Algebra. Warming up exam
Name:

1. Define **basis of a vector space**.
2. Define **eigeivalue** and **eigenvector** of a matrix A.
3. Consider the matrix $A = \begin{bmatrix} 1 & b & 0 \\ 0 & b & 0 \\ 2 & 0 & 2 \end{bmatrix}$. Determine for which values of b the matrix A is diagonalizable. For the case $b = 3$ compute an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$. Using this expression, compute a formula for the powers A^n, where n is a natural number.
4. Suppose that A is the matrix
$$A = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 0 & 3 & 8 & 7 \\ 0 & 0 & 4 & 2 \end{bmatrix}.$$
   a) Compute a basis of the column subspace of A (that is, $Col(A)$). Compute its dimension. Compute also a basis of the row space of A^t ($Row(A^t)$).
   b) Without doing any calculation, write the dimension of the row subspace of A ($Row(A)$) and the rank of A. **Justify your answers**.
   c) Compute a basis of $Row(A)$.
   d) Write a formula relating the following numbers: "number of columns of A", "rank of A" and "dimension of the kernel of A". Apply this formula to compute $\dim \ker(A)$.
   e) Compute a basis of $\ker(A)$.
   f) Compute implicit equations of $Row(A)$.
   g) Does the vector $\vec{b} = (0, 1, 1, 1)$ belong to $Row(A)$? **Justify your answer**.
   h) Compute a basis of the intersection $Col(A) \cap span((1, 5, 4), (-1, 1, 1))$.
   i) Compute a basis of the sum $Col(A) + span((1, 5, 4), (-1, 1, 1))$.