RES PRAC AYED

$$T(n) = \begin{cases} 2, & n = 1 \\ T(n-1) + n, & n \geq 2 \end{cases}$$

Suponiendo un $n \geq 2$:

$P_1 = T(n-1) + n$

$P_2 = [T(n-2) + n-1] + n = T(n-2) + n - 1 + n$

$P_3 = [T(n-3) + n-2] + n-1 + n = T(n-3) + n-2 + n-1 + n$

$P_4 = [T(n-4) + n-3] + n-2 + n-1 + n$

$P_i = T(n-i) + \sum_{k=0}^{i-1}(n-k) = T(n-i) + \sum_{k=0}^{i-1} n - \sum_{k=1}^{i-1} k = T(n-i) + (i-1-0+1)n - \frac{i\cdot(i-1)}{2}$

$T(n-i) + in - \frac{i^2-i}{2} = T(n-i) + in - \frac{i^2}{2} + \frac{i}{2}$

- Ahora debo encontrar el valor de i que haga $T(1)$:

$n - i = 1$

$n - 1 = i$

- Reemplazo en el paso general:

$T(n-(n-1)) + (n-1)\cdot n - \frac{(n-1)^2}{2} + \frac{(n-1)}{2} = T(1) + n^2 - n - \frac{(n^2-2n+1)}{2} + \frac{(n-1)}{2} =$

$T(1) + n^2 - n - \frac{n^2}{2} + n - \frac{1}{2} + \frac{n}{2} - \frac{1}{2} = T(1) + n^2 - \frac{n^2}{2} - 1 + \frac{n}{2} = 2 + \frac{n^2}{2} + \frac{n}{2} - 1$

$\boxed{1 + \frac{n^2}{2} + \frac{n}{2}}$ $O(n^2/2)$

- Existen constante c y n_0 tales que: $1 + n^2/2 + n/2 \leq c \cdot n^2/2$

$1 \leq c \cdot n^2/2$ si $n_0 = 1 \Rightarrow c = 2$ ✓

$1 \leq 2 \cdot 1/2$

$1 \leq 1$

$n^2/2 \leq c \cdot n^2/2$ si $n_0 = 1 \Rightarrow c = 1$ ✓

$1/2 \leq 1/2$

$n/2 \leq c \cdot n^2/2$ si $n_0 = 1 \Rightarrow c = 1$ ✓

$1/2 \leq 1/2$

$1 + \frac{n^2}{2} + \frac{n}{2} \leq (2+1+1) \cdot \frac{n^2}{2}$

$1 + \frac{n^2}{2} + \frac{n}{2} \leq 4 \cdot \frac{n^2}{2} \in O(n^2/2)$ ← ¿Está bien en $O(n^2)$?

NOTA

HOJA Nº

FECHA

b)-

$$T(n) = \begin{cases} 2 & , n=1 \\ T(n-1)+n/2 & , n \geq 2 \end{cases}$$

1- Encontrar paso general

$$P_1 = T(n-1)+n/2$$

$$P_2 = [T(n-2)+(n-1)/2] + n/2 = T(n-2) + (n-1)/2 + n/2$$

$$P_3 = [T(n-3)+(n-2)/2] + (n-1)/2 + n/2 = T(n-3) + (n-2)/2 + (n-1)/2 + n/2$$

$$P_i = T(n-i) + \sum_{k=0}^{i-1} n-k = T(n-i) + \sum_{k=0}^{i-1} n - \sum_{k=1}^{i-1} k = T(n-i) + (i-1+1)n - \left(\frac{(i-1)i}{2}\right)$$

$$T(n-i) + in - \left(\frac{i^2}{2} - \frac{i}{2}\right) = T(n-i) + in - \frac{i^2}{2} + \frac{i}{2}$$

$$n-i=1$$
$$n-1=i$$

$$T(n-(n-1)) + (n-1).n - \frac{(n-1)^2}{2} + \frac{(n-1)}{2} = T(1) + n^2 - n - \left(\frac{n^2-2n+1}{2}\right) + (n-1)/2$$

$$2 + n^2 - n - \frac{n^2}{2} + n - \frac{1}{2} + \frac{n}{2} - \frac{1}{2} = 2-1 + \frac{n^2}{2} + n - n + \frac{n}{2} = 1 + \frac{n^2}{2} + \frac{n}{2}$$

Scanned with CamScanner

c-

$$T(n) = \begin{cases} 1 & n = 1 \\ 2T(n/4) + \sqrt{n} & n \geq 2 \end{cases}$$

$P_1 = 2T(n/4) + \sqrt{n}$

$P_2 = 2\left[2T(n/4/4) + \sqrt{(n/4)}\right] + \sqrt{n} = 4T(n/16) + 2\sqrt{(n/4)} + \sqrt{n}$

$P_3 = 4\left[2T(n/16/4) + \sqrt{n/16}\right] + 2\sqrt{(n/4)} + \sqrt{n} = 8T(n/64) + 4\sqrt{n/16} + 2\sqrt{(n/4)} + \sqrt{n}$

$P_i = 2^i T(n/4^i) + \sum_{k=0}^{i-1} 2^k \sqrt{n/4^k} = 2^i T(n/4^i) + \sum_{k=0}^{i-1} 2^k . \sqrt{n/(2^2)^k}$

$P_i = 2^i T(n/4^i) + \sum_{k=0}^{i-1} 2^k . \frac{1}{2^k} . \sqrt{n} = 2^i T(n/4^i) + \sum_{k=0}^{i-1} \sqrt{n} = 2^i T(n/4^i) + i . \sqrt{n}$

$n/4^i = 1$

$n = 4^i$

$\log_4(n) = \log_4(4^i)$

$\log_4(n) = i . \log_4(4)$

$\log_4(n) = i$

$2^{\log_4(n)} T(n/4^{\log_4(n)}) + \log_4(n) . \sqrt{n} = \sqrt{n} . T(n/n) + \log_4(n) . \sqrt{n}$

$\sqrt{n} . T(1) + \log_4(n) . \sqrt{n} = \sqrt{n} . 1 + \log_4(n) . \sqrt{n} = \sqrt{n} + \log_4(n) . \sqrt{n}$

El órden de la función $\sqrt{n} + \log_4(n) . \sqrt{n}$ es $\log_4(n) . \sqrt{n}$

Existen constantes c y n_0 tales que:

$\sqrt{n} \leq c . \log_4(n) . \sqrt{n}$

$\frac{\sqrt{n}}{\sqrt{n}} \leq c . \log_4(n)$

$1 \leq 1 . \log_4(4)$ con $n_0 = 4$, $c = 1$

$\log_4(n) . \sqrt{n} \leq c . \log_4(n) . \sqrt{n}$ con $n_0 = 4$, $c = 1$

$\log_4(4) . \sqrt{1} \leq 1 . \log_4(4) . \sqrt{1}$

$1 \leq 1$

Por lo tanto:

$\sqrt{n} + \log_4(n) . \sqrt{n} \leq (1+1) . \log_4(n) . \sqrt{n}$

$\leq 2 . \log_4(n) . \sqrt{n}$ Para todo $c \geq 2$ y $n_0 \geq 4$

Scanned with CamScanner

d-

$$T(n) = \begin{cases} 1 & n=1 \\ 4T(n/2) + n^2 & n \geq 2 \end{cases}$$

$P_1 = 4T(n/2) + n^2$

$P_2 = 4[4T(n/2/2) + (n/2)^2] + n^2 = 16T(n/4) + 4.\left(\frac{n}{2}\right)^2 + n^2 = 16T\left(\frac{n}{4}\right) + 2n^2$

$P_3 = 16[4T(n/4/2) + (n/4)^2] + 2n^2 = 64T(n/8) + 16\left(\frac{n}{4}\right)^2 + 2n^2 = 64T\left(\frac{n}{8}\right) + 3n^2$

$P_i = 4^i T(n/2^i) + i n^2$

$n/2^i = 1$

$n = 2^i$

$\log_2(n) = \log_2(2^i)$

$\log_2(n) = i$

$4^{\log_2(n)} T(n/2^{\log_2(n)}) + \log_2(n).n^2 = n^2.T(1) + \log_2(n) = n^2 + \log_2 n$

$O(n^2)$

Existen constantes c y n_0 tales que:

$n^2 \leq c n^2 \quad n_0 = 1 \quad c = 1$

$1 \leq 1$

$\log_2(n) \leq c.n^2 \quad n_0 = 1 \quad c = 1$

$0 \leq 1$

Entonces

$n^2 + \log_2(n) \leq c.(n)^2$

$\quad " \quad \leq (1+1) n^2$

$\quad " \quad \leq 2.n^2$ Para todo $n_0 \geq 1$ y $c \geq 2$