MATHEMATICAL ANALYSIS

L4 - Recurrences **LIST 2**

1. Solve the recurrence and put it in explicit form

$$\begin{cases} a_{n+1} = \frac{a_n}{3} \quad , \quad n \geq 1 \\ a_1 = 2 \end{cases}$$

2. Solve the Hanoi's towers problem: $a_{n+1} = 2a_n + 1$, $a_1 = 1$, with the characteristic equation and check the result using the direct resolution.

3. Solve the linear second order recurrence (non homogeneous): $a_{n+2} = 2a_n + 3n$, with the characteristic equation.

4. Let the recurrence:

$$\begin{cases} a_{n+2} = a_{n+1} + 2a_n \\ a_1 = 0, a_2 = 3 \end{cases}$$

   a) Find a_5
   b) Solve the recurrence finding the general value of a_n

5. Let $a_{n+2} - 2a_{n+1} - 3a_n = 0,\ with\ a_1 = a_2 = 1$ a second order linear recurrence

   a) Solve it, using the characteristic equation.
   b) Solve the same non homogeneous problem with the right hand side of the equation $t_n = 2$
   c) Study the order of magnitude of the solutions of this exercise.

6. Let $a_{n+2} - 4a_{n+1} + 4a_n = 0,\ with\ a_1 = 2, a_2 = 8$ a second order linear non homogenous recurrence

   a) Solve it, using the characteristic equation.
   b) Solve the same non homogeneous problem with the right hand side of the equation $t_n = 2$
   c) Study the order of magnitude of the first and second solutions of this exercise.
   d) Solve the same non homogeneous problem with the right hand side of the equation $t_n = n - n^2$

7. Find the explicit form of the recurrence defined by

$$\begin{cases} a_{n+2} + 2a_{n+1} + a_n = 0 \\ a_1 = 0 \quad , \quad a_2 = \frac{3}{4} \end{cases}$$

8. Let the recurrence defined by

$$\begin{cases} a_{n+2} - 2a_{n+1} + a_n = 0 \\ a_1 = 0, a_2 = 1 \end{cases}$$

   a) Solve the recurrence a_n.
   b) Study the order of magnitude $\{a_n\}$ and $\{b_n\} = \{\log(n)\}$. Compare both recurrences.
   c) Now, redo a) y b) with the right hand side of the equation, 2^n.

9. Let the sequence

$$\begin{cases} 2a_{n+2} = 3a_{n+1} + 2a_n \\ a_1 = 0, a_2 = 1 \end{cases}$$

   a) Write the a_3, a_4, a_5.
   b) Solve the recurrence calculating a_n.
   c) Solve the same exercise with the right hand side of the equation, $6n$.

10. Find the particular solution of the recurrence

$$\begin{cases} a_{n+2} + 2a_{n+1} + a_n = 1 \\ a_1 = 1 \quad , \quad a_2 = 1 \end{cases}$$

11. Someone give you three stamps as a present. You decided to collect stamps as a hobby. The next year you buy three stamps for your collection. If you buy each year the double of stamps than the previous year. How many years do you need to have more than 10^6 stamps?