Unit 3

Direct current and electric resistance

Objectives

- Define intensity of electric current, drift speed, density of current and resistance.
- State Ohm's law.
- Define the resistivity, and to know its dependence on temperature.
- Compute the equivalent resistance of associations of resistors.
- To know the energetic effects of the electric current and the Joule heating.

3.1 Introduction

Until now, on electrostatics, we have studied the electric charges at rest. In this unit we'll begin to analyse the phenomena related with the trip of electric charges between two points, what is defined as *electric current*. An electric current can be produced when the electrostatic equilibrium is not allowed inside a conductor, avoiding this equilibrium can be reached.

They exist different ways to produce the movement of electric charges. The most known case is that of the conductor metals, but it is not the only: in a tube of cathode rays of a T.V. screen there is a movement of electric charges, in this case through the vacuum; also in a conductive dissolution like sodium chloride in water or through a nerve, a movement of charges can occur.

However, from a technological point of view, the more important case is the movement of electric charges on metallic conductors. In this case, the electrical conduction is due to a difference of potential between two points, or an electrical field, what produces a force, and a trip of the electric charges.

3.2 Direct current and alternating current

The electric current can be, according its evolution on time:

- Direct (DC), if the different quantities related with the current (potential, intensity…), remain invariable on time, so in magnitude as in sense. This type of current is produced by the electrochemical generators (batteries, or power supplies) photovoltaic devices, or rectifier circuits.
- Alternating (AC), if the different magnitudes related with the current periodically evolve on time, changing alternatively of sense. A particular case is the sinusoidal alternating current that varies on time in accordance with a sinusoidal function. This is the usual current for domestic use, and it's produced by generators of alternating current running under the electromagnetic induction bases.
- General variable current. This is the case of any temporary variation of electrical magnitudes; for example, the current arriving to a speaker.

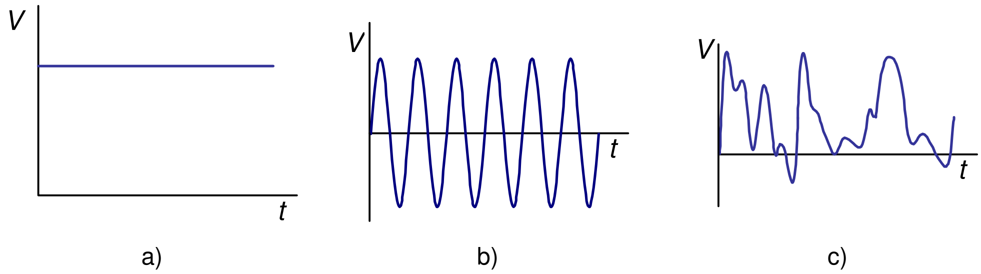


Figure 3-1. Difference of potential as a function of time between two points of a circuit (a) direct current, b) sinusoidal alternating current, and c) variable current

In this unit we'll deal with direct current, although almost all the topics can be extrapolated to any kind of current.

3.3 Current and movement of charges

As we told on section 2.2, the mobile particles on metallic conductors are free electrons. These free electrons are shared by the group of ions on metal, through a type of linking between the different atoms, known as metallic bond. In this way, in a metal there is a big quantity of free electrons, (around 10^{29} e/m^3), chaotically moving as a consequence of thermal agitation. The metal always remains electrically neutral, as it does not experience loss of electrons.

When applying an electric field to conductor, or what it is the same, a difference of potential (d.d.p.), each charged particle will be subjected to an electric force as:

$$\vec{F} = q\vec{E}$$

Drift speed

If the charges were on vacuum, this force would produce an acceleration that would promote them to high speeds, such as it occurs on a vacuum tube of a T.V. screen. However, on a metallic conductor, an orderly atomic network exists, more or less interfering in this movement; a similar situation can be seen on Figure 3.2: the marbles are subjected to the acceleration of gravity and bump with the nails, in each crash losing the kinetic energy that the gravitational field gave them. In the same way, the accelerated electrons due to the electric field lose a part of the speed as a consequence of the crashes or interactions with the solid network. The lost kinetic energy due to these interactions produces a warming in the conductor that we'll carefully study later (Joule heating). This increasing on speed due to the electric forces is cancelled by the crashes with the atomic cores of solid network, reaching the free electrons an average speed called **drift speed**. This speed is very low, of the order of only a few mm by minute (see example 3-1).

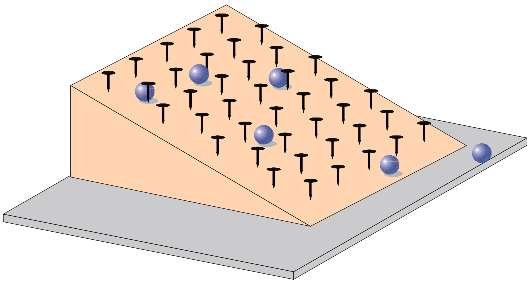


Figure 3-2.Mechanical analogy of movement of free electrons in a conductor

3.4 Intensity and density of current

The magnitude quantifying the greater or lower electric charge flowing along a conductor is the intensity of current. In the same way it is useful and necessary quantify the flow of water carrying a river, and do it expressing the volume of water that crosses by second a cross section to the movement of the water, we define the **intensity of the electric current** as *the electric charge flowing through a cross section of conductor by unit of time:*

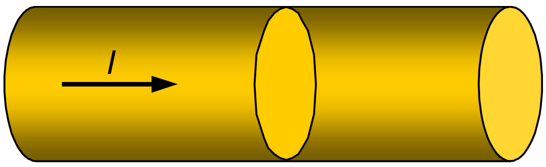


Figure 3-3. Intensity of current

$$I = \frac{dQ}{dt} \qquad \text{Equation 3-1}$$

The intensity of current is a base quantity on the IS, being its unit the **ampere** (A). Its definition will be given on a next unit, but from equation 3-1, can be deduced that 1C= 1A·1s.

Although the electric current is owed to the movement of electrons, by historical reasons, the sense of the electrical current has always been adopted as a flow of positive charges. That is, will consider the intensity of current as if it was been due to movement of positive charges. This apparent confusion doesn't must carry consequences if we take in account that the movement of particles with negative charge in a sense is equivalent to the movement of positive particles in opposite sense. In this way, always we are dealing with conductor materials, we'll consider the sense of the electric current opposite to the movement of the electrons. This equivalence won't be true for semiconductor materials, and then we'll need study it with more detail.

On the other hand, in this study we'll consider that the cross section of conductor softly varies along the conductor. Then, when applying an electric field to an homogeneous conductor (or a d.d.p. between the ends of conductor), we can suppose that both the electric field applied as the speed of the electric charges are perpendicular to the cross section. And all points of any cross section have the same electric potential.

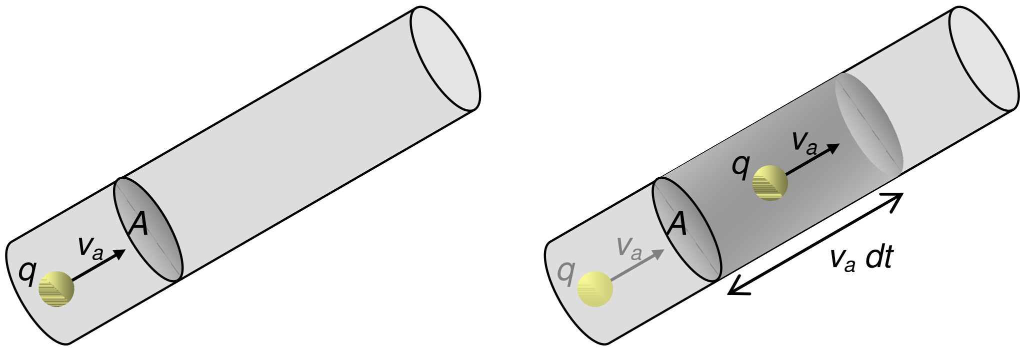


Figure 3-4

In order to get some applicable results to any material and type of current, let's consider a piece of conductor with an electric current flowing along it, as it appears on Figure 3-4 4 (a), of cross section *A*. After an interval of time *dt* the electric charges have moved a distance equal to its drift speed multiplied by the time. So, the dark volume on Figure 3-4 4 (b) corresponds to the occupied volume by the charges have crossed *A* along a time *dt*. The quantity of charge, *dQ*, that has crossed the cross section S_n along time *dt* will be equal to the density of free charge, ρ, multiplied by the volume of the cylinder of Figure 3-4 4 (b), $A\ v_a\ dt$

$$dQ = \rho\ A\ v_a\ dt$$

Calling *n* the density of charge carriers, the density of charge will be *n* multiplied by the charge of each one of the particles $|q|$. In this way, finally:

$$dQ = n\ |q|\ A\ v_a\ dt$$

And the intensity, therefore:

$$I = \frac{dQ}{dt} = n|q|Av_a \qquad \text{Equation 3-2}$$

We can see as the intensity of current is related with drift speed and with the density of charge carriers.

Density of current

It's defined the **density of current** $\vec{J}$ as *a vector having the direction and sense of the movement of positive charges in each point of the conductor; its magnitude equals the electric charge crossing the unit of normal surface to speed by unit of time*.

In this way, $\vec{J}$ is expressed in differential form as: $\vec{J} = \frac{dI}{dA}\vec{u}$

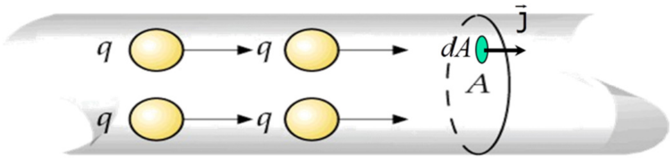


Figure 3-5. Density and intensity of current

dA is a differential element of surface of cross section of conductor, and $\vec{u}$ the perpendicular unit vector to such element of surface (parallel to the movement of charges). From such expression, can be easily checked that the units of the density of current in the IS are A/m^2.

Known the density of current, the intensity can be easily computed by integrating $\vec{J}$ across the whole cross section of conductor:

$$\int_S \vec{J} \cdot d\vec{A} = \int_S J\,dA = \int_S dI = I$$

On the other hand, since the intensity of current flowing across a cross section *A* of a conductor comes from: $I = n|q|v_a A$

The differential current flowing acrosss an element of surface *dA*, will be:

$$dI = n|q|v_a\,dA$$

And so, the density of current can be written as:

$$\vec{J} = \frac{dI}{dA}\vec{u} = \frac{nqv_a dA}{dA}\vec{u} = nqv_a\vec{u} = nq\vec{v}_a$$

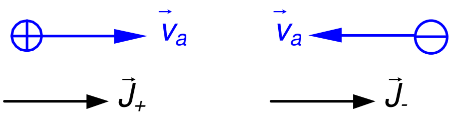
($\vec{v}_a = v_a\vec{u}$, since $\vec{u}$ it represents the unit vector in the direction of the movement of charges).

$$\vec{J} = nq\vec{v}_a \qquad \text{Equation 3-3}$$

Note that any reference to the magnitude of electric charges has disappeared, and *q* will have the sign of charge carriers. For positive charge carriers, $\vec{J}$ will have the same direction and sense that $\vec{v}_a$, but for negative charge carriers, as it occurs in metallic conductors, $\vec{J}$ will have the same direction but opposite sense than $\vec{v}_a$.

By this reason, on a material with two types of charge carriers with opposite sign, as it happen on semiconductor materials, when applying an external electric field, the charge carriers would move with drift speed of opposite sense, but however both densities of current vector would have the same sense, adding their effects.

$$\vec{J}_+ = n(+q_+)\vec{v}_a \qquad \vec{J}_- = n(-q_-)\vec{v}_a$$



Example 3-1

Along a conductor of 1,3 mm of radius is flowing a current of 20 A. Which is the density of current, and which is the drift speed? How long would take the electrons covering a metre of distance? Data: the electronic density is $1{,}806 \cdot 10^{29}$ electrons/m^3, and the charge of the electron $q = 1{,}6 \cdot 10^{-19}$ C.

Solution:

Considering that the density of current is uniform, it comes from:

$$J = \frac{I}{\pi r^2} = \frac{20}{\pi(1{,}3 \cdot 10^{-3})^2} = 3{,}77 \cdot 10^6 \, \frac{\text{A}}{\text{m}^2}$$

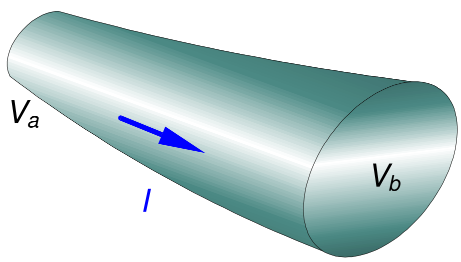
And the drift speed:

$$v_a = \frac{J}{nq} = \frac{3{,}77 \cdot 10^6 \, \dfrac{\text{A}}{\text{m}^2}}{1{,}806 \cdot 10^{29} \, \dfrac{\text{e}^-}{\text{m}^3} \cdot 1{,}6 \cdot 10^{-19} \, \dfrac{\text{C}}{\text{e}^-}} = 0{,}13 \text{ mm/s}$$

The time an electron would take in covering 1 m of distance would be $t = 1/v_a = 7692$ s. That is, they take more than two hours in covering 1 m of distance, and however the lights light immediately when lighting a switch. This apparent contradiction is due to the fact that the density of charge (ρ) inside a conductor is constant, and all the electrons are moving at the same time (like a log). So, when an electron starts to move on an end of conductor, almost immediately (300000 km/s on vacuum) are moving the electrons at the other end, but this movement is very slow, as this example shows.

3.5 Ohm's law. Electrical resistance

As it was before said, the electrical current is produced by an electric field inside a conductor; but this is equivalent to a difference of potential between its ends (a fall of tension between the extremes). In energetic terms, the electric charges entering on conductor through section "*a*" will have an energy qV_a and when exiting through "*b*" section, will have a lower energy qV_b as a consequence of the collisions in the solid net-



work. About the difference of potential $\Delta V=V_a-V_b$ the greater is the intensity of current, the greater is. This experimental result is known as **Ohm's law**, that can be stated as:

The difference of potential in the ends of a conductor is directly related to the intensity of current flowing along it.	$V_a - V_b = \Delta V = IR$

The constant relating difference of potential and intensity of current is called **electrical resistance.**

Ohm's law can be taken as the definition of resistance of a conductor, as the quotient between the difference of potential applied to a conductor and the intensity of current flowing along it.

The unit of resistance on IS is the **ohm** (Ω), and their dimensions:

$$[R]=ML^2T^{-3}I^{-2}$$

The inverse quantity of resistance is the **conductance**, measured in **siemens** (S), 1 S $\equiv$ 1 Ω^{-1}.

Ohm's law is not a universal rule, but only verified by a very specific (but very usual) type of conductors; such conductors are called ohmic conductors, in order to tell them from the non ohmic conductors. In an ohmic conductor, the difference of potential is directly related to the intensity; that is, on an intensity–voltage diagram, we'll see a straight line. In the case of not ohmic conductors this relation is not linear. For example, in a diode, for very low voltages, the intensity is almost zero, but for higher voltages, the intensity grows quickly (see characteristic curve voltage–intensity of a diode on a further unit). In this case we say that the element is not linear or not ohmic.

OHMIC		NON OHMIC	
V (V)	*I* (mA)	*V* (V)	*I* (mA)
2	6	2	3
4	12	4	11
6	18	6	34
8	24	8	111
10	30	10	360

Table 3-1. Examples of relation between voltage and intensity for an ohmic and for a non ohmic material

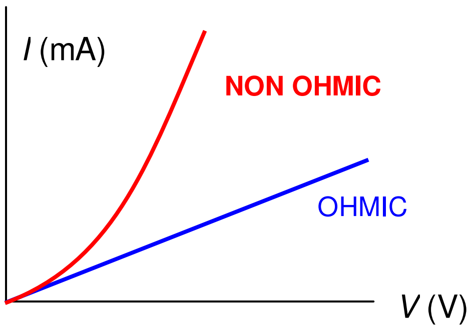
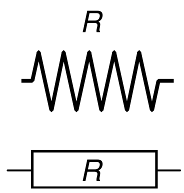


Figure 3-6. Drawing of data of Table 3-1

When we want to represent a device having this behaviour on an electric circuit or electric network, we use any of these symbols to right. This element showing an electrical resistance is called a **resistor**.



Microscopic Ohm’s law

The previous statement of Ohm’s law is the usual way we use it when analysing circuits or measuring resistors. It relates macroscopic magnitudes, involving an enormous number of particles, and so being average values. The resistance R is a parameter depending both on material and the geometry (shape and size) of conductor. Nevertheless, Ohm’s law can be also written relating two vector quantities on a point of conductor (the density of current and the electric field) through a parameter only depending on the characteristics of the conductor material. This is what we know as **microscopic Ohm’s law**, stated as:

The density of current on a point is directly related to the electric field in such point.	$\vec{J} = \sigma\vec{E}$

The constant σ, called **conductivity**, is a characteristic parameter of each material, being measured in $(\Omega m)^{-1}$. In the same way than before, this law is only valid for ohmic materials, and we’ll state below that both statements of Ohm’s law are equivalent.

The inverse of conductivity is the **resistivity**: $\rho = \frac{1}{\sigma}$, measured in Ωm. Both quantities can either be used, since they are totally equivalent.

Relation between microscopic and macroscopic Ohm’s law. Resistance of a homogeneous of constant cross section conductor.

Let’s consider a straight cylindrical conductor, homogeneous and of constant cross section, whit an intensity of current I flowing along it (in longitudinal sense). This intensity has been got by applying potentials V_1 and V_2 to the ending cross sections of conductor, as can be seen on Figure 3-7. We’ll prove that starting on microscopic Ohm’s law, we can arrive to macroscopic Ohm’s law.

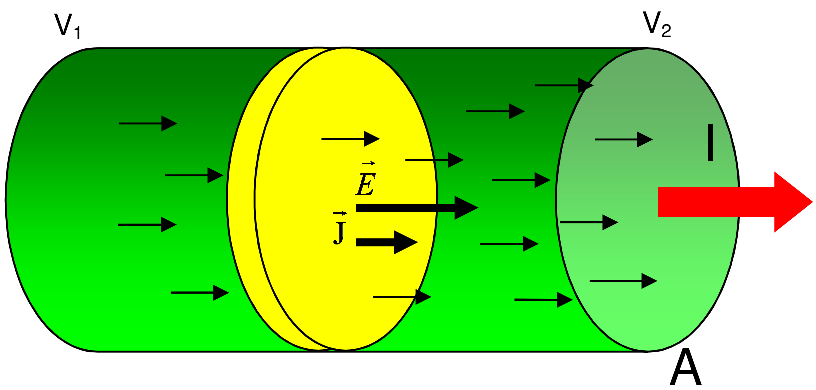


Figure 3-7. Straight and homogeneous cylindrical conductor. Electrocinetical magnitudes.

The intensity of current is the same for any cross section of conductor, and as conductor is homogeneous, the density of current will be constant across a cross section. So, the density of current vector $\vec{J}$ will have a magnitude (see example 3-1): $J = \frac{I}{A}$ where A is the cross section of conductor. $\vec{J}$ is parallel to the axis of conductor, and with the same sense than intensity.

On the other hand, if movement of charges is parallel to the axis of conductor, electric field and electric field lines are also parallel to this axis. From $\vec{J} = \sigma\vec{E}$ (σ constant), magnitude of E will be also constant on conductor. Computing the difference of potential V_1 - V_2 along a field line:

$$V_1 - V_2 = \int_L \vec{E}\cdot d\vec{l} = \int_L Edl = E\int_L dl = EL \Rightarrow E = \frac{V_1 - V_2}{L}$$ L is the length of conductor

Writing microscopic Ohm's law with the magnitudes of computed J and E, it comes:

$$J = \sigma E \Rightarrow \frac{I}{A} = \sigma\frac{V_1 - V_2}{L} \Rightarrow V_1 - V_2 = I\frac{\rho L}{A}$$

This resulting equation is the macroscopic Ohm's law ($\Delta V = IR$) if we consider the resistance of conductor $R = \frac{\rho L}{A}$.

As can be seen in this equation, the resistance of a conductor depends on two type of parameters: geometrical parameters, as area of cross section and length of conductor, and structural parameters, as **resistivity**.

It is necessary to have in mind that the resistance is a magnitude of a piece of conductor, whereas the resistivity is a magnitude of a substance. So, it will be correct talk about the resistance of a piece of a wire, and it will be correct talk about the resistivity of copper, but it's wrong talk about resistivity of a wire or resistance of copper.

About drift speed, from Equation 3-33, we can relate drift speed and electric field:

$$\vec{v}_a = \frac{1}{ne}\vec{J} = \frac{\sigma}{ne}\vec{E} = \mu\vec{E}$$

The constant $\mu = \frac{\sigma}{ne}$ is called **mobility**, being a quantity giving the ability to move the charge carriers on a material when an electric field is applied. E is the magnitude of electric charge of an electron.

Resistivity of some substances at 20 ºC

On table 3-2, magnitudes for resistivity of several usual substances at 20 ºC is shown. Resistivity is used to classify the substances in three types: conductors, isolators and semiconductors. Conductors show low magnitudes of resistivity (in the order of 10^{-8} Ωm), isolators show high magnitudes (higher than 10^{10} Ωm), and intermediate magnitudes for semiconductors. On table it also appears the temperature coefficient, useful for characterize the changes on resistivity with temperature that will be explained on next section.

	Substance	ρ (Ωm)	Temperature coefficient (K^{-1})
Conductors	Silver	$1,59 \cdot 10^{-8}$	$3,8 \cdot 10^{-3}$
	Copper	$1,67 \cdot 10^{-8}$	$3,9 \cdot 10^{-3}$
	Gold	$2,35 \cdot 10^{-8}$	$3,4 \cdot 10^{-3}$
	Aluminium	$2,65 \cdot 10^{-8}$	$3,9 \cdot 10^{-3}$
	Volframio	$5,65 \cdot 10^{-8}$	$4,5 \cdot 10^{-3}$
	Nickel	$6,84 \cdot 10^{-8}$	$6,0 \cdot 10^{-3}$
	Iron	$9,71 \cdot 10^{-8}$	$5 \cdot 10^{-3}$
	Platinum	$10,6 \cdot 10^{-8}$	$3,93 \cdot 10^{-3}$
	Lead	$20,65 \cdot 10^{-8}$	$4,3 \cdot 10^{-3}$
Semiconductors	Silicon	4300	$-7,5 \cdot 10^{-2}$
	Germanium	0,46	$-4,8 \cdot 10^{-2}$
Insulators	Glass	$10\ 10 - 10^{14}$	
	Quartz	$7,5 \cdot 10^{17}$	
	Sulphur	10^{15}	
	Teflón	10^{13}	
	Rubber	$10\ 13 - 10^{16}$	
	Wood	$10\ 8 - 10^{11}$	
	Diamante	10^{11}	

Table 3-2. Resistivities and temperature coefficients of usual substances at 20 ºC

Example 3-2

A wire of copper has a radius of 0,5 mm. Which length should the wire measure to get a resistance of 10 Ω?. Magnitude of resistivity can be taken from Table 3-22 .

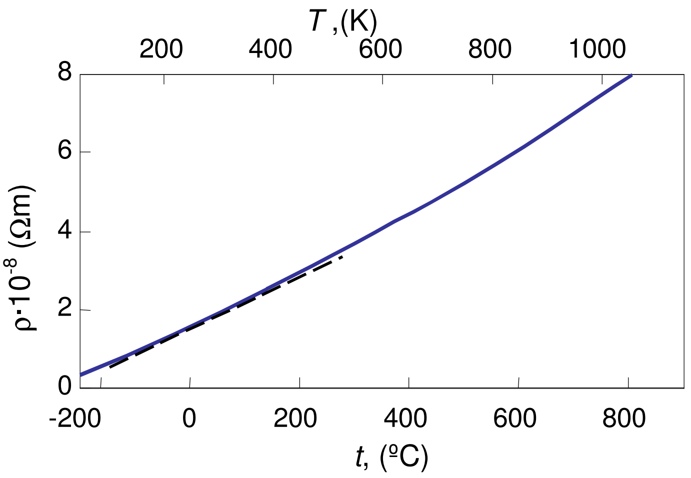
Solution:

Taking a temperature of 20 ºC, $L = \dfrac{RA}{\rho} = \dfrac{10 \cdot \pi(0{,}5 \cdot 10^{-3})^2}{1{,}7 \cdot 10^{-8}} = 462\ \text{m}$

Change on resistivity with temperature

Increasing the temperature on a conductor results in an increasing of thermal agitation, producing a greater number of collisions between electrons and nucleus of atoms, and so a lower drift speed of electrons. So, a decreasing on conductivity (or an increasing on resistivity) is expected when temperature increases.

On Figure 3-8, dependency of resistivity of copper with temperature can be observed. It shows a no linear trend, but if we take a narrower range (for example, between -200 and 200 ºC), it can be supposed a linear trend, being the slope of this straight line the before called **temperature coefficient (α)**; its unit is **K^{-1}**. This coefficient simplifies computations if we want to know the resistivity of a material at a temperature when it's known at another temperature. On tables is usual the resistivity at 20º C appears.



$$\rho = \rho_{20}(1 + \alpha(t - 20))$$

Figure 3-8. Variation of resistivity of copper with temperature

Equation 3-4

The change on resistivity with temperature for other conductor materials is similar to described for the copper.

When designing electrical circuits must be taken into account the running temperature, since conductors modify their resistance, and intensity and difference of potential will be consequently modified. On the other hand, a useful application of the variation of resistance with temperature is the building of thermometers.

Example 3-3

Which increasing on temperature should we have, from 20º C, to increase 50% the resistivity of copper?

Solution:

Solving the equation $\rho = \rho_{20}(1 + \alpha(t - 20))$, we'll have:

$$\frac{\rho}{\rho_{20}} = 1{,}5\ ;\ \ \Delta t = \frac{1{,}5 - 1}{\alpha} = \frac{0{,}5}{3{,}9 \cdot 10^{-3}} = 128\ ºC$$

Superconductivity

Some metals show resistivity close to zero for temperatures below some value called **critical temperature**. This phenomenon is called **superconductivity**, and it was discovered in 1911 by the Dutch physicist H. Kamerlingh Ornnes. The superconductivity involves resistance zero and therefore an intensity of current in a circuit without a generator. On Figure 3-9 9, the sudden fall of the resistance of a piece of mercury at 4,2 K is shown. Other metals like the niobium have a critical temperature of 9,2 K. However, these temperatures are too low to give utility to this phenomenon. From 1987, however, a ceramic oxide with higher critical temperature was discovered, achieving superconductor alloys at temperatures around 90 K; being still a low temperature, it's above the boiling temperature of liquid nitrogen, being possible some applications where high intensities must be achieved.

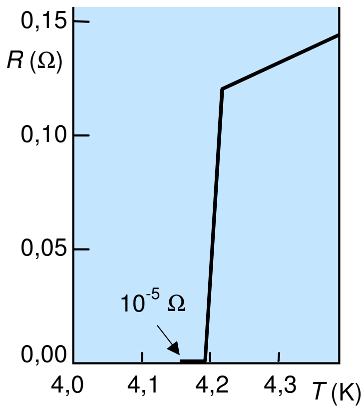


Figure 3-9. Superconductivity in mercury. (Source P.A. Tipler)

3.6 Association of resistors

Resistors can be associated in series, in parallel, or in mixed associations. It's defined the equivalent resistor of a set of resistors like that resistor flowed by the same intensity of current that the set of resistors when subjected to the same difference of potential.

Association in series

A set of elements with two terminals are associated in series when they are placed one next to another, so that the current can't find any junction.

Resistors in series are flowed by the same intensity of current, since electric charges can't find any junction and they can't neither create, nor destroy nor accumulate them on any point. On the other hand, the difference of potential is additive, that is, the energy of the charges decreases from *A* to *C* in the same way that would do it the energy of a stone rolling down along a slope from A to C. So, the d.d.p. between points *A* and *C* is:

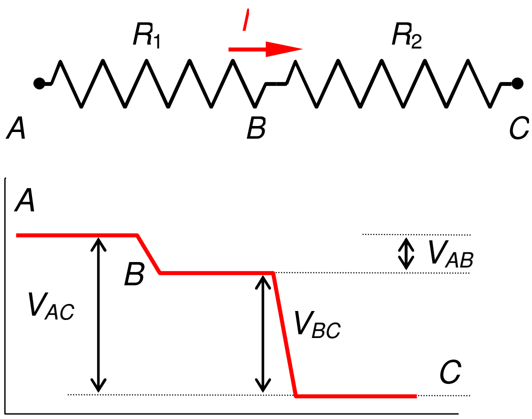


Figure 3-10. Resistors associated in series

$V_{AC} = V_A - V_C = (V_A - V_B) + (V_B - V_C) = V_{AB} + V_{BC}$

If we take in account that $V_{AC} = I\,R_{eq}$ $\quad V_{AB} = I\,R_1$ $\quad V_{BC} = I\,R_2$ substituting and solving for R_{eq}, it comes $R_{eq} = R_1 + R_2$. In general, if we had n resistors in series:

$$R_{eq} = \sum_1^n R_i \qquad \text{Equation 3-5}$$

Association in parallel

A set of elements with two terminals are connected in parallel when they are placed between only two points in such way that we can go from a point to the other point through anyone of the elements, without going through any other.

In the association in parallel, the difference of potential is the same in all the resistors, and instead, the intensity of current is different as a consequence of flowing electric charges along different paths to go from *A* to *B*. So, the total intensity, or intensity entering on the set of resistors equals the addition of intensities in each branch:

$$I = I_1 + I_2$$

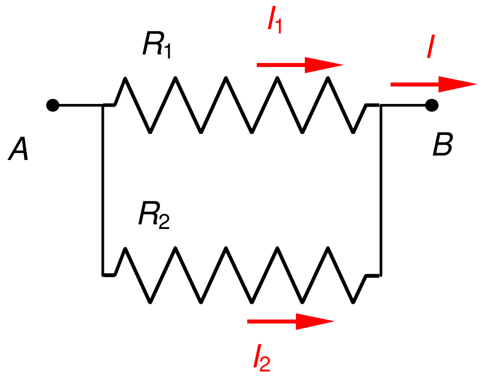


Figure 3-11.Resistors associated in parallel

By applying Ohm's law for resistors and substituting in this equation:

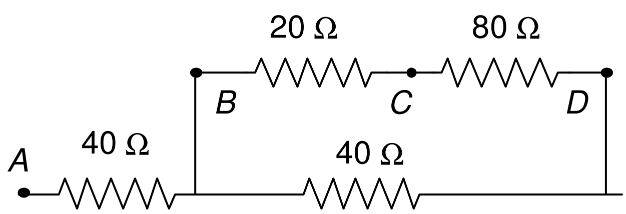
$$\frac{V_{AB}}{R_{eq}} = \frac{V_{AB}}{R_1} + \frac{V_{AB}}{R_2}\,; \qquad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$, and in general, for n resistors:

$$\frac{1}{R_{eq}} = \sum_{1}^{n} \frac{1}{R_i} \qquad \text{Equation 3-6}$$

There also are mixed associations in series and in parallel. In these cases, we'll try to identify sets in series and in parallel whenever it was possible, and we'll solve each of these associations. There are some cases where it is not possible to do it, but these cases remain out of the aims of this subject.

Example 3-4

Given the association on figure, find V_{AB}, if V_{CD} = 4 V.



Solution:

Applying Ohm's law to the 80 Ω resistor, we get the intensity of current flowing along the upper branch:

$$i_{CD} = \frac{V_{CD}}{R} = \frac{4}{80} = \frac{1}{20}\,\text{A}$$

This intensity is the same flowing along the 20 Ω resistor, and so

$$V_{BC} = 20{\cdot}1/20 = 1\ \text{V}.$$

In this way, $V_{BD} = V_{BC} + V_{CD} = 5$ V

Knowing, V_{BD}, we can compute the intensity flowing between B and D along the lower branch:

$$i_{BD} = \frac{V_{BD}}{R} = \frac{5}{40} = \frac{1}{8}\,\text{A}$$

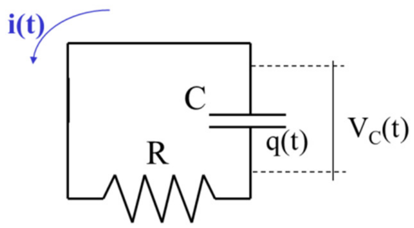
And the total intensity flowing along the two branches is:

$$I = \frac{1}{20} + \frac{1}{8} = \frac{7}{40}\,\text{A}$$

And finally, the tension V_{AB} between A and B is: $V_{AB} = IR = 7/40 \cdot 40 = 7$ V

3.7 Discharging process of a capacitor

When a capacitor is charged (capacitance C, voltage V_0, charge $Q_0 = CV_0$), if the terminals of capacitor are disconnected, the capacitor will remain charged forever. But if we connect both terminals of capacitor through a wire with resistance R (or directly through a resistor R), then the electrons on the negative plate will flow through the resistor to the positive plate, until the net charge at any plate was zero (or the difference of potential between the plates of capacitor was null). The capacitor will be now discharged and as we'll see on next unit, the **stored energy on capacitor has been lost on resistor due to Joule heating**, being this loosing of energy faster the lower is the resistor R.



The transferring of charge from a plate to the other plate produces an intensity of current on circuit. If q(t) is the charge of capacitor on time t the intensity on circuit equals the decreasing on charge at any plate (negative because it's decreasing):

$$i(t) = -\frac{dq(t)}{dt}$$

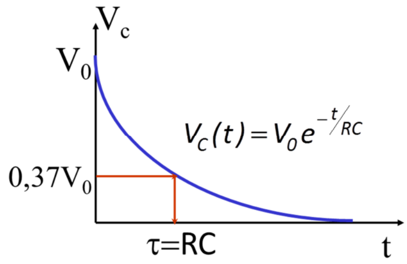
Moreover, as the difference of potential between two points can't have two different values, the voltage on terminals of capacitor $V_c(t)$ must be equal to the voltage on terminals of resistor i(t)R:

$$V_C(t) = \frac{q(t)}{C} = i(t)R \Rightarrow \frac{q(t)}{C} = -\left[\frac{dq(t)}{dt}\right]R \Rightarrow \frac{dq(t)}{q(t)} = -\frac{1}{RC}dt$$

By integrating this equation from time t=0 (when charge on capacitor is CV_0) until a time t when the charge on capacitor is q(t):

$$\int_{CV_0}^{q(t)} \frac{dq(t)}{q(t)} = -\int_0^t \frac{1}{RC} dt \Rightarrow q(t) = V_0 C e^{-t/RC} \Rightarrow \boxed{V_c(t) = \frac{q(t)}{C} = V_0 e^{-t/RC}}$$

If we graph this equation (V_c against the time), we get a decreasing exponential as that on picture.

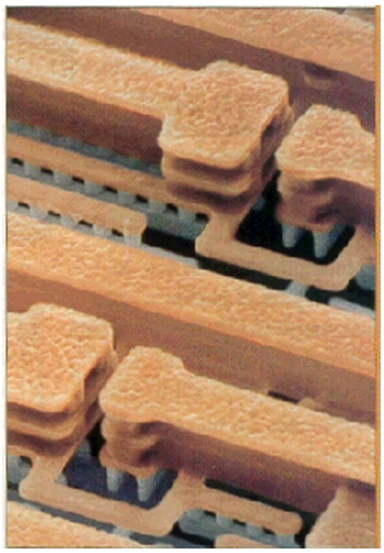


Product **RC** is known as **time constant** of circuit **τ=RC** (you can check that RC is dimensionally a time, so being measured in seconds). As $V_c(t=\tau)=V_0(e^{-1})=0{,}37V_0$ the time constant can be experimentally measured as the **time when the voltage** (or charge) on capacitor is a **37% of initial voltage** (or initial charge). **Time constant** give us an idea about the **speed of discharging process**.

Discharging process is theoretically an infinite process, but **it's usually assumed that capacitor is fully discharged after five times the time constant**, because after this time the remaining voltage (or charge) is almost zero: $V_c(t=5\tau)=V_0(e^{-5})=0{,}007V_0$

Copper for junctions

On September, microelectronic gave a big jump with the apparition in the market of the first microprocessors with junctions of copper. These microprocessors Power PC, commercialized by IBM, will begin to run at a frequency of 400 MHz against the 350 MHz of the actually most advanced Power Pc, to reach 1.000 MHz in two or three years, according IBM. The technical progress is much more remarkable not only for the performances increasing and the decreasing size of the components (the width of the grid of transistors of the new Power PC is 0,18 microns) but also for a modification of the material joining each other. The aluminium now used reaches, in fact, its limit since the more miniaturized the fast transistors are. This gain in speed and, therefore, in performance must be preserved in the interconnections of aluminium joining transistors between them. However, at this level of miniaturization, the conductivity of the aluminium does not increase more and the isolation between connectors does not result perfect. The connectors, the feeblest link of the chain, become on a brake for the processor. The copper is a suitable substitute, twice more conductive that its neighbour on periodic table, but it's considered as the poison of the integrated circuits: it migrates across the microprocessor degrading the transistors.



Detail of a microprocessor with connectors of copper. These are rodetotwo of an insulating layer that prevents the degradation of the transistors by the copper. (Photography *tdr*).

It was necessary to find new procedures to avoid this problem. IBM has developed a new isolating layer coating the copper connectors, made by a sheet of 0,02 microns of tungsten nitrite and another sheet, even more thin, of a material now kept as a secret.
Scientific world. Number 196, December of 1998. Page 11.

3.8 Problems

1. A direct current 1 A sized flows along a conductor wire.

a) How much charge flows through a cross section of wire along 1 minute?

b) If current is due to movement of electrons, how many electrons will cross this section along 1 minute?

Sol: a) 60 C; b) $3{,}75 \cdot 10^{20}$ electrons.

2. On a fluorescent tube 4,0 cm of diameter sized, $2{,}0 \cdot 10^{18}$ electrons and $1{,}0 \cdot 10^{17}$ positive ions (with charge +e) flow through a cross section every second. Which is the intensity of current flowing along the tube?

Sol: 0,336 A

3. A ring of radius R has a linear density of charge λ. If the ring turns with an angular speed ω around its axis, compute the intensity of current on ring.

Sol: $I = \lambda\omega R$

4. A disk of radius R, charged with a surface density of charge σ, turns with an angular speed ω around its axis. Compute the intensity of current on disk.

Sol: $I = \sigma\omega R^2/2$

5. The current flowing along a wire varies according the equation $I = 20 + 3t^2$, where I is expressed in A and t in s.

a) Which is the carried charge along wire between $t = 0$ and $t = 10$ s?

b) Which direct current would carry the same charge in the same interval of time?

Sol: a) 1200 C, b) 120 A

6. The charge flowing through the cross section of a metallic wire is given by $Q(t) = 6{,}5\, t^2 + 3{,}5$ C, t € [0,0 , 8,0] s.

a) Which is the equation for intensity of current $I(t)$ in this interval of time?

b) Which is the intensity of current on t = 3 s?

Sol: a) $I = 13t$ b) 39 A

7. The density of current on a conductor with circular cross section of radius R varies according the equation $J = J_0 r/R$, being r the distance to the axis and J_0 a constant. Compute the intensity of current on conductor.

Sol: $I = 2\pi\, J_0 R^2/3$

Ohm’s law and electrical resistance

8. Along a conductor having 10 m of length, 1 mm^2 of cross section and 0,2 Ω of resistance, flow a current of 5 A.

a) Which is the difference of potential between the endings of conductor?
b) Which is the electric field inside this conductor?
c) Which are the magnitudes for density of current and conductivity?

Sol: a) 1 V b) 0,1 V/m c) $J = 5 \cdot 10^6$ Am^{-2} $\sigma = 5 \cdot 10^7$ $(\Omega\text{m})^{-1}$

9. Could you explain the difference between resistance and resistivity? Is it correct to talk about resistance of copper or resistivity of copper? Is it correct to talk about resistance of one euro coin or resistivity of one euro?

10. A bar of tungsten (wolframium) has a length of 1 m and a cross section of 1 mm^2. A difference of potential of 10 V between its endings is applied.

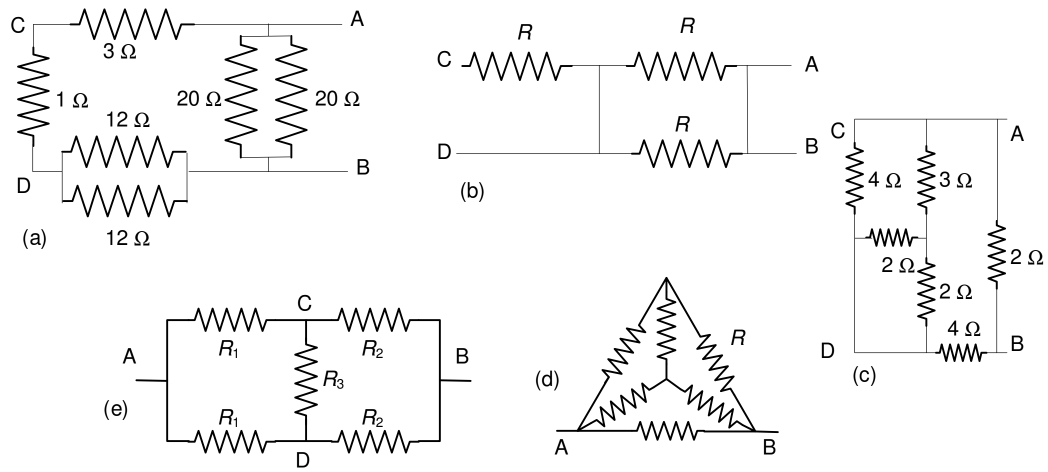
a) Which is its resistance at 20 ºC?
b) Which is its resistance at 40 ºC?
c) Which is the intensity of current at 20 ºC?

Sol: a) 0,056 Ω, b) 0,062 Ω, c) 177 A

11. At which temperature will be the resistance of a conductor of copper 10% higher than when it is at 20 ºC?

Sol: 45,6 ºC.

12. Compute the equivalent resistance between points *A* and *B* and between *C* and *D* on circuits on picture:



Sol: a) R_{AB}= 5 Ω, R_{CD}=19/20 Ω ;
b) R_{AB}= 0, R_{CD}=R;
c) R_{AB}= 3/2 Ω=R_{CD};
d) R_{AB}= R/2;
e) R_{AB}= $(R_1+R_2)/2$, $R_{CD} = 2R_1R_2R_3/(2R_1R_2+R_1R_3+R_2R_3)$

GLOSSARY

Drift speed: Average speed reached by the electric charges when they are accelerated by an electric field, and braked by the crashes with the atomic network.

Density of current. It is a vector having the direction of drift speed in each point of conductor and the magnitude equal to the quantity of charge flowing through a cross section of conductor by unit of time.

Intensity of current. It is the charge crossing through a cross section of conductor by unit of time.

$$I = \frac{dQ}{dt}$$

Ohm's law. The difference of potential between the endings of a conductor is directly related to the intensity flowing along it. The constant of equation is called resistance of conductor.

$$\Delta V = IR$$

Ohm. It is the resistance having a conductor when applying it a difference of potential of 1 volt, an intensity of current of 1 A flows. It's represented by Ω.

Ohmic conductor. Conductor following Ohm's law.

Mobility. Characteristic of material constant relating drift speed and electric field.

Conductivity. Characteristic of material constant relating density of current and electric field. It's defined in Ohm's law.

Resistivity. Inverse of conductivity.

Joule heating. Lost energy as heating produced when an electric current flows along a conductor.

Temperature coefficient. Parameter relating changes in resistivity with temperature on a conductor when a linear approximation can be done.

Density of charge carriers. Number of charge carriers by unit of volume.