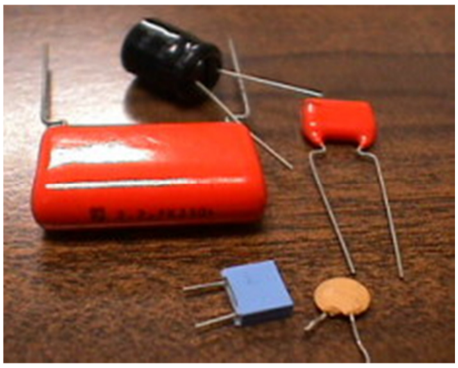
Unit 2

Electrostatic properties of conductors and dielectrics



Objectives

- To know the features of charged conductors in equilibrium: electric field inside and near the surface, electric potential and distribution of electric charges.
- To know the phenomena of electrostatic influence between conductors.
- Define the capacitance of a capacitor and to be able to compute the equivalent capacitance of associations of capacitors in series and in parallel.
- Understand the charging process of a capacitor and compute its stored energy.
- To know the effects of inserting a dielectric on a capacitor. Capacitance, charge, energy, difference of potential and electric field.

Previous unit was devoted to the study of the electric field on vacuum. This unit, being a part of electrostatics, is devoted to the study of electric field on conductor and isolator materials. From now, we'll call dielectric to isolators (from greek prefix "dia-", meaning through, and electric). On this unit, phenomena of electrostatic influence on conductor materials and polarization on dielectrics (as a consequence of electric fields), are described. As an application of such phenomena, electric shields and capacitors are studied. Electric shields are used to electrically isolate different areas on the space; for example, to avoid electric interferences on a laboratory performing accurate measurements, or to protect the signal on wires of an oscilloscope, TV, etc… Capacitors can be found on almost all boards of printed circuits on computers and many other electronic devices. They are useful storing electrostatic energy, as filters of electrical signals, etc…

2.1 Introduction

According to its ability to conduct the electricity, allowing the motion of its electrical charge, materials can be classified as:

◊ Conductors (metals, aqueous solutions of acids and bases,...)
◊ Semiconductors (Ge, Si,...)
◊ Dielectrics (rubber, glass,...)

The different electrical behaviour is mainly due to the atomic and molecular structure of the matter. For example: the copper (Cu) as a typical conductor, the germanium (Ge) as a semiconductor and the air as a dielectric.

➢ The electronic structure of copper (conductor) is:

^{29}Cu: $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}4s^1$

Due to the fact that each atom of copper has entirely occupied their three first electronic layers, and a lonely electron on last one, when the atoms are joined to form a crystal, atomic stability is increased if the atoms remain with the last complete layer (the third in this case); so, they let almost free the electron of the outer layer. These free electrons can easily move through the crystal of copper when an electric field is acting; so the copper is a good conductor of the electricity.

➢ The electronic configuration of germanium (semiconductor), instead, is:

^{32}Ge: $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}4s^2\ 4p^2$

Germanium has occupied the first three layers like the copper, and four electrons on fourth one. In this case, when forming the crystal, atomic stability increases when they are eight electrons on outer layer; it can be done if each atom shares the four electrons of its last layer with its four neighbors, forming covalent bonds. These electrons are more tied to their nucleus and they need a contribution of energy to get free electrons and conduct the electrical current.

➢ The air is mainly made up by Nitrogen and Oxygen. Even though both elements don't have completed their outer layers, they are very stable, being the air a good isolator. Anyway, if a strong electric field acts on the air, may be some electron was pull out (remaining the atom ionized), becoming the air a conductor. When it happens, the ionization produces a light emission, and a light can be seen. Besides, the motion of free electrons due to the electric field produces an electric current in the air. This phenomenon occurs not only on the air, but also inside any isolator, and it's known as **dielectric breaking**. In the dry air is needed an electric field around 3 KV/mm (**dielectric strength**), decreasing with increasing moisture.

2.2 Conductor in electrostatic equilibrium

In this unit, when speaking of conductors, we'll only refer to metallic conductors.

Model of metal: The positive ions (nucleus + tied electrons) are periodically distributed forming the crystalline network, and the electrons of last layer of the atoms (the $4s^1$ of the example of Cu) move freely among the ions, but without going out of the metal; they are forming a cloud or electronic gas, and these electrons are called free electrons. This mobility of free electrons is what characterizes the metals as good conductors.

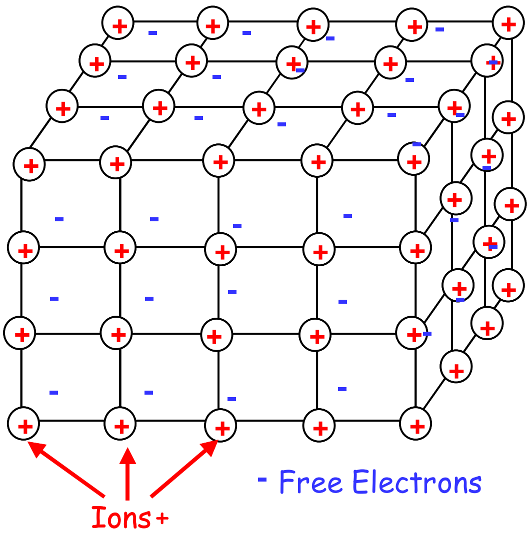


Figure 2-1. Model of metal

Conductor in electrostatic equilibrium (solid conductors): When on a conductor, both neutral as charged, there is no net movement of electric charges, we say that it is in electrostatic equilibrium. In this case, fulfils:

a) **Electric Field:** If there is not movement of free electrons, the net force acting on them must be zero. As electrostatic forces on electrons are much more intense than gravitational forces, the equilibrium on a conductor means that the electric field at any point of the conductor must be zero, $\vec{E}=0$.

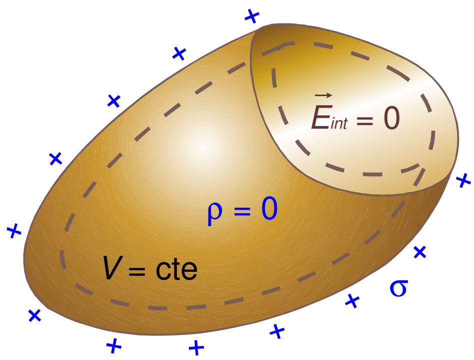


Figure 2-2. Distribution of charge on a conductor in electrostatic equilibrium. Electric flux through any Gauss's surface (dashed line) is zero

b) **Location of charge:** If the electric field is zero inside the conductor, by applying Gauss's law to any closed surface inside the conductor, the electric flux will be zero, and therefore the enclosed charge will also be zero. So, the volumetric density of charge inside conductor will be zero (ρ = 0). If the conductor is charged, the charge is distributed across its surface. In the same way, if an external electric field is applied to conductor, electric charges are distributed in such way that the electric field created by this distribution of charges, cancel the external one.

c) **Electrostatic potential:** As electric field is zero inside a conductor in equilibrium, the difference of potential between two any points of the conductor will also be zero, and therefore the potential will be constant in all conductor.

d) **Electrical field in an external point next to conductor. <u>Coulomb's theorem:</u>**

Electric field on points outside the conductor but very close to its surface is perpendicular to surface of conductor; if it wasn't, the tangent component of electric field to the surface, would move the charges on the surface, not being the conductor in equilibrium (but our hypothesis is that the conductor is in equilibrium).

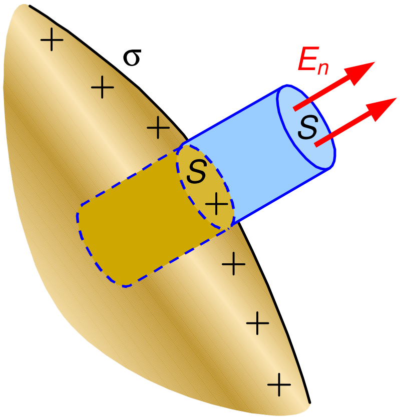


Figure 2-3. Gaussian surface for the computation of the electric field in the surface of a conductor

To compute the electric field at any point outside and close to the surface of a charged conductor in electrostatic equilibrium, we can apply Gauss's law, considering a surface of a small cylinder shaped with both bases parallel to the surface of conductor (see Figure 2-3). One of the bases is outside the conductor and the other base is inside. There isn't electric flux through internal base, because the electric field is zero inside the conductor. Outside the conductor, the electric field is perpendicular to its surface, being zero the electric flux through the lateral surface of the cylinder. Therefore, the net flux through the gaussian surface will be the flux through the base of the cylinder external to conductor:

$$\Phi = \int_S \vec{E}_n \cdot d\vec{S} = \int_S E_n dS = E_n \int_S dS = E_n S$$

E_n is the module of the electrical field near the surface S of the base of cylinder.

Applying now Gauss's law we get: $\Phi = \frac{q_{int}}{\varepsilon_0} = \frac{\sigma S}{\varepsilon_0}$

Equalizing both equations we finally get the electric field on points near the surface of a charged conductor

$$E_n = \frac{\sigma}{\varepsilon_0} \qquad \text{Equation 2-1}$$

This electric field is pointing outside the conductor if the surface density of charge is positive. This result is only correct on points very close to the surface. For a conductor without a flat surface, the field lines are curve lines, and modulus of electric field falls when we move away from the surface.

But if the conductor is flat and infinite, with uniform surface density of charge σ, field lines are straight lines perpendicular to the surface, and modulus of electric field is always the same for any distance to conductor:

$$\forall d \rightarrow E_n = \frac{\sigma}{\varepsilon_0}$$

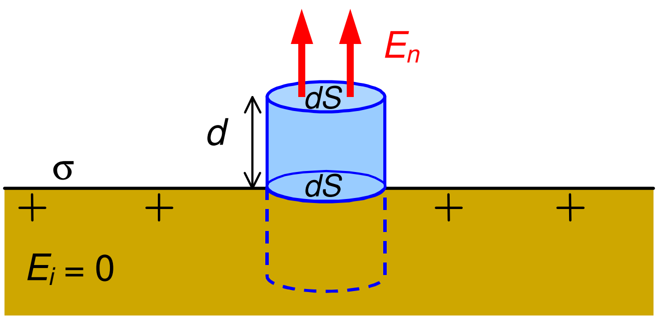


Figure 2-4. Gaussian surface for computation of electric field near the surface of a flat and infinite conductor

Hollow conductor in electrostatic equilibrium

The behaviour of a hollow conductor without any charge on the cavity, is exactly the same as if it was a solid conductor, as can be seen on Figure 2-5. As it's demonstrated on next paragraph, there aren't charges on inner surface of conductor, being all the charge placed on the outer surface. Electric field on cavity is zero, and electric potential on cavity equals the potential on conductor (constant).

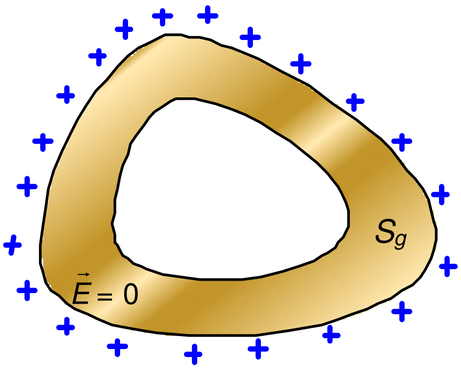
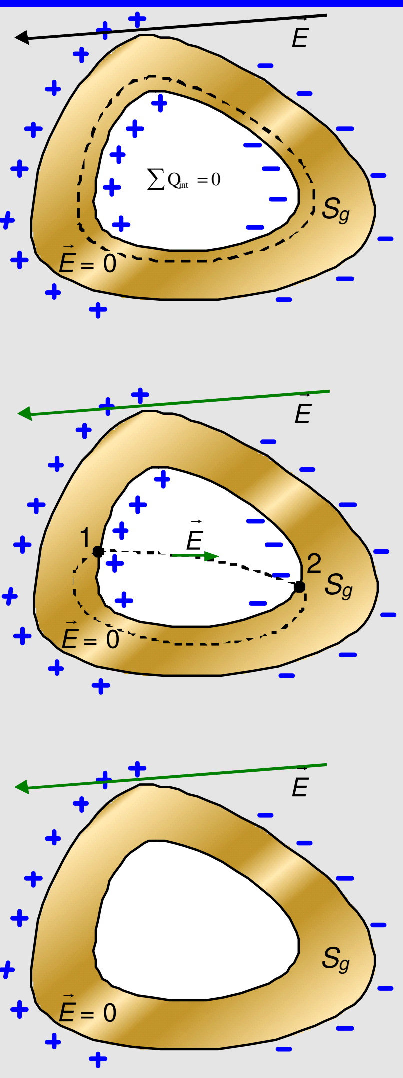


Figure 2-5. Hollow and charged conductor

Charge in the inner surface of a hollow conductor

By applying Gauss's law to the dashed line allows us to deduce that the net charge on inner surface of the conductor is zero. But this solution is not incompatible with the idea of a distribution of positive and negative charges on this surface, being zero its total sum. We´ll prove that this solution is not possible, and none charge can be placed on inner surface.



Let's consider equals positive and negative charge in the inner surface, creating an electrical field in the cavity, and a field line going from the distribution of positive charges until the distribution of negative charges, between points 1 and 2. The difference of potential between 1 and 2 will give us a not zero value:

$$V_1 - V_2 = \int \vec{E} \cdot d\vec{r} \neq 0$$

However, if we follow for the calculation a way inside the conductor, where the electric field is zero, result is different:

$$V_1 - V_2 = \int \vec{E} \cdot d\vec{r} = 0$$

The difference of potential between two points can not be different according the path to compute it, and the only possible solution is the no existence of distribution of charges in the inner surface of a hollow conductor charged or subjected to the action of an applied external electric field.

Example 2-1

The Earth is a conductor negatively charged. If the average electric field at its surface equals 100 N/C, compute the total electric charge of Earth and its electric potential.

Solution:

The Earth can be taken as an isolated spherical conductor in the space, with approximated radius of R=6.371.000 m.

If we suppose a Q charge on the Earth, this charge will be distributed across its surface, with a density of charge σ related to the electric field near the surface: $E_s = \dfrac{\sigma}{\varepsilon_0} \Rightarrow \sigma = E_s \varepsilon_0 = 100 \cdot 8{,}85 \cdot 10^{-12} = 8{,}85 \cdot 10^{-10}\ C/m^2$

The surface of Earth is: $S = 4\pi R^2 = 4\pi(6{,}371 \cdot 10^6)^2 = 5{,}1 \cdot 10^{14}\ m^2$

And the total charge of Earth: $Q = \sigma S = 8{,}85 \cdot 10^{-10} \cdot 5{,}1 \cdot 10^{14} = 4{,}51 \cdot 10^5\ C$

To compute the electric potential, every points of Earth will have equal potential, and so it will be enough computing the potential at only one point. Choosing the center of the sphere, this point is placed at the same distance R from any point of its surface. The potential created on the center of the sphere by a little charge q_i corresponding to a surface S_i (taking q_i as a point charge), is:

$$V_i = K\frac{q_i}{R}$$

If we consider all the charges q_i, placed on all surfaces S_i, the total potential will be the addition of all the potentials created by these charges:

$$V = \sum_i V_i = \sum_i K\frac{q_i}{R} = \frac{K}{R}\sum_i q_i = \frac{K}{R}Q = \frac{9 \cdot 10^9}{6{,}371 \cdot 10^6} 5{,}51 \cdot 10^5 = 6{,}4 \cdot 10^8\ V$$

This potential could be looked as very high, but we must remember that the zero potential has been chosen at an arbitrary point (the infinite). As we'll see above, for practical applications, zero potential is usually taken on Earth.

Example 2-2

A spherical conductor, with radius R_1 and charge Q is joined through a conductor wire (the charge on wire can be neglected), to another sphere of radius R_2 ($R_2<R_1$), initially discharged. Supposing that the spheres are far enough so the phenomena of influence are negligible, compute:
a) Charges Q_1 and Q_2 on each sphere; b) Potential of both spheres; c) sur-

face density of charge on each sphere; d) What happen if $R_2 >> R_1$?
Solution

a) When joining the two spherical conductors through a conductor wire, the total charge Q is preserved and distributed between both spheres in such way that two spheres are equipotential:

$$\left.\begin{array}{l} Q = Q_1 + Q_2 \\ V = \dfrac{Q_1}{4\pi\varepsilon_0 R_1} = \dfrac{Q_2}{4\pi\varepsilon_0 R_2} \end{array}\right] \Rightarrow Q_1 = \frac{QR_1}{R_1 + R_2}, \quad Q_2 = \frac{QR_2}{R_1 + R_2}$$

b) And the electrostatic potential:

$$V = V_1 = V_2 = \frac{Q_1}{4\pi\varepsilon_0 R_1} = \frac{Q_2}{4\pi\varepsilon_0 R_2} = \frac{Q}{4\pi\varepsilon_0 (R_1 + R_2)}$$

c) The surface density of charge is:

$$\sigma_1 = \frac{Q_1}{S_1} = \frac{QR_1}{4\pi R_1^2 (R_1 + R_2)} = \frac{Q}{4\pi R_1 (R_1 + R_2)} \quad , \quad \sigma_2 = \frac{Q_2}{S_2} = \frac{Q}{4\pi R_2 (R_1 + R_2)}$$

d) If $R_2 >> R_1$ the conductor of radius R_2 will have the total charge, remaining discharged the sphere of radius R_1; it can be proved from results of a):

$$\lim_{R_2 \to \infty} Q_1 = \lim_{R_2 \to \infty} \left(\frac{QR_1}{R_1 + R_2} \right) = 0 \qquad \lim_{R_2 \to \infty} Q_2 = \lim_{R_2 \to \infty} \left(\frac{QR_2}{R_1 + R_2} \right) = Q$$

On the other hand the potential of both conductors will be canceled :

$$lim_{R_2 \to \infty} V = lim_{R_2 \to \infty} \left(\frac{Q}{4\pi\varepsilon_0 (R_1 + R_2)} \right) = 0$$

An interesting consequence of this example (2-2) is the case of a conductor similar to that shown on Figure 2-6, with a sharp tip and the other rounded. We can imagine this conductor as two spherical conductors (radius R_1 and R_2) connected through a wire, as on example 2-2; from point c):

$$\sigma_1 R_1 = \sigma_2 R_2$$

This result means that the tip with higher radius will have lower surface density of charge; and higher den-

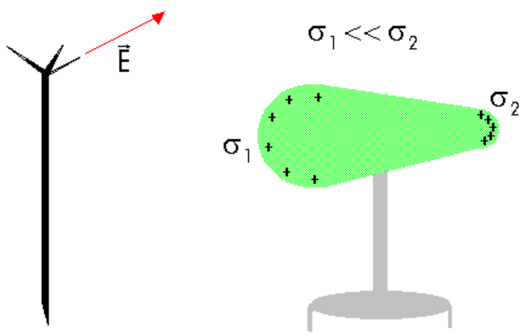


Figure 2-6. The surface density of charge depends on the radius of curvature of conductor. In the lightning rod we get points with high electric field enabling the electrical discharge, and avoiding this discharge can occur through non desired areas.

sity on tip with lower radius. As electric field on points near the surface is proportional to surface density of charge ($E=\sigma/\varepsilon_0$), electric field will be higher near the tip with lower radius (R_2); in this way, if dielectric breaking occurs, it always happen on sharp tip. Lightning of a storm goes to a lightning rod because it has a sharp tip, and a high electric field around.

2.3 Ground connection

The Earth is usually taken as origin of potentials (electric potential zero). Besides, as the radius of Earth is so big compared with any other conductor we can usually handle, even though an amount of electric charge can be taken or given by the Earth, its potential will remain almost constant. This situation can be compared with the sea level: even if you pour the water of a glass in the sea, the increasing on sea level isn't noticeable; in the same way, if you add or remove some quantity of charge from Earth, its potential doesn't change.

So, a conductor linked to ground (grounded) will have two important features:

- Its electric potential will be always zero.
- Through the ground connection some charge can pass from or to ground, to satisfy the features of a conductor in electrostatic equilibrium.

Linking an electrical facility to ground is a safety measure to avoid electrical discharges, since the potential of the frames of all devices linked to ground is zero.

2.4 Phenomena of electrostatic influence. Electrostatic shields.

Let's suppose an electric charge *Q* placed near a conductor; electric field produced by such charge causes a movement of charges inside the conductor, by effect of Coulomb's forces, producing a redistribution of charges (the charges of different sign than external charge will move towards surface near the external charge, and charges with the same sign will move towards far away surfaces); so, areas with negative and positive surface density of charge will appear, but the net charge of conductor will be the same than before applying the external charge. Such movement of charges finishes when new equilibrium conditions are reached: electric field inside the conductor is zero (the conditions of electrostatic equilibrium are reached on a very small time, of the order of 10^{-17} s). In this way, a phenomenon of *electrostatic influence* on the conductor has occurred, as a consequence of the external electric field.

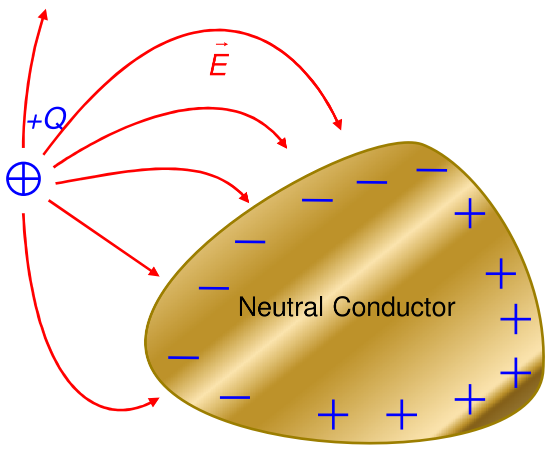


Figure 2-7. The external electric field produces on conductor a separation of charges, creating an electric field opposite to the external. The phenomenon disappears when external charge is removed.

This phenomenon of electrostatic influence bring us a way to charge a conductor: let's place two neutral conductors in contact, inside the electric field created by a near electric charge, as it shown in the Figure 2-. When acting the electric field created by the charge, charges on conductors are distributed until the electric field inside conductors was zero (a). Keeping the near electric charge, if we separate the two conductors (b), both conductors remain charged, one positively, and the other negatively (c). If both conductors were initially discharged, both charges (positive and negative) are equal in magnitude.

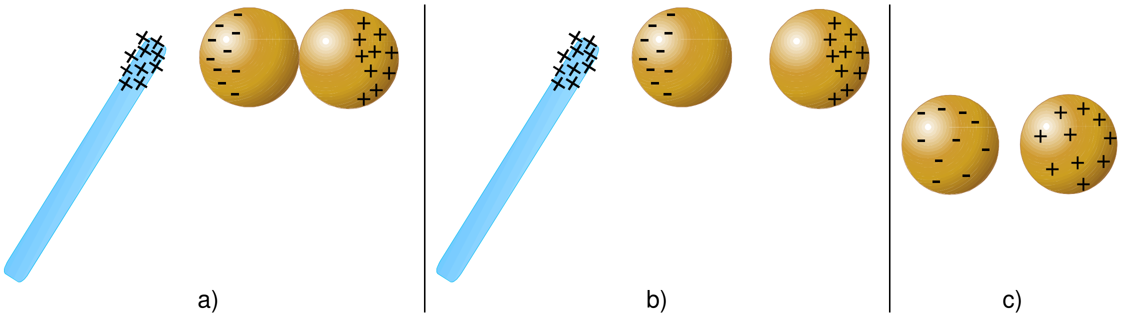


Figure 2-8. Charging two conductors by electrostatic influence. A charged object causes the conductors in contact take opposite charges. If we separate the conductors in presence of the object, they will remain charged,

On a general way, two conductors show electrostatic influence when the electric field created by one of them electrically influences on the other. Given two conductors as shown on Figure 2-9, if we draw a tube of current from an element of surface of the conductor 1, dS_1, to an element of surface dS_2 (surface made up by the field lines going out from the border of dS_1 and arriving to the conductor 2), surfaces dS_1 and dS_2 are **corresponding elements**.

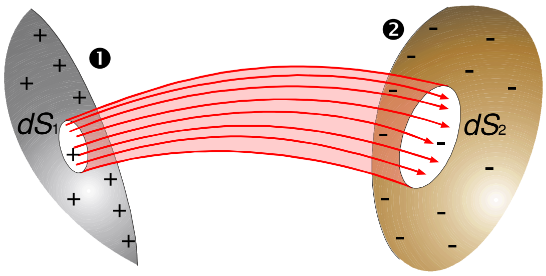


Figure 2-9. Corresponding elements in two conductors that exert influence electrostática

Let's apply Gauss's law to an enclosed surface made up by the tube of current, so that the bases of the tube are placed inside of two conductors, where the electric field is zero. Electric field along the lateral surface of the tube is tangent to this surface; so, the flux of the electric field through this enclosed surface will be zero:

$$\Phi = \int \vec{E}\cdot d\vec{S} = \int_{\text{Lateral surface}} \vec{E}\cdot d\vec{S} + \int_{\text{Bases}} \vec{E}\cdot d\vec{S} = 0$$

According Gauss's law, such flux will be equal to the enclosed charge divided into ε_0:

$$\Phi = \int \vec{E}\cdot d\vec{S} = 0 = \frac{Q_{inside}}{\varepsilon_0} \Rightarrow Q_{inside} = 0$$

As the charge inside the enclosed surface can only be placed on surfaces dS_1 and dS_2, calling dq_1 and dq_2 to the electric charges on such surfaces, must be satisfied:

$$Q_{inside} = dq_1 + dq_2 = 0 \Rightarrow dq_1 = -dq_2$$

This result is the theorem of corresponding elements: "**Corresponding elements have equal but opposite charges**".

Not all the *dS* of a conductor have always their corresponding elements on the other conductor; in this case we speak of partial influence. There will be ***total electrostatic influence*** when all the surface of a conductor has his corresponding on the other, or, in a graphic way, when all the field lines starting on surface of a conductor finish on the other conductor.

The two more common examples of total influence are the cases of a conductor entirely surrounding the other, and two flat parallel plates face to face with a little distance between them (Figure 2-10).

Two conductors with total electrostatic influence between them, necessarily have the same charge but of different sign. This result is of direct application on capacitors.

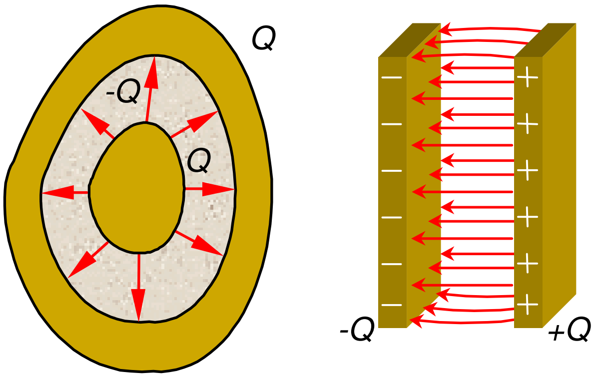


Figure 2-10. Examples of total influence. On left, a conductor entirely surrounding to another. On right, two parallel plates conductors face to face.

Electric shields: Faraday's cage.

The *electric shields* are devices avoiding the phenomena of electrostatic influence. The simplest example consists in that we call Faraday's cage: a conductor with an internal cavity (hollow conductor) linked to ground. From the electrostatic point of view, such system insulates the outside and the inner cavity of conductor (Figure 2-):

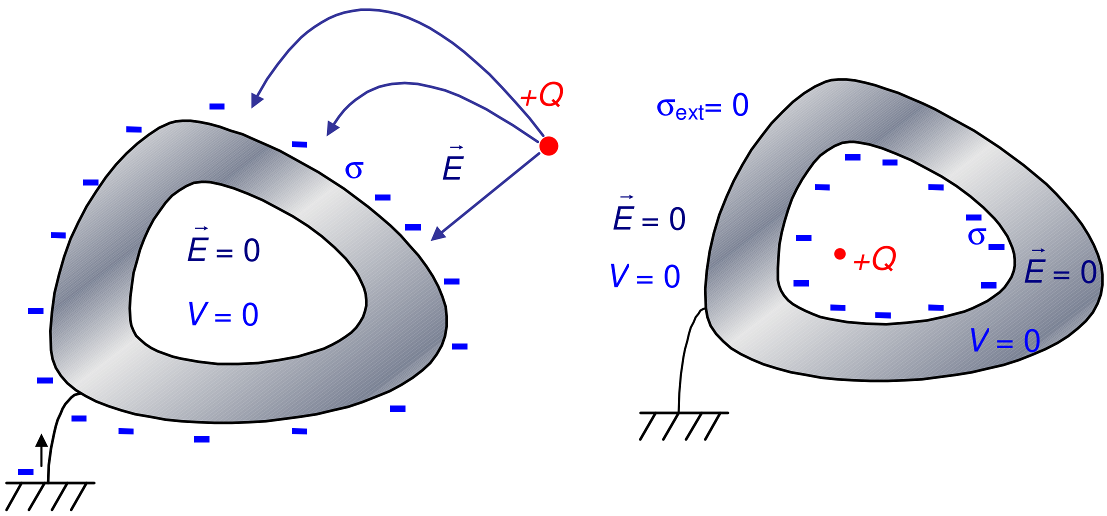


Figure 2-11. Electric shields. a) On left, shielding of inner area: external charges don't influence the inner area of the hollow conductor linked to ground. b) On right, shielding of outer area: internal charges on hollow conductor don't influence outer area of the hollow conductor linked to ground.

- a) Electric field and potential in the cavity due to the external charges are zero. The explanation of this phenomenon was already given on section 2-2 when speaking about the location of the charges in a hollow conductor. The behaviour of a hollow conductor without charges on its cavity is the same than a solid conductor and, therefore, values of electric field inside (zero) and potential (zero, due to linkage to ground) also are they.

- b) Electric field and potential outside the conductor due to the internal charges (on cavity) are zero. On internal surface of cavity, due to electrostatic influence, it appears the same charge but of opposite sign, coming from ground. This distribution of charge will cancel the electric field and the potential inside the conductor. Therefore, electric field and potential outside the conductor will be zero.

As an example of electric shield, it can be quoted the coaxial wire. A coaxial wire is made up by a conductor, surrounded by a dielectric material, being this set surrounded by another conductor (Figure 2-). In this way, the signal traveling across inner conductor remains protected by the shield effect produced by the external conductor.

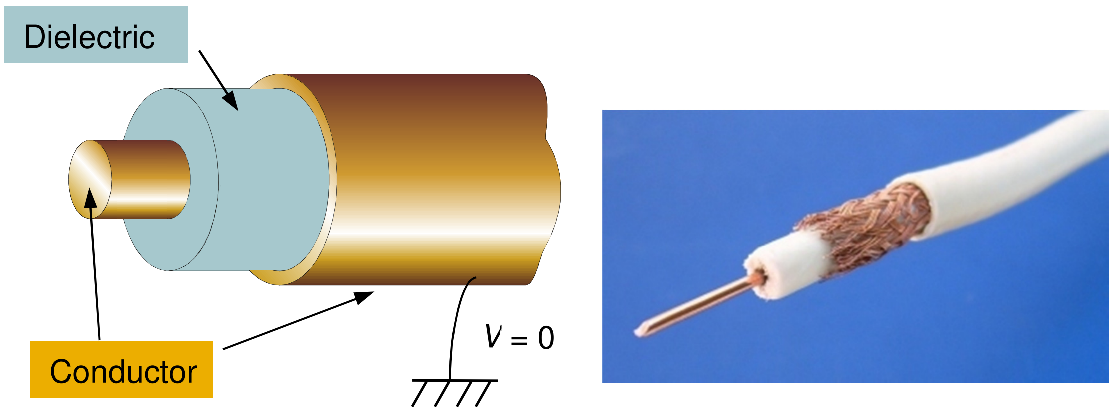


Figure 2-12. Diagram and photograph of a coaxial wire

2.5 The capacitor. Capacitance of a capacitor and a conductor

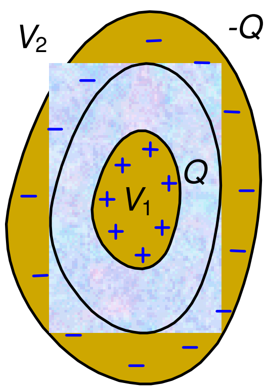


Figure 2-33. Capacitor

A capacitor is a system of two conductors (plates), isolated one of another, exerting total influence between them. As a consequence of total influence, both conductors have equal and opposite charges. Capacitors are used to store electric charge and energy and they have a lot of applications on electrical circuits.

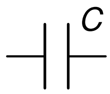
When a difference of potential $V=V_1-V_2$ is applied to the plates of a capacitor, a movement of charges is produced from a conductor to another, until the difference of potential between both plates equals the applied difference of potential. Therefore, the quantity of charge Q on the capacitor depends on the applied difference of potential V, on geometry and size of capacitor and on the material between plates.

The capacitance of a capacitor is defined as the rate between the absolute value of the charge Q on each plate and the difference of potential between them. The value of the capacitance is neither depending on the charge Q and on the difference of potential; only depends on geometry and size of capacitor and on insulating material between plates.

$$C = \frac{Q}{V_1 - V_2} = \frac{Q}{V_{12}} \qquad \text{Equation 2-2}$$

Capacitance of a capacitor is measured in S.I. in **Farads** (F). In the practice a farad is a too big unit, being used their submultiples as microfarad (µF), nanofarad (nF) and picofarad (pF).

Its graphic representation on the circuits is:



Dimensions of the capacitance are: $[C] = M^{-1}L^{-2}T^{4}I^{2}$

The definition of capacitance can also be extended to a only conductor. **The self-capacitance of a conductor** is defined as the needed charge to increase 1 V the potential of conductor:

$$C = \frac{\Delta Q}{\Delta V}$$

It give us idea about the quantity of charge can the conductor store at a given potential. The self-capacitance of a conductor depends only on its geometry and size, being therefore not depending on the charge or potential of conductor.

Parallel plate flat capacitor

A parallel plate flat capacitor has their plates parallel of surface S at a distance d, very low compared with S. In this situation, we can suppose total influence between plates. We'll consider the vacuum between plates (Figure 2-14).

When it is charged, both conductors are in electrostatic equilibrium, with the charge uniformly distributed across its surface. The surface density of charge will be:

$$\sigma = \frac{Q}{S}$$

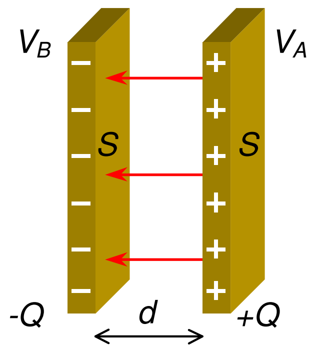


Figure 2-14. Parallel plate capacitor

Given the geometry of the capacitor, on the space between plates we are near the surface of an infinite flat conductor, being the electric field perpendicular to the plates, uniform, and with a magnitude:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{S\varepsilon_0}$$

So, the difference of potential between plates can be computed as:

$$V_A - V_B = \int_0^d \vec{E} \cdot d\vec{\ell} = Ed = \frac{Qd}{\varepsilon_0 S}$$

And the capacitance of parallel plates flat capacitor:

$$C = \frac{Q}{V_A - V_B} = \frac{Q}{\frac{Qd}{\varepsilon_0 S}} = \frac{\varepsilon_0 S}{d}$$

$$C = \frac{\varepsilon_0 S}{d} \qquad \text{Equation 2-3}$$

The capacitance depends exclusively on geometrical parameters. Physical limits for capacitors come from the fact that if the distance between plates is very short and very high the stored charge, the magnitude of electric field in the space between plates can reach a high magnitude that ionizes the air, producing a spark (and a charge) passing from a plate to another, and so discharging the capacitor.

The cylindrical capacitor

A cylindrical capacitor is made up by two cylindrical conductors, coaxial of radii R_1 and R_2 and length L much greater than the space between conductors ($L >> R_2$-R_1). This last condition will allow us to neglect the effects of edge and suppose the capacitor like an indefinite system. This type of capacitor will help us to understand the features of the coaxial wires used in the transmission of signals of TV, in the oscilloscopes, etc...

As there is total influence between conductors, the will have the same quantity of charge and this will distribute uniformly across their surfaces with surface density of charge σ_1 positive and σ_2 negative:

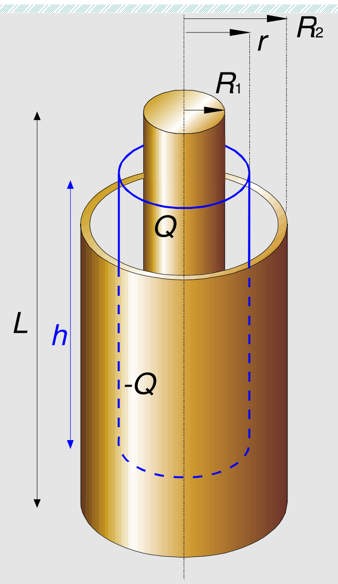
$$\sigma_1 = \frac{Q}{S_1} = \frac{Q}{2\pi R_1 L} \qquad \text{And} \qquad \sigma_2 = \frac{-Q}{S_2} = \frac{-Q}{2\pi R_2 L}$$

The electric field in the space between two conductors can be computed by applying Gauss's law to an enclosed cylindrical surface (blue cylinder on picture) of radius r (R_1<r<R_2) and height h<L. By symmetry of the problem, the electric field will have radial direction on each point, being perpendicular to cylindrical surfaces:

$$\Phi = \int_{S_{gaussian}} \vec{E}\cdot d\vec{S} = \int_{S_{lateral}} EdS = ES = E2\pi rh$$

$$\Phi = \frac{Q_{inside}}{\varepsilon_0} = \frac{\sigma_1 S_1}{\varepsilon_0} = \frac{\frac{Q}{2\pi R_1 L}2\pi R_1 h}{\varepsilon_0} = \frac{Qh}{L\varepsilon_0}$$

$$E = \frac{Qh}{L\varepsilon_0 2\pi rh} = \frac{Q}{2\pi rL\varepsilon_0}$$



The difference of potential between conductors can be computed following the radial path between both surfaces of conductors, along a field line:

$$V_1 - V_2 = \int_{R_1}^{R_2} \vec{E}d\vec{r} = \int_{R_1}^{R_2} Edr = \int_{R_1}^{R_2} \frac{Q}{2\pi L\varepsilon_0 r}dr = \frac{Q}{2\pi L\varepsilon_0} \ln r\Big|_{R_1}^{R_2} = \frac{Q}{2\pi L\varepsilon_0}\ln\frac{R_2}{R_1}$$

And capacitance

$$C = \frac{Q}{V_1 - V_2} = \frac{Q}{\frac{Q}{2\pi L\varepsilon_0}\ln\frac{R_2}{R_1}} = \frac{2\pi L\varepsilon_0}{\ln\frac{R_2}{R_1}}$$

As it happened on parallel plate capacitor: the capacitance only depends on the geometry, on size, and on ε_0.

2.6 Stored energy on a capacitor.

To charge a capacitor (initially discharged), a difference of potential V can be applied to the plates of capacitor, producing a flow of electrons, from the plate with positive potential to the plate with negative potential (it can be done using a specific device). The charging process progress from charge 0 to charge $Q = CV$. At an intermediate state, being q the charge of the capacitor, the difference of potential v between plates will be

$$v = \frac{q}{C}$$

At this moment, to add an additional charge dq to capacitor, we'll need an energy:

$$dU = vdq = \frac{q}{C}dq$$

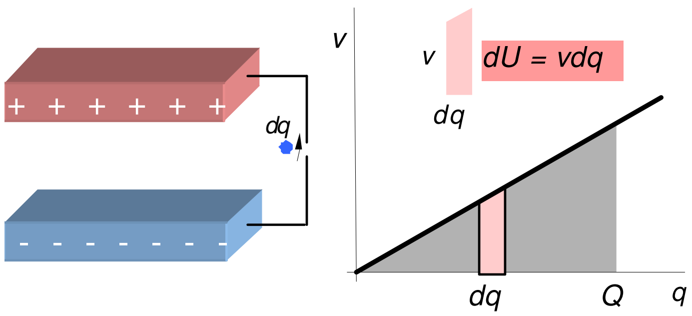


Figure 2-45. When the charge on capacitor increases *dq*, its energy increases *dU*. This energy increasing equalizes the area of the pink trapezium on picture. The energy of the charged capacitor is the area of the triangle

So, needed energy to charge the capacitor from discharged to charge Q and difference of potential V is:

$$U = \int_0^Q v dq = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C} = \frac{QV}{2} = \frac{V^2 C}{2}$$

$$U = \frac{CV^2}{2} = \frac{QV}{2} = \frac{Q^2}{2C} \qquad \text{Equation 2-4}$$

This result can be geometrically seen on Figure 2-15, since equals the area of grey triangle. This energy remains stored in the capacitor on the electric field between plates.

2.7 Association of capacitors

It's defined the **equivalent capacitance** of an association of capacitors as the capacitance of a lonely capacitor that having the same difference of potential between plates that the association, stores the same quantity of charge. We'll study the two more frequent types of association of capacitors: in series and in parallel. But we must have in mind that a set of capacitors can't be associated neither in series nor in parallel, and so more complex methods must be accomplished in order to compute the equivalent capacitance.

Association in series

The association of capacitors in series is made connecting a plate of a capacitor to a plate of another capacitor and so on, as it's shown in Figure 2-6. We suppose the capacitors are initially discharged, and to charge them, we apply a difference of potential V between the free plates of first and last capacitor (*1* and *n+1*). One plate will take a charge *+Q* and the other plate, *-Q*. As the charge has been displaced between both plates has been Q, this is the charge of the set of capacitors. But due to the total influence between the plates on all capacitors, they will appear charges of opposite sign on plates of all capacitors. As the electrical neutrality must be preserved on the internal plates, all the capacitors show the same charge *Q*, the same that the set of capacitors.

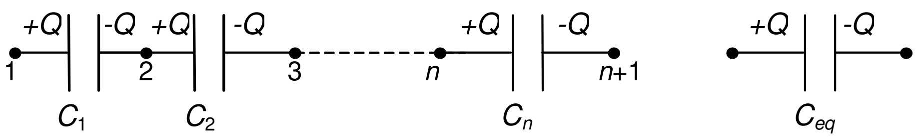


Figure 2-16. Association *of* n capacitors in series and its equivalent capacitor

The difference of potential V between terminals of the association can be written as the addition of the differences of potential in each one of the capacitors in series: $V_1 - V_{n+1} = \sum_{i=1}^{n} (V_i - V_{i+1})$

For each capacitor: $V_i - V_{i+1} = \dfrac{Q}{C_i} \qquad i = 1,2,...,n$

And therefore, $V = V_1 - V_{n+1} = \sum_{i=1}^{n} \dfrac{Q}{C_i}$.

On the equivalent capacitor to the association, when applying the same difference of potential V, it's verified $V = \frac{Q}{C_{eq}}$, and comparing both expressions:

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$$ Equation 2-5

The equivalent capacitance of a set of capacitors associated in series is the inverse of the addition of the inverses of the capacitances of associated capacitors. So, when n capacitors are associated in series, the equivalent capacitance is lower than the capacitance of each one of the associated capacitors.

Association in parallel

A set of capacitors are associated in parallel when all the capacitors are connected to the same difference of potential, as it's shown in Figure 2-17. When applying a difference of potential V_A-V_B, the charge is distributed on plates of all capcitors, being the charge of the set of capacitors, the addition of charge on each one: $Q_t = \sum_{i=1}^{n} Q_i$

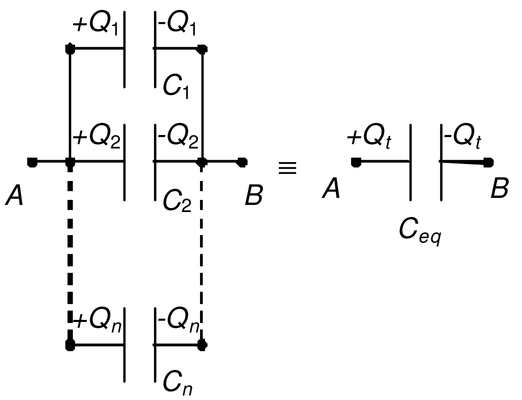


Figure 2-57. Association of capacitors in parallel and equivalent capacitor

For each capacitor $Q_i = (V_A - V_B)C_i$

And so: $Q_t = (V_A - V_B)\sum_{i=1}^{n} C_i$

In the equivalent capacitor, when applying the same difference of potential V_A-V_B, it will take the same charge:

$$Q_t = (V_A - V_B)\, C_{eq}$$

Comparing both equations:

$$C_{eq} = \sum_{i=1}^{n} C_i$$ Equation 2-6

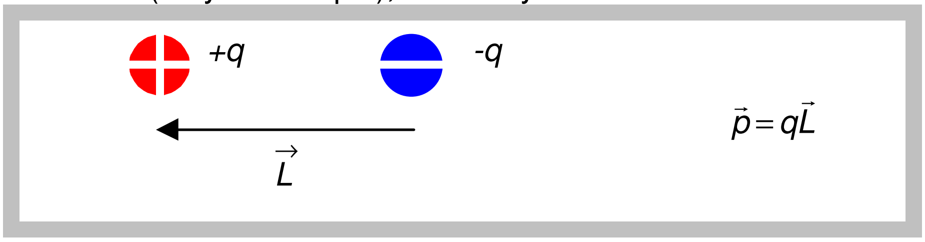
The equivalent capacitance of a set of capacitors associated in parallel equals the addition of the capacitances. So, when n capacitors are associated in parallel, the equivalent capacitance is greater than the capacitance of each one of the associated capacitors.

2.8 Dielectrics. Electric dipole. Polarization

Dielectric materials are poor conductors of the electric current. Compared with conductors, the dielectrics haven't free charges inside; all the electrons are linked to the atoms or molecules. Although they are electrically neutral, this doesn't mean that dielectrics can have local electrostatic forces since, at molecular level, the distribution of positive and negative electric charges is not uniform. As an example, on a covalent bond, the more electronegative atom will attract the shared electrons near it, remaining negatively charged, whereas the less electronegative atoms will remain with positive charge: the consequence is that the bond will have two centers of charge, one positive and another negative with a distance between them, that is, an electric dipole.

Electric dipole

An **electric dipole** is a set of two electric charges of the same magnitude, q, but opposite sign, having a distance L between them. It is characterized by the **dipolar electric torque** $\vec{p}$, (dipolar torque) vector magnitude whose modulus is the product of the charge q by the distance between charges L, and pointing to the positive charge. The dipolar electric torque is measured in C·m, but for molecules (very low torque), is usually used e·nm instead.



Electric dipoles are interesting due to its presence in the matter, since a high number of molecules are polar molecules, showing asymmetry in its spatial distribution of electric charge, being considered as electric dipoles. A characteristic example is the case of the molecule of water, with a high dipolar torque.

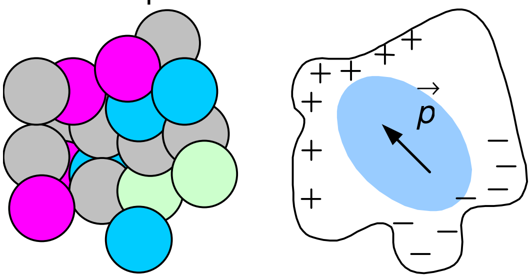


Figure 2-68. The asymmetry in the spatial distribution of charge creates a dipolar torque in a lot of molecules

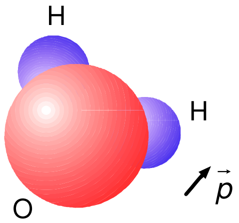
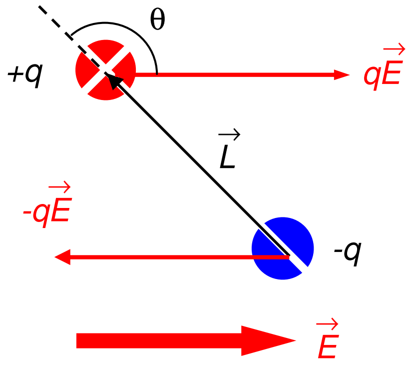


Figure 2-79. Dipolar torque of the molecule of water. The dipolar torque vector points to the atoms of hydrogen, where the electric density of charge is lower

Dipolar electric torques of some substances in e ·nm			
Hydrogen chloride	0,021	Ethyl alcohol	0,023
Carbon monoxide	0,0025	Sodium chloride	0,021
Water	0,039	Hydrogen	0
Ammonia	0,031	Methane	0

The importance of electric dipoles in the matter comes from the response when an external electric field is applied to a material. As a consequence of this electric field, two parallel forces with opposite sense act on both charges of dipole, giving a zero resulting force and a **mechanical torque** ($\vec{\tau}$) given by the vector product between any vector joining the two lines of action of the forces and the force vector.



$$\vec{\tau} = \vec{L} \times q\vec{E} = \vec{p} \times \vec{E}$$

The effect of the torque will be, therefore, to turn the dipole trying to align the dipolar electric torque with the applied electric field.

Polarization

Given a dielectric material, when an external electric field is applied, the electric field inside the material is lower than the electric field that would be on vacuum. That means, for example, that a capacitor filled with a dielectric between their plates will have higher capacitance than the same capacitor without dielectric. This phenomenon is not justified enough with a model in which the electrons are linked to the atoms, and without free charges. But can be understood if we consider that the electrons can move slightly due to Coulomb's forces. The consequences of these trips are the called *phenomena of polarization* of dielectrics, being two models in order to explain them:

a) Dielectrics with atoms or molecules in which the distribution centers of positive and negative charges match up on the same point. In this case when applying an electric field, the positive charges are displaced in the sense of the field, and the negative charges on opposite sense, making up a dipole with a dipolar torque $\vec{p}$. This type of polarization is called *polarization by distortion or induced polarization* and it's a very fast phenomenon.

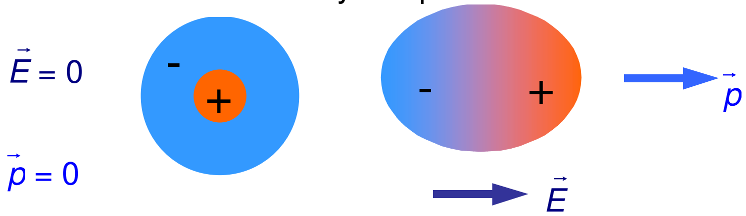


Figure 2-20. Polarization by distortion

b) Dielectrics with atoms or molecules in which the distribution centers of positive and negative charges don't match up, already having small dipoles randomly oriented. In this case when applying an electric field, the dipoles are pointed with their dipolar torques in the same sense than the applied electric field. It's called *polarization by orientation*. This phenomenon can be more or less fast depending on the interaction of the material with the surrounding molecules, and strongly depends on temperature.

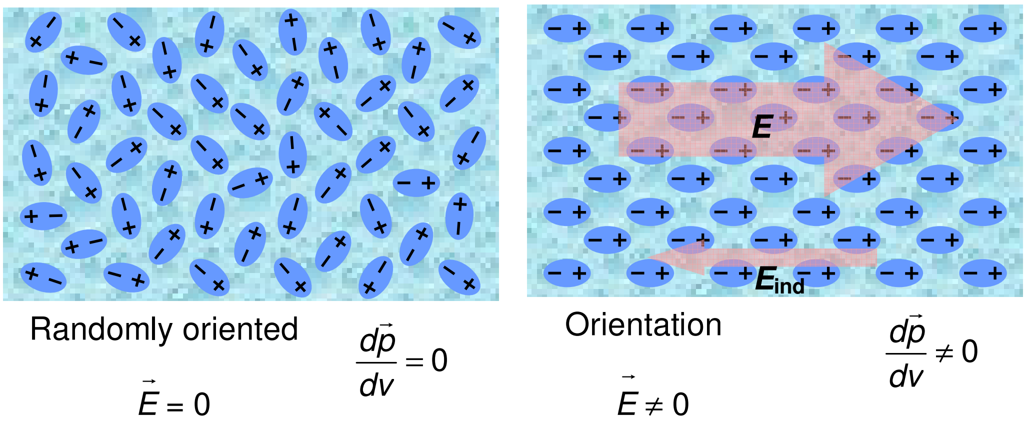


Figure 2-21. Polarization by orientation

Both polarizations, by distortion and by orientation, can happen at the same time in a lot of materials, and suppose a microscopic justification of that happening in dielectric materials when an electric field is applied.

From a macroscopic point of view, to study the effect of the electric field on dielectrics, we can consider the following experience:

- Let's take a parallel plates flat capacitor connected to a difference of potential V_0 (it can be done with a power supply); the capacitor takes a charge Q. Now, we disconnect the power supply (the charge on plates must be preserved) and the space between plates is filled up with a dielectric or insulating material. Measuring the difference of potential between plates, we find now a value $V < V_0$. The rate V_0/V is a characteristic constant for each dielectric material, called **relative dielectric constant** or relative permittivity of the material, ε_r always being greater or equal than one.

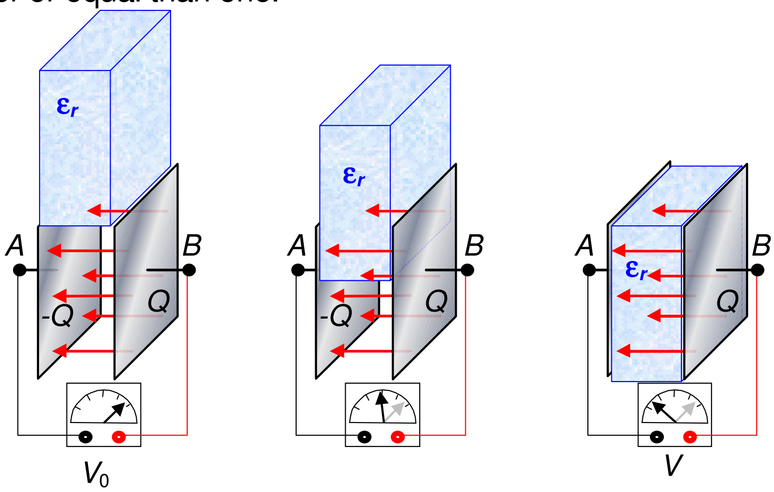


Figure 2-82. Changes on V on a capacitor when the space between plates is filled up with a dielectric, preserving the charge on plates

If $V = V_0/\varepsilon_r$, then $E = V/d = V_0/(d\varepsilon_r) = E_0/\varepsilon_r$, that is, when entering the dielectric in the capacitor, preserving the charge, both the electric field as the potential diminish in the factor ε_r .

On the other hand, the capacitance initially was $C_0 = Q/V_0$; when filling up the capacitor with dielectric, increases up to $C = Q/V = \varepsilon_r C_0$

- If the capacitor is filled up of dielectric *keeping the power supply connected*, i.e., keeping *V constant*, then the electric field keeps also constant and, since the capacitance of the capacitor with dielectric is greater: $C = \varepsilon_r C_0$, then the charge increases. Without dielectric, the charge was $Q_0 = C_0 V$, and with dielectric will be: $Q = CV = \varepsilon_r C_0 V = \varepsilon_r Q_0$.

We can redefine the laws we have seen until here for dielectrics, only substituting the magnitude of the electric field on vacuum by the equivalent for homogeneous and isotropic dielectrics (dividing the electric field in vacuum into the relative dielectric permittivity of the dielectric), and working as if the medium was vacuum.

On Table 2-1 magnitudes of relative permittivity ε_r for some usual dielectric materials are shown. Water has a relative permittivity very high due to the high polar character of the molecule, and to the ease to its orientation on an electric field. But it isn't used as dielectric due to the ease to dissolve the salts, becoming conductor with a small quantity of impurities.

Material	ε_r
Oil	2,24
Water at 20 ºC	80
Air	1,0006
Bakelite	4,9
Mica	5,4
Neoprene	6,9
Paper	3,7
Paraffin	2,3
Plexiglás	3,4
Porcelane	7
Pyrex glass	5,6

Table 2-1. Relative permittivity of some materials

The result of the phenomenon of polarization in a homogeneous dielectric material due to the application of an electric field is the appearance of a surface density of charge σ', called **bound charge density** because it's bonded to the molecules of the dielectric, instead the free charges that can move inside the dielectric. This bound charge produces an electric field E_{ind} (see Figure 2-3) of opposite sense to the applied electric field, producing a lower electric field inside the dielectric, as we are going to prove:.

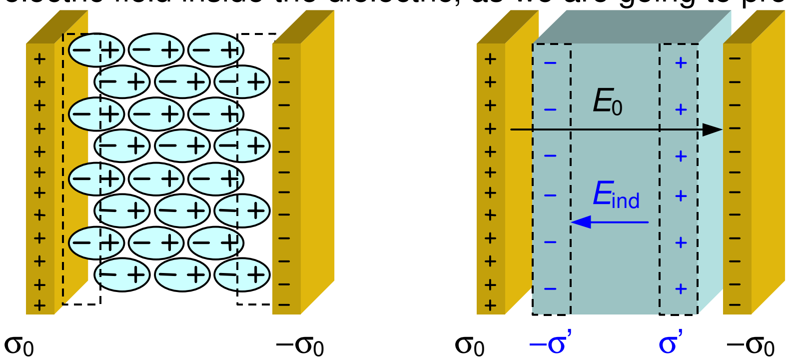


Figure 2-23. The polarization of the dielectric produces a bound charge as a surface density of charge σ' in the surface of dielectric

The electric field due to the surface density of charge on conductor (σ_0) is (pointing to right): $E_0 = \dfrac{\sigma_0}{\varepsilon_0}$

The electric field due to the bound density of charge on dielectric (σ') is (pointing to left): $E_{ind} = \dfrac{\sigma'}{\varepsilon_0}$

And the resulting electric field, taking in account the effect of polarization of dielectric (pointing to right): $E = E_0 - E_{ind} = \dfrac{\sigma_0 - \sigma'}{\varepsilon_0}$ This effect is the same than consider a surface density of charge σ_0 - σ' on surface of conductor and suppose vacuum on the space between plates.

But we have seen that the effect of dielectric can be also taken in account through the relative dielectric permittivity: $E = \dfrac{E_0}{\varepsilon_r} = \dfrac{\sigma_0}{\varepsilon_0 \varepsilon_r} = \dfrac{\sigma_0}{\varepsilon}$

where the parameter $\varepsilon = \varepsilon_0 \varepsilon_r$ is the permittivity of dielectric material. So, equalizing both equations and solving for σ', we can relate the bound charge density to the free charge on plates of capacitor and the dielectric permittivity:

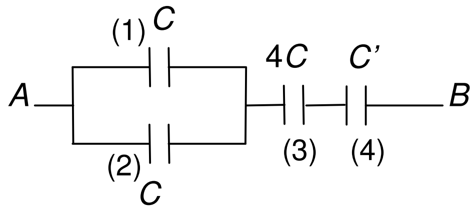
$$\frac{\sigma_0}{\varepsilon} = \frac{\sigma_0 - \sigma'}{\varepsilon_0} \Rightarrow \sigma' = \sigma_0 (1 - \frac{\varepsilon_0}{\varepsilon_r})$$

$$\sigma' = \sigma_0 \left(1 - \frac{1}{\varepsilon_r} \right) \qquad \text{Equation 2-7}$$

Note that the bound charge density has opposite sign than the surface density of free charge.

Example 2-3

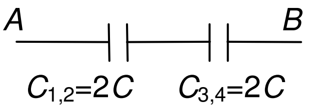
Between points A and *B* of the association of capacitors on figure, a difference of potential *V* is applied. The capacitor 4 had a capacitance *C* before to be filled up with a dielectric of $\varepsilon_r = 4$. Find the capacitance *C'* of this capacitor, the charge and the difference of potential on each capacitor.



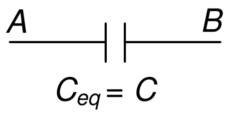
Solution

$C' = 4C$

Finding the equivalent capacitance of capacitors 1 and 2 in parallel by one hand, and 3 and 4 in series by other hand, the resulting system is:



The equivalent capacitance is:



The total charge of the association when applying the difference of potential V is:

$Q_T = VC = Q_{3,4} = Q_{1,2} = Q_3 = Q_4$; $Q_1 = Q_2 = Q_{1,2}/2 = VC/2$

The difference of potential on terminals of each capacitor will be:

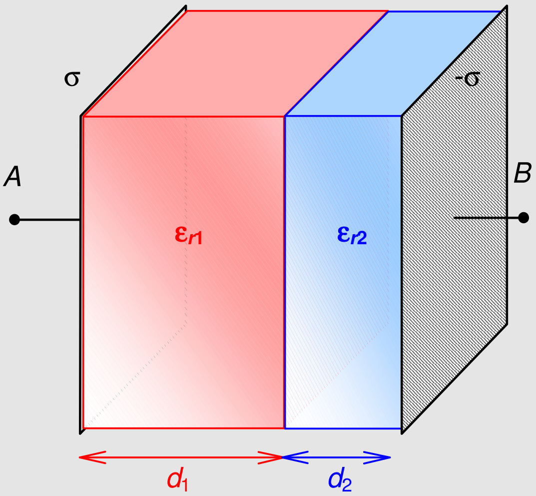
$V_1 = V_2 = Q_1/C = VC/2C = V/C$; $V_3 = Q_3 / 4C = V/4 = V_4$

Writing these values in a table:

	Q	**V**
(1)	$VC/2$	$V/2$
(2)	$VC/2$	$V/2$
(3)	VC	$V/4$
(4)	VC	$V/4$

Capacitance of a flat capacitor with several layers of dielectric

Let's have a parallel plate flat capacitor whose plates are charged with a surface density $\pm\sigma$, and it has been filled up with two layers of dielectric, one of them with thickness d_1 and relative dielectric permittivity ε_{r1} and the other with thickness d_2 and relative dielectric permittivity ε_{r2}.



The capacitance of such capacitor comes from

$$C = \frac{Q}{V}$$

The difference of potential between the plates of capacitor can be written as the addition of the difference of potential in the first dielectric and the difference of potential in second dielectric:

$$V_{AB} = V_1 + V_2 = E_1 d_1 + E_2 d_2$$

The electric field in each one of the dielectric comes from:

$$E_1 = \frac{\sigma}{\varepsilon_{r1}\varepsilon_0} \qquad E_2 = \frac{\sigma}{\varepsilon_{r2}\varepsilon_0}$$

And the capacitance of capacitor will be:

$$C = \frac{Q}{V_{AB}} = \frac{Q}{E_1 d_1 + E_2 d_2} = \frac{Q}{\dfrac{d_1\sigma}{\varepsilon_{r1}\,\varepsilon_0} + \dfrac{d_2\sigma}{\varepsilon_{r2}\,\varepsilon_0}} = \varepsilon_0 \frac{S}{\dfrac{d_1}{\varepsilon_{r1}} + \dfrac{d_2}{\varepsilon_{r2}}}$$

In general, for n layers of dielectrics:

$$C = \varepsilon_0 \frac{S}{\sum_{i=1}^{n} \dfrac{d_i}{\varepsilon_{ri}}}$$

Tactile pointer

George Gerpheide

Investigation and Science. September of 1998

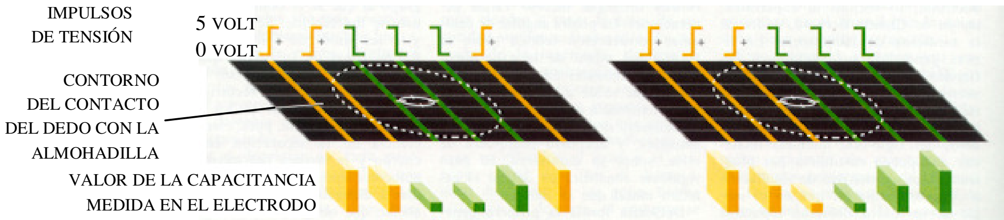
The device of pointer more common in the new portable computers is the tactile pad, a black or grey rectangle always placed in front of the keyboard. The trip of a finger above it does that the cursor describes an analogous movement on the screen.

The tactile pads initiated its history not more than four years ago, but already have displaced the mouse of ball integrated like standard pointer of the portable computers, that offer them now in more than the 66%. (The rest, in its majority models of IBM and Toshiba, employ the small pointer, similar to a control of games and that remembers to the rubber to erase a pencil, installed in the keyboard between the keys "G", "H" and "B".) The handling of the tactile pads is much more convenient for a lot of people, between them those who suffer of arthritis. As it treats of devices entirely hermetical, in its interior do not penetrate the dust neither the odd substances, what does them more adapted for difficult environments, such like workshops, factories and garages.

The type of tactile pad more extended is the one of capacitance, that works measuring its variations when the finger of the user alters the tiny electric fields existent in the top of the pad.

The electrodes are placed in two layers, orthogonally placed, and separated by a thin plate of fiber of glass, acting as insulator or "dielectric".

An electric field acts when an impulse of tension is applied between an upper electrode and another inferior, what has as result that the two electrodes, the dielectric material interposed and even the surrounding air work like a capacitor. This electric field is modified in front of the presence of a finger, distortion generating a decreasing of the capacitance between the electrodes and of the electric charge.



THE DISEQUILIBRIUM BETWEEN THE TOTAL CAPACITANCE (THE ONE OF THE ORANGE GROUP IS GREATER THAT THE ONE OF THE GREEN) MEANS THAT THE FINGER COVERS MORE THE GREEN ELECTRODES

THE BALANCE BETWEEN THE VALUES OF BOTH GROUPS INDICATES A EQUIDISTANT POSITION OF THE FINGER CONCERNING THE ELECTRODES

The location of the finger requires the trip of two groups of impulses of tension. If it was treated to evaluate the capacitance in each one of the points of crossing of the electrodes, it would take too time and the reaction of the pointer to the movement of the finger would be lazy. Thus what it's done is to apply two groups of impulses, positive *(in orange)* and negative *(in green; right and left diagrams)* to the electrodes, measuring the resulting charge of its capacitance. The situation of the finger related to the limit between the regions of positive and negative impulses is determined by means of calculations realised with the total loads measures. Both groups of impulses must to displace according the finger is moved, so that its border keeps near the centre of the finger. The picture does not show more than the corresponding impulses to the group of the parallel and vertical electrodes; the same must be done with the set of transversal electrodes. In this way can be followed the two-dimensional movement of the finger, until speeds around of 100 centimeters by second.

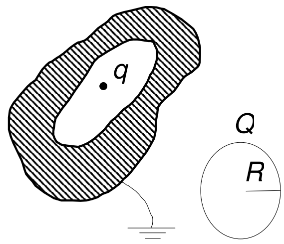
2.9 Questions and problems

Conductors

1. Which is the direction of the field lines on points close to a charged conductor in equilibrium? Why?

Sol: Perpendicular to the surface of conductor, because if not, the charges on the surface of conductor would be moving, and the conductor wouldn't be in equilibrium.

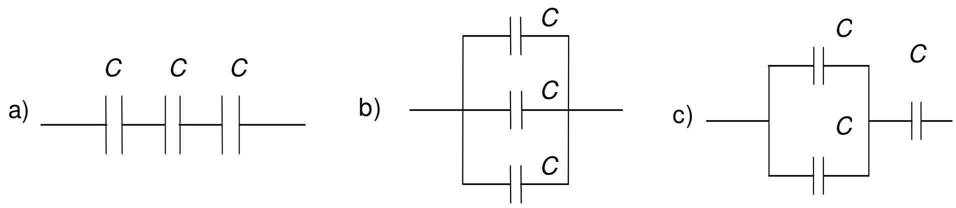
2. The picture shows a hollow conductor connected to ground, with a charge q in its cavity. There is a charged sphere with charge Q outside to conductor. Which is the effect of the charge q in the distribution of charge Q in the surface of the sphere of radius R? Justify the answer.



Sol: None, because the hollow conductor connected to ground acts as a Faraday's cage.

Capacitors and dielectrics

3. Given three equal capacitors with capacitance C, compute the equivalent capacitance of the system in each case

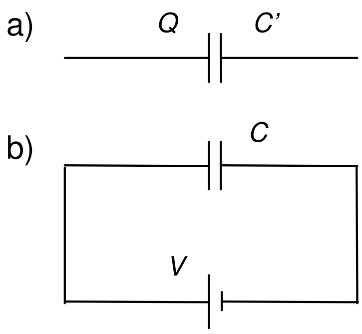


Sol: a) C/3 b) 3C c) 2C/3

4. Two capacitors with capacitances 2,4 and 3,1 µF are connected in series and the set is connected to a battery of 6,1 V. a) Which is the equivalent capacitance of the set? b) Which is the charge of each capacitor? c) Which is the difference of potential between the plates of each capacitor?

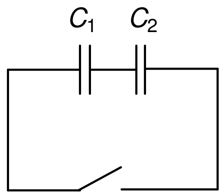
Sol: a) 1,35 µF b) $Q_{2,4}=Q_{3,1}=8{,}25\ \mu C$ c) $V_{2,4}=3{,}44$ V $V_{3,1}=2{,}66$ V

5. Given the two flat capacitors on picture:
a) isolated with charge Q;
b) connected to a power supply of difference of potential V.
If we move apart the plates of both capacitors, say how the stored energy changes on each capacitor (increases, decreases, or remains constant.)



Sol: a) Increases b) Decreases

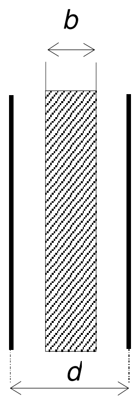
6. A capacitor of capacitance C_1, with charge Q, is connected to another, with capacitance C_2, initially discharged, such as it appears on picture. Compute the charge on each capacitor before and after closing the switch.



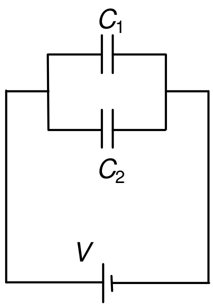
Sol:

	Q_1	Q_2
Before	Q	0
After	$QC_1/(C_1+C_2)$	$QC_2/(C_1+C_2)$

7. A plate of copper of thickness b is placed inside a capacitor of surface S, such as shown in the figure. Which is the capacitance of the capacitor before and after to place the copper plate?



Sol: before $C_0=\varepsilon_0 S/d$ afterwards $C = \varepsilon_0 S/(d-b)$

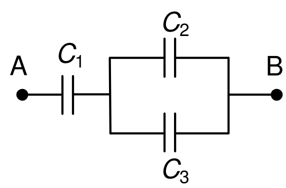


8. Two capacitors with capacitances C_1 and C_2 are connected in parallel, and a difference of potential V is applied. Compute the charge taken by each capacitor (Q_1 and Q_2) as well as the difference of potential between the plates of each one. (V_1 and V_2).

Sol: $Q_1=C_1V$ $Q_2=C_2V$ $V_1= V_2=V$

9. In the association of capacitors on picture, when a difference of potential is applied between A and B, answer which capacitor stores:

a) the greater charge
b) the lower charge,

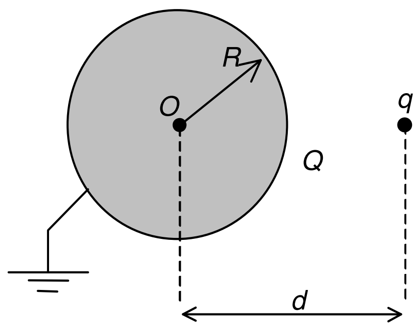


($C_1 = C$; $C_2 = C/3$; $C_3 = C(2/3)$).

Sol: a) C_1 b) C_2

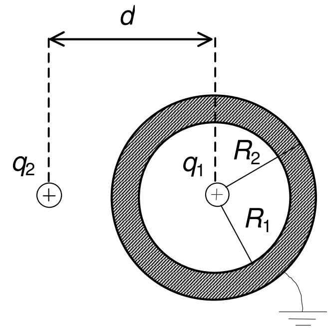
Conductors

10. Let's have a spherical conductor, centered in O and with radius R. Such sphere is connected to ground (potential zero), and it's influenced by a point charge q, placed at a distance d from O ($d>R$). Compute the charge on the sphere as a function of q , R and d.



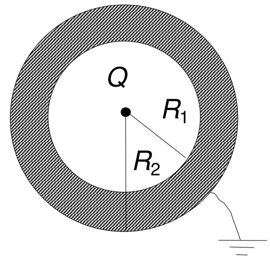
Sol: $Q = -q\dfrac{R}{d}$

11. Given the system of the figure, compute the total charge Q of the sphere.



Sol: $-q_1 - \dfrac{q_2}{d}R_2$

12. The figure shows a hollow metallic sphere of inner and outer radii R_1 and R_2. The sphere is connected to ground with a positive point charge, Q, in its centre.
a) Which is the distribution of charges on inner and outer surfaces of the sphere?
b) Obtain the expressions for V(r) for $r \le R_1$, $R_1 \le r \le R_2$, $r \ge R_2$.



Sol:

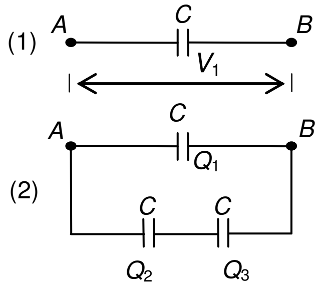
a) on R_1, -Q; on R_2, zero; b) $r \le R_1$, $V = \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{1}{r} - \dfrac{1}{R_1}\right)$; ($R_1 \le r \le R_2$; $r \ge R_2$) $V = 0$

Capacitors and dielectrics

13. Two parallel plate flat capacitors 1 and 2 of equal capacitance C are connected in parallel to a d.d.p. V. After disconnect the association from the power supply, the distance between plates of capacitor 1 is decreased to a half of the initial distance. Which will be the charge on each capacitor?

Sol: $Q_1 = \dfrac{4}{3}CV;\;\; Q_2 = \dfrac{2}{3}CV$

14. A difference of potential V_1 is applied to a capacitor with capacitance C (1). Two equal capacitors in series are connected in parallel with the first one (2); these new capacitors are initially discharged. Compute the charge taken by each capacitor, Q_1, Q_2, and Q_3.



Sol: $Q_1 = \dfrac{2}{3}V_1C; Q_2 = Q_3 = \dfrac{1}{3}V_1C$

15. Two metallic parallel plates are separated a distance d on vacuum. A d.d.p. V is applied and then the power supply removed. A layer of glass (dielectric) of thickness d/2 and relative permittivity ε_r is placed between the two plates. Which is the new value of the d.d.p. between plates? Which must be the separation between the plates to get the d.d.p. was the same that in the beginning?

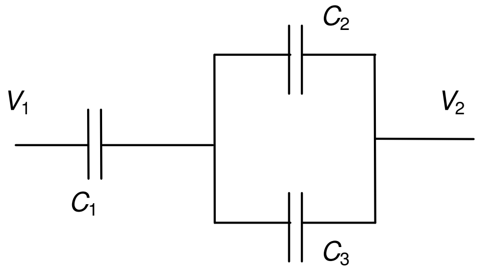
Sol: $V' = \dfrac{V}{2}\left(1 + \dfrac{1}{\varepsilon_r}\right);\quad d' = d\left[1 + \dfrac{1}{2}\left(1 - \dfrac{1}{\varepsilon_r}\right)\right]$

16. Two metallic parallel plates with surface S are separated a distance d on vacuum. A d.d.p. V is applied and then the power supply removed. A layer of

dielectric of thickness b ($b<d$) and relative permittivity ε_r is placed between the two plates. Compute:
a) capacitance before entering the dielectric.
b) free load on plates.
c) electric field in the space between plates where there is vacuum.
d) electric field in the space between plates where there is dielectric.
e) d.d.p. between plates after entering the dielectric.
f) Capacitance with the dielectric.

Sol: to) $C_0 = \varepsilon_0 \dfrac{S}{d}$ b) $Q = \varepsilon_0 V \dfrac{S}{d}$ c) $E_0 = \dfrac{V}{d}$ d) $E = \dfrac{V}{d\,\varepsilon_r}$ and) $V' = \dfrac{V}{d}\left(\dfrac{b}{\varepsilon_r} + d - b\right)$

f) $C = \dfrac{\varepsilon_0 S}{\dfrac{b}{\varepsilon_r} + d - b}$



17. The picture shows a set of equal capacitors (capacitance C) connected to a d.d.p. $V=V_1-V_2$.
a) Compute the stored energy on capacitor 2.
Capacitor 2 is filled up 2 with a dielectric of relative permittivity ε_r
b) Compute the total energy stored on all capacitors.
c) Which is the factor should we multiply by the distance between plates of capacitor 3 in order the equivalent capacitance wouldn't be modified?

Sol: a) $W_2 = \dfrac{CV^2}{18}$; b) $W_T = \dfrac{C(1+\varepsilon_r)V^2}{2(2+\varepsilon_r)}$; c) $x = \dfrac{1}{2-\varepsilon_r}$

GLOSSARY

Theorem of Coulomb: The electric field near the surface of a charged conductor in electrostatic equilibrium is perpendicular to its surface, pointing to outside if the charge is positive and opposite if it's negative, and with magnitude:

$$E_n = \frac{\sigma}{\varepsilon_0}$$

Ground: point of linking assuring the electric potential is zero.

Electrostatic shield: Device dividing the space in two regions electrostatically independent.

Capacitor: system of two conductors, isolated one of another, exerting total electrostatic influence between them and storing electric charge.

Capacitance of a capacitor: is the rate between the absolute value of charge Q on each plate and the difference of potential between both plates.

Equivalent capacitance of an association of capacitors: is the capacitance that would have a lonely capacitor so that when applying it the same difference of potential that to the association, it would take the same quantity of charge.

Electric dipole: set made up by two electric charges of the same value but opposite sign, separated a distance.

Polarization: Apparition or orientation of electric dipoles in the matter as a consequence of an external electric field.

Dielectric constant: Factor of decreasing of the electric field inside a dielectric as a consequence of polarization.

Bound charge: Induced electric charge in the surface of a dielectric as a consequence of the polarization of dielectric.