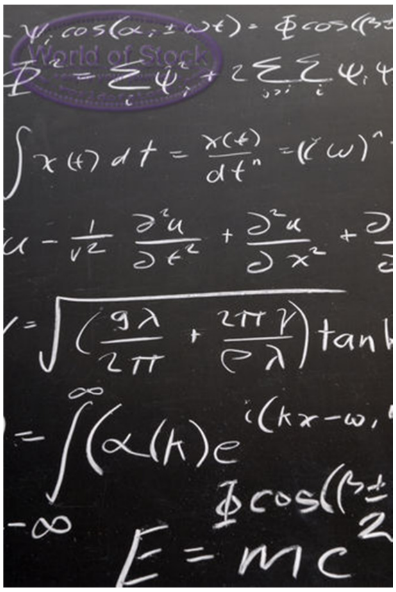
Unit 0

Vectors, derivatives, integrals and trigonometry



Objectives

- To know the basic rules to operate with quantities and physic laws.

0.1 Scalar and vector quantities

There are in physics some quantities being enough to give a number to specify it in a complete way. These quantities are scalar quantities: for example mass of an object, temperature on a point in the space, or kinetic energy of a body.

Scalar quantities are not depending on the chosen reference system, having the same value on any other reference system.

On the other hand, there are quantities that need more information to be completely defined. Quantities as velocity, force, or magnetic field need not only a scalar giving its size, but they also need specify its direction. For example, a speed of 20 Km/h hasn't meaning without the direction (for example, North direction). These quantities are called vector quantities.

0.2 Reference systems

In order to work with vector quantities, a reference system to work with must be defined. On three-dimensional space, such reference system can be defined in some different ways, but it's usual to take a cartesian reference system, consisting on three perpendicular axes crossing on a point known as origin of the reference system (O). Such axes are usually called as X, Y and Z axes, having a positive and a negative orientation. To express a vector in this reference system, three coordinates or scalar components are needed, being each of these components the projection of vector along each axis. These components are usually written as (if vector is $\vec{v}$) v_x v_y and v_z. To be able to measure each of these projections, a unit must be defined. Obviously, if the reference system is changed, the components of vector change too.

To simplify the way in which a vector can be expressed, vectors with a unit of length on each of axis are called vectors $\vec{i}$, $\vec{j}$ and $\vec{k}$ (corresponding to X, Y and Z axes). For example, vector $\vec{v}$ starting on the origin of coordinates (O) and finishing on point A (2,3,-1), could be expressed as $\vec{v} = \overrightarrow{OA} = 2\vec{i} + 3\vec{j} - \vec{k}$.

But if vector $\vec{\mathbf{v}}$ starts on point B (-1,2,0) and finishes on point C (2,3,-2), then $\vec{v} = \overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = (2\vec{i} + 3\vec{j} - 2\vec{k}) - (-\vec{i} + 2\vec{j}) = 3\vec{i} + 1\vec{j} - 2\vec{k}$

A very interesting question is to notice that three perpendicular axes have only two ways in which they can be arranged, as it's shown on figure:

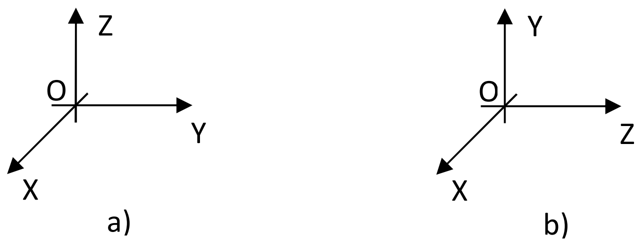


Figure 1: The two ways to arrange a reference system. a) Right-handed reference system b) Left-handed reference system

Any other arrangement of the axes or is a) type or is b) type, only turning the system on the space. It exist an agreement in order to use right-handed reference system as (a), and left-handed reference systems as b) shouldn't be used in Physics.

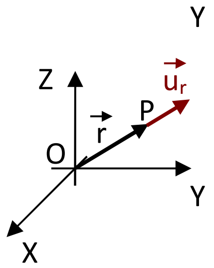
0.3 Multiplying a scalar by a vector. Magnitude of a vector. Unit vector.

Product of a scalar b by a vector $\vec{v}$ is a new vector k times the vector $\vec{v}$. That is, if $\vec{v} = 2\vec{i} + 3\vec{j} - \vec{k}$, then $b \cdot \vec{v} = 2b\vec{i} + 3b\vec{j} - b\vec{k}$.

Magnitude of a vector ($v = |\vec{v}|$) give us idea about the length of vector. It's defined as

$$v = |\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

If a vector $\vec{v}$ is divided into its magnitude, we get a vector with magnitude equal to one, called unit vector of $\vec{v}$ ($\vec{u}_v$): $\vec{u}_v = \frac{\vec{v}}{v}$ Obviously, a vector can be written by multiplying its magnitude by its unit vector.



Sometimes, depending on the nature of problem, can be useful to write a vector on not cartesian coordinates. For example, let's consider a point P(x,y,z); vector starting on origin of coordinates and finishing on P is called position vector $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, being their components the coordinates of P. Unit vector of $\vec{r}$, $\vec{u}_r = \frac{\vec{r}}{r}$ is always pointing from O to P, and $\vec{r} = r \cdot \vec{u}_r$. Both $\vec{r}$ and $\vec{u}_r$ are always perpendicular to a sphere of radius r and centred on O.

Example 0-1

Write a vector $\vec{c}$ with magnitude 3 and in the same direction but opposite orientation than vector $\vec{v} = 2\vec{i} + 3\vec{j} - \vec{k}$.

Solution:

Magnitude of $\vec{v}$ is $v = |\vec{v}| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}$ and so, unit vector of $\vec{v}$ is $\vec{u}_v = \frac{2\vec{i} + 3\vec{j} - \vec{k}}{\sqrt{14}}$

Unit vector of $\vec{c}$ will be opposite to $\vec{u}_v$, $\vec{u}_c = -\frac{2\vec{i} + 3\vec{j} - \vec{k}}{\sqrt{14}}$ and $\vec{c} = 3 \cdot \vec{u}_c = -\frac{6\vec{i} + 9\vec{j} - 3\vec{k}}{\sqrt{14}}$

0.4 Operating vectors.

Some operations can be defined working with vectors. More usual are:

a. Adding/subtracting vectors:

Addition/subtraction of vectors is a new vector whose components are the addition/subtraction of components of such vectors:

$$\vec{u} \pm \vec{v} = (u_x\vec{i} + u_y\vec{j} + u_z\vec{k}) \pm (v_x\vec{i} + v_y\vec{j} + v_z\vec{k}) = (u_x \pm v_x)\vec{i} + (u_y \pm v_y)\vec{j} + (u_z \pm v_z)\vec{k}$$

b. Multiplying vectors:

Two vectors can by multiplied in two different ways:

i. Inner product: Result of inner product of two vectors $\vec{u}\cdot\vec{v}$ is a scalar:

$$\vec{u}\cdot\vec{v} = u\cdot v\cdot cos(angle \quad between \quad \vec{u} \; and \; \vec{v})$$

From this definition is easy to see that $\vec{i}\cdot\vec{i} = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = 1$ and $\vec{i}\cdot\vec{j} = \vec{j}\cdot\vec{i} = \vec{i}\cdot\vec{k} = \vec{k}\cdot\vec{i} = \vec{j}\cdot\vec{k} = \vec{k}\cdot\vec{j} = 0$. So, inner product of two vectors can be computed as:

$$\vec{u}\cdot\vec{v} = (u_x\vec{i} + u_y\vec{j} + u_z\vec{k})\cdot(v_x\vec{i} + v_y\vec{j} + v_z\vec{k})j = u_xv_x + u_yv_y + u_zv_z$$

ii. Cross product (reading appendix A and B needed): Result of cross product of two vectors $\vec{u}\times\vec{v}$ is a vector, whose magnitude is: $\left|\vec{u}\times\vec{v}\right| = u\cdot v\cdot sin(angle \quad between \; \vec{u} \; and \; \vec{v})$. Cross product of two vectors is a new vector perpendicular to the plane made up by $\vec{u}$ and $\vec{v}$ and orientation given by the right hand (or screw) rule.

From this definition is easy to see that $\vec{i}\times\vec{i} = \vec{j}\times\vec{j} = \vec{k}\times\vec{k} = 0$, $\vec{i}\times\vec{j} = \vec{k}$, $\vec{j}\times\vec{i} = -\vec{k}$, $\vec{i}\times\vec{k} = -\vec{j}$, $\vec{k}\times\vec{i} = \vec{j}$, $\vec{j}\times\vec{k} = \vec{i}$ and $\vec{k}\times\vec{j} = -\vec{i}$. So, cross product of two vectors can be computed as:

$$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix}$$

c. Deriving a vector (reading appendix C needed):

If the components of a vector are a function of some variable or variables, derivative of such vector can be computed deriving each component of vector.

For example, a vector can depend on time, so having a vector variable on time. For example, $\vec{v} = 2t\vec{i} + 3\vec{j} - 2t^2\vec{k}$ is a vector changing on time (as an example $\vec{v}(t=0) = 3\vec{j}$ or $\vec{v}(t=1) = 2\vec{i} + 3\vec{j} - 2\vec{k}$) , whose derivate with respect to time is

$$\frac{d\vec{v}}{dt} = 2\vec{i} - 4t\vec{k}$$

But a vector can also depend on coordinates of the point where vector is applied; for example $\vec{v} = 2xy\vec{i} + 3\vec{j} - 2z^2\vec{k}$. In this case, the vector has not only a variable but three, and we can compute three different partial derivates; to derive a function with respect to a variable means deriving the function by supposing the other variables as constants:

for example, the derivative of before function $\vec{v}$ with respect to variable x (symbol ∂ is used instead d) $\frac{\partial \vec{v}}{\partial x}$ is:

$\frac{\partial \vec{v}}{\partial x} = 2y\vec{i}$ (variables y and z have been taken as constants)

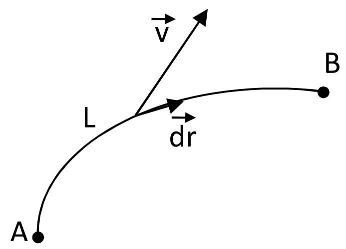
And partial derivatives with respect variables y and z:

$$\frac{\partial \vec{v}}{\partial y} = 2y\vec{i} \qquad \frac{\partial \vec{v}}{\partial z} = -4z\vec{k}$$

Vectors not depending on time are called stationary vectors, and vectors not depending on coordinates where they are applied, are called uniform vectors.

d. Integrating a vector along a path (reading appendix D needed):

To integrate a vector along a path, we need a line defining the path between two points A and B on the line, and a vector having a value on each point of the line. This vector can depend, or not, on the point where it's applied. It's defined the integral of vector $\vec{v}$ along line L between points A and B, as



$$\int_A^B \vec{v} \cdot d\vec{r}$$

This definition means that on each point of the line between A and B, we must compute inner product of $\vec{v}$ and a little (differential) displacement $d\vec{r}$ along the line, and add all these quantities starting on A and finishing on B.

Calculation of this integral can be very difficult on a general case, if $\vec{v}$ is changing or L isn't a straight line. But there are some cases where this integral can be easily computed. For example, let's consider $\vec{v} = 2xy\vec{i} + 3\vec{j} - 2z^2\vec{k}$ and we want to integrate it along a straight line parallel to Y axis from point A(1,1,1) to point B(1,3,1). As this line is parallel to Y axis, a differential displacement $d\vec{r}$ along the line will be $d\vec{r} = dy\vec{j}$, and so

$$\int_A^B \vec{v} \cdot d\vec{r} = \int_A^B (2xy\vec{i} + 3\vec{j} - 2z^2\vec{k}) \cdot dy\vec{j} = \int_1^3 3dy = 3y\Big|_1^3 = 3(3-1) = 6$$

In a general way, integral of a vector along a line is called circulation of a vector along a line between two points.

If vector $\vec{v}$ is a force, then the circulation is the work done by the force acting along the line between two points.

e. Integrating a vector across a surface:

A vector can be integrated across a surface in a similar way that it can be integrated along a line. But in this case, we must define the surface vector featuring the surface. To do it, we take a little (infinitesimal) element of surface dS, and we create vector $d\vec{S}$, having this vector the magnitude of chosen surface, and direction perpendicular to it (orientation is arbitrary if surface isn't closed); if the surface is closed, surface vector must always exit from inside to outside of the volume enclosed by the surface. So, to integrate $\vec{v}$ across S surface, inner product $\vec{v} \cdot d\vec{S}$ must be computed on each point of the surface, and added for the whole surface: $\int_S \vec{v} \cdot d\vec{S}$

This integral must be computed across surface S, and usually it can be very difficult to compute, but in some cases can be easily solved. For example, let's consider $\vec{v} = 2xy\vec{i} + 3\vec{j} - 2z^2\vec{k}$ and we want to integrate it across a square of side h placed on plane z=1, parallel to XY plane. If we take a little element of surface dS, its surface vector will be perpendicular to surface, and so parallel to Z axis, being $d\vec{S} = dS\vec{k}$. So

$$\int_S \vec{v} \cdot d\vec{S} = \int_S (2xy\vec{i} + 3\vec{j} - 2z^2\vec{k}) \cdot dS\vec{k} = \int_S -2z^2 dS = -2 \cdot (1)^2 \int_S dS = -2h^2$$

To complete calculations, we have taken in account that z=1 (the square is placed on plane z=1), and surface of the square is h^2.

In general way, integral of a vector across a surface is called flux of a vector across the surface.

0.5 Torque of a vector related to a point.

Let's consider a vector $\vec{v}$ applied on a point A, and another point O in the space. It's defined the torque of $\vec{\mathbf{v}}$ related to point O, as: $\vec{\tau}_O(\vec{v}) = \overrightarrow{OA} \times \vec{v}$

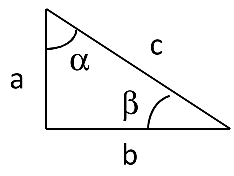
Torque is a new vector, resulting of a cross product, so being perpendicular to plane of $\vec{v}$ and $\overrightarrow{OA}$ and orientation according right hand rule (see appendix B). Torque of a vector is depending on magnitude and direction of $\vec{v}$ and distance $\overrightarrow{OA}$.

0.6 Trigonometry.

Most important part of trigonometry is applied to state relations on a right triangle between sides and angles. Pythagoras's theorem states that: $a^2 + b^2 = c^2$

And we can define trigonometric rates for α angle as:

$$sin\alpha = \frac{b}{c} \qquad cos\alpha = \frac{a}{c} \qquad tg\alpha = \frac{b}{a}$$



In the same way, rates for β angle will be:

$$sin\beta = \frac{a}{c} \qquad cos\beta = \frac{b}{c} \qquad tg\beta = \frac{a}{b}$$

For any angle, taking in account Pythagoras's theorem, it's verified that

$$sin^2\varphi + cos^2\varphi = 1 \qquad \text{and} \qquad tg\varphi = \frac{sin\varphi}{cos\varphi}$$

0.7 Base quantities. Dimension of a quantity. Systems of units.

Physics try to find laws relating between themselves the different physical quantities. In Mechanics, they exist three physical magnitudes more than independent physical laws; so, from three quantities properly chosen, any other quantity can be expressed as function of them. These quantities are known as base quantities. It exist a general agreement using mass (M) length (L) and time (T) as base quantities. In Electromagnetism, the number of base quantities is four, and we need add intensity of current (I) to M, L and T. Any other quantity (derived quantities) can be expressed as a function of base quantities, obtaining the dimensions of such quantity. Dimensions of a quantity are written in square brackets [ ]. For example, dimensions of speed will be:

$$[v] = \frac{L}{T} = LT^{-1}$$

The same thing can be done with quantities not related to base quantities in so evident way. For example, knowing that the consumed power by an electric device (P) can be computed multiplying the voltage of device (V) by the intensity of current flowing through the device (I), we can get dimensions of voltage. To do it, it's necessary to take in account that power is the work (W) done by unit of time, and that the work is a force (F) multiplied by a length (L):

$$[V] = \frac{[P]}{I} = \frac{\frac{[W]}{T}}{I} = \frac{\frac{[F]L}{T}}{I} = \frac{[F]L}{IT}$$

But force is a mass multiplied by an acceleration (a), being the last one a length divided into a squared time, resulting

$$[F] = M[a] = M\frac{L}{T^2} = MLT^{-2}$$

Dimensions of voltage will be, finally:

$$[V] = \frac{[P]}{I} = \frac{\frac{[W]}{T}}{I} = \frac{\frac{[F]L}{T}}{I} = \frac{[F]L}{IT} = \frac{MLT^{-2}L}{IT} = ML^2T^{-3}I^{-1}$$

Dimensions of a quantity are useful because they put of self-evident the relations of the quantity with the base quantities, and can help us to detect mistakes when writing physical laws, since the two members of a physical law must always have the same dimensions. This feature is known as homogeneity of the physical laws.

It is known that a physical magnitude can be measured in different units, even using multiples and submultiples of them. With the aim of standardize the use of the units, like general criterion has been adopted the use of the International System of units (S.I.); In it, the base units for the base quantities are the kilogram (kg), the metre (m), the second (s) and the ampere (A).

Returning to the previously obtained dimensions of the voltage, it will be measured in $kg \cdot m^2 \cdot s^{-3} \cdot A^{-1}$. In some cases the units have proper noun, as the voltage, giving the name of Volt (V) (from Alessandro Volta) to this quantity:

$$V = kg \cdot m^2 \cdot s^{-3} \cdot A^{-1}$$

But in other cases, this proper noun doesn't exist, and units must be written as a function of the units of the base quantities.

The units whose appoint honour to some scientist must be written with capital letters, writing the rest in lower cases.

0.8 Appendices.

APPENDIX A: Computing a determinant

Determinant of a matrix can be computed in two ways (with equal results):

i. Sarrus's rule: $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = aei + dhc + gbf - gec - afh - dbi$

ii. Operating minors: $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei - hf) - b(di - gf) + c(dh - ge)$

APPENDIX B: Right hand or screw rule: This useful to convert a rotating movement into a linear movement.

If you consider the needles of a clock rotating in the natural direction (clockwise), and we are faced up to the clock, this movement is converted (according this rule) into a linear movement going from us to the clock. But if needles would turn in opposite direction (counterclockwise), this rule converts it into a linear movement coming from clock to us.

In the same way, if a screw is turning on a direction, the advancing movement of the screw corresponds with this rule.

Or, if you also consider the fingers of your right hand turning around the thumb, the direction pointed by the thumb corresponds with this rule.

For these reasons, this rule that converts a rotating movement on a linear movement is called “right hand rule” or “screw rule”.

This rule can be applied to obtain the orientation of cross product of two vectors. If we want multiply $\vec{u}\times\vec{v}$, we must imagine $\vec{u}$ and $\vec{v}$ vectors with their origins together. So, we turn $\vec{u}$ as far as it be coincident with $\vec{v}$ (along shorter way), and this turning direction, taking in account screw rule, say us the direction of cross product vector $\vec{u}\times\vec{v}$.

Obviously, cross product $\vec{v}\times\vec{u}=-\vec{u}\times\vec{v}$, because turning direction to go from $\vec{v}$ to $\vec{u}$ is opposite to direction to go from $\vec{u}$ to $\vec{v}$.

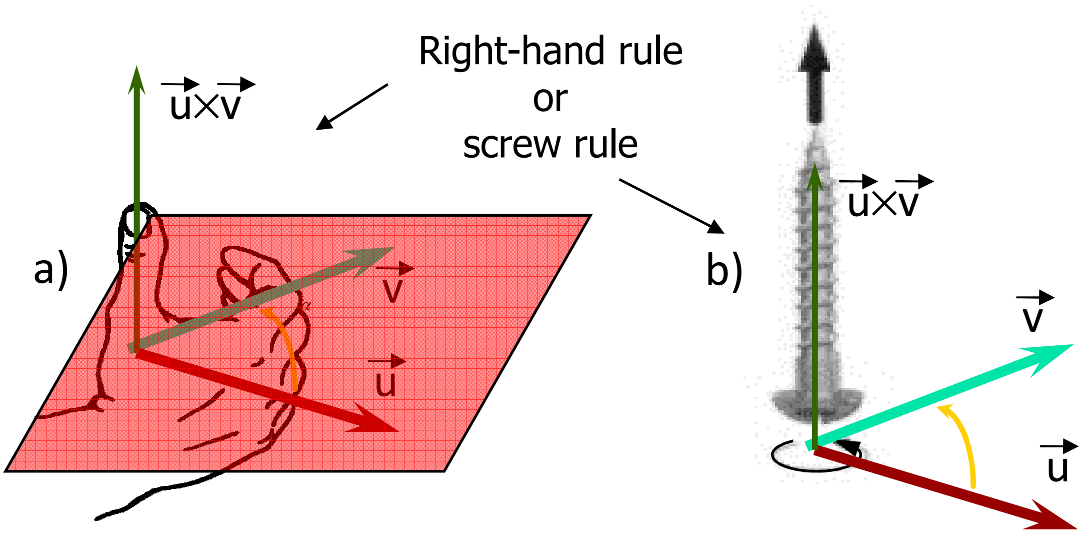


Figure 2: a) Right hand rule or b) screw rule. Useful to state the direction of cross product vector

APPENDIX C: Most used derivatives.

k, a and n are real numbers x is the not depending variable

u and v are functions of x, u(x) and v(x)

$y = k \Rightarrow y' = 0$ $y = x \Rightarrow y' = 1$ $y = u + v \Rightarrow y' = u' + v'$

$y = x^n \Rightarrow y' = nx^{n-1}$ $y = u^n \Rightarrow y' = nu^{n-1}u'$ $y = u \cdot v \Rightarrow y' = u \cdot v' + u' \cdot v$

$y = \frac{u}{v} \Rightarrow y' = \frac{vu' - uv'}{v^2}$ $y = lnu \Rightarrow y' = \frac{u'}{u}$ $y = a^u \Rightarrow y' = a^u u' lna$

$y = sinu \Rightarrow y' = u'cosu$ $y = cosu \Rightarrow y' = -u'sinu$

APPENDIX D: Basic Integrals. Most used integrals are:

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$ $\int u'u^n dx = \frac{u^{n+1}}{n+1} + C$ $\int \frac{u'}{u} dx = lnu + C$

$\int \frac{u'}{u^2} dx = -\frac{1}{u} + C$ $\int u'cosudx = sinu + C$ $\int u'sinudx = -cosu + C$

Really, computation of a definite integral means to obtain an addition of infinite number of terms. It can be understood if we think about a function y=u(x); product ydx represents the surface of rectangle with sides y and dx. So, $\int_a^b ydx$ means the addition of surfaces of all these rectangles from x=a to x=b. The value of a definite integral can be computed according Barrow's law:

$$\int_a^b ydx = F(b) - F(a) \qquad \text{being } F(x) = \int ydx$$

APPENDIX E: Lengths, surfaces and volumes of simple figures:

<u>Lengths</u>

Length of a circumference: $L = 2\pi R$

<u>Surfaces</u>

Circle: $S = \pi R^2$ Sphere: $S = 4\pi R^2$ Cylinder (side surface): $S = 2\pi RH$

Cube: $S = 6l^2$

<u>Volumes</u>

Sphere: $V = \frac{4}{3}\pi R^3$ Cylinder: $V = \pi R^2 H$ Cube: $V = l^3$

0.9 Problems

1. Compute following products between vectors:

a) $(3\vec{i}+5\vec{j}-2\vec{k})\cdot(-3\vec{j}+\vec{k})$; b) $(2\vec{i}+4\vec{j}-2\vec{k})\times(3\vec{i}-\vec{k})$

Sol: a) -17 b) $-4\vec{i}-4\vec{j}-12\vec{k}$

2. Obtain the unit vector corresponding to the solution of previous exercise.

Sol: $\dfrac{-\vec{i}-\vec{j}-3\vec{k}}{\sqrt{11}}$

3. Obtain a unit vector perpendicular to vectors $2\vec{i}+3\vec{j}-6\vec{k}$ and $\vec{i}+\vec{j}-\vec{k}$

Sol: $\pm\dfrac{3\vec{i}-4\vec{j}-\vec{k}}{\sqrt{26}}$

4. Compute a vector of magnitude 4 making 60º with the positive direction of *OX* axis and 45º with the positive direction of *OZ* axis.

Sol: $2\vec{i}\pm 2\vec{j}+2\sqrt{2}\vec{k}$

5. Write vector going from point (2,1) to point (5,7), and compute angle making this vector with *OX* axis *OY* axis.

Sol: a) $3\vec{i}+6\vec{j}$ b) $\alpha_{OX}=cos^{-1}(\dfrac{1}{\sqrt{5}})$ $\alpha_{OY}=cos^{-1}(\dfrac{2}{\sqrt{5}})$

6. Derive vector $\vec{r}=3t^4\vec{i}+5t^3\vec{j}-\dfrac{2}{t}\vec{k}$ with respect to t to get $\vec{v}=\dfrac{d\vec{r}}{dt}$ and $\vec{v}(2)$.

Sol: a) $\vec{v}=\dfrac{d\vec{r}}{dt}=12t^3\vec{i}+15t^2\vec{j}+\dfrac{2}{t^2}\vec{k}$ b) $\vec{v}(2)=96\vec{i}+60\vec{j}+\dfrac{1}{2}\vec{k}$

7. Given the vector $\vec{r}=3xyt^2\vec{i}-2xz\vec{j}+t^2x\vec{k}$ compute $\dfrac{\partial\vec{r}}{\partial x}$, $\dfrac{\partial\vec{r}}{\partial y}$, $\dfrac{\partial\vec{r}}{\partial z}$ and $\dfrac{\partial\vec{r}}{\partial t}$.

Sol: $\dfrac{\partial\vec{r}}{dx}=3yt^2\vec{i}-2z\vec{j}+t^2\vec{k}$ $\dfrac{\partial\vec{r}}{dy}=3xt^2\vec{i}$ $\dfrac{\partial\vec{r}}{dz}=-2x\vec{j}$ $\dfrac{\partial\vec{r}}{dt}=6xyt\vec{i}+2tx\vec{k}$

8. Compute integrals:

a) $\int \frac{3}{x}dx$; b) $\int_2^4 \frac{3}{1-2x}dx$; c) $\int_0^4 3t^2 dt$; d) $\int_2^3 \frac{2}{r^2}dr$;

e) $\int 4\cos 500t\,dt$; f) $\int_0^2 \left(30+4t-t^2\right)dt$ g) $\int_2^3 \frac{2}{(4-x)^2}dx$ h) $\int_0^L \frac{kdx}{(r+mx)^2}$

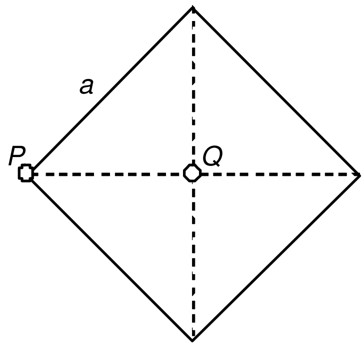
Sol: a) $3\ln x + C$ b) $-\frac{3}{2}\ln\frac{7}{3}$ c) 64 d) $\frac{1}{3}$ e) $\frac{1}{125}sen 500t + C$

f) $\frac{196}{3}$ g) 1 h) $-\frac{k}{m}(\frac{1}{r+mL}-\frac{1}{r})$

9. A spherical surface area has 1000 cm^2. Compute its volume in litres.

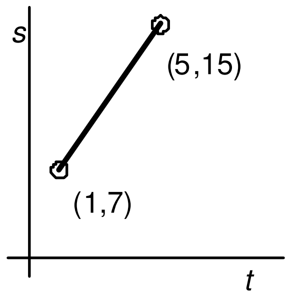
Sol: 2,97

10. Obtain the distance between points *P* and *Q* on drawing.



Sol: $\frac{a}{\sqrt{2}}$

11. Write equation of the straight line on the figure and find *s*(3).



Sol: a) s=2t+5 b) s(3)=11

12. Compute the line integral (circulation) of vector $\vec{v} = 2\vec{j}$ along straight line joining the origin with point A(3,3).

Sol: 6

13. Let's have a spherical surface with radius 2 units centred on origin. In each point of such surface is defined a vector $\vec{v}$ of magnitude 3 perpendicular to spherical surface and pointing outside the sphere, $\vec{v} = 3\vec{u}_r$. Compute the surface integral (flux) of vector $\vec{v}$ through spherical surface.

Sol: 48π