UD 3

PROBABILITY

The object of STATISTICS is the study of random phenomena like:

- To throw a dice (results: 1, 2, 3, 4, 5, 6)

- To measure the weight of something (results: continuous scale)

- To choose a piece produced in a factory (results: correct / defective)

- SAMPLE SPACE (E): Set of results that can be obtained when conducting a random experiment.

Examples:

- obtain a head when throwing a coin. E finite
- number of times that we should throw the dice until obtaining a six. E infinite (discrete)
- length of a piece in cm. E infinite (continuous)

DICE throwing: unpredictability of results, BUT statistical regularity

- The statistical regularity observed in a random experiment is the pattern that allows to MODEL the results.

- **PROBABILITY**:

  Measure of the certainty (objective or subjective) of any proposal or of an event associated to a random experiment. Varies from 0 (null certainty, impossible event) to 1 (certain event).

PROBABILITY CALCULATIONS

Mathematical model used to build statistical inference

- Approaches of probability:

WHAT IS THE PROBABILITY OF...

- obtaining 6 when throwing a dice?
- obtaining a defective piece if one is chosen at random?
- having rain tomorrow? www.rtve.es/eltiempo
- existence of extraterrestrial life?
- a big meteorite to crash with Earth?

www.20minutos.es/noticia/203639/0/nasa/asteroide/tierra/

CLASSICAL APPROACH (FREQUENTIALIST):

- It is the result of a random experiment that can be repeated (drawback: restricted to repetitive experiences)
- probability = relative frequency of occurrence of an event.
- Laplace: probability = favorable cases / possible cases

BAYESIAN APPROACH (SUBJETIVIST):

- Not the result of a random experiment
- probability = degree of “truth” or centainty that a given individual has about a proposal

EXERCISE: Bayes' theorem

- Event B: Accident in a road
- Type of vehicle:

  event A_1=car, A_2=motorcycle, A_3=truck

- *A priori* probabilities $P(A_i)$

  $P(A_1)$=P(car)=0.50 (50% of vehicles are cars)

  $P(A_2)$=P(motorcycle)=0.20

  $P(A_3)$=P(truck)=0.30

- Probability of having an accident according to the type of vehicle:

  $P(B/A_1)$=P(accident/car)=0.15

  $P(B/A_2)$=P(accident/motorcycle)=0.30

  $P(B/A_3)$=P(accident/truck)=0.10

If there is an accident, what is the probability of being an accident caused by motorcycle?

- If there is an accident, what is the probability of being an accident caused by motorcycle?

$$P(A_i / B) = \frac{P(A_i).P(B / A_i)}{\sum_{i=1}^{n} P(A_i).P(B / A_i)}$$

$$P(A_2 / B) = \frac{P(A_2)P(B / A_2)}{P(A_1)P(B / A_1) + P(A_2)P(B / A_2) + P(A_3)P(B / A_3)} =$$

$$\frac{0.20 \cdot 0.30}{(0.50 \cdot 0.15) + (0.20 \cdot 0.30) + (0.30 \cdot 0.10)} = 0{,}3636 \quad \Rightarrow 36{,}36\%$$

P(car/accident)=0.454

P(truck/accident)=0.182

EXERCISES

(after finishing “teacher notes” file)

EXERCISE:

A random experiment consists of throwing a dice. Considering the events:

A = obtain an even number (2, 4, 6)

B = obtain a number higher than 4.

C = obtain 6.

Are A and B independent?

Are A and C independent?

Are A and B exclusive?

Are A and C exclusive?

Are independent “women” and “weight≤55”?

weight / gender	40 55	55 70	70 85	85 99	Row Total
MEN	0	42	41	5	88
	.0	73.7	97.6	100.0	67.7
WOMEN	26	15	1	0	42
	100.0	26.3	2.4	.0	32.3
COLUMN	26	57	42	5	130
TOTAL	20.0	43.8	32.3	3.8	100.0

¿P (woman with weight ≤ 55)?

P(woman/weight≤55)=1

P(woman)=0.323

P(weight≤55)=0.2

Example

In one pack with 10 pieces, 4 of them are defective. If 3 pieces are taken out by random, what is the probability of being correct the three pieces?

Calling C_i=piece i correct, the event requested is $(C_1 \cap C_2 \cap C_3)$ and the probability is:

Solution:

$P(C_1 \cap C_2 \cap C_3) = P(C_1) \cdot P(C_2/C_1) \cdot P(C_3/C_2 \cap C_1) =$

$6/10 \cdot 5/9 \cdot 4/8 = 120/720 =$ **1/6**

A given population is formed by N=50 pieces in a box, being 10% defective. One piece is extracted, and the event A “the piece is defective” is proposed. A second piece is extracted next (without replacement of the first one), and the event B “the second piece is defective” is proposed.

Calculate P(B) and P(B/A).

Are A and B independent?

If N= 100.000, can A and B considered as independent events, despite not replacing the first piece?