EQUIVALENCES AND INFERENCE RULES

The symbol (*) means that the "dual rule" must also be considered (that is, the one obtained replacing $\vee$ by $\wedge$ and vice versa).

1. Equivalences

Name	**Rule**	**Short name**
Associative	$P \vee (Q \vee R) \equiv (P \vee Q) \vee R$ (*)	A
Commutative	$P \vee Q \equiv Q \vee P$ (*)	C
Distributive	$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$ (*)	D
Identity element	$P \vee \phi \equiv P$; $P \wedge \tau \equiv P$	Id
Inverse element	$P \vee \neg P \equiv \tau$; $P \wedge \neg P \equiv \phi$	Inv
Absorption	$\tau \vee P \equiv \tau$; $\phi \wedge P \equiv \phi$	Abs
Simplification	$P \vee (P \wedge Q) \equiv P$ (*)	Simp
Idempotent law	$P \vee P \equiv P$; $P \wedge P \equiv P$	Idemp
De Morgan laws	$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$ (*)	DM
Double negation law	$\neg(\neg P) \equiv P$	DN
Conditional-disjunction	$P \to Q \equiv \neg P \vee Q$	CD
Conditional-biconditional	$(P \to Q) \wedge (Q \to P) \equiv P \leftrightarrow Q$	CB
Transposition	$P \to Q \equiv \neg Q \to \neg P$	T
Exportation law	$(P \wedge Q) \to R \equiv P \to (Q \to R)$	E
Negation of quantifiers	$\neg \forall x\ P(x) \equiv \exists x\ \neg P(x)$; $\neg \exists x\ P(x) \equiv \forall x\ \neg P(x)$	NQ

2. Inference rules (or implications)

Name	**Rule**	**Short name**
Conjunction	$\{P, Q\} \vdash P \wedge Q$	Conj
Simplification	$\{P \wedge Q\} \vdash P$; $\{P \wedge Q\} \vdash Q$	Simp
Addition	$\{P\} \vdash P \vee Q$; $\{Q\} \vdash P \vee Q$	Add
Modus ponens	$\{P, P \to Q\} \vdash Q$	MP
Modus tollens	$\{\neg Q, P \to Q\} \vdash \neg P$	MT
Disjunctive syllogism (or Modus Tollendo Ponens)	$\{\neg P, P \vee Q\} \vdash Q$	DS (or MTP)
Hypothetical syllogism	$\{P \to Q, Q \to R\} \vdash (P \to R)$	HS
Universal specification	$\forall x\ P(x) \vdash\ P(y)$ (y arbitrary)	US
Existential specification	$\exists x\ P(x) \vdash\ P(a)$ (a specific)	ES
Universal generalization	$P(y)$ for **any** $y\ \vdash \forall x\ P(x)$	UG
Existential generalization	$P(a)$ for **certain** $a\ \vdash \exists x\ P(x)$	EG