13a-

$$T(n)=\begin{cases} 1 & ,n\leq 1 \\ T(n-1)+c & ,n\geq 2 \end{cases}$$

Paso 1: $T(n-1)+c$

Paso 2: $[T(n-1-1)+c]+c = T(n-2)+2c$

Paso 3: $[T(n-2-1)+c]+2c = T(n-3)+3c$

Paso i: $T(n-i)+ic$

$n-i=1$
$n-1=i$

$T(n-(n-1))+(n-1)c = T(1)+nc-c = 1+nc-c$

$T(n) = 1+nc-c \quad O(n)$

b-

$$T(n)=\begin{cases} 1 & ,n\leq 1 \\ T(n/2)+c & ,n\geq 2 \end{cases}$$

Paso 1: $T(n/2)+c$

Paso 2: $[T(n/2/2)+c]+c = T(n/4)+2c$

Paso 3: $[T(n/4/2)+c]+2c = T(n/8)+3c$

Paso i: $T(n/2^i)+ic$

Cuando $n/2^i=1$ termina

$n=2^i$
$\log_2(n)=\log_2(2^i)$
$\log_2(n)=i$

Reemplazo en el paso i

$T(n)=T(2^i/2^i)+\log_2(n).c = T(1)+\log_2 n.c = 1+\log_2(n).c$

$T(n)=1+\log_2(n).c \quad O(\log_2(n))$

c-

$$T(n)=\begin{cases} 1 & ,n\leq 1 \\ 2T(n/2)+c & ,n\geq 2 \end{cases}$$

Paso 1: $T(n)=2T(n/2)+c$

Paso 2: $T(n)=2[2T(n/2/2)+c]+c = 4T(n/4)+3c$

Paso 3: $T(n)=4[2T(n/4/2)+c]+2c = 8T(n/8)+7c$

Paso i: $T(n)=2^iT(n/2^i)+(2^i-1)c$

Termina cuando $n/2^i = 1$
$$n = 2^i$$
$$\log_2(n) = \log_2(2^i)$$
$$\log_2(n) = i$$

Reemplazo en el paso i:

$T(n) = 2^{\log_2(n)} T(2^i/2^i) + (2^{\log_2(n)} - 1) \cdot c = n \cdot T(1) + (n-1) \cdot c$

$T(n) = n + nc - c \quad O(n)$

d-
$$T(n) = \begin{cases} 1 & , n \leq 5 \\ T(n-5) + c, & n \geq 6 \end{cases}$$

Paso 1: $T(n) = T(n-5) + c$

Paso 2: $T(n) = [T(n-5-5) + c] + c = T(n-10) + 2c$

Paso 3: $T(n) = [T(n-10-5) + c] + 2c = T(n-15) + 3c$

Paso i: $T(n-5i) + ic$

Termina cuando $n - 5i = 5$
$$n - 5 = 5i$$
$$(n-5)/5 = i$$
$$n/5 - 1 = i$$

Reemplazo en el paso i:

$T(n) = T(n - 5 \cdot (n/5 - 1)) + (n/5 - 1) \cdot c = T\left(n - \dfrac{5n}{5} + 5\right) + \dfrac{nc}{5} - c = T(5) + \dfrac{nc}{5} - c$

$T(n) = 1 + \dfrac{nc}{5} - c \quad O(n)$

e-
$$T(n) = \begin{cases} 1 & n = 1 \\ 2T(n-1) + c, & n \geq 2 \end{cases}$$

Paso 1: $T(n) = 2T(n-1) + c$

Paso 2: $T(n) = 2 \cdot [2T(n-1-1) + c] + c = 4T(n-2) + 3c$

Paso 3: $T(n) = 4[2T(n-2-1) + c] + 3c = 8T(n-3) + 7c$

Paso i: $T(n) = 2^i T(n-i) + (2^i - 1)c$

Termina cuando $n - i = 1$
$$n - 1 = i$$

Scanned with CamScanner

Reemplazo en el paso i:

$$T(n) = 2^{n-1} T(n-n+1) + (2^{n-1}+1)c = 2^{n-1} \cdot T(1) + 2^{n-1}c + c$$

$$T(n) = 2^{n-1} + c 2^{n-1} + c \qquad O(2^{n-1})$$

b-
$$T(n) = \begin{cases} 1 & , n \leq 7 \\ T(n/8) + c, & n \geq 8 \end{cases}$$

Paso 1: $T(n) = T(n/8) + c$

Paso 2: $T(n) = [T(n/8/8) + c] + c = T(n/64) + 2c$

Paso i: $T(n) = T(n/8^i) + ic$

Termina cuando $n/8^i = 1$
$$n = 8^i$$
$$\log_8(n) = \log_8(8^i)$$
$$\log_8(n) = i$$

Reemplazo en el paso i:

$$T(n) = T(8^i/8^i) + \log_8(n) \cdot c = T(1) + \log_8(n)c$$

$$T(n) = 1 + \log_8(n)c \qquad O(\log_8(n))$$