Scalar Charges and the First Law of Black Hole Thermodynamics in Asymptotically Flat Spacetime

Romina Ballesteros, Francisco Gómez, Jorge Maggiolo

Pontificia Universidad Católica de Valparaíso, Instituto de Física

INSTITUTO DE FÍSICA

PONTIFICIA UNIVERSIDAD CATÓLICA DE VALPARAÍSO

Abstract

We present a variational formulation of Einstein-Maxwell-dilaton theory in flat spacetime, when the asymptotic value of the scalar field is not fixed. We obtain the boundary terms that make the variational principle well posed and then compute the finite gravitational action and corresponding Brown-York stress tensor. We show that the total energy has a new contribution that depends on the asymptotic value of the scalar field and discuss the role of scalar charges for the first law of thermodynamics.

Introduction

Scalar fields play a central role in particle physics and cosmology and arise naturally in the high energy physics unification theories. It is then important to understand generic properties of gravity theories coupled to scalars (and other matter fields), particularly the role of scalars for black hole physics.

The first law of thermodynamics is modified as follow

$$dM = TdS + \Psi dQ + \Upsilon dP + \left(\frac{\partial M}{\partial \phi^a_\infty}\right) d\phi^a_\infty. \tag{1}$$

One problem with the first law of thermodynamics (1) for stringy black holes is that the scalar charges are not conserved charges. They correspond to degrees of freedom living outside the horizon (the 'hair') and are not associated to a new independent integration constant, that is why it is called 'secondary hair'. In string theory, the scalar fields (moduli) are interpreted as local coupling constants and so a variation of their boundary values is equivalent to changing the couplings of the theory.

In this work, we investigate the role of non-trivial boundary conditions of the scalar field in Einstein-Maxwell-dilaton theory. We are interested in asymptotically flat hairy black hole solutions for which the asymptotic value of the scalar can vary. In flat spacetime, we obtain a well-posed variational principle by adding a new boundary term to the action, which permits us to compute the correct total energy, and clarify the role of the (non-conserved) scalar charges to the first law of thermodynamics [1].

Asymptotically flat hairy black holes

In this section, we propose a variational principle for asymptotically flat hairy black holes when the boundary values of the scalar fields can vary and show that the total energy has a new contribution that is relevant for thermodynamics. The goal of this section is to discuss this issue concretely in the simplest possible non-trivial setting, namely we are going to use the quasilocal formalism of Brown and York for a theory with only one scalar field that is coupled to the gauge field.

The first law of hairy black hole thermodynamics

We start with a brief review of [1] and, for clarity, we explicitly obtain the scalar charge term in the first law for an exact hairy black hole solution. Besides the graviton, every string theory contains another universal state, a massless scalar called the dilaton . We consider the Einstein-Maxwell-dilaton action

$$I[g_{\mu\nu}, A_\mu, \phi] = \frac{1}{2\kappa}\int_{\mathcal{M}} d^4x\sqrt{-g}\left(R - e^{\alpha\phi}F_{\mu\nu}F^{\mu\nu} - 2\partial_\mu\phi\partial^\mu\phi\right) + \frac{1}{\kappa}\int_{\partial\mathcal{M}} d^3x\sqrt{-h}K, \tag{2}$$

where $\kappa = 8\pi G_N$ and, with our conventions, $G_N = 1$. The second term is the Gibbons-Hawking boundary term and K is the trace of the extrinsec curvature K_{ab} defined on the boundary $\partial\mathcal{M}$ with the induced metric h_{ab}.
The coupling between the scalar field and gauge field in the action (2) appears in the low energy actions of string theory for particular values of α, though in our analysis we are going to keep α arbitrary. The equations of motion for the metric, scalar, and gauge field are

$$E_{\mu\nu} := R_{\mu\nu} - 2\partial_\mu\phi\partial_\nu\phi - 2e^{\alpha\phi}\left(F_{\mu\alpha}F^\alpha_\nu - \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}\right) = 0, \tag{3}$$

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right) - \frac{1}{4}\alpha e^{\alpha\phi}F_{\mu\nu}F^{\mu\nu} = 0, \tag{4}$$

$$\partial_\mu\left(\sqrt{-g}e^{\alpha\phi}F^{\mu\nu}\right) = 0. \tag{5}$$

The general metric ansatz for a static dyonic hairy black hole solution is

$$ds^2 = -a^2dt^2 + a^{-2}dr^2 + b^2(d\theta^2 + sin^2\theta d\varphi^2), \tag{6}$$

where the metric functions have only radial dependence, $a = a(r)$ and $b = b(r)$. The gauge field compatible with this ansatz and the equations of motion is

$$F = -\frac{qe^{-\alpha\phi}}{b^2}dt\wedge dr - p\sin\theta d\theta\wedge d\varphi. \tag{7}$$

To concretely check the steps for obtaining (1), we are going to use an exact ($\alpha = -2$) four dimensional hairy black hole solution [2], and the exact black hole solution is

$$a^2 = \frac{(r-r_+)(r-r_-)}{r^2-\Sigma^2}, \quad b^2 = r^2 - \Sigma^2, \quad \phi = \phi_\infty + \frac{1}{2}\ln\left(\frac{r+\Sigma}{r-\Sigma}\right), \tag{8}$$

where

$$r_- = -\Sigma, \quad r_+ = \Sigma - \frac{(qe^{\phi_\infty})^2}{\Sigma}. \tag{9}$$

The ADM mass is obtained by expanding the g_{tt} component of the metric, which leads to the following expression

$$M = -\frac{(qe^{\phi_\infty})^2}{2\Sigma} \tag{10}$$

with the scalar charge negative, $\Sigma < 0$. The physical (conserved) electric charge Q is computed, as usual, by integrating the equation of motion for the electric field and, with our conventions, the result is

$$Q = \frac{2}{\kappa}\oint e^{-2\phi}\star F = \frac{2}{\kappa}\oint e^{-2\phi}\left(-\frac{1}{4}\sqrt{-g}\epsilon_{\alpha\beta\mu\nu}F^{\alpha\beta}dx^\mu\wedge dx^\nu\right) = q \tag{11}$$

At this point, it is important to observe that, when evaluating on an exact solution, Σ is not an independent integration constant and the solution is regular only if $2M^2 - Q^2e^{2\phi_\infty} > 0$.

Total energy and Brown-York formalism

For Einstein-Maxwell theory in four dimensions, the action (2) should be supplemented with a counterterm that cancel the IR divergencies,

$$I = I_{bulk} + I_{GH} + I_{ct}\,, \qquad I_{ct} = -\frac{1}{\kappa}\int_{\partial\mathcal{M}} d^3x\sqrt{-h}\sqrt{2\mathcal{R}^{(3)}}, \tag{12}$$

where $\mathcal{R}^{(3)}$ is the Ricci scalar of the 3-dimensional boundary metric h_{ab}.
Considering a general boundary condition of the form

$$\Sigma \equiv \frac{dW(\phi_\infty)}{d\phi_\infty}, \tag{13}$$

the general boundary term is now

$$\tilde{I}^\phi_{ct} = -\frac{2}{\kappa}\int_{\partial\mathcal{M}} d^3x\sqrt{-h}\left[\frac{(\phi-\phi_\infty)^2}{\Sigma^2}W(\phi_\infty)\right]. \tag{14}$$

To get the free energy F of a hairy black hole, we shall compute the on-shell action on the Euclidean section:

$$I^E = \beta F = \beta\left(M - TS - Q\Psi - P\Upsilon + \Sigma\phi_\infty\right), \tag{15}$$

where the periodicity of the Euclidean time is related to the black hole temperature by $\beta = 1/T$.
We observe that there is an extra term $\Sigma\phi_\infty$ that is, in fact, coming from the scalar field counterterm $\tilde{I}^\phi_{ct}$. With all the terms required for a correct variational principle, we obtain the regularized quasilocal stress tensor with a contribution from the scalar field

$$\tau_{ab} = \frac{1}{\kappa}\left[K_{ab} - h_{ab}K - \Phi(\mathcal{R}^{(3)}_{ab} - h_{ab}\mathcal{R}^{(3)}) - h_{ab}\Box\Phi + \Phi_{;ab}\right] + \frac{2h_{ab}}{\kappa}\left[\frac{\phi_\infty}{\Sigma}(\phi-\phi_\infty)^2\right] \tag{16}$$

where

$$\Phi = \sqrt{\frac{2}{\mathcal{R}^{(3)}}} \tag{17}$$

Considering the Brown York formalism to compute conserved quantities [3]:

$$E = \oint_\Xi d^2\Xi\sqrt{\sigma}n^a\xi^b\tau_{ab} \tag{18}$$

Here, Ξ is a two dimensional closed surface with the unit normal n^a and the induced metric

$$\sigma_{ij}dx^idx^j = b^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{19}$$

Evaluating this conserved quantity, (18), at the spatial infinity, we obtain the following expression for the total energy:

$$E_{\text{total}} = M + \phi_\infty\Sigma \tag{20}$$

and this leads to the following first law of thermodynamics

$$dE_{\text{total}} = TdS + \Psi dQ + \Upsilon dP \tag{21}$$

with the $\Sigma d\phi_\infty$ reabsorbed in the total energy of the spacetime, which is different from the ADM mass.

Conclusion

In this work [4], using ideas arising from the quasilocal formulation of energy and counterterm method, we have revised the first law of hairy black hole thermodynamics and shown that the (non-conserved) scalar charges can not appear as independent terms.

References

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