UNIVERSITAT POLITÈCNICA DE VALÈNCIA

Introduction to Lab Work and Probabilistic Reasoning

DSIC
Departamento de Sistemas Informáticos y Computación

Formative objectives

- To introduce the lab work
- To apply concepts and techniques from probabilistic reasoning

Contents

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1 Introduction to lab work: Octave

- Octave is a high-level language for numerical computations
- It can be used interactively or as a batch-oriented language
- Free open-source version of the commercial software Matlab
- Available at `http://www.gnu.org/software/octave`
- Reference manual
- Introduction to Octave with exercises for probabilistic reasoning
- To start an Octave session run from a terminal: `octave -q`

2 Probabilistic representation

The knowledge to diagnose cavity can be probabilistically represented with a joint probability distribution for the random variables.

Dentist's example: knowledge to diagnose cavity

Random variables of interest:

$$\begin{aligned} Toothache: &\quad T \in \{0,1\} \\ Catch^{1}: &\quad H \in \{0,1\} \\ Cavity: &\quad C \in \{0,1\} \end{aligned}$$

Representation:

$$P(T=t, H=h, C=c)$$

Table:

t	h	c	P
0	0	0	0.576
0	0	1	0.008
0	1	0	0.144
0	1	1	0.072
1	0	0	0.064
1	0	1	0.012
1	1	0	0.016
1	1	1	0.108
	Sum:		1.000

[1] The dentist's nasty steel probe catches in my tooth.

Dentist's table in Octave

t	h	c	P
0	0	0	0.576
0	0	1	0.008
0	1	0	0.144
0	1	1	0.072
1	0	0	0.064
1	0	1	0.012
1	1	0	0.016
1	1	1	0.108

Introduce Dentist's table in Octave:

```
T = [0 0 0 .576; 0 0 1 .008; 0 1 0 .144; 0 1 1 .072;
     1 0 0 .064; 1 0 1 .012; 1 1 0 .016; 1 1 1 .108];
```

Element in row 1, column 4:

```
T(1,4)
```

```
ans = 0.57600
```

Element in row 1, last column:

```
T(1,end)
```

```
ans = 0.57600
```

Elements from row 1 to 4 in the last column:

```
T(1:4,end)
```

```
ans = 0.5760000
      0.0080000
      0.1440000
      0.0720000
```

Elements (from all rows) in the last column:

```
T(:,end)
```

```
ans = 0.5760000
      0.0080000
      0.1440000
      0.0720000
      0.0640000
      0.0120000
      0.0160000
      0.1080000
```

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t	h	c	P
0	0	0	0.576
0	0	1	0.008
0	1	0	0.144
0	1	1	0.072
1	0	0	0.064
1	0	1	0.012
1	1	0	0.016
1	1	1	0.108

Elements from rows 1, 2, 5 y 6 in the last column:

```
T([1 2 5 6],end)
```

```
ans =
   0.5760000
   0.0080000
   0.0640000
   0.0120000
```

Sum over all elements in the last column:

```
sum(T(:,end))
```

```
ans =  1.00000
```

Indicators of rows with null elements in column 3:

```
T(:,3)==0
```

```
ans = 1
      0
      1
      0
      1
      0
      1
      0
```

Rows with non-null elements in column 2:

```
find(T(:,2))
```

```
ans = 3
      4
      7
      8
```

Rows with null elements in columns 2 and 3:

```
find(T(:,2)==0 & T(:,3)==0)
```

```
ans = 1
      5
```

3 Probabilistic inference

From a joint probability distribution we can compute the probability of any *event* (*proposition*) by applying:

Sum rule:

$$P(x) = \sum_y P(x, y)$$

Product rule:

$$P(x, y) = P(x)\, P(y \mid x)$$

In general, it is not necessary to know the full joint probability table to compute the probability of a given event.

t	h	c	P
0	0	0	0.576
0	0	1	0.008
0	1	0	0.144
0	1	1	0.072
1	0	0	0.064
1	0	1	0.012
1	1	0	0.016
1	1	1	0.108

Elements in last col. from rows with zero in cols. 2 and 3:

```
T(find(T(:,2)==0 & T(:,3)==0),end)
```

```
ans = 0.576000
      0.064000
```

Prob. of observing catch and cavity at the same time:

$$P(H=1, C=1) = \sum_{T=0,1} P(T, H=1, C=1) = 0.180$$

```
Ph1c1=sum(T(find(T(:,2)==1 & T(:,3)==1),end))
```

```
Ph1c1 =  0.18000
```

Probability of observing cavity:

$$P(C=1) = \sum_{T=0,1} \sum_{H=0,1} P(T, H, C=1) = 0.200$$

```
Pc1=sum(T(find(T(:,3)==1),end))
```

```
Pc1 =  0.20000
```

Probability of observing catch after observing cavity:

$$P(H=1 \mid C=1) = \frac{P(H=1,C=1)}{P(C=1)} = \frac{0.180}{0.200} = 0.900$$

```
Ph1Gc1 = Ph1c1/Pc1
```

```
Ph1Gc1 =  0.90000
```

Probability of observing toothache after observing catch:

$$P(T=1 \mid H=1) = \frac{P(T=1,H=1)}{P(H=1)} = \frac{0.124}{0.340} = 0.365$$

```
Pt1h1=sum(T(find(T(:,1)==1 & T(:,2)==1),end))
Ph1=sum(T(find(T(:,2)==1),end))
Pt1Gh1=Pt1h1/Ph1
```

```
Pt1h1  =  0.12400
Ph1    =  0.34000
Pt1Gh1 =  0.36471
```

4 Exercise: applying Bayes' theorem

Bayes' theorem allows to update our knowledge on a hypothesis y after observing a new evidence x:

$$P(y \mid x) = \frac{P(x, y)}{P(x)} = P(y) \frac{P(x \mid y)}{P(x)}$$

Alternatively: $P(y \mid x)$ is the probability of the effect y to happen after observing the cause x.

Exercise: compute the prob. of cavity after observing toothache:

$$P(C = 1 \mid T = 1) = \frac{P(T = 1 \mid C = 1) \, P(C = 1)}{P(T = 1)}$$