

3: Functional paradigm (III)

Programming Languages, Technologies and Paradigms

Summary



Introduction to Functional Programming

PART I: Types in Functional Programming

1. Functional types. Algebraic types.
2. Predefined types.
3. Polymorphism: genericity, overloading and coercion. Inheritance in Haskell.

PART II: Models of computation in functional programming.

4. Operational model.

PART III: Advanced features

5. Anonymous functions and composition.
6. Iterators and compressors (foldl, foldr).

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Anonymous functions

- Anonymous (or nameless) functions.
 - ▣ In Haskell we can define anonymous functions of the form $\lambda x \rightarrow e$

Example: The function `square x = x*x` can be defined in this way as follows: `square = (\ x -> x*x)`

- ▣ In general, $\lambda x_1 x_2 \dots x_n \rightarrow e$ is equivalent to $\lambda x_1 \rightarrow (\lambda x_2 \rightarrow (\dots \rightarrow (\lambda x_n \rightarrow e) \dots))$

Example:

`sumOfSquares = \x y -> x*x + y*y`

is equivalent to:

`sumOfSquares = \x -> (\y -> x*x + y*y)`

Function composition

□ Function composition

□ $(.) :: (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow a \rightarrow c$

Higher-order

polymorphic

$$(f . g) x = f (g x)$$

- Function composition is defined in the **Haskell prelude** as follows:

$$(f . g) = \backslash x \rightarrow f (g x)$$

- Function composition is a frequent computation pattern. The solution of a problem consists of several steps each of which can be independently addressed by using independent functions that can be then *composed* to solve the problem.

Function composition

Example:

$\text{twice } f \ x = (f . f) \ x$

point-wise

Equivalently, $\text{twice } f = f . f$

point-free

in lambda notation: $\text{twice} = \lambda f \ x \rightarrow f (f \ x)$

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Iterators and compressors

□ Iterators

- Iterators can be used to save memory and time when dealing with *iterable types* like lists or sequences.

- `iterate f x` returns an infinite list of repeated applications of `f` to `x`: `iterate f x` is `[x, f x, f (f x), ...]`

`iterate :: (a -> a) -> a -> [a]`

`iterate f x = x : iterate f (f x)`

Example: The Haskell prelude function `from` is defined by:

`from = iterate (1+)`

Iterators and compressors

□ Compressors

Many functions over lists follow a recursive scheme

$$f :: [a] \rightarrow b$$

$$f [] = z$$

$$f (x:xs) = x \otimes f xs$$

transforming a list $x_1:(x_2:(x_3:(x_4:[])))$ into $x_1 \otimes (x_2 \otimes (x_3 \otimes (x_4 \otimes z)))$

Example: $sum :: [Int] \rightarrow Int$

$$sum [] = 0$$

$$sum (x:xs) = x + sum xs$$

$$product :: [Int] \rightarrow Int$$

$$product [] = 1$$

$$product (x:xs) = x * product xs$$

Iterators and compressors

- We can introduce a function “foldr” that implements this kind of transformation:

$\text{foldr} :: (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b$

$\text{foldr } \text{op } z [] = z$

$\text{foldr } \text{op } z (x:xs) = x \text{ `op` } (\text{foldr `op` } z xs)$

- The previously considered function f can be just define as follows $f = \text{foldr } (\otimes) z$

And similarly for specific functions like:

- $\text{sum} = \text{foldr } (+) 0$

- $\text{product} = \text{foldr } (*) 1$

Iterators and compressors

- ▣ Example: the function `sum` in a previous example (*length of a path*) can be given by using `foldr` or `foldl`

```
sum :: [Float] -> Float
```

```
sum = foldr (+) 0.0
```

- ▣ Exercise: Define `concat`, `and`, `or`, and `map` by using `foldr`.

The Mapreduce scheme

- The combined use of optimized functions like **map** and **fold** inspired **an efficient style of sequence processing**, the **MapReduce** scheme, which has been successfully used in massive dataprocessing ($> 1\text{Tb}$), with thousands of processors and around 100.000 HDs.
- The functional scheme MapReduce was promoted by Google and has a number of relevant applications:
 - ▣ Information retrieval
 - ▣ cloud computing (computation services delivered by companies like Google, Yahoo!, etc., to external clients
 - ▣ ...

Use of Mapreduce

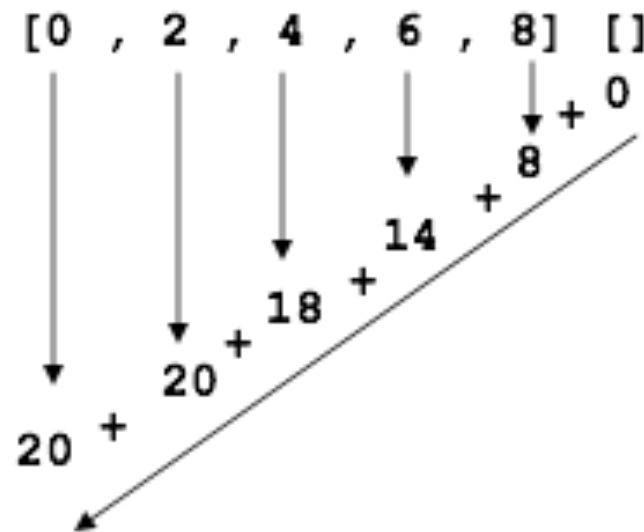


- In **Google:**
 - ▣ Building indices for Google Search
 - ▣ Classifying notices for Google News
 - ▣ Automatic translation
- In **Yahoo!:**
 - ▣ Yahoo! Search
 - ▣ Spam detection in Yahoo! Mail
- In **Facebook:**
 - ▣ Data mining
 - ▣ Optimization of publicity
 - ▣ Spam detection

Use of Mapreduce

The inspiration:

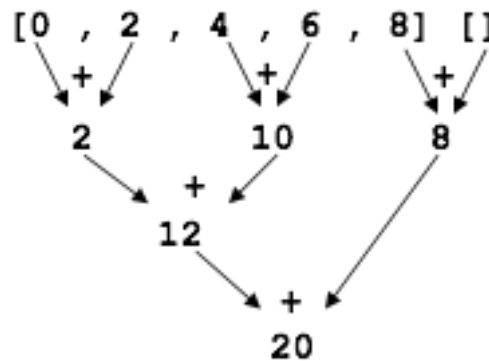
`sumList = foldr (+) 0`



- The computation proceeds from left to right; the number of steps is equal to the length of the list
- *But (+) is associative and commutative!*
-> we can (automatically) parallelize the process...
...and distribute the workload over hundreds/thousands processors!

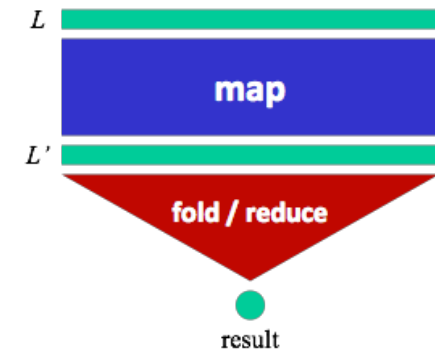
Use of Mapreduce

- The cost becomes $O(\log n)$ if we proceed as follows:

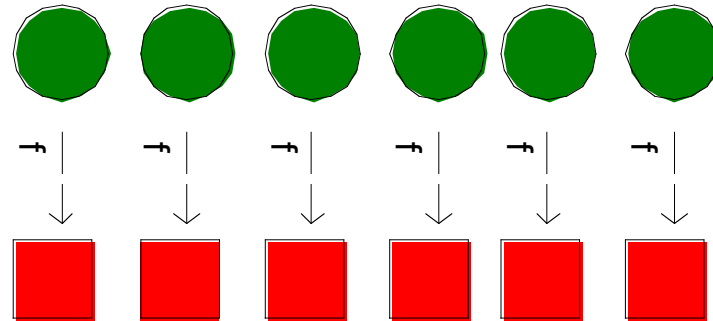


- We can do it by appropriately combining **map** and **fold**.

- We can generalize idea:
this is the “secret” of the MapReduce scheme

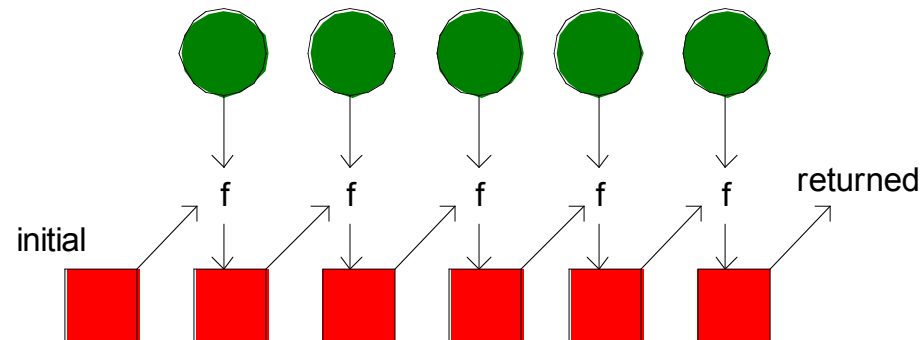


- **map** f : build a new list by applying f to the input list



- **fold** $f\ z\ xs$: applies f to the elements of a list and carries an *accumulator*.

The function f returns the new value of the accumulator, which is combined with the next element of the list



Use of Mapreduce



- The MapReduce scheme is a useful abstraction that simplifies and optimizes heavy computations
- MapReduce has inspired the design of libraries for other languages:
 - ▣ There is now a C++ library MapReduce where map() is divided into 64 MB blocks (of the same size that the chunks of Google's File System).
 - ▣ There is a similar library for Java.
 - ▣ Advantages: we can focus on the problem, and leave the management details (organization, keys, access, etc.) to the library.

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