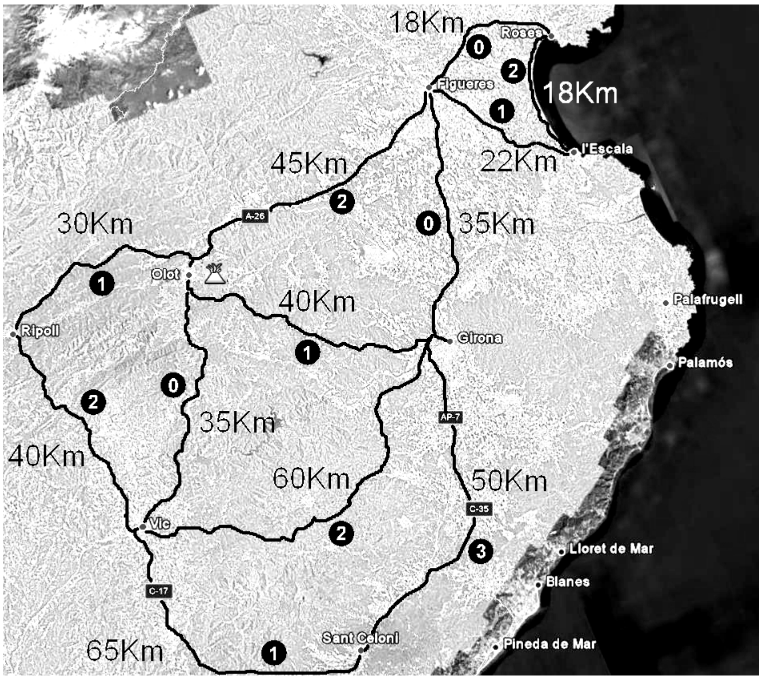
Discrete Mathematics (Lab)

Activities of session 1

September 4, 2017

Exercise 1. Let us consider the following road map (the distances are given in Km and the numbers inside circles denote the number of tolls):



1. Represent this road map (without distances and number of tolls) by means of a graph whose edges correspond with roads and whose vertices denote the cities and towns. Write a label for each vertex and each edge.
2. Describe the incidence map of this graph
3. Write its adjancency matrix
4. Write the degrees of the vertices.
5. Compare the sum of all degrees with the number of edges. Which is the relationship between both numbers?

Exercise 2. Is the following matrix the incidency matrix of a graph?

$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

Exercise 3. Represent a graph whose adjacency matrix is

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & 3 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}.$$

Exercise 4. Let $G = (V, A, \delta)$ the graph with $V = \{a, b, c, d, e, f, g\}$ and $A = \{a_1, \ldots, a_7\}$ and whose incidency map is given by:

$$\delta(a_1) = \{a, b\}, \quad \delta(a_2) = \{b, c\}, \quad \delta(a_3) = \{b, e\}, \quad \delta(a_4) = \{c, e\}$$
$$\delta(a_5) = \{d, f\}, \quad \delta(a_6) = \{e, g\}, \quad \delta(a_7) = \{f, g\}$$

1. Is this a simple graph? Is there a loop? Is there an isolated vertex?
2. Draw a representation of G.
3. Compute the adjacency and incidency matrices of G.

Exercise 5. Consider the graph given in Exercise 1 and weight its edges with the distances of the roads. Enter this weighted graph in *SWGraphs*. The algorithm *Dijkstra* (which appears in the menu bar of *SWGraphs*) computes a "shortest path" between two vertices of the graph (that is, a"path" between two vertices such that the sum of the weights is the minimum possible). Although in a subsequent practice we will study in detail this algorithm, we will use it here to give an answer to the following question: what is the shortest route between Sant Celoni and L'Escala? (Obviously this question can be answered, in this case, without using a computer; however, in the case of very large and complex graphs, it is necessary to have an algorithm to solve such type of problems).

Exercise 6. (*) In the same situation as above use the program *SWGraphs* to give an answer to the following questions:

(a) A traveler wants to go from Sant Celoni to L'Escala. It does not matter the number of kilometers, but he wants to go through the fewest possible number of tolls. Find a route with this condition.

(b) The same traveler wants to go from Sant Celoni to L'Escala so that the money that he spends in gas and tolls is the lowest possible. Taking into account that his car spends six liters of gas per 100 kilometers, that one liter of gas costs 1.5 euros and that in each toll he must pay 3 euros, compute the most economical route.