5 \times 10^{-d}\right]$$
$$\leq \mathbf{6.16026 \times 10^{-6} + 0.698704 \times 10^{-d}}$$

EJERCICIO #3

$E_{X \times X} = XE_X + XE_X + R_1$
$= 2XE_X + R_1$

$E_{4xX^2} = X^2E_4 + 4E_X^2 + R_2$
$= X^2E_4 + 4(2XE_X + R_1) + R_2$
$= X^2E_4 + 8XE_X + 4R_1 + R_2$



$E_V = E_{4 X X^2} - E_Z + R_3$
$= X^2E_4 + 8XE_X + 4R_1 + R_2 - E_Z + R_3$
$|V| = |8XEX + 4R1 + R2 - EZ + R3|$
$\leq |8XEX| + |4R1| + |R2| + |EZ| + |R3|$
$\leq 8XEX + EZ + [(4 \times 1 + 1 + 1) \times .5 \times 10^{-d}]$
$\leq 0.80746 \times 10^{-2} + 3 \times 10^{-d}$

$R1 \leq 0.5 \times 10^{-d}$
$R2 \leq 0.5 \times 10^{-d}$
$R3 \leq 0.5 \times 10^{-d}$

EJERCICIO #4

$V = (X + Y) + Z$

X + Y R_2 Z

X R1 Y

$E_{X+Y} = E_X + E_Y + R_1$

$R1 \leq 0.5 \times 10^{-d}$
$R2 \leq 0.5 \times 10^{-d}$

$E_V = E_{X+Y} + E_Z + R_2$
$= E_X + E_Y + E_Z + R_1 + R_2$

$$
\begin{aligned}
|V| &= |E_X + E_Y + E_Z + R_1 + R_2| \\
&\leq |E_X| + |E_Y| + |E_Z| + |R_1| + |R_2| \\
&\leq E_X + E_Y + E_Z + .5 \times 10^{-d} + .5 \times 10^{-d} \\
&\leq E_X + E_Y + E_Z + [(1+1) \times .5 \times 10^{-d}]
\end{aligned}
$$

$= 0.531 \times \mathbf{10^{-4}} + \mathbf{1} \times \mathbf{10^{-d}}$