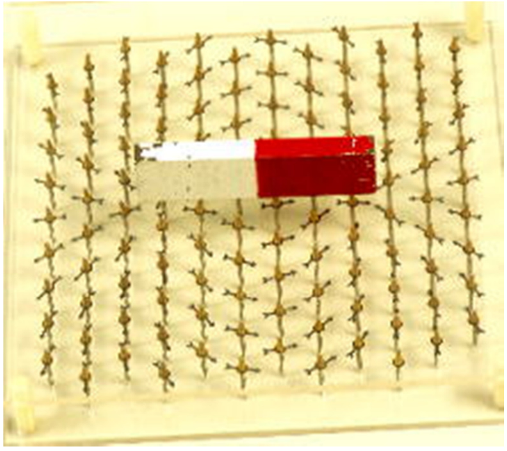
Unit 6

Magnetic forces



Objectives

- Describe the effects of a magnetic field on a moving electric charge.
- Calculate the magnetic force acting on a conductor with current and a loop inside a magnetic field.
- Find the magnetic moment of a flat current-carrying loop.
- Calculate the torque appearing on a flat current-carrying loop inside a uniform magnetic field.
- Explain the Hall Effect.

6.1 Introduction

Although the knowledge of magnetic properties of some minerals traces back to the ancient Greece, until 13th century any systematic study of their properties is carried out. In this period, Pierre of Maricourt experiences with magnets knowing two of their properties:

- The existence of two **magnetic poles rejecting each other if they are equal**, and **attracting each other if they are different**.
- The **persistence of both poles after breaking the magnet**.

The use of magnets for the bearings (compass needle) gave name to the poles of a magnet since both poles orient according the north–south terrestrial poles. The pole that orients to the north terrestrial pole was called North Pole, and a similar situation occurs for the South Pole.

This behavior allowed identify the Earth as a magnet and, since the poles of the same name reject each other and the ones of different name are attracted; a consequence of this fact is that **in the north geographic pole there is a magnetic south pole**, and **in the south geographic pole there is a north magnetic pole**.

In the same way that it happen in electric fields, the area of the space where there are magnetic properties is called a **magnetic field** and an observable consequence of its existence is that it acts on a moving charge, as we'll see in the following section.

6.2 Magnetic force acting on moving charges. Magnetic field

When electrostatic phenomena were studied, it was observed that an electric force $\vec{F} = q\vec{E}$ acts on an electric charge q when an electric field $\vec{E}$ exists.

However, the force on electric charge q doesn't depend only of its position but also of its velocity $\vec{v}$. Added to electric force, another component exists, that we'll call **magnetic force**, having the following features:

- It only acts if charge is moving
- It's perpendicular to velocity of the charge
- There is a fixed direction (for each point of space) in such way that when charge is moving in this direction, no one force acts on charge.
- The magnitude of force is directly related to the value of charge, to the value of speed and to the value of a fixed quantity for each point of the space

These features carry us to define the **magnetic field** $\vec{B}$ as the quantity gathering the direction and fixed module in each point that we have quoted. Therefore, we can express the magnetic force as:

Magnetic force acting on an electric charge inside a magnetic field	$\vec{F} = q\vec{v}\times\vec{B}$	Equation 6-1

It is necessary to remember that $\vec{v}\times\vec{B}$ is a cross product, and so the magnetic force will have the following features:

- It's a perpendicular vector to the two multiplied vectors, $\vec{v}$ and $\vec{B}$.
- Its magnitude is $F = q\ v\ B \sin\alpha$, being α the angle between $\vec{B}$ and $\vec{v}$. From this feature, when charge is moving parallel to magnetic field on a point, magnetic force is zero.
- Its sense can be obtained from the right hand rule (or screw rule).
- If the charge is negative, the sense of the force is the opposite.

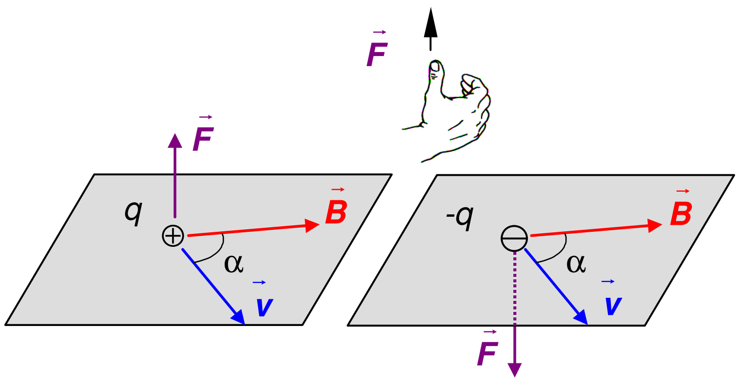
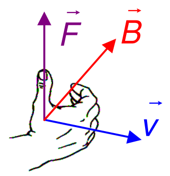


Figure 6-1. Magnetic force is perpendicular to the plane containing the velocity and the magnetic field vector

The right hand rule or the screw rule has been explained on Unit 0. It's useful to convert a rotating movement into a linear movement, determining the corresponding senses.
Refer to Unit 0 in order to find a more detailed explanation about these rules.



The unit of magnetic field $\vec{B}$ in the International System is the **tesla** (T), honoring Croatian scientist Nikola Tesla (1856-1943). From Equation 6-2 it can be stated that *there is a magnetic field of one tesla on a point of the space when moving a point charge of one coulomb perpendicularly to magnetic field with a speed of one meter by second, on charge acts a force of one newton*. For the majority of applications, 1T is a too big unit, as usual magnetic fields are in range of mT, being usually used the gauss (G): 1 G = 10^{-4} T.

As an example, the terrestrial magnetic field magnitude is roughly 0,6 G = 60 μT, and the magnetic field used in spectroscopy of magnetic nuclear resonance is the order of 0.5 to 30T.

As magnetic field is a vector quantity, lines being parallel to magnetic field on each point of the space are the magnetic field lines. These lines enable us "visualize" the direction of magnetic field at any point of space. An example can be seen in Figure 6-2.

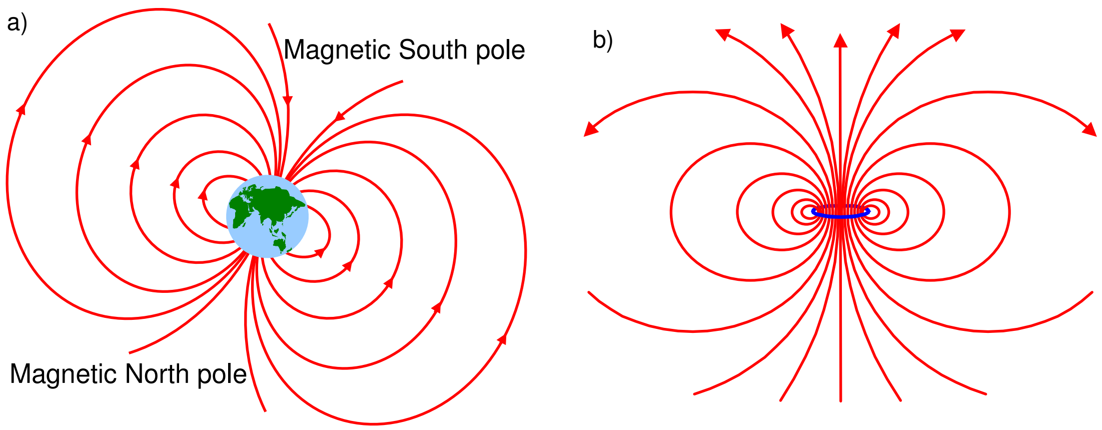


Figure 6-3. a) Lines of the magnetic field in the vicinities of the Earth and b) lines of the magnetic field produced by a circular current

There are two characteristics of magnetic field lines:

- The magnetic field lines are perpendicular to magnetic force on each point.
- The magnetic field lines are closed lines, since an only type of magnetic pole (monopole) can't be isolated. Until now, any one has been able to isolate a north or a south magnetic pole. If it would be possible, magnetic field lines starting on a monopole and finishing on another monopole would exist, but as a monopole can't be obtained, magnetic field lines are closed lines.

If we take in account not only the electrostatic forces but also the magnetic forces, the total force acting on a charge q with velocity $\vec{v}$ inside an electric and a magnetic field is

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}, \qquad \text{Equation 6-2}$$

called **Lorentz's Force**, expressed as a function of $\vec{E}$ and $\vec{B}$.

Movement of point charges inside a uniform magnetic field

Case a) If a point charge q have a parallel velocity $\vec{v}$ to the uniform magnetic field $\vec{B}$, the magnetic force acting on charge will be zero; so, the charge will move with Uniform Linear Motion.

Case b) If a point charge has a velocity $\vec{v}$ and enters inside a uniform magnetic field $\vec{B}$ perpendicular to $\vec{v}$ (no electric field exists), a magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ appears on charge. As this force is perpendicular to movement (no work produces), the speed (modulus of velocity) will be constant, and a circular path of radius r will be described by the charge. So, only a centripetal acceleration acts on charge $a_N = \frac{v^2}{r}$

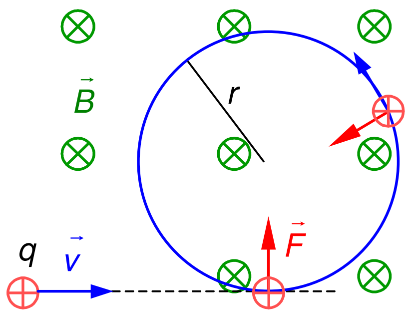


Figure 6-4. Charged particle moving in a magnetic field perpendicular to plane of paper[1]

If we call m the mass of charged particle, magnetic force and centripetal acceleration can be related through the third Newton's law, $\vec{F} = m\vec{a}$, being possible express the angular frequency of movement as a function of magnetic field, electric charge and mass:

$$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB} \Rightarrow \omega = \frac{v}{r} = \frac{qB}{m}.$$ A Uniform Circular Motion occurs.

[1] The perpendicular vectors to plane of paper are represented with symbol $\odot$ when they are going out from paper to reader and with symbol $\otimes$ when they are going in from reader to paper.

Case c) When velocity of particle is neither perpendicular nor parallel to magnetic field, velocity can be split in a component in the same direction than magnetic field and another component perpendicular. The first one produces a uniform circular motion (as it has been explained) and the second one, a uniform linear motion, resulting in a helicoidally path; both movements occur at the same time.

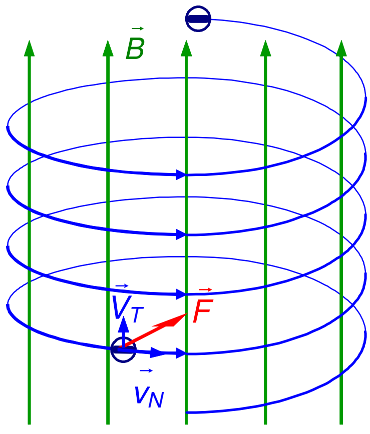


Figure 6-4. Negative charged particle with a helicoidally movement inside a magnetic field

Selector of speed

In a selector of speed, a lot of charged particles with different speed enter inside a magnetic field. The objective is to select an only speed, or a narrow range of speeds, separating those particles not being in the wished range. For this purpose, the particles with a wide range of speeds, fast, slow, etc., enter in an area with an electrical field and a perpendicular (to the electric field) magnetic field (Figure 6-5) . The electric field in the figure produces a force pointing to down, whose magnitude qE is not depending on the speed. The magnetic field produces a pointing up force depending on the speed, qvB. In this way, the force pointing to up is directly related to the speed, and particles with high speed will be deflected to up. Particles with low speed will be deflected to down due to the force of electric field. Only those particles verifying that $qE = qvB$, will not be deflected, and will pass horizontally, without deflection, along the selector of speed. Then, this device “classifies” the particles according their speed, not producing any deflection on particles with a speed $v=E/B$.

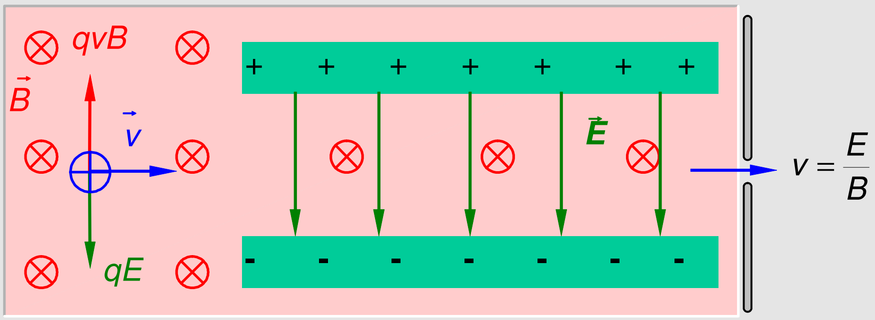


Figure 6-5. Charged particles in a selector of speed. They aren’t deflected those particles having a speed $v = E/B$

Varying the applied electric field (it’s easy to do with only change the difference of potential between plates), charged particles with different speeds can be selected, and particles faster or slower can be produced.

Example 6-1

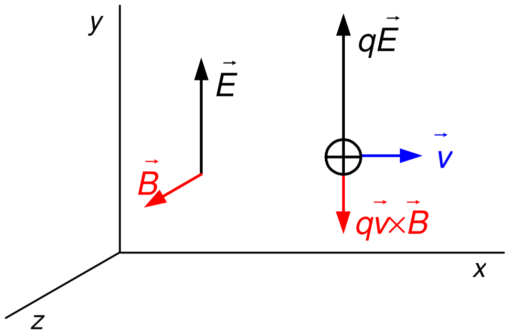
A proton moves in the direction of the axis x and positive sense in a region where the electric and magnetic field are perpendicular between them. If the electric field is $3\vec{j}$ kV/m sized, and the magnetic field $50\vec{k}$ mT sized, Which is the speed of the no deflected protons? If the protons are moving slower, where will be they deflected to?

Solution

The no deflected protons are those bearing an electric force qE equal to the magnetic force qv. So:

$$v = \frac{E}{B} = \frac{3 \cdot 10^3}{50 \cdot 10^{-3}} = 60 \text{ km/s}$$

For lower speeds, magnetic force qvB will be lower, and therefore they will be deflected to the y positive axis.



6.3 Force acting on an element of current

In previous section we have seen that a force acts on an electric charge moving inside a magnetic field. So, on a conductor flowed by an electric current inside a magnetic field, will also act a magnetic force. This force will be the resulting force acting on each charge is moving on conductor.

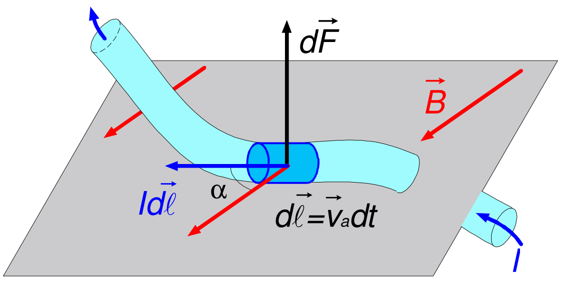


Figure 6-6. Force on an element of current

Let's consider a conductor flowed by a current I, placed inside a magnetic field $\vec{B}$, as on Figure 6-. In an interval of time dt, the electric charges move along the conductor a length $d\vec{\ell}$ equal to the product of drift speed by the time

$$d\vec{\ell} = \vec{v}_a dt$$

The electric charge contained inside the taken piece of conductor (length $d\vec{\ell}$) could be computed as the total charge has crossed a cross section of conductor in a time dt , and so $dq=Idt$. Then, the elementary force $d\vec{F}$ acting on the element $d\vec{\ell}$ will be:

$$d\vec{F} = dq(\vec{v}_a \times \vec{B}) = Idt(\vec{v}_a \times \vec{B}) = I(d\vec{\ell} \times \vec{B})$$

The product $Id\vec{\ell}$ is called an **element of current**. It's necessary to note that this equation only provides the elementary force acting on an element of current. To calculate the force acting on a piece of conductor flowed by an electric current, it will be necessary to integrate this expression.

Force acting on a conductor carrying electric charge inside a magnetic field	$\vec{F} = I\int_{\vec{\ell}_A}^{\vec{\ell}_B}\left(d\vec{\ell}\times\vec{B}\right)$ Equation 6-3

In the case of **currents in a uniform magnetic field**, the magnetic field can go out of the integral, so this expression becomes

$$\vec{F} = \int d\vec{F} = I\left(\int_{\vec{\ell}_A}^{\vec{\ell}_B} d\vec{\ell}\right)\times\vec{B} = I\vec{\ell}\times\vec{B}$$

being $\vec{\ell}$ the vector joining the starting point of conductor and the finishing point, as it's shown on Figure 6-7 .

Force acting on a conductor carrying electric charge inside a uniform magnetic field	$\vec{F} = I\vec{\ell}\times\vec{B}$ Equation 6-4

A consequence of this result is that the force acting on a closed conductor (with any shape) inside a uniform magnetic field is zero, because $\vec{\ell}$ vector is also zero.

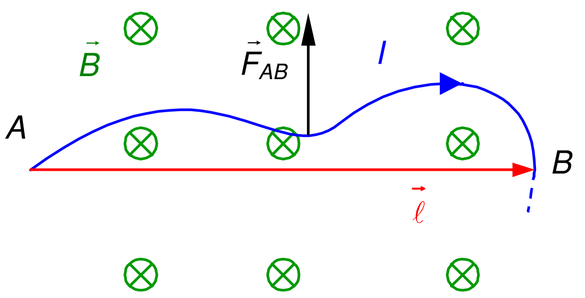
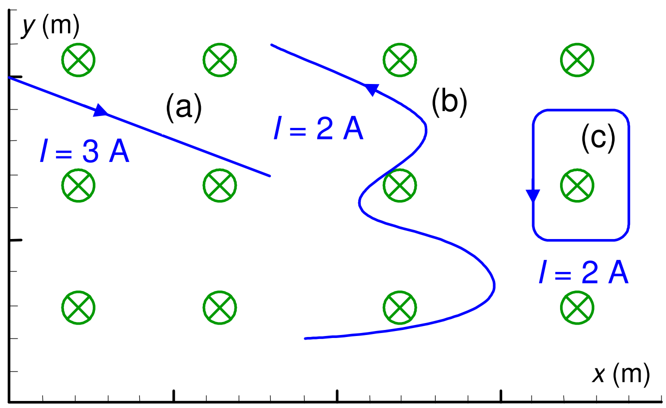


Figure 6-7 Inside a uniform magnetic field doesn't matter the shape of conductor to calculate the net force on conductor.

Example 6-2

Calculate the force acting on conductors on figure, placed inside a magnetic field $-2\vec{k}$ T.



Solution

a) The current goes from (0,10) to (8,7),

$$\vec{\ell} = (8,7) - (0,10) = (8,-3)$$

and using the Equation 6-4

$$\vec{F}=I\vec{\ell}\times\vec{B}=3\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 8 & -3 & 0\\ 0 & 0 & -2\end{vmatrix}=18\vec{i}+48\vec{j}\ \text{N}$$

b) In the same way, for any shape of conductor, $\vec{\ell}=(8,11)-(9,2)=(-1,9)$ and

$$\vec{F}=I\vec{\ell}\times\vec{B}=2\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ -1 & 9 & 0\\ 0 & 0 & -2\end{vmatrix}=-36\vec{i}-4\vec{j}\ \text{N}$$

c) Since it's a closed conductor, $\vec{\ell}=(0,0)$ and $F=0$.

6.4 Action of a uniform magnetic field on a flat loop. Magnetic moment

A particular case of force on a conductor flowed by a current is that of a flat loop with a current inside a magnetic field. Let's consider a rectangular loop like that of Figure 6-8, with sides of length *a* and *b*. An intensity of current *I* is flowing along the loop, and it's placed inside a uniform magnetic field *B*. The force acting on each side of loop comes from Equation 6-4, but the net force acting on loop is zero, since a loop is a closed conductor. Since the vector length for two parallel sides is the same but with opposite sign, forces on sides of loop are cancelled by couples:

$$\vec{F}_1=I\vec{a}\times\vec{B} \qquad \vec{F}_2=I(-\vec{a})\times\vec{B}=-\vec{F}_1$$

$$\vec{F}_3=I\vec{b}\times\vec{B} \qquad \vec{F}_4=I(-\vec{b})\times\vec{B}=-\vec{F}_3 \qquad \sum\vec{F}=0$$

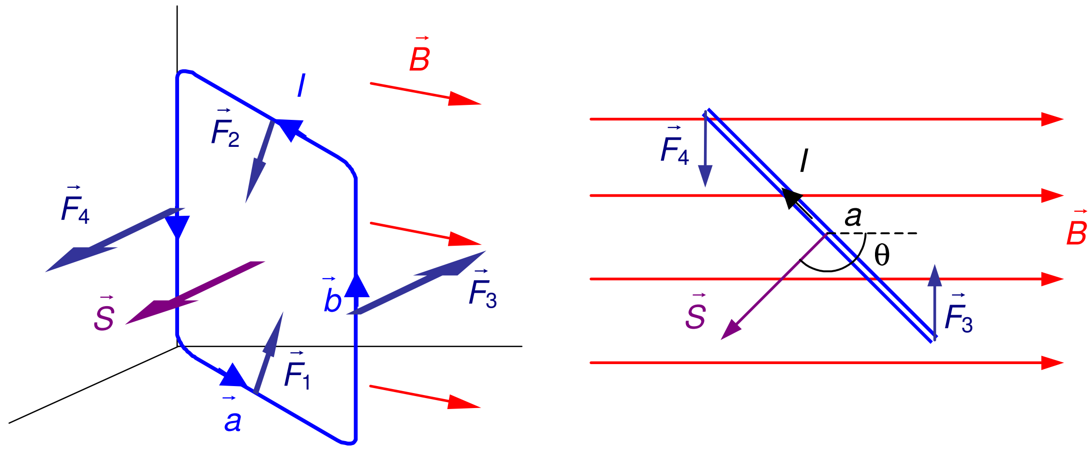


Figure 6-8. Rectangular loop of surface *S*, inside a uniform magnetic field *B*

In these equations, $\vec{a}$ and $\vec{b}$ vectors have been chosen in such way that the sense of their cross product $\vec{a}\times\vec{b}$ is related to the sense of the intensity of current according with the right hand or screw rule. Besides, this area vector is perpendicular to the loop and its magnitude equals the area of the loop $\vec{a}\times\vec{b}=\vec{S}$.

Although the net force is zero, the loop is subjected to a torque[2]; if we compute the torque due to each force with respect the center of loop:

$$\vec{\tau}_1 = -\frac{\vec{b}}{2}\times\vec{F}_1 \qquad \vec{\tau}_2 = \frac{\vec{b}}{2}\times\vec{F}_2 \qquad \vec{\tau}_3 = \frac{\vec{a}}{2}\times\vec{F}_3 \qquad \vec{\tau}_4 = -\frac{\vec{a}}{2}\times\vec{F}_4$$

The net torque, due to the set of four forces, taking in account that these forces are cancelled by couples and the equation for the double cross product:

$$\begin{aligned}
\vec{\tau} = \vec{\tau}_1+\vec{\tau}_2+\vec{\tau}_3+\vec{\tau}_4 &= -\frac{\vec{b}}{2}\times\vec{F}_1+\frac{\vec{b}}{2}\times\vec{F}_2+\frac{\vec{a}}{2}\times\vec{F}_3-\frac{\vec{a}}{2}\times\vec{F}_4 = \vec{b}\times\vec{F}_2+\vec{a}\times\vec{F}_3\\
&= \vec{b}\times I(-\vec{a})\times\vec{B}+\vec{a}\times I\vec{b}\times\vec{B} = I\left(-\vec{b}\times\left(\vec{a}\times\vec{B}\right)+\vec{a}\times\left(\vec{b}\times\vec{B}\right)\right)=\\
&= I\left(-\left(\vec{b}\cdot\vec{B}\right)\vec{a}+\left(\vec{b}\cdot\vec{a}\right)\vec{B}+\left(\vec{a}\cdot\vec{B}\right)\vec{b}-\left(\vec{a}\cdot\vec{b}\right)\vec{B}\right)=\\
&= I\left(\left(\vec{a}\cdot\vec{B}\right)\vec{b}-\left(\vec{b}\cdot\vec{B}\right)\vec{a}\right)= I\vec{B}\times\left(\vec{b}\times\vec{a}\right)= I\left(\vec{a}\times\vec{b}\right)\times\vec{B}
\end{aligned}$$

It's necessary to remember that as well as the result of a force on a system is an acceleration, the result of a torque on a system is an angular acceleration, and therefore, the loop will turn around its axis due to the computed net torque $\vec{\tau}$. Since $\vec{a}\times\vec{b}=\vec{S}$:

$$\vec{\tau} = I\vec{S}\times\vec{B}$$

The product of intensity of current by the area vector is called ***magnetic moment*** of the loop:

$$\vec{\mu} = I\vec{S} \qquad \text{It's measured in Am}^2 \qquad \text{Equation 6-5}$$

To finish, the **torque** acting on loop remains finally:

$$\vec{\tau} = \vec{\mu}\times\vec{B} \qquad \text{It's measured in Nm} \qquad \text{Equation 6-6}$$

Therefore, when the magnetic moment vector or the area of the loop vector are parallel to the applied magnetic field, the torque will be zero and the loop will be in dynamic equilibrium (stable balance if they are parallel and unstable balance if they are antiparallel). If the loop is placed with its magnetic moment in any other direction, a torque will appear turning the loop until magnetic moment vector remains parallel to applied magnetic field.

Equations for a squared loop can be extrapolated to any other loop with any shape, with the same result. The only requirement is that the loop must be a flat loop (it should be possible to place the loop in a plane). These equations fail if the loop wasn't flat.

Anyway, these equations are also applicable to a coil of N turns, only multiplying by N: magnetic moment of coil $\vec{\mu} = NI\vec{S}$, and torque $\vec{\tau} = NI\vec{S}\times\vec{B} = \vec{\mu}\times\vec{B}$

[2] On unit 0, definition of torque can be found.

Example 6-3

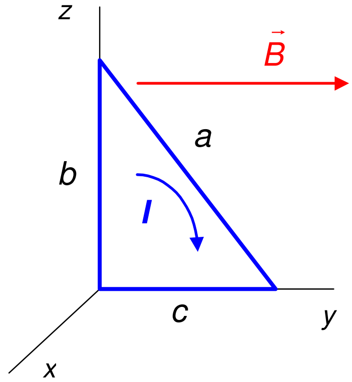
Along the loop of the figure of sides *a*, *b* and *c*, flows an intensity *I* in the shown direction. The loop is placed inside a magnetic field $\vec{B} = B\vec{j}$.

Find:

a) Magnetic forces on sides of the loop.

b) Magnetic moment of the loop.

c) Torque acting on loop.



Solution

a) The force on a conductor of length $\vec{\ell}$ with a current *I* inside a magnetic field $\vec{B}$ is $\vec{F} = I(\vec{\ell} \times \vec{B})$. Therefore on each side of loop we'll have:

$$\vec{F}_a = I(\vec{a} \times \vec{B}) = I((-b\vec{k} + c\vec{j}) \times B\vec{j}) = IbB\vec{i}$$
$$\vec{F}_b = IbB(\vec{k} \times \vec{j}) = -IbB\vec{i}$$
$$\vec{F}_c = IbB(-\vec{j} \times \vec{j}) = 0$$

b) Magnetic moment of a loop is defined as $\vec{\mu} = I\vec{S}$. As $\vec{S} = \frac{1}{2}cb(-\vec{i}) = -\frac{1}{2}cb\vec{i}$

$$\vec{\mu} = I\vec{S} = I\frac{1}{2}cb(-\vec{i}) = -\frac{1}{2}Icb\vec{i}$$

c) And the torque

$$\vec{\tau} = \vec{\mu} \times \vec{B} = -\frac{1}{2}Icb\vec{i} \times B\vec{j} = -\frac{1}{2}IcbB\vec{k}$$

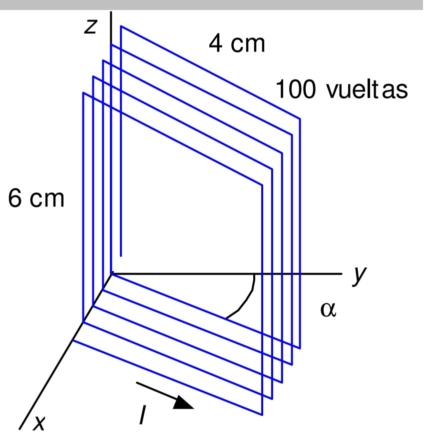
Therefore, the torque will try to turn the loop around the OZ axis (in clockwise sense when it's seen from upside).

Example 6-4

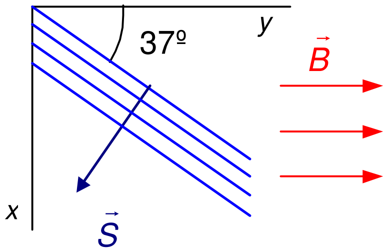
A coil is made up by 100 rectangular loops having 6x4 cm^2 of area. It's turned as it appears on picture, forming a 37º angle with the *y* axis. A 2,5 A current flows along coil.

a) Which is the magnetic moment of the coil?

b) Which is the torque acting on coil when a 2 T magnetic field is applied in the direction of the *y* axis in positive sense?



Solution



a) The area vector is perpendicular to the plane of coil, being its sense according to the screw rule for the intensity of current.

$\vec{S} = S(\cos 37\vec{i} - sen37\vec{j}) = 24(0.8\vec{i} - 0.6\vec{j})\text{cm}^2$

And magnetic moment

$$\vec{\mu} = NI\vec{S} = 250 \cdot 24 \cdot 10^{-4}(0.8\vec{i} - 0.6\vec{j}) = (0.48\vec{i} - 0.36\vec{j})\text{Am}^2$$

b) $\vec{\tau} = \vec{\mu} \times \vec{B} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0{,}48 & -0{,}36 & 0 \\ 0 & 2 & 0 \end{vmatrix} = 0{,}96\vec{k}\,\text{Nm}$

Therefore, the torque will turn the coil in counterclockwise sense.

Electric engine

An electric engine is made up by a coil (N loops) able to turn around a fixed axis. When a current flows along this coil and a magnetic field is applied to coil, a torque acts

$$|\vec{\tau}| = NISB\,\text{sen}\alpha$$

Magnetic field can be applied in some different ways, but for little engines a permanent magnet can be used; so, magnetic field can be supposed as uniform.

This torque will produce a turning to align the area vector parallel to magnetic field (α=0 and τ=0); equilibrium position would be reached after a few oscillations due to the inertia of coil. But when equilibrium position is reached, the sense of current on coil is changed through the brushes (two half rings at the end of coil, where terminals of battery are gliding). If sense of current is inverted, area vector is inverted, and magnetic moment

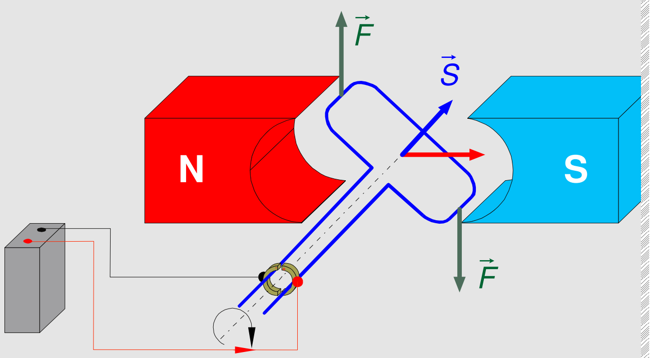
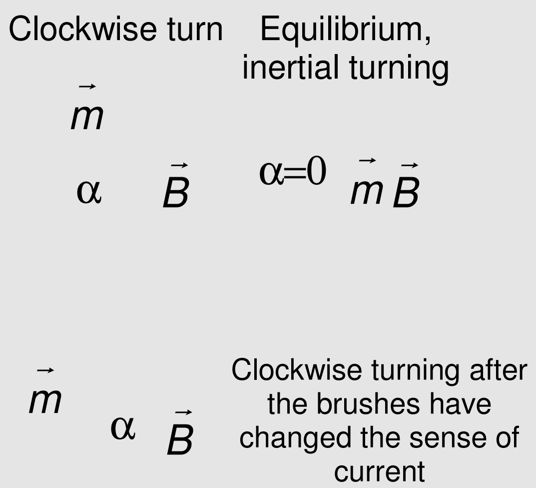
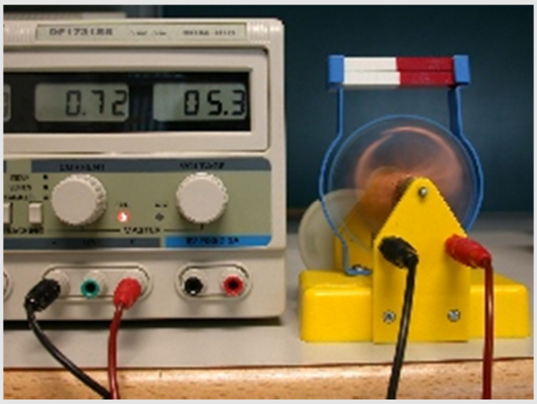


Figure 6-5. Electric engine. The magnetic field doesn't change its direction, performing the area vector a whole turn



remains opposite to magnetic field (antiparallel) (α=180º and τ=0). In this situation, the coil turns a half of turn in order to get magnetic moment parallel to magnetic field. When this position is reached, the brushes change the sense of current, and the process is repeated, in such way that equilibrium is never reached, and engine is always turning. The inertia of coil enables that the engine don't stop at the exact position where torque is zero.



The average torque along a half of turn can be calculated as

$$\tau_m = \frac{1}{\pi}\int_0^{\pi} NISBsen\,\alpha\, d\alpha = \frac{2NISB}{\pi}$$

And the transformed power by the engine is

$$P = \tau_m \omega = \frac{2NISB}{\pi}\omega$$

From this equation, assuming there aren't mechanical losses and using the given definition for contraelectromotive force of a receptor

$$\varepsilon' = \frac{dW}{dq} = \frac{dW}{dt}\frac{dt}{dq} = \frac{P}{I} = \frac{2NS}{\pi}B\omega = kB\omega$$

Where k is a constant only depending on the geometry and number of turns of engine, $k = \frac{2NS}{\pi}$. Then, contraelectromotive force is linearly related to B and ω (angular speed).

6.5 Hall Effect

Hall Effect is produced when a conductor carries an electric current in a magnetic field. As we already know, this situation produces a force acting on conductor (section 6.3), but also produces the Hall Effect. Hall Effect is very difficult to detect, since it produces differences of potential in the order of µV (10^{-6} V) and usually it's neglected. But in some cases, Hall Effect is used to build measurement instruments, as happen with instruments to measure magnetic fields (teslameters). Besides, Hall Effect demonstrates that it isn't true that positive charges moving in a sense was completely equivalent to negative charges moving in opposite sense. Even this equivalence has been useful when studying direct current, Hall Effect leads to different results when positives or negatives charges are supposed. Hall Effect is the resulting equilibrium state between a magnetic and an electric field, as we are going to study.

Let's consider a conductor flowed by a current in a magnetic field perpendicular to current, as can be seen on Figure 6-7. In accordance with Equation 6-3, a force perpendicular to magnetic field and velocity of charges appears ($F_M=qv_aB$); if we suppose that electrons (negative charge) are moving, this force is pointing to down, and electrons will be deflected to down when moving. So, in lower side of conductor we'll find a lot of electrons (negative charge), and a lack of them in upper side (positive charge). So, an electric field (E) appears on conductor, pointing to down (from positive charge to negative charge), and the electrons moving on conductor will suffer an electric force to up ($F_E=qE$). This electric field produces a difference of potential between the upper and the lower side of conductor (in the perpendicular direction both to current as to magnetic field), called **Hall voltage (V_H)**. If we suppose electric field as uniform, $V_H = -\int_A^C \vec{E}\cdot d\vec{\ell} = Ea$.

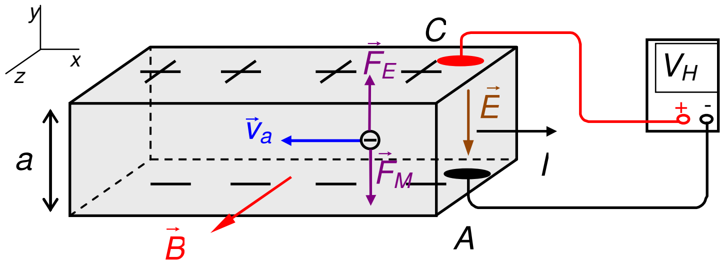


Figure 6-6. Hall Effect on a conductor

This voltage is very little (of the order of μV), and can be detected with only very accurate instruments (its polarity is that shown on picture). In the equilibrium state, magnetic force and electric force are in equilibrium and the electrons move without deflection. Then, it'll be: $qv_aB = qE \Rightarrow B = \frac{E}{v_a} = \frac{V_H}{av_a}$

As drift speed (v_a) is related to intensity of current in accordance to equation 3.2 ($I=nqSv_a$), being S the cross section of conductor perpendicular to intensity of current, then

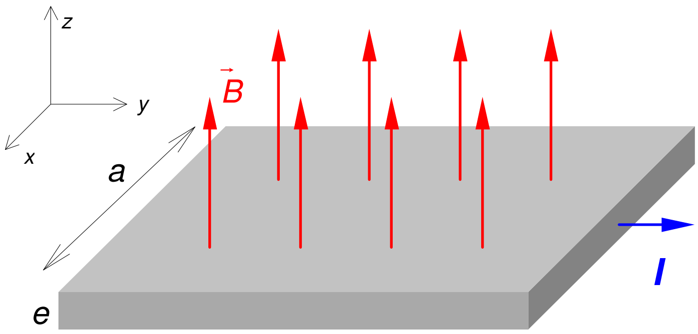
$$B = \frac{V_H}{av_a} = \frac{V_H nqS}{aI} \qquad \text{Equation 6-7}$$

If we take a piece of conductor, n, q, S and a are constants. The intensity of current I can be measured with an ammeter and so, magnetic field is directly related to V_H. Measuring this Hall voltage with an accurate voltmeter, magnetic field can be computed, and a teslameter can be built.

But if we consider that charges moving inside conductor are positive charges moving in opposite sense (to right), then the magnetic force also acts to down, appearing positive charge on lower side of conductor and negative charge on upper side. So, polarity of Hall voltage is opposite to that found when negative charges moving on conductor were supposed. This difference supposing the charge carriers are positive or negative charges moving in opposite sense will be useful when we study semiconductor materials.

Example 6-5

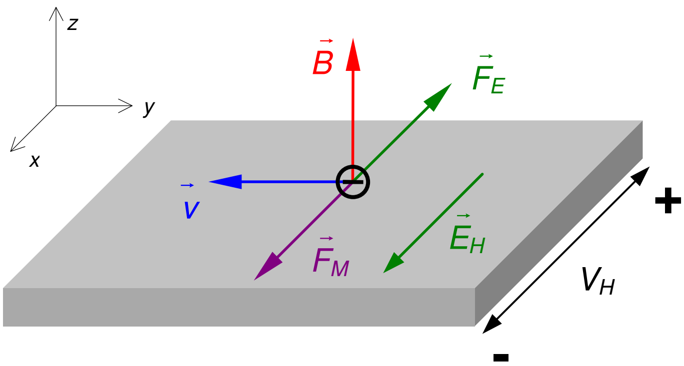
A piece of silver with 5 cm of width (a) and 0,5 mm of thickness (e) is placed in a 2 T magnetic field, such as it appears on the figure. Which is the Hall voltage if 200 A are flowing along conductor? You can suppose that silver has 0,65 free electrons by atom (average value).
Data: Density of silver 10,5 g/cm^3; molar mass of silver 107,9 g/mole; Avogadro's number $6{,}022 \cdot 10^{23}$ atom/mole.



Solution

As we are dealing with a conductor, negatives charges will move inside, and the magnetic and electric forces acting on electrons are those drawn on picture:

$$\vec{F}_M + \vec{F}_E = 0 \quad \rightarrow q(\vec{v} \times \vec{B}) = -q\vec{E}$$



In order to apply Equation 6-7, the density of electrons by unit of volume (n) on silver must be computed. It can be done from data of density (ρ), molar mass (m_m) of silver and Avogadro's number (Av):

$$n\left(\frac{e^-}{cm^3}\right) = 0{,}65\left(\frac{e^-}{atom}\right)Av\left(\frac{atom}{mol}\right)\frac{\rho\left(\frac{g}{cm^3}\right)}{m_m\left(\frac{g}{mol}\right)} = \frac{0{,}65 \cdot 6{,}022 \cdot 10^{23} \cdot 10{,}5}{107{,}9} = 3{,}8 \cdot 10^{28}\ \frac{e^-}{m^3}$$

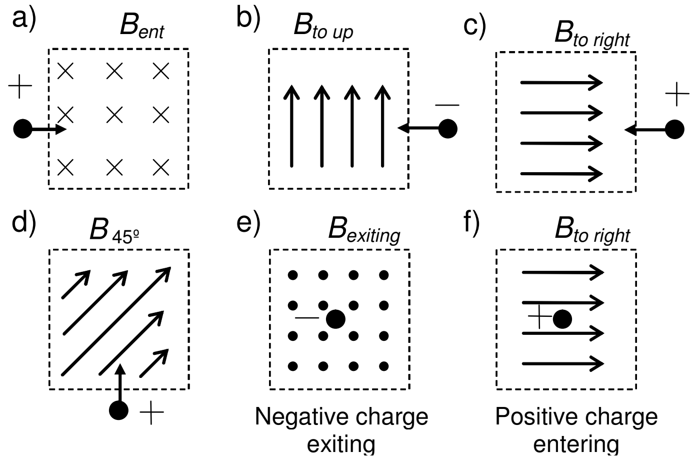
And from Equation 6-7:

$$V_H = \frac{BaI}{nqS} = \frac{2 \cdot 5 \cdot 10^{-2} \cdot 200}{3{,}8 \cdot 10^{28} \cdot 1{,}6 \cdot 10^{-19} \cdot 5 \cdot 10^{-2} \cdot 0{,}5 \cdot 10^{-3}} = 131\,\mu V$$

6.6 Problems

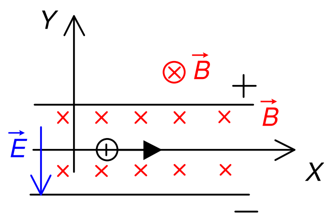
1. Find the magnetic force acting on a proton moving at 4·10 6 m/s in the positive sense of the *X* axis, inside a 2 T magnetic field in the positive sense of *Z* axis.
(Data: $q(p)$ = 1,6·10^{-19} C).
Sol: $\vec{F} = -1{,}28 \cdot 10^{-12}\ \vec{j}$ N

2. Point the initial direction of the deflection of charged particles on picture.



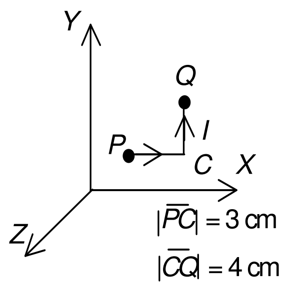
Sol: a) upwards; b) perpendicularly to paper, going out; c) doesn't deflect; d) perpendicularly to paper, entering; e) doesn't deflect; f) downwards.

3. A bundle of electrons move between the plates of a capacitor with a difference of potential *V*. Between plates there is a uniform magnetic field perpendicular to the electric field. If plates of capacitor are separated a distance *d*, calculate the speed of the electrons not deflecting when they move between plates.
Sol: $v = V/Bd$

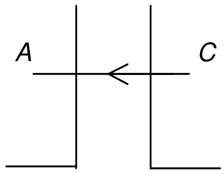


4. A long conductor, parallel to the *X* axis, carries a current of 10 A in the positive sense of *X* axis. There is a uniform magnetic field of 2 T in the direction and sense of the *Y* axis. Find the force by unit of length acting on conductor.
Sol: $20\vec{k}$ N/m

5. By the segment of conductor in the figure flows a current *I* = 2 A from *P* to *Q*. It exists a magnetic field $\vec{B} = 1\vec{k}$ T. Find the total force acting on conductor and prove that it is the same that if the entire conductor was a straight segment from *P* to *Q*.
Sol: $\vec{F} = \left(8\vec{i} - 6\vec{j}\right) \cdot 10^{-2}$ N



6. Along conductor *AC* of figure flows a current of 10 A (it's a part of an electric circuit), being able to glide along two vertical rods.

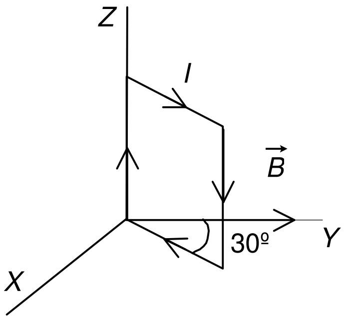


Compute the necessary uniform magnetic field, perpendicular to the plane of the figure, in order that the magnetic force on conductor could equilibrate the gravitational force. Which should be the sense of magnetic field?
The length of conductor is 10 cm and its mass, 20 g.
Sol: $B = 0{,}196$ T exiting from paper to reader.

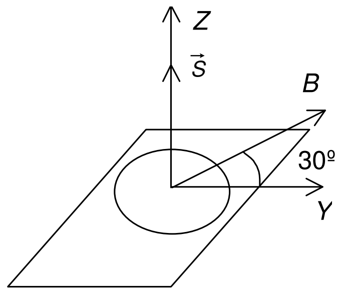
7. Along a flat conductor of arbitrary shape flows a current *I*, being the conductor inside a uniform magnetic field $\vec{B}$ perpendicular to the plane of conductor. Prove that the total force acting on the piece of conductor going from point *a* to another *b* is $\vec{F} = I\vec{L}x\vec{B}$,where $\vec{L}$ is the vector going from *a* to *b*.

8. The figure shows one of the rectangular loops of 10 cm by 5 cm of a coil with 20 turns. The coil has hinges on side along *Z* axis, and a 0,1 A current is flowing. Which is the torque acting on coil (modulus, direction and sense) if it is forming 30º regarding the direction of a uniform magnetic field $\vec{B} = 0{,}5\vec{j}$ T ?

Sol: $-4{,}3 \cdot 10^{-3}\ \vec{k}$ Nm

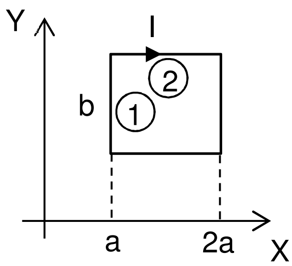


9. In order to measure a magnetic field, a coil with 200 turns of 14 cm^2 of area and forming an angle of 30º with the field is used. When flowing an intensity of 0,7 A, a torque of $980 \cdot 10^{-6}$ Nm is measured. Compute *B*.
Sol: $B = 5{,}7 \cdot 10^{-3}$ T.



10. Let's consider the rectangular loop on picture, with sides *a* and *b*, and flowed by an intensity *I* in the shown sense. The loop is in a no uniform magnetic field $\vec{B} = B_0 \dfrac{a}{x}\vec{k}$. Calculate the forces acting on sides 1 and 2.

Sol: $\vec{F}_1 = IB_0 b\vec{i} \quad \vec{F}_2 = IB_0 a \ln 2(-\vec{j})$



GLOSSARY

Magnetic force on a moving charge. It's the force acting on a charge q when moving with a speed v in a magnetic field B.

$$\vec{F} = q\vec{v} \times \vec{B}$$

Tesla: On a point of the space there is a magnetic field of 1 tesla if a point charge of 1 coulomb moving perpendicularly to the magnetic field with a speed of one meter by second, suffer a force of one newton perpendicularly to magnetic field and velocity.

Force on an element of current: Force acting on a conductor of length $d\vec{l}$ flowed by a current I in a magnetic field $\vec{B}$

$$d\vec{F} = I(d\vec{\ell} \times \vec{B})$$

Magnetic moment of a loop: it's the product of the intensity flowing along the loop by the area vector. Its sense is that of the area vector, that is, the resulting to apply the rule of the screw or the right hand to the sense of intensity of current on loop.

$$\vec{m} = I\vec{S}$$

Torque on a loop: It's the cross product between the magnetic moment of the loop and the magnetic field

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Hall Effect. Voltage appearing when an electric current flows along a conductor in a magnetic field. This voltage is very little and it's directly related to intensity of current and magnetic field

$$V_H = v_a B a$$

Being v_a the drift speed of charge carriers, B the magnetic field and a the width of conductor.