Chapter 3: Nerode

U.D. Computación

DSIC - UPV

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Definitions

- Given a DFA $A = (Q, \Sigma, \delta, q_0, F)$ and $q \in Q$, the *Right language* of q in L is defined as:

$$R_q = \{x \in \Sigma^* \;:\; \delta(q, x) \in F\}$$

- Given a language $L \subseteq \Sigma^*$ and a DFA $A = (Q, \Sigma, \delta, q_0, F)$ such that $L(A) = L$, we say that A is *reduced* if for every $p, q \in Q$ such that $p \neq q$ it follows that $R_p \neq R_q$
- Given $L \subseteq \Sigma^*$, the *Nerode's equivalence* R_L is defined as:

$$x \equiv_{R_L} y \iff x^{-1}L = y^{-1}L$$

A condition for non regularity (1/2)

- If there exists an infinite sequence $(x_i)_{i \in \mathbb{N}}$ of words over Σ such that $\forall i, j,\ i \neq j$

$$\exists z \in \Sigma^* \ : \ x_i z \in L \ \mathit{iff} \ x_j z \notin L$$

then L is not regular.
Proof:
  - Suppose that L is regular and let A be a reduced DFA accepting L.
  - For every $i \neq j$, the right lenguage of states $\delta(q_0, x_i)$ and $\delta(q_0, x_j)$ are different.
  - Thus, if the condition holds, A DFA A for L would have an infinite number of states.

A condition for non regularity (2/2)

- R_L is of infinite index if and only if there exists an infinite sequence $(x_i)_{i \in \mathbb{N}}$ of words over Σ such that $\forall i, j,\ i \neq j$, $\exists z \in \Sigma^* \ : \ x_i z \in L \ iff \ x_j z \notin L$
- For the previous result, this condition of non regularity, can be established as:

A condition for non regularity (2/2)

- R_L is of infinite index if and only if there exists an infinite sequence $(x_i)_{i\in\mathbb{N}}$ of words over Σ such that $\forall i, j,\ i \neq j$, $\exists z \in \Sigma^* \ :\ x_i z \in L \textit{ iff } x_j z \notin L$
- For the previous result, this condition of non regularity, can be established as:

  Given a language $L \subseteq \Sigma^*$, if R_L is of infinite index, then L is not regular

Nerode's Theorem

- Given a language $L \subseteq \Sigma^*$, the lenguaje L es regular if and only if R_L is of finite index.

  Proof

  1. For the non regularity condition, we have that if L is regular, then R_L is of finite index.
  2. Suppose that R_L is of finite index. The fact that L is regular can be seen giving an algorithm to construct a DFA that accepts L from the classes of R_L:

     $$A = (\Sigma^*/R_L, \Sigma, \delta, [\lambda]_{R_L}, L/R_L)$$

     where:

     $$\delta([x]_{R_L}, a) = [xa]_{R_L}$$