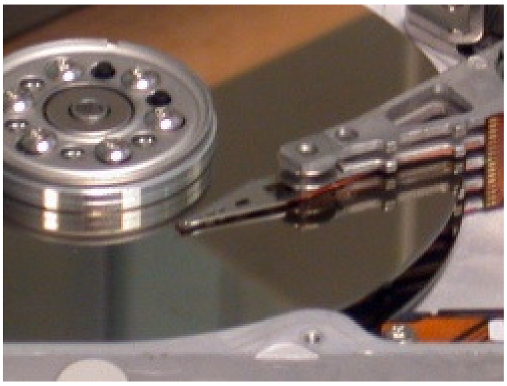
Unit 8

Electromagnetic induction



Objectives

- Introduce the electromagnetic induction phenomena.
- State Faraday's and Lenz's law and apply them to compute the induced electromotive force (e.m.f.) due to variable magnetic fluxes.
- To know some applications of induction phenomena.
- To define self-inductance and mutual inductance.
- To analyse circuits with inductors and resistors.
- To define magnetic energy.

8.1 Introduction

On before units they have been studied the forces created by magnetic fields acting on moving electric charges and on wires carrying currents. Oersted's experiment also revealed that an electric current creates a magnetic field (Biot and Savart's law), and around 1830, Michael Faraday and Joseph Henry (independently one of each other) stated that a variable magnetic field produced an electric current (and then an electric field). These phenomena are called "electromagnetic induction phenomena", and they set up a clear link between electricity and magnetism, stating that the origin of both things is the same.

Later, in 1873, James Clerk Maxwell summarized in mathematical form the experimental rules of Gauss, Biot and Savart, Ampère and Faraday. He wrote Ampère's law in a general way, introducing the idea of displacement currents, stating the electromagnetic theory of light (but he couldn't imagine it).

The applications of the electromagnetic induction phenomena are the basis of our electric generating system and a lot of devices run under their rules: alternating current generator, telephone, telegraph, induction brakes, induction stove, transformer, reading of information recorded on magnetic supports are some examples.

8.2 Electromagnetic induction phenomena

Let's suppose a magnet near a conductor loop (Figure 8-1). If we approach the magnet to the loop, being the loop connected to a galvanometer, we detect an electric current flowing along the loop (Figure 8-1 a). But if we move away the magnet from the loop, a new current appears on the loop, but in this case in opposite sense (Figure 8-1 b). If we hold the magnet and we move the loop, the phenomenon is exactly the same, but if there is no relative movement between magnet and loop, the current doesn't appear.

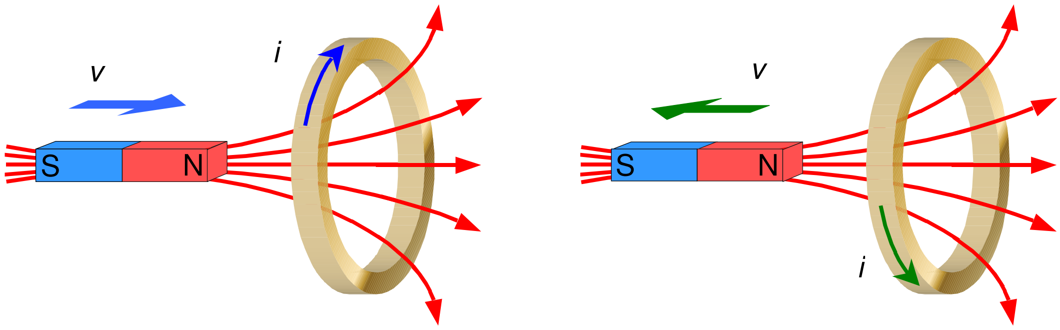


Figure 8-1. Movement of a magnet near a conductor loop

Let's now consider, according Figure 8-2, a conductor loop connected to a circuit with a battery and a varying resistor, and a second loops near the first one. If along the circuit and the first loop flows a constant current, none current can be detected on the upper loop; but if we modify the intensity of current flowing along the circuit with the variable resistor, a current can be detected on the upper loop meanwhile the variable resistor is handled; but this current disappears when the current on the circuit is steady. If the current on circuit decreases (modifying the position of varying resistor), the current on upper loop appears again meanwhile the resistor is moving, but in the opposite sense to the first one.

To finish, Figure 8-3 shows a conductor wire U shaped with a gliding conductor rod, inside a uniform magnetic field $\vec{B}$. When the rod glides along the U shaped conductor, a current flow along the circuit made up by the conductor and the rod. But if the rod glides in the other sense, the current flows also in the opposite sense. If the rod doesn't glide, no current can be detected.

As we'll see on next paragraph, the common pattern of these experiments is that an induced current appears when a change in the magnetic flux happens.

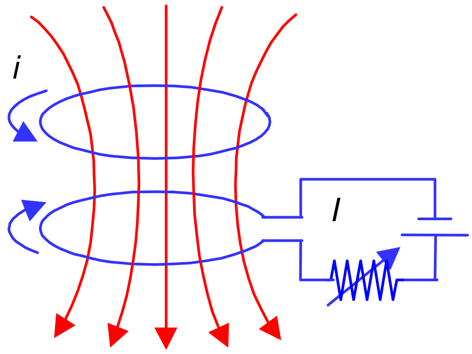


Figure 8-2. A varying current induces an electric current

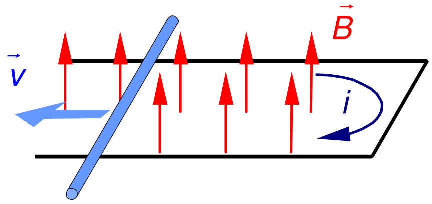


Figure 8-3. U shaped conductor

8.3 Faraday's law. Lenz's law

From the equation of magnetic flux through a circuit, can be seen that changes on magnetic flux can be produced both by a change on magnitude of magnetic field (B), by a change on the area of circuit (S), or by a change on the angle made up by the magnetic field and the surface (α). On any of these cases, a change on magnetic flux through the circuit occurs, and Faraday's law relates this change with an induced electromotive force (e.m.f.) appearing on circuit.

$$\Phi = \int_S \vec{B} \cdot d\vec{S} = \int_S B \cos \alpha dS$$

Faraday's law is an experimental law, found by Michael Faraday on 1829. It states that:

The induced electromotive force, ε, appearing in a circuit is directly related to the varying speed of magnetic flux through the circuit.

$$\varepsilon = -\frac{d\Phi}{dt}$$

Equation 8-1

This electromotive force acts as a battery "distributed" along the circuit, producing an intensity of current flowing along the circuit.

The negative sign of this equation means that the sense of induced current (and then of the electromotive force) is always opposite to that is producing it (Lenz's law), but this statement will be later discussed.

The e.m.f. was defined as the work done by the forces of electric field by unit of charge; if we consider the before circuit:

$$\varepsilon = \frac{W}{q} = \oint \frac{\vec{F}}{q} \cdot d\vec{\ell} = \oint \vec{E} \cdot d\vec{\ell}$$

The electric field in this equation is created by the induced electromotive force, and it is a no conservative electric field (its circulation along a closed path is not zero). This is a very important difference with the electric field produced by resting charges studied in Electrostatics ($\oint \vec{E} \cdot d\vec{\ell} = 0$) (conservative electric field). Farady's law can then be written as:

$$\varepsilon = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi}{dt} \qquad \text{Equation 8-2}$$

And taking in account the equation for magnetic flux:

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{S} \qquad \text{Equation 8-3}$$

Maxwell's equations on vacuum for no steady fields

On before unit were introduced the famous Maxwell's equations for steady fields. For no steady fields, we have just seen what happens when a change on magnetic flux through a circuit occurs, and that the circulation of an electric field along a closed path isn't zero (equation 8-3).

In the same way, Ampère's law must be modified when no stationary electric fields are acting, and a new term must be added. This new term (involving the called "displacement currents") was the original idea of Maxwell.

So, Maxwell's equations for no steady fields can be written as:

$$\oint_s \vec{E} \cdot d\vec{S} = \frac{\sum Q}{\varepsilon_0}$$

(a)

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{\partial}{\partial t}\int \vec{B} \cdot d\vec{S}$$

(b)

$$\oint_s \vec{B} \cdot d\vec{S} = 0$$

(c)

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \sum I + \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\int \vec{E} \cdot d\vec{S}$$

(d)

(a) and (c) are not modified with respect to those for steady fields; (b) has been modified according Faraday's law: *the circulation of electric field along a closed path is the varying speed of magnetic flux crossing any surface bordered by the path.* (d) is Ampère's law with a new term including the varying speed of electric flux across a surface.

Example 8-1

A coil with 100 turns, circular cross section having 2 cm diameter and axis parallel to *OZ* axis is placed inside a magnetic field $\vec{B}(t) = 2t\,\vec{k}\ \mathrm{T}$. Compute the e.m.f. induced on coil.

Solution

The magnetic flux through the coil is 100 times the magnetic flux through one turn of coil:

$$\Phi = N\int \vec{B}\cdot d\vec{S} = N\int BdS = NB\pi R^2 = 0{,}08\pi t \text{ Wb}$$

According Faraday's law, the induced electromotive force on coil is:

$$|\varepsilon| = \frac{d\Phi}{dt} = 0{,}08\pi \text{ V}$$

Lenz's law

In the experiments shown on paragraph 8.2 about electromagnetic induction, the sense of induced current changed depending if the magnet was approached or moved away, if the intensity increased or decreased, or if the rod was gliding over the U shaped conductor to right or to left. The negative sign of Faraday's law is the mathematical expression of **Lenz's law**:

The sense of induced current is always opposite of that is producing such current.

It's very important to note that the induced current is due to a change in the magnetic flux, and the **induced current must be opposite to such change on flux, but not necessarily to the existing flux.**

In the first experiment (Figure 8-1) the magnetic flux crosses the loop from left to right, because the North Pole of magnet produces a magnetic field pointing to right. When the magnet is approached to the loop, the magnetic flux through the loop increases, and then the induced current (blue current) must be opposite to such increasing of magnetic flux; it is, the induced current must create a magnetic field opposite to that already existing (in order to avoid it increases); a current flowing in the sense shown on picture creates a magnetic field to left, opposite to the increasing of magnetic flux.

When the magnet is moved away from the loop, the magnetic field and then the magnetic flux crossing the loop from left to right decreases, and then the induced current (green current) must reinforce the existing magnetic field. The induced current shown on picture creates a magnetic field pointing to right, reinforcing the already existing magnetic field.

In second case, when the intensity on circuit is increased handling the varying resistor, the magnetic flux through the loop increases, and then the induced current must flow in opposite sense to that of the current on circuit, creating a magnetic field opposite to that already existing. If the intensity on circuit decreases, the induced current must create a magnetic field reinforcing that already existing (Figure 8-2).

In third experiment (Figure 8-3) if the rod glides to left, increasing the area of circuit and then the magnetic flux, the induced current flows in clockwise sense, creating a new magnetic field opposite to that already existing. If the rod would glide to right, the induced current would have counterclockwise sense, in order to reinforce the flux.

To finish, the fourth example is displayed on Figure 8-4; a circuit with a coil, a battery, a resistor and a switch is placed near a second circuit with a coil and a resistor (in order to limit the induced current). When the switch is on, an induced current appears on second circuit meanwhile the current is increasing to reach its steady value. When the switch is off, an induced current appears on second circuit meanwhile the intensity is decreasing. None intensity is induced on second circuit when the intensity on first circuit is zero or constant.

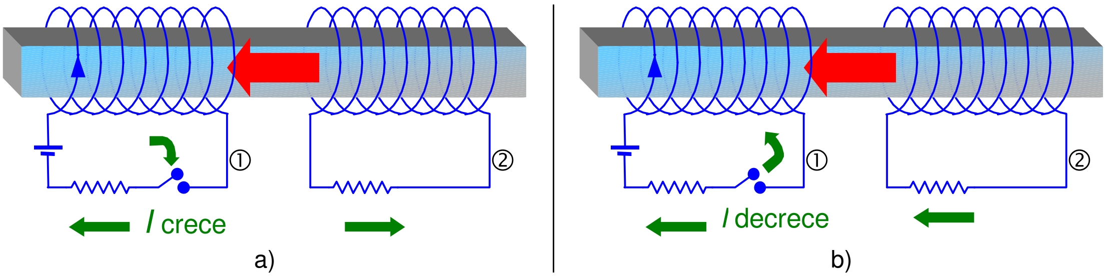
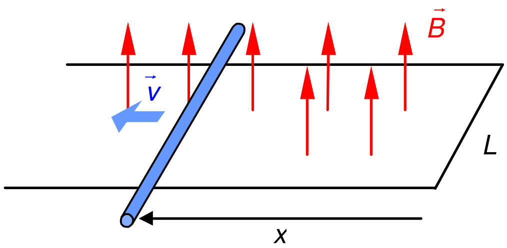


Figure 8-4. a) Swicth is ON, the intensity along circuit 1 increases, magnetic flux increases, and a second current trying to decrease the flux is induced on circuit 2; this second current flows opposite to that of circuit one. b) Swicth is OFF, the intensity along circuit 1 decreases, the flux decreases, and a second current trying to preserve the magnetic flux is induced on circuit 2; this current flows in the same sense that the current along circuit 1.

Lenz's law must obviously verify the law of conservation of energy.
Next, we'll see some examples about Faraday's and Lenz's law.

Example 8-2

Let's consider a conductor rod with resistance R and length L gliding without friction and speed v constant, over a U shaped conductor placed inside a uniform magnetic field B, as can be seen on picture. On time t=0, x=0.

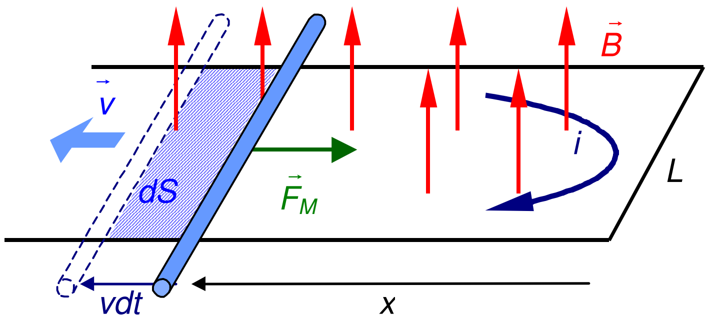


Compute: a) magnetic flux through the circuit made up by conductor and rod; b) induced electromotive force on circuit and intensity of current flowing along rod; c) magnetic force acting on rod produced by the induced current and the magnetic field; d) the force needed to keep constant the speed of rod; e) demonstrate that the work done by the external force computed on before paragraph is lost as heating in the rod.

Solution

This problem was already described on Figure 8-3. Magnetic flux increases as a consequence of increasing of area produced on circuit. This area can be related to time:

$S(t) = Lx = Lvt$



a) $\vec{B}$ and $d\vec{S}$ are parallel vectors, and B is uniform. Then:

$$\phi = \int \vec{B} \cdot d\vec{S} = \int BdS = BS = BLx = BLvt$$

b) induced electromotive force is:

$$|\varepsilon| = \frac{d\Phi(t)}{dt} = BLv$$

and intensity of current:

$$i = \frac{\varepsilon}{R} = \frac{BLv}{R}$$

Magnetic flux is crossing the circuit from down to up, and as the area of circuit is increasing on time, this flux is also increasing. The induced current should be opposite to this increasing of flux, and then must have clockwise sense (in this way, this intensity creates a magnetic field and a flux pointing to down).

c) When the induced current appears, the rod is a conductor flowed by a intensity inside a magnetic field; then, a magnetic force $F_M = iLB$ acts on rod:

$$F_M = iLB = \frac{B^2L^2v}{R}$$

The sense of this force is opposite to the movement of rod, and then:

$$\vec{F}_M = -\frac{B^2L^2}{R}\vec{v}$$

d) To get a constant speed on rod (acceleration zero), is needed a external force acting on rod, opposite to magnetic force:

$$\vec{F}_{Ext} = -\vec{F}_M = \frac{B^2L^2}{R}\vec{v}$$

e) Along a time dt, the work done by this external force is:

$$dW = F_{Ext}dx = F_{Ext}vdt = \frac{B^2L^2v^2}{R}dt$$

Along this time, the energy lost on rod by Joule heating is:

$$dW = i^2Rdt = \frac{B^2L^2v^2}{R}dt$$

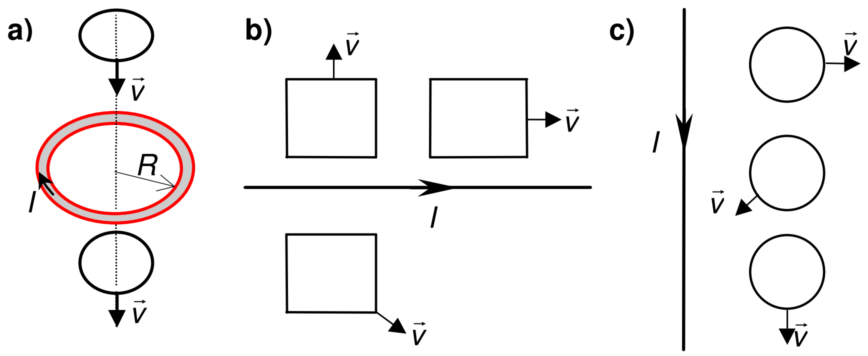
The work done by the external force to keep constant the speed of rod is the same that the lost energy by Joule heating on circuit.

This fact is a consequence of law of conservation of energy, as we have already said. Really, Lenz's law is a consequence of this law of conservation of energy.

Example 8-3

Drawn the sense of induced current in next cases, by applying Lenz's law:

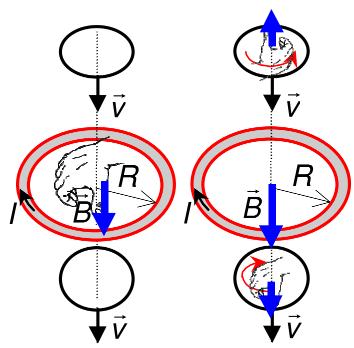
a) On two little circular loops.
b) On three rectangular loops.
c) On three circular loops.



The sense of induced current is given by Lenz's law, and must always be **opposite to <u>changes</u>** on flux (**not to the flux**).

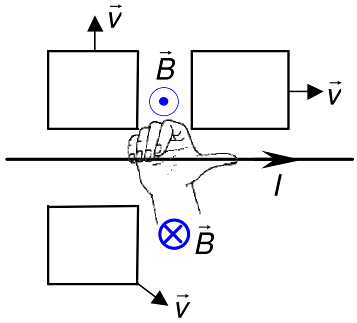
Then, to know the sense of induced current, we must on first understand the sense of flux, if this flux is increasing or decreasing, and how should the induced intensity flow in order to create a flux opposite to these changes.

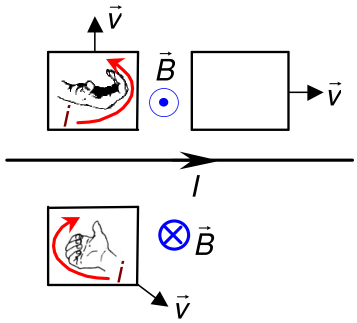
a) In this case, the current on big loop creates a magnetic field pointing to down (because of the sense of this current). Focusing our attention on the upper little loop, the flux crossing this loop points to down, and when this loop moves to down, the flux increases. Then, the induced current appearing in this little loop must flow in counterclockwise sense, in order to create a flux pointing to up, trying to avoid that the flux going to down increases.



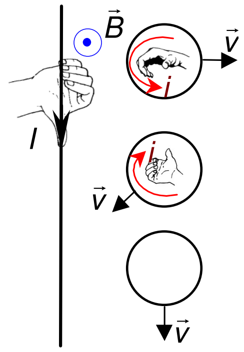
Focusing our attention on the lower little loop, the flux through this loop goes to down, but when this loop moves to down, this flux is decreasing. Then, the induced current in this loop must flow in clockwise sense, in order to reinforce the flux.

b) In this case, according the sense of current on wire, the magnetic field due to this current is exiting from paper on points above the wire, and entering on paper in points under the wire.
Focusing our attention on upper left loop, as this loop moves to up, the flux exiting from paper decreases, and then the induced current must flow in counterclockwise sense, in order to reinforce the already existing flux.
About the upper right loop, as this loop moves horizontally, the magnetic flux doesn't change, and none intensity is induced.
On lower loop, when it moves, the flux entering on paper decreases, and then the induced current must reinforce this flux, flowing in clockwise sense.



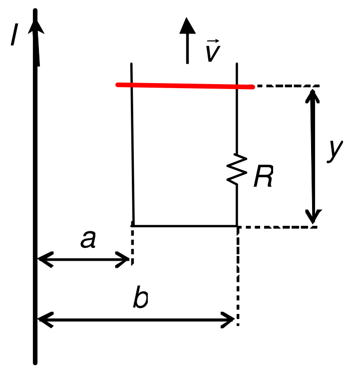


c) Now, the magnetic field created by the intensity on wire exits from paper on points to right of wire.
As the upper loop moves to right, the flux crossing this loop decrases, and then the induced current must reinforce this flux, flowing in counterclokwise sense.
About the middle loop, when it moves, the flux increases, and the induced current must flow in clockwise sense, to create a flux opposite to that existing.
On the lower loop, as it moves parallel to the wire, the magnetic field due to this wire doesn't change, and none induced current appears.



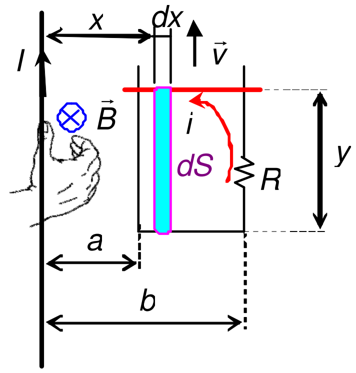
Example 8-4

Along an infinite straight carrying current wire flows an intensity of current *I* as it's show on picture. In the same plane is placed a loop with resistance *R*; the upper side of this loop moves to up at a constant speed *v*. Compute:
a) Th magnetic flux crossing the loop due to I, as a function of y (position of upper side related to lower side).
b) e.m.f. and intensity of current induced on loop, giving its sense.
c) Magnetic force acting on moving side of loop,



giving its direction and sense.

a) The magnetic field created by the wire where the loop is placed, is perpendicular to the plane of paper and entering on it, according the right hand rule. Its modulus in a point placed at a distance x from wire will be:



$$B = \mu_0 \frac{I}{2\pi x}$$

To compute the flux through the loop, we should take a differential surface where the magnetic field can be taken as uniform.

If we consider a rectangle with height y and thickness dx, magnetic field can be supposed constant across this area. The area of this rectangle is $dS = ydx$ and the flux across the whole loop can be computed integrating this elementary flux from the left side of loop (x=a) to the right side (x=b):

$$\Phi = \int_S \vec{B} \cdot d\vec{S} = \int_S BdS = \int_a^b \frac{\mu_0 Iz}{2\pi} \frac{dx}{x} = \frac{\mu_0 Iy}{2\pi} \ln\left(\frac{b}{a}\right)$$

b) The induced electromotive force can be computed from Faraday's law, taking in account that $dy/dt=v$:

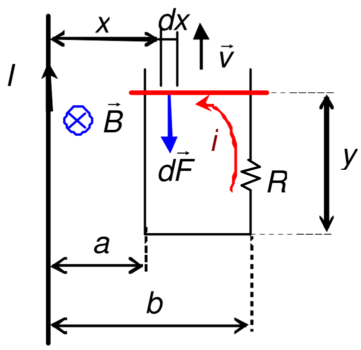
$$|\varepsilon_i| = \frac{d\Phi}{dt} = \frac{\mu_0 I}{2\pi} \ln\left(\frac{b}{a}\right) \frac{dy}{dt} = \frac{\mu_0 I}{2\pi} \ln\left(\frac{b}{a}\right) v$$

And the induced current, according Ohm's law:

$$i = \frac{\varepsilon_i}{R} = \frac{\mu_0 I}{2\pi R} \ln\left(\frac{b}{a}\right) v$$

The flux through the loop is entering on paper, and when the upper side of loop moves, this flux increases. In order to avoid that this flux increases, the induced current must flow in counterclockwise sense, producing a flux exiting from paper.

c) The force acting on moving side can be computed from the equation giving the force acting on a wire flowed by a current inside a magnetic field. But in this case, the magnetic field isn't uniform (it isn't constant on points of moving side), and then we must integrate the force acting on a little (elementary) piece of side (length dl):



$$d\vec{F} = i d\vec{l} \times \vec{B}$$

This force goes to down, according the right hand rule and the senses of i and B. The modulus of force acting on the whole side is

$$F = i \int_C Bdx = i \int_a^b \frac{\mu_0 I}{2\pi} \frac{dx}{x} = \left(\frac{\mu_0 I}{2\pi} \ln\left(\frac{b}{a}\right)\right)^2 \frac{v}{R}$$

This force will act to down. It will be opposite to the increasing of area of circuit, the origin of flux increasing.

8.4 Mutual Inductance. Self-inductance

When a circuit is flowed by an electric current, this current produces a magnetic field and a magnetic flux through the circuit itself or through other circuits near the first one. If current changes, there are also changes on magnetic fluxes, and induced currents can appear. These processes of interaction over the circuit itself or between circuits near each other can be characterized using the self-inductance or the mutual inductance coefficients.

Mutual Inductance

Let's suppose that a current I_1 flows along the curcuit 1 of Figure 8-5. This current creates a magnetic field and a magnetic flux through the circuit 2; we'll call Φ_{21} the magnetic flux through circuit 2 due to intensity flowing along circuit 1. This flux linearly depends on magnetic field created by I_1 and then linearly depends on I_1. So, we can write:

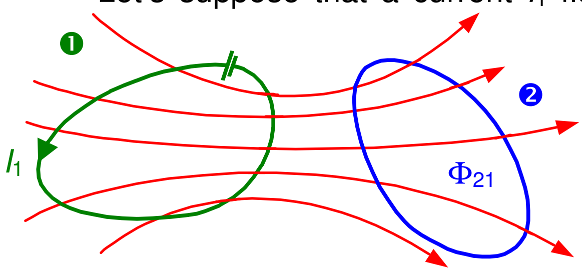


Figure 8-5. The current on circuit 1 produces a magnetic flux through circuit 2

$$\Phi_{21} = M_{21} I_1 \qquad \text{Equation 8-1}$$

The coefficient M_{21} is the **mutual inductance coefficient** and it only depends on geometry and relative position of both circuits.

In the same way, if a current I_2 would flow along circuit 2 a magnetic flux would appear through circuit 1, Φ_{12}, linearly depending on I_2:

$$\Phi_{12} = M_{12} I_2$$

Can be demonstrated that both coefficients are equal, and then:

$$M_{21} = M_{12} = M$$

M is called the mutual inductance coefficient between both circuits.

If the current is variable, magnetic flux will be variable and according Faraday's law, an induced current will appear on the other circuit. For example, if I_1 is variable, on circuit 2 will appear an induced electromotive force:

$$\varepsilon_2 = -\frac{d\Phi_{21}}{dt} = -M\frac{dI_1}{dt} \qquad \text{Equation 8-2}$$

The unit of M in the International System of Units is **henry** (H), which can be defined from Equation 8-2 as: there is a mutual inductance of 1 henry between two circuits when a rate of variation of 1 A/s in one circuit produces an e.m.f. of 1 V in the second circuit.

Self-inductance

In the same way that on before paragraph about mutual inductance, when we have only one circuit, flowed by an intensity of current *I*, the magnetic field created by this intensity produces a magnetic flux ϕ through the circuit itself; this magnetic flux is directly related to the intensity of current (Figure 8-6). It is $\Phi = LI$

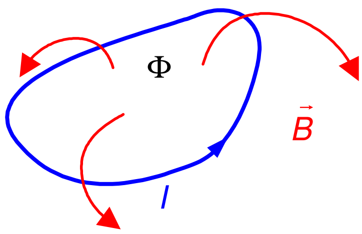


Figure 8-6. Current on a circuit produces a magnetic flux through the circuit itself

The rate *L* between flux and intensity is called **self-inductance coefficient**

$L = \dfrac{\Phi}{I}$	Equation 8-6

It only depends on geometry of circuit. Its unit on I.S. is the same that the unit for mutual inductance (Henry). The physical device producing a significant self-inductance is called an **inductor**; for example, a coil shows a high self-inductance. Its symbol in a scheme of a circuit is that of Figure 8-7.

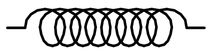


Figure 8-7. Symbol for an inductor

If the intensity *I* on circuit is varying on time, also the magnetic flux through the circuit is varying on time, and according Faraday's law, an induced e.m.f. appears in the circuit itself:

$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}$$

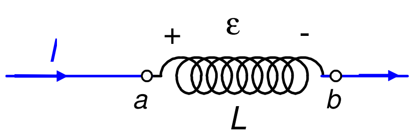
Self-inductance coefficient *L* is always positive.

We are going to analyse how we can determine the drop of potential in terminals of an inductor:

a) If intensity *I* increases, according Lenz's law, the induced current is opposite to this increasing of intensity; then, the inductor behaves as a generator with the positive terminal on point *a*, and point *b* with lower potential; difference of potential between *b* and *a* will come from

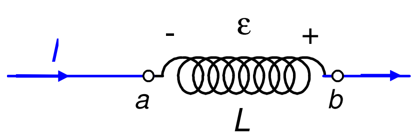
$$V_b - V_a = -L\frac{dI}{dt} < 0$$

because *L* and *dI*/*dt* are both positive.



I increases

$$V_b - V_a = -L\frac{dI}{dt} < 0$$



I decreases

$$V_b - V_a = -L\frac{dI}{dt} > 0$$

b) If intensity *I* decreases, according Lenz's law, the induced current will have the same sense tan *I* and the inductor will behave as a generator, with the positive terminal on point *b*, and the negative terminal on point *a*; the difference of potential between *b* and *a* will come from

$$V_b - V_a = -L\frac{dI}{dt} > 0$$

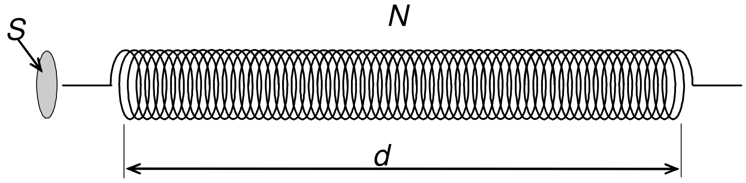
In this case, as dI/dt is negativo, point *b* has greater potential than *a*.

As a conclusion, difference of potential between *b* (point where intensity is exiting from inductor) and *a* (point where intensity is entering on inductor) will be given by the same equation:

$$V_b - V_a = -L\frac{dI}{dt} \qquad \text{or} \qquad V_a - V_b = L\frac{dI}{dt} \qquad \text{Equation 8-7}$$

Example 8-6

Find the self-inductance coefficient of coil on picture assuming that its length is very large compared with its radius (d>>r), the number of turns *N* is high, and that the magnetic field created inside of coil by an intensity of current *I* flowing along the coil is $B = \mu_0 NI/d$. Apply the result to a coil having 500 turns, 5 cm of radius, and length 50 cm.



The self-inductance coefficient is defined, according Equation 8-6, as

$$L = \frac{\Phi}{I}$$

As d>>r, the magnetic field can be supposed as uniform (constant) inside the solenoid, and this magnetic field goes in the direction of axis of coil. The magnetic flux through the coil is

$$\Phi = N\int_S \vec{B}\cdot d\vec{S} = N\int_S BdS = NB\int_S dS = NBS = N\frac{\mu_0 NI}{d}S = \frac{\mu_0 N^2 S}{d}I$$

and

$$L = \frac{\mu_0 N^2 S}{d} = \mu_0 S n^2 d$$

being *n* the number of turns by unit of length n=N/d

Applying this equation to the given coil:

$$L = \frac{\mu_0 N^2 S}{d} = \frac{4\pi\cdot 10^{-7}\cdot 500^2 \pi \cdot 0{,}05^2}{0{,}5} = 4{,}93\ \text{mH}$$

8.5 *LR* Circuit

A *LR* circuit is a circuit having resistors and inductors. If the intensity on circuit is a direct current (not varying on time), the inductor hasn't any effect on circuit, because the induced electromotive force on inductor is zero *di/dt=0* and then the inductor acts exactly as a short-circuit.

But if the intensity changes on time, the induced electromotive force on inductor, according Lenz's law, produces transient (no steady) effects on circuits, in a similar way that it happens in circuits with capacitors. In order to analyse this transient behaviour, we'll analyse the circuit on Figure 8-7 when the switch is turned on and off. This circuit is made up by a resistor, an inductor, a battery and a switch.

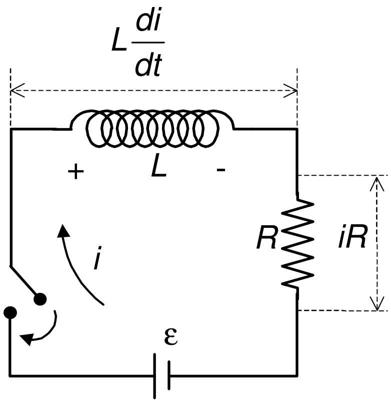


Figure 8-7. *LR* circuit when switch is turned on (closed)

When the **<u>switch is turned on</u>**, the intensity of current flows from none intensity to its steady magnitude i_e; when this steady magnitude is reached, there isn't e.m.f. on inductor and then

$$i_e = \frac{\varepsilon}{R}$$

But while the intensity is increasing from zero to i_e, e.m.f. on inductor must be taken in account, and the second Kirchhoff's law for this circuit is:

$$\varepsilon - L\frac{di}{dt} - iR = 0$$

It is an equation where intensity (*i*) and its derivative can be found; in order to get how the intensity evolves on time while the intensity reaches its steady magnitude, both variables (*i* and *t*) must be separated and then integrate the resulting equation; from before equation:

$$\int \frac{di}{\frac{\varepsilon}{R} - i} = \int \frac{R}{L} dt \Rightarrow \ln\left(\frac{\varepsilon}{R} - i\right) = -\frac{R}{L}t + C$$

the constant of integration can be easily computed taking in account that on time $t=0 \rightarrow i=0$. Result is $C = \ln\frac{\varepsilon}{R}$ and then, substituting and reordering the before equation it comes:

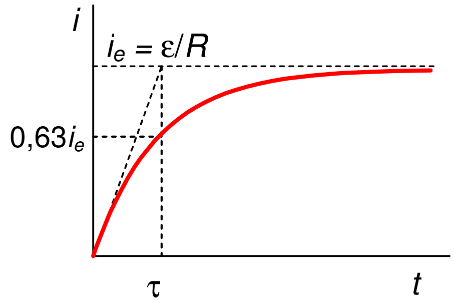
$$i(t) = \frac{\varepsilon}{R}\left(1 - e^{-\frac{R}{L}t}\right) \quad \text{or} \quad i(t) = i_e\left(1 - e^{-\frac{t}{\tau}}\right)$$

Equation 8-8

remember that $i_e = \frac{\varepsilon}{R}$ is the steady intensity, and $\tau = \frac{L}{R}$ is known as **time constant** of circuit. Really, τ has dimensions of a time, and then it's measured in seconds. τ represents the time when the 63% of steady intensity is reached, because substituting $t=\tau$ on equation 8-8:

$$i(\tau) = i_e\left(1 - e^{-\frac{\tau}{\tau}}\right) = i_e\left(1 - \frac{1}{e}\right) = 0{,}63 i_e$$

Drawing of intensity against time is:



If the **switch is turned off** when the steady current is flowing, and the circuit is closed through a second wire (removing the battery, Figure 8-9), the energy stored on the inductor is released as Joule heating on resistor, and the intensity decreases from the steady value, i_e ,to zero.

The second Kirchoff's rule for this circuit at any time *t* is:

$$-L\frac{di}{dt} - iR = 0$$

Integrating this equation in the same way that we have already done, and taking in account that intensity is $i_e = \frac{\varepsilon}{R}$ on time *t=0* (when the switch is turned off), it comes:

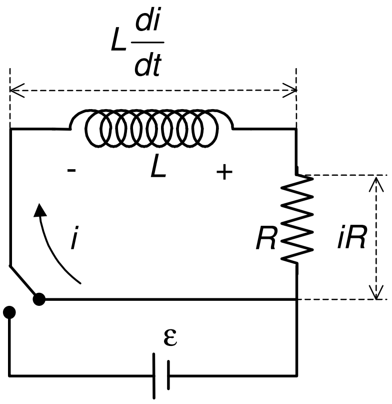


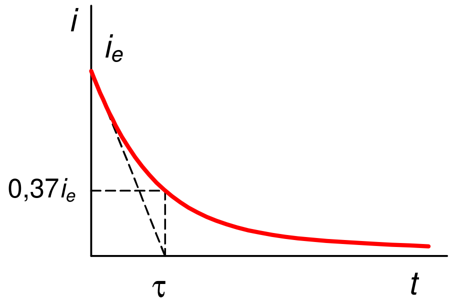
Figure 8-8. *LR* circuit when switch is turned off (open the circuit with the battery)

$$i(t) = i_e e^{-\frac{t}{\tau}} \qquad \text{Equation 8-9}$$

Now, the time constant is the time when the intensity is 37% of i_e :

$$i(\tau) = i_e e^{-\frac{\tau}{\tau}} = \frac{i_e}{e} = 0{,}37 i_e$$

And drawing of intensity against time:



Obviously, the time to completely reach the steady state (both for switch on or for switch off) is infinite, but this steady state can be supposed when a 99% of steady intensity (on switch on) or 1% (on switch off) is reached:

for switch on: $$i(t = 5\tau) = i_e\left(1 - e^{-5}\right) \approx 0{,}99 i_e$$

for switch off $$i(t = 5\tau) = i_e e^{-5} \approx 0{,}01 i_e$$

The inductors on circuits of variable currents produce an effect similar to that of capacitors, introducing a transient state. Any circuit shows a certain self-inductance, not only the coils; even the circuits made up by straight conductors show self-inductive behaviour. This fact must be taken in account when a variable electric signal, digital or analogic, is transmitted along a circuit (it behaves as a *RL* circuit).

As an example of behaviour of variable signals on any circuit, we'll do a brief study about the opening and closing of a switch in an electric circuit; we'll see how the speed of opening/closing affects the behaviour of circuit.

On first, we'll suppose that the speed of opening/closing of switch is low; for example, with a period (T) 100 times higher than the time constant of circuit: T = 100 τ. In this case, the time taken by the circuit to reach the steady state (5τ) is very low compared with the period of switch on/switch off, and the drawing of intensity on circuit against time is box shaped. If this circuit is trannsmitting a digital signal instead opening or closing a switch, the intensity along the circuit is very similar to the digital signal (Figure 8-9 a):

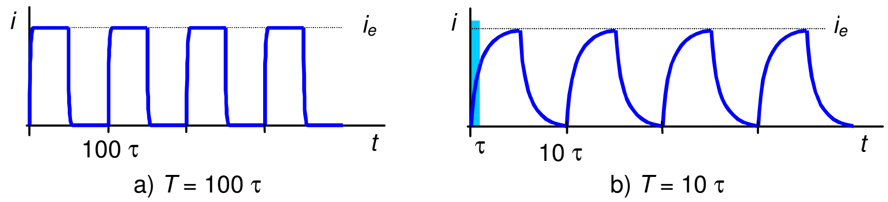
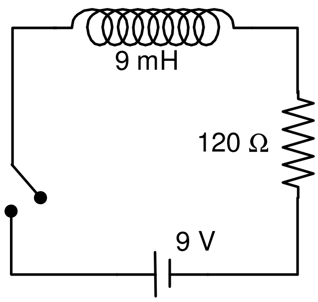


Figure 8-9. a), intensity on a *RL* circuit with a period 100 times the time constant of circuit. b), intensity on a circuit with a period 10 times the time constant of circuit.

But if the speed of opening/closing of switch is fast, for example T = 10 τ, then the intensity on circuit doesn't almost reach the steady intensity, and the signal doesn't correspond to a digital signal. This circuit isn't able to transmit digital signals of high frequency (Figure 8-9 b). This phenomenon must be taken in account when designing of circuit transmitting electrical signals.

Example 8-7

Determine the time constant of a circuit with a 9 mH coil and a 120 Ω resistor, as can be seen on picture. ¿Which is the steady current when the switch is on? ¿How much time is needed to get a 99% of the steady current?



Solution

$$\tau = L/R = 9*10^{-3}/120 = 75\ \mu s$$

The steady current will be $i_e = \varepsilon/R = 75$ mA

The time needed to get a 99% of steady current will come from:

$$0{,}99 i_e = i_e \left(1 - e^{-\frac{t}{75*10^{-6}}}\right)$$

Solving for t: t=375 µs.

This magnitude is near $5\tau=5*75*10^{-6}=345$ µs. The difference is only due to the rounding.

8.6 Stored energy on an inductor.

Let's suppose a circuit (Figure 8.10) made up by a battery, a resistor and an inductor, with an intensity of current *i* flowing along it. The second Kirchoff's law for this circuit can be written as:

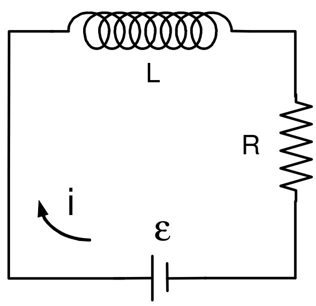


Figure 8.10

$$iR - \varepsilon + L\frac{di}{dt} = 0$$

By multiplying all the equation by the intensity *i* and rearranging it comes:

$$\varepsilon i = i^2 R + Li\frac{di}{dt}$$

The first term on left is the supplied power by the ideal generator to the circuit (P_s); the first term on right is the power lost by Joule heating on resistor (P_R), and then the second term on right must be the consumed power on inductor (P_L):

$$P_s = P_R + P_L \qquad \text{being} \qquad P_L = Li\frac{di}{dt}$$

It's interesting to note that the inductor doesn't consume power ($P_L=0$) when the circuit is in the steady state (intensity of current constant). If the intensity is increasing ($\frac{di}{dt} > 0$) P_L is positive and the inductor "takes" energy from the circuit, storing this energy in the magnetic field created by the inductor. If the intensity is decreasing ($\frac{di}{dt} < 0$) P_L is negative and the inductor "returns" energy to the circuit.

The total stored energy by the inductor since the intensity began to flow (*i=0*) until the intensity *I* is reached (W_L) will be:

$$W_L = \int_T P_L dt = \int_T Li\frac{di}{dt}dt = \int_0^i Lidi = \frac{1}{2}Li^2$$

In this equation, *T* is the time taken to reach the intensity *i*, and the constant of integration has been computed considering that intensity was zero on time *t=0*. It's interesting to note that the stored energy is not depending on the increasing or decreasing of intensity (even if the intensity is constant), and it only depends on the magnitude of the intensity.

An inductor behaves in a similar way that a capacitor. Capacitor stores energy in the electric field between its plates, and this energy is returned to the circuit when the intensity changes its sense. A coil stores energy in the magnetic field created when an intensity of current flows along it, and this energy is returned to the circuit when the intensity decreases.

As an example, we can compute the energy stored in the coil of example 8-6:

$$W = \frac{1}{2}LI^2 = \frac{1}{2}\mu_0 Sn^2 dI^2$$

In this equation, can be identified several magnitudes:

Magnetic field inside the coil: $B = \mu_0 nI$
Volume of coil: V=Sd

The, the energy stored on coil can be written as:

$$W = (\mu_0 nI)^2 \frac{Sd}{2\mu_0} = \frac{1}{2\mu_0}B^2V$$

And the stored energy by unit of volume of coil: $\frac{W}{V} = \frac{1}{2\mu_0}B^2$

This equation of stored energy by unit of volume is general for any magnetic field, not only for a coil.

8.7 Applications of electromagnetic induction phenomena

Reading of information recorded on a magnetic support

In the before unit we dealt with the information recorded in ferromagnetic supports by using magnetic discs or magnetic tapes. A coil flowed by an intensity varying according the information to be recorded records the information in the ferromagnetic material.

In order to read this information, the disc or tape must now be moved near a reading head, containing a nucleus of iron with a coil around it. When the magnetic material is moved near the reading head, a variable magnetic field (and a variable magnetic flux) induces a current in the reading head, according the information recorded in the ferromagnetic material. This foundation is good both for analogic information as for digital information.

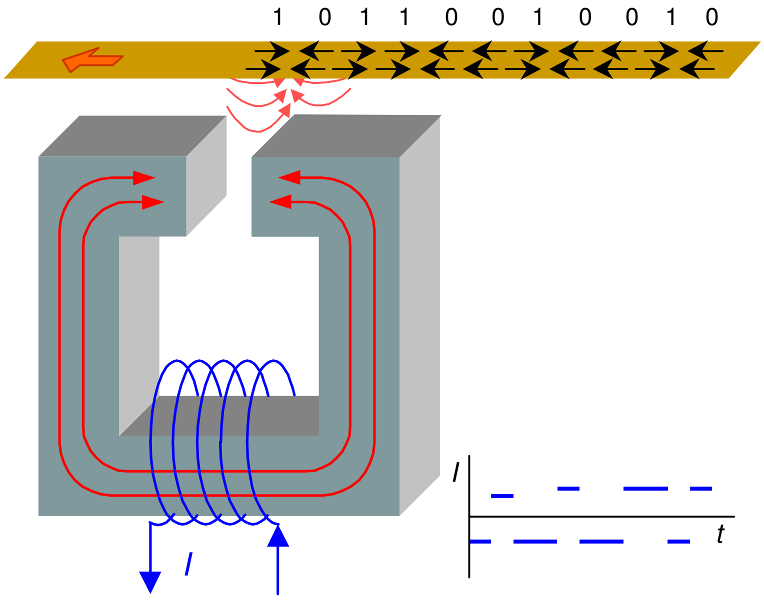


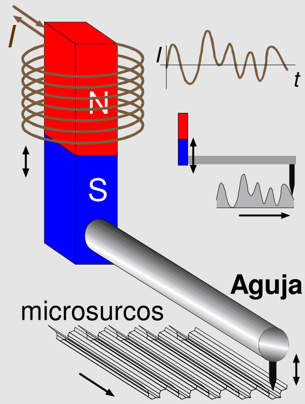
Figure 8-11. Reading of information in magnetic support

Lectura por inducción

La tecnología para la grabación y reproducción de información ha evolucionado a lo largo de todo el siglo XX, aunque los fundamentos físicos apenas han variado hasta la llegada de la tecnología óptica en 1979 con la aparición de los primeros CD.

Hasta la llegada de la era de la informática a mediados del siglo XX, en que comenzó a utilizarse también para datos informáticos, esta tecnología ha estado asociada a la industria musical.

Así, principios del siglo XX se utilizaba el gramófono para grabar y reproducir voz y música. Éste se transformó en tocadiscos con fonocaptor electromagnético en la década de 1930. Para almacenar la información musical, se utiliza un soporte plástico (discos de vinilo) que gira a 33 revoluciones por minuto. En este soporte, una aguja recorre los microsurcos grabados vibrando de acuerdo con las irregularidades de los microsurcos, al igual que haría una bicicleta en un camino montañoso. Las vibraciones de la aguja, son transmitidas a un imán, que induce de este modo un flujo magnético variable en un bobina, transformándose así la información "topográfica" del microsurco en corriente eléctrica inducida que reproduce las irregularidades del "terreno". Puede hacerse con imán móvil (figura), o con bobina móvil.



Detalle de un fonocaptor electromagnético de imán móvil de un tocadiscos. La aguja, al pasar por las irregularidades del microsurco, vibra y transmite la vibración al imán.

A lo largo del siglo XX, el procedimiento evolucionó dirigido hacia su miniaturización, alcanzándose un gran hito con el desarrollo de los casetes portátiles a partir de 1961, que permitió por primera vez grabar y reproducir música fácilmente de un modo económico. El soporte en este caso es una cinta de material ferromagnético donde la información está grabada, al igual que en un disquete, en forma de dominios magnéticos, que son leídos al pasar la cinta cerca de un cabezal férrico con una bobina donde se induce la corriente, y por tanto, se lee la información.

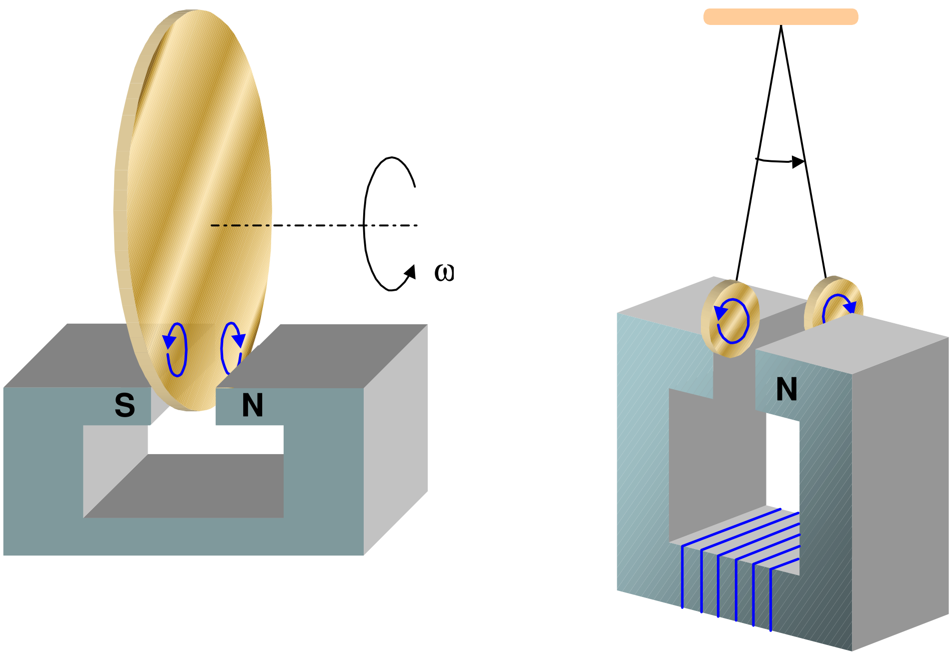
En 1951 se usó la cinta magnética por primera vez para datos informáticos. En 1976 aparecen los primeros disquetes de 5,25 pulgadas. A finales del siglo XX, con la llegada de la era informática, se alcanzaron tamaños de cabezales muy pequeños capaces de leer y grabar. Sin embargo, el fundamento básico siguió siendo el mismo: un soporte con información, magnética en este caso, induce corriente en un cabezal lector con una bobina arrollada a su alrededor.

Eddy currents (Corrientes de Foucault)

On the examples above we have analysed the electromagnetic induction phenomena produced in well-defined circuits, but sometimes the induced currents are produced in no so evident circuits.

As an example, on Figure 8-12 (a), a conductor disc is turning around an axis with a part of disc inside the magnetic field produced by a magnet. Through the surface bordered by any path or closed line we can imagine on disc, the magnetic flux is varying when this surface is entering and exiting on magnetic field; according Faraday's law, an induced current will flow along the closed line.

According Lenz's law, this current will be opposite to that is producing it, and a braking force appears on disc opposite to the movement. On Figure 8-12 (b), a force appears on disc opposite to the movement of disc. This phenomenon is used to build magnetic brakes on trucks, high speed trains, etc…



(a) (b)

Figure 8-12. Eddy currents

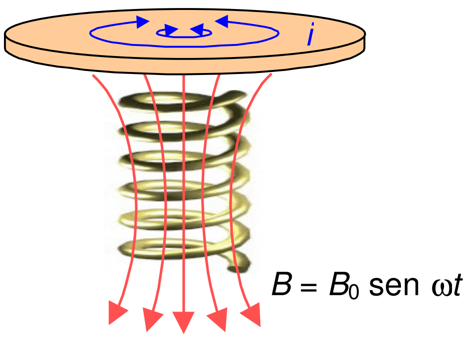


Figure 8-13. Eddy currents in induction cook tops and ovens

But these currents produce a second effect on conductor: due to Joule heating, the conductor is heated, producing heat. In some cases can be taken advantage of this effect, for example building induction cook tops: a high frequency sinusoidal magnetic field is produced by a coil placed under the cook top. A cookware made with a conductor material is placed over the cook top, and the Eddy currents induced in the cookware produce the heating of the meal.

But this heating effect can be also undesirable, because of the loss of energy. This effect can be reduced avoiding the paths for the induced currents, as we'll see on next paragraph speaking about the transformer.

Transformer

A very important application of electromagnetic induction phenomena is the transformer (Figure 8-14), used to increase or reduce the voltage of an electric current. As we'll see on next unit, the transformer is responsible for commercially using alternating current instead direct current.

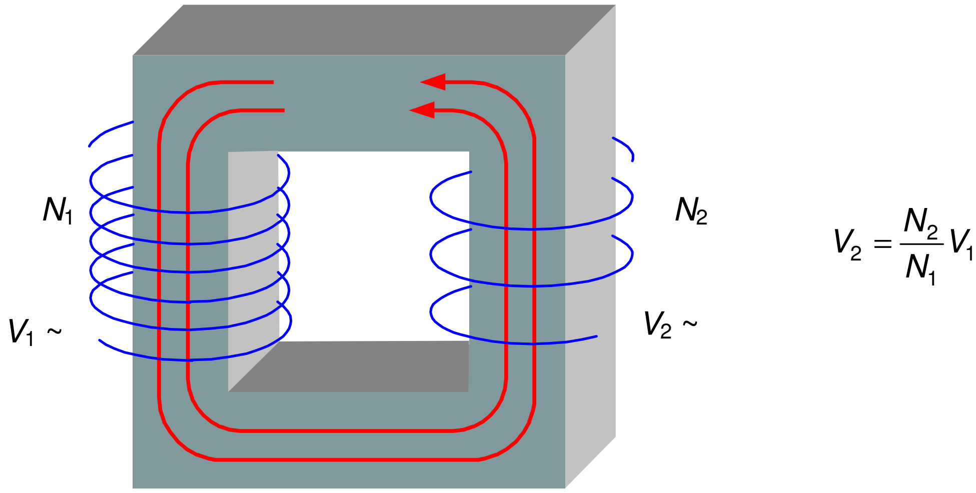


Figure 8-14. The transformer

A nucleus of a soft ferromagnetic material is turned by two coils, the primary (N_1 turns) and the secondary (N_2). When a voltage is applied to one of them (for example to the primary), the current flowing along it produces a magnetic field and then a magnetic flux through the primary. But this magnetic flux is driven by the nucleus and crosses the coil of secondary. If voltage applied to primary is constant, then the magnetic flux is constant and nothing happens (the transformer doesn't work on direct current). But if the voltage applied varies on time, magnetic flux varies on time and an e.m.f. is induced on secondary. According Faraday's law:

$$\varepsilon_1 = V_1 = N_1 \frac{d\Phi_u}{dt}; \qquad \varepsilon_2 = V_2 = N_2 \frac{d\Phi_u}{dt}$$

As magnetic flux per loop of coil ϕ_u crossing primary and secondary is the same:

$$\frac{V_1}{N_1} = \frac{V_2}{N_2} \quad \Rightarrow \frac{V_2}{V_1} = \frac{N_2}{N_1},$$

the rate between voltages on primary and secondary is exactly the same that the rate between turns, and this rate is known as rating of transformer.

The nucleus is made with a soft ferromagnetic material in order to minimize the loss by magnetic hysteresis.

In order to also minimize the loss due to Eddy currents, the nuclei of transformers are not built with solid material, but with a lot of sheets isolated between them with a thin layer of an insulator. In this way, only low Eddy currents are produced in the nucleus and only a small energy is lost by Joule heating (Figure 8-15).

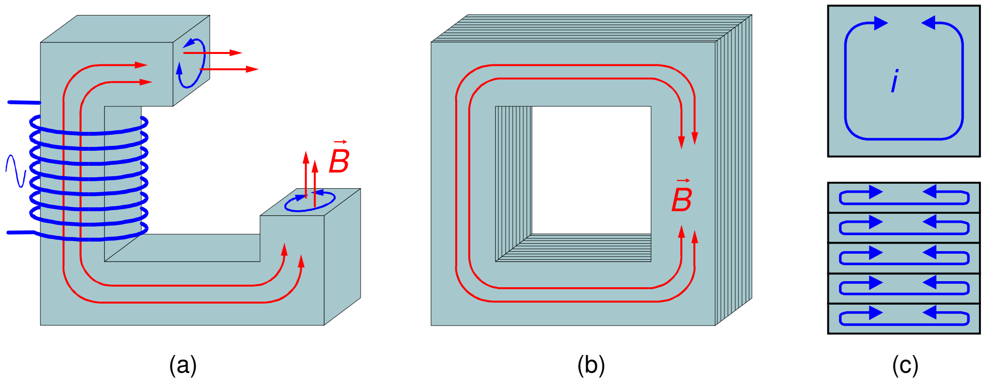


Figure 8-15. In order to minimize the loss due to Eddy currents on a transformer, the nuclei are laminated as thin layers isolated between them, avoiding the big circuits with high induction currents.

Alternating current (A.C.) generation

The foundation of A.C. generation is a loop turning inside a magnetic field, as can be seen on Figure 8-16.

When the loop turns inside a uniform magnetic field (in this case created by a magnet), the magnetic flux across the loop depends on angle between loop and magnetic field, according the equation:

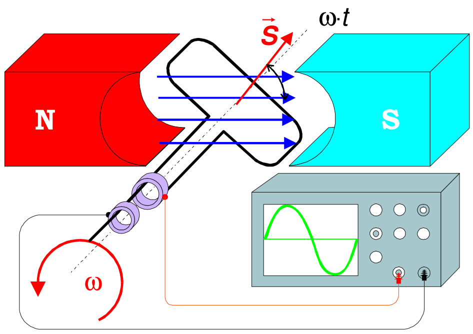


Figure 8-16. Sinusoidal A.C. generation

$$\Phi_l = \int_S \vec{B}\cdot d\vec{S} = \vec{B}\cdot\vec{S} = BS\cos\varphi$$

where φ is the angle between magnetic field $\vec{B}$ and surface vector $\vec{S}$. If we have not only one loop but a coil (N loops), magnetic flux will be multiplied by N:

$$\Phi = NBS\cos\varphi$$

if the angular speed of coil (ω) is constant, the turned angle depends on time according the equation:

$$\varphi = \omega t + \varphi_0$$

applying Faraday's law to magnetic flux, we get the induced electromotive force on coil:

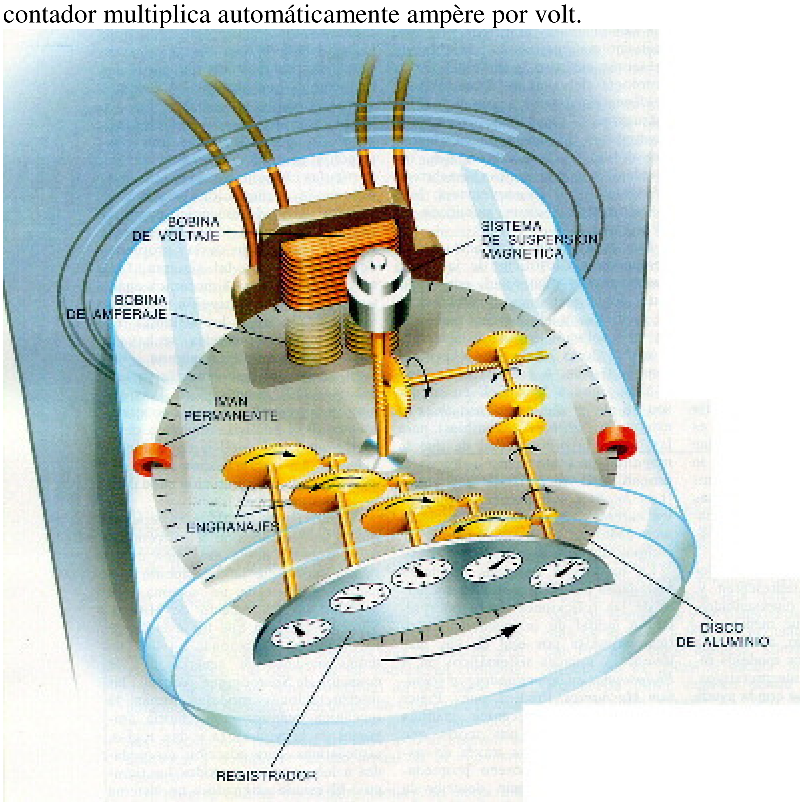
$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{d(NBS\cos(\omega t+\varphi_0))}{dt} = NBS\omega sen(\omega t+\varphi_0) = NBS\omega\cos(\omega t+(\varphi_0+\frac{\pi}{2}))$$

It is a sinusoidal electromotive force (alternating current), that we can found on sockets at home. But the physics foundations of producing alternating current is the turning a loop inside a magnetic field, both if this turning is got from hydraulic, thermal or nuclear energy. On next unit will be specifically studied alternating current.

Contadores de electricidad

Les Rosenau Investigación y Ciencia, mayo de 2000

El contador encerrado en una carcasa de vidrio que pende de una pared de su casa, en el sótano o de un poste cercano al aire libre, registra la energía que fluye a su domicilio procedente de una planta de la compañía de luz. Ese aparato mide la corriente (flujo de electrones, que se expresa en ampère) y el voltaje, o tensión que impulsa a los electrones por el hilo conductor. Para determinar el consumo, el contador multiplica automáticamente ampère por volt.



Un contador es, a grandes rasgos, un motor movido por las fuerzas magnéticas creadas por el paso de una corriente eléctrica a través de bobinas. Los conductores de entrada están conectados a una bobina de voltaje; la corriente fluye entonces por la bobina de amperaje hacia el circuito del domicilio. Cuando la corriente atraviesa las dos bobinas, el campo magnético inducido hace que un disco de aluminio gire a una velocidad proporcional a la cantidad de watt consumidos.

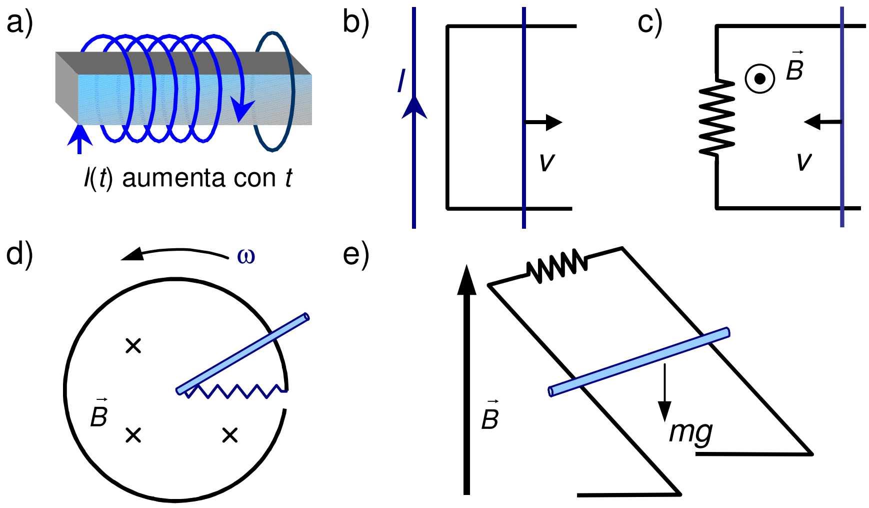
Sobre ambas caras del disco de aluminio hay montados imanes permanentes para asegurar la precisión de su movimiento; otro campo magnético mantiene suspendidos en el aire el disco y su eje, eliminando así los rozamientos que pudieran estorbar una lectura correcta.

Cada revolución del disco suele equivaler a 7,2 watt-hora. (Como referencia, una bombilla consume 100 watt-hora de electricidad por hora.) Cuanto más potencia consuma la casa, más rápido gira el disco. Como las compañías de electricidad miden los consumos en grandes unidades, o sea, en kilowatt-hora, cada 138,88 revoluciones del disco indican un consumo eléctrico de 1 Kilowatt-hora (1000watt-hora). Por consiguiente cada 1000 vueltas del disco indican un consumo de 7,2 kilowatt-hora.

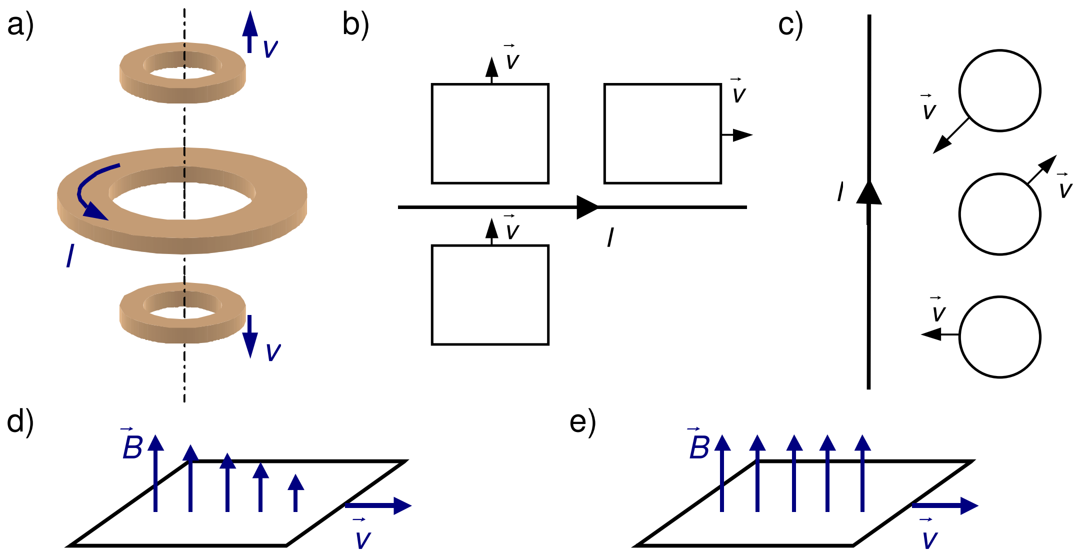
Un tren de engranajes transfiere la información sobre el numero de revoluciones del disco al conjunto de diales de un registrador; el numero de diales depende del tipo de contador. El lector del contador registra la posición del dial de kilowatt-hora y determina el consumo del mes por sustracción de la lectura anterior. La implantación de una nueva técnica permite que los contadores comuniquen las lecturas de kilowatt-hora a una instalación central mediante radioondas, líneas telefónicas o incluso mediante la misma línea de distribución de electricidad.

8.8 Problems

1. State the sense of induced current on circuits on picture; also state the sense of magnetic force appearing on mobile side (on loop of case a).

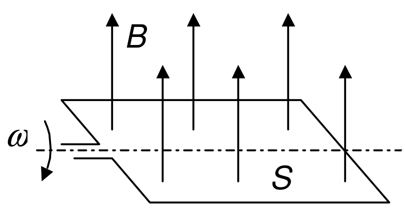


2. Drawn the sense of induced current on loops of pictures:

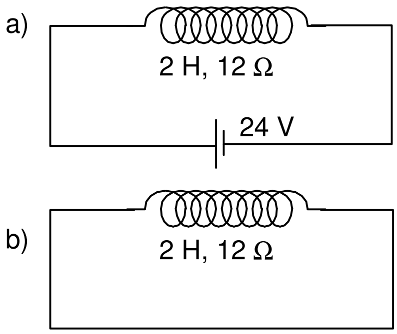


3. Compute the magnetic flux and the e.m.f. on the squared loop of picture; this loop has an area S and it's turning at constant angular speed ω inside a uniform magnetic field B.

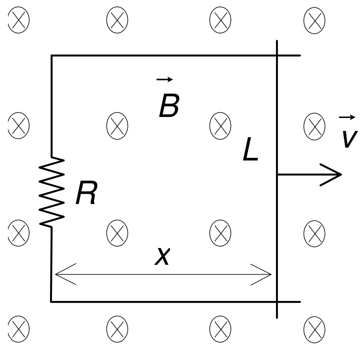
Sol: $\Phi = BS\cos\omega t$ $\quad \varepsilon = BS\omega\ \mathrm{sen}\,\omega t$



4. Let's take a coil with a self-inductance coefficient $L = 2$ H and a resistance $R = 12\ \Omega$ connected to an ideal generator with $\varepsilon = 24$ V (fig.(a)).
When the steady state has been got:
a) Compute the intensity of current flowing along the circuit.
b) Compute the stored energy on coil.
c) If the coil is short-circuited and generator is removed (fig. (b)) ¿Which is the lost energy as heating on coil due to its resistance?
Sol: a) $I = 2$ A , b) $W = 4$ J , c) $W_Q = 4$ J

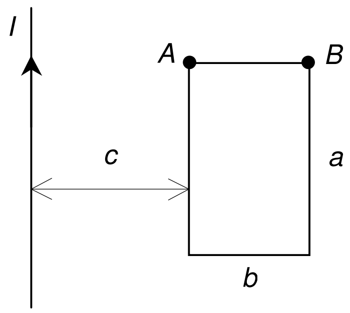


5. A conductor rod with resistance negligible and length L glides without friction and constant speed v on a conductor U shaped. The U shaped conductor has a resistance R and it's placed inside a uniform magnetic field $\vec{B}$ perpendicular to conductor. Compute:
a) Magnetic flux crossing the loop as a function of x.
b) Induced current on the loop, showing its sense.
c) Force should act on the rod in order it be displaced at constant speed v.



Sol: a) $\Phi = BLx$ b) $I = \dfrac{BLv}{R}$ c) $F = \dfrac{B^2 L^2 v}{R}$

6. Along an infinite straight carrying current conductor flows an intensity $I = Kt$ (K is a positive constant). A rectangular loop with sides a and b is placed in the same plane than the conductor, as can be seen on picture. Compute:
a) Induced e.m.f. on loop ε_i.
b) If the loop has a resistance R, compute the induced current i, showing its sense.
c) Magnetic force acting on side AB as a function of time, $\vec{F}(t)$.
d) Mutual inductance coefficient M between conductor and loop.



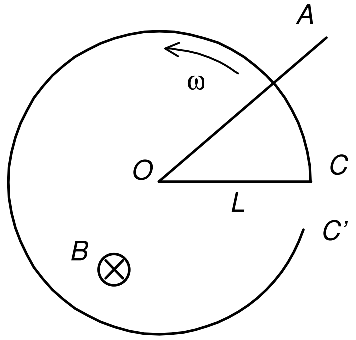
Sol:

a) $\varepsilon_i = \dfrac{\mu_0 K a}{2\pi} \ln\left(\dfrac{c+b}{c}\right)$ b) $i = \dfrac{\mu_0 K a}{2\pi R} \ln\left(\dfrac{c+b}{c}\right)$

c) $F = \left[\dfrac{\mu_0 K}{2\pi}\right]^2 \ln^2\left(\dfrac{c+b}{c}\right)\dfrac{at}{R}$ perpendicular to AB and pointing to down

d) $M = \dfrac{\mu_0 a}{2\pi} \ln\left(\dfrac{c+b}{c}\right)$

7. A ring shaped conductor having radius L and negligible resistance is opened between C and C'; it's placed inside a uniform magnetic field B, perpendicular to the plane of ring, entering on paper. A copper rod OA turns around its ending O with constant angular speed ω, in such way that the ending A is always in contact with the ring. Between O and C there is a conductor wire with resistance R. Compute:



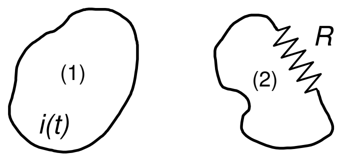
a) Magnetic flux through closed circuit $OACO$ as a function of time.

b) Induced e.m.f. on such circuit.

c) Induced current flowing across resistor R.

Sol: a) $\Phi = \dfrac{BL^2\omega t}{2}$ b) $\varepsilon = \dfrac{BL^2\omega}{2}$ c) $I = \dfrac{BL^2\omega}{2R}$

8. The mutual inductance coefficient between circuits on picture is M. If a current $i(t) = I_0\cos(\omega t + \phi)$ flows along circuit 1, ¿which is the intensity flowing along circuit 2?

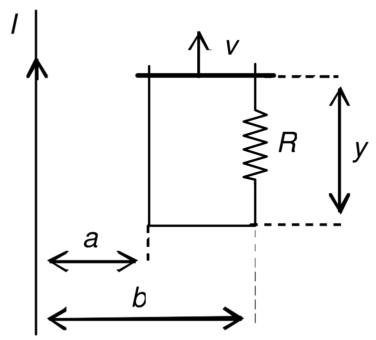


Sol: $\dfrac{MI_0\omega}{R}\mathrm{sen}(\omega t + \phi)$

9. Along the infinite straight carrying current wire on picture flows an intensity of current 2 A sized in the shown sense.

There is a loop of resistance R in the same plane that the wire; the upper side of loop is moving to up with constant speed v.

Compute:



a) Magnetic flux crossing the loop due to the 2 A current as a function of y.

b) The induced e.m.f. on loop.

c) The induced current on loop, showing its sense.

d) The force acting on moving side of loop.

Sol: a) $\Phi = \dfrac{\mu_0 Iy}{2\pi}\ln(b/a)$ b) $\varepsilon = \dfrac{\mu_0 I}{2\pi}\ln(b/a)v$ c) $i = \varepsilon / R$ d) $F = \left[\dfrac{\mu_0 I\ln(b/a)}{2\pi}\right]^2 \dfrac{v}{R}$

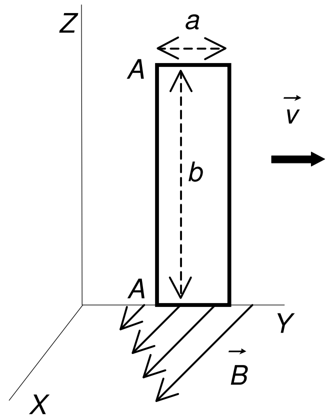
10. A rectangular loop (sides a and b) having resistance R is placed on plane X=0. The whole loop is moving with constant speed v to right, inside a (no uniform) magnetic field $\vec{B} = Cy\vec{i}$ T. On time t = 0 the side AA' was over OZ axis. Compute, for a time t:

a) Mgnetic flux through the loop.

b) E.m.f. on loop.

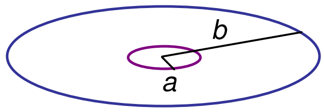
c) Intensity of current flowing along the loop, showing its sense.

d) Resulting force acting on loop, showing its sense.



Sol: a) $\Phi = \dfrac{Cb\left(a^2 + 2avt\right)}{2}$ b) $\varepsilon = avCb$ c) $i = avCb/R$ d) $\vec{F} = -\dfrac{a^2 b^2 C^2 v}{R}\vec{j}$

11. Two ring shaped loops, having radii a = 1 cm and b = 50 cm, are placed concentric in the same plane. If can be supposed that a<<b (magnetic field on loop a due to current on b can be supposed uniform) compute:



a) Mutual induction coefficient between both loops.
b) Magnetic flux across loop b when an intensity I = 5 A sized flows along a.

Sol: a) $M = \dfrac{\mu_0 \pi a^2}{2b} = 4 \cdot 10^{-11}\pi^2$ H b) $\Phi = 2\pi^2 \cdot 10^{-10}$ Wb

GLOSARIO

Faraday's law: Rule quantifying the electromagnetic induction phenomena: *"The induced electromotive force (e.m.f.) on a circuit, ε, es directly related to the varying speed of magnetic flux through the circuit"* ($\varepsilon = -\dfrac{d\phi}{dt}$).

Lenz's law: *The sense of induced current is always opposite to that is producing it.*

Eddy currents: Electric currents appearing inside the conductors when changes on magnetic flux through them are produced.

Self-inductance coefficient (L): Constant relating the magnetic flux crossing a circuit to the intensity flowing along it ($\Phi = LI$).
Its magnitude only depends on the geometry of circuit.

Mutual inductance coefficient (M): Constant relating the magnetic flux crossing a circuit to the intensity flowing along another circuit ($\Phi_2 = MI_1$).
Its magnitude depends on the geometry of both circuits and their relative position.

***Henry (H)*:** I.S. unit of inductance. There is a 1 henry mutual inductance between two circuits when an e.m.f. of 1 *V* is induced on a circuit if 1 *A/s* is the varying speed of intensity on another circuit. The same definition can be applied for the self-inductance coefficient.

Time constant of a RL circuit **($\tau = L/R$):** When the switch of a *RL* circuit is on, the time constant is the time taken to reach the 63% of the steady intensity.