PRACTICE 2. ERRORS ON MEASUREMENT. DIRECT MEASUREMENTS.

1. ERRORS ON MEASUREMENT. ABSOLUTE AND RELATIVE ERRORS.

Always that a measurement is carried out an error is done, due to different causes. For this reason, when expressing a measurement, we must always enclose the magnitude of measurement with a value giving us information about the uncertainty of measurement. For example, if we measure a length, the correct way to express the measurement would be: L=23,45 ± 0,02 m (in general form, **symbol X ± ΔX is used**). It doesn't mean that the true measurement certainly was on range between 23,43 and 23,47 m, but true measurement will be **on such range whit some probability**. This range is known as **absolute error or uncertainty** of a measurement (**ΔX**), depending on some factors, as the quality of instrument, the operator, the measuring method, the hoped probability, etc…

There are **two main rules** to correctly write the result of a measurement:

a) The number of **meaningful figures of absolute error must be only one**, and in exceptional cases up to two (provided that both figures make up a value <25). So, absolute error should be rounded in order to verify this rule.

   Note: There are meaningful figures:
   i. Any figure not being zero.
   ii. Zeros placed between two figures not being zero.
   iii. On any value >1, zeros placed to right than comma are meaningful figures.

b) The **last meaningful figure of measurement and absolute error must be of the same decimal order**. That is, hasn't meaning that measurement has higher accuracy than error.

   Examples: We must always round the error on first, and then round the measurement according the meaningful figures of error:

Incorrect measurements	**Correct measurements**
48,721 ± 0,32 V	48,7 ± 0,3 V
4,6 ± 0,018 V	4,600 ± 0,018 V
563 ± 30 cm	560 ± 30 cm
$872\cdot10^{-6} \pm 0{,}86\cdot10^{-4}$ N	$8{,}7\cdot10^{-4} \pm 0{,}9\cdot10^{-4}$ N
$4{,}678\cdot10^{-8} \pm 4{,}6\cdot10^{-10}$ A	$(4{,}68 \pm 0{,}05)\cdot10^{-8}$ A
0,23±3 ºC	0±3 ºC

Absolute error isn't a useful parameter to compare between two measurements, but the relative error is useful to do it. **The relative error is the quotient between the absolute error and the measurement**: $\varepsilon_x = \frac{\Delta x}{x}$. Besides, relative error is important because on color code of resistors and capacitors, the color of last strip give us the relative error (in %) of nominal value given by the rest of color strips.

The way to compute absolute error on a measurement depends on how the measurement has been carried out: if it's a **direct measurement** or an **indirect measurement**.

2. DIRECT AND INDIRECT MEASUREMENTS

a) **Direct measurements** are those **directly obtained from measuring instruments**. For example, measuring a length with a ruler, or measuring an electric resistor using an ohmmeter are direct measurements.

b) **Indirect measurements** are those **obtained through computations from other measurements**. That is, they are not directly obtained from measuring instruments. For example, when measuring voltage between terminals on a resistor and intensity of current flowing along it, quotient between voltage and intensity V/I (Ohm's law) give us the magnitude of resistor, but in this case in indirect way. Another example of indirect measurement would be calculating the area of a square from the measurement of its side: $S = a^2$.

3. COMPUTATION OF ABSOLUTE ERROR. GENERAL RULE

As general thumb, both on direct as on indirect measurements, the absolute error of a measurement can be computed from the statistical theory. To do it, it's necessary perform at least **three measurements** (x_1,x_2,x_3) of that magnitude we are looking for and:

a) Take the **measurement as the average value** of measurements: $\overline{x} = \dfrac{\sum\limits_{i=1,2,3} x_i}{3}$

b) Take the **absolute error as the standard deviation**: $\Delta x = \sqrt{\dfrac{\sum\limits_{i=1,2,3} (\overline{x} - x_i)^2}{3}}$

c) Verify that the **dispersion of three measurements** $D = \dfrac{x_{max} - x_{min}}{\overline{x}} 100 < 2\%$

d) If **dispersion is < 2%** then the three measurements are enough and the result of our measurement (after rounding error and measurement) is: $x = \overline{x} \pm \Delta x$

If **dispersion is > 2%** then we must perform more measurements (3 additional measurements) and verify that the dispersion is < 8 %.

If **dispersion is > 8 %** then we must increase up to 15 measurements (D < 12 %), or 50 measurements.

Usually, all **this process has already been done by the manufacturer** of the measurement device, and **is unnecessary repeat it**. In these cases, on **technical data sheet** of user's manual, the manufacturer gives the features to directly compute the error on measurement.

4. COMPUTATION OF ABSOLUTE ERROR ON DIRECT MEASUREMENTS

Even though the way for computing the absolute error on direct measurements is general for any kind of measurement device, we'll focus now on **devices for measuring electrical magnitudes**, as ohmmeters, voltmeters, ammeters. **Absolute error** of a direct measurement **is always made up by two different parts**:

- The **error made by the instrument** during measurement. This error is computed from the **accuracy or class** of measurement device (given on technical data of device).

- The **reading error** made when measurement is shown from measuring instrument to operator. It depends on the number of figures on display, or the reading scale.

Both errors can be computed in different way according if measurement instrument is a **digital** or an **analog** instrument; but **absolute error is always the addition of both errors**.

Note: Digital instrument is that showing measurement through some figures on a display. **Analog instrument** is that showing measurement through a needle on a calibrated scale:

Figure 1. Digital and analog instruments

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a) **Absolute error on digital instruments**.

On digital instruments, the **error made by the instrument** is called **accuracy error**, being computed as **a percentage of measurement**. The accuracy can be obtained from data sheet, often depending on the used scale and/or the measurement speed, as well as other factors.

The **Reading error** is due to the rounding of instrument when the result of measurement is shown on the display. It corresponds to a **certain number of units of the last figure appearing on the display** of instrument. The number of units of this figure giving the reading error comes on data sheet of instrument.

As an example, let's suppose we look up the data sheet of digital multimeter Fluke 45 shown on figure 1; this figure shows the measurement of an intensity of current (1,436 mA) in d.c., and with this information, the data sheet tell us that **accuracy is** (range lower than 30 mA) **0,05% + 3 d** (3 d means that reading error is three units of last figure on display). Last figure on display corresponds to a thousandth of miliampere, 0,001 mA. So, absolute error of this measurement would be:

Accuracy error: $$Measurement * \frac{Accuracy}{100} = 1{,}436\frac{0{,}05}{100} = 0{,}000718\ mA$$

Reading error: *Units reading error*lowest unit on display*=3*0,001=0,003 mA

Absolute error: *Accuracy+Reading error*=0,000718+0,003=0,003718, rounded to 0,004

So, the correct writing of this measurement would be: **1,436±0,004 mA**

Note that, in this case, accuracy error is very low compared with reading error. For this reason, **on next practices, we'll neglect the accuracy error** against the reading error on digital devices. We'll also consider on next practices (if nothing is said) the **reading error as only one unit** of last figure on the display.

b) **Absolute error on analog instruments**

On analog instruments, the **error made by the instrument** is called **accuracy class (or class)**, being computed as a **percentage of full scale** (full scale is the maximum value that the instrument can measure on the scale). Such percentage usually comes on the calibrated scale of measuring instrument.

Reading error is due to the difficulty of operator to exactly read the position of needle. It depends on the operator's eyesight and on the amplitude of divisions on calibrated scale of instrument. Nevertheless, **reading error is usually chosen as one of smaller division** appearing on calibrated scale; this is equivalent to say that operator can only distinguish which is the division on the scale closer to the needle of instrument.

Let's suppose we have performed a measurement like that shown on figure. The analog voltmeter is 2,5 accuracy class, full scale is 15 V, and each division on scale corresponds to 0,5 V, being the reading error (one division) 0,5 V. So, absolute error of this measurement would be:

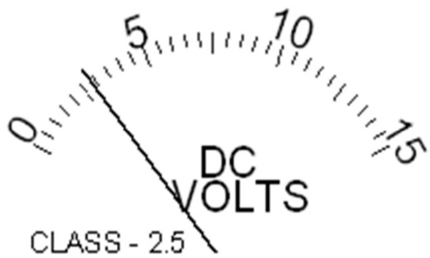


Figure 2: Example of measurement with an analog voltmeter.

Accuracy class error: $15\dfrac{2{,}5}{100}=0{,}375\ V$

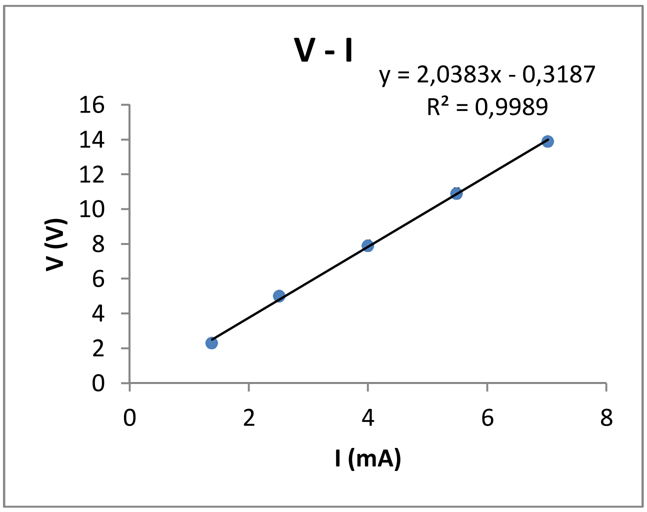
Reading error: $0{,}5\ V$

Absolute error: 0,375+0,5=0,875 rounded to 0,9 V

So, correct writing of this measurement would be: **2,9±0,9 V**.

5. ABSOLUTE ERROR OF A LINEAR FITTING

When a magnitude is **indirectly measured using a linear fitting with the Excel** spreadsheet, the spreadsheet gives us directly the error of that magnitude we are computing.



Remembering the example given on before session, about the linear fitting done to compute a resistance, we got the resistance R=2,0383 KΩ; but we didn't know the error of this resistance or even the number of meaningful figures.

In order to compute the error of such resistance, we must use the function **ESTIMACION.LINEAL** on Excel:

- We select an empty block having three rows and two columns (6 cells).
- On function bar we write "=" and on left, we select the function "ESTIMACION.LINEAL"
- A window is open, and we must introduce on "Conocido_y" and "Conocido_x", the range of cells on vertical and horizontal axes. On "Constante" and "Estadística" we must introduce "1" and "1", and then (**don't press Accept**) press at one time **"Control"+"Shift"+"Enter"**.
- On the selected block appears

2,038302	-0,3187181
0,03853002	0,1754285
0,99892918	0,17389656

The first row corresponds to the slope of linear fitting and the y intercept.
The second row gives us the errors of slope and y intercept.
The first number of third row gives us the correlation coefficient R^2.

Then, the resistance of this resistor, after rounding error and measurement is:

R=2,04 $\pm$ 0,04 KΩ

It is noticeable that when this command is used, the resulting absolute error doesn't depend on the error of experimental measurements; only the dispersion of measurements (if they lie or not along a straight line) determines the error of linear fitting.

6. SYSTEMATIC AND RANDOM ERRORS

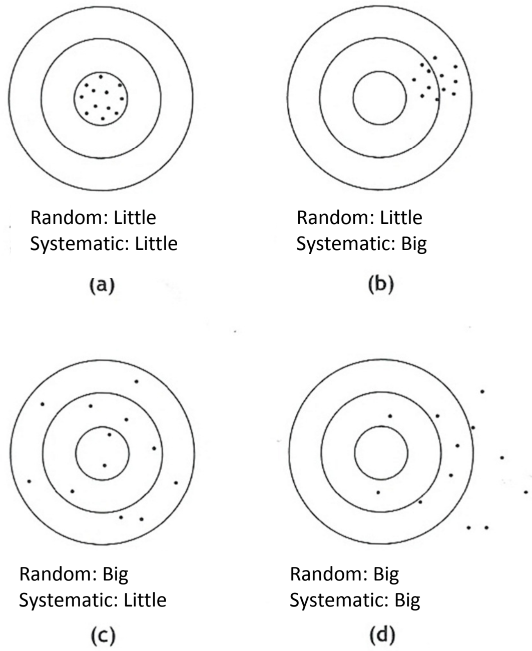
Any error on a measurement can be classified on one of following classes:

a) **Systematic errors**: They are those **errors produced systematically** when a measurement is done. Deviation is always produced by over measuring or by under measuring, **not being useful to perform a lot of measurements to correct it with statistical methods**. They usually come from a wrong calibration of measuring instrument (for example if zero is wrongly adjusted), or because the own measuring process affects its result (for example, when a temperature is measured with a thermometer, the own thermometer changes the temperature we want to measure). Systematic errors must be removed always it was possible, previously **calibrating the measuring instruments or by correcting the error** due to measuring method, both analytically as using a calibration method. As we don't know previously the correct measurement, we need a reference to calibrate it.

b) **Random errors**: **They are those errors produced randomly** when a measurement is done, sometimes by over measuring and sometimes by under measuring. Doing **a set of measurements and statistically analyze them, is possible minimize these errors**. The linear fitting is a very good technique to do it.

Example of systematic and random errors when shooting to a bull's eye:

Random and Systematic errors



PRACTICE 2. ERRORS ON MEASUREMENT. PRACTICE CARRYING OUT

1. OBJECTIVES

The objective of this practice is **learning to handle and correct** (when it was possible) the **errors** on measurements.

To do it, a resistor will be measured in three different ways, computing in each case the corresponding errors.

2. MATERIAL

- Color code of resistors and technical data sheet of measurement devices. You'll find these data on web site/Practices.
- Gold Source d.c. power supply
- 2 digital Fluke 45 multimeter
- Analog Demestres voltmeter
- One 15 KΩ resistor

3. CARRYING OUT

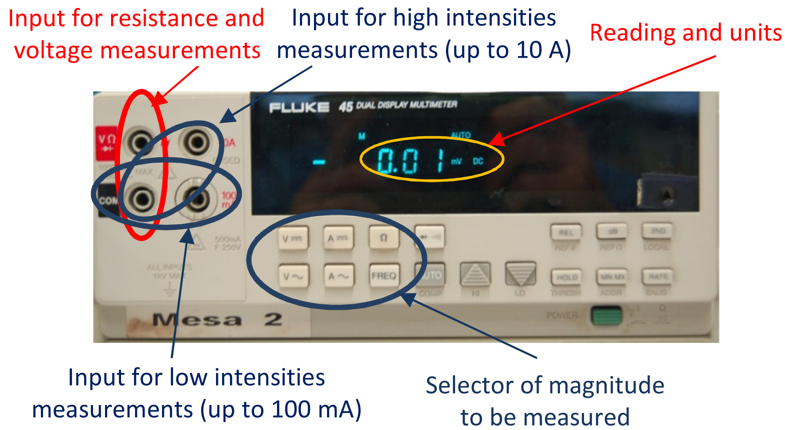
a) Take the 15 KΩ resistor, verifying its **nominal value** through the **color code** (if colors can't be properly seen, you can use the ohmmeter to verify it). From last strip placed to right (it gives us the relative error), **compute the absolute error** of resistance.

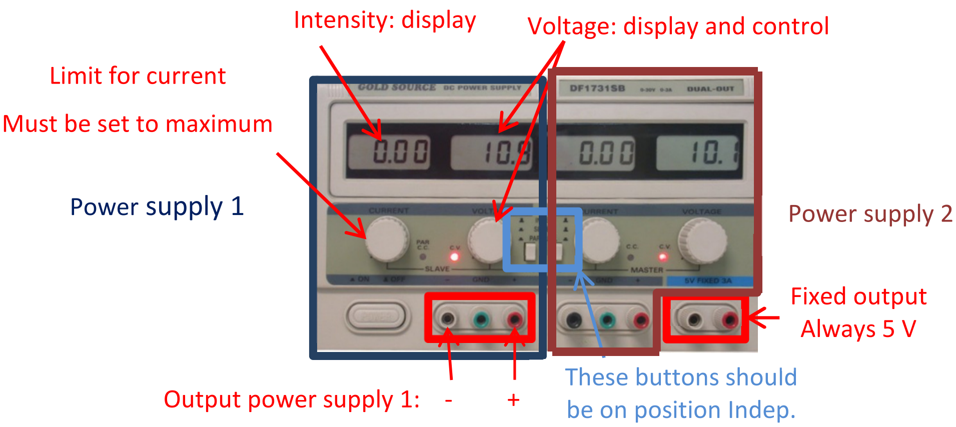
b) By using **Fluke 45 multimeter as an ohmmeter**, measure the **resistance with its absolute error**. Accuracy and reading error to compute absolute error must be got **from technical data sheet**, taking in account that instrument is being used as an ohmmeter in the corresponding range.

To measure:

i. Switch on the Fluke 45 and **select the button to measure Ω**.

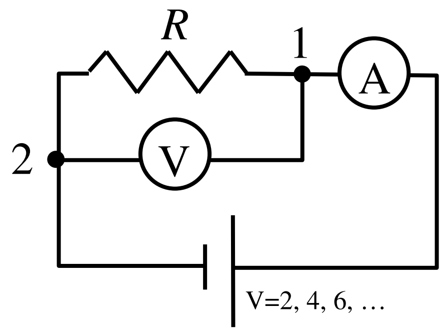
ii. **Connect the resistor** to be measured between the inputs of Fluke 45.





c) By using the same procedure than on b), **measure the internal resistance of analog Demestres voltmeter**. Only replace the resistor by the terminals of voltmeter, and on the display you will read the resistance of Demestres voltmeter. Let you write down this resistance **with its error**.

d) Assembly the circuit on picture with the 15 KΩ resistor. As voltmeter will be used the analog Demestres, and digital Fluke 45 as ammeter.



Cautions on Gold Source DC power supply before switch on:

i. **Verify** that both central **buttons** are on position **"Indep"** (both buttons unpressed).

ii. Verify that **"CURRENT"** button of selected output is turned to the **maximum** (clockwise direction) in order to avoid the intensity was limited. Verify that **"VOLTAGE"** button of selected output is turned to the **minimum** (counterclockwise direction) in order to avoid that when connecting the wires, the current can damage any device.

iii. **Connect** the wires on terminals **+ (red) and – (black)** of selected output. The terminal between black and red terminals is the ground connection, not being necessary connect it.

iv. **Switch on** the Gold Source DC power supply.

Adjust the **Gold Source** power supply to give **successively 2, 4, 6, 8 and 10 V**. For each applied voltage, measure the **voltage** on terminals of resistor and **intensity** flowing along it, **with their absolute errors**.

Write all the measurements and features of measurement devices on the spreadsheet.

4. REPORT

When all measurements are finished, **each group** should do a report from the data on spreadsheet in **PDF** format. In such report must appear:

- 1. Resistance with error computed from color code (section a).
- 2. Resistance with error measured with ohmmeter (section b).
- 3. Measurements and results corresponding to section d).

V source (V)	V (V)	ΔV (V)	I (mA)	ΔI (mA)
2				
4				
6				
8				
10				

- 4. Draw the data of before table on a graph V versus I and perform a linear fitting to compute the resistance of resistor. With the function

Estimacion.Lineal, compute the error of resistance. Remind correctly format this graph.

- 5. Compare the measurements you have got by different methods. ¿Why do you think result of d) is so different from results of a) and b)? Dou you think then internal resistance of voltmeter Demestres measured on c) could be responsible for it?

  Look for the internal resistance (Input Impedance on data sheet) of Fluke 45 when acting as a voltmeter. ¿Do you think that you would get better results if you had used this voltmeter instead the analog Demestres voltmeter?

The report must be **sent through Poliformat/Tasks** by one of the members of group. You'll find there the deadline to upload your report. After this time, only in absolutely justified cases will be the report accepted.