PROBLEMS TO BE SOLVED IN CLASSROOM

Unit 0. Prerrequisites

0.1. Obtain a unit vector perpendicular to vectors $2\vec{i}+3\vec{j}-6\vec{k}$ and $\vec{i}+\vec{j}-\vec{k}$

0.2 a) Find the integral of vector $\vec{v}=2xy\vec{i}+3\vec{j}-2z^2\vec{k}$ along the straight line parallel to Y axis from point A(1,1,1) to point B(1,3,1) (circulation of a vector along a line).

b) Repeat the above exercise with vector $\vec{v}=2xy\vec{i}+3x\vec{j}-2z^2\vec{k}$.

c) If possible, repeat the exercise b) but along the straight line going from A to point C (3,3,1).

0.3 Find the integral of vector $\vec{v}=2xy\vec{i}+3\vec{j}-2z^2\vec{k}$ through a square with side *h* parallel to the plane XY and placed on plane *z=1* (integral of a vector through a surface).

0.4 Calculate:

a) The circulation of vector $\vec{v}=r\vec{u}_r$ along any circumference having radius *2* and centred at the origin of coordinates.

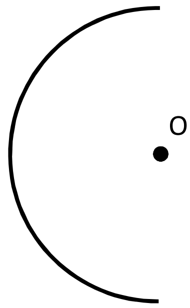
b) The integral of vector $\vec{v}=r\vec{u}_r$ through a sphere having radius *2* and centred at the origin of coordinates.

c) The circulation of vector $\vec{v}=r\vec{u}_r$ along any radius of before sphere between points with *r=0* and *r=2*.

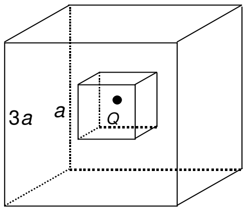
Unit 1: Electrostatics

1.1 a) Calculate the electric field produced at point (4,0) m by two point charges: 2 µC at (0,0) m and -2 µC at (0,3) m. Calculate the force acting over -5 µC placed at point (4,0) m.

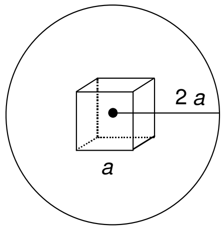
b) Repeat the before exercise but at point (4,4) m instead point (4,0) m.



1.2 A uniform charged rod of length 14 cm is bent into the shape of a semicircle as shown on picture. The rod has a total charge of -7,5 µC, homogeneously distributed along the rod . Find the magnitude and the direction of the electric field at point O, the centre of the semicircle.



1.3 Let's take the point charge *Q* and the two cubic surfaces, parallel, centered in *Q*, and with sides *a* and 3*a*, shown in the figure. Compute the rate between fluxes of the electric field through both surfaces ($\Phi a/\Phi 3a$). Justify the answer.



1.4 A cube of edge *a* and uniform volumetric density of charge, ρ, is placed on vacuum. It's surrounded by a spherical surface of radius 2*a*. Compute the flux of the electric field through the spherical surface.

1.5 Calculate the electric field produced by:

a) A spherical surface having radius *R* charged with a homogeneous density of charge σ; calculate the electric field at a distance *r* from the centre of sphere for r<R and r>R.

b) An infinite plane charged with σ homogeneous.

c) Two infinite and parallel planes charged with a homogeneous density of charge σ, inside the space between both planes and outside the space (applying superposition). Also consider the case when the charges on both planes have different sign.

d) The same exercise as a) but adding a new point charge -Q at the centre of sphere. ¿Could be the electric field zero at any point of the space? ¿In which circumstances?

1.6 Calculate:

a) The electric potential at points *A (4,0)* and *B (4,3)* produced by the charges of exercise 1.1: $2\ \mu C$ at *(0,0)* and *-2* μC at *(0,3)*.

b) The work needed to carry a charge of *-3* μC from *A* to *B*.

1.7 Calculate the d.d.p. (difference of potential) between two points of the electric field created by an infinite plane.

1.8 A spherical surface (radius *R*) is charged with a homogeneous surface density of charge σ. Calculate:

a) Electric field on a point placed at a distance *r* from the centre of the sphere for r<R and for r>R.
b) Difference of potential between A(2R) and B(3R).
c) Electric potential on point B.
d) Difference of potential between C(R/3) and D(R/2).
e) Electric potential on points C and D.
f) Work done by the electric field to carry a charge *q* from B to A.
g) Work done by the electric field to carry a charge *q* from A to E(2R) (being E a point different than A).

h) If σ is positive and we add a negative point charge Q at the center of sphere, ¿is possible to find a point where the electric field was zero?

1.9 Planes $y=-1$ and $y=1$ have surface densities of charge respectively, $1\ \mu C/m^2$ and $2\ \mu C/m^2$.

Calculate the difference of potential (d.d.p) between points $A(0,3,0)$ and $B(0,5,0)$, as well as the work needed to move a $2\ \mu C$ charge from point A to point B. ¿Who is doing this work, the forces of the electric field or an external force against the electric field?

Unit 2: Conductors in electrostatic equilibrium. Dielectrics

2.1 Calculate the electric potential created by a conductor sphere (radius R) charged with charge Q at a point placed at a distance $r>R$ from the centre of sphere. Calculate the electric potential of sphere.

2.2 Two conductor spheres with radii R_1 and R_2 ($R_1 > R_2$), the first one with charge and the second one without charge are joined with a conductor wire without capacitance (the electric influence between both spheres can be neglected). Calculate the charge and the electric potential of both spheres after the joining.

Exercises 2.3 to 2.7 must be solved as pieces of the same exercise:

2.3 A point charge q is placed at a distance d from the centre of a conductor sphere with radius R and discharged (d>R). Calculate the potential of sphere.

2.4 A point charge q is placed at the centre of a hollow and conductor sphere (radii R_1 and R_2) discharged. Calculate the electric field at the different areas of the space: $r<R1$, $R1<r<R_2$, $r>R_2$. Calculate the electric potential of sphere.

2.5 A point charge q is placed at a distance d from the centre of a conductor sphere with radius R (d>R) linked to ground. Calculate the charge of sphere.

2.6 A point charge q is placed at the centre of a hollow and conductor sphere (radii R_1 and R_2) linked to ground. Calculate the electric field at the different areas of the space: $r<R1$, $R1<r<R_2$, $r>R_2$. Calculate the electric potential of sphere.

2.7 A hollow sphere (radii R_1 and R_2) is linked to ground. A point charge q is placed at the centre of sphere, and another point charge Q is placed outside of sphere at a distance d from its centre (d>R). Calculate the total charge of sphere. (In order to solve this exercise, apply the superposition principle taking in account the results of exercises 2.5 and 2.6).

2.8 A Van de Graaf generator is made up by a *10 cm* of radius conductor sphere. The generator is transferring electric charge to this sphere from a second smaller sphere having *5 cm* of radius. The distance between both spheres is *5 mm*. By assuming that electric field between the spheres is uniform (really it only changes around 13%) and that the dielectric breaking of the air is produced when the electric field reaches *1 KV/mm*, calculate the charge of each sphere when the spark is produced between the spheres.

2.9 A plane conductor (surface S and negligible thickness) is charged with a charge Q:
a) Let you say how the charge is distributed on conductor, and give its surface density of charge.
b) A second equal but discharged conductor plate is approached to the first one up to a little distance compared with the magnitude of both plates (we can then assume total influence between plates). Explain how the charges are distributed on both plates, and give their respective surface densities of charge.

2.10 A capacitor (capacitance C) is connected to a power supply (d.d.p. V between its plates). Next is disconnected from power supply and it is connected to a second capacitor having capacitance $2C$, initially discharged.
a) Calculate the charge and potential of each capacitor after connecting them.

b) Plates of second capacitor are approached up to a distance a half of the initial distance. Compute the charge and the difference of potential between plates of each capacitor.

2.11 Repeat before exercise but keeping the power supply connected every time.

2.12 Two capacitors (capacitance *2* and *3* μF) are connected in series to a *10 V* power supply. Compute the charge and the difference of potential between the plates of each capacitor:
a) Without using the idea of equivalent capacitance of set of capacitors.
b) By using the idea of equivalent capacitance of set of capacitors.

2.13 A capacitor with capacitance C_0 is connected to a power supply giving a difference of potential V. Next, a dielectric with relative dielectric permittivity ε_r is inserted between the plates of capacitor. Compute the charge and the difference of potential of capacitor after inserting the dielectric. ¿Has the energy stored on capacitor increased or decreased when dielectric is inserted?

2.14 A capacitor with capacitance C_0 is connected to a power supply giving a difference of potential V; capacitor is disconnected from power supply and a dielectric with relative dielectric permittivity ε_r is inserted between the plates of capacitor. Compute the charge and the difference of potential of capacitor after inserting the dielectric. ¿Has the energy stored on capacitor increased or decreased when dielectric is inserted?

2.15 A capacitor with capacitance C_0 is filled with a dielectric having a relative dielectric permittivity *3*. Compute the new capacitance of capacitor

a) If it's filled only the left half of space between plates.

b) If it's filled only the lower half of space between plates.

c) If it's completely filled with a conductor.

Unit 3: Electric current

3.1 A ring of radius *R* has a linear density of charge λ. If the ring turns with an angular speed ω around its axis, compute the intensity of current of ring.

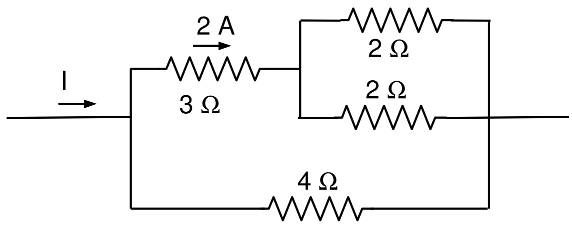
3.2 Along a conductor of Cu with a radius of *1,3 mm* and length *1 m*, flows a *20 A* intensity of current.

a) Compute the density of current J, the drift speed V_d, and the time taken by the electrons to cover *1 m* of distance. Data: $n=1{,}806\cdot10^{29}\ e^-/m^3$, $q = 1{,}6\cdot10^{-19}\ C$.

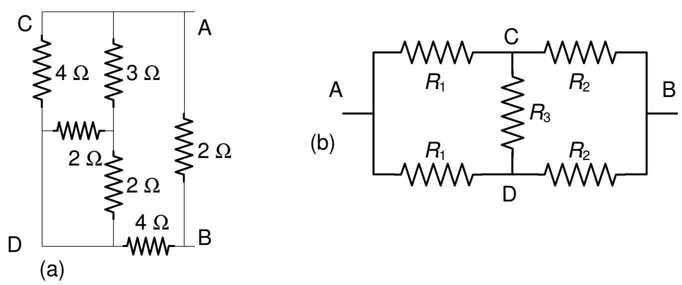
b) By knowing the resistivity of Cu, $\rho_{Cu}=1{,}7*10^{-8}\ \Omega m$, compute the electric field E inside the conductor, its resistance R and the d.d.p. between its endings.

3.3 Two *3* and *5* Ω resistors are connected in parallel; along the set of resistors is flowing a total intensity of current I = *10 A*. Find how much current flows along each resistor.

3.4 Given the set of resistors on picture, find the total intensity I.



3.5 For both circuits on right, compute the equivalent resistance between terminals *A* and *B* (R_{AB}) and between terminals *C* and *D* (R_{CD}).



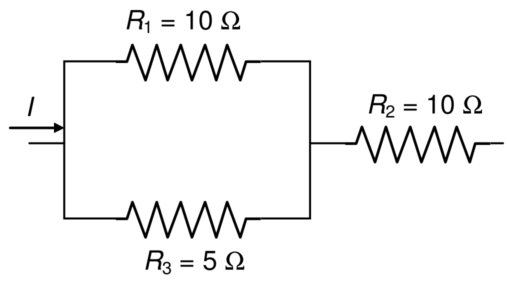
Unit 4: Energy and power

4.1 In the circuit of figure, compute:

a) Which resistor dissipates more power due to Joule heating?

b) Which resistor dissipates less power due to Joule heating?

Justify the answers.



4.2 In the circuit of picture, $\varepsilon = 6\ V$ and $r = 0{,}5\ \Omega$. Lost power by Joule heating in r is $8\ W$. Find:

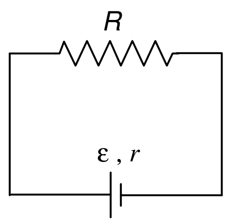
a) The intensity of current on circuit.

b) The difference of potential between terminals of *R.*

c) *R*.

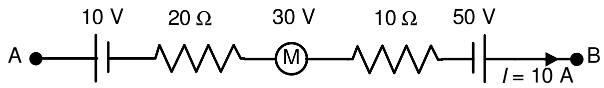
d) The generated power, the supplied power, and the efficiency of generator.

e) Verify that the supplied power equals the lost power by Joule heating on R.

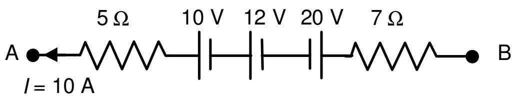


4.3 A resistor *R* is connected to a generator of electromotive force ε and internal resistance *r*. Which should be the value of *R* in order the lost power on this resistor was maximum?

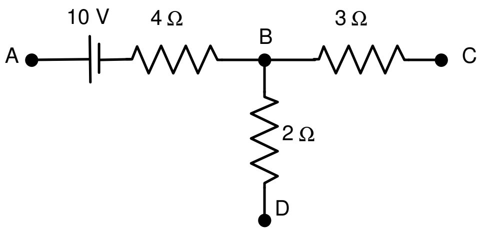
4.4 Compute the difference of potential between points *A* and *B* on next pictures:



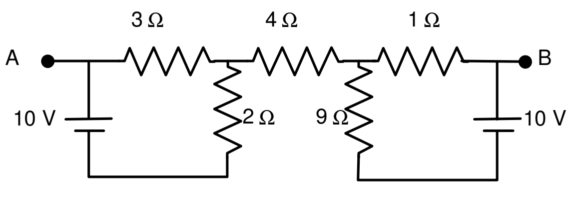
a)



b)



c)



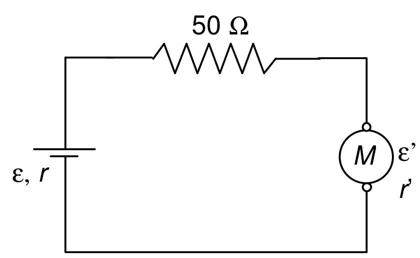
d)

4.5 The engine of the drawn circuit consumes *50 W*, being a *20% by Joule heating*. If the generator supplies *100 W* to the circuit, compute:

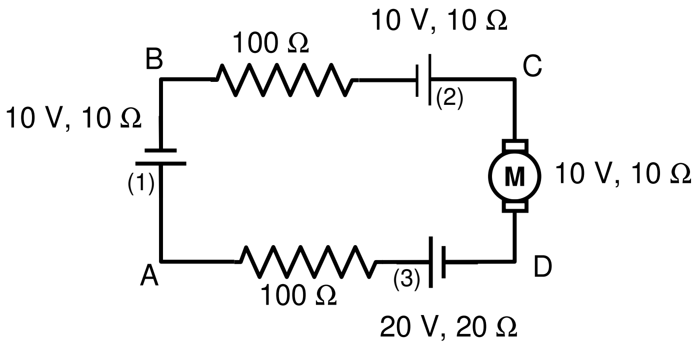
a) Consumed power on 50 Ω resistor.

b) If the generator generates a power of 110 W, compute their characteristic parameters ε and *r.*

c) The characteristic parameters of engine, ε' and *r'*.



4.6 Given the circuit of figure:



a) Compute the magnitude and sense of intensity flowing along circuit.

b) Compute the difference of potential between points *A* and *C* (V_A-V_C), both along path *ABC* as along path *ADC*.

c) Which devices supply energy to the circuit? Compute the value of supplied power by each device.

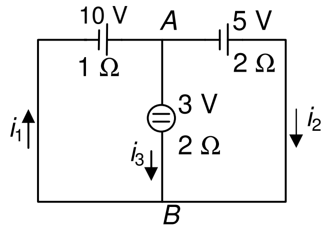
d) Which devices consume energy from circuit? Compute the value of consumed power by each device.

e) Which are efficiencies of engine and power supply (2)?

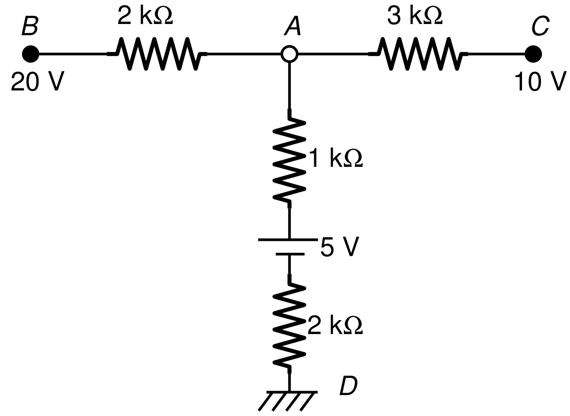
f) If we modify the electromotive force of power supply (1), which must be its magnitude for the difference of potential between points *A* and *C* equals zero? Which is the intensity on circuit in this case?

Unit 5: Networks

5.1 In the circuit on picture, calculate the intensities in the three branches and the difference of potential between terminals of engine.



5.2 By using Kirchoff's rules, compute the potential on point *A* and the intensities of current along the branches of circuit on picture.

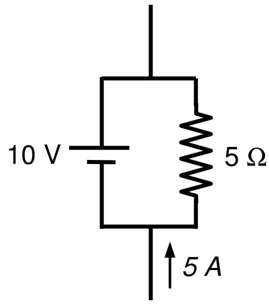


5.3 Along a wire flows an intensity of current *5 A*. This wire is divided into two branches: one of them with an ideal generator having a electromotive force *10 V*, and another branch with a *5 Ω* resistor. After these devices, both branches are joined.

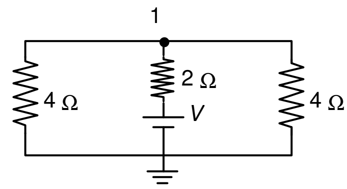
a) Compute the intensity of current flowing along each branch.

b) Repeat the calculations after inverting the polarity of generator.

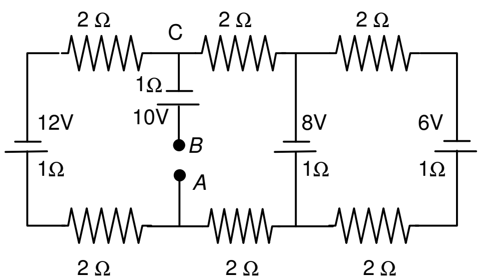
c) Repeat calculations of points a) and b) by changing the ideal generator by a real generator with internal resistance *10 Ω*.



5.4 In the network on picture, find the voltage *V* so that voltage on junction 1 was 50 V.



5.5 Find the difference of potential between *A* and *B*.

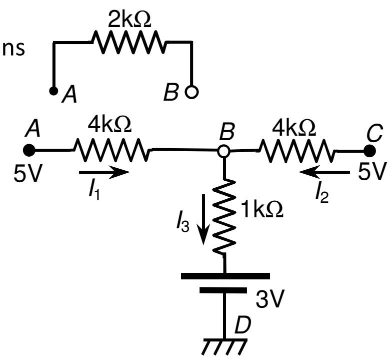


5.6 Given the network on picture:

a) Compute the intensities of branches I_1, I_2, and I_3 by means of Kirchhoff's rules.

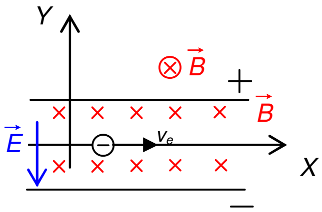
b) Find the Thevenin's equivalent generator between *A* and *B*, clearly showing its polarity.

c) In parallel to points *A* and *B* of network, a new 2 kΩ resistor is connected. Compute the intensity would flow along this resistor, clearly showing its direction.

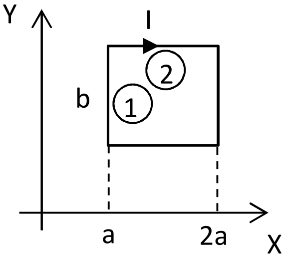


Unit 6: Magnetic forces

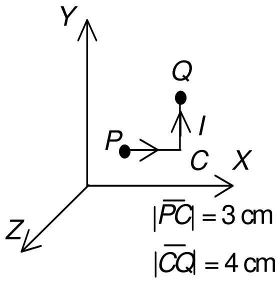
6.1 A bundle of electrons move between the plates of a capacitor with a difference of potential *V*. Between plates there is a uniform magnetic field perpendicular to the electric field. If plates of capacitor are separated a distance *d*, calculate the speed of the electrons not deflecting when they move between plates.



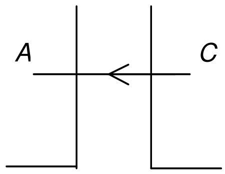
6.2 Let's consider the rectangular loop on picture, with sides *a* and *b*, and flowed by an intensity *I* in the shown direction. The loop is inside a no uniform magnetic field $\vec{B} = B_0 \frac{a}{x} \vec{k}$. Calculate the forces acting on sides 1 and 2.



6.3 By the segment of conductor in the figure flows a current *I* = 2 A from *P* to *Q*. It exists a magnetic field $\vec{B} = 1\vec{k}\,T$. Find the total force acting on conductor and prove that it is the same that if the entire conductor was a straight segment from *P* to *Q*.

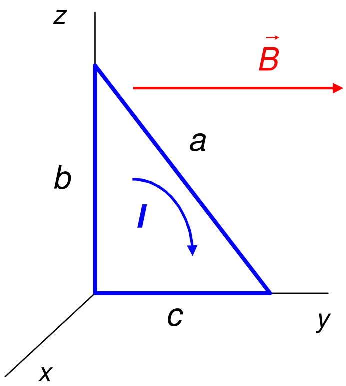


6.4 Along conductor *AC* of figure flows a current of 10 A (it's a part of an electric circuit), being able to glide along two vertical rods.
Compute the necessary uniform magnetic field, perpendicular to the plane of the figure, in order that the magnetic force on conductor could equilibrate the gravitational force. Which should be the direction of magnetic field?



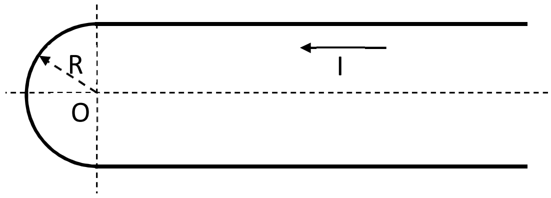
The length of conductor is 10 cm and its mass, 20 g.

6.5 Along the loop of the figure of sides *a*, *b* and *c*, flows an intensity *I* in the shown direction. The loop is placed inside a magnetic field $\vec{B} = B\vec{j}$.
Find:
a) Magnetic forces on sides of the loop.
b) Magnetic moment of the loop.
c) Torque acting on loop.



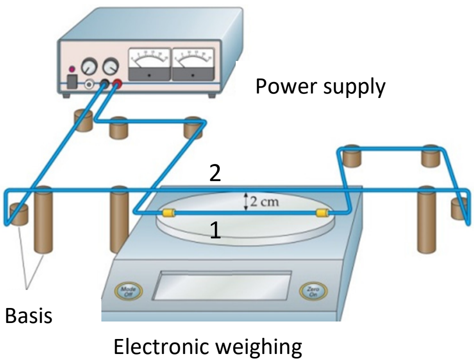
Unit 7: Sources of Magnetic field

7.1 Two parallel semiinfinite conductors are joined by a semicircumference, as can be seen on picture. Along the set of conductors flows an intensity of current I. Compute at point O (centre of semicircumference), always giving its direction:



a) The magnetic field produced by one of the straight conductors.
b) The magnetic field produced by the semicircumference.
c) The total magnetic field produced by the set of conductors.

7.2 A current weighing scale is made up as is shown on picture:

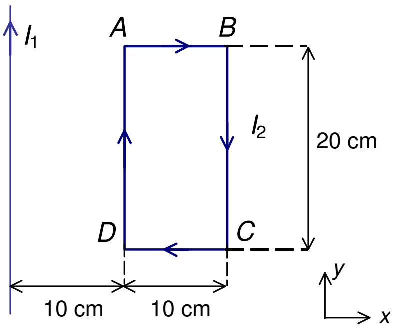


An horizontal straight conductor 10 cm sized (1) is over the plate of an electronic weighing scale and connected to a second horizontal straight conductor (2), at a distance 2 cm from before conductor (the thickness of conductors can be neglected). Both conductors are connected to a D.C. power supply, making up a circuit. When the power supply is switched on, the reading of weighing scale increases 5.0 mg. (with respect to the reading with the power supply switched off).

a) Explain why the reading of weighing scale increases when power supply is switched on.
b) Compute the magnitude of intensity flowing along the circuit when power supply is switched on.
c) Red and black terminals of power supply correspond to its positive and negative terminals. ¿Which would be the reading of weighing scale if we invert the polarity of circuit? (it is, if the intensity would flow in opposite direction).
d) If the power supply can be taken as ideal, and it is supplying 12 V to the circuit, compute the lost power by Joule heating on all the resistors of circuit.

$\mu_0=4\pi\cdot10^{-7}$ (I.S. units)

7.3 An infinite and straight line conductor is flowed by an intensity I_1 *=30 A*. The rectangle *ABCD*, whose sides *BC* and *DA* are parallel to conductor is in the same plane than straight conductor, and it's flowed by an intensity I_2*=10 A*. Compute:



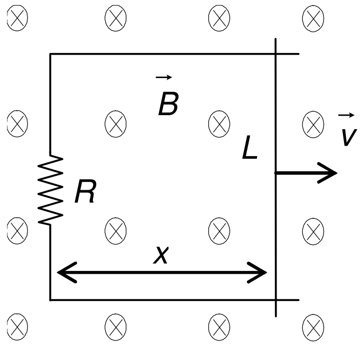
a) The magnetic flux produced by I_1 through the rectangle.

b) The force acting on each side of rectangle because of the magnetic field created by I_1.

c) Should the resulting force acting on four sides of rectangle be null?

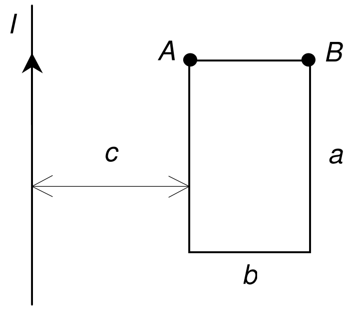
Unit 8: Electromagnetic induction

8.1 A conductor rod with resistance negligible and length L glides without friction and constant speed v over a conductor U shaped. The U shaped conductor has a resistance R and it's placed inside a uniform magnetic field $\vec{B}$ perpendicular to conductor. Compute:



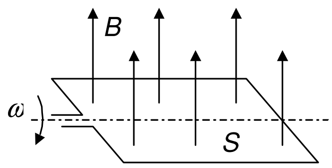
a) Magnetic flux crossing the loop as a function of x.
b) Induced current on the loop, showing its direction.
c) Force should act on the rod in order it be displaced at constant speed v.
d) Verify that the power produced by the force computed on c) is lost on resistance as Joule heating.

8.2 Along an infinite straight carrying current conductor flows an intensity $I = Kt$ (K is a positive constant). A rectangular loop with sides a and b is placed in the same plane than the conductor, as can be seen on picture. Compute:

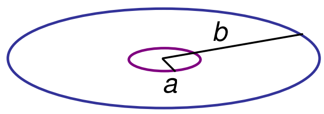


a) Induced e.m.f. on loop ε_i.
b) If the loop has a resistance R, compute the induced current i, showing its sense.
c) Magnetic force acting on side AB as a function of time, $\vec{F}(t)$.
d) Mutual inductance coefficient M between conductor and loop.

8.3 Compute the magnetic flux and the e.m.f. on the square loop of picture; this loop has an area S and it's turning at constant angular speed ω inside a uniform magnetic field B.

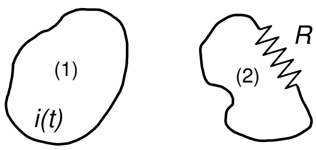


8.4 Two ring shaped loops, having radii a = 1 cm and b = 50 cm, are placed concentric in the same plane. If can be supposed that $a \ll b$ (magnetic field on loop a due to current on b can be supposed uniform) compute:



a) Mutual induction coefficient between both loops.
b) Magnetic flux across loop b when an intensity I = 5 A sized flows along a.

8.5 The mutual inductance coefficient between circuits on picture is M. If a current $i(t) = I_0\cos(\omega t + \phi)$ flows along circuit 1, ¿which is the intensity flowing along circuit 2?

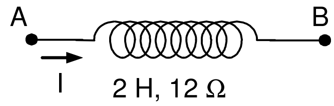


8.6 Let's consider two coaxial coils having the same length l but different cross section (S_1 and S_2) and different number of turns (n_1 and n_2). Coil 2 is placed inside coil 1 ($S_1 > S_2$).
a) By supposing an intensity of current is flowing along coil 1, compute the magnetic flux through coil 2 and then the mutual inductance coefficient between both coils.
b) By supposing an intensity of current is flowing along coil 2, compute the magnetic flux through coil 1 and then the mutual inductance coefficient between both coils.
c) Verify that mutual inductance coefficients computed on a) and b) matches.

8.7 Compute the self inductance coefficient of a coil having length *l*, cross section *S* and *n* turns.

8.8 Let's take a coil with a self-inductance coefficient $L = 2\ H$ and a resistance $R = 12\ \Omega$.

a) If an intensity of current $I=3t$ (t is the time) is flowing from A to B, compute the difference of potential V_A-V_B at time $t=1\ s$.

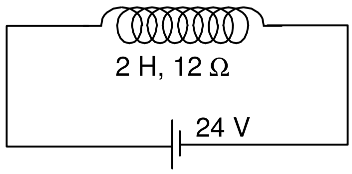


b) If an intensity of current $I=10\text{-}3t$ (t is the time) is flowing from A to B, compute the difference of potential V_A-V_B at time $t=1\ s$.

The coil is connected to an ideal generator with an electromotive force ε = 24 V (fig.(a)):

c) When the steady state is got, remaining constant the intensity of current, compute the intensity of current flowing along the circuit.

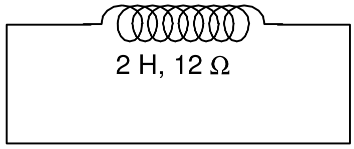
a)



d) Compute the stored energy on coil.

e) If the coil is short-circuited and generator is removed (fig. (b)) ¿Which is the lost energy as heating on coil due to its resistance?

b)



Unit 9: Alternating current

9.1 A resistor 5 Ω sized, an inductor 10 mH sized, and a capacitor 50 µF sized, are connected in series. The voltage on terminals of inductor is $u_L(t) = 100\,cos(1000t + \frac{\pi}{4})\ V$. Compute the intensity of current flowing along three dipoles, the voltage on terminals of resistor and capacitor.

9.2 A circuit is made up by two basic dipoles in series. The terminals of this circuit are connected to an A.C. generator giving a voltage $u(t)$ = 150 cos(500 t+ 10º) V, and flowing along the circuit an intensity of current $i(t)$ = 13,42cos(500t - 53,4º) A. Determine the two basic dipoles and their magnitudes.

Unit 10 Resonance and filters.

10.1 Represent on a graph the drawing of intensity of current, voltage on resistor, voltage on inductor, voltage on capacitor and voltage on terminals of a *RLC* series circuit having a resistor 5 Ω sized, a 10 mH inductor and a 100 µF capacitor. The amplitude of intensity is 10 A, initial phase of intensity can be taken zero, and as angular frequency must be used that corresponding to the resonant angular frequency.

10.2 Using the circuit of exercise 10.1, represent on a graph the active and reactive powers on resistor, inductor and capacitor, as well as their average values in resonance.

Unit 11: Semiconductor materials

10.1 Find the density of electrons and holes on Ge on following circumstances:

a) Pure Ge at 300 K (n_i (300 K) = $2{,}36\cdot10^{19}$ m^{-3})

b) At 300 K doped with Sb (antimonium) with a concentration of $4\cdot10^{22}$ m^{-3}

c) At 300 K doped with In (indium) with a concentration of $3\cdot10^{22}$ m^{-3}

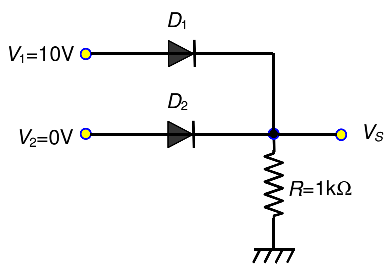
d) Pure Ge at 500 K (n_i (500 K) = $2{,}1\cdot10^{22}$ m^{-3})

e) At 500 K doped with Sb with a concentration of $3\cdot10^{22}$ m^{-3}.

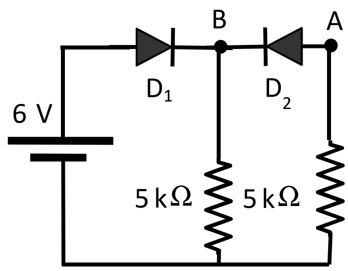
f) At 500 K doped with In with a concentration of $4\cdot10^{22}$ m^{-3}

Unit 12: Semiconductor devices

11.1 On circuit on picture, compute the voltage at output (V_s) as well as the intensities of current flowing along both diodes.
Diodes D_1 and D_2 are diodes built with Si (drop forward voltage V_u=0,7 V).



11.2 On circuit on picture both diodes have drop forward voltage V_u=*0,7 V*, and internal resistance can be neglected. Compute:



a) Intensities I_1 e I_2 flowing along diodes D_1 and D_2.
b) Potential difference V_A-V_B between terminals of diode D_2.