MATHEMATICAL ANALYSIS (AMA)

L3 - Review. RIEMANN'S INTEGRALS

A) IMMEDIATE INTEGRALS:

1. $\int_1^2 \frac{dx}{x^4}$ $\left(Tip: \frac{1}{x^4} = x^{-4}; \quad Sol: \frac{7}{24}\right)$

2 $\int_1^2 \frac{x^3+1}{x^5} dx$ $\left(Tip: \frac{x^3+1}{x^5} = x^{-2} + x^{-5}; \quad Sol: \frac{47}{64}\right)$

3. $\int_0^1 \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx$ $\left(Tip: \sqrt{x} = x^{1/2}; \quad Sol: -\frac{4}{3}\right)$

4. $\int_0^1 \frac{dx}{(4x+1)^2}$ $\left(Tip: (4x+1)' = 4; \quad Solución: \frac{1}{5}\right)$

5. $\int_0^{\pi^2/4} \frac{\sin(\sqrt{x})}{\sqrt{x}} dx$ $\left(Tip: (\sqrt{x})' = \frac{1}{2\sqrt{x}}; \quad Sol: 2\right)$

B) INTEGRATION BY PARTS:

1. $\int_1^2 x^2 \ln(x) dx$ $\left(Tip: u = \ln(x), dv = x^2 dx; \quad Sol: \frac{8\ln(2)}{3} - \frac{7}{9}\right)$

2. $\int_1^2 x \ln(\sqrt{x}) dx$ $\left(Tip: u = \ln(\sqrt{x}), dv = x dx; \quad Sol: \ln(2) - \frac{3}{8}\right)$

3. $\int_1^2 \frac{\ln(x)}{x^2} dx$ $\left(Tip: u = \ln(x), dv = \frac{1}{x^2} dx; \quad Sol: \frac{1}{2} - \frac{\ln(2)}{2}\right)$

4. $\int_0^1 x e^{-\frac{x}{2}} dx$ $\left(Tip: u = x, dv = e^{-\frac{x}{2}} dx; \quad Sol: 4 - \frac{6}{\sqrt{e}}\right)$

5. $\int_0^{\pi/2} x^2 \cos(x) dx$ $\left(Tip: u = x^2, dv = \cos(x) dx; \quad Sol: \frac{\pi^2}{4} - 2\right)$

C) INTEGRATION CHANGING THE VARIABLE:

1. $\int_{-1}^1 \left(\frac{x^2}{5} - 1\right)^3 x dx$ $\left(Tip: \frac{x^2}{5} - 1 = t; \quad Sol: 0\right)$

2. $\int_0^2 x^2 \sqrt{4 + x^3} dx$ $\left(Tip: \sqrt{4 + x^3} = t; \quad Sol: \frac{16\sqrt{3}}{3} - \frac{16}{9}\right)$

3. $\int_1^2 x\sqrt{x+1} dx$ $\left(Tip: \sqrt{x+1} = t; \quad Sol: \frac{8\sqrt{3}}{5} - \frac{4\sqrt{2}}{15}\right)$

4. $\int_1^2 \frac{(x+3)}{\sqrt{x}} dx$ $\left(Tip: \sqrt{x} = t; \quad Sol: \frac{22\sqrt{2}}{3} - \frac{20}{3}\right)$

5. $\int_{1/2}^1 \frac{e^{1/x}}{x^3} dx$ $\left(Tip: \frac{1}{x} = t; \quad Sol: e^2\right)$