

## Unit 2: DIGITAL DESIGN FUNDAMENTALS

2016

### **EXERCISES**

2.1-Getting the truth table	page 2
2.2-equivalence between logic and circuit function	page 2
2.3-Circuit analysis (reverse engineering)	page 3
2.4-Obtaining the logic function: canonical forms	page 3
2.5-Simplification of functions: Karnaugh maps	page 5
2.6-Circuit Implementation	page 6

### **Extension**

2.7-Truth Tables	page 7
2.8-Canonical Forms	page 10
2.9-Simplification by Karnaugh	page 12

## EXERCISES

---

### 2.1 – Obtaining the truth table

---

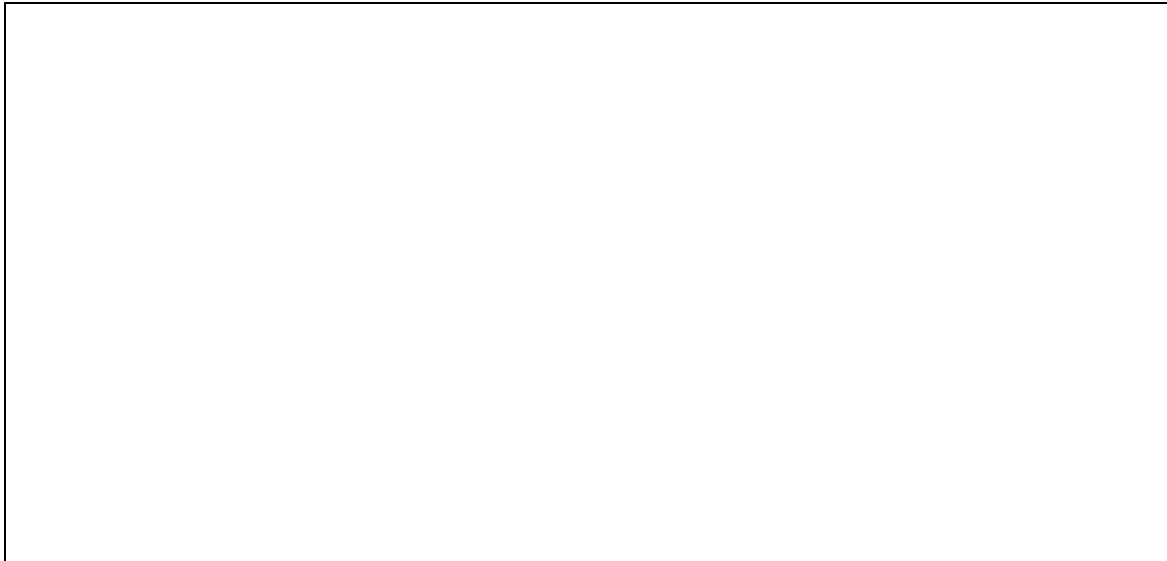
2.1.1. In a farm want to perform a quality control for the classification of the qualities of the oranges. To do this, there are 3 types of sensors: a camera, a caliper and a scale. These sensors provide input information on the color (C), diameter (D) and weight (P) of the orange. Orange is a premium quality (X) when color is enabled and at least one of the other two parameters. An orange is of normal quality (N) activated if the weight and / or diameter and yet the color does not work.

Get the truth table of the digital circuit that regulates the quality controller.

SOLUTION:

2.1.2. We are accountable, control the level of a water irrigation. This deposit has two water level sensors, one indicating the lowest level (Nm) and a maximum (NM), these sensors are active at high level. Fresh water enters the tank through an inlet tap water pouring from the top level, and that can be opened by activating the Ge signal (high level opens and closes at low level). The water leaves the tank through a tap at the bottom, and that can be opened by activating the Gs signal (high level opens and closes at low level). Finally there is an emergency entrance E (active high), which closes the entrance and exit is opened regardless of water levels that have the deposit). If not checked the emergency signal, the system should maintain the level between the minimum and maximum. When the minimum level is reached, you must close the output valve and open the inlet valve. If you reach the maximum level, the need to close the inlet valve and open the exit. If the level is between the minimum and maximum should be close both taps.

SOLUTION:



---

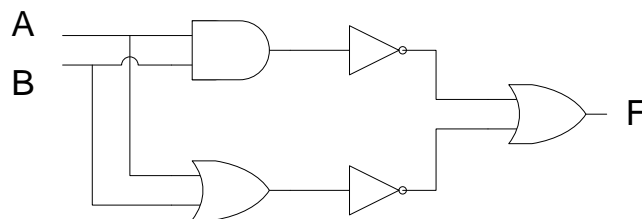
---

## 2.2 – Equivalence between logic and circuit function

---

---

2.2.1. Given the following circuit, obtain the equivalent logical function:



SOLUTION:



2.2.2. Given the following logic function, obtain the equivalent circuit:

$$F = (A \cdot B + C) \cdot \overline{D}$$

SOLUTION:



---

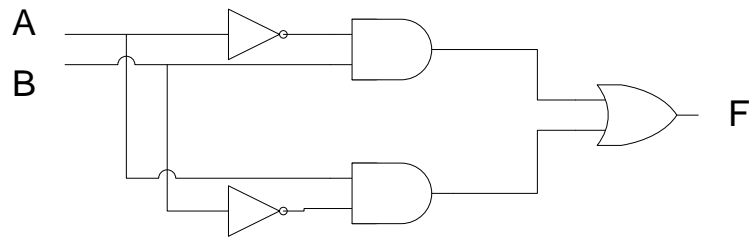
---

## 2.3 – Circuit analysis (reverse engineering)

---

---

2.3.1. Given the following circuit, obtain its truth table:



**SOLUTION:**

## 2.4 – Obtain the logic function: canonical forms

2.4.1. Given the following truth table:

Input				Output
D	C	B	A	S
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Enter the conjunctive and disjunctive canonical equations of the output S.

**SOLUTION:**

2.4.2. A company dedicated to manufacturing packaging boxes. The boxes are stored based on their size and color, so they are assigned two codes, first serve to indicate the size and the second to indicate the color. Four bits are used, two for code size and two for the color code. The boxes are manufactured in 4 different sizes (A, B, C and D) and three colors, codes of sizes and colors below.

Size	Code (T1 T0)
A	00
B	01
C	10
D	11

Colour	Code (C1 C0)
Red	00
Blue	01
Black	11

The boxes are stored in three different stores, according to the following criteria:

- The first store kept the boxes of any size to color.
- The second store will keep the red boxes with size C or D.
- In the third store will put the remaining boxes.

The information on the type of box and its color is written in a bar code that attaches to each of the boxes so that, for storage, the boxes are placed on a conveyor belt which divides into three paths, from the bar code information is forwarded to be taken to store them properly. To read the code uses a gun that reads the barcode and sends the information to the combinational circuit that bypasses the boxes to their destination.

Write the truth table of the combinational circuit which is responsible for diverting the boxes, consider that a '1' indicates the path to follow.

Colour		Size		trayectory 1	trayectory 2	trayectory 3
C1	C0	T1	T0			
0	0	0	0			
...	...	...	...	...	...	...
1	1	1	1			

(It is considered that when the circuit is activated so that the box follow the path 1, the case is brought to the warehouse, in the same way, when you select the path 2, the box is taken to the second store and when you turn path 3 the box leads to the third store). From the truth table you want to know what is the canonical equation for the path 1?

**SOLUTION:**

2.4.3. Picture yourself using the sum function the following function:

$$f = \prod_{D,C,B,A}(0, 2, 3, 14) \cdot \prod_{\Phi}(1, 10, 15)$$

SOLUTION:

2.4.4 Productorio Represent the function using the following function:

$$f = \sum_{D,C,B,A}(1, 2, 11, 12, 15) + \sum_{\Phi}(5, 7, 8)$$

SOLUTION:

## 2.5 –Simplification of functions: Karnaugh maps

2.5.1. Given the following truth table:

<i>Entradas</i>				<i>Salida</i>
<i>D</i>	<i>C</i>	<i>B</i>	<i>A</i>	<i>S</i>
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Write the two equations of the S obtained by simplifying (by ones and zeros by) using Karnaugh maps.

SOLUTION:

2.5.2. Get the logic function output **simplified** "segment **G**" Circuit BCD display "if the truth table corresponding to that output is:

<i>Input</i>				<i>Output</i>
<i>D</i>	<i>C</i>	<i>B</i>	<i>A</i>	<i>Segment G</i>
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

**SOLUTION:**

## 2.6 – Circuit Implementation



2.6.1. In the assembly of a car manufacturing plant is to embed a digital circuit that is able to control the opening and closing of two gates (S1, S2) through which vehicles must pass. The gates are controlled in terms of three characteristic parameters of the vehicle (C, S, P). Where: C: Quality control of the vehicle. S: indicates whether the vehicle has been a soldier or not. P: indicates whether the vehicle has been painted or not. S1 gate must be opened whenever the vehicles are also welded or painted and have passed quality control. S2 gate opens whenever the vehicles are not soldiers, regardless of meeting the quality control.

Implement a simplified digital circuit.

SOLUTION:

2.6.2. Want to implement a circuit that controls the lighting of the flashing lights of a car. The system has a lever that when in raised position generates a signal "PS" to activate the flashing lights on the right ("LD"). If you are in lowered position generates a signal "PB" to activate the flashing lights on the left ("LI"). If the lever is in an intermediate position (not rise or fall) does not generate any signal and therefore will not turn on any lights.

For the system to have the operation described is necessary to introduce an engine ignition key and is in the contact position, thereby generating the signal "C".

The system has an additional input fault is active when the signal generated by "A" by turning the four indicators regardless of whether the contact is given or not.

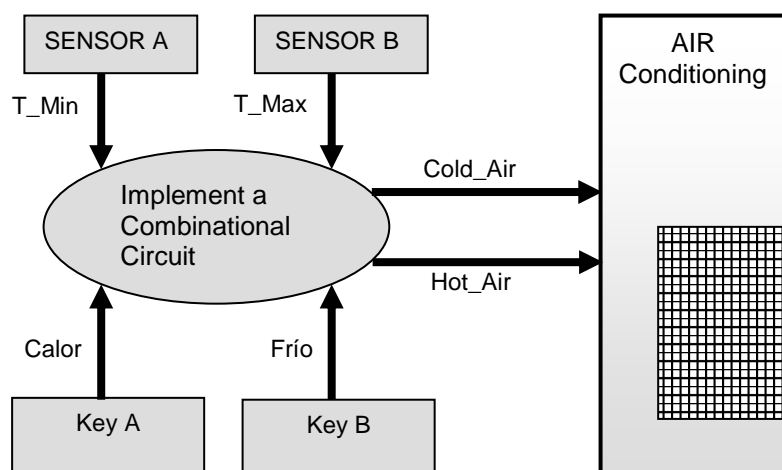
Implement a simplified circuit.

SOLUTION:

2.6.3. Want to implement a combinational circuit to try to keep between two values, the temperature of a room. This will have an air conditioner, two manually operated switch, and two temperature sensors.

Each sensor has an associated signal that is activated when temperature is reached in that sensor. A sensor activates the signal "**T\_Min**" when the ambient temperature is **higher** than the **minimum** required, and active sensor signal B "**T\_Max**" when the ambient temperature is **above** the required **maximum**.

Similarly, each switch also has an associated signal. When the switch is actuated signal is activated to "**heat**" and when pressed the switch B signal is activated "**cold**."



The circuit to implement indicate the operating mode of the air conditioner through the activation of the signals "**Cold\_Air**" (indicating that cold air to be ejected) or "**Hot\_Air**" (indicating that hot air is expelled). They must meet the following criteria:

- ❑ When the ambient temperature **does not exceed** the minimum temperature hot air must be expelled except when the signal is active "**cold**" that in that case, do not expel or heat or cold air
- ❑ When the ambient temperature is between the allowed values will not eject or hot or cold air except when the signal is active "**cold**" cold air to be ejected or the signal "**heat**" hot air is expelled
- ❑ When the ambient temperature **exceeds** the maximum temperature will blow cold air required except when the signal is active "**heat**" which in that case, do not expel or hot or cold air
- ❑ In the event that signals "**cold**" and "**heat**" is activated **at the time** should be ignored, the mode of operation will be the same if not enabled

Implement the circuit at minimal cost.

SOLUTION:

## Extension

---

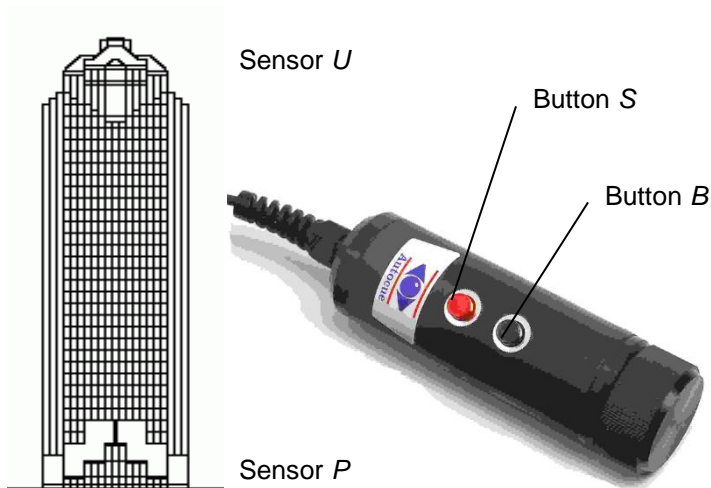
### 2.7 – Truth Tables

---

2.7.1. The pallet of glass cleaner to the Empire State Building has capacity to reach each of the 256 floors of this landmark building. So that workers can access each of the floors, the platform includes a driver that allows scrolling up and down.

The engine control mechanism has:

- Two sensors  $U$  and  $Q$ . The  $U$  sensor is active high only if the platform is located on the top floor (floor 256). The  $P$  sensor is active high only if the platform is on the first floor (floor 1). In the event that the dais is situated between two floors, both sensors are disabled.
- Two buttons  $S$  and  $B$ : Workers strike  $S$  and  $B$  buttons to raise and lower, respectively, the stage. The control signals generated by these buttons are active high.



Depending on the sensors and buttons, the motor control mechanism triggers two signals  $M$  and  $S/\bar{B}$ .  $M$  must be activated high when you want to put the engine running. In that case the signal  $S/\bar{B}$  indicates if you want top ( $S/\bar{B} = 1$ ) or Down ( $S/\bar{B} = 0$ ) the stage. When  $M$  is deactivated, the engine is stopped and the state  $S/\bar{B}$  is considered immaterial.

Fill in the table below, corresponding to the function performed by the motor control mechanism, taking into account the following specifications.

- If workers only pressed the  $S$  button, the platform must rise unless it is on the top floor, in which case the platform should not travel.
- If workers only pressed the button  $B$ , the platform must be lowered unless it is on the first floor, in which case the platform should not travel.
- If workers strike  $S$  and  $B$  buttons at the same time, the platform must not move.
- Otherwise, the platform must not move.

U	P	S	B	M	$S/\bar{B}$

2.7.2. The Dry River bridge has one lane in each direction for the passage of motor vehicles. However, due to recent rains, the structure has deteriorated, and each lane can support only the weight of three vehicles maximum. Moreover, adding the number of vehicles in both lanes, it should never exceed four.

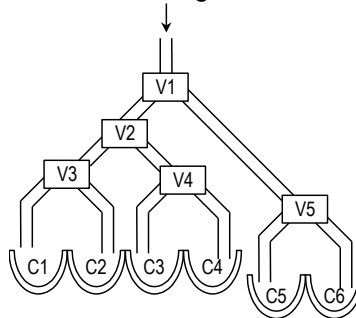
If for some reason, were housed in the bridge five or more vehicles, the bridge would collapse instantaneously (ie there can be no more than four vehicles on the bridge). To prevent this catastrophe from happening, two lights were installed, one at the entrance to each lane (right lane  $S_R$ , and if left lane).

It also has a timer for each lane vehicles, natural binary indicating the number of cars at all times ( $I_1, I_0$  to the left,  $D_1, D_0$  to the right lane).

Indicate what the truth table represent the problem, assuming that the semaphore is 1 for the light is red, and when is 0, the light is green.

---

2.7.3 A firm fruit sorting machine has the following 6-baskets, C1 to C6:



To set the path of the fruit to the racks, the valves V1 to V5 should take the value 0 if the fruit should go to the left of the figure and the value 1 if you must go to the right. The information available about each piece of fruit is:

- a) The weight. With two bits called P1P0, indicates that the fruit is: 00 - Extra Large, 01 - large, 10 - medium, 11 – small
- b) The existence of defects. With a bit D, is indicated by the value 0 that the piece is flawless, and with it the value 1 is

We have been asked to establish the truth table of the valves that govern the way the pieces of fruit sorting machine in that, taking into account the characteristics of the fruit from each basket:

- The basket C1 must be extra-large parts without defects
- The basket C2 should be large pieces without defects
- The basket C3 must be flawless medium-sized pieces
- The basket C4 small parts must be flawless
- The basket C5 must be defective parts with extra-large or large
- The basket C6 should be the other parts

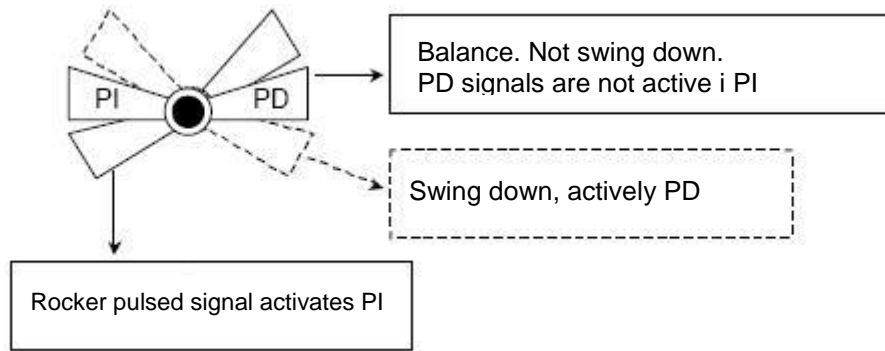
**NOTE:** The order of the variables in the truth table must be D, P1 and P0

---

2.7.4. We want to build the circuit controlling the movement of a conveyor that can move indefinitely in both directions (left and right).

To determine if the tape has to move and the direction of movement, the operator has a rocker button that generates two mutually exclusive signals (see figure below).

- A)** If the signal indicates that PD is on the rocker switch is in the position of moving to the right.
- B)** If the signal is activated PI indicates that the rocker switch is in the position of moving to the left.
- C)** When the rocker switch is released, none of the two signals is active. Both signals are **active low**.



To allow movement of the tape without the operator is continually pressing the swing have added a couple of sensors that indicate whether the tape is already in motion, / CD i / CI, with the following meanings:

**A)** If the signal / CD (respectively / CI) is active means that the tape is currently moving to the right (respectively left)

**B)** Both signals are **active low**

To govern the tape, the control circuit to be designed has to generate two outputs:

**A)** Signal M / P (Move = 1 / Stop = 0), I

**B)** Signal D / I (Right = 1 / Left = 0). Obviously, if the signal M / P = 0 the value of the signal D / I is indifferent.

The operation of the system must follow these rules:

1. If the tape is stopped, pressing the saw must start the tape running in the desired direction.
2. If the tape is currently in progress (indicated by the activation of a sensor / CD / CI), the film has to continue moving in the current sense if the rocker is pressed in the same direction of travel or not pressed.
3. 3. If the tape is currently in progress, pressing the swing in the opposite direction to the current motion is to stop the tape.

Required: Describe how the previous system using a truth table. The order of the variables (highest to lowest weight) must be / CD, / CI, / i PD / PI. The order of the outputs has to be K / P i D / I.

---

## 2.8 – Canonical Form

---

2.8.1. Obtain the canonical form follows function by maxterms product:

$$f(D, C, B, A) = (c + \bar{d} + a) \bullet (a + b + c + \bar{d}) \bullet (d + a + \bar{b} + \bar{c}) \bullet (\bar{b} + c + d + a)$$

2.8.2. What is the disjunctive canonical equation represents the following function?

$$f(d, c, b, a) = 1$$

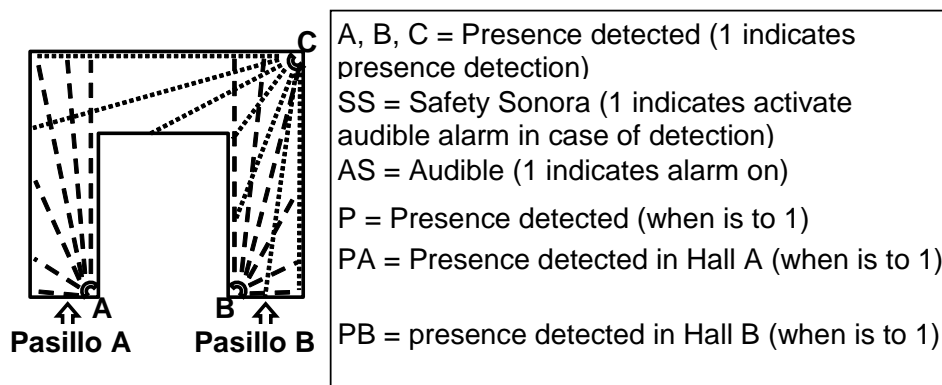
2.8.3. Let the function  $f(c, b, a) = \sum_{c, b, a} (2, 4, 6) + \sum_{\emptyset} (1)$ . Enter the conjunctive canonical equation representing the function.

2.8.4. Obtain the canonical form follows function by sum of minterms:

$$f(c, b, a) = \bar{c} \cdot a$$

2.8.5. In the building of the figure below, have installed three presence sensors (A, B and C). These sensors are always in operation and each covers an area (not exclusively) of detection. The system has a function selection input SS (Security Sonora), which determines whether or not an output enable ACE (buzzer) should detect the presence inside the building. The system must indicate whether there is presence (activating the output P), and if possible, indicate whether it is in the hall A (activating the output PA), or in the hall B (activating the output PB), or both in case several subjects).

**NOTE: By the provision of sensors, it is impossible to sensor B activation without activation of the sensor C.**



Enter the disjunctive canonical form correctly referred to the previous system.

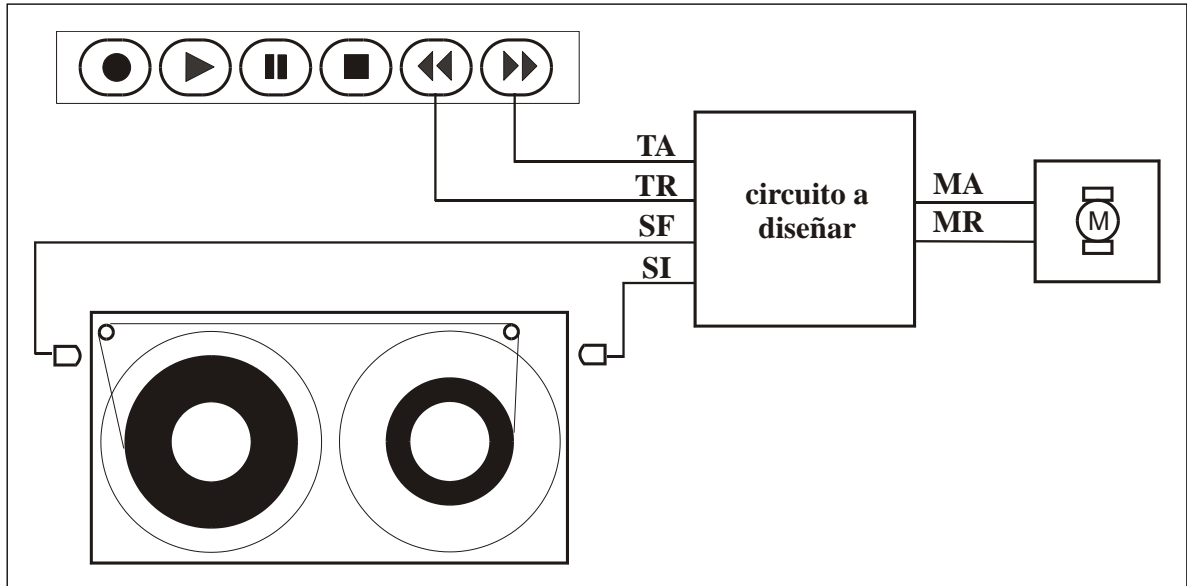
2.8.6. You want to design a control circuit part of a VCR. The circuit will be responsible for the activation of two binary signals MA and MR to begin the process of fast forward or rewind the tape (respectively) in response to pressing keys fast forward or reverse direction by the user. As shown in the figure, pressing the fast forward button activates a signal TA and pressing the backspace key activates a signal TR. The unit also has two sensors that detect when you've reached or trailing edge of the tape, activating the signal SI or SF, respectively. The circuit operation is as follows:

- Press a key forward or reverse should lead to the activation of the appropriate motor signal (MA for advancement, MR for reverse), except when one of the sensors indicate that the tape is the extreme position that is prevented (SF activation prevents the progress, prevents back SI), in which case they should turn the engine to prevent damage to the tape or the drive mechanism.



- If the user presses keys simultaneously forward and back, no MA or MR is activated, except when the tape is in one of two extreme positions, in which case that signal must be activated to start the engine drag the tape in the sense that there are thwarted.

All signals are active high ("1").



- Make the truth table of the proposed circuit. Observe the following order for the input variables: SF, SI, TA, TR.
- Get the disjunctive and conjunctive canonical functions for the function MA (summation notation and productorio). Observe the order in the previous section for the input variables: SF is more variable weight.

## 2.9 – Karnaugh Simplification

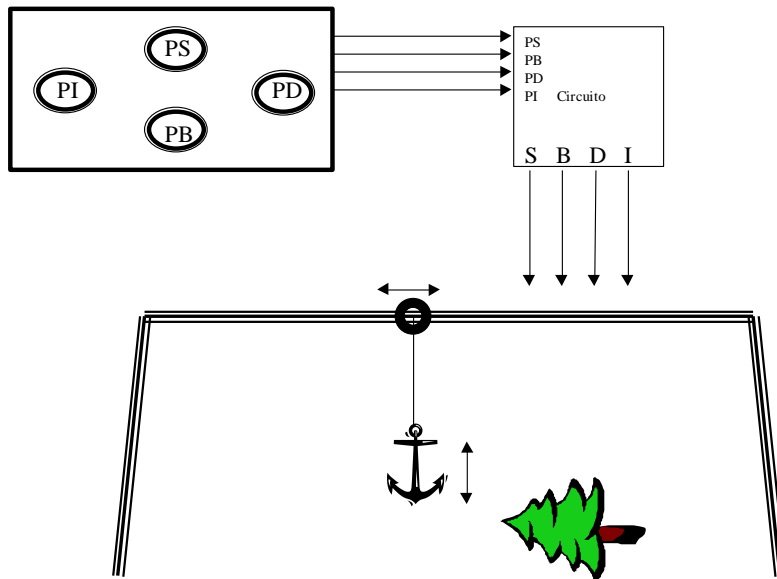
2.9.1. Simplify both zeros by some as the following function:

$$f = \sum_{C,B,A} (0,1,2,3)$$

2.9.2. In the old factory has installed a crane to move trees from one area to another plant. This crane has a hook that can make four moves: Up, Down, Right and Left. You can also make four combined movements: up and right, up and left, down and right, down and left. The hook has four signals: S, B, D, and R to indicate the movement to be performed. To operate the bridge crane has a control panel with four buttons, each of them to move the hook to suit the operator. The buttons are called PS, PB, PI and PD. To press a button it takes a whole hand. Only one operator can handle the panel, and the operator can only use two hands. Want to make a circuit that governs the bridge crane in accordance with the following specifications:

- If you press one of four buttons, the hook should move in the direction as indicated by the button, activating the signal.
- If both buttons are pressed simultaneously indicating one of the four movements combined, the two signals to be activated for the hook to make the right move.

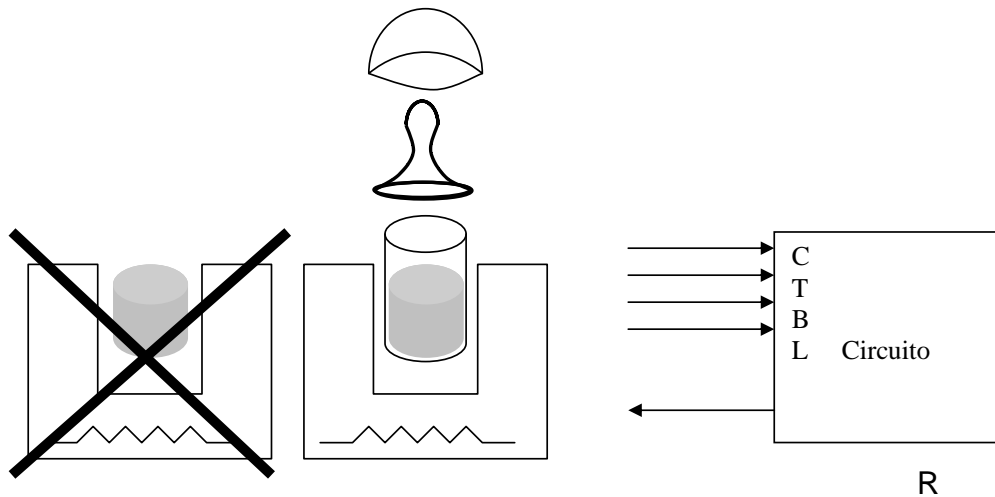
- If both buttons are pressed simultaneously indicating contradictory movement (eg, up and down or left and right) hook should not move.
- Note: All inputs and outputs are active with 1, and inactive with a 0.



- Make the truth table logic function. Please follow the following order for entries: PS, PB, PI, PD, and the following order for the outputs: S, B, I, D.
- Indicate the disjunctive and conjunctive canonical forms of the logic function from the truth table of the above statement to output B.
- Get the minimum expression of the logic function by simplifying Karnaugh, for just as zeros to the output B.

2.9.3. Applying to the control circuit of a bottle warmer. This circuit has four inputs: C, T, B and L. C entry indicates that the bottle is capped. The entry T indicates that the bottle is attached to the nipple. The entry B indicates a bottle in the correct position for heating, ie, the bottle is within or bottle warmer. Finally, the input L indicates that there is milk in the warmer. The circuit has an output R, which when activated put into operation a resistance that heats the milk. The circuit operation is as follows:

- If there is a bottle in the correct position and has milk must be heated with or without placing the nipple and as long as the bottle is not clogged.
- The bottle may have nipple and / or be covered but not in the correct position, so do not be heated.
- Unable to enter the milk in a bottle warmer if not in the correct position.
- In any other case scenario, the bottle not be heated.
- Note: All inputs and outputs are active with 1, and inactive with a 0.



- A) Make the truth table logic function  $R = f(C, T, B, L)$ . Please follow the following order for tickets: CTBL
- B) Indicate the disjunctive and conjunctive canonical forms of the logic function R from the truth table of the above statement.
- C) Get the minimum expression of the logic function R by simplifying Karnaugh, for just as zeros.

2.9.4. In an assembly of ceramic is desired to incorporate a quality control process of production. The pieces are composed of 3 basic products: A, B and C. The pieces can be classified into two classes, class1 and class2. One piece will be of the class 1 if it is true that consists of product A and at least one of the other two products. One piece will be of the class2 if satisfied that it is composed of at least 2 of the three products.

What is the minimum expression of the output functions class1 and class2?

2.9.5. Want to implement a circuit with 4 inputs (DCB and A) and two outputs (S1 and S0). The canonical equations of the output circuits are:

$$S0 = \sum_{DCBA} (2,3,6,7,14,15) + \sum_{\phi} (8,9,10,11)$$

$$S1 = \sum_{DCBA} (1,5,13) + \sum_{\phi} (8,9,10,11)$$

Get the minimum circuit equations for the outputs S0 and S1 respectively.

2.9.6. Let  $A = a_1a_0$  and  $B = b_1b_0$  two natural numbers expressed in binary by two bits. Get the comparison logic functions  $(A \geq B)$  and  $(A \leq B)$  simplified.

2.9.7. Let  $A = a_1a_0$  and  $B = b_1b_0$  two natural numbers expressed in binary and simplified logic function  $F = (a_1 + a_0 + b_1 + b_0)(a_1 + \bar{a}_0 + b_1 + \bar{b}_0)(\bar{a}_1 + \bar{a}_0 + \bar{b}_1 + \bar{b}_0)(\bar{a}_1 + a_0 + \bar{b}_1 + b_0)$  that implements a comparison function of two numbers. Indicate which performs that function comparison: "F=(A>B)", "F=(A<B)", "F=(A=B)" o "F=(A≠B)".