Unit 5

Electric networks analysis

Objectives

- Know Kirchhoff's rules and how use them in order to analyse basic networks of direct current.
- To be able to substitute a circuit for its equivalent circuit, according superposition principle and Thèvenin's theorem.

5.1 Introduction. Definitions

Until now, in previous unit, we have studied how voltage and intensity are related in different devices in d.c., as generators, receptors and resistors. We have learnt how compute the difference of potential between two points of a circuit, and the intensity in a simple circuit made up by these devices.

The simple circuits we have handled until this moment only had a intensity of current, or they could be transformed to get only an intensity of current. In this unit we'll learn to compute the intensities of current flowing along more complex circuits constituted also by the basic devices in d.c. These more complex circuits (we'll call them electric networks), have different intensities of current flowing along different pieces of circuit. On first, we'll see some definitions will be used along this entire unit.

Dipole: It's any electronic device with two terminals (also called poles); a resistor, a generator, a diode, etc… are examples of dipoles. A transistor, with three terminals, isn't a dipole. They can be distinguished between active dipoles and passive dipoles.

Active dipoles are those dipoles supplying energy to the system: generators.

Passive dipoles are those dipoles consuming energy: resistors, receptors, coils (inductances), capacitors …., etc.

Linear dipole: a linear dipole shows a linear relation between voltage and intensity of current. For example, a resistor is a linear dipole: $V=IR$.

Electric network: It is a set of dipoles interconnected between them with wires without resistance making up a network with different intensities of current. In Figure 5-11 an example is shown.

A **linear network** is a network made up by linear dipoles. In this unit won't be studied electric networks with no linear devices, as diodes or transistors. Electric networks with receptors will be studied only in some cases.

Although we only will work with electric networks in direct current, the explained methods are also useful for alternating current circuits.

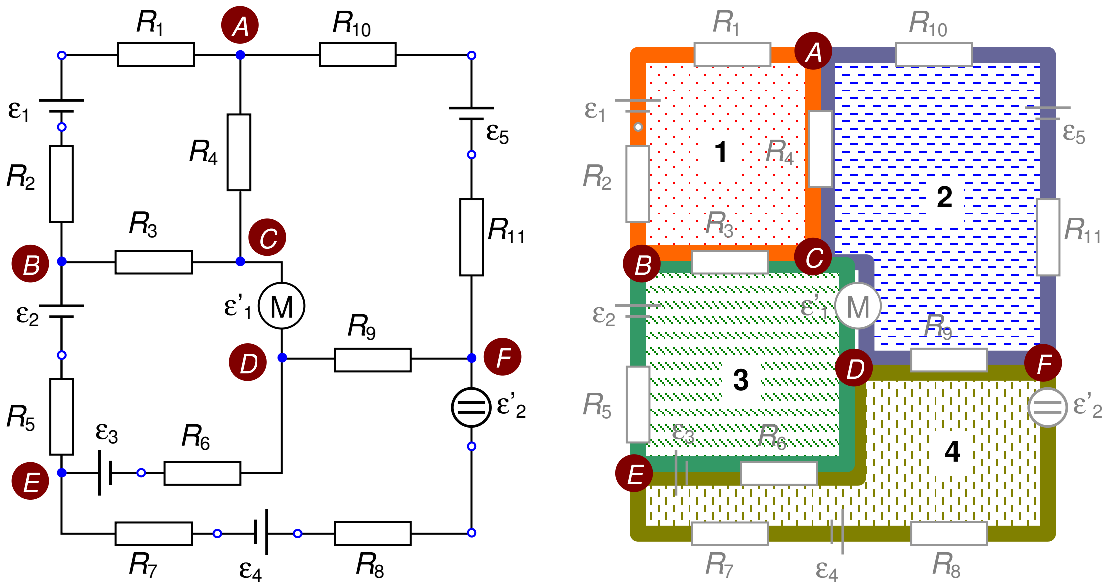


Figure 5-1. Flat electric network: junctions and loops

Junction is a point of a network where three or more dipoles are joined. When two different junctions are connected through a wire without resistance, we'll consider that they are the same junction, because their potentials will be equal. The network in Figure 5-11 has 6 junctions, named from *A* to *F*.

Branch is a piece of circuit between two consecutive junctions. Each branch has its own intensity of current flowing along it. In example of Figure 5-1, 9 branches can be seen.

Loop is an enclosed circuit made up by branches, in such way that starting on a point and following a sense, can be returned to the same point crossing only once each branch. Although a loop can contain smaller loops inside, we'll consider that **a loop doesn't contain any other inside**. In the example of the Figure 5-11, there are 4 loops numbered from 1 to 4.

Flat network is a network with any branch not belonging to more than two loops. This definition comes from the fact that a flat network always can be built in a plane, whereas to build a not flat network, a three-dimensional circuit is necessary.

In this unit we'll study some general methods for analysing flat and linear networks of direct current in steady state (not depending on time).

5.2 Kirchhoff's rules

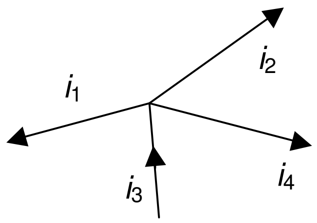
Gustave Kirchhoff, a German physicist (1824-1887), stated the basis to solve electric networks, by means two rules known as Kirchhoff's rules: the rule of junctions and the rule of loops.

Rule of junctions: It is a consequence of the preservation of the electric charge, that is, the addition of charges going in a junction equals the addition of charges going out in the junction. Since the intensity of electric current is the quantity of charge crossing a cross section of a conductor by unit of time, this involves that **the sum of the intensities going in a junction equals the sum of the intensities going out of such junction**. Usually, intensities entering on a junction are taken with opposite sign that intensities exiting from that junction. In this way, Kirchoff's rule of junction can be written as:

The algebraic addition of intensities (i) on a junction (k) is zero.

$$\sum i_k = 0 \qquad \text{Equation 5-1}$$

In Figure 5- 5- 2, an example of such rule is shown (intensities entering on junction are taken as positives, and intensities exiting from junction are negatives).



$$-i_1 - i_2 + i_3 - i_4 = 0$$

Figure 5-2. Application of rule of junctions

Rule of loops: The second rule is a consequence of the fact that a point can't have two different potentials at the same time. As a consequence,

The addition of the drop of potential in all the branches along a loop (k) is zero.

$$\sum V_k = 0 \qquad \text{Equation 5-2}$$

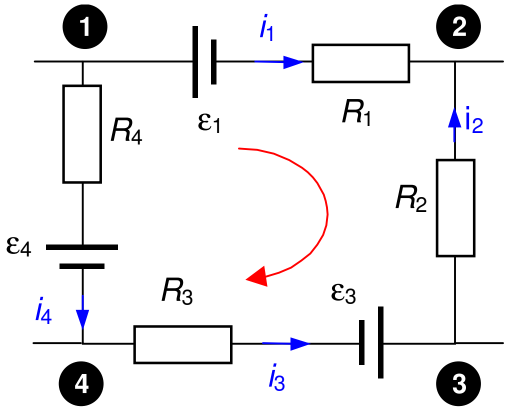
In order to use this rule, an arbitrary sense to go round the loop must be taken. According to this sense, difference of potential between each two consecutive junctions must be computed, applying the known equation

$$V_A - V_B = I\sum R_i - \sum \varepsilon_j$$

being I the intensity of current flowing along the branch between A and B, and R_i and ε_j all the resistors, electromotive and contralectromotive forces between A and B.

So, the addition of all these differences of potential along the loop must be zero.

In the following example is shown how this rule is applied; we choose the clockwise sense to go from point 1 to the same point 1:



$$V_{12} = i_1R_1 - (-\varepsilon_1)$$
$$V_{23} = -i_2R_2$$
$$V_{34} = -i_3R_3 - (-\varepsilon_3)$$
$$V_{41} = -i_4R_4 - \varepsilon_4$$

$$\sum IR_i - \sum \varepsilon_j = 0$$

$$i_1R_1 - i_2R_2 - i_3R_3 - i_4R_4 + \varepsilon_1 + \varepsilon_3 - \varepsilon_4 = 0$$

Figure 5-3. Application of rule of loops

The rule of loops can be applied to any enclosed path of circuit, not only to the loops.

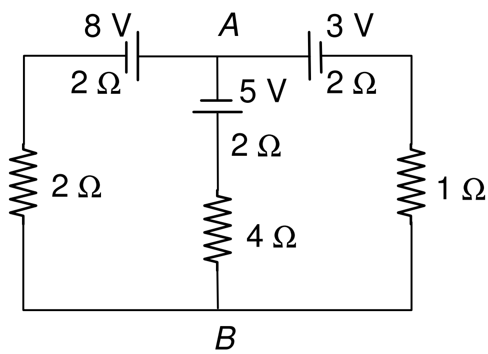
Applying in a circuit the rule of junctions to all the junctions (let's suppose n junctions), the equation for the last junction will be a linear combination of the equations for preceding junctions, and so, only $n - 1$ equations linearly independent could be written. On the other hand, applying the rule of loops (let's suppose m loops), we'll be able to write equal number of independent equations that loops. To finish, we can say that $n - 1 + m$ linearly independent equations can be written for a flat network with n junctions and m loops.

From these equations, we can solve for the intensities of circuit.

Example 5-1

By using Kirchhoff's rules, compute intensities of current flowing along each branch of network on picture.

Compute the difference of potential between points A and B.



Solution

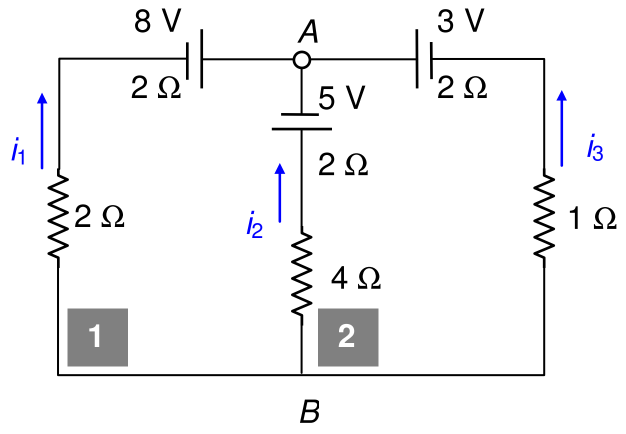
This circuit is made up by two junctions (A and B), two loops and three branches. So, we can write one independent equation for rule of junctions and two independent equations for rule of loops. We'll call i_1, i_2 and i_3 to the intensities on branches 1, 2 and 3, giving them an arbitrary sense, as it's shown on next picture. The loops have also been numbered.

Writing the rule of junctions on junction A, taking entering intensities as positives:

$i_1 + i_2 + i_3 = 0$ [1]

Obviously, if we try to write the rule of junctions on junction B, we get the same equation.

Now, we are going to write the rule of loops for loops *1* and *2*. If, on loop *1*, we start on point B and we go round loop *1* to the same point B (for example, in clockwise sense):



$$V_B - V_B = i_1(2+2) - (8) - i_2(2+4) - (5) = 0 \qquad [2]$$

Note that i_1 appears with positive sign and i_2 with negative sign according the sense taken to go round loop *1*. And the same can be said for electromotive and counter-electromotive forces.

Repeating the rule of loops on loop *2* (from A to A in clockwise sense), it becomes:

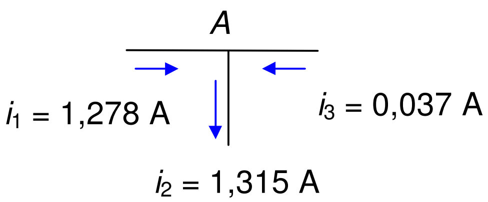
$$V_A - V_A = -i_3(2+1) - (-3) + i_2(4+2) - (-5) = 0 \qquad [3]$$

Simplifying [1], [2] and [3] we can write the system of equations:

$$\left.\begin{array}{llll} i_1 & +i_2 & +i_3 & =0 \\ 4i_1 & -6i_2 & & =13 \\ & 6i_2 & -3i_3 & =-8 \end{array}\right\}$$

Solving for this system: $i_1 = 1{,}278$ A $i_2 = -1{,}315$ A and $i_3 = 0{,}037$ A

From this result, we can see that supposed senses for currents i_1 and i_3 were correct, whereas supposed sense for i_2 was wrong. So, the real senses for intensities are:

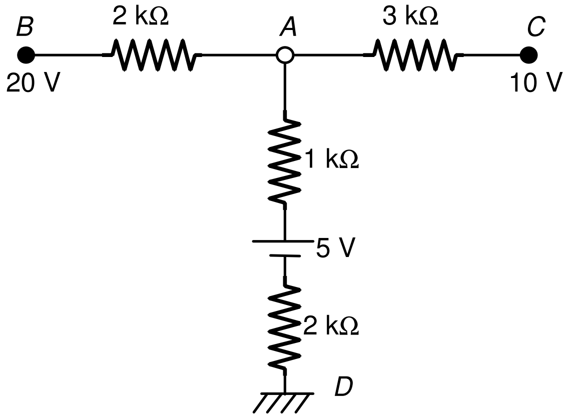


Finally, the difference of potential between points A and B (going from A to B) along the middle branch:

$$V_A - V_B = \sum IR - \sum \varepsilon = -(-1{,}315)(4+2)-(5) = 2{,}89 \text{ V}$$

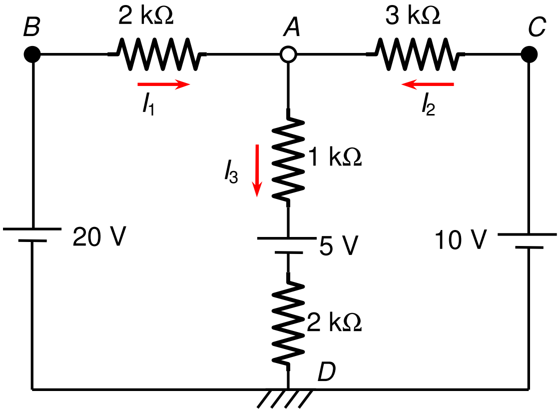
Example 5-2

By using Kirchoff's rules, compute the potential on point *A* and the intensities of current along the branches of circuit on picture.



Solution

In order to solve this circuit, is very important to note that this circuit is very similar to circuit of example 5.1. Now, the symbol of ground on point *D* means that potential zero has been taken for point *D*. Points *B* and *C* have fixed potentials, and so, this circuit could be drawn connecting two ideal generators between *B* and *D* and between *C* and *D*, as it's shown on picture on right.



In it, we have supposed a sense for intensities of current on each branch, and we could solve for these intensities with three equations as we have done on example 5-1. Working with resistors in KΩ, computed intensities will be given in mA. The rule of junctions on junction *A* would be:

$$I_1 + I_2 - I_3 = 0 \quad [1]$$

But in this example, as we already know potentials on points *B*, *C* and *D*, is easier to write the rule of loops for both loops as two differences of potential between points of known potential. For example:

$$V_B - V_D = 20 - 0 = 20 = (V_B - V_A) + (V_A - V_D) = I_1 2 + I_3(1+2) - (-5) \Rightarrow 2I_1 + 3I_3 = 15 \quad [2]$$

$$V_C - V_D = 10 - 0 = 10 = (V_C - V_A) + (V_A - V_D) = I_2 3 + I_3(1+2) - (-5) \Rightarrow 3I_2 + 3I_3 = 5 \quad [3]$$

Note that we could have also computed V_B–V_C instead any of before equation. From equations [1], [2] and [3] we can solve for intensities, resulting:

$$I_1 = \frac{25}{7}\text{ mA} \qquad I_2 = -\frac{20}{21}\text{ mA} \qquad I_3 = \frac{55}{21}\text{ mA}$$

And potential on *A* can be computed knowing that potential on *D* is zero:

$$V_A = V_A - V_D = I_3(1+2) - (-5) = 3\frac{55}{21} + 5 = \frac{90}{7} \quad V$$

This network can also be solved taking in account that the only equation for rule of junctions can be written as a function of unknown potential on point A (V_A). If we compute the difference of potential between the endings of each branch, and assuming the senses for intensities shown on picture:

Branch AB: $V_B - V_A = V_{BA} = 2I_1 = 20 - V_A \quad \rightarrow I_1 = \dfrac{20 - V_A}{2}$

Branch CA: $V_C - V_A = V_{CA} = 3I_2 = 10 - V_A \quad \rightarrow I_2 = \dfrac{10 - V_A}{3}$

Branch AD: $V_A - V_D = V_{AD} = V_A = 3I_3 - (-5) \quad \rightarrow I_3 = \dfrac{V_A - 5}{3}$

Now, writing the rule of junctions for junction A:

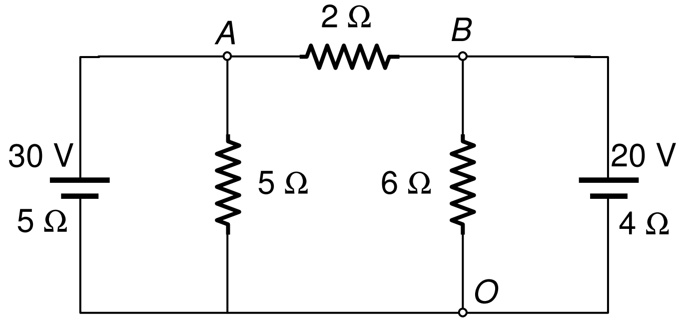
$$I_1 + I_2 - I_3 = 0 \Rightarrow \frac{20 - V_A}{2} + \frac{10 - V_A}{3} - \frac{V_A - 5}{3} = 0 \Rightarrow V_A = \frac{90}{7} \quad V$$

And intensities of current, substituting the found value for V_A :

$$I_1 = \frac{25}{7}\,\text{mA}\,; \qquad I_2 = -\frac{20}{21}\,\text{mA}\,; \qquad I_3 = \frac{55}{21}\,\text{mA}$$

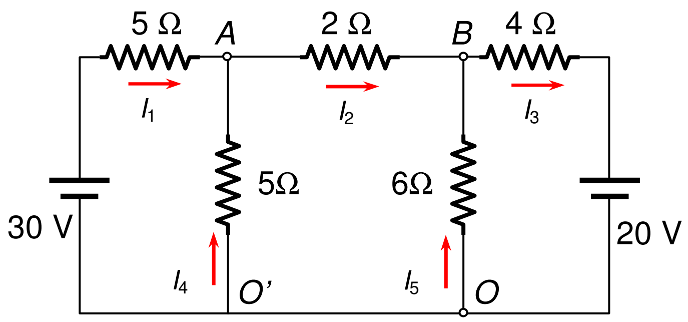
Example 5-3

Compute the intensity of current flowing from A to B (along branch AB) I_{AB}, and intensity from B to O, I_{BO}, by using Kirchoff's rules.



Solution

This network is made up by four junctions and three loops; but two junctions connected through a wire without resistance (named O and O') have



equal potentials, and both junctions can be taken as the same junction, being zero the intensity of current flowing along this wire without resistance.

So, 3-1=2 equations for rule of junctions and 3 equations for rule of loops could be written. Giving an arbitrary sense for intensities of current on branches, as can be seen on picture, the rule of junctions on *A* and *B* are:

$I_1 + I_4 - I_2 = 0$ [1] and $I_2 + I_5 - I_3 = 0$ [2]

And the three equations of rule of loops (taking clockwise sense on each one):

Loop on left: $5I_1 - 30 - 5I_4 = 0$ [3]
Loop on middle: $5I_4 + 2I_2 - 6I_5 = 0$ [4]
Loop on right: $6I_5 + 4I_3 - (-20) = 0$ [5]

From system of equations [1], [2], [3], [4] and [5], by using any method for solving it, we can solve for intensities of current:

I_1 = 3,217 A I_2 = 0,434 A I_3 = -1,739 A I_4 = -2,783 A I_5 = -2,174 A

And $I_{AB} = I_2$ = 434 mA and $I_{BO} = -I_5$ = 2,174 A

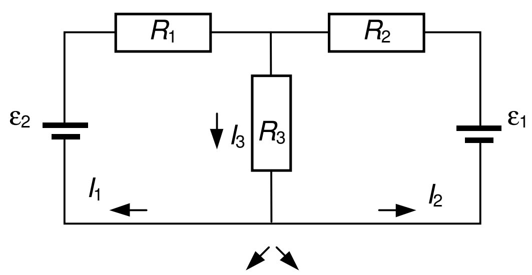
5.3 Superposition principle

When a linear network has several generators, intensities on different branches can be computed by solving the network in simple steps, applying the different generators one by one to the circuit (removing all the other generators). At the end, intensity on a branch can be computed as the addition of intensities due to each generator separately. This equality is the superposition principle, which can be stated as:

The intensity in a branch of a linear circuit having two or more generators equals the addition of intensities on such branch obtained for each one of the generator separately.

We must be careful, because they must be only removed ideal generators. If the circuit has a real generator (ideal generator with an internal resistance), only the ideal generator must be removed, but not the internal resistance.

In this way, the network of figure can be solved in two stages (a) and (b):



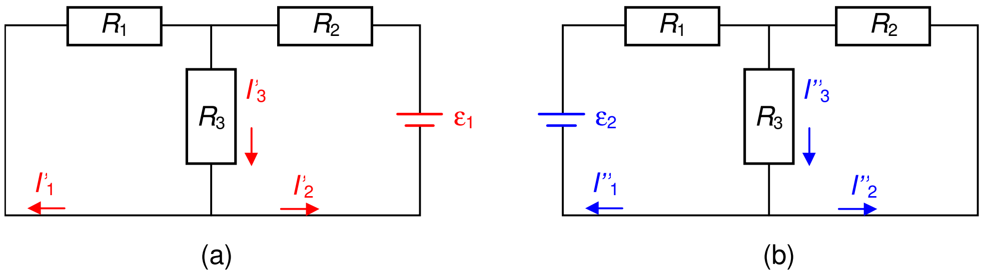


Figure 5-4. Superposition principle

Once calculated the intensities when each generator is acting, the total intensities can be obtained, by adding the partial intensities of (a) and (b):

$$I_1 = I'_1 + I''_1$$
$$I_2 = I'_2 + I''_2$$
$$I_3 = I'_3 + I''_3$$

Example 5-4

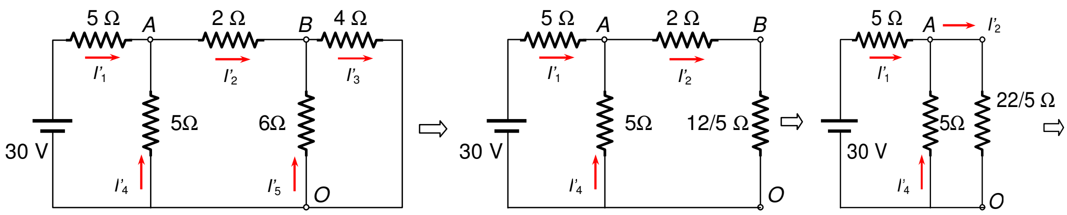
Applying the superposition principle, repeat example 5-3, checking that results are the same.

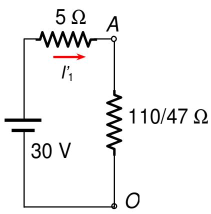
Solution

In this network we can find two generators. If we remove the 20 V generator, and after the 30 V generator, maintaining the same names for intensities on branches, we get:

Network with only 30 V generator

In order to calculate intensities on branches, resistors can be associated, as can be seen on next pictures:



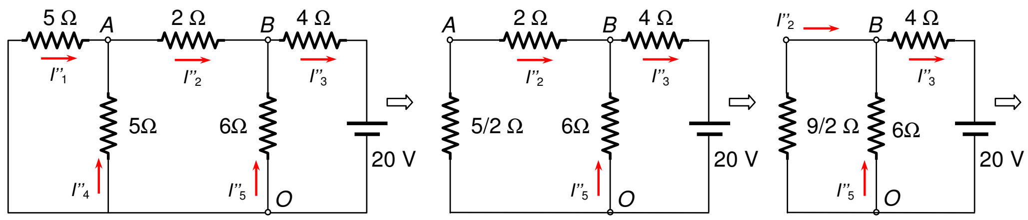


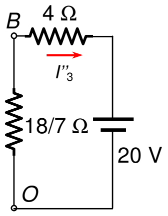
From this simple circuit, $I'_1 = \dfrac{30}{\dfrac{110}{47}+5} = 4{,}087\ \text{A}$

$$V'_{AO} = I'_1 \frac{110}{47} = 9{,}56\ V \qquad I'_2 = \frac{V'_{AO}}{\dfrac{22}{5}} = 2{,}174\ \text{A} \qquad I'_4 = \frac{-V'_{AO}}{5} = -1{,}913\ \text{A}$$

$$V'_{BO} = I'_2 \frac{12}{5} = 5{,}22\ V \qquad I'_3 = \frac{V'_{BO}}{4} = 1{,}304\ \text{A} \qquad I'_5 = \frac{-V'_{BO}}{6} = -0{,}870\ \text{A}$$

Network with only 20 V generator





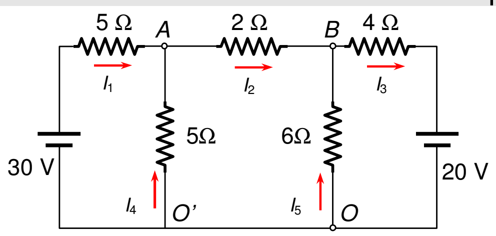
From this simple circuit,

$$I''_3 = \frac{-20}{\frac{18}{7}+4} = -3{,}043\ \text{A}$$

$$V''_{BO} = -I''_3 \frac{18}{7} = 7{,}83\ V \qquad I''_2 = \frac{-V''_{BO}}{\frac{9}{2}} = -1{,}739\ \text{A} \qquad I''_5 = \frac{-V''_{BO}}{6} = -1{,}304\ \text{A}$$

$$V''_{AO} = -I''_2 \frac{5}{2} = 4{,}347\ V \qquad I''_1 = I''_4 = \frac{-V''_{AO}}{5} = -0{,}870\ \text{A}$$

Complete network



$$I_{AB} = I_2 = I'_2 + I''_2 = 2{,}174 - 1{,}739 = 0{,}434\ \text{A}$$

$$I_{OB} = -I_5 = -I'_5 - I''_5 = 0{,}870 + 1{,}304 = 2{,}174\ \text{A}$$

5.4 Thevenin's theorem

Any active linear network with terminals between A and B is equivalent to a generator of electromotive force ε_T and internal resistance R_T, being ε_T the difference of potential between A and B and R_T the equivalent resistance of the passive network (after remove all generators) between A and B.

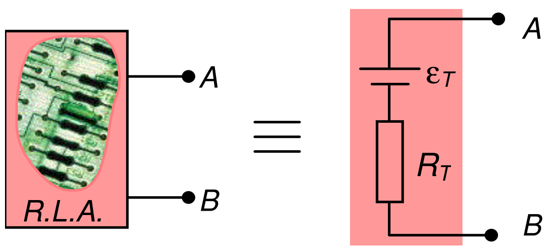
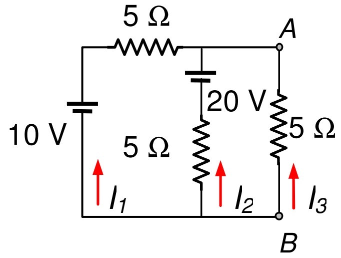


Figure 5-5 . Thevenin's theorem

This equivalent generator is called Thevenin's equivalent generator. The polarity of Thevenin's equivalent generator must be according with the sign of the computed V_A - V_B. The positive terminal must correspond to point where greater is the potential. (On picture, V_A - V_B should be > 0).

Example 5-5

Calculate Thevenin's equivalent generator between A and B on circuit of figure.



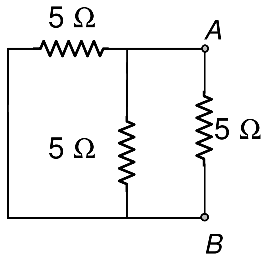
Solution

On first, we must assign a intensity of current with a sense for each branch of network; on picture can be seen this assignment. Application of Kirchoff's rules lead us to:

$$I_1 + I_2 + I_3 = 0 \qquad I_2 5-(20)-I_3 5=0 \qquad I_1 5-(10)-I_2 5-(-20)=0$$

Solving for intensities and V_{AB}: $I_1 = 0$ $I_2 = 2$ A $I_3 = -2$ A $V_{AB} = -I_3 5 = 10\,V$

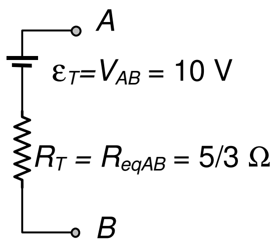
Equivalent resistance of passive network between A and B con be got after remove all generators and compute the equivalent resistance between A and B:



Three resistors are connected in parallel between points A and B. So, equivalent resistor will be:

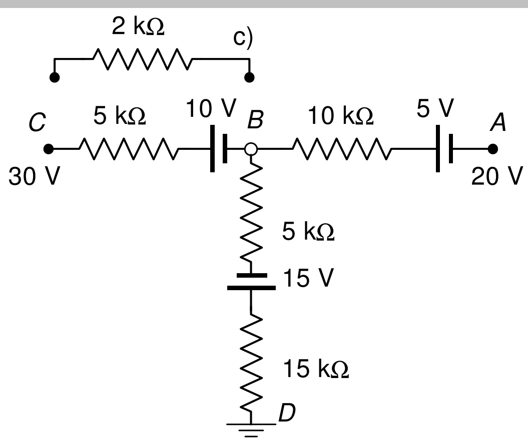
$$\frac{1}{R_{eqAB}} = \frac{1}{5}+\frac{1}{5}+\frac{1}{5} = \frac{3}{5} \Rightarrow R_{eqAB} = \frac{5}{3}\,\Omega$$

As potential on point A is higher than potential on B, positive terminal of Thevenin's equivalent generator must be connected to A, as can be seen on picture.



Example 5-6

Given the circuit on figure,



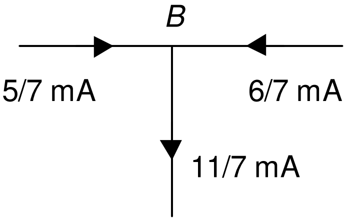
a) Calculate the intensities of current by means of Kirchoff's rules.
b) Calculate the equivalent resistance between C and B.
c) Determine Thevenin's equivalent generator between C and B, and compute the intensity of current that would flow through a 2 kΩ resistor added between C and B.

Solution

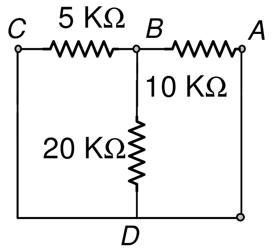
a) Writing equations of Kirchoff's rules (intensities on three branches have been taken going to point *B*:

$$I_1 + I_2 + I_3 = 0$$
$$V_C - V_D = 30 - 0 = 30 = I_1 5 - (-10) - I_3(5+15) - (15) \Rightarrow 5I_1 - 20I_3 = 35$$
$$V_A - V_D = 20 - 0 = 20 = I_2 10 - (5) - I_3(5+15) - (15) \Rightarrow 10I_2 - 20I_3 = 40$$

Resulting: $I_1 = \frac{5}{7}\text{mA}$ $\quad I_2 = \frac{6}{7}\text{mA}$ $\quad I_3 = -\frac{11}{7}\text{mA}$ $\quad V_{CB} = I_1 5 - (-10) = \frac{95}{7} V$



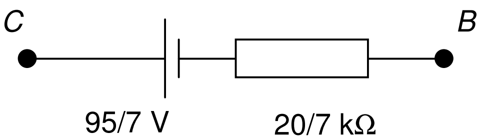
b) Removing all generators on network, passive network is:



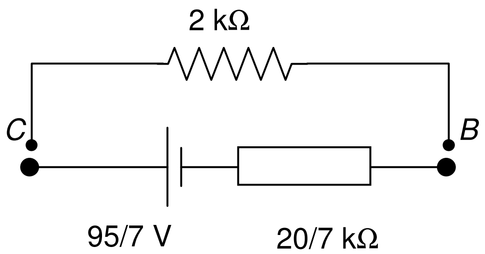
Between *C* and *B*, three resistors are connected in parallel, being its equivalent resistor:

$$\frac{1}{R_{eqCB}} = \frac{1}{5} + \frac{1}{20} + \frac{1}{10} = \frac{7}{20} \Rightarrow R_{eqCB} = \frac{20}{7} K\Omega$$

c) Then, Thevenin's equivalent generator is:



By adding a 2 kΩ resistor between *C* and *B*, the circuit is:

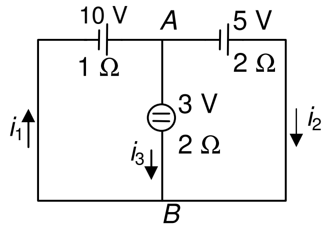


Being the intensity flowing along this circuit (clockwise sense):

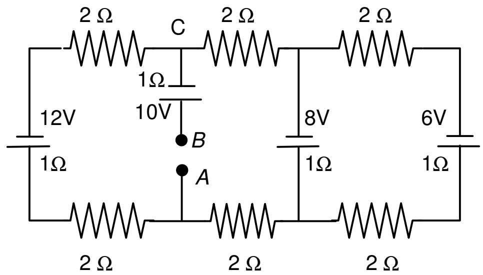
$$i = \frac{95/7}{20/7 + 2} = \frac{95}{34}\text{ mA} = 2{,}79\text{ mA}$$

5.5 Problems

1. In the circuit on picture, calculate the intensities in the three branches and the difference of potential between terminals of engine.
Sol: i_1 = 5,5 A; i_2 = 4,75 A; i_3 = 0,75 A; V_{AB} = 4,5 V

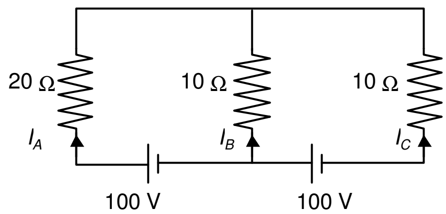


2. Find the difference of potential between A and B.
Sol: -0,20 V

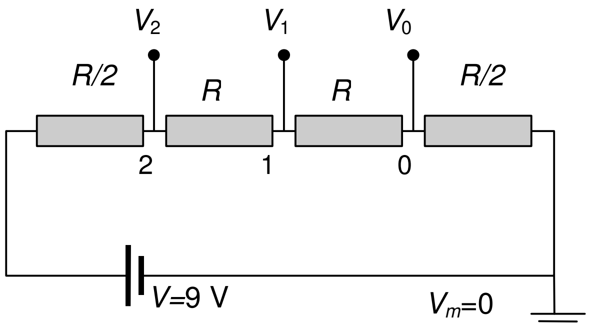


3. Find I_A, I_B and I_C in the network.

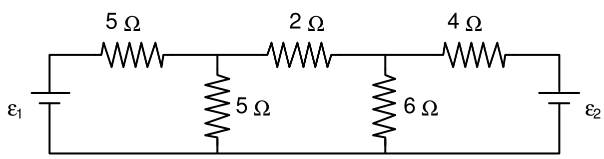
Sol: I_A = 6 A I_B = 2 A I_C = -8 A



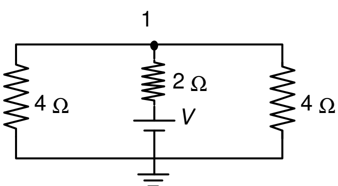
4. The diagram on picture represents the resistor divider of an analogical-digital converter. Calculate the intermediate voltages V_0, V_1, V_2.
Sol: V_2 = 7,5 V, V_1 = 4,5 V, V_0 = 1,5 V



5. On network, ε_1 = 30 V. Find ε_2 so the intensity flowing through the 2 Ω resistor was zero.
Sol: ε_2 = 25 V

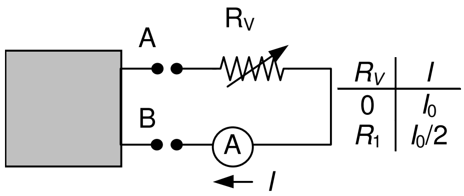


6. In the network on picture, find the voltage V so that voltage on junction 1 was 50 V.
Sol: V = 100 V



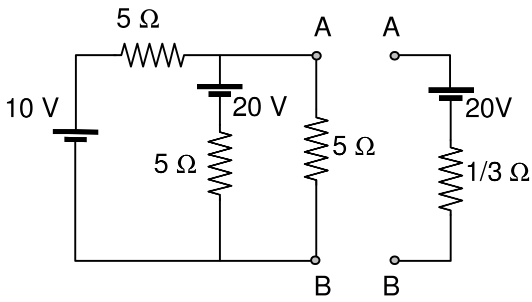
Equivalence theorems (Supperposition and Thèvenin)

7. Let's have an active linear circuit with terminals A and B: a branch with a variable resistor R_V and an ammeter with negligible internal resistance is connected between A and B. When R_V = 0, $I = I_0$, and when $R_V = R_1$, $I = I_0/2$. Justifying the answer, give values for:



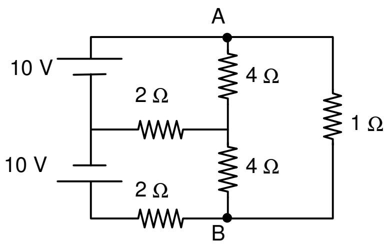
a) ε_T and R_T of Thevenin's equivalent generator.
Sol: a) $\varepsilon_T = I_0 R_1$, $R_T = R_1$

8. In network of figure:
a) Compute the Thevenin's equivalent generator between points A and B (left picture).
b) The branch drawn on right is added to network, connecting it between A and B. Explain if the generator with electromotive force 20 V on new branch consumes or supplies power, computing its magnitude.

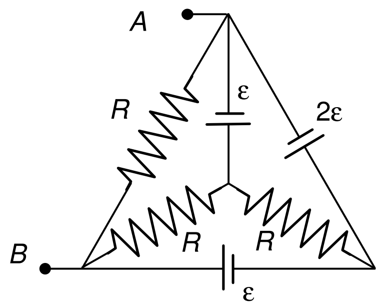


Sol: a) ε_T = 10 V , R_T = 5/3 Ω ;
b) Generated power = 100 W

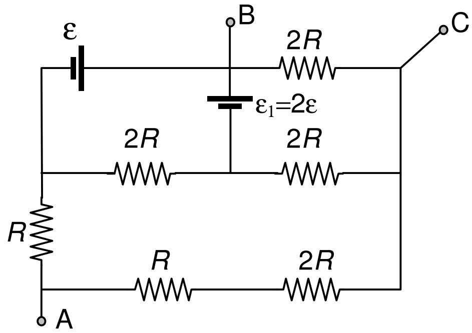
9. In network of figure, find the intensity flowing along resistor R = 1 Ω, applying Thevenin's theorem between A and B.
Sol: I = 20/27 A



10. In network of figure, calculate: a) Thevenin's equivalent generator (ε_T, R_T) between A and B.
b) Supplied or consumed power by a real generator of fem ε and internal resistance R, when connecting its negative pole to terminal A and its positive pole to terminal B of previous network.
Sol: a) $\varepsilon_T = 3\varepsilon$, ($B$ is the positive pole), $R_T = 0$;
b) $P = 6\varepsilon^2 / R$



11. In network on picture, calculate:
a) The difference of potential between A and B,
b) Equivalent resistor between A and C,
c) Power on device ε_1 (say if it is generated or consumed).
Sol: a) $V_{AB} = -\varepsilon$, b) $R = 6R/5$, c) $P = 2\varepsilon^2/R$



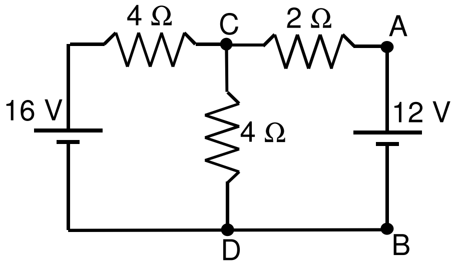
12. Given the network on picture:

a) Calculate the intensity of current flowing along branch from A to B.

b) Calculate lost power on resistor on branch CD.

c) Find Thevenin's equivalent generator between C and D, clearly showing its polarity.

d) If a new 9 Ω resistor is added between points C and D, by using the previous Thevenin's equivalent generator, calculate the intensity would flow along such resistor.



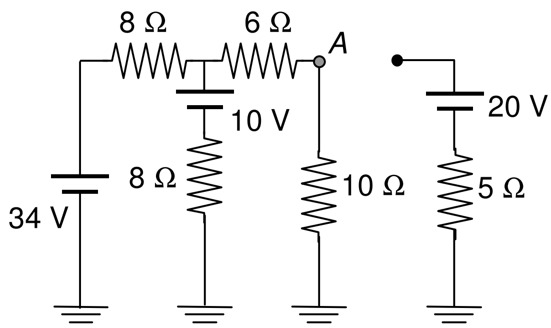
Sol: a) 1 A, b) 25 W, $\varepsilon_T = 10$ V $R_T = 1$ Ω, d) 1 A

13. In network on picture, calculate:

a) The equivalent resistor of passive circuit between points A and ground.

b) Voltage on point A and the Thevenin's equivalent generator of the circuit between points A and ground.

c) The intensity that would flow along branch of 20 V generator if positive terminal of such generator is connected to point A .



Sol: a) 5 Ω , b) $\varepsilon_T = 11$ V, $R_T = 5$ Ω, c) 0,9 A

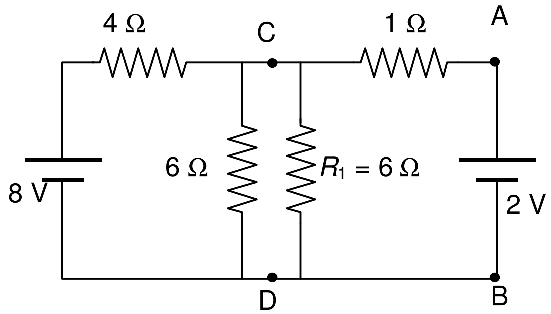
14. Given the network of picture:

a) Calculate the intensity of current would flow from point A to point B.

b) Calculate lost power on resistor R_1.

c) Find the Thevenin's equivalent generator between C and D, clearly showing its polarity.

d) If a new 5 Ω resistor is added between points C and D, by using the previous Thevenin's equivalent generator, calculate the intensity would flow along such resistor.



Sol: a) 26/19 A, b) 1,06 W, c) $\varepsilon_T = 48/19$ V, $R_T = 12/19$ Ω, d) 48/107 A.

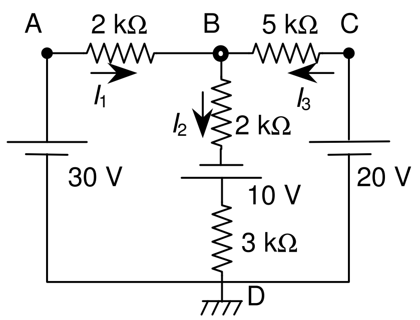
15. Given the network on picture:

a) Compute the intensities of branches I_1, I_2, and I_3 by means of Kirchhoff's rules.

b) Compute voltage on point B.

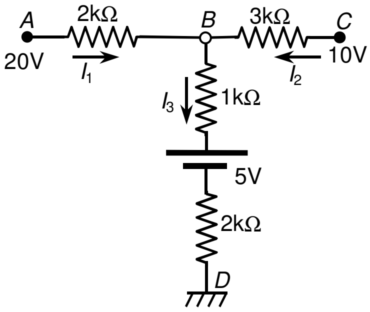
c) Calculate lost power on resistors of network.

d) Determine if the 10 V generator acts as a generator or as a receptor, computing the generated or the consumed power.



Sol: a) 5,56 mA, 5,78 mA, 0,22 mA b) 18,9 V

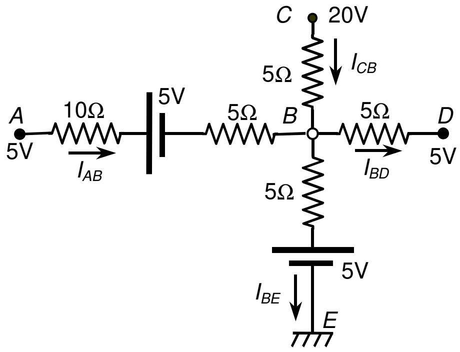
16. Given the network on picture:
a) Compute the intensities of branches I_1, I_2, and I_3 by means of Kirchhoff's rules.
b) Compute voltage on point B.



Sol: a) $I_1 = \dfrac{25}{7}$ mA $\quad I_2 = \dfrac{-20}{21}$ mA $\quad I_3 = \dfrac{55}{21}$ mA

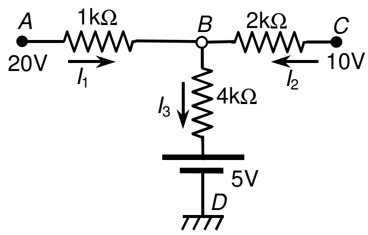
b) $V_B = \dfrac{90}{7}$ V

17. Given the network on picture:
a) Compute the intensities of branches I_{AB}, I_{CB}, I_{BD}, and I_{BE} by means of Kirchhoff's rules.
b) Compute voltage on point B.



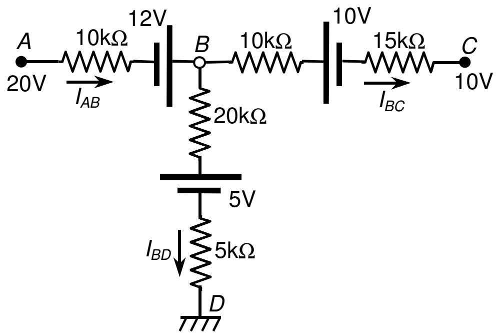
Sol: a) $I_{AB} = -3/5$ A $\quad I_{CB} = 11/5$ A
$I_{BD} = 4/5$ A $\quad I_{BE} = 4/5$ A $\quad$ b) $V_B = 9$ V

18. Given the network on picture:
a) Compute the intensities of branches I_1, I_2, and I_3 by means of Kirchhoff's rules.
b) Compute voltage on point B.



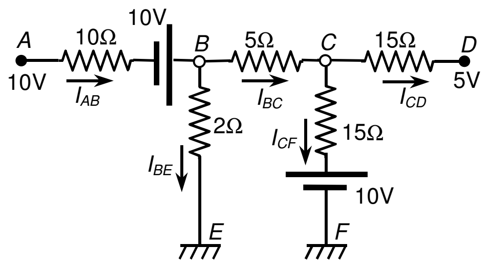
Sol: a) $I_1 = 5$ mA $\quad I_2 = -5/2$ mA $\quad I_3 = 5/2$ mA
b) $V_B = 15$ V.

19. Given the network on picture:
a) Compute the intensities of branches I_{AB}, I_{BC} and I_{BD}, by means of Kirchhoff's rules.
b) Compute voltage on point B.



Sol: a) $I_{AB} = 13/15$ mA $\quad I_{BC} = 2/15$ mA
$I_{BD} = 11/15$ mA $\quad$ b) $V_B = 70/3$ V.

20. Given the network on picture:
a) Compute the intensities of branches I_{AB}, I_{BE}, I_{BC}, I_{CF} and I_{CD} by means of Kirchhoff's rules.
b) Compute voltage on point B.



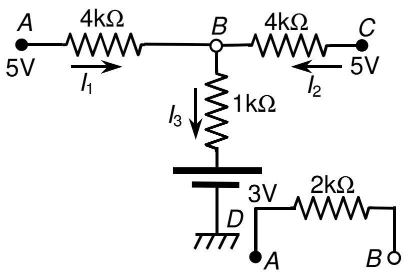
Sol:
a) $I_{AB} = 55/34$ A $\quad I_{BE} = 65/34$ A $\quad I_{BC} = -5/17$ A $\quad I_{CF} = -16/51$ A $\quad I_{CD} = 1/51$ A
b) $V_B = 65/17$ V $\quad V_C = 90/17$ V

21. Given the network on picture:

a) Compute the intensities of branches I_1, I_2, and I_3 by means of Kirchhoff's rules.

b) Find the Thevenin's equivalent generator between *A* and *B*, clearly showing its polarity.

c) In parallel to points *A* and *B* of network, a new 2 kΩ resistor is connected. Compute the intensity would flow along this resistor, clearly showing its sense.



Sol:a) I_1 = 0,333 mA; I_2 = 0,333 mA; I_3 = 0,666 mA;
b) V_{AB} = 4/3 V R_{eq} = 2/3 kΩ c) I = 0,5 mA.

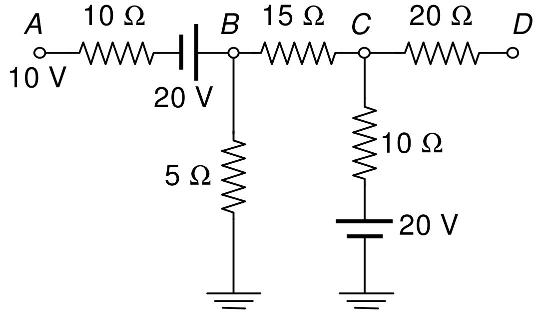
22. Given the network on picture:

a) Compute the intensities of branches I_{AB}, $I_{B\text{-}GROUND}$, I_{BC}, $I_{C\text{-}GROUND}$ and I_{CD}.

b) Compute voltage on point *D*.

c) Find the Thevenin's equivalent generator between *D* and *Ground*, clearly showing its polarity.

d) Which intensity would flow along a 10 V of counter-electromotive force receptor connected between *D* and *Ground*?



Sol: a) I_{AB} = 32/17 A; $I_{BC} = I_{C\text{-}GROUND}$ = -6/17 A; $I_{B\text{-}GROUND}$ = 38/17 A; I_{CD} = 0;
b) V_D = 280/17 V; c) ε_T = 280/17 V; R_T = 450/17 Ω; d) I = 11/45 A

GLOSSARY

Dipole. Any electrical device with two terminals.

Electric network. Set of dipoles connected between them making up interconnected loops.

Junction. Point of a network where three or more dipoles are joined.

Branch. Piece of a circuit between two consecutive junctions.

Loop. Enclosed circuit made up by branches, in such way that starting on a point and following a sense, can be returned to the same point crossing only once each branch. A loop doesn't contain any other inside.

Rule of junctions. The algebraic addition of intensities on a junction is zero.

Rule of loops. The addition of the drop of potential in all the branches along a loop is zero.

Thevenin's theorem. Any active linear network with terminals between A and B is equivalent to a generator of electromotive force ε_T and internal resistance R_T, being ε_T the difference of potential between A and B and R_T the equivalent resistance of the passive network between A and B.