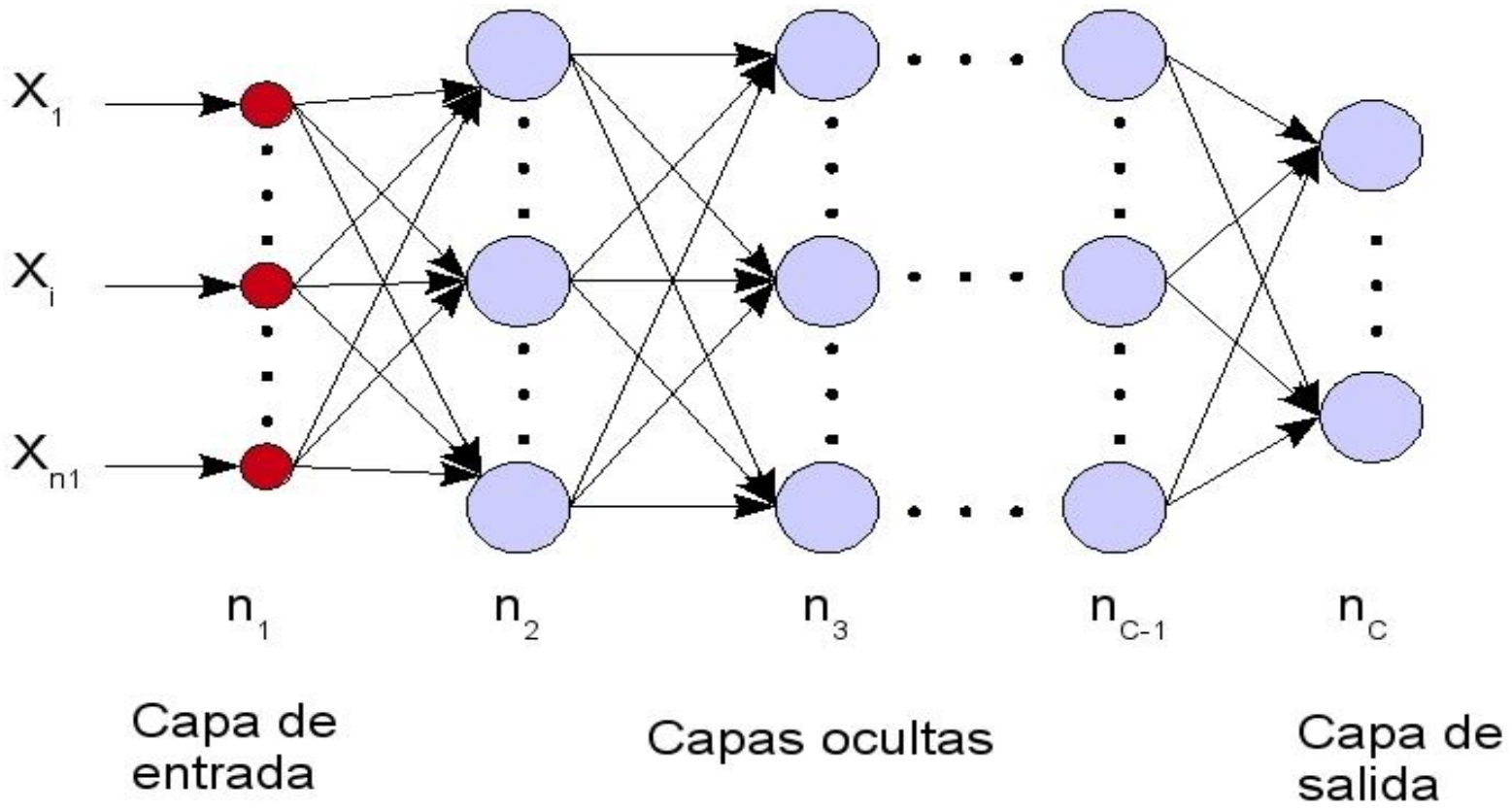
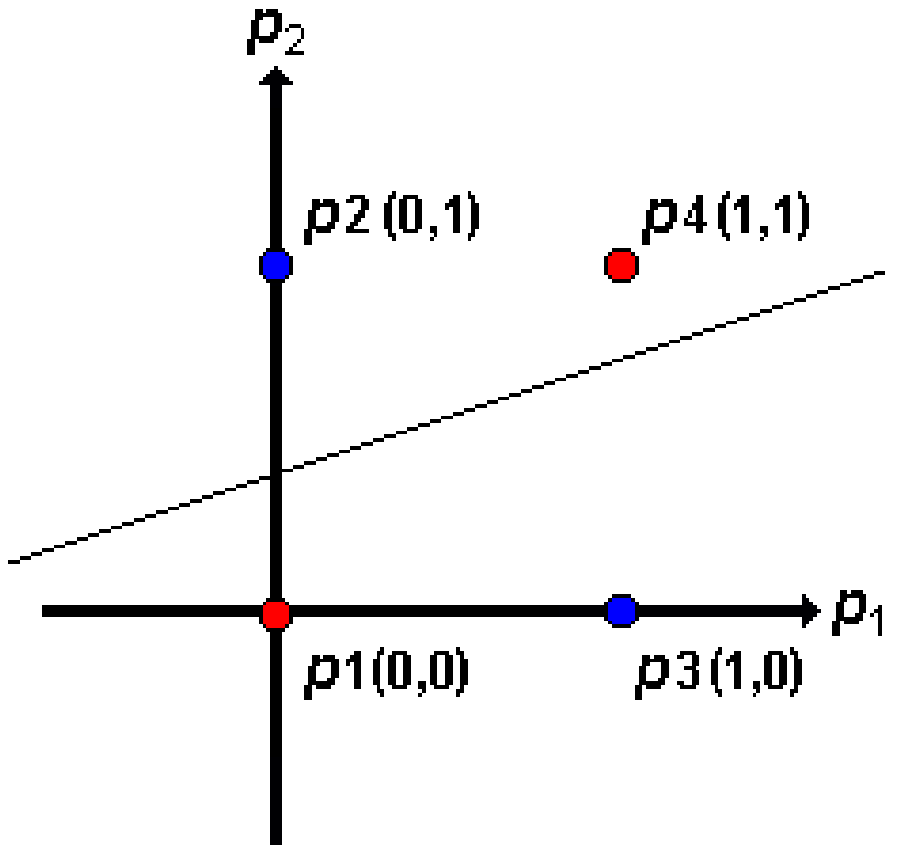
MULTIPERCEPTRÓN Y EL ALGORITMO BACKPROPAGATION

Multiperceptrón - Arquitectura

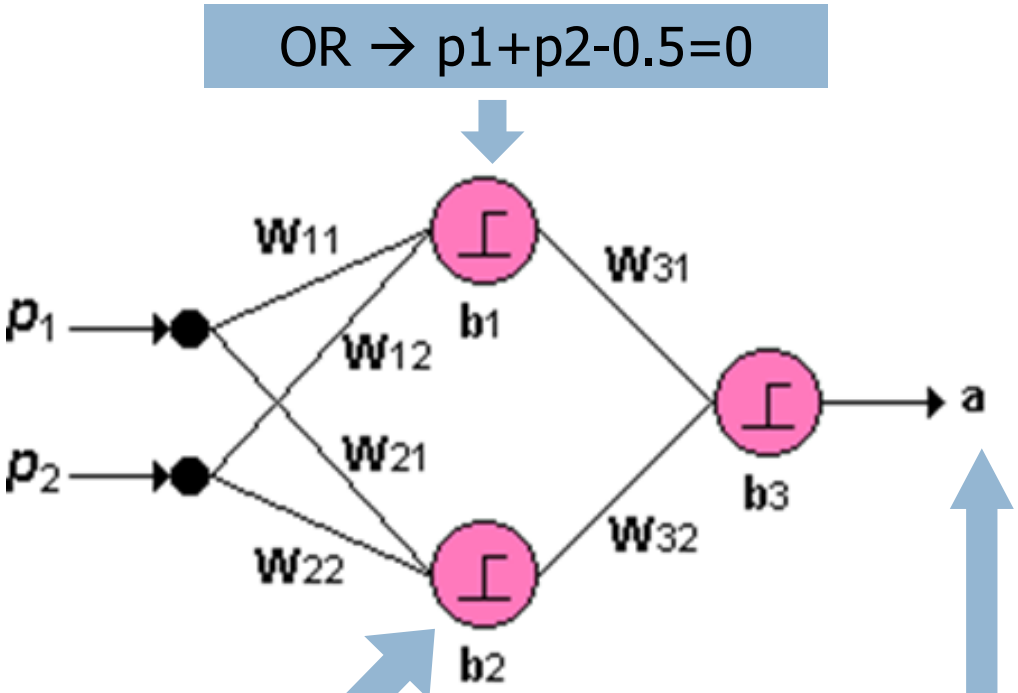


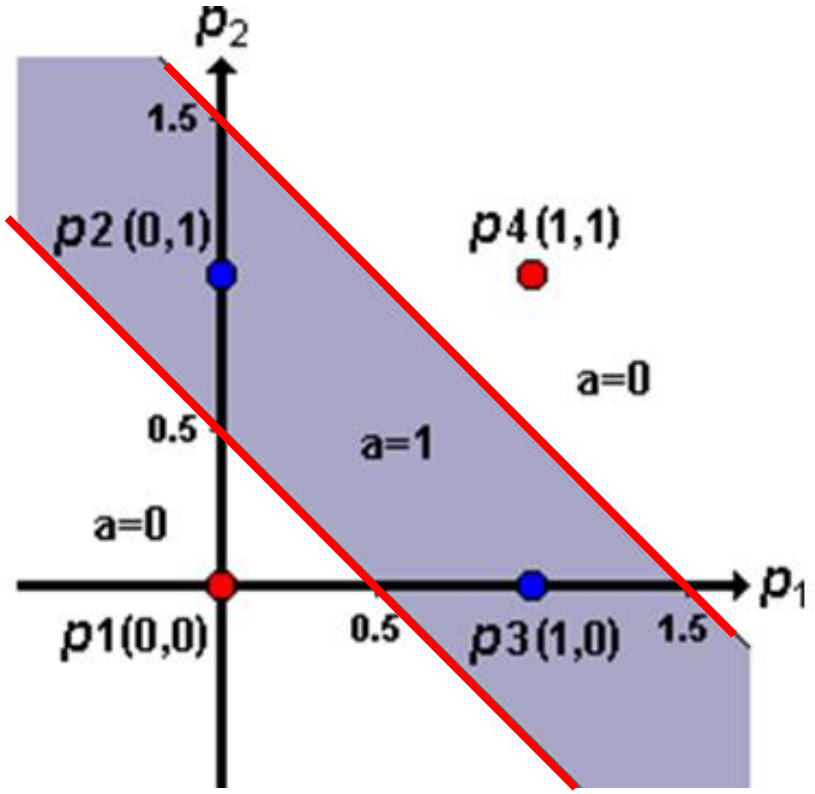
Redes multicapa

- Con una sola neurona no se puede resolver el problema del XOR porque no es linealmente separable.



XOR



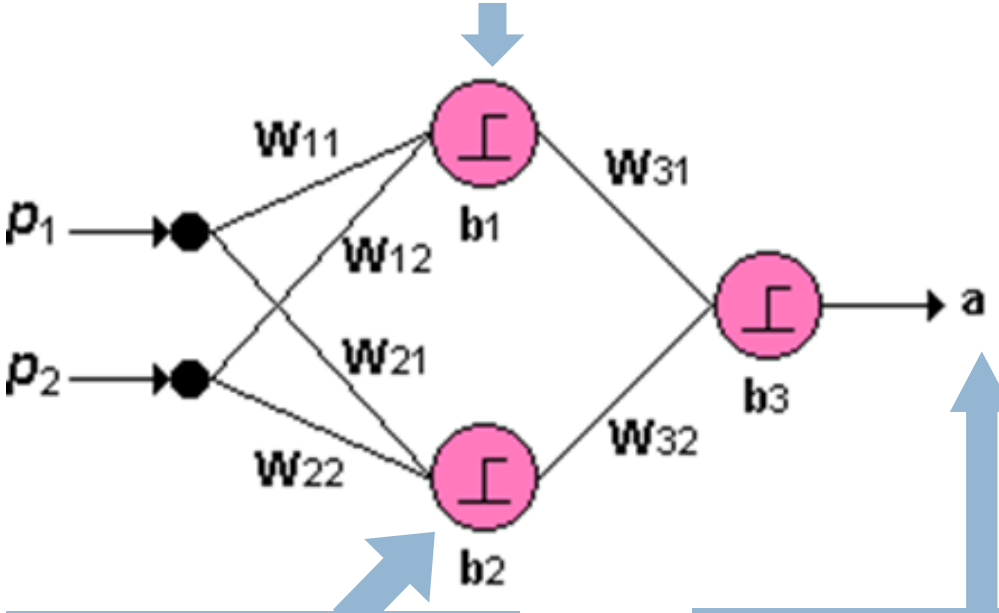


$w_{11}=1 \quad w_{12}=1 \quad b_1=-0.5 \; ; \quad w_{21}=1 \quad w_{22}=1 \quad b_2=-1.5 \; ; \; w_{31}=1 \quad w_{32}=-1.5 \quad b_3=-0.5$

XOR

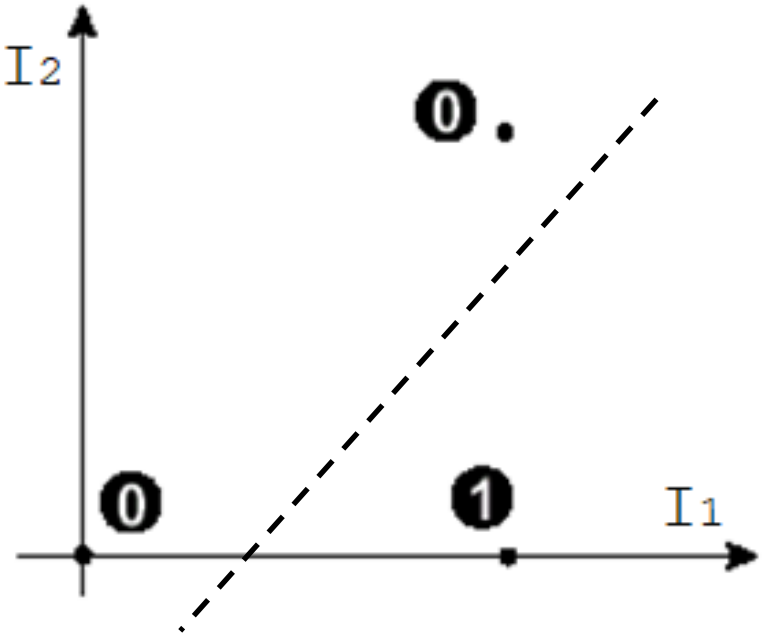
p1	p2	I1 (or)	I2 (AND)	a
1	0	1	0	1
1	1	1	1	0
0	0	0	0	0
0	1	1	0	1

OR → p1+p2-0.5=0

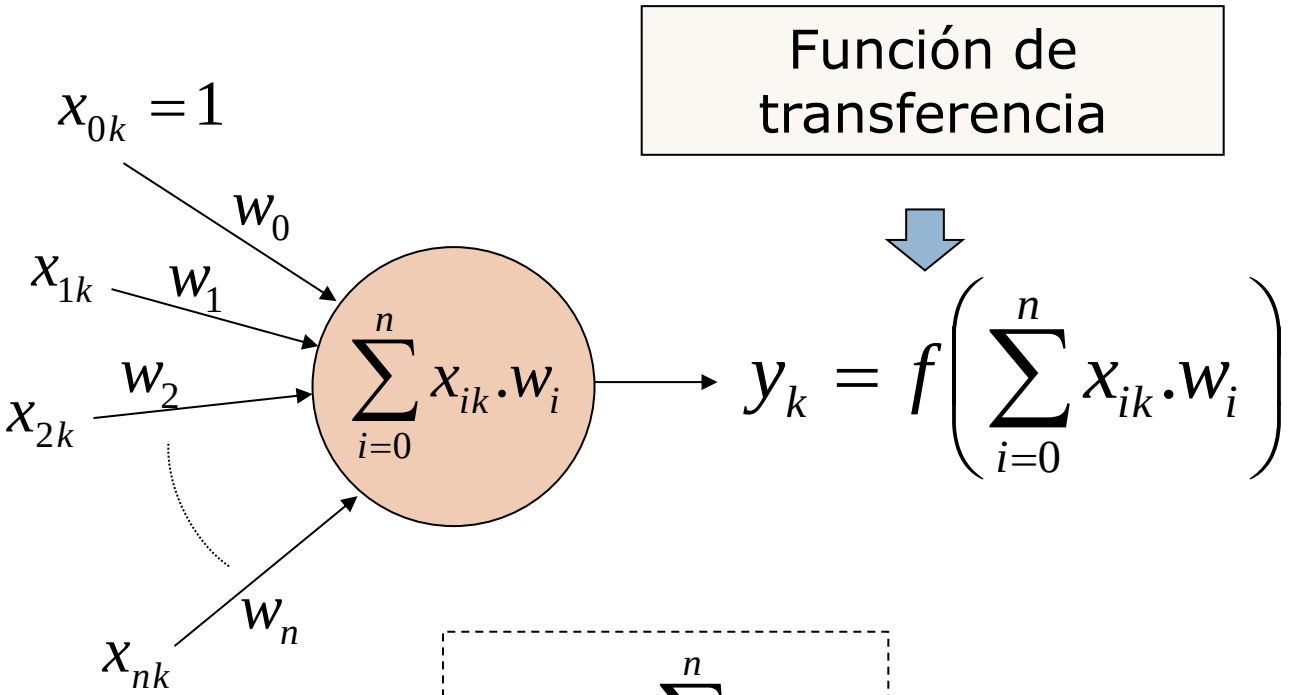


AND
p1+p2-1.5=0

I1 - 1.5 * I2 - 0.5=0



Neurona artificial



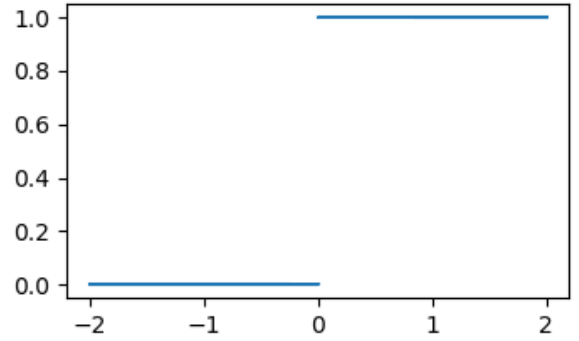
Función de transferencia

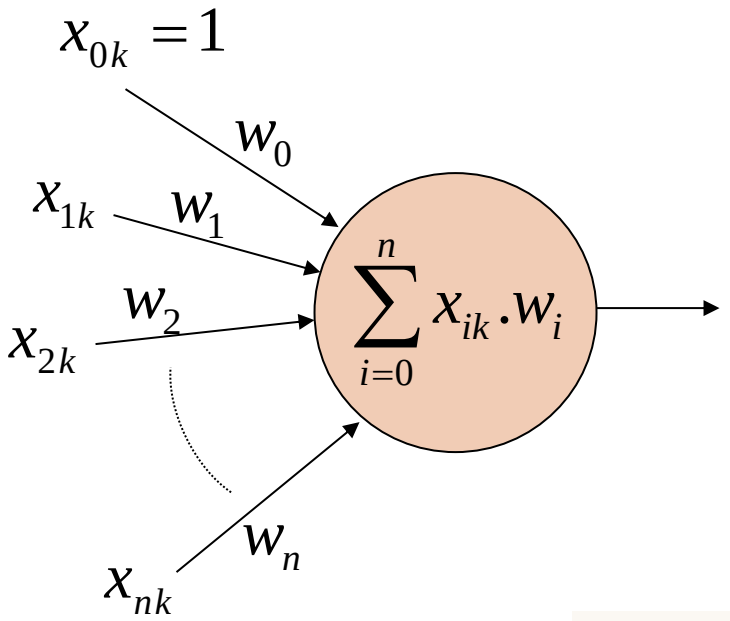
$$y_k = f\left(\sum_{i=0}^{n} x_{ik}.w_i\right)$$

$$neta_k = \sum_{i=0}^{n} x_{ik}.w_i$$

Perceptrón

$$f(x) = \begin{cases} 1 & si\ x \geq 0 \\ 0 & si\ x < 0 \end{cases}$$



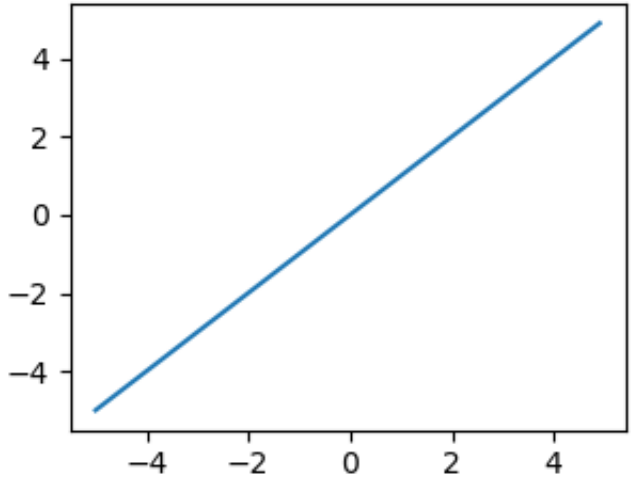


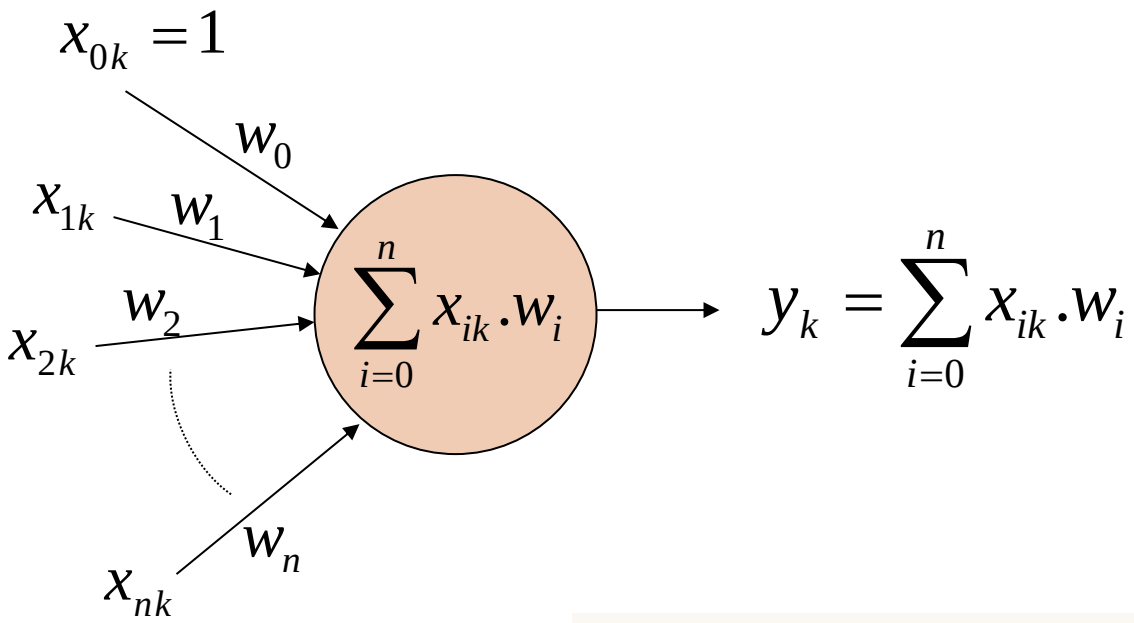
$$y_k = \begin{cases} 1 & si\ \sum_{i=0}^{n} x_{ik}.w_i \geq 0 \\ 0 & si\ \sum_{i=0}^{n} x_{ik}.w_i < 0 \end{cases}$$

- Actualiza el vector W modificando el ángulo que forma con cada ejemplo X_k

Combinador lineal

$$f(x) = x$$



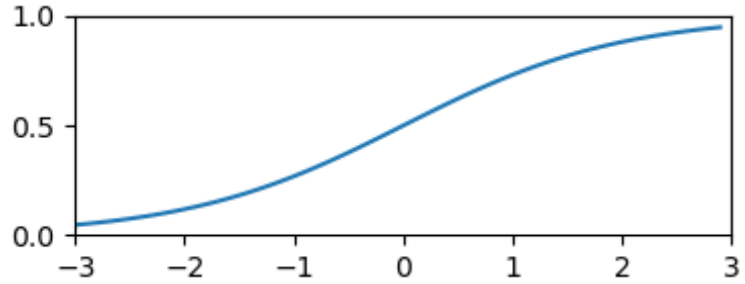


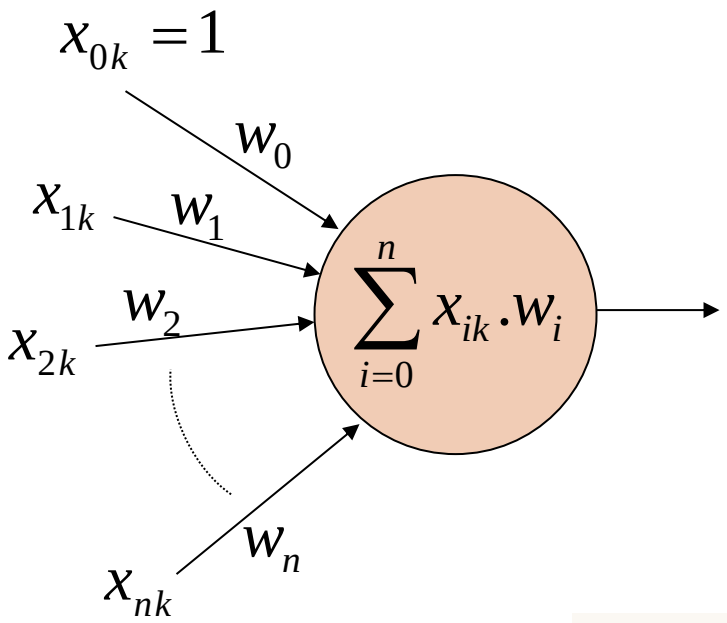
$$y_k = \sum_{i=0}^{n} x_{ik}.w_i$$

- Utiliza descenso por gradiente para actualizar el vector de pesos.

Neurona no lineal (logsig)

$$f(x) = \frac{1}{1+e^{-x}}$$



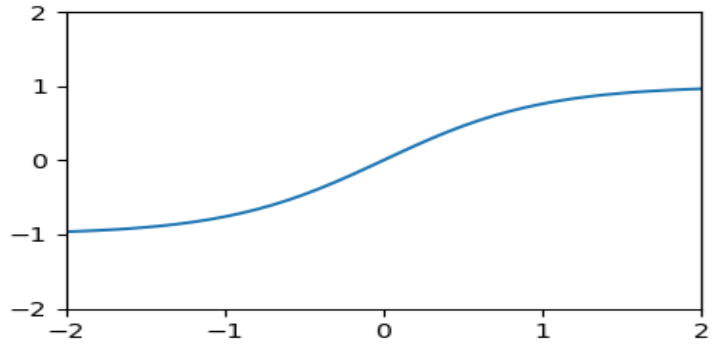


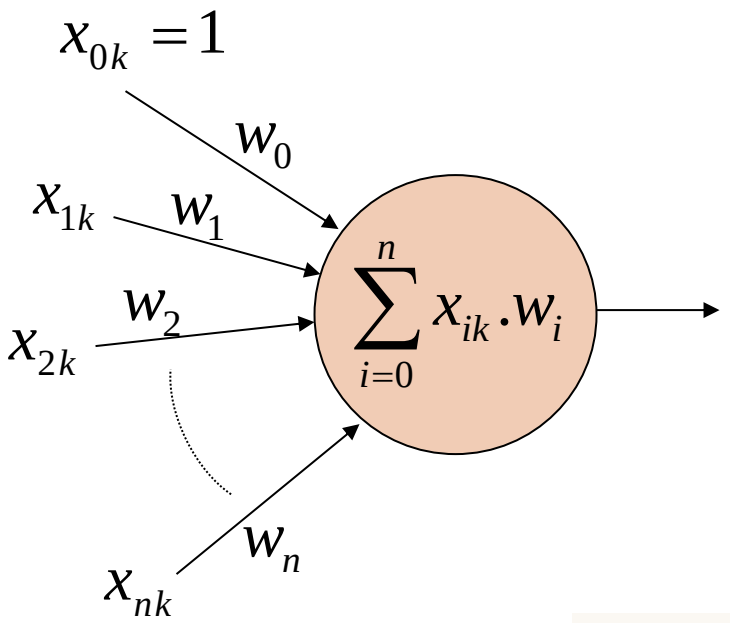
$$y_k = \frac{1}{1+e^{-\sum_{i=0}^{n} x_{ik}.w_i}}$$

- Utiliza descenso por gradiente para actualizar el vector de pesos.

Neurona no lineal (tansig)

$$f(x) = \frac{2}{1+e^{-2x}} - 1$$



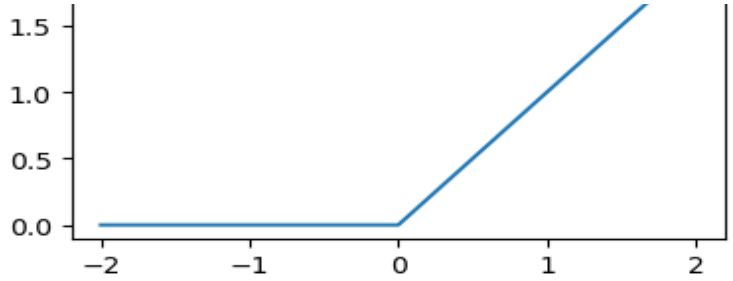


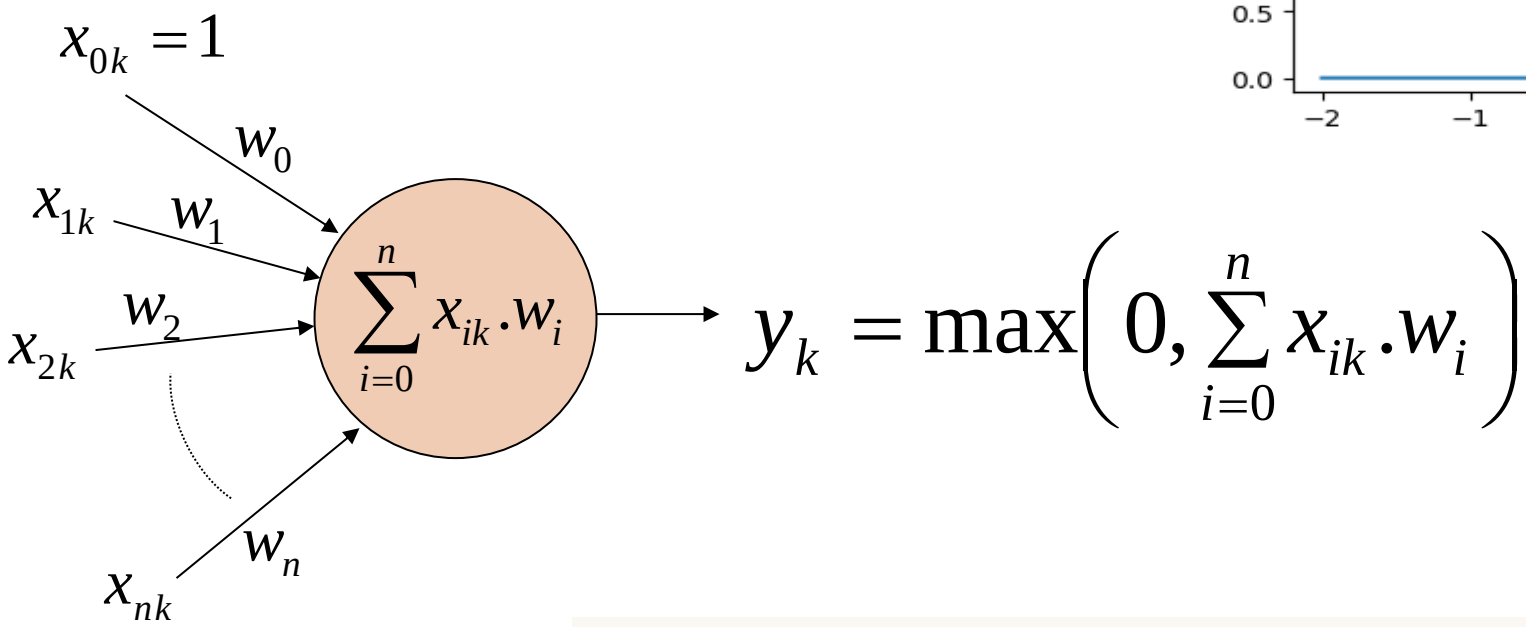
$$y_k = \frac{2}{1+e^{-2\sum_{i=0}^{n} x_{ik}.w_i}} - 1$$

- Utiliza descenso por gradiente para actualizar el vector de pesos.

Neurona no lineal (Relu)

$$f(x) = \max(0, x)$$

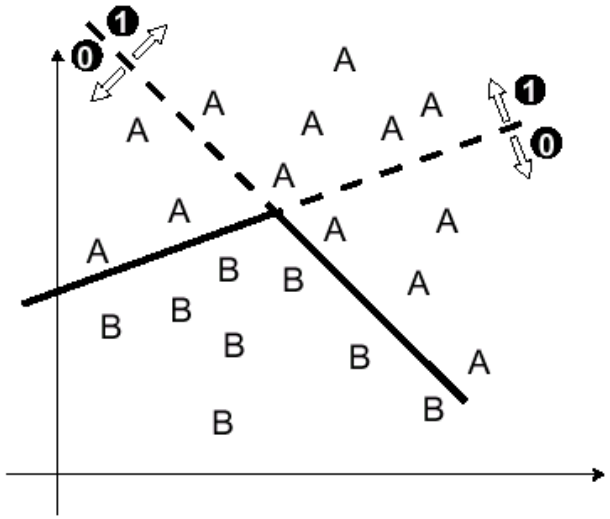


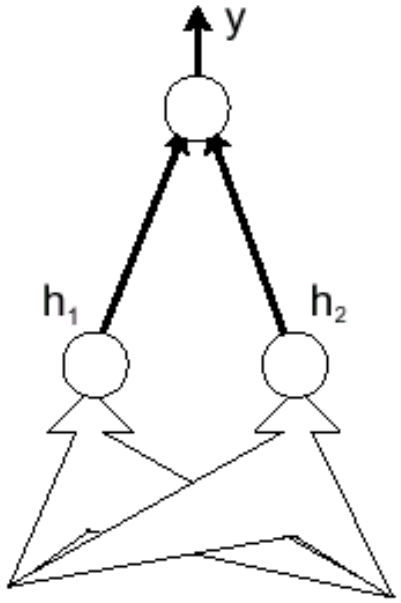


$$y_k = \max\left(0, \sum_{i=0}^{n} x_{ik}.w_i\right)$$

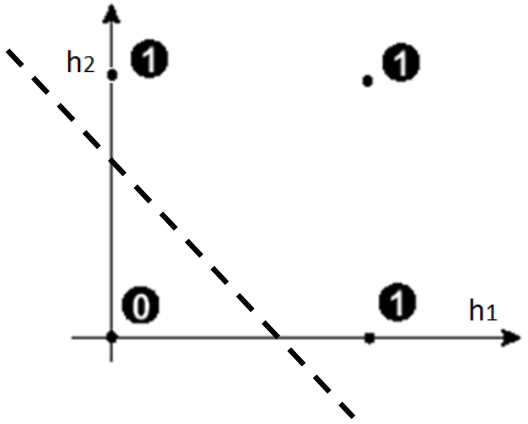
- Utiliza descenso por gradiente para actualizar el vector de pesos.

Problema no separable linealmente





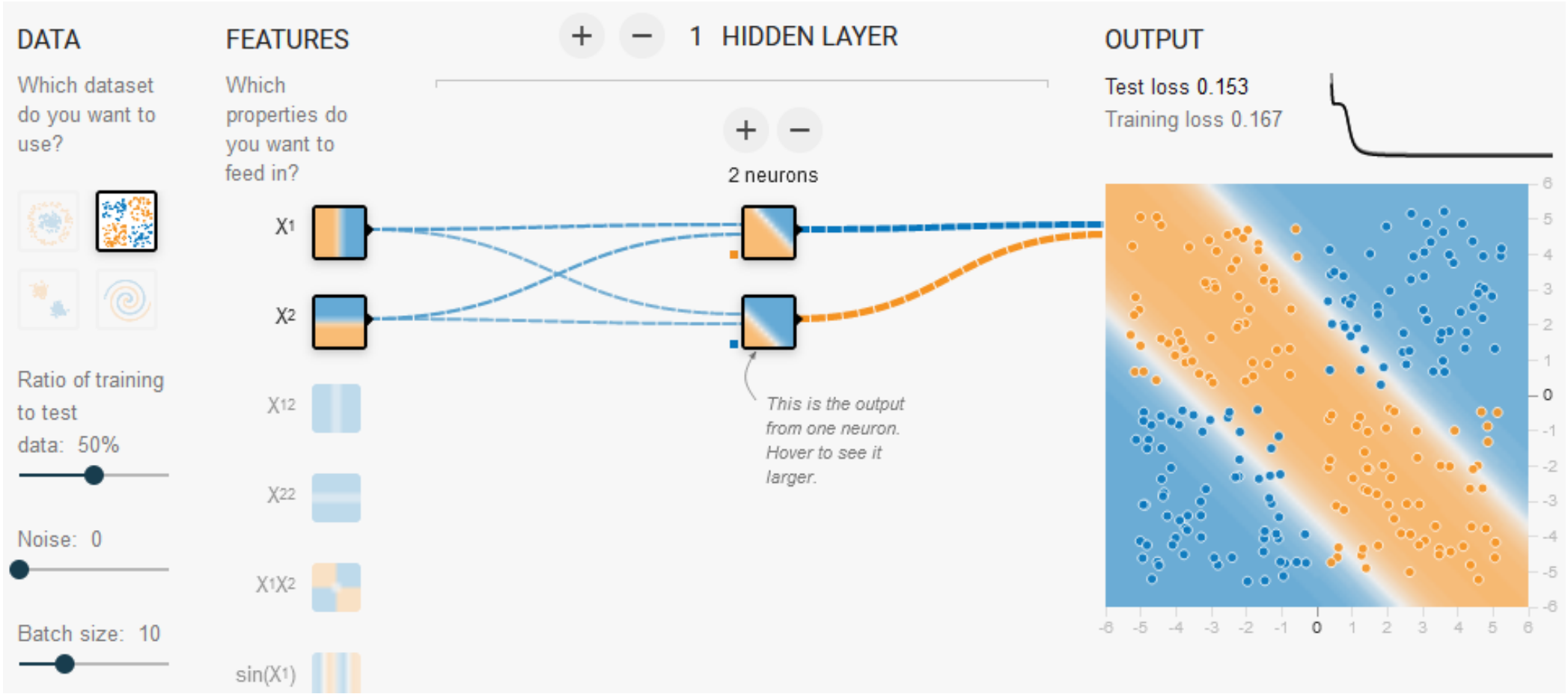
h_1	h_2	y	
0	0	0	
0	1	1	A ↔ 1
1	0	1	
1	1	1	



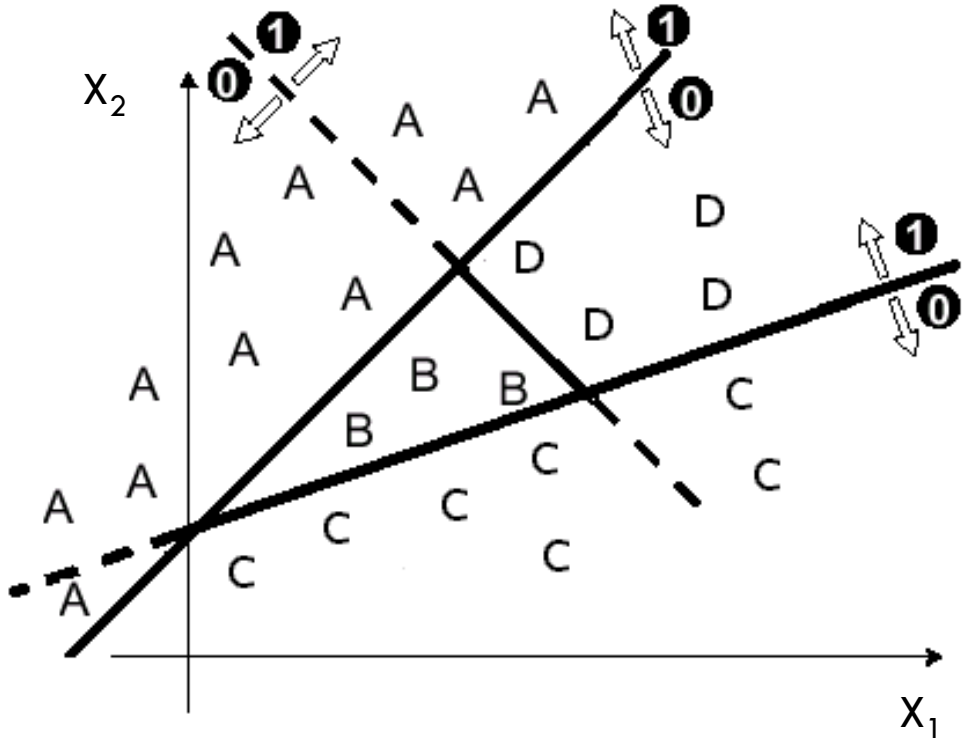
- Se busca obtener un algoritmo más general que permita integrar el aprendizaje entre las dos capas.

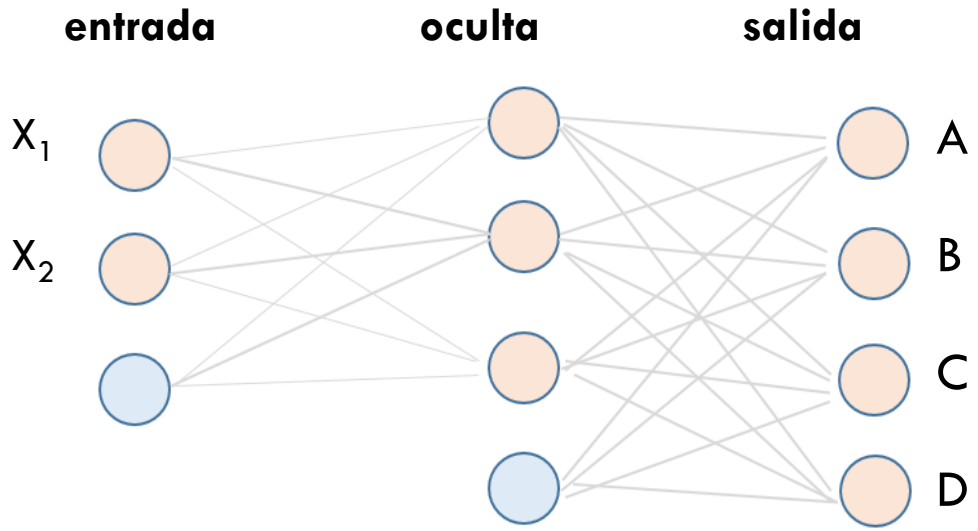
Animación de una RN

[Tinker With a Neural Network Right Here in Your Browser]



Problema no separable linealmente



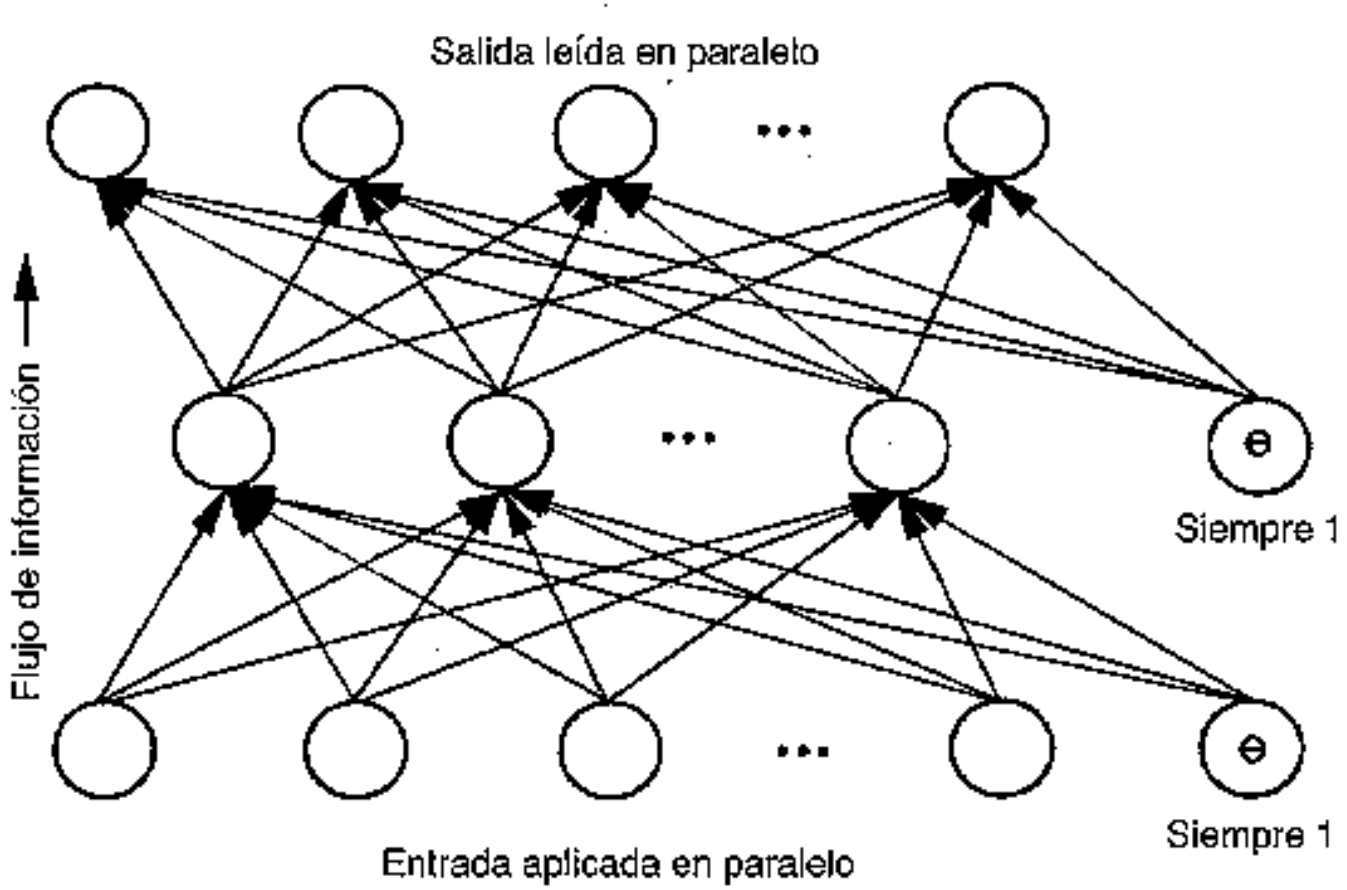


- ¿Cuál es el tamaño de cada capa?

Problema no separable linealmente

- La idea es aplicar un descenso en la dirección del gradiente sobre la superficie de error expresada como una función de los pesos.
- Deberán tenerse en cuenta los pesos de los arcos que unen AMBAS capas.
- Dado que el aprendizaje es supervisado, para los nodos de salida se conoce la respuesta esperada a cada entrada. Por lo tanto, puede aplicarse la **regla delta** vista para el Combinador Lineal y la Neurona No Lineal.

Multiperceptrón - Arquitectura



Algoritmo backpropagation

Dado el siguiente conjunto de vectores

$$\{(x_1, y_1), ..., (x_p, y_p)\}$$

que son ejemplos de correspondencia funcional

$$\mathrm{y} = \phi(\mathrm{x}) \qquad \mathrm{x} \in R^N, y \in R^M$$

se busca entrenar la red para que aprenda una aproximación

$$y' = \phi'(x)$$

Backpropagation. Capa Oculta

- Ejemplo de entrada

$$x_p = (x_{p1}, x_{p2}, \ldots, x_{pN})^t$$

- Entrada neta de la j-ésima neurona de la capa oculta

$$neta_{pj}^h = \sum_{i=1}^{n} w_{ji}^h x_{pi} + \theta_j^h$$

- Salida de la j-ésima neurona de la capa oculta

$$i_{pj} = f_j^h(neta_{pj}^h)$$

Backpropagation. Capa de Salida

- Entrada neta de la k-ésima neurona de la capa de salida

$$neta_{pk}^{o} = \sum_{j=1}^{L} w_{kj}^{o} i_{pj} + \theta_{k}^{o}$$

- Salida de la k-ésima neurona de la capa de salida

$$o_{pk} = f_{k}^{o}(neta_{pk}^{o})$$

Actualización de pesos

- Error en una sola unidad de la capa de salida

$$\delta_{pk} = (y_{pk} - o_{pk})$$

donde

- y es la salida deseada
- o es la salida real.
- p se refiere al p-ésimo vector de entrada
- k se refiere a la k-ésima unidad de salida

Actualización de pesos

- Se busca minimizar

$$E_p = \frac{1}{2}\sum_{k=1}^{M} \delta_{pk}^2$$

$$E_p = \frac{1}{2}\sum_{k=1}^{M} (y_{pk} - o_{pk})^2$$

se tomará el valor negativo del gradiente

Actualización de pesos

$$\frac{\partial E_p}{\partial w_{kj}^o} = -(y_{pk} - o_{pk}) \frac{\partial f_k^o}{\partial (neta_{pk}^o)} \frac{\partial (neta_{pk}^o)}{\partial w_{kj}^o}$$

$$f_k^{o\,'}(neta_{pk}^o)$$

$$\frac{\partial}{\partial w_{kj}^o}\left(\sum_{j=1}^{L} w_{kj}^o i_{pj} + h_k^o\right) = i_{pj}$$

Salida de la neurona oculta j

$$\frac{\partial E_p}{\partial w_{kj}^o} = -(y_{pk} - o_{pk}) f_k^{o\,'}(neta_{pk}^o) i_{pj}$$

Peso del arco que une la neurona j de la capa oculta y la neurona k de la capa de salida

Actualización de pesos

- Por lo tanto, para la capa de salida se tiene

$$\delta_{pk}^{o} = (y_{pk} - o_{pk})\, f_k^{o}{}'(neta_{pk}^{o})$$

$$w_{kj}^{o}(t+1) = w_{kj}^{o}(t) + \alpha\, \delta_{pk}^{o}\, i_{pj}$$

Actualización de pesos

- Corrección para los pesos de los arcos entre la capa de entrada y la oculta

$$\Delta_p w_{ji}^h(t) = \alpha \delta_{pj}^h x_{pi}$$

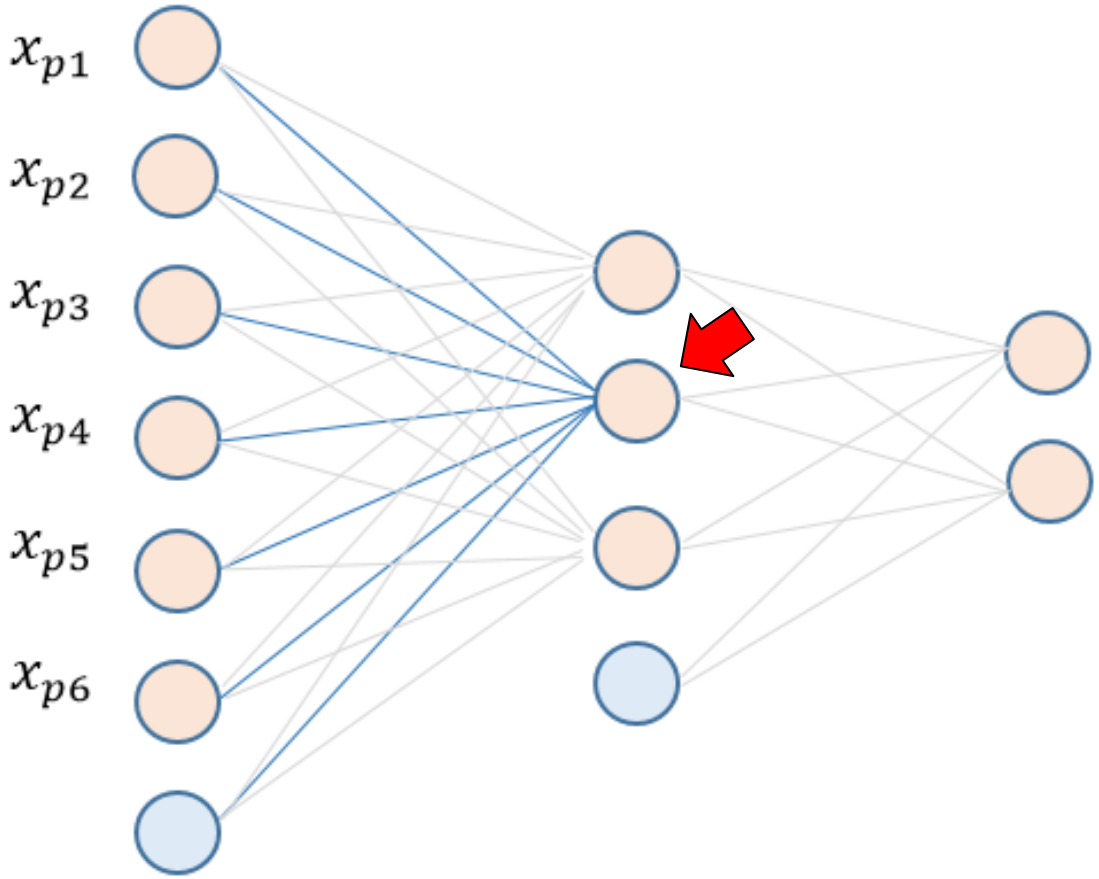
serán de la forma:

$$\delta_{pj}^h = f_j^h{}'(neta_{pj}^h) \sum_k \delta_{pk}^o w_{kj}^o$$

Backpropagation. Entrenamiento

- Aplicar un vector de entrada y calcular su salida.
- Calcular el error.
- Determinar en qué dirección (+ o -) debe cambiarse los pesos para reducir el error.
- Determinar la cantidad en que es preciso cambiar cada peso.
- Corregir los pesos de las conexiones.
- Repetir los pasos anteriores para todos los ejemplos hasta reducir el error a un valor aceptable.

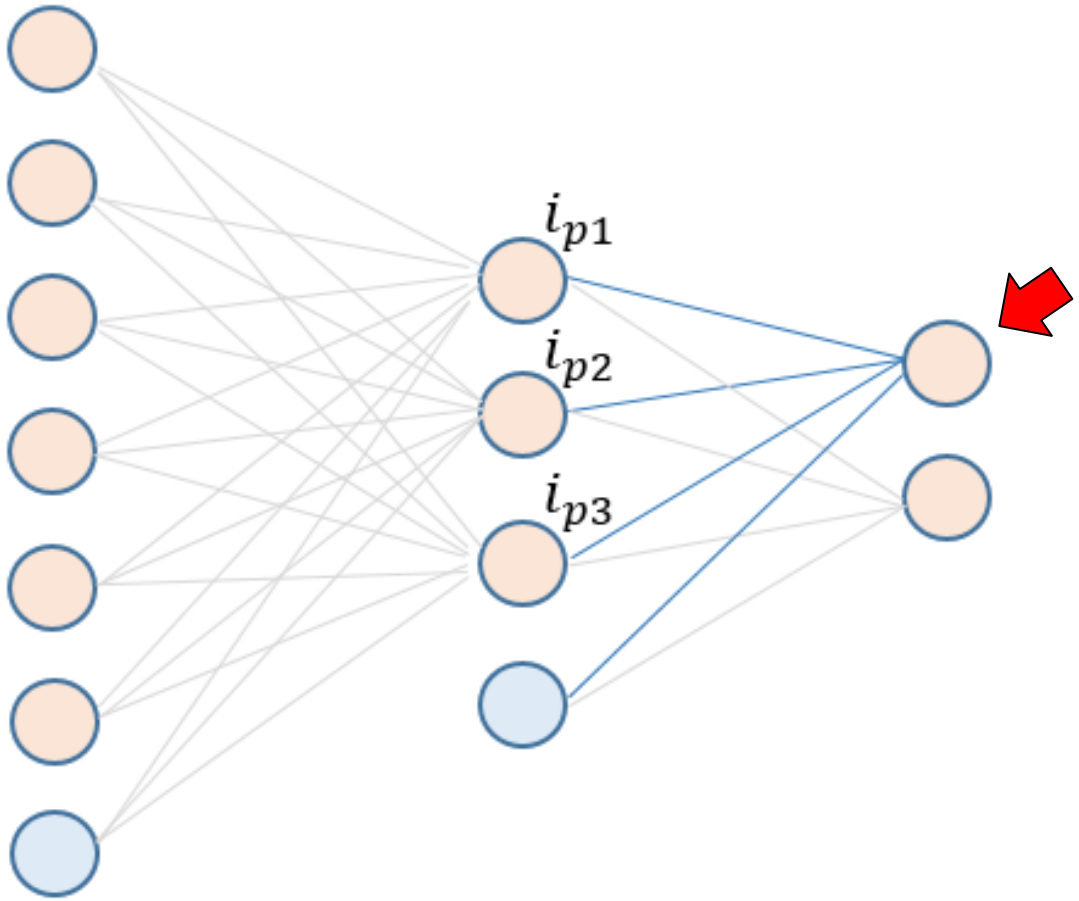
Propagar el ejemplo de entrada a través de la capa oculta



$$neta_{pj}^{h} = \sum_{i=1}^{n} w_{ji}^{h} x_{pi} + \theta_{j}^{h}$$

$$i_{pj} = f_{j}^{h}(neta_{pj}^{h})$$

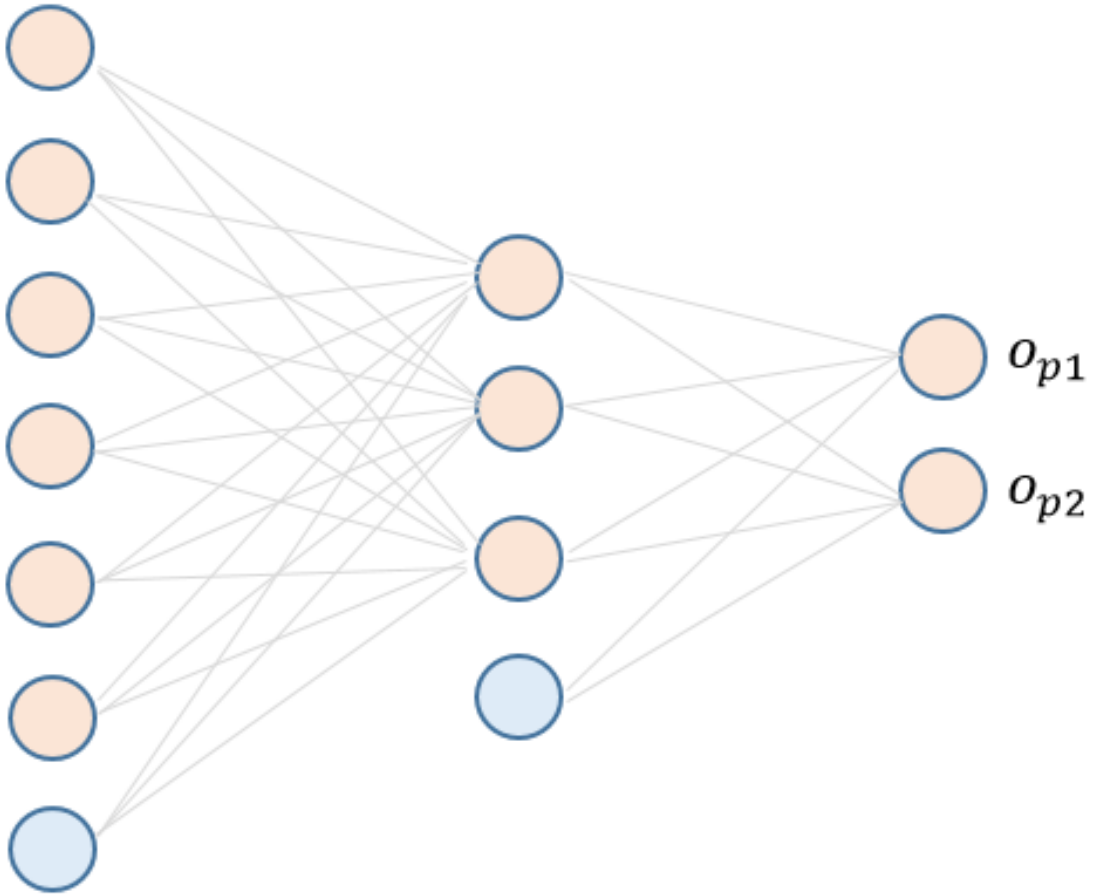
- Propagar las salidas de la capa oculta hacia la capa de salida



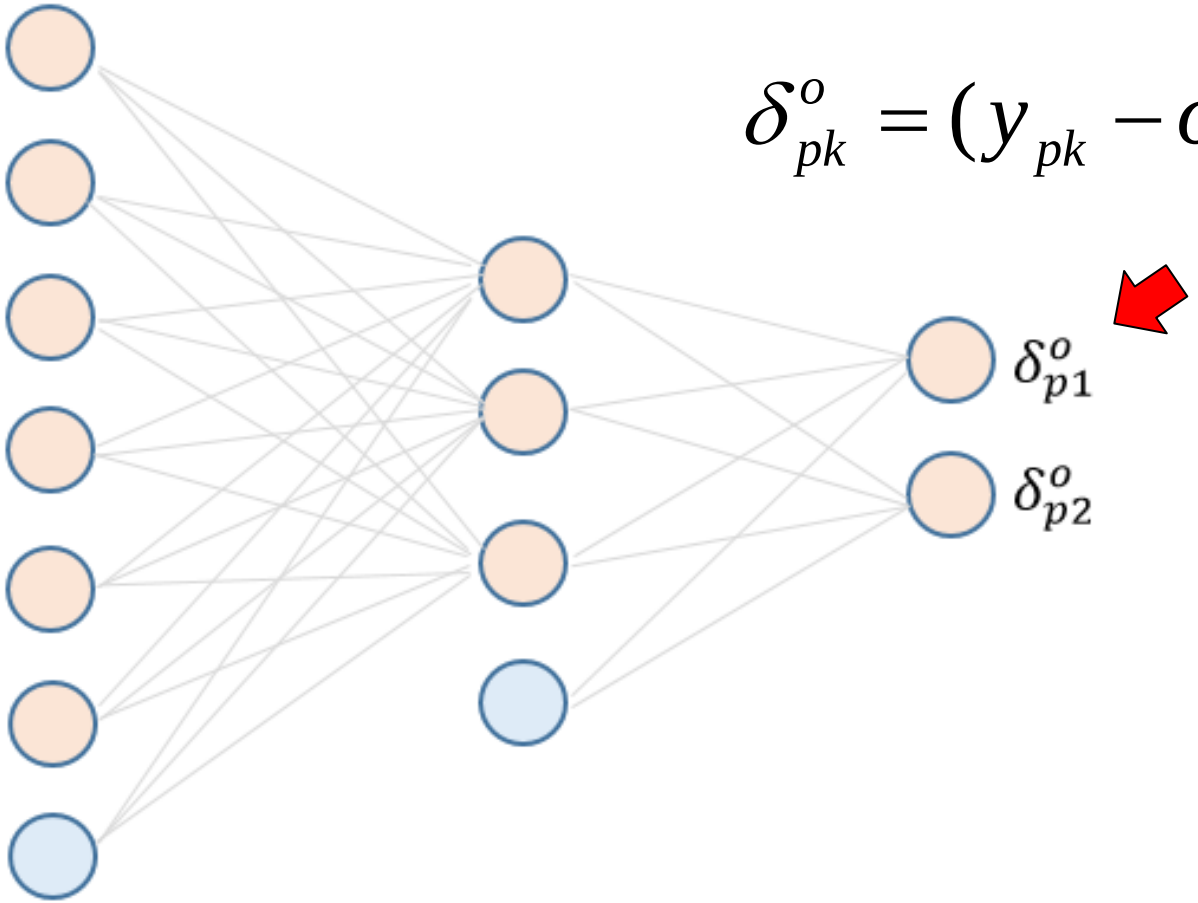
$$neta_{pk}^{o} = \sum_{j=1}^{L} w_{kj}^{o} i_{pj} + \theta_{k}^{o}$$

$$o_{pk} = f_{k}^{o}(neta_{pk}^{o})$$

- Se obtienen los valores de salida



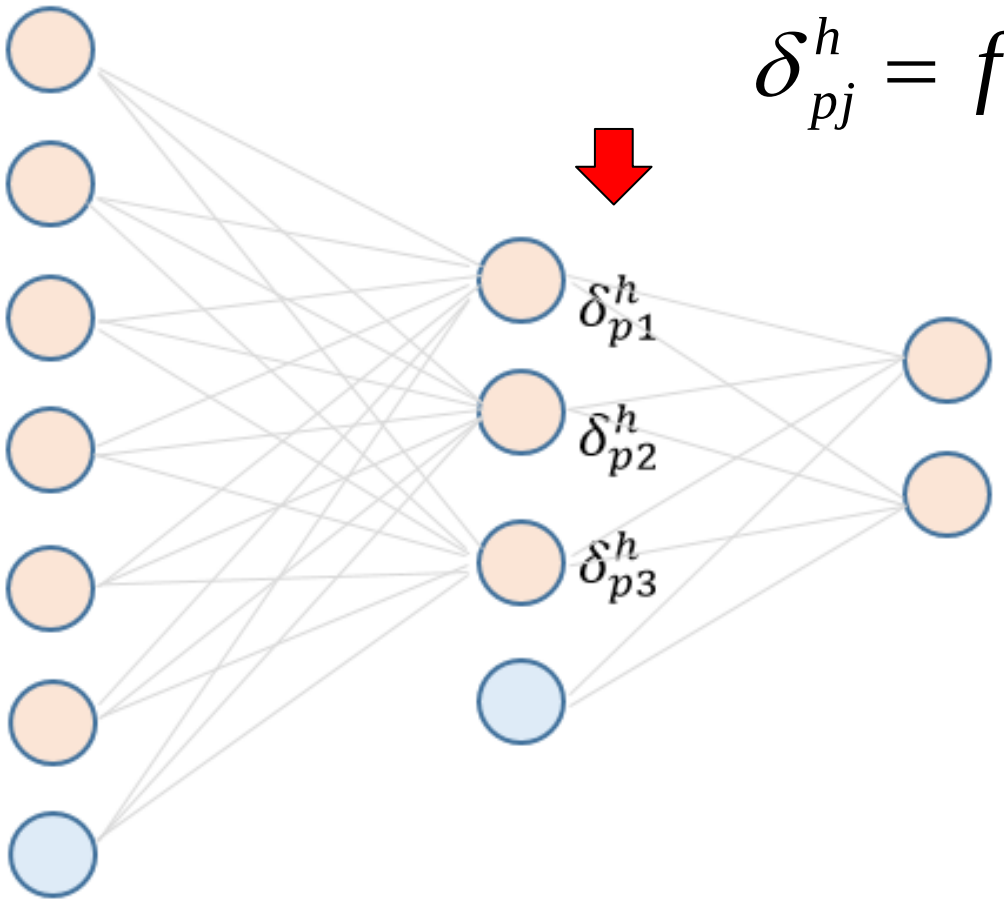
- Se calcula la corrección que se realizará al vector de pesos que llega a cada neurona de salida



$$\delta^o_{pk} = (y_{pk} - o_{pk})\, f^{o}_{k}{}'(neta^o_{pk})$$

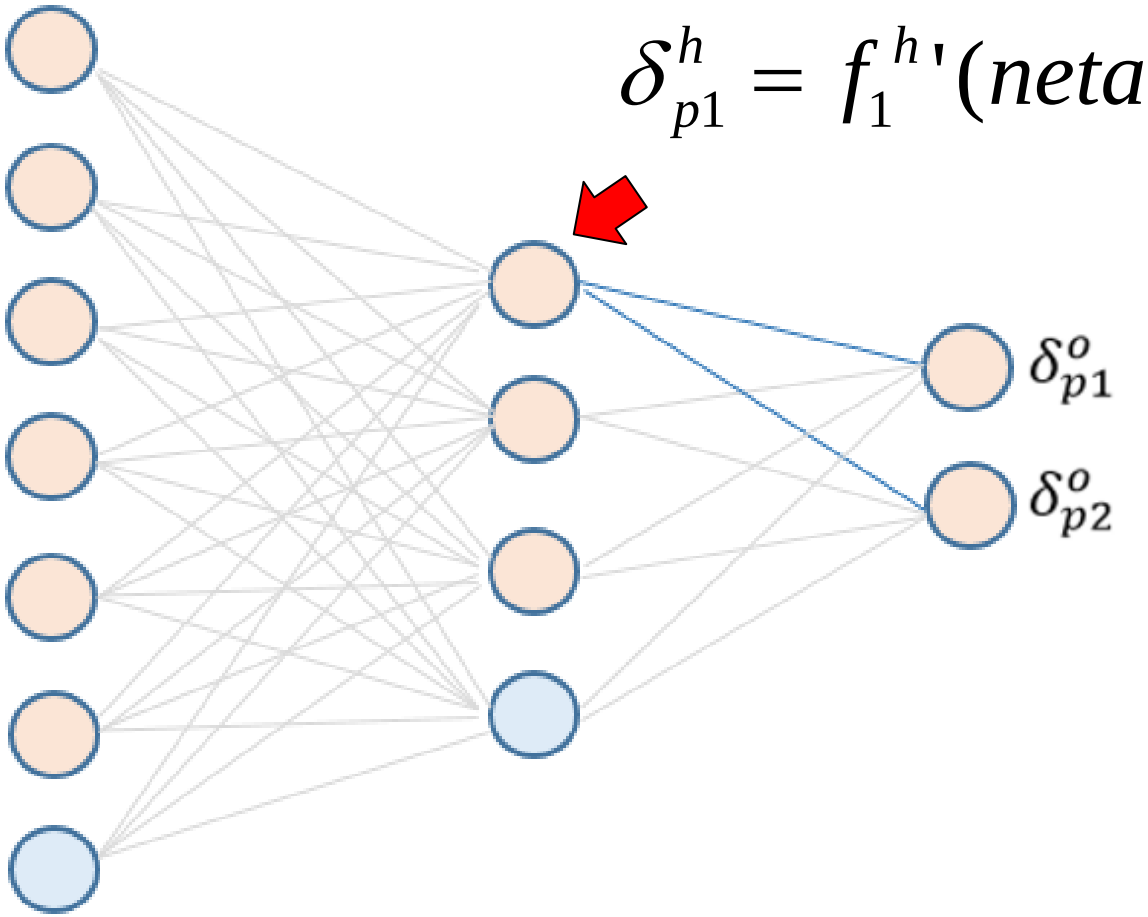
- Se calcula la corrección que se realizará al vector de pesos que llega a cada neurona oculta

$$\delta_{pj}^{h} = f_{j}^{h}{}'(neta_{pj}^{h}) \sum_{k} \delta_{pk}^{o} w_{kj}^{o}$$

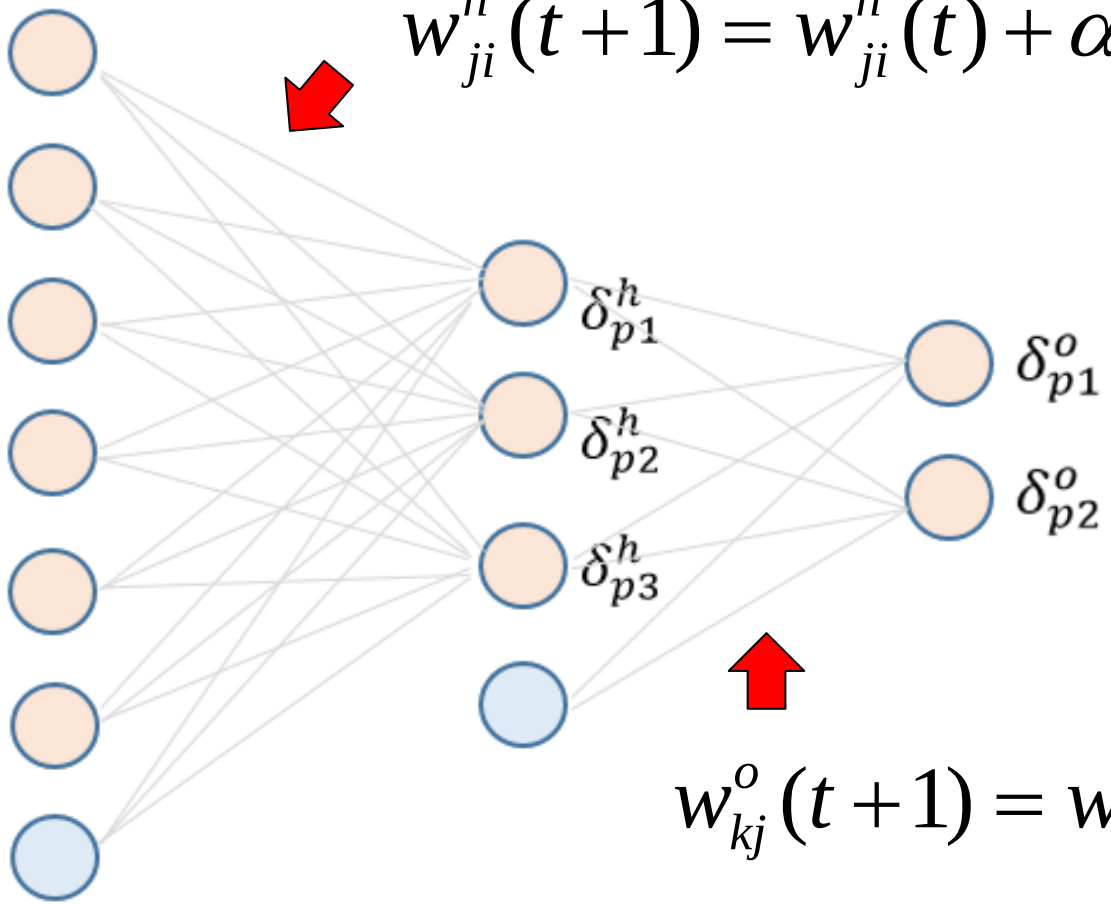


- Se calcula la corrección que se realizará al vector de pesos que llega a cada neurona de la capa oculta

$$\delta_{p1}^{h} = f_1^{h}{}'(neta_{p1}^{h}) \sum_k \delta_{pk}^{o} w_{k1}^{o}$$



- Se actualizan ambas matrices de pesos



Backpropagation. Resumen

- Aplicar el vector de entrada

$$x_p = (x_{p1}, x_{p2}, \ldots, x_{pN})^t$$

- Calcular los valores netos de las unidades de la capa oculta

$$neta_{pj}^h = \sum_{i=1}^{n} w_{ji}^h x_{pi} + \theta_j^h$$

- Calcular las salidas de la capa oculta

$$i_{pj} = f_j^h(neta_{pj}^h)$$

Backpropagation. Resumen

- Calcular los valores netos de las unidades de la capa de salida

$$neta_{pk}^{o} = \sum_{j=1}^{L} w_{kj}^{o} i_{pj} + \theta_{k}^{o}$$

- Calcular las salidas

$$o_{pk} = f_{k}^{o}(neta_{pk}^{o})$$

Backpropagation. Resumen

- Calcular los términos de error para las unidades de salida

$$\delta_{pk}^{o} = (y_{pk} - o_{pk})\, f_k^{o}{}'(neta_{pk}^{o})$$

- Calcular los términos de error para las unidades ocultas

$$\delta_{pj}^{h} = f_j^{h}{}'(neta_{pj}^{h}) \sum_k \delta_{pk}^{o} w_{kj}^{o}$$

Backpropagation. Resumen

- Se actualizan los pesos de la capa de salida

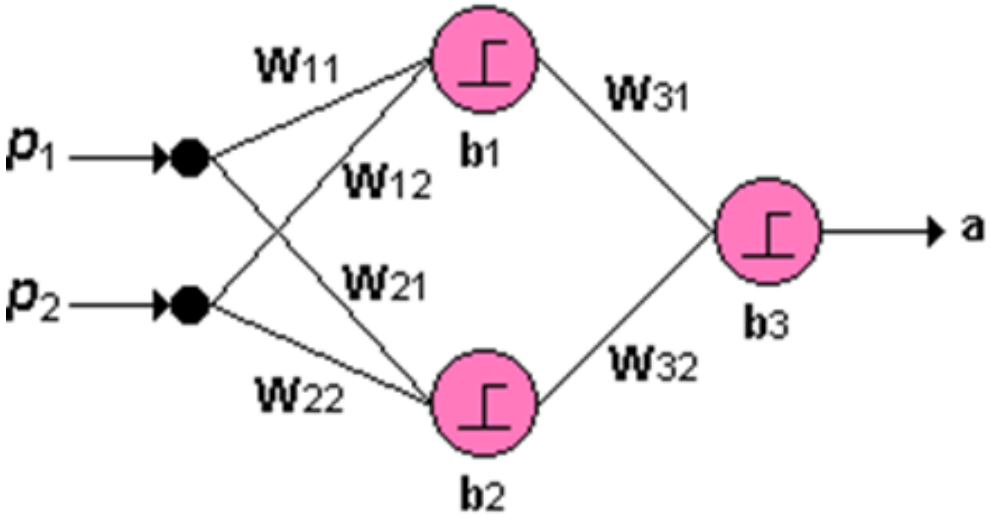
$$w_{kj}^{o}(t+1) = w_{kj}^{o}(t) + \alpha\, \delta_{pk}^{o}\, i_{pj}$$

- Se actualizan los pesos de la capa oculta

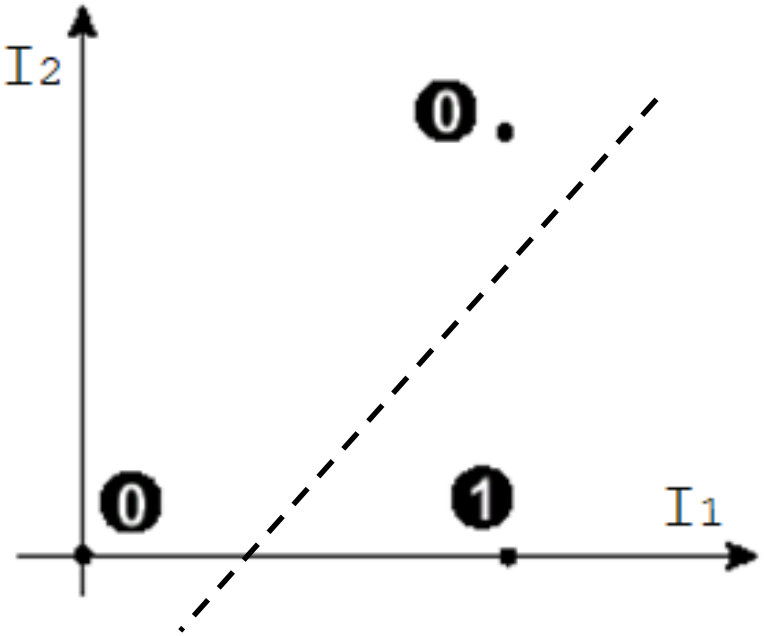
$$w_{ji}^{h}(t+1) = w_{ji}^{h}(t) + \alpha\, \delta_{pj}^{h}\, x_{i}$$

- Repetir hasta que el error resulte aceptable

XOR



p1	p2	l1 (or)	l2 (AND)	a
1	0	1	0	1
1	1	1	1	0
0	0	0	0	0
0	1	1	0	1



MLP_XOR.ipynb

Problema del XOR

MLP_XOR.ipynb


```
import numpy as np
from graficaMLP import dibuPtos_y_2Rectas
from Funciones import evaluar, evaluarDerivada

X = np.array([ [-1, -1], [-1, 1], [1, -1], [1, 1]])
Y = np.array([-1, 1, 1, -1]).reshape(-1,1)

entradas = X.shape[1]
ocultas = 2
salidas = Y.shape[1]
```

```
In [13]: X
Out[13]:
array([[-1, -1],
       [-1,  1],
       [ 1, -1],
       [ 1,  1]])
```

```
In [14]: Y
Out[14]:
array([[ 1],
       [-1],
       [-1],
       [ 1]])
```

Pesos iniciales

MLP_XOR.ipynb

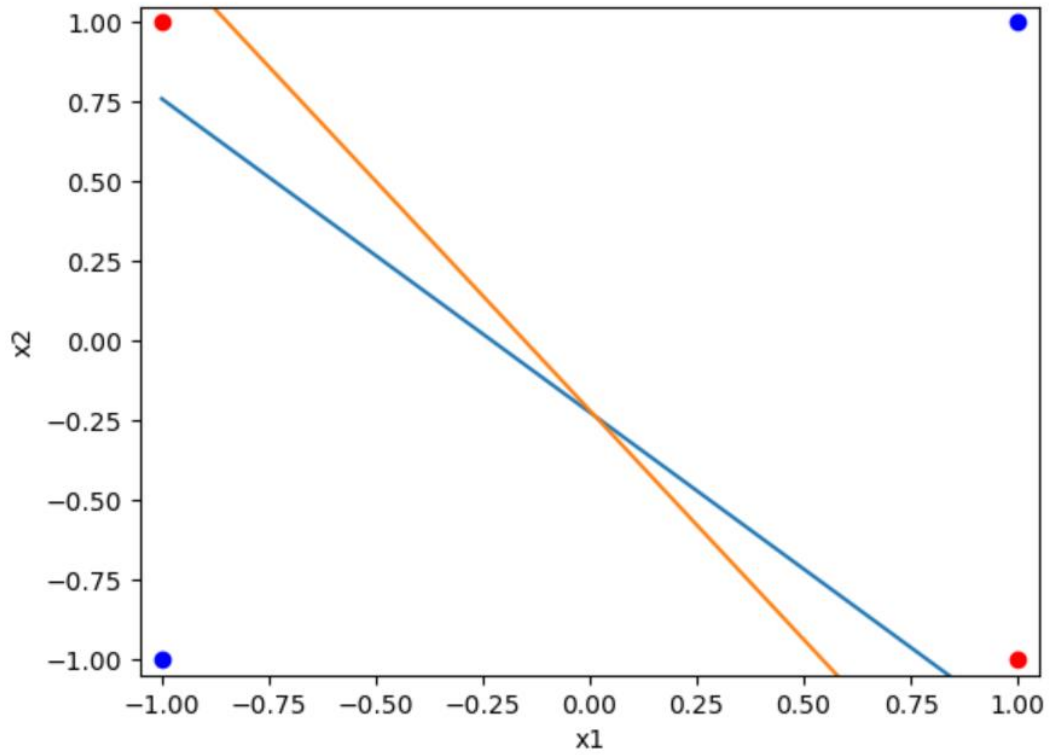

```
W1 = np.random.uniform(-0.5,0.5,[ocultas, entradas])
b1 = np.random.uniform(-0.5,0.5, [ocultas,1])
W2 = np.random.uniform(-0.5,0.5,[salidas, ocultas])
b2 = np.random.uniform(-0.5,0.5, [salidas,1])
```

```
In [17]: W1
Out[17]:
array([[-0.09705477, -0.48156505],
       [ 0.14081924,  0.17185576]])

In [18]: b1
Out[18]:
array([[ 0.28208473],
       [ 0.07973888]])

In [19]: W2
Out[19]: array([[-0.48098277,  0.45515249]])

In [20]: b2
Out[20]: array([[ 0.15708682]])
```

Graficar W1 y b1

MLP_XOR.ipynb


```
ph = dibuPtos_y_2Rectas(X,Y, W1, b1, ph)
```



```
alfa = 0.15
CotaError = 0.001
MAX_ITERA = 300
ite = 0
while ( abs(ErrorAVG-ErrorAnt)>Cota) and ( ite < MAX_ITERA ):
    for p in range(len(P)):   #para cada ejemplo
        # propagar el ejemplo hacia adelante
        # calcular los errores en ambas capas
        # corregir los todos los pesos

    # Recalcular AVGError
    ite = ite + 1
    print(ite, AVGError)
    # Graficar las rectas
```

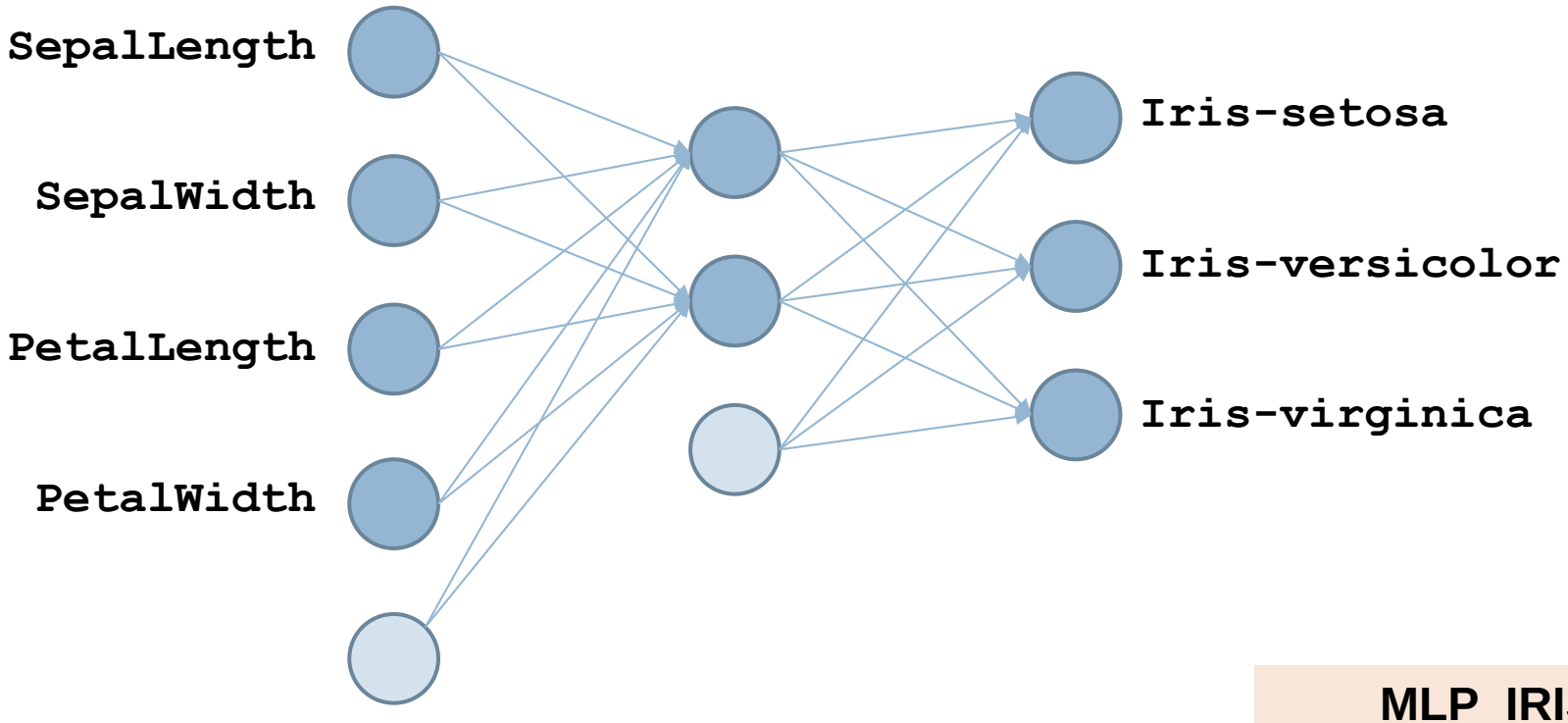
Ver
MLP_XOR.ipynb

Ejemplo: Clasificación de flores de Iris

Id	sepallength	sepalwidth	petallength	petalwidth	class
1	5,1	3,5	1,4	0,2	Iris-setosa
2	4,9	3,0	1,4	0,2	Iris-setosa
…	…	…	…	…	…
95	5,6	2,7	4,2	1,3	Iris-versicolor
96	5,7	3,0	4,2	1,2	Iris-versicolor
97	5,7	2,9	4,2	1,3	Iris-versicolor
…	…	…	…	…	…
149	6,2	3,4	5,4	2,3	Iris-virginica
150	5,9	3,0	5,1	1,8	Iris-virginica

https://archive.ics.uci.edu/ml/datasets/Iris

Ejemplo: Clasificación de flores de Iris



MLP_IRIS.ipynb

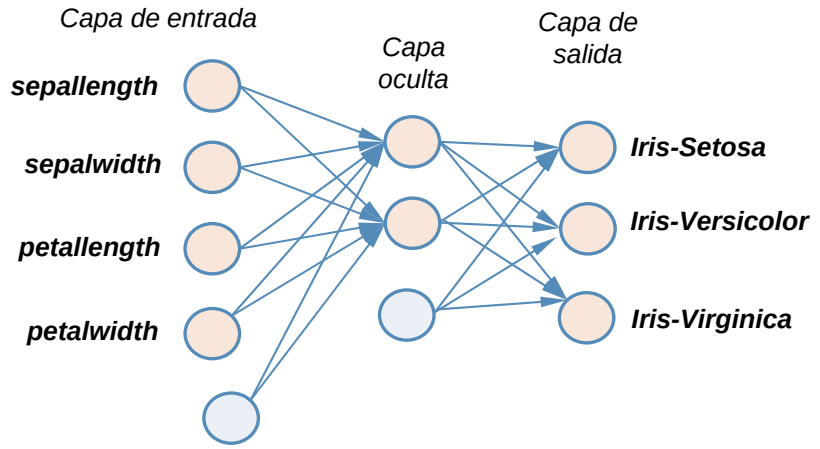
Clasificación de flores de Iris

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



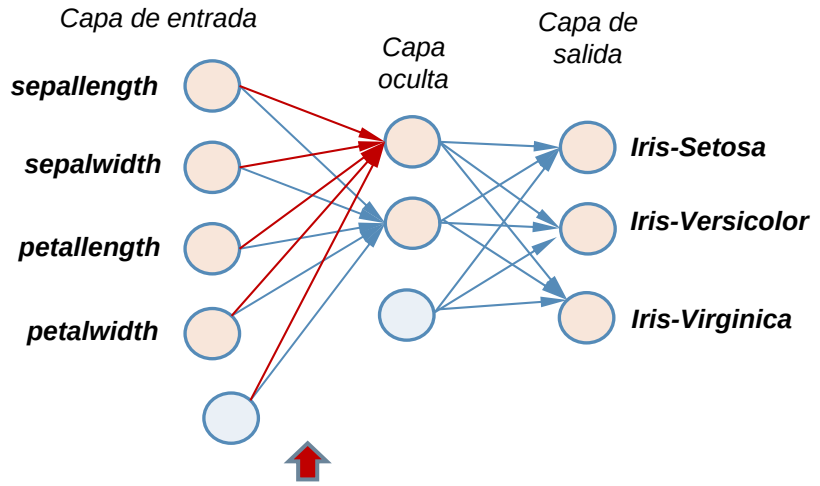
Clasificación de flores de Iris

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```




```
[[ 0.15, -0.13,  0.23, -0.45],
 [-0.29, -0.41, -0.19,  0.37]]
```

W1

```
[[-0.45],
 [-0.10]]
```

b1

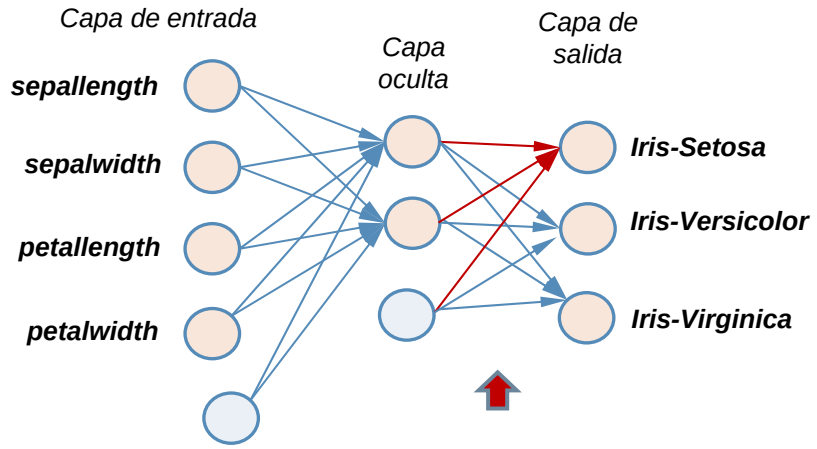
Clasificación de flores de Iris

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```




```
[[-4.79, -0.99],
 [ 0.38,  0.09],
 [ 2.40, -0.38]]
```

W2

```
[[-0.29],
 [-0.17],
 [-0.27]]
```

b2

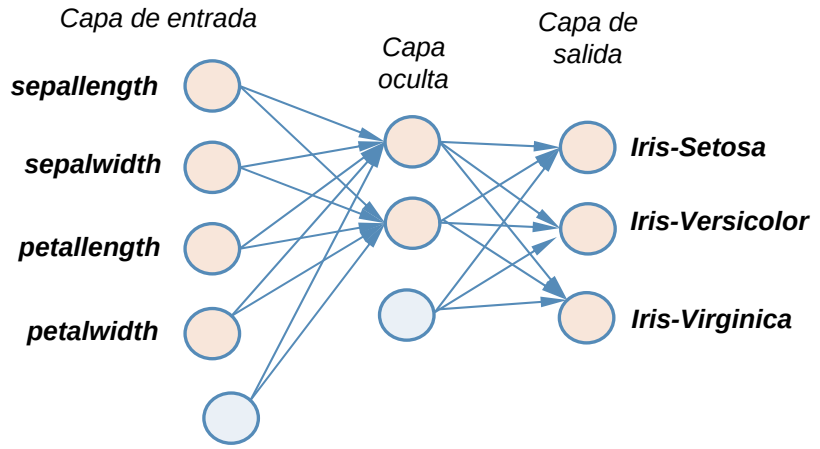
Clasificación de flores de Iris

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```




```
FunH='tanh' ; FunO='sigmoid'
```

Ingresar el primer ejemplo a la red y calcular su salida

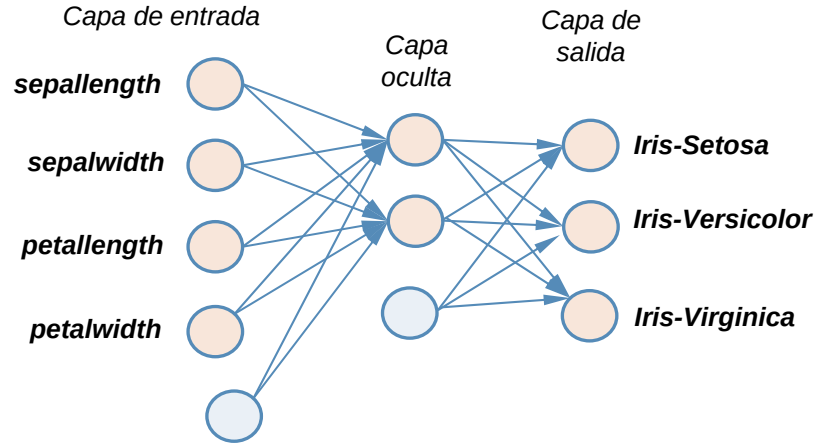
Calculando la salida de la capa oculta

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- Salida de la capa oculta

```
netasH = W1 * x.T + b1
```

FunH='tanh' ; FunO='sigmoid'

x^T

```
[[ 0.15, -0.13,  0.23, -0.45],
 [-0.29, -0.41, -0.19,  0.37]]
```
W1

*

```
[[-1.73],
 [-0.05],
 [-1.38],
 [-1.31]]
```

\+

```
[[-0.45],
 [-0.10]]
```
b1

=

```
[[-0.4309]
 [ 0.1997]]
```

$$neta_{pj}^{h} = \sum_{i=1}^{n} w_{ji}^{h} x_{pi} + \theta_{j}^{h}$$

$$i_{pj} = f_{j}^{h}(neta_{pj}^{h})$$

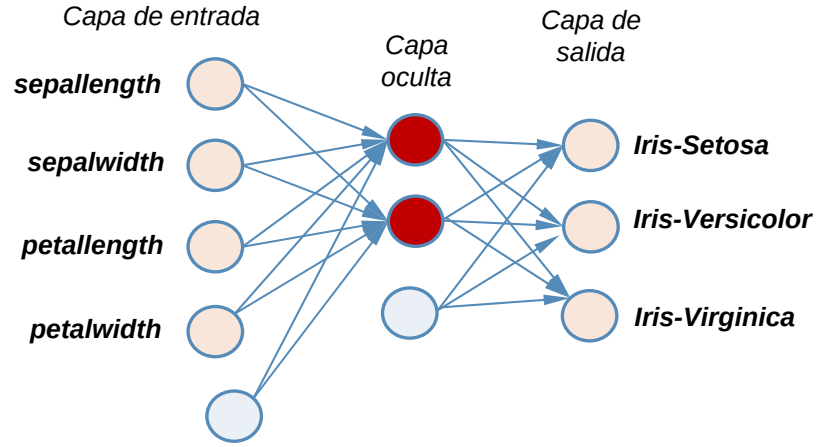
Calculando la salida de la capa oculta

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- Salida de la capa oculta

```
netasH = W1 * x.T + b1
```

FunH='tanh' ; FunO='sigmoid'

W1

```
[[ 0.15, -0.13,  0.23, -0.45],
 [-0.29, -0.41, -0.19,  0.37]]
```

*

x^T

```
[[-1.73],
 [-0.05],
 [-1.38],
 [-1.31]]
```

\+

b1

```
[[-0.45],
 [-0.10]]
```

=

```
[[-0.4309]
 [ 0.1997]]
```

$$neta_{pj}^{h} = \sum_{i=1}^{n} w_{ji}^{h} x_{pi} + \theta_{j}^{h}$$

```
salidasH = 2 / (1+np.exp(-netasH))- 1  =
```

```
[[-0.21217712]
 [ 0.09951948]]
```

$$i_{pj} = f_{j}^{h}(neta_{pj}^{h})$$

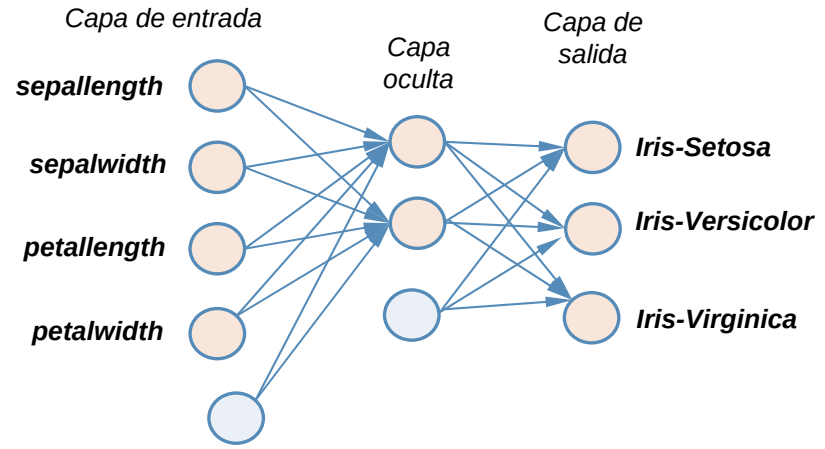
Calculando la salida de la red (capa de salida)

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- Salida de red

```
netasO = W2 * salidasH + b2
```

FunH='tanh' ; FunO='sigmoid'

salidasH

netasO

```
[[-4.79, -0.99],
 [ 0.38,  0.09],
 [ 2.40, -0.38]]
```

W2

*

```
[[-0.21217712]
 [ 0.09951948]]
```

+

```
[[-0.29],
 [-0.17],
 [-0.27]]
```

b2

=

```
[[ 0.62780411]
 [-0.24167055]
 [-0.81704249]]
```

$$neta_{pk}^{o} = \sum_{j=1}^{L} w_{kj}^{o} i_{pj} + \theta_{k}^{o}$$

$$o_{pk} = f_{k}^{o}(neta_{pk}^{o})$$

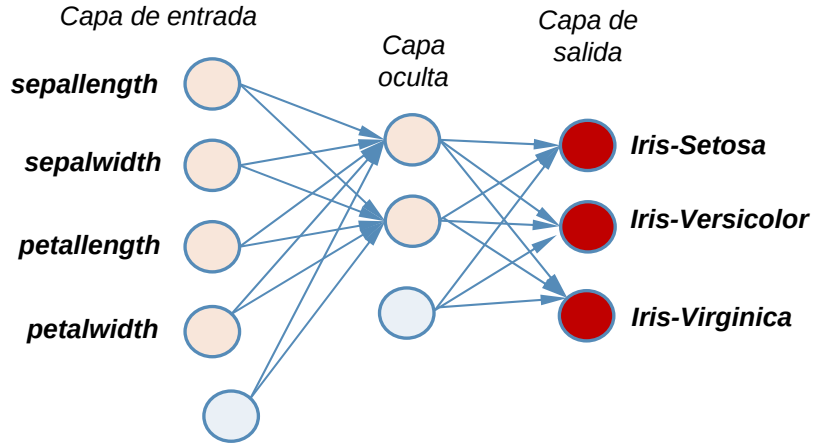
Calculando la salida de la red (capa de salida)

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- Salida de red

```
netasO = W2 * salidasH + b2
```

FunH='tanh' ; FunO='sigmoid'

salidasH

netasO

```
[[-4.79, -0.99],
 [ 0.38,  0.09],
 [ 2.40, -0.38]]
```

W2

*

```
[[-0.21217712]
 [ 0.09951948]]
```

+

```
[[-0.29],
 [-0.17],
 [-0.27]]
```

b2

=

```
[[ 0.62780411]
 [-0.24167055]
 [-0.81704249]]
```

$$neta^o_{pk} = \sum_{j=1}^{L} w^o_{kj} i_{pj} + \theta^o_k$$

$$o_{pk} = f^o_k(neta^o_{pk})$$

```
salidasO = 1 / (1+np.exp(-netasO))  =
```

```
[[0.65199139]
 [0.43987471]
 [0.30639182]]
```

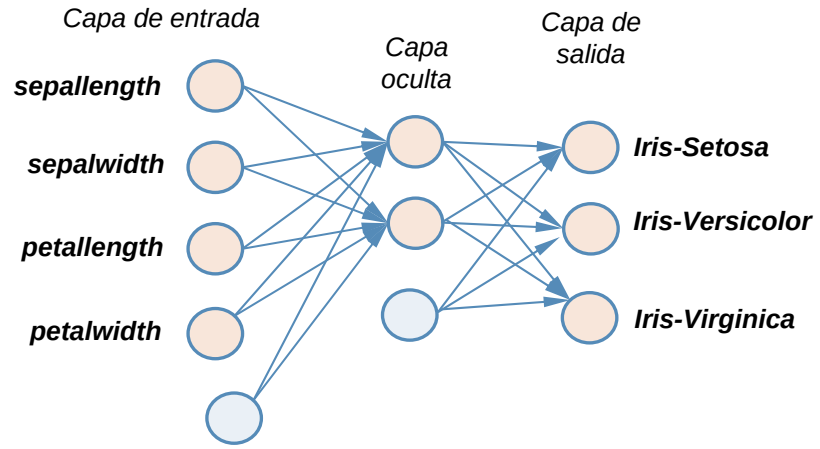
Error de la capa de salida

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- Error en la respuesta de la red para este ejemplo

```
ErrorSalida =  y.T - salidasO
```

```
ErrorSalida =  [[1.0]      [[0.65199139]      [[ 0.34800861]
                [0.0]   -   [0.43987471]   =   [-0.43987471]
                [0.0]]      [0.30639182]]      [-0.30639182]]
```

salidasO

```
FunH='tanh' ; FunO='sigmoid'
```

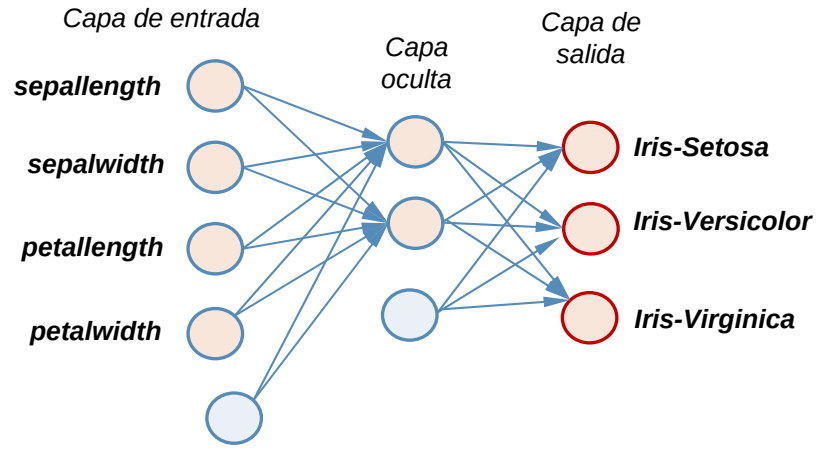
Factores de corrección de los pesos

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- **Factores para corregir W2 y b2**

`salidasO*(1-salidasO)`

```
deltaO =  ErrorSalida   .*  derivada_FunO

                [[ 0.34800861]        [[0.22689862]        [[ 0.07896267]
deltaO =         [-0.43987471]   .*    [0.24638495]    =    [-0.10837851]
                 [-0.30639182]]        [0.21251587]]        [-0.06511312]]
```

FunH='tanh' ; FunO='sigmoid'

$$\delta^{o}_{pk} = (y_{pk} - o_{pk})\, f_k^{o}{}'(neta^{o}_{pk})$$

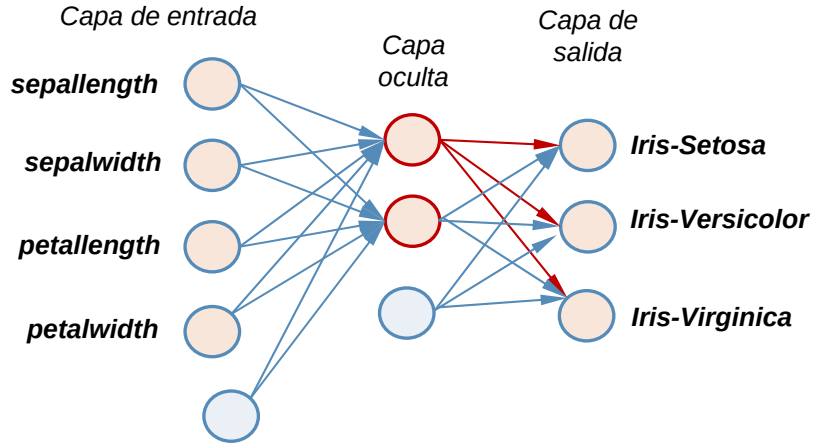
Factores de corrección de los pesos

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- **Factores para corregir W1 y b1**

`(1-salidasH**2)`

```
deltaH = deriv_FunH .* (W2.T @ deltaO)
```

```
deltaH =  [[0.95498087]
           [0.99009587]]  .*  [[-4.79  0.38  2.4 ]
                               [-0.99  0.09 -0.38]]  @  [[ 0.07896267]
                                                         [-0.10837851]
                                                         [-0.06511312]]
```

```
deltaH =  [[-0.54976963]
           [-0.06255834]]
```

FunH='tanh' ; FunO='sigmoid'

$$\delta_{pj}^{h} = f_{j}^{h}{}'(neta_{pj}^{h}) \sum_{k} \delta_{pk}^{o} w_{kj}^{o}$$

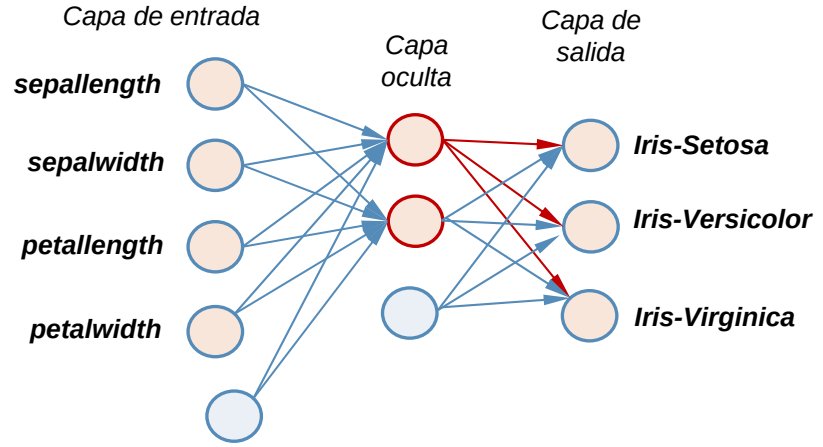
Factores de corrección de los pesos

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- **Factores para corregir W1 y b1**

```
(1-salidasH**2)
```

```
deltaH = deriv_FunH .* (W2.T @ deltaO)
```

```
deltaH =  [[0.95498087]      .*  [[-4.79  0.38  2.4 ]   @   [[ 0.07896267]
           [0.99009587]]           [-0.99  0.09 -0.38]]       [-0.10837851]
                                                              [-0.06511312]]
```

```
deltaH =  [[-0.54976963]
           [-0.06255834]]
```

FunH='tanh' ; FunO='sigmoid'

*Note que las derivadas de ambas funciones de activación sigmoides (**derivada_FunO** y **deriv_FunH**) son siempre positivas y actúan como factor de escala*

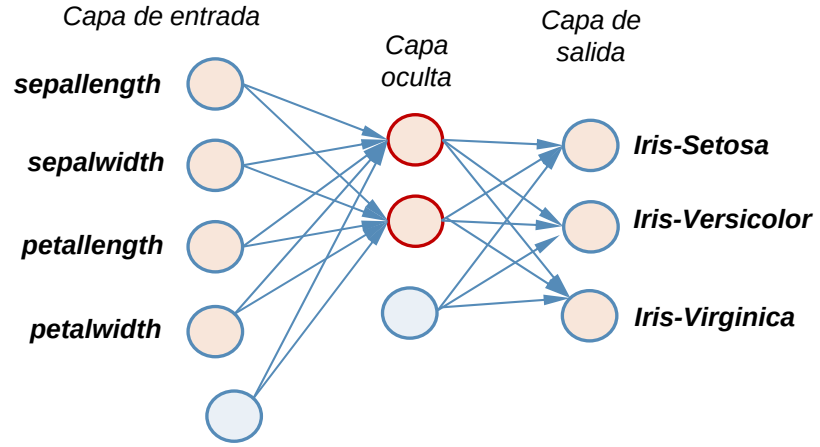
Corrigiendo de los pesos

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- **Modificación de W2 y b2**

```
FunH='tanh' ; FunO='sigmoid'
```

```
W2 = W2 + alfa * deltaO @ salidasH.T
```

```
W2 = [[-4.79, -0.99],
      [ 0.38,  0.09],   + alfa * ( [[ 0.07896267]
      [ 2.40, -0.38]]              [-0.10837851]  @ [[-0.212177],[0.099519]] )
                                   [-0.06511312]]

   = [[-4.79, -0.99]
      [ 0.38,  0.09]
      [ 2.4 , -0.38]]
```

```
b2 = b2 + alfa * deltaO = [[-0.29],
                           [-0.17],   + alfa * [[ 0.07896267]
                           [-0.27]]            [-0.10837851]
                                               [-0.06511312]]

                        = [[-0.28],
                           [-0.18],
                           [-0.28]]
```

Corrigiendo de los pesos

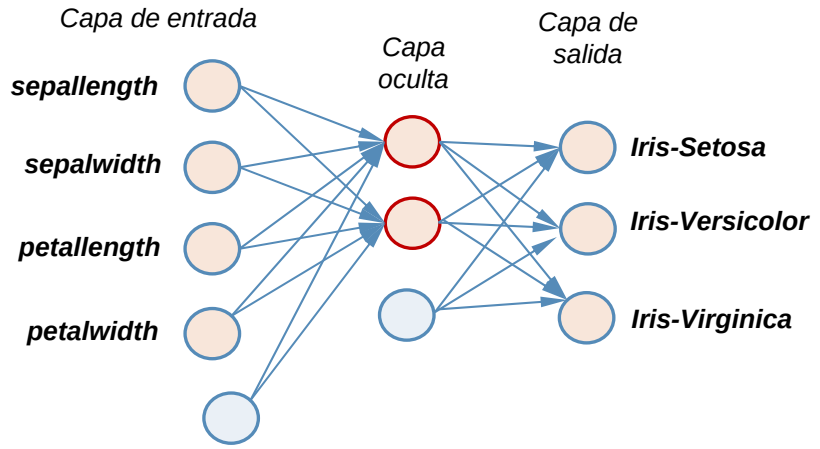
MLP_IRIS_algBPN.ipynb

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- **Modificación de W1 y b1**

```
W1 = W1 + alfa * deltaH @ x
```

```
FunH='tanh' ; FunO='sigmoid'
```

```
W1 = [[ 0.15,-0.13,0.23,-0.45],
      [-0.29,-0.41,-0.19,0.37]] + alfa * [[-0.54976963]  @  [[-1.73,-0.05,-1.38,-1.31]
                                          [-0.06255834]]
```

```
W1 = [[ 0.25 -0.13  0.31 -0.38]
      [-0.28 -0.41 -0.18  0.38]]
```

```
b1 = b1 + alfa * deltaH = [[-0.45],   + alfa * [[-0.54976963]   =  [[-0.5],
                           [-0.10]]             [-0.06255834]]     [-0.11]]
```

Si se ingresa el mismo ejemplo luego de modificar los pesos de la red …

```
1  netasH = W1 @ xi.T + b1
2  salidasH = 2.0/(1+np.exp(-netasH))-1
3  netasO =  W2 @ salidasH + b2
4  salidasO = 1.0/(1+np.exp(-netasO))
5  print("salidaO = \n", salidasO)
6  ErrorSalidaNew = yi.T-salidasO
7  print("ErrorSalida = \n", ErrorSalidaNew)
```

```
salidaO =
 [[0.82694546]
 [0.41887838]
 [0.21955387]]
ErrorSalida =
 [[ 0.17305454]
 [-0.41887838]
 [-0.21955387]]
```

```
1  print("Error inicial = ", np.sum(ErrorSalida**2))
2  print("Error luego de la correccion = ", np.sum(ErrorSalidaNew**2))
```

```
Error inicial =  0.40847570307169573
Error luego de la correccion =  0.25361086753371476
```

Antes de modificar los pesos de la red

```
salidaO =
 [[0.65199139]
 [0.43987471]
 [0.30639182]]
ErrorSalida =
 [[ 0.34800861]
 [-0.43987471]
 [-0.30639182]]
```

Ver
MLP_IRIS.ipynb

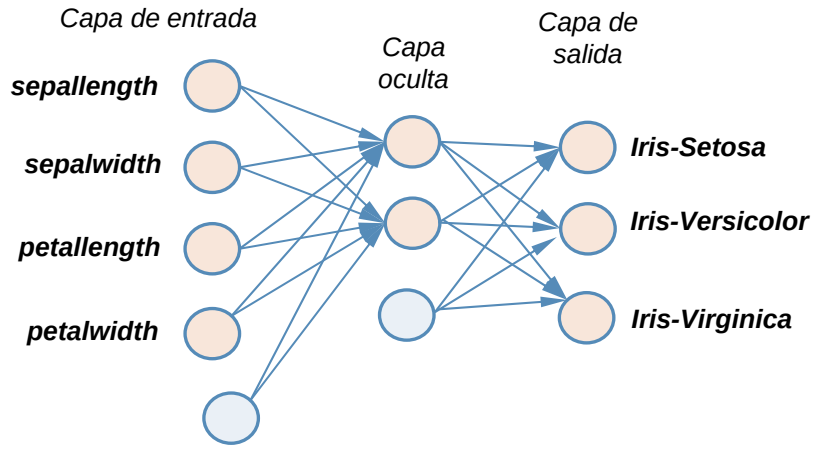
Clasificación de flores de Iris

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



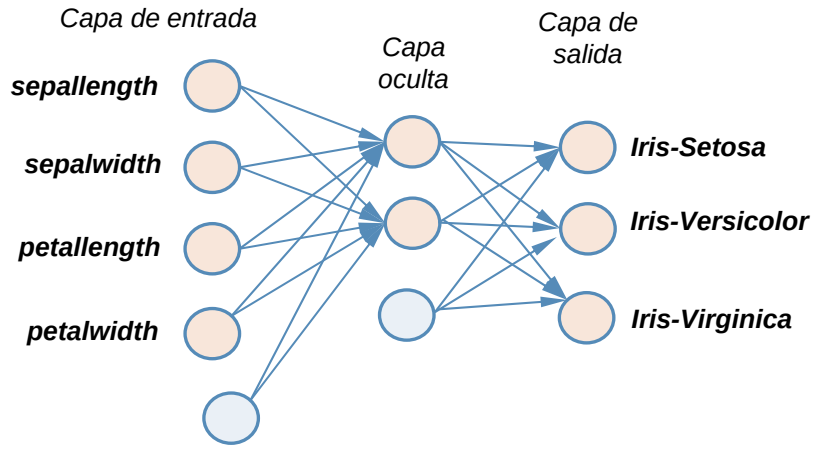
Clasificación de flores de Iris

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```




```
FunH='tanh' ; FunO= 'softmax'
```

Ingresar el primer ejemplo a la red y calcular su salida

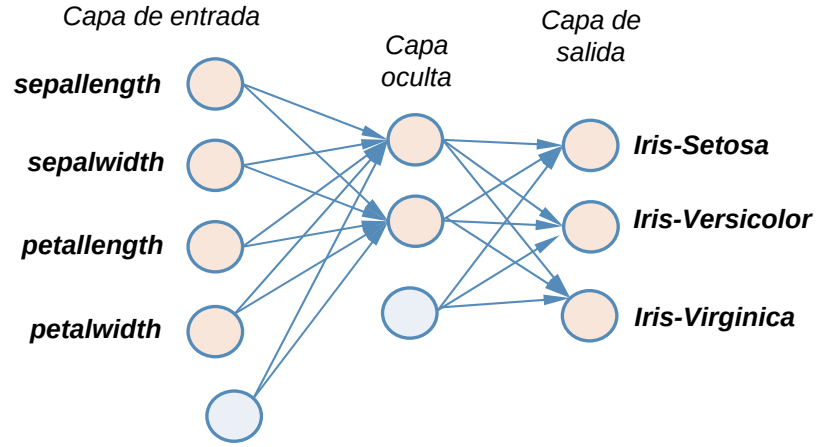
Calculando la salida de la capa oculta

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- Salida de la capa oculta

$$netasH = W1 * x^T + b1$$

W1

```
[[ 0.15, -0.13,  0.23, -0.45],
 [-0.29, -0.41, -0.19,  0.37]]
```

x^T

```
[[-1.73],
 [-0.05],
 [-1.38],
 [-1.31]]
```

b1

```
[[-0.45],
 [-0.10]]
```

=

```
[[-0.4309]
 [ 0.1997]]
```

```
FunH='tanh' ; FunO='softmax'
```

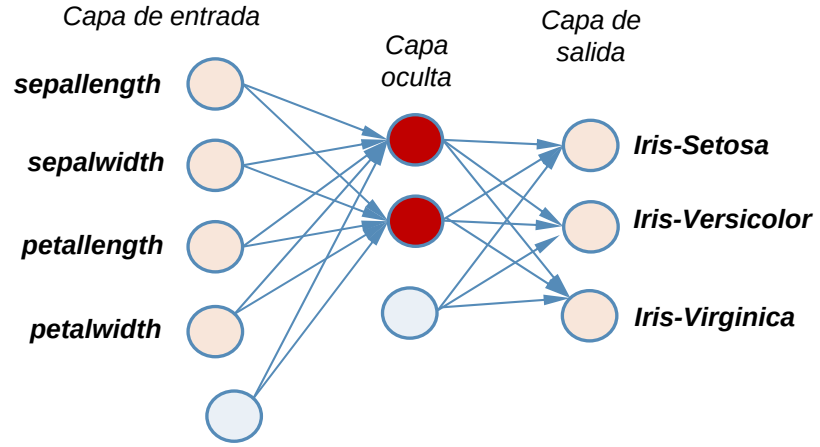
Calculando la salida de la capa oculta

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- Salida de la capa oculta

```
FunH='tanh' ; FunO='softmax'
```

```
netasH = W1 * x.T + b1
```

x^T

```
[[ 0.15, -0.13,  0.23, -0.45],
 [-0.29, -0.41, -0.19,  0.37]]   *  [[-1.73],
                                    [-0.05],
                                    [-1.38],
                                    [-1.31]]   +  [[-0.45],
                                                   [-0.10]]   =  [[-0.4309]
                                                                  [ 0.1997]]
```

W1 **b1**

```
salidasH = 2 / (1+np.exp(-netasH))- 1  =  [[-0.21217712]
                                           [ 0.09951948]]
```

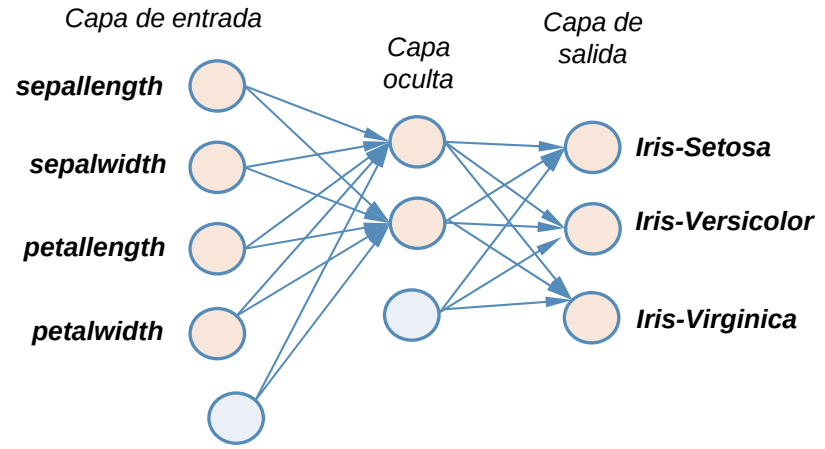
Calculando la salida de la red (capa de salida)

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- Salida de red

```
netasO = W2 * salidasH + b2
```

FunH='tanh' ; FunO='softmax'

W2

```
[[-4.79, -0.99],
 [ 0.38,  0.09],
 [ 2.40, -0.38]]
```

*

salidasH

```
[[-0.21217712]
 [ 0.09951948]]
```

+

b2

```
[[-0.29],
 [-0.17],
 [-0.27]]
```

=

netasO

```
[[ 0.62780411]
 [-0.24167055]
 [-0.81704249]]
```

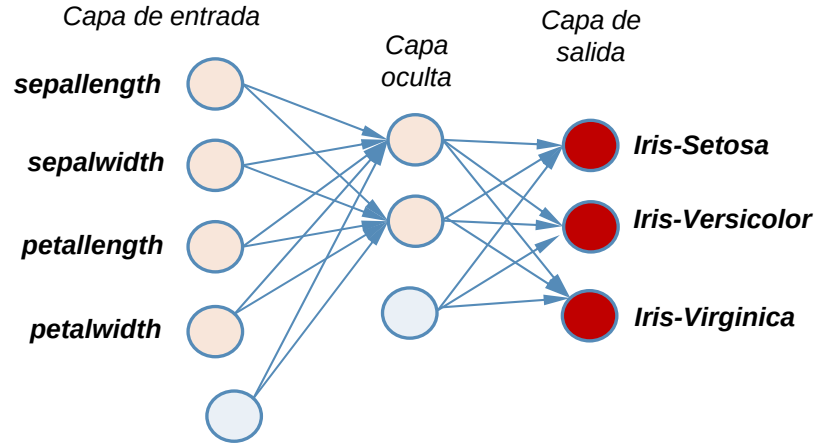
Calculando la salida de la red (capa de salida)

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- Salida de red

```
FunH='tanh' ; FunO='softmax'
```

```
netasO = W2 * salidasH + b2
```

W2

```
[[-4.79, -0.99],
 [ 0.38,  0.09],
 [ 2.40, -0.38]]
```

*

salidasH

```
[[-0.21217712]
 [ 0.09951948]]
```

\+

```
[[-0.29],
 [-0.17],
 [-0.27]]
```

=

netasO

```
[[ 0.62780411]
 [-0.24167055]
 [-0.81704249]]
```

b2

```
salidasO = np.exp(netasO)/(np.sum(np.exp(netasO)))
```

=

```
[[0.60424642]
 [0.253283  ]
 [0.14247058]]
```

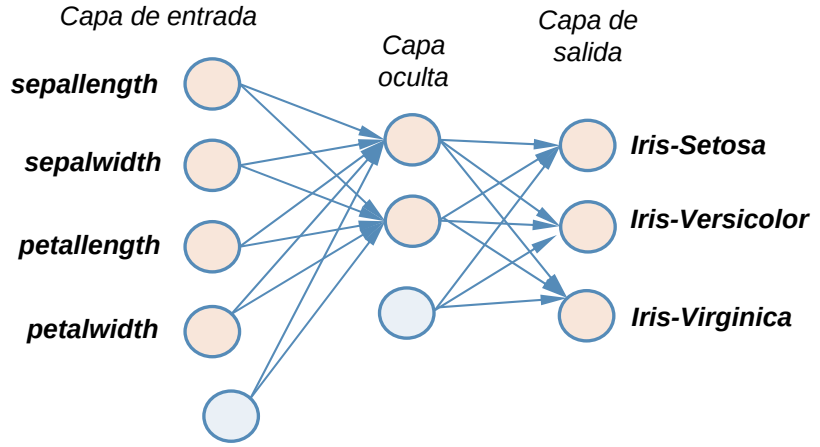
Error de la capa de salida

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- Error en la respuesta de la red para este ejemplo

```
ErrorSalida =  - y.T * np.log(salidasO)
```

```
ErrorSalida =   [[1.0]          [[-0.50377318]        [[ 0.50377318]
                 [0.0]   .*      [-1.37324784]    =    [-0.         ]
                 [0.0]]          [-1.94861978]]        [-0.         ]]
```

`np.log(salidasO)`

FunH='tanh' ; FunO='softmax'

$$C = -\sum_{k=1}^{3} y_k \ln \hat{y}_k$$

$$C = -\ln \hat{y}_s$$

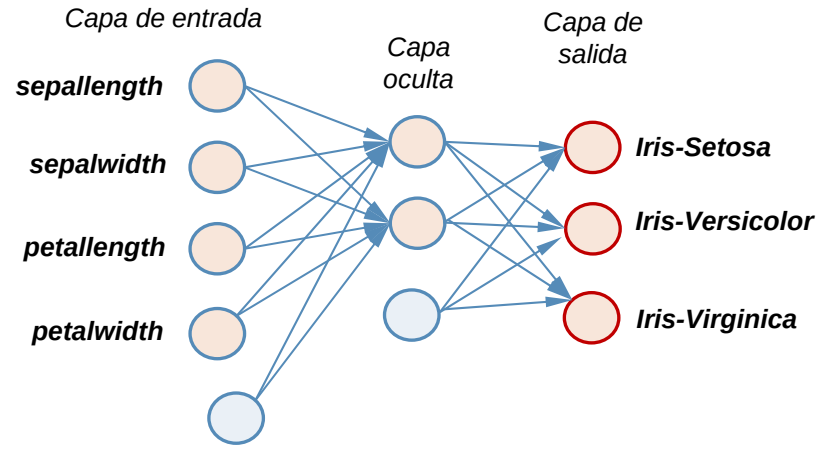
Factores de corrección de los pesos

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- **Factores para corregir W2 y b2**

FunH='tanh' ; FunO='softmax'

```
deltaO =  (y.T - salidasO)

deltaO =  [[1.0]      [[0.60424642]      [[ 0.39575358]
           [0.0]   -   [0.253283  ]   =   [-0.253283  ]
           [0.0]]      [0.14247058]]      [-0.14247058]]
```

$$\frac{\partial C}{\partial w_{jk}} = -\left(y_j - \hat{y}_j\right) x_k$$

$$\frac{\partial C}{\partial b_j} = -(y_j - \hat{y}_j)$$

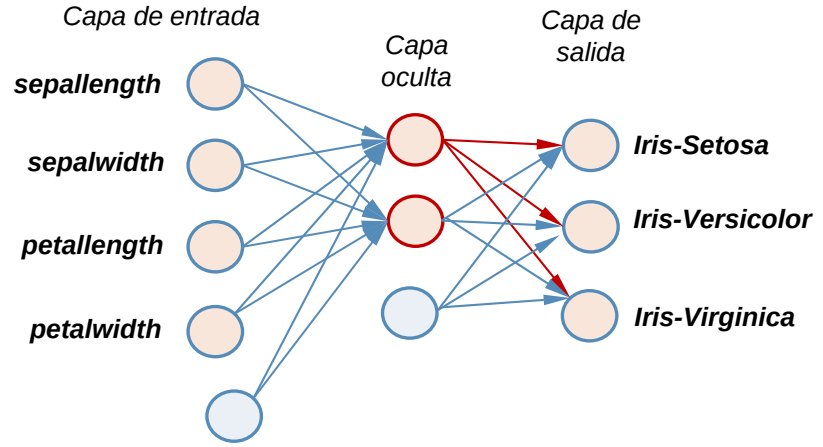
Factores de corrección de los pesos

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- **Factores para corregir W1 y b1**

```
(1-salidasH**2)
```

```
deltaH = deriv_FunH .* (W2.T @ deltaO)
```

```
deltaH =  [[0.95498087]      .*  [[-4.79  0.38  2.4 ]   @   [[ 0.39575358]
           [0.99009587]]           [-0.99  0.09 -0.38]]       [-0.253283  ]
                                                              [-0.14247058]]
```

```
deltaH =  [[-2.22876926]
           [-0.35688272]]
```

```
FunH='tanh' ; FunO='softmax'
```

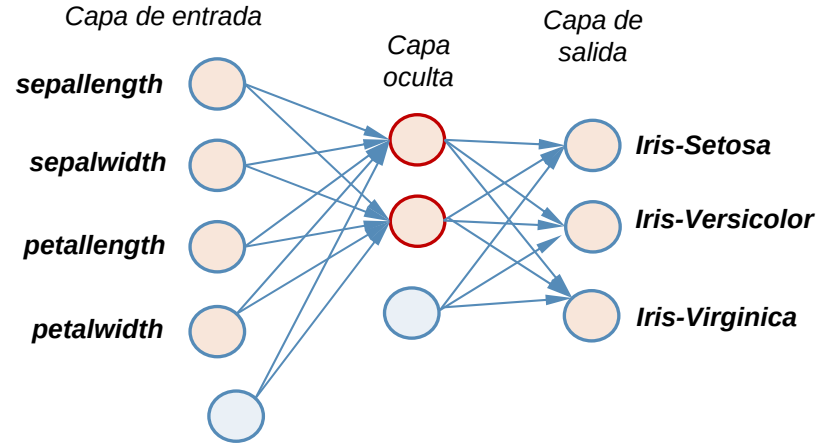
Corrigiendo de los pesos

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- **Modificación de W2 y b2**

```
FunH='tanh' ; FunO='softmax'
```

```
W2 = W2 + alfa * deltaO @ salidasH.T
```

```
W2 = [[-4.79, -0.99],
      [ 0.38,  0.09],  + alfa * ( [[ 0.39575358]
      [ 2.40, -0.38]]              [-0.253283  ]  @ [[-0.212177],[0.099519]] ) =
                                   [-0.14247058]]

     [[-4.8  -0.99]
      [ 0.39  0.09]
      [ 2.4  -0.38]]
```

```
b2 = b2 + alfa * deltaO =  [[-0.29],
                            [-0.17],  + alfa * [[ 0.39575358]
                            [-0.27]]           [-0.253283  ]
                                               [-0.14247058]]  =

     [[-0.25]
      [-0.2 ]
      [-0.28]]
```

Corrigiendo de los pesos

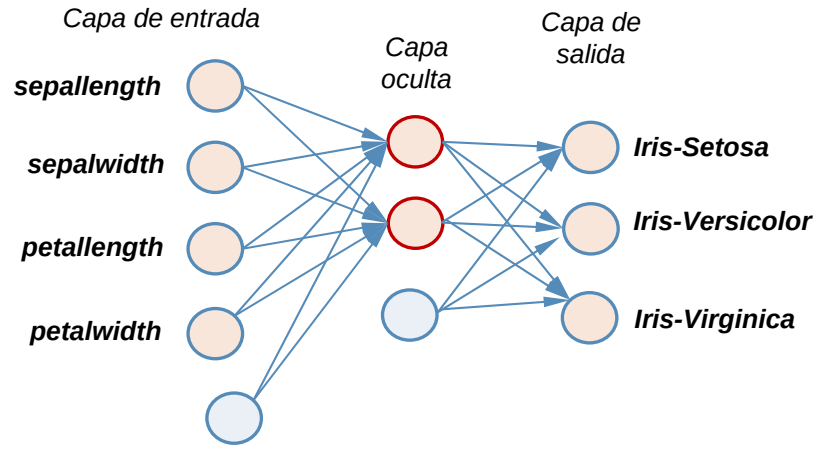
MLP_IRIS_algBPN_SoftMax.ipynb

X

```
[[-1.73,-0.05,-1.38,-1.31],
 [-0.37,-1.62, 0.22, 0.18],
 [ 1.11,-0.05, 0.93, 1.54],
 [-0.99, 0.39,-1.44,-1.31],
 [ 1.73, 1.29, 1.46, 1.81]]
```

Y

```
[[1,0,0],
 [0,1,0],
 [0,0,1],
 [1,0,0],
 [0,0,1]]
```



- **Modificación de W1 y b1**

```
W1 = W1 + alfa * deltaH @ x
```

```
FunH='tanh' ; FunO='softmax'
```

```
W1 = [[ 0.15,-0.13,0.23,-0.45],
      [-0.29,-0.41,-0.19,0.37]] + alfa * ( [[-2.22876926]  @  [[-1.73,-0.05,-1.38,-1.31]
                                            [-0.35688272]] )
```

```
W1 = [[ 0.54 -0.12  0.54 -0.16]
      [-0.23 -0.41 -0.14  0.42]]
```

```
b1 = b1 + alfa * deltaH = [[-0.45],  + alfa * [[-2.22876926]  =  [[-0.67]
                           [-0.10]]            [-0.35688272]]     [-0.14]]
```

Si se ingresa el mismo ejemplo luego de modificar los pesos de la red …

```
1  netasH = W1 @ xi.T + b1
2  salidasH = 2.0/(1+np.exp(-netasH))-1
3  netasO =  W2 @ salidasH + b2
4  salidasO = np.exp(netasO)/(np.sum(np.exp(netasO)))
5  print("salidaO = \n", salidasO)
6  ErrorSalidaNew = -yi.T*np.log(salidasO+EPS)
7  print("ErrorSalida = \n", ErrorSalidaNew)
```

```
salidaO =
 [[0.9799741 ]
 [0.01682763]
 [0.00319826]]
ErrorSalida =
 [[ 0.02022913]
 [-0.        ]
 [-0.        ]]
```

```
1  print("Error inicial = ", np.max(ErrorSalida))
2  print("Error luego de la correccion = ", np.max(ErrorSalidaNew))
```

```
Error inicial =  0.5037731768932103
Error luego de la correccion =  0.02022913433699978
```

Antes de modificar los pesos de la red

```
salidaO =
 [[0.60424642]
 [0.253283  ]
 [0.14247058]]
ErrorSalida =
 [[ 0.50377318]
 [-0.        ]
 [-0.        ]]
```

Ver
MLP_IRIS_SoftMax.ipynb

Multiperceptrón en Python

MLP_IRIS_RN.ipynb

```
from sklearn.neural_network import MLPClassifier

clf = MLPClassifier( solver='sgd',
                     learning_rate_init=0.15,
                     hidden_layer_sizes=(5),
                     max_iter=700, verbose=False,
                     tol=1.0e-09, activation = 'tanh')

clf.fit(X_train,T_train)
y_pred= clf.predict(X_train)
```

Matriz de Confusión

	Predice Clase 1	Predice Clase 2	Recall
True Clase 1	**A**	**B**	**A/(A+B)**
True Clase 2	**C**	**D**	**D/(C+D)**
Precision	**A/(A+C)**	**D/(B+D)**	**(A+D)/(A+B+C+D)**
			accuracy

- Los **aciertos** del modelo están sobre la **diagonal** de la matriz.
- ***Precision***: la proporción de predicciones correctas sobre **una clase.**
- ***Recall***: la proporción de ejemplos de **una clase** que son correctamente clasificados.
- ***Accuracy***: la performance general del modelo, sobre **todas las clases**. Es la cantidad de aciertos sobre el total de ejemplos.

sklearn_metrics.ipynb

Sklearn.metrics.accuracy_score

```
from sklearn import  metrics

Y_train = [0, 1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3]
Y_pred  = [0, 2, 1, 3, 0, 1, 2, 0, 0, 1, 2, 3]

aciertos = metrics.accuracy_score(Y_train,Y_pred)
print("%% accuracy = %.3f" % aciertos)
```

sklearn_metrics.ipynb

Sklearn.metrics.accuracy_score

```
from sklearn import  metrics

Y_train = [0, 1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3]
Y_pred  = [0, 2, 1, 3, 0, 1, 2, 0, 0, 1, 2, 3]

aciertos = metrics.accuracy_score(Y_train,Y_pred)
print("%% accuracy = %.3f" % aciertos)
```

Rtas esperadas

Rtas obtenidas

- De los 12 valores sólo 9 fueron identificados correctamente.
- La tasa de aciertos es 9/12 = **0.75**

sklearn_metrics.ipynb

Sklearn.metrics.confusion_matrix

```
Y_train = [0, 1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3]
Y_pred  = [0, 2, 1, 3, 0, 1, 2, 0, 0, 1, 2, 3]

MM = metrics.confusion_matrix(Y_train,Y_pred)
print("Matriz de confusión:\n%s" % MM)
```

```
Matriz de confusión:
[[3 0 0 0]
 [0 2 1 0]
 [0 1 2 0]
 [1 0 0 2]]
```

TRUE: 0, 1, 2, 3

0 1 2 3

PREDICE

Esperaba obtener 1 como respuesta pero la red respondió 2

sklearn_metrics.ipynb

Sklearn.metrics.confusion_matrix

```
Y_train = [0, 1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3]
Y_pred  = [0, 2, 1, 3, 0, 1, 2, 0, 0, 1, 2, 3]

MM = metrics.confusion_matrix(Y_train,Y_pred)
print("Matriz de confusión:\n%s" % MM)
```

```
Matriz de confusión:
[[3 0 0 0]
 [0 2 1 0]
 [0 1 2 0]
 [1 0 0 2]]
```

TRUE: 0, 1, 2, 3

PREDICE: 0 1 2 3

La respuesta correcta es 2 pero la red respondió 1

sklearn_metrics.ipynb

Sklearn.metrics.confusion_matrix

```
Y_train = [0, 1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3]
Y_pred  = [0, 2, 1, 3, 0, 1, 2, 0, 0, 1, 2, 3]

MM = metrics.confusion_matrix(Y_train,Y_pred)
print("Matriz de confusión:\n%s" % MM)
```

```
Matriz de confusión:
[[3 0 0 0]
 [0 2 1 0]
 [0 1 2 0]
 [1 0 0 2]]
```

TRUE: 0, 1, 2, 3

PREDICE: 0 1 2 3

Esperaba un 3 pero la red respondió 0

sklearn_metrics.ipynb

Sklearn.metrics.confusion_matrix

```
Y_train = [0, 1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3]
Y_pred  = [0, 2, 1, 3, 0, 1, 2, 0, 0, 1, 2, 3]

MM = metrics.confusion_matrix(Y_train,Y_pred)
print("Matriz de confusión:\n%s" % MM)
```

```
Matriz de confusión:
TRUE
0 [[3 0 0 0]
1  [0 2 1 0]
2  [0 1 2 0]
3  [1 0 0 2]]
    0 1 2 3
   PREDICE
```

Los valores fuera de la diagonal principal son errores

sklearn_metrics.ipynb

Sklearn.metrics.classification_report

```
Y_train = [0, 1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3]
Y_pred  = [0, 2, 1, 3, 0, 1, 2, 0, 0, 1, 2, 3]
report = metrics.classification_report(Y_train,Y_pred)
print("Resultado de la clasificación:\n%s" % report)
```

```
Resultado de la clasificación:
              precision    recall  f1-score   support

           0       0.75      1.00      0.86         3
           1       0.67      0.67      0.67         3
           2       0.67      0.67      0.67         3
           3       1.00      0.67      0.80         3

    accuracy                           0.75        12
   macro avg       0.77      0.75      0.75        12
weighted avg       0.77      0.75      0.75        12
```

```
Matriz de confusión:
[[3 0 0 0]
 [0 2 1 0]
 [0 1 2 0]
 [1 0 0 2]]
```

sklearn_metrics.ipynb

Sklearn.metrics.classification_report

```
Y_train = [0, 1, 2, 3, 0, 1, 2, 3, 0, 1, 2, 3]
Y_pred  = [0, 2, 1, 3, 0, 1, 2, 0, 0, 1, 2, 3]
report = metrics.classification_report(Y_train,Y_pred)
print("Resultado de la clasificación:\n%s" % report)
```

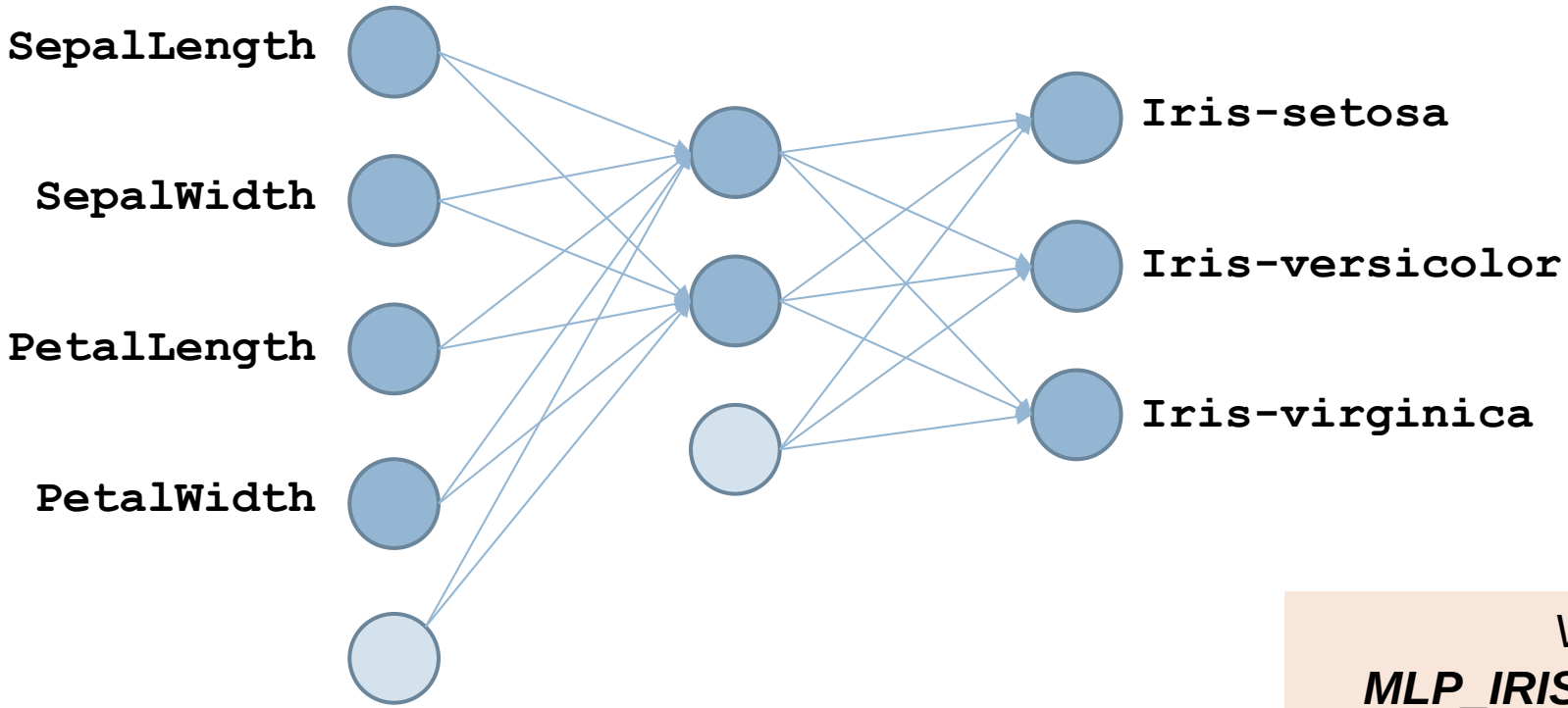
Resultado de la clasificación:

	precision	recall	f1-score	support
0	0.75	1.00	0.86	3
1	0.67	0.67	0.67	3
2	0.67	0.67	0.67	3
3	1.00	0.67	0.80	3
accuracy			0.75	12
macro avg	0.77	0.75	0.75	12
weighted avg	0.77	0.75	0.75	12

F1-score

$$F1 = 2 * \frac{precision * recall}{precisión + recall}$$

Ejemplo: Clasificación de flores de Iris



Ver
MLP_IRIS_RN.ipynb

Reconocedor de dígitos escritos a mano

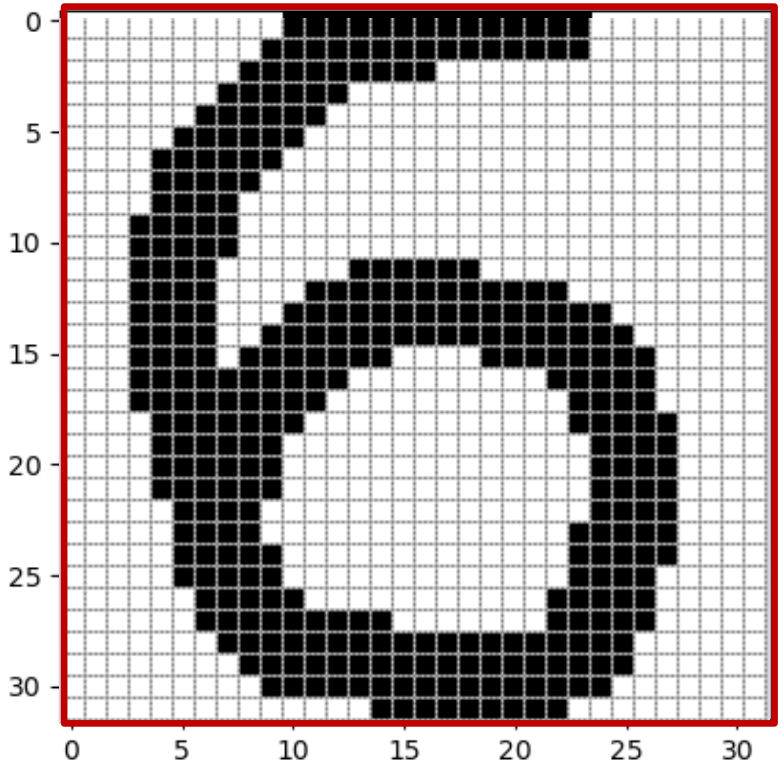
- Se desea entrenar un multiperceptrón para reconocer dígitos escritos a mano. Para ello se dispone de los mapas de bits correspondientes a 3823 dígitos escritos a mano por 30 personas diferentes en el archivo "optdigits_train.csv".

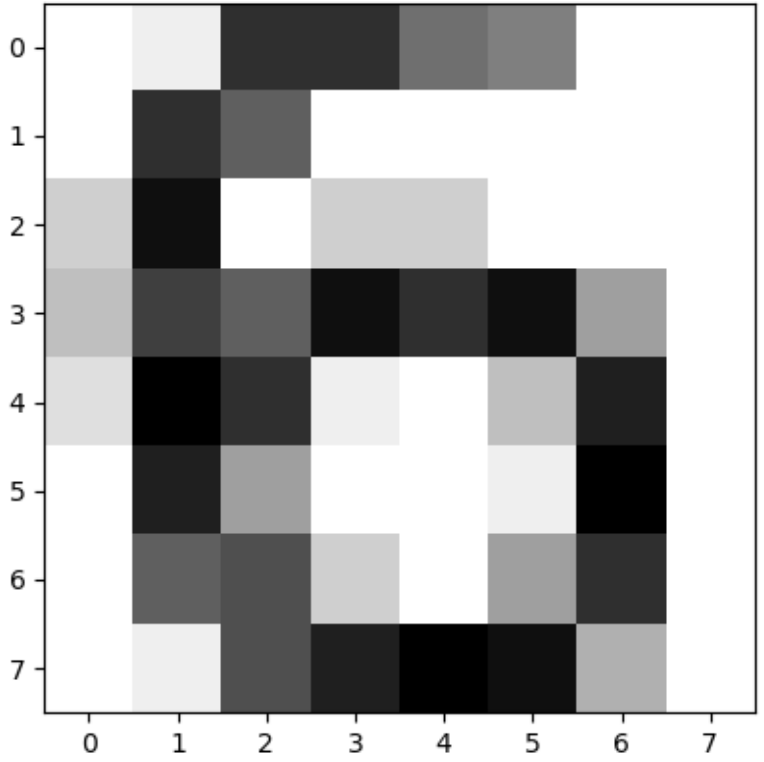
- El desempeño de la red será probado con los dígitos del archivo "optdigits_test.csv" escritos por otras 13 personas.

https://archive.ics.uci.edu/ml/datasets/optical+recognition+of+handwritten+digits

“optdigits_train.csv” y “optdigits_test.csv”

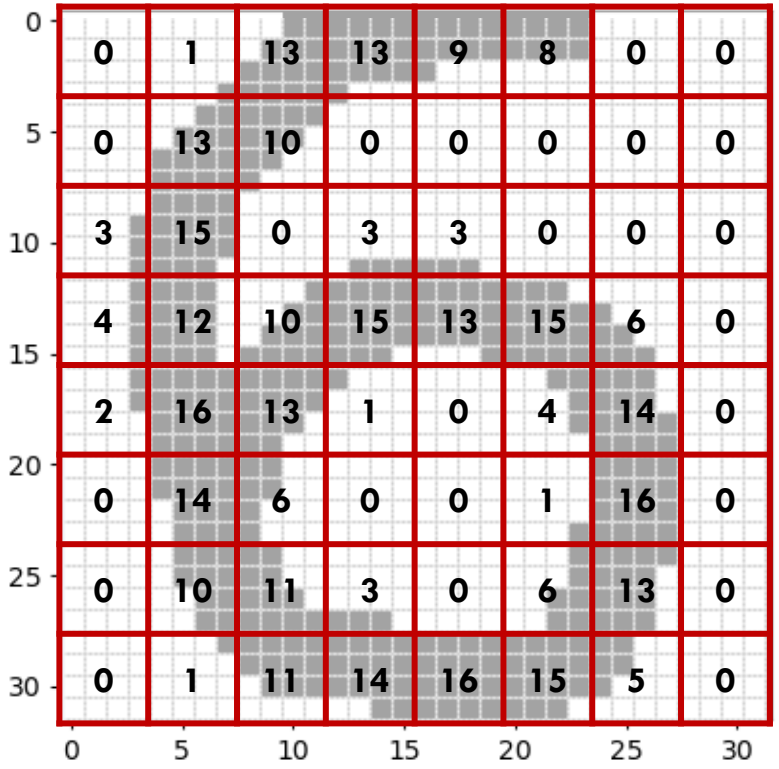
- Cada dígito está representado por una matriz numérica de 8x8

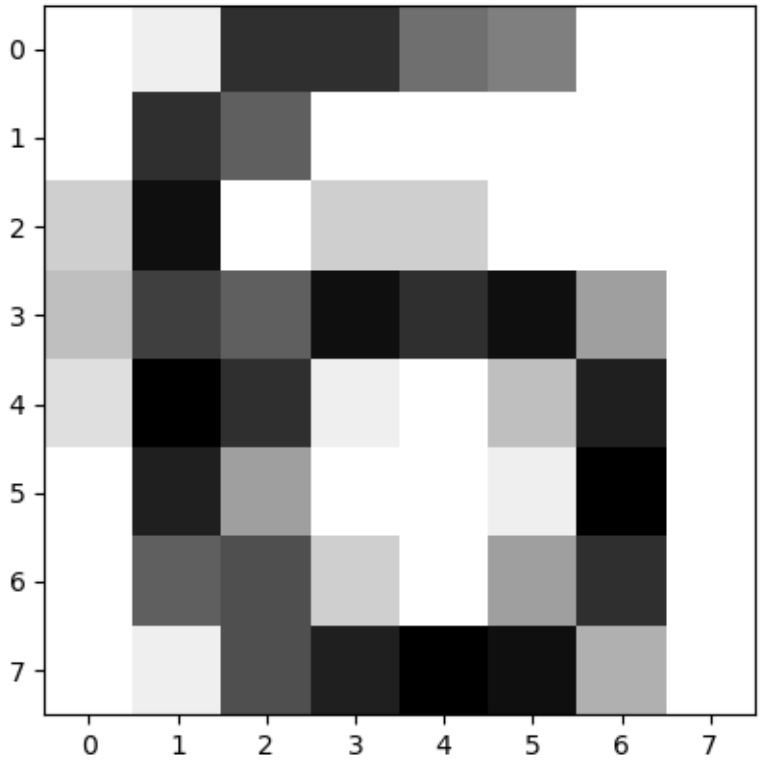




“optdigits_train.csv” y “optdigits_test.csv”

- Cada dígito está representado por una matriz numérica de 8x8





im_32x32_a_8x8.ipynb

RN para reconocer dígitos manuscritos

```
In [90]: print("ite = %d  %% aciertos X_train : %.3f" % (ite,
metrics.accuracy_score(Y_train,Y_pred)))
ite = 200  % aciertos X_train : 0.982

In [91]: MM = metrics.confusion_matrix(Y_train,Y_pred)
    ...: print("Matriz de confusión TRAIN:\n%s" % MM)
Matriz de confusión TRAIN:
[[375   0   0   0   0   0   0   0   1   0]
 [  7 382   0   0   0   0   0   0   0   0]
 [  2   0 378   0   0   0   0   0   0   0]
 [  7   0   0 380   0   2   0   0   0   0]
 [  3   0   0   0 383   0   1   0   0   0]
 [  6   0   0   1   0 369   0   0   0   0]
 [  2   2   0   0   0   0 373   0   0   0]
 [  0   0   0   1   0   0   0 386   0   0]
 [ 17   2   0   0   0   0   0   0 361   0]
 [ 13   1   0   1   0   1   0   0   0 366]]
```

im_32x32_a_8x8.ipynb

Entrenamiento muy lento ¿Se puede acelerar?

MLP_MNIST_8x8.ipynb