

Tarea #2 Procesos numéricos

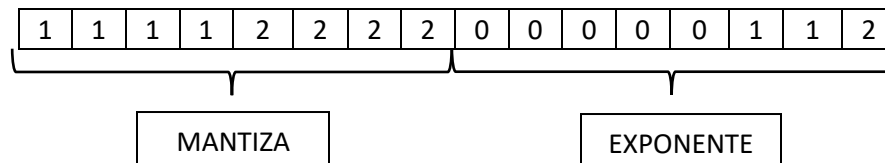
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2. Cambie cada uno de los siguientes números a base 10.

- $235_6 \rightarrow 2 \cdot 6^2 + 3 \cdot 6^1 + 5 \cdot 6^0 = 95_{10}$
- $65_9 \rightarrow 6 \cdot 9^1 + 5 \cdot 9^0 = 59_{10}$
- $1111222220000_3 \rightarrow 1 \cdot 3^{13} + 1 \cdot 3^{12} + 1 \cdot 3^{11} + 1 \cdot 3^{10} + 2 \cdot 3^9 + 2 \cdot 3^8 + 2 \cdot 3^7 + 2 \cdot 3^6 + 2 \cdot 3^5 + 2 \cdot 3^4 + 0 \cdot 3^3 + 0 \cdot 3^2 + 0 \cdot 3^1 + 0 \cdot 3^0 = 2420928_{10}$
- $555ABC_{13} \rightarrow 5 \cdot 13^5 + 5 \cdot 13^4 + 5 \cdot 13^3 + A \cdot 13^2 + B \cdot 13^1 + C \cdot 13^0 = 2012100_{10}$
- $A1B2C3D4E5F6_{16} \rightarrow A \cdot 16^{11} + 1 \cdot 16^{10} + B \cdot 16^9 + 2 \cdot 16^8 + C \cdot 16^7 + 3 \cdot 16^6 + D \cdot 16^5 + 4 \cdot 16^4 + E \cdot 16^3 + 5 \cdot 16^2 + F \cdot 16^1 + 6 \cdot 16^0 = 177789162812934_{10}$

6. Considere una máquina de 16 bits que trabaja en base tres. Los bits los tienen distribuidos por partes iguales para mantisa y exponente (ignore los signos). ¿Cómo se almacena cada uno de los siguientes números?

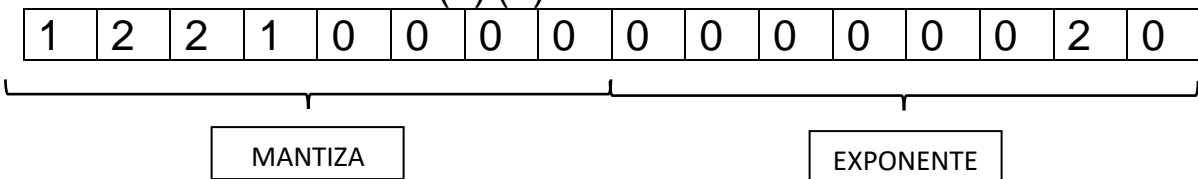
- 1111222220000_3



- $222222.111111113 \rightarrow 2730.333339691162_{10}$

- $468 \rightarrow 122100_3 \rightarrow 0.122100 \times 3^{20}$

$$\begin{array}{r}
 468 \overline{)3} \\
 (0) \ 156 \overline{)3} \\
 \quad (0) \ 52 \overline{)3} \\
 \quad \quad (1) \ 17 \overline{)3} \\
 \quad \quad \quad (2) \ 5 \overline{)3} \\
 \quad \quad \quad \quad (2) \ (1)
 \end{array}$$

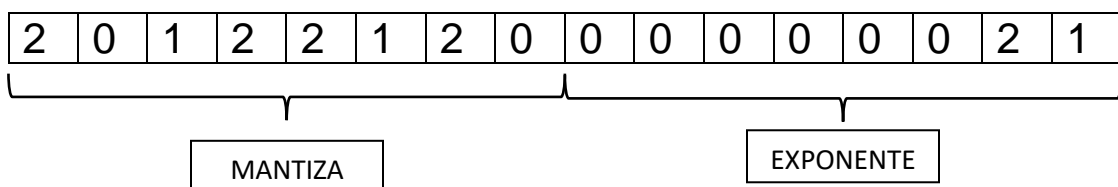


- 2354.217

- 11000111000.0001111_2

- $123A11 \rightarrow 1616_{10} \rightarrow 2012212_3 \rightarrow 0.2012212 \times 3^{21}$

$$\begin{array}{r}
 1616 \overline{)3} \\
 (2) \ 538 \overline{)3} \\
 \quad (1) \ 179 \overline{)3} \\
 \quad \quad (2) \ 59 \overline{)3} \\
 \quad \quad \quad (2) \ 19 \overline{)3} \\
 \quad \quad \quad \quad (1) \ 6 \overline{)3} \\
 \quad \quad \quad \quad \quad (0) \ (2)
 \end{array}$$



- $222222.111111113_4 \rightarrow 2730.3333339691162_{10} \rightarrow 10202010_3 \rightarrow 0.10202010 \times 3^{22}$

$$2730 \div 3$$

$$(0) \quad 910 \div 3$$

$$(1) \quad 303 \div 3$$

$$(0) \quad 101 \div 3$$

$$(2) \quad 33 \div 3$$

$$(0) \quad 11 \div 3$$

$$(2) \quad 3 \div 3$$

$$(0) \quad (1)$$

$$0.10202010 \times 3^{22}$$

1	0	2	0	2	0	1	0	0	0	0	0	0	0	2	2
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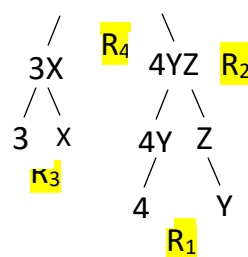
13. Con base en los datos del ejercicio 11 hallar el máximo error absoluto

$$\text{Datos: } X = 23,35 \pm 0,43 \times 10^{-4} \quad Y = 75,35 \pm 0,23 \times 10^{-5}$$

$$Z = 31.1356 \pm 0,78 \times 10^{-5}$$

EJERCICIO #1

$$V = 3X - 4YZ$$



$$\begin{aligned} R_1 &\leq 0,5 \times 10^{-5} \\ R_2 &\leq 0,5 \times 10^{-5} \\ R_3 &\leq 0,5 \times 10^{-4} \end{aligned}$$

$$E_{4Y} = YE_4 + 4E_Y + r_1$$

$$\begin{aligned} E_{4YZ} &= ZE_{4Y} + 4YE_Z + R_2 \\ &= Z(YE_4 + 4E_Y + R_1) + 4YE_Z + r_2 \\ &= ZYE_4 + 4ZE_Y + ZR_1 + 4YE_Z + R_2 \end{aligned}$$

$$E_{3X} = XE_3 + 3E_X + R_3$$

$$E_V = E_{3X} - E_{4YZ}$$

$$E_V = XE_3 + 3E_X + R_3 - ZYE_4 - 4ZE_Y - ZR_1 - 4YE_Z + R_2$$

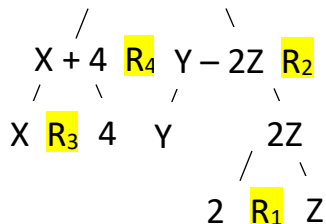
$$E_V = \cancel{XE_3} + 3\cancel{E_X} - \cancel{ZYE_4} - 4\cancel{ZE_Y} - 4YE_Z + R_2 + R_3 + ZR_1$$

$$E_V \leq 0,250837 \times 10^{-2} + (R_2 + R_3 + ZR_1)$$

$$E_V \leq 0,401015 \times 10^{-2}$$

EJERCICIO #2

$$V = (X + 4) / (Y - 2Z)$$



$$E_{2Z} = ZE_2 + 2E_Z + R_1$$

$$\begin{aligned} E_{Y-2Z} &= E_Y - E_{2Z} + R_2 \\ &= E_Y - ZE_2 + 2E_Z + R_1 + R_2 \end{aligned}$$

$$E_{X+4} = E_X + E_4 + R_3$$

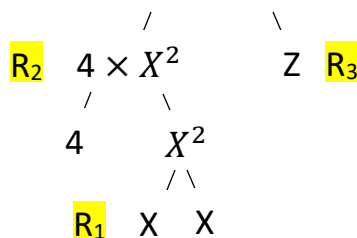
$$\begin{aligned} E_V &= \frac{1}{Y-2Z} E_{X+4} - \frac{X+4}{(Y-2Z)^2} E_{Y-2Z} + R_4 \\ &= \frac{1}{Y-2Z} (E_X + E_4 + R_3) - \frac{X+4}{(Y-2Z)^2} (E_Y - ZE_2 + 2E_Z + R_1 + R_2) + R_4 \\ &= \frac{1}{Y-2Z} E_X - \frac{X+4}{(Y-2Z)^2} E_Y - \frac{X+4}{(Y-2Z)^2} 2E_2 + \left(\frac{R_3}{Y-2Z} - \frac{X+4}{(Y-2Z)^2} (R_1) - \frac{X+4}{(Y-2Z)^2} (R_2) \right) + R_4 \end{aligned}$$

$$E_V \leq 0,419282 \times 10^{-5}$$

$$\begin{aligned} R_1 &\leq 0,5 \times 10^{-10} \\ R_2 &\leq 0,5 \times 10^{-5} \\ R_3 &\leq 0,5 \times 10^{-4} \\ R_4 &\leq 0,5 \times 10^{-5} \end{aligned}$$

EJERCICIO #3

$$v = 4 \times X^2 - Z$$



$$\begin{aligned} E_{X \times X} &= XE_X + XE_X + R_1 \\ &= 2XE_X + R_1 \end{aligned}$$

$$\begin{aligned} E_{4 \times X^2} &= X^2 E_4 + 4XE_X^2 + R_2 \\ &= X^2 E_4 + 4(2XE_X + R_1) + R_2 \\ &= X^2 E_4 + 8XE_X + 4R_1 + R_2 \end{aligned}$$

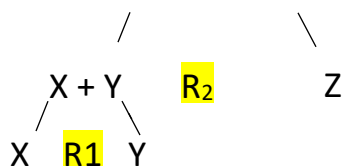
$$\begin{aligned} R_1 &\leq 0,5 \times 10^{-4} \\ R_2 &\leq 0,5 \times 10^{-4} \\ R_3 &\leq 0,5 \times 10^{-5} \end{aligned}$$

$$\begin{aligned} E_V &\leq E_{4 \times X^2} - E_Z + R_3 \\ &\leq X^2 E_4 + 8XE_X + 4R_1 + R_2 - E_Z + R_3 \end{aligned}$$

$$E_V \leq 0,8314 \times 10^{-2}$$

EJERCICIO #4

$$V = (X + Y) + Z$$



$$E_{X+Y} = E_X + E_Y + R_1$$

$$\begin{aligned} E_V &\leq E_{X+Y} + E_Z + R_2 \\ &\leq E_X + E_Y + E_Z + R_1 + R_2 \end{aligned}$$

$$E_V \leq 0,631 \times 10^{-4}$$

$$\begin{aligned} R_1 &\leq 0,5 \times 10^{-5} \\ R_2 &\leq 0,5 \times 10^{-5} \end{aligned}$$