Unit 4

Energy and Power

Objectives

- To know the energetic effects of the electric current and the Joule heating.
- To know the behavior of generators and receptors on a circuit.
- Be able to prove the preservation of energy on a circuit.
- Compute the difference of potential between two points of a circuit.
- Be able to solve simple circuits.

4.1 Introduction

In this unit we are going to deal with the energetic effects of the electric current on a circuit. They are some devices releasing (supplying) energy to the circuit and other devices absorbing (consuming) energy from the circuit. By taking in account that the power is the rate of energy by unit of time, we'll be able to balance energy on a circuit, doing what we'll call a power balance.

4.2 Energy of the electric current. Joule heating

When Ohm's law was stated, we said that the electric current involves a fall of potential between two points, and therefore a lost energy on charge carriers. If we consider any device with an intensity of current I flowing along it, the charges dq entering on this device will have an energy dqV_1, and a lower energy dqV_2 when exiting from the device. The difference of energy is:

$$\Delta U = dq\,(V_2 - V_1)$$

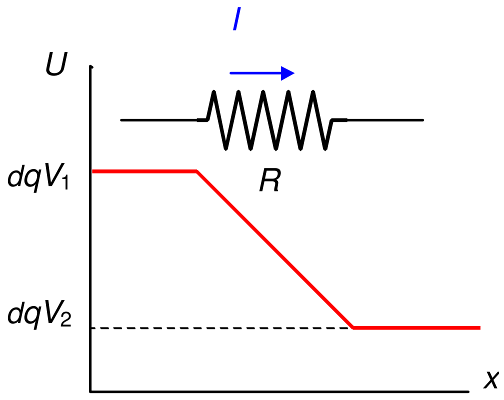


Figure 4-1. Lost energy on a resistor

The rate of lost energy by unit of time is the consumed power

$$P = \frac{dU}{dt} = \frac{dq}{dt}(V_2 - V_1) = IV$$

If the device we are considering is a resistor, by applying Ohm's law this equation becomes:

$$P = VI = I^2R = \frac{V^2}{R} \qquad \text{Equation 4-1}$$

We must remember that the power is measured in **watts** (W).

On a resistor, this consumed energy is converted in heat, and so heating the surroundings of resistor. This physical phenomenon of transformation of electric energy on heating energy is called **Joule heating**. This phenomenon can be sometimes undesirable, due to the loss of energy, but in other cases can be used to produce heating on a stove.

4.3 Generator

Let's imagine we have two conductors at different electric potentials, and we link both conductors through a conductor wire. As a consequence of the difference of potential and the electric field created inside the wire, a movement of electrons from conductor with lower potential to conductor with higher potential is produced, until both conductors reach the same potential and the electrostatic equilibrium is produced. When it happens, electric current stops. But to get a durable electric current is necessary to keep the difference of potential, avoiding the electrostatic equilibrium can be reached; it can be achieved with a device called generator. At this point, it's necessary to have in mind that when talking about electrical circuits, we always consider as positives the charge carriers, and so this positive charges will move from higher potentials to lower potentials.

A generator is a device keeping the difference of potential between their terminals, also called poles, and therefore, supplying the needed energy in order the charges can flow along the circuit.

A similar situation is produced on a water circuit. In the same way than a pump can take up the water giving it energy as a pressure, the generator gives energy to the charges increasing their electric potential. The pump takes water at low pressure giving water at high pressure, and the generator takes charges with low potential giving charges with high potential. If we call V_1 the electric

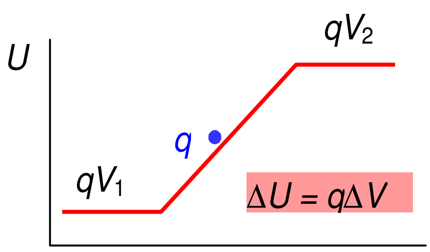
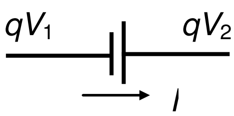


Figure 4-2

potential of a charge q entering on generator, its energy will be qV_1, and calling V_2, the potential of this charge when exiting from generator, its energy will be qV_2. The supplied energy to this charge is $q\Delta V$ (Figure 4-2).

On Figure 4- 3 can be seen the symbol for a generator on a circuit, where the vertical greater line corresponds to the high potential terminal (positive pole), and the shorter line to the low potential terminal (negative pole). Unlike it happen on a resistor, the intensity of current flows on a generator from negative pole to positive pole.



$$\Delta U = q\,(V_2 - V_1)$$

Figure 4-3. On a generator, the current goes out by the positive pole

There are different types of generators, depending on the kind of energy (primary energy) is converted to electric energy; for example, chemical type (batteries), mechanical (alternators and electric generators) or photovoltaic, among others. It is also necessary to note that an alternating current can be generated, as it occurs on an electric power station, both being hydroelectric, thermal, or nuclear. Nevertheless, in this unit we'll only study generators of direct current, also called power supply.

Generated energy is that energy the generator converts from primary energy into electric energy. This energy is used to increase the potential energy of electric charges are flowing through generator. The energy given to charges crossing generator by unit of charge is called ***electromotive force (f.e.m.),*** ε, of generator:

Electromotive force of a generator (f.e.m.) is the generated energy by unit of charge crossing through generator

$$\varepsilon = \frac{dU}{dq}$$

Equation 4-2

Equation 4-2 can be also defined in terms of power, by dividing numerator and denominator into the time dt the charge dq takes crossing the generator:

$$\varepsilon = \frac{\frac{dU}{dt}}{\frac{dq}{dt}} = \frac{P_g}{I} \Rightarrow P_g = \varepsilon I$$

where P_g is the generated power. The dimensions of electromotive force are that of electric potential and, therefore, its unit in the I.S. is the **volt** (V).

Linear generator

If any loss was produced inside a generator, all the generated power would be supplied to the external electric circuit, and so the generated power P_g would be equal to the **supplied power, P_s**. In this case, we would be talking about an **ideal generator**.

But any generator having physical existence shows loss inside the generator, and a heating of generator can be noted. This effect can be modeled supposing that inside the generator, we have a resistor loosing energy (Joule heating) when an intensity of current is flowing. This resistor is called **internal resistor r** of generator, and a generator having an internal resistor is called a **real generator** (Figure 4-4). A real generator is an ideal generator with a resistor in series, but really ideal generator and internal resistor can't be distinguished.

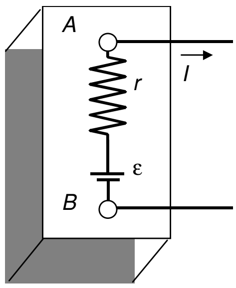


Figure 4-4 . Model of real generator

On a real generator, there is a drop of potential on internal resistor, and the difference of potential on terminals of real generator won't be ε (Figure 4-5). If I is the intensity of current:

$$V_A - V_B = -Ir + \varepsilon \qquad \text{Equation 4-3}$$

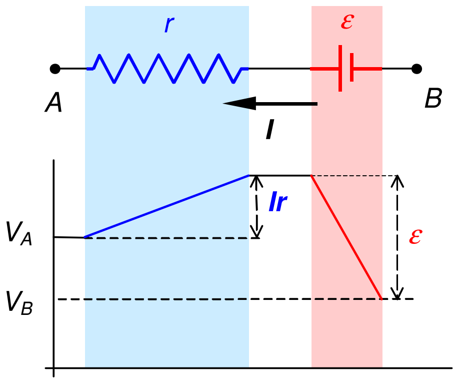


Figure 4-5. Drop of potential on a real generator. Electromotive force and d.d.p. on internal resistor.

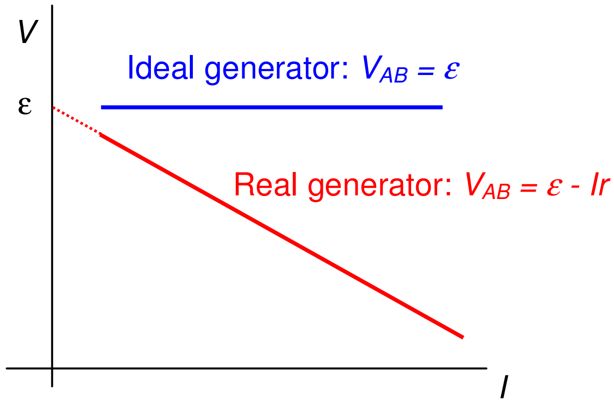


Figure 4-6. Characteristic curves for an ideal and real generator.

If ε and r are characteristic parameters of generator, the difference of potential between terminals is depending on the intensity of current flowing through generator, being a linear dependence between $V_A - V_B$ and I (drawing $V = V_A - V_B$ against I we get the Figure 4-6). But this situation is not produced on the whole running range of a generator, and for some values of intensity, there isn't linear behavior between voltage and intensity. In the range where both quantities are directly related, we can talk about a **linear generator**, being ε and r their characteristic parameters. Obviously, an ideal generator is a particular case of a real generator, with $r=0$. In this case, the difference of potential between terminals is always the electromotive force of generator: $V_A - V_B = \varepsilon$. The d.d.p. between terminals of generator also equals electromotive force ε when there isn't any device connected between terminals of generator (open circuit); it corresponds with the ordinate at the origin on Figure 4-6.

It will be also a value of intensity cancelling the d.d.p. between terminals of generator. In this case we say that generator is in shortcircuit (both terminals connected without resistance). This intensity is called intensity of shortcircuit, being the maximum intensity that can produce a generator:

$$\Delta V \to 0 \; ; \; I \to \; I_C = \frac{\varepsilon}{r}$$

A power balance on generator arises if we multiply by I the equation 4-3:

$$\varepsilon I - I^2 r = (V_A - V_B) I$$

First term in this equation, εI, is the generated power (P_g); second term, $I^2 r$, is the lost power on internal resistor (P_r); so, their difference $(V_A\text{-}V_B)I$ is the power supplied to the circuit connected between terminals of generator (P_s) , and this equation can be written as:

$$P_g - P_r = P_s$$

The consumed power on internal resistor can be seen as a undesirable effect, because it can't be made use of this power. So, the rate between power supplied to external circuit and generated power is called **efficiency of generator (η_g)** , being a dimensionless parameter:

$$\eta_g = \frac{P_s}{P_g} = \frac{\varepsilon I - r I^2}{\varepsilon I} = \frac{V_{AB}}{\varepsilon} \leq 1$$

Efficiency of a generator equals one only for an ideal generator, and it's is usually written as a percentage.

4.4 Receptor

A receptor is a device transforming electric energy taken from a circuit in any type of energy different than heat (electric energy is transformed on heat by resistors). A very common case of receptors are the engines, transforming electric energy in mechanics energy; but there are systems wasting energy for chemical uses, as storage battery or electrolytic cells, to name a few. A battery taking energy from circuit and storing it as chemical energy is also a receptor.

The characteristic parameter of any receptor is the **contraelectromotive force (f.c.e.m.) ε'**, defined as:

The contraelectromotive force of a receptor is the energy transformed by receptor on mechanical energy or other type of energy different than heat, by unit of charge flowing through receptor.

$$\varepsilon' = \frac{dU}{dq}$$

Equation 4-4

Equation 4-4 can be also defined in terms of power, by dividing numerator and denominator into the time dt the charge dq takes crossing the generator:

$$\varepsilon' = \frac{\frac{dU}{dt}}{\frac{dq}{dt}} = \frac{P_t}{I} \Rightarrow P_t = \varepsilon' I$$

where P_t is the transformed power. The dimensions of contraelectromotive force are that of electric potential and, therefore, its unit in the I.S. is the **volt** (V).

Linear receptor

If there wasn't any loss inside a receptor, all the consumed power by receptor from external circuit would be transformed, and so the transformed power P_t would be equal to the **consumed power, P_c**. In this case, we would be talking about an **ideal receptor**.

The symbol of a receptor is represented in the Figure 4-7. The intensity crossing the receptor always enters thorough the higher potential terminal, in such way the electric charges loos energy, falling down their electric potential. For this reason, a generator with the intensity flowing from positive to negative terminal behaves as a receptor. This device is the only can behaves both as a generator and a receptor, depending on the sense the intensity of current is flowing.

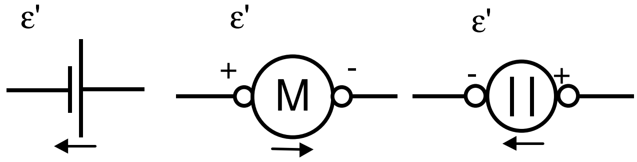


Figure 4-7. Symbols for receptors. In all cases the positive charges enter on receptor through the higher potential terminal, loosing energy.

As it happened on generators, all receptors are heated when they are running, being lost this energy. This behavior is modeled supposing that inside the receptor, we have a resistor loosing energy by Joule heating when an intensity of current is flowing. This resistor is called **internal resistor r'** of receptor, and a receptor having an internal resistor is called a **real receptor**. A real receptor can be represented as an ideal receptor with a resistor in series (Figure 4-8), but really ideal receptor and internal resistor can't be distinguished.

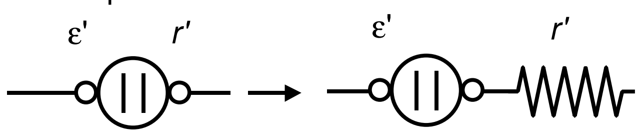


Figure 4-8. Model of real receptor

On a real receptor, there is a drop of potential on internal resistor, and the difference of potential on terminals of real receptor won't be ε', but a higher value (Figure 4-9). If I is the intensity of current:

$$V_A - V_B = Ir' + \varepsilon' \qquad \text{Equation 4-5}$$

If ε' and r' are characteristic parameters of receptor, the difference of potential between terminals is depending on the intensity of current flowing through receptor, being a linear dependence between $V_A - V_B$ and I (drawing $V = V_A - V_B$ against I we get the Figure 4-10). But this situation is not produced on the whole running range of a receptor, and for some values of intensity, there isn't linear behavior between voltage and intensity. In the range

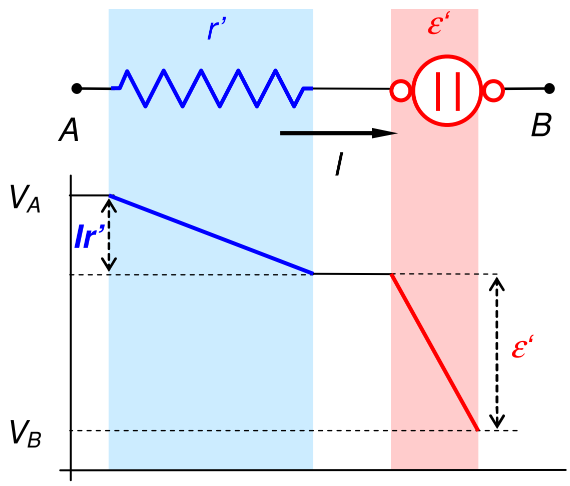


Figure 4-9. Drop of potential on a real receptor. Contraelectromotive force and d.d.p. on internal resistor.

where both quantities are directly related, we can talk about a **linear receptor**, being ε' and r' their characteristic parameters. Obviously, an ideal receptor is a particular case of a real receptor, with $r'=0$. In this case, the difference of potential between terminals is always the contraelectromotive force of receptor: $V_A - V_B = \varepsilon'$.

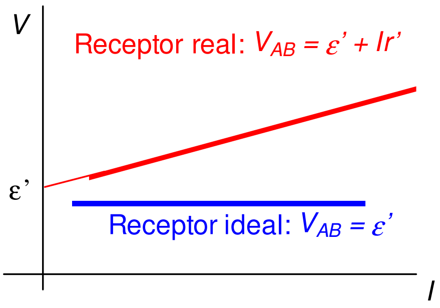


Figure 4-10 . Characteristic curves of an ideal and a real receptor

A power balance on receptor arises if we multiply by I the equation 4-5. By doing this operation:

$$\varepsilon' I + I^2 r' = (V_A - V_B) I$$

First term in this equation, $\varepsilon' I$, is the transformed power on any type of energy different than heat (P_t); second term, $I^2 r'$, is the lost power on internal resistor ($P_{r'}$); so, their addition $(V_A - V_B)I$ is the consumed power from the electric circuit connected between terminals of receptor (P_c) , and this equation can be written as:

$$P_c = P_t + P_{r'}$$

The power on internal resistor can be seen as a undesirable effect, because it can't be made use of this power. So, rate between power transformed and consumed power from external circuit is called **efficiency of receptor (η_r)** , being a dimensionless parameter:

$$\eta_r = \frac{P_t}{P_c} = \frac{\varepsilon' I}{(V_A - V_B) I} = \frac{\varepsilon'}{(V_A - V_B)} \leq 1$$

Efficiency of a receptor equals one only for an ideal receptor, and it's usually written as a percentage.

4.5 Difference of potential between two points of a circuit

Let's consider a piece of a circuit, with generators, receptors, and resistors, as it's drawn on Figure 4-11. I is the intensity of current flowing along this piece of circuit, from point A to point B. According the sense of I, device 1 behaves as a receptor (intensity is entering by positive terminal), device 2 behaves as a generator (intensity is entering by negative terminal), third device is a receptor (the symbol corresponds to a receptor) and so it positive terminal is that where the intensity is entering by; the last device is a resistor.

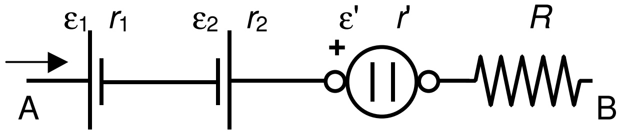


Figure 4-11. Piece of a circuit between points A and B, with generators, receptors and a resistor.

By using the rules for computing the difference of potential between terminals on generators, receptors and resistors, and taking in account in which way behaves each device, the difference of potential between points A and B becomes:

$$V_A - V_B = (Ir_1 + \varepsilon_1) + (Ir_2 - \varepsilon_2) + (Ir' + \varepsilon') + IR$$

This equation can be written in a general form as:

$$V_A - V_B = \sum_i IR_i - \sum_j \varepsilon_j \qquad \text{Equation 4- 6}$$

But in order to use this equation, some rules for signs must be obeyed:

1. $V_A - V_B$ represents the difference of potential between point A and point B. To calculate such difference of potential, the branch of circuit from A to B must be covered.

2. In the summation $\sum_i IR_i$, the intensity is positive if it goes from point A to B, and negative otherwise. It's very important to note that the main sense to compare any other quantity is the sense going from point A to B, but not the sense of intensity of current. R_i involves all the resistors between points A and B (also internal resistors of generators and receptors), being always positives.

3. $\sum_j \varepsilon_j$ involves all the electromotive and contraelectromotive forces of all generators and receptors being between points A and B. Sign of each ε_j will be positive or negative according the sense in which we cross the device going from A to B (here, the sign of ε_j is not depending on the sense of intensity of current). When we go out from a device through its positive terminal, ε will be positive, but when we go out from the device through its negative pole, ε will be negative. Moreover, a minus sign must be written on equation.

If we multiply the above equation by intensity of current I, we get an equation giving us a power balance on circuit:

$$(V_A - V_B)I = (I^2 r_1 + \varepsilon_1 I) + (I^2 r_2 - \varepsilon_2 I) + (I^2 r' + \varepsilon' I) + I^2 R = I^2 (r_1 + r_2 + r' + R) + \varepsilon_1 I - \varepsilon_2 I + \varepsilon' I$$

The term $I^2(r_1 + r_2 + r' + R)$ (positive) is the power lost on all resistors (consumed power); terms $\varepsilon_1 I$ and $\varepsilon' I$ (positives) are the consumed powers on generator 1 (behaves as a receptor) and receptor; $\varepsilon_2 I$ (negative) is the generated power on generator 2 (behaves as a generator). The difference between consumed and generated power in this piece of circuit is the power taken from the circuit connected between A and B $(V_A\text{-}V_B)I$.

An example of application of this rule can be seen on Figure 4-12. In order to compute the d.d.p. between A and B, V_A-V_B we must take in account that:

- Intensity is negative, because its sense is opposite to sense going from A to B.
- Resistance between A and B is R, because all the other devices are ideal devices (ideal generators and ideal receptors), no having any internal resistor.
- Positive terminal on receptor is terminal to right, because on a receptor, the intensity must necessarily enter through positive terminal. As we go out from this device through positive terminal when we move from A to B, then ε' is negative.

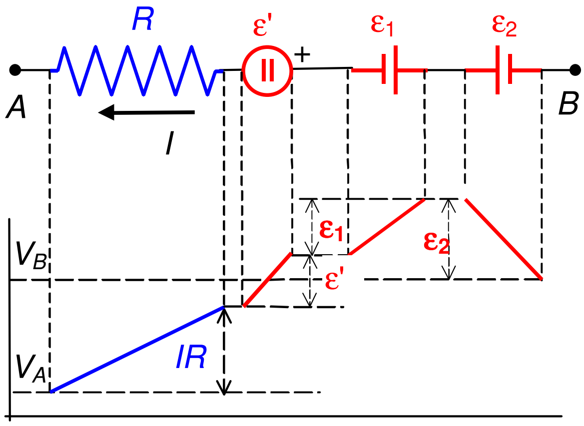


Figure 4-12. Difference of potential between two points on a circuit.

- ε_1 is positive because we go out through positive terminal on this device when going from A to B.
- ε_2 is negative because we go out through negative terminal on this device when going from A to B.

In this way: $$V_A - V_B = -IR - (\varepsilon' + \varepsilon_1 - \varepsilon_2) = \varepsilon_2 - \varepsilon' - \varepsilon_1 - IR$$

On Figure 4-12 are represented all the drops of potential on each device.
We could have computed d.d.p. between B and A, obtaining a result opposite to d.d.p. between A and B; now, intensity would be positive, ε_1 and ε' negatives, and ε_2 positive:

$$V_B - V_A = IR - (-\varepsilon' - \varepsilon_1 + \varepsilon_2) = -(V_A - V_B)$$

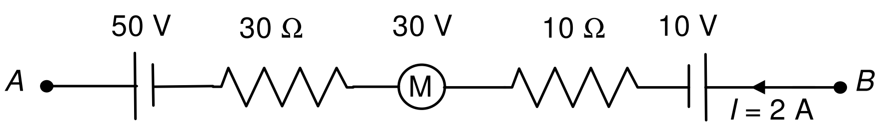
If we perform a power balance on this circuit:

- generated power: $\varepsilon_2 I$
- consumed powers:
  - On resistors: I^2R
  - On f.c.e.m: $(\varepsilon_1 + \varepsilon')I$
  - Consumed power on external circuit connected between A and B: $(V_A - V_B)I$.

So: $$\varepsilon_2 I = I^2 R + (\varepsilon' + \varepsilon_1)I + (V_A - V_B)I$$

Example 4-1

Compute the difference of potential between points A and B on branch of the figure. Is the branch consuming or supplying energy to the rest of the circuit?



Solution

Applying Equation4-6 , becomes:

$$V_{AB} = V_A - V_B = \Sigma IR - \Sigma\varepsilon = -2(40) - (-50 + 30 + 10) = -80 + 10 = -70 \text{ V}$$

As $V_B > V_A$, a positive charge moving from *B* to *A* along this branch will lose energy, energy that will be consumed on different devices. This energy is supplied by the rest of circuit between *A* and *B*. So, this branch consumes energy from the rest of circuit.
The consumed power by this branch can be easily computed by doing a power balance on it, taking in account the 50 V generator is the only device acting as a generator:

- Generated powers: 50x2=100 w
- Consumed powers: 2^2x(30+10)+10x2+30x2=240 w

As consumed power in this branch is higher than generated power, the difference 240-100=140 w must be supplied by the rest of circuit connected between *A* and *B*. Moreover, this supplied power could be easily computed multiplying the difference of potential between B and A by the intensity of current: 70x2=140 w.

4.6 Equation of the circuit

A simple circuit is a set of devices connected in such way that there is only one intensity of current flowing along it. On such circuits it's possible to compute the intensity of current. Let's suppose the circuit on Figure 4-13, and let's choose two points *A* and *B*, directly joined by a wire without resistance; by this reason, the difference of potential V_{AB} between them is zero. Let's suppose an intensity of current *I* flowing from *A* to *B* along the long way (clockwise sense), as can be seen on Figure 4-13. The d.d.p. V_{AB} will be:

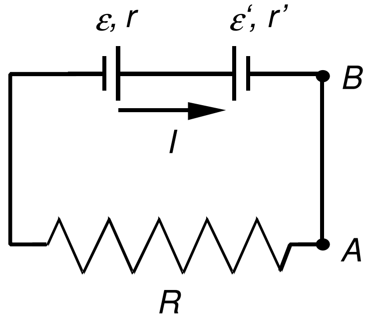


Figure 4-13. Simple circuit with a generator, a receptor and a resistor

$$V_A - V_B = I(R + r + r') - (\varepsilon - \varepsilon') = 0 \Rightarrow I = \frac{\varepsilon - \varepsilon'}{R + r + r'}$$

This equation can be written in general form as: $I = \dfrac{\sum \varepsilon}{\sum R}$ Equation 4-7

When applying Equation 4-7, a sense for intensity of current must be previously supposed, being electromotive and contraelectromotive forces positives or negatives according this supposed sense for intensity. If there is any receptor on circuit, its polarity comes from the supposed sense for the intensity (the positive pole is that where intensity is entering through). All resistances, both internal as external, are positives.

If the computed intensity of current results negative, it means that the supposed sense for intensity was wrong, and computations must be repeated as it's below explained.

About the power on different devices of circuit, the only device supplying power to the circuit is the generator of electromotive force ε, acting all the other devices as power consumers, both on resistors as charging the battery of con-traelectromotive force ε' (transforming electric energy on chemical energy). On Figure 4-14, a power diagram can be seen, corresponding to the circuit on Figure 4-13:

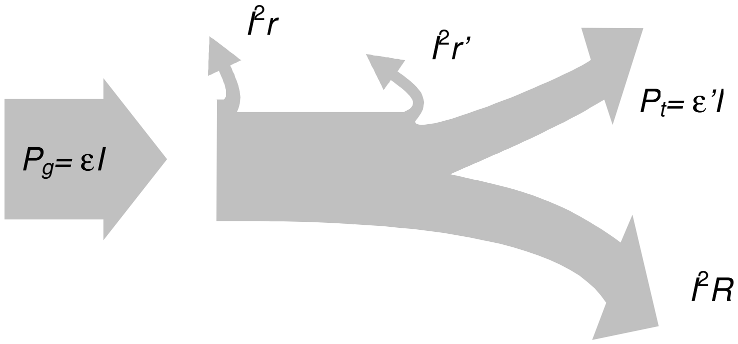


Figure 4-14. Drawing of a power balance

Obviously, as power must be preserved, generated power must equal consumed power:

$$\varepsilon I = I^2 r + I^2 r' + \varepsilon' I + I^2 R$$

Determination of the sense of the intensity

Usually, solving a circuit involves to compute the intensity of current when are known the components of circuit. Sometimes, the batteries indicates clearly which is the sense of intensity, but in other cases (when they are batteries and receptors) can't be evident the sense for intensity.

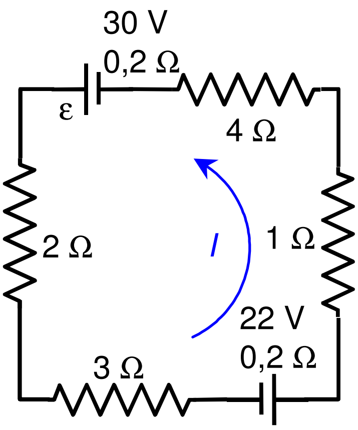
In case of doubt on the sense of the intensity of current it's necessary to proceed as it's explained:

1. Suppose an arbitrary sense for intensity.
2. By using equation 4-7, compute the intensity of current, verifying if it's positive or negative.
3. If result is positive, the sense initially supposed is correct and the problem is solved.
4. If intensity is negative, the sense initially supposed in point 1 is not correct. Then, we must change to the opposite sense.
5. We compute again the intensity, checking its sign.
6. If intensity is positive, the sense we have supposed on point 4 is correct and the problem becomes solved.
7. If intensity is negative, this sense we have supposed isn't correct neither. So, any of senses for intensity are possible, which indicates that there is not current in the circuit: the intensity is zero. It can happen in case where there is some engine on circuit, not having the batteries enough power to make work the engine.

It's necessary to note that in **absence of receptors**, if intensity is negative, can be deduced that intensity goes in opposite sense than supposed but with the same absolute value. This is due to the fact that Equation 4-7 is symmetrical (the change of sense for intensity equally affects the sign of both terms) in absence of receptors. The presence of a receptor breaks this symmetry and forces to repeat again the computations after change the sense of intensity.

Example 4-2

Compute the intensity of current flowing along the circuit on picture.



Solution

In this circuit there isn't any receptor, and both generators are connected in such way that their effects are added. So, the sense for intensity will be as shown on picture. This intensity can be got by applying equation of circuit.

$$I = \frac{30+22}{10{,}4} = 5\text{ A}$$

Example 4-3

Given the circuit of figure, answer the following questions:

a) Compute the magnitude and sense of intensity flowing along circuit.

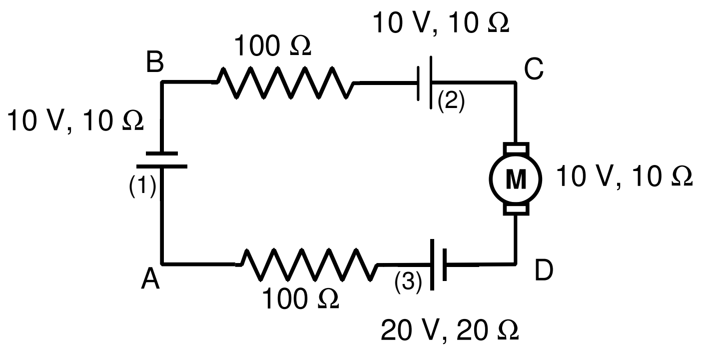
b) Compute the difference of potential between points A and C (V_A-V_C), both along path *ABC* as along path *ADC*.

c) Which devices supply energy to the circuit? Compute the value of supplied power by each device.

d) Which devices consume energy from circuit? Compute the value of consumed power by each device.

e) Which are efficiencies of engine and power supply (2)?

f) If we modify the electromotive force of power supply (1), which must be its magnitude for the difference of potential between points A and C equals zero? Which is the intensity on circuit in this case?



Solution:

a) According the polarities of power supplies, can be expected that the intensity flows in clockwise sense; computing its magnitude:

$$I = \frac{\sum(\varepsilon_i - \varepsilon'_i)}{\sum R_i} = \frac{30-20}{250} = 0{,}04 \text{ A}$$

As $I > 0$, the supposed sense for intensity is correct.

If we had supposed counterclockwise sense, the intensity would be:

$$I = \frac{\sum(\varepsilon_i - \varepsilon'_i)}{\sum R_\mathrm{i}} = \frac{-30-10+10}{250} = -0{,}12 \text{ A}$$

And so, we should to compute again the intensity by changing the sense for intensity, as we have already done. Note that absolute values of both intensities are not equal, due to the engine.

b) Computing V_A-V_C along path *ABC*:

$$V_A - V_C = I_{AC}\sum R_{AC} - \sum(\varepsilon_{AC} - \varepsilon'_{AC})$$

I_{AC} is the intensity going from *A* to *C* (through B), with the corresponding sign (positive because the computed intensity flows from *A* to *C*). Then:

$$V_A\text{-}V_C = 0{,}04(10+100+10) - (-10 + 10) = 4{,}8 \text{ V}$$

Computing the d.d.p. between *A* and *C* through point *D*, and being clockwise the supposed sense, the intensity flows from *C* to *A*, and so the positive pole of engine (receptor) will be the upper pole (on a receptor, the intensity must go out through negative pole), So:

$$V_A - V_C = -0{,}04(20+100+10) - (-20 + 10) = 4{,}8 \text{ V}$$

The same value previously found.

c) Generators are the only elements capable to supply energy to the circuits. To do it, positive charges must enter on generator through negative pole, and exiting through positive pole, increasing in this way the energy of charges. This energy supplied by the generator will be after consumed by the different devices on circuit. In this exercise, as intensity flows in clockwise sense, devices (2) and (3) act as generators, but (3) is acting as a receptor.

The power supplied to the circuit for each generator will be the result to subtract the lost power by Joule heating on internal resistor to the generated power:

$$P_s = P_g - P_r = \varepsilon I - rI^2$$

On generator (2):

$$P_s = 10\cdot 0{,}04 - 10\cdot 0{,}04^2 = 0{,}384 \text{ W}$$

On generator (3):

$$P_s = 20\cdot 0{,}04 - 20\cdot 0{,}04^2 = 0{,}768 \text{ W}$$

The total power supplied to the circuit is the sum of both values = 1,152 W

d) Devices consuming energy are all resistors, the engine and the generator (1):

- On each resistor:

$$P_R = RI^2 = 100 \cdot 0{,}04^2 = 0{,}16 \text{ W}$$

- On engine:

$$P_{ce} = P_t + P_r = \varepsilon' I + rI^2 = 10 \cdot 0{,}04 + 10 \cdot 0{,}04^2 = 0{,}416 \text{ W}$$

- On generator (1):

$$P_{cg} = P_t + P_r = \varepsilon I + rI^2 = 10 \cdot 0{,}04 + 10 \cdot 0{,}04^2 = 0{,}416 \text{ W}$$

The total power consumed on circuit (without considering the internal resistors of generators acting as generators), is:

$P_c = 2P_R + P_{ce} + P_{cg} = 2 \cdot 0{,}16 + 0{,}416 + 0{,}416 = 1{,}152$ W

This value equals the power supplied to the circuit. The power balance equals the supplied and consumed power on a circuit.

e) Efficiency of a receptor is the rate between the transformed power and the total consumed power:

$$\eta = \frac{P_t}{P_{c_e}} = \frac{\varepsilon' I}{\varepsilon' I + r I^2} = \frac{0{,}4}{0{,}416} = 0{,}962$$

Giving the result in %: $\eta = 96{,}2$ %

From 100 watts consumed by the engine, 96,2 are transformed on mechanical power, losing the rest by warming.

On the other hand, efficiency of a generator is the rate between the supplied power to the circuit and the generated power:

$$\eta = \frac{P_s}{P_g} = \frac{\varepsilon I - r I^2}{\varepsilon I} = \frac{0{,}384}{0{,}4} = 0{,}96$$

Given in %: $\eta = 96$ %

Then, from 100 watts generated by the source (2), 4 watts are lost by Joule heating in its internal resistor.

f) The presence of the engine gives the possibility that they exist two different solutions giving V_A - V_C = 0, since the equation determining this difference of potential along the path *ADC* is different depending on the sense of intensity in the circuit.

If we consider that the intensity keeps its clockwise sense, the d.d.p. $V_A - V_C$

along path *ADC* will be:

$$V_A - V_C = -130\ I - (-20 + 10) = 0$$

Becoming

$$I = \frac{1}{13}\ \text{A}$$

As intensity is positive, the supposed sense is correct. So, the needed electromotive force of generator (1) can be computed writing the equation for d.d.p. between *A* and *C* through path *ABC*:

$$V_A - V_C = 120\frac{1}{13} - (-\varepsilon + 10) = 0$$

Resulting

$$\varepsilon = 10 - \frac{120}{13} = \frac{10}{13}\ \text{V}$$

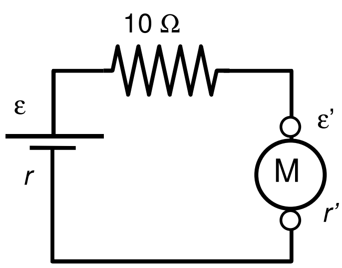
Example 4-4

The engine of the circuit in picture consumes 20 W, being a 10 % by Joule heating. The generator supplies 60 W to the circuit. Compute:

a) consumed power on 10 Ω resistor.

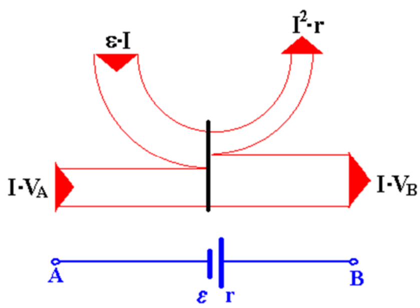
b) ε and r, knowing that generator generates 64 W.

c) ε' and r'.



Solution:

Before solving this problem, we are going to remember some questions about power on electric circuits and their devices:

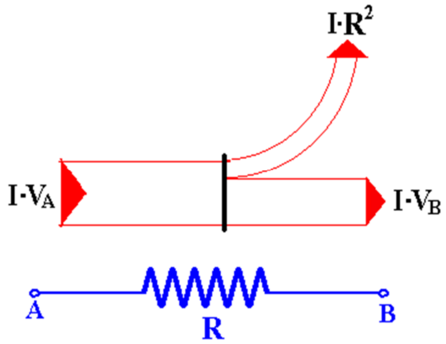


The generator is the only device capable to supply energy to the circuit. The energy that generates by unit of time, the generated power, is proportional to the intensity flowing along the circuit: $P_g = \varepsilon I$. A part of this energy is lost as heat in the own generator ($I^2 r$), and the rest is supplied to the circuit:

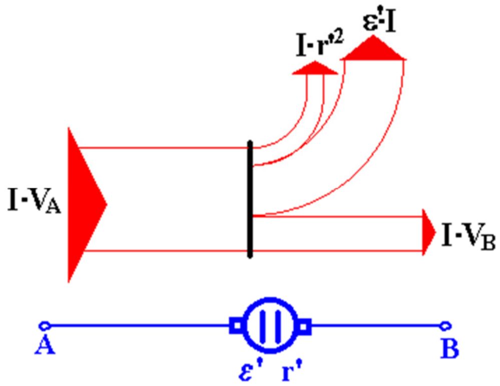
$$P_r = I^2r;\quad P_s = IV_B - IV_A = \varepsilon{\cdot}I - I^2r$$

On a resistor the charges always lose energy, according Joule's law, being the lost power:

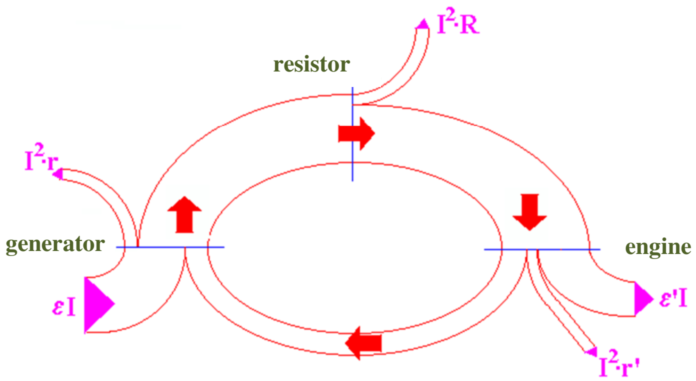
$$P_R = I^2R$$



Finally, on engines, a part of the energy of electric charges is transformed in mechanical energy ($\varepsilon' I$), and another part is lost as heating in the own engine (I^2r'). Therefore, the consumed energy by the engine by unit of time (the power) is the sum of the transformed plus the lost by Joule heating in its internal resistance: $P_c = \varepsilon' I + I^2r'$



In this way, a circuit can be considered as a system where exchanges of energy are continuously produced, and can be represented using the following power diagram:



Taking in account all these considerations, we can already solve this problem.

On engine, consumed power (20 w) is used in producing mechanical power (90%) and in losses by Joule heating on internal resistor (10%)

$\varepsilon' I + I^2 r' = 20$ W 10 % (20 W) = 2 W = $I^2 r'$ 18 W = $\varepsilon' I$

The power supplied by generator to the circuit (60 W) will be equal to the consumed power on engine (20 W) plus the lost power due to Joule heating in resistor R:

$$60 \text{ W} = 20 \text{ W} + I^2R \rightarrow P_R = I^2R = 40 \text{ W} = I^2 \cdot 10 \rightarrow I = 2 \text{ A}$$

From the 64 W generated by generator, 60 W are supplied to the circuit and the rest are lost by Joule heating in the own generator:

$$\varepsilon I = 64 \text{ W} = 60 \text{ W} + I^2 r$$

Now, we already can solve all the points of problem:

a) $P_R = I^2R = 40$ W

b) $\varepsilon I = 64 \text{ W} \rightarrow \varepsilon = 32$ V $I^2 r = 4 \text{ W} \rightarrow r = 1\ \Omega$

c) $\varepsilon' I = 18 \text{ W} \rightarrow \varepsilon' = 9$ V $I^2 r' = 2 \text{ W} \rightarrow r' = 0{,}5\ \Omega$

4.7 Problems

1. A 10 Ω resistor can dissipate up to 5,0 W without damaging.

a) What is the maximum intensity can tolerate this resistor?

b) Which voltage between its terminals will produce this current?

Sol: a) 0,707 A b) 7,07 V

2. If the energy costs 8 cents by kilowatt–hour, how much will cost to do working a computer along 4 hours if its resistance is 120 Ω and it's connected to a voltage of 220 V?

Sol: 12,91 cents

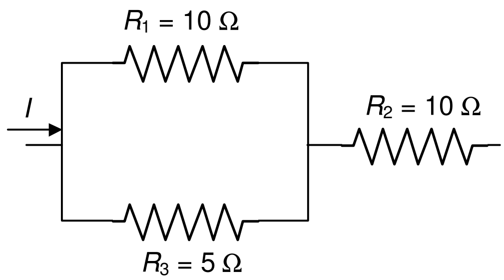
3. In the circuit of figure, compute:

a) Which resistor dissipates higher power due to Joule heating?

b) Which resistor dissipates lower power due to Joule heating?

Justify the answers.

Sol: a) R_2 b) R_1

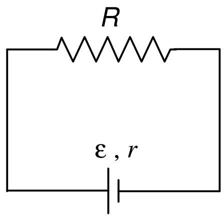


4. Two equal resistors are connected in series to a voltage V. Later, they are connected in parallel and the same voltage V is applied. In which of the two settings is dissipated a lower power?

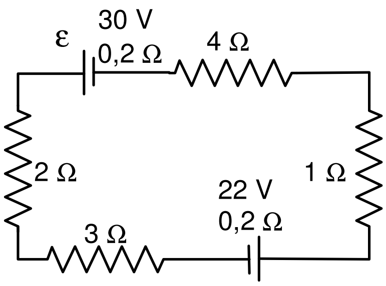
Sol: $P_s < P_p$

5. A variable resistor of resistance *R* is connected to a generator of electromotive force ε. For a value of R = R_1 the current is 6 A. When *R* increases until $R = R_1 + 10\ \Omega$, the current falls until 2 A. Find: a) R_1, b) ε.
Sol: a) 5 Ω, b) 30 V

6. A battery has an electromotive force ε and an internal resistance *r*. When it's connected a resistance of 5 Ω between their terminals, the current is 0,5 A. When resistor is changed by a 11 Ω resistor, the current is 0,25 A. Find: a) The electromotive force ε and b) the internal resistance *r*.
Sol: a) 3 V, b) 1 Ω



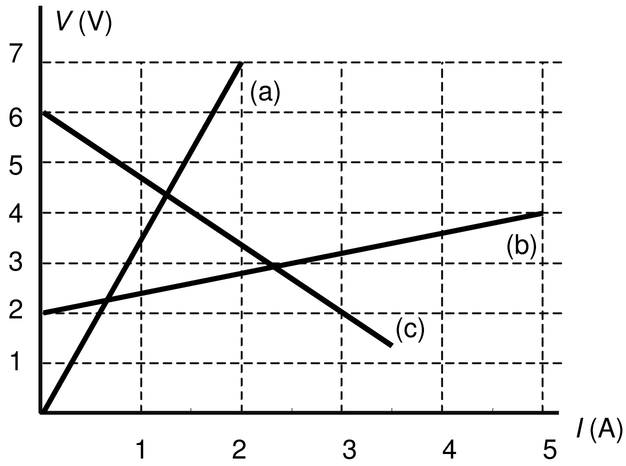
7. In the circuit of picture, $\varepsilon = 6$ V and $r = 0{,}5\ \Omega$. Lost power by Joule heating in *r* is 8 W. Find: a) the intensity of current, b) the difference of potential between terminals of *R*, c) *R*.
Sol: a) 4 A, b) 4 V, c) 1 Ω



8. Find the difference of potential between terminals of generator ε.
Sol: 29 V

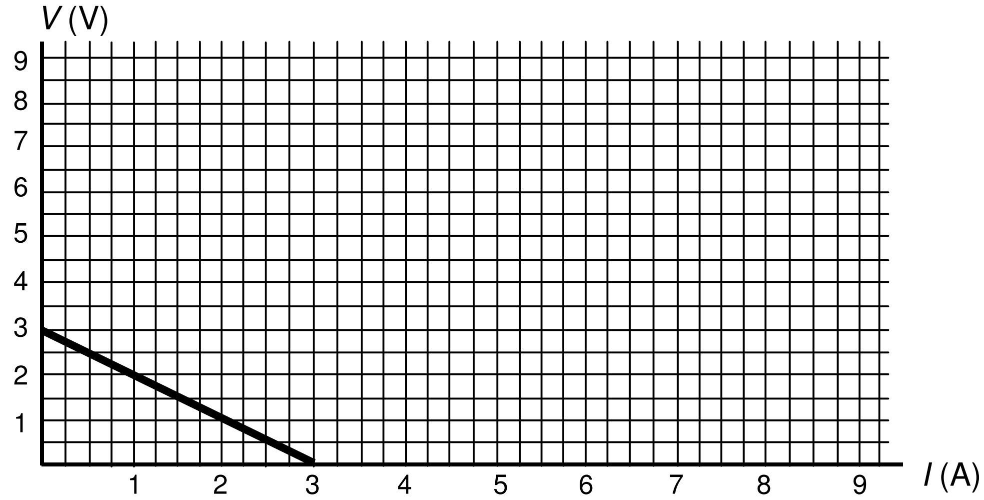
9. A resistor *R* is connected to a generator of electromotive force ε and internal resistance *r*. Which should be the value of *R* in order the lost power on this resistor was maximum?
Sol: $R = r$

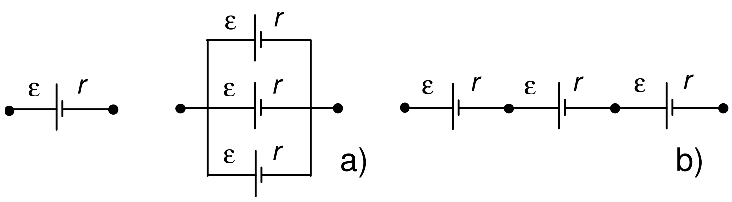
10. On picture are represented the curves intensity-voltage corresponding to different devices of a d.c. circuit. Identify each curve corresponds to each device and determine the characteristic parameters of each device.



	Curve	Parameters
Generator		
Receptor		
Resistor		

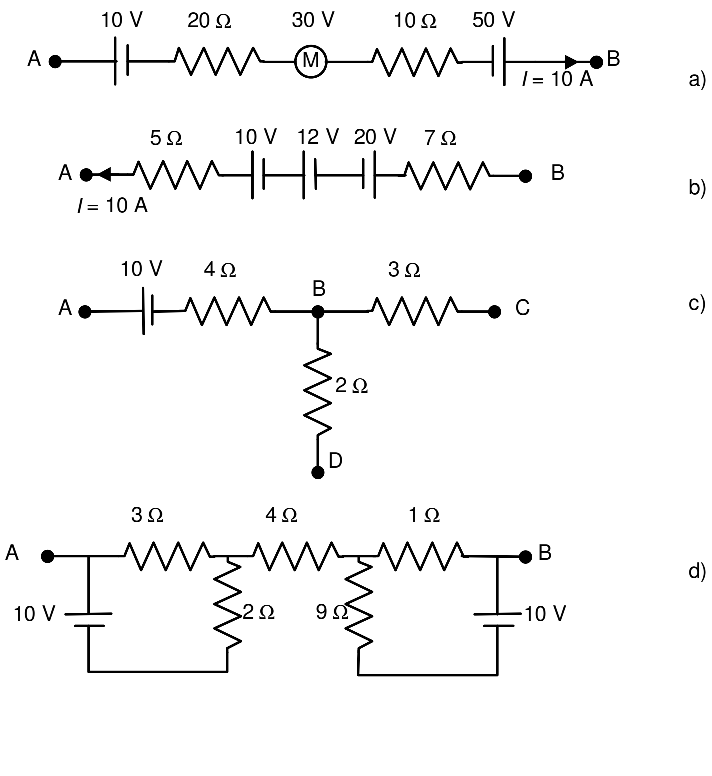
11. On picture is represented the curve intensity-voltage for a generator. In the same picture, draw the curves corresponding to three generators equal to the previous generator: a) connected in parallel, b) connected in series.





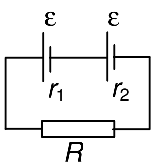
12. Compute the difference of potential between points *A* and *B* on next pictures:

Sol: a) 290 V, b) -118 V, c) 10 V, d) 5 V

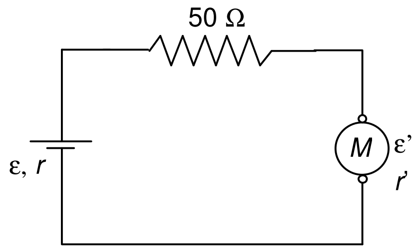


13. A set of N identical generators with electromotive force ε and internal resistance *r* are connected in series, closing the circuit with a wire without resistance. Compute: a) intensity of current flowing along the circuit, b) difference of potential between two anyone points *j* and *k*.
Sol: a) ε/r , b) 0

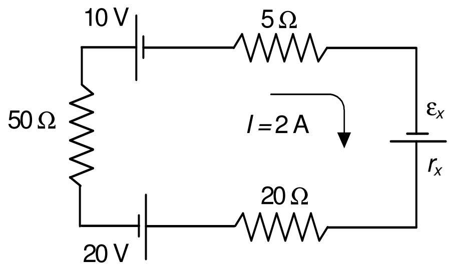
14. Given the circuit of figure, being $r_1 > r_2$, compute the value of R for the difference of potential between terminals of one generator was zero. Say in which generator it happens.
Sol: $R = r_1 - r_2$. In generator 1.



15. The engine of the drawn circuit consumes 50 W, being a 20% by Joule heating. If the generator supplies 100 W to the circuit, compute: a) consumed power on 50 Ω resistor b) If the generator generates a power of 110 W, compute their characteristic parameters ε and *r.* c) the characteristic parameters of engine ε' and r'.
Sol: a) 50 W, b) 110 V, $r = 10$ Ω, c) 40 V, 10 Ω.



16. Along the circuit on picture an intensity of current $I = 2$ A, in the shown sense, is flowing. Efficiency of generator ε_x is 80%. Find values of ε_x and R_x.
Sol: $\varepsilon_x = 225$ V and $R_x = 22{,}5$ Ω.



GLOSSARY

Joule heating. Dissipation of energy, as heating, produced when a current flows along a conductor.

Electromotive force of a generator is the energy produced by generator by unit of electric charge.

Linear generator. Device that supplies energy to a circuit, producing a difference of potential between their terminals linearly decreasing with the intensity of current, as:

$$V_A - V_B = \varepsilon - Ir$$

Efficiency of a generator is the rate between the power supplied to the circuit and the power generated.

Receptor is a device transforming electric energy in other forms of energy different than heat.

Contraelectromotive force of a receptor is the energy transformed by the receptor in mechanical energy or other forms of energy different than heat, by unit of charge flowing along the receptor.

Efficiency of a receptor is the rate between the transformed power and the consumed power.