Mathematical Analysis

L1 - Exercises: Real numbers

1. Find $x \in \mathbb{R}$ such as:

   a) $2x + 3 < 3x - 8$

   b) $(x + 2)(x - 2) \geq -3$

   c) $\frac{2x-1}{x+1} \geq 1$

   d) $(2x + 1)^4 (x - 2)(x + 3) \leq 0$

2. Find $x \in \mathbb{R}$ such as:

   a) $|x - 3| \leq 8$

   b) $|x - 1||x + 2| < 3$

   c) $||x| + 4| \leq 5$

   d) $(x - 2)^2 \geq 4$

   e) $|2 - |x|| \geq 1$

   f) $|x - 1| > \frac{2x+1}{x+1}$ *(tip:$|x - a| > b$)*

   g) $\left|\frac{x-1}{x+1}\right| = \frac{x-1}{x+1}$

   h) $|x| - |x + 1| > 1$ *(tip:$|x + 1| < |x| - 1$; sol : $\emptyset$)*

   i) $|x^2 - 2| \leq 1$

   j) $||x - 1| + 2| < 3$

   k) $|2 - x^2| \leq 1 + 2x^2$.

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L1 -Additional exercises: Real numbers

1. Find $x \in \mathbb{R}$ such as $x^3 + 2x^2 - 5x > 6$.

2. Show that: $0 < a < b \Rightarrow a < \sqrt{ab} < \frac{a+b}{2} < b$.

3. If a, b and c are real numbers, show that:

   a) $0 < a < b \Rightarrow \frac{1}{b} < \frac{1}{a}$

   b) $0 < a < 1 \Rightarrow a^2 < a$. What about $a > 1$?

4. Find $x \in \mathbb{R}$ such as

   a) $\left|x^2 + 6x + 5\right| \geq 2\left|x - 3\right|$ *(tip: div; $x \neq 3$)*

   b) $2 < |x| + |2x + 2| < 4$.
   (tip: consider the regions; $x < -1$, $-1 < x < 0$, $x > 0$;
   sol: $]-2, -\frac{4}{3}\left[\cup \right]0, \frac{2}{3}[$)

5. a) Show that: if $x \in \mathbb{R} - \{0\}$, then $\left|x + \frac{1}{x}\right| \geq 2$

   b) Show that if $x, y \in \mathbb{R}$,

   $$\max(x, y) = \frac{1}{2}(x + y + |x - y|)$$
   $$\min(x, y) = \frac{1}{2}(x + y - |x - y|)$$

6. a) Show that: If $x, y \in \mathbb{R}$, then $|xy| \leq x^2 + y^2$

   b) When $|x + y| = |x| + |y|$?

7. Find —you can help yourself with a graph— the set $(x, y) \in \mathbb{R}^2$ such as:

   a) $|x| = |y|$

   b) $|x \cdot y| = 2$

   c) $|x| \leq |y|$

   d) $|x| - |y| \geq 2$

   e) $|x| + |y| \leq 1$

   f) $\max(|x|, |y|) < 2$