Unit 9

Alternating current

Objectives

- To know the main features of alternating current (A.C.) and their effect on resistors, capacitors and inductors.
- Understand the phase lag between drop of potential and intensity of current on A.C. circuits.
- Compute the rate between drop of potential and intensity of current in *RLC* series dipoles.
- Define the impedance of a circuit.
- Analyze a *RLC* series circuit from the point of view of energy.
- To know the meaning of power factor on A.C.

9.1 Introduction

Along 19th century, the electricity was developed around of direct current (D.C.). This type of electricity was useful for a lot of applications, on electric engines, heating, lighting, telegraph, telephone, etc… One of the most important businessmen was Thomas Alva Edison, owner of General Electric Company. But when Edison tried to send the electric energy massively to a large distance, almost all the energy was lost by Joule heating along the transmission line.

In order to understand the problem of sending electric energy with direct current, let's imagine we want to send, for example, a power of 1000 W at a voltage of 100 V (an usual voltage), to a little distance, for example 3 Km; the current along the line will be I=1000/100=10 A. The resistance of a copper wire having a cross section of 50 mm^2 is around 0,4 Ω/Km; as the current must go and return, we need 6 Km of wire, with a total resistance of 0,4*6=2,4 Ω. The total power lost by Joule heating on transmission line will be:

Lost Power=10^2*2,4=240 W ¡ 24% of sent power ¡

To solve this problem, the consumers of electricity should be placed near the generating factory and a big project to build a lot of factories in Niagara Falls (to profit its water jumping), was presented.

But they were some scientists, engineers and businessmen supporters of alternating current (A.C.) because of its easy generation (on before unit we saw that a loop turning inside a magnetic field produces sinusoidal alternating current) and the low quantity of energy lost on the transmission line. We know that the transformer doesn't work with direct current, but it works on alternating current, and a voltage of 100 V can be easily converted in, for example, 10000 V. The lost power sending 1000 W at 10000 V (intensity of current = 1000/10000=0,1 A) to a distance of 3 Km with the same wire we have used for direct current will be:

Lost Power=$(0,1)^2$*2,4=0,024 W only a 0,0024% of sent power

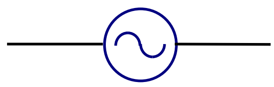
Nikola Tesla, a Croatian engineer, and George Westinghouse were two of the most active supporters of A.C., and their disagreement with the wide use of D.C. produced the "War of currents" on last decade of 19^{th} century. They won this war and Niagara Falls was preserved.

From then, the electric energy can be sent at very large distances at very high voltages, even 100000 V, and only a few generating factories of electricity are needed. Of course, this voltage is not useful at home (it would be very dangerous) and it must be reduced with a second transformer; this is the reason why we can found transformers along any town.

Besides its easy transportation, Fourier's theorem states that any periodic function can be written as an addition of sinusoidal functions, and then the results for alternating current can be extrapolated to any type of periodic function, even digital signals.

9.2 Features of an a.c.

We saw on before unit that a sinusoidal alternating current can be produced by turning a loop inside a uniform magnetic field. The symbol for the A.C. generators on circuits is that on right, and the mathematical expression for the difference of potential on terminals of generator can be written as:



$$u(t) = U_m \cos(\omega t + \varphi_u)$$

this equation has be written as a cosine, but it could be also written as a sine, only adding 90º (π/2 rad) to the phase:

$$u(t) = U_m \cos(\omega t + \varphi_u) = U_m \sin(\omega t + \varphi_u + \frac{\pi}{2})$$

The main features of this sinusoidal function are:

- The **amplitude**, U_m, is the maximum value reached by the function (Figure 9-1). The units of amplitude are the same that the magnitude we are representing; in this case, volts.
- The **period,** T is the time taken for a whole cycle of function (Figure 9-1). Its unit in the I.S. is the second.

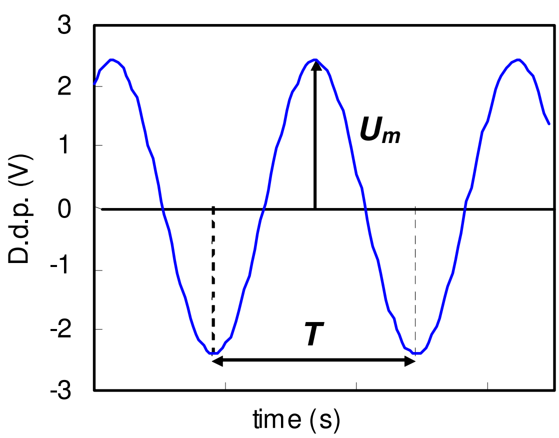


Figure 9-1. Amplitude and period of an a.c.

- The **frequency,** *f,* is the number of cycles by second. It’s the inverse of period: $f = \frac{1}{T}$. Its unit is Hertz (Hz): 1 Hz = 1 s^{-1} .

- The **angular frequency, angular speed or pulsatance**, ω, is the angle (in radians) turned on a second. Its unit is rad/s. We must remember that the radian is dimensionless, and then the dimensions of angular frequency are T^{-1}. As a whole cycle equals 2π radians and the period, *T*, is the time taken by a cycle: $\omega = 2\pi \frac{1}{T} = 2\pi f$

- The **phase** $\omega t + \varphi_u$, expressed in radians.

- The **initial phase** φ_u, is the magnitude of phase on time t=0. For an easier reading it is usual to give the magnitude ▪$_u$ in degrees instead radians, but when it’s operated, it must be converted to radians. The initial phase, really, locates the sinusoidal wave along the X axis. On right (Figure 9-2) the initial phase of several sinusoidal functions can be seen. It’s important to note that this figure represents the function $u(t) = U_m cos(\omega t + \varphi_u)$; if the sinus function is used, as 90º must be added, the initial phases would be different.

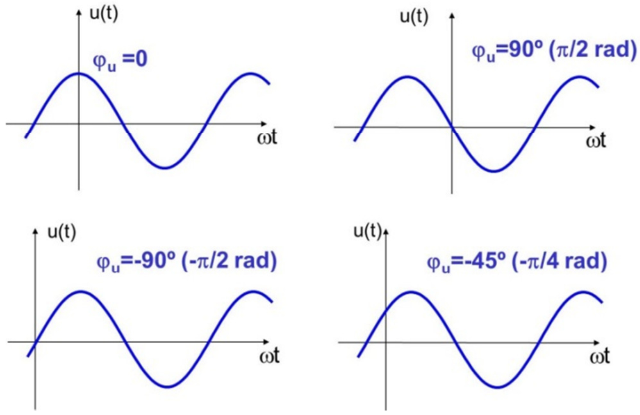


Figure 9-2. Initial phase of several sinusoidal functions (expressed as cosinus)

- **Phase lag** is defined for two sinusoidal functions having the same angular speed. Phase lag is the difference between phases of both functions at the same time; as the angular frequency is the same for both functions, the difference between phases equals the difference between initial phases.

  Let’s consider, for example, a voltage $u(t) = U_m cos(\omega t + \varphi_u)$ and an intensity of current $i(t) = I_m cos(\omega t + \varphi_i)$. The phase lag between voltage and intensity is:

$$\varphi_{ui} = (\omega t + \varphi_u) - (\omega t + \varphi_i) = \varphi_u - \varphi_i$$

  For this example, if phase lag is positive (Figure 9-3), voltage goes ahead intensity (or intensity goes behind voltage). But if phase lag is negative (Figure 9-4), voltage goes behind intensity (or intensity goes

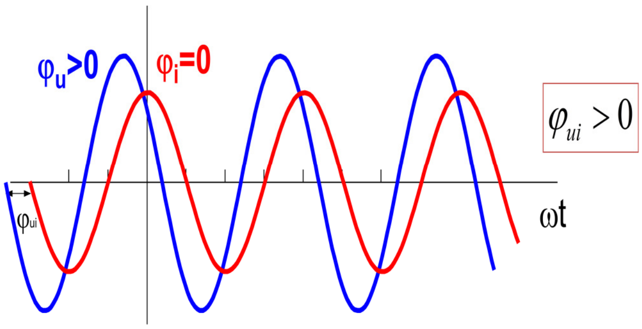


Figure 9-3 **Positive phase lag** between voltage and intensity; voltage goes ahead intensity (or intensity goes behind voltage)

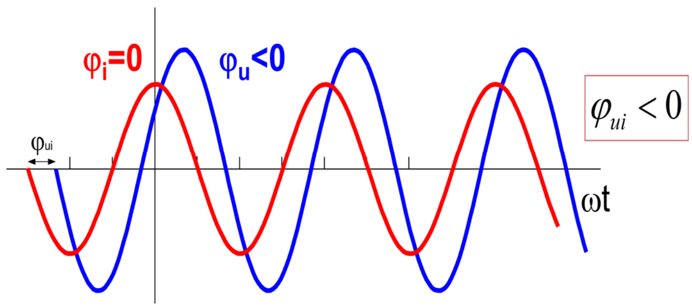


Figure 9-4 **Negative phase lag** between voltage and intensity; voltage goes behind intensity (or intensity goes ahead voltage)

ahead voltage). If phase lag is zero, both functions go **in phase**.

If we had defined phase lag between intensity and voltage (φ_{iu} instead φ_{ui}) the sign of phase lag would be opposite. But we always will refer to a phase lag between voltage and intensity: $\varphi = \varphi_u - \varphi_i$

- **Root mean square**: Root mean square (rms) of a sinusoidal function is defined as the amplitude divided into $\sqrt{2}$. For example (for a voltage and for a intensity):

$$U_{rms} = \frac{U_m}{\sqrt{2}} \qquad\qquad I_{rms} = \frac{I_m}{\sqrt{2}}$$

The root mean square of a sinusoidal function is very important in alternating current because this is the magnitude measured by the measuring devices (voltmeters, ammeters, multimeters, etc…). Its physical meaning is that of a direct current dissipating the same power that an alternating current of amplitude U_m on a resistor. The demonstration of his equivalence will be done when we deal with power on A.C., at the end of this unit. The current we can found in the wall sockets at our homes is a current having a voltage U_{rms}=220 V, and then the amplitude of this voltage is U_m=220*$\sqrt{2}$ =310 V.

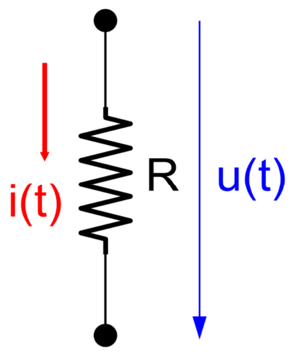
9.3 Behavior of basic dipoles facing an A.C.

At this point we'll study the behavior of basic dipoles (resistor, inductor and capacitor) when an alternating current is flowing along them. To do it, we'll study two parameters:

- the rate between the amplitude of voltage on terminals of dipole and the intensity of current
- the phase lag between voltage on terminals and intensity of current

1. Resistor

Let's suppose a resistor (resistance R) flowed by a intensity of current $i(t)$ and having a difference of potential $u(t)$ between its terminals. In order to simplify the computations, we'll take initial phase zero for intensity ($\varphi_i=0$):



$$i(t) = I_m cos(\omega t)$$

At any time, on a resistor must be verified that $u(t) = Ri(t)$ and then

$$u(t) = RI_m cos(\omega t)$$

The difference of potential on terminals of resistor is a new cosine function,

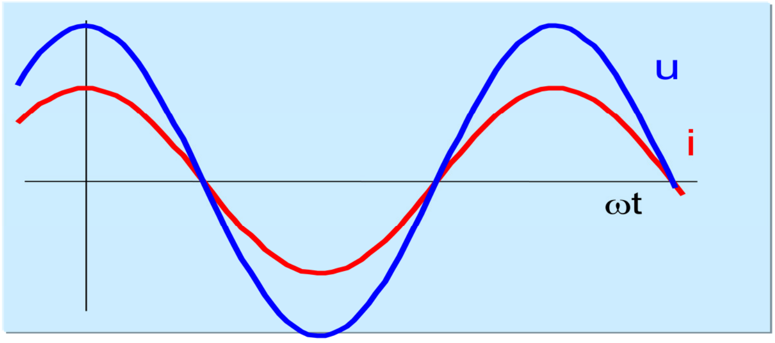
$$u(t) = U_m cos(\omega t + \varphi_u) \qquad \text{being} \qquad U_m = RI_m \qquad \text{and} \qquad \varphi_u=0$$

Obviously, if we had taken a initial phase not zero for intensity, the initial phase of voltage would have been the same of intensity.

On a resistor, then

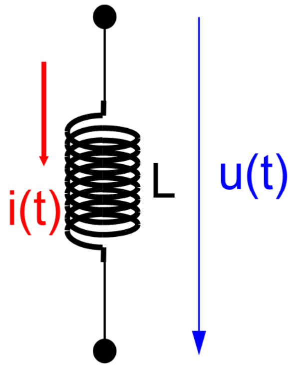
$$\frac{U_m}{I_m} = R$$
$$\varphi = \varphi_u - \varphi_i = 0$$

Voltage and intensity go on phase



2. Inductor

On an inductor, the main parameter is the self inductance coefficient *L*. If we take initial phase zero for intensity ($\varphi_i=0$):



$$i(t) = I_m \cos(\omega t)$$

At any time, on an inductor must be verified that $u(t) = L\dfrac{di(t)}{dt}$ and then

$$u(t) = L\frac{d(I_m \cos(\omega t))}{dt} = -L\omega I_m \sin(\omega t) = L\omega I_m \cos(\omega t + \frac{\pi}{2})$$

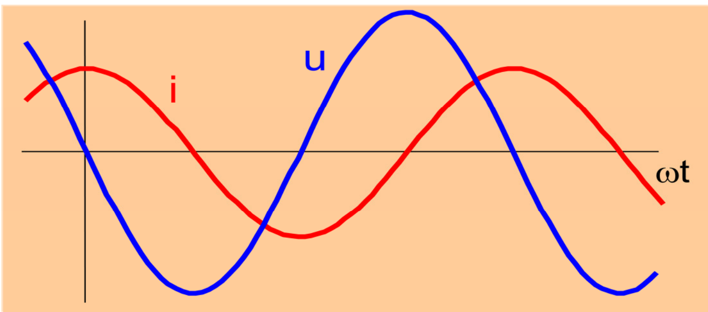
The difference of potential on terminals of inductor is a new cosine function,

$$u(t) = U_m \cos(\omega t + \varphi_u) \quad \text{being} \quad U_m = L\omega I_m \quad \text{and} \quad \varphi_u = \pi/2$$

On an inductor, then

$$\frac{U_m}{I_m} = L\omega$$
$$\varphi = \varphi_u - \varphi_i = \frac{\pi}{2}$$

Voltage goes 90º ahead intensity, or Intensity goes 90º behind voltage.

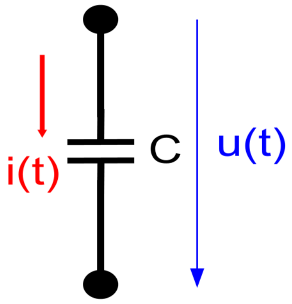


The factor *Lω* is known as **Inductance or Inductive Reactance** ($X_L = L\omega$); it has dimensions of a resistance, and thus is measured in Ohms. Really, the inductance on inductor acts as the resistance on a resistor, but the most important difference is that the inductance depends not only on the self inductance coefficient, but also on the frequency. So, on a circuit, the rate between amplitudes of voltage and intensity on inductor can change by only changing the frequency of current.

3. Capacitor

In this case, for an easier computation, we'll suppose the voltage on terminals of capacitor having initial phase zero ($\varphi_u=0$):

$$u(t)=U_m\cos(\omega t)$$



For a capacitor must be remembered that

$$C=\frac{q(t)}{u(t)} \qquad \text{and} \qquad i(t)=\frac{dq(t)}{dt}$$

Therefore

$$i(t)=\frac{dq(t)}{dt}=\frac{dCu(t)}{dt}=\frac{Cd(U_m\cos(\omega t))}{dt}=-C\omega U_m\sin(\omega t)=U_mC\omega\cos(\omega t+\frac{\pi}{2})$$

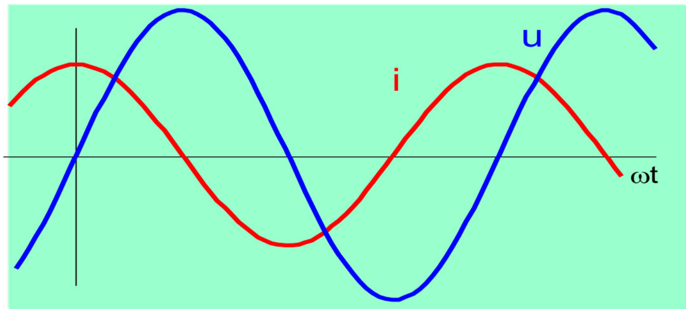
The intensity of current on a capacitor is a new cosine function,

$i(t)=I_m\cos(\omega t+\varphi_i)$ being $I_m=U_mC\omega$ and $\varphi_i=\pi/2$

On a capacitor, then

$$\frac{U_m}{I_m}=\frac{1}{C\omega}$$

$$\varphi=\varphi_u-\varphi_i=-\frac{\pi}{2}$$



Voltage goes 90º behind intensity, or Intensity goes 90º ahead voltage

The factor $1/C\omega$ is known as **Capacitance or Capacitive Reactance** ($X_C=1/C\omega$); it has dimensions of a resistance, and thus is measured in Ohms. Really, the capacitance on a capacitor acts as the resistance on a resistor, but the most important difference is that the capacitance not only depends on the capacitance, but also on the frequency. So, in a circuit, the rate between amplitudes of voltage and intensity on capacitor can change by only changing the frequency of current.

Example 9-1

A resistor 5 Ω sized, an inductor 10 mH sized, and a capacitor 50 µF sized, are connected in series. The voltage on terminals of inductor is $u_L(t)=100\cos(1000t+\frac{\pi}{4})\ V$. Compute the intensity of current flowing along three dipoles, the voltage on terminals of resistor and capacitor, and plot all these magnitudes on a graph.

Solution:

As we know voltage on inductor, it's easy to get the intensity of current on inductor. The angular speed of current is ω=1000 rad/s, and then:

$$X_L=L\omega=10\cdot10^{-3}\cdot1000=10\ \Omega\Rightarrow I_m=\frac{U_{Lm}}{X_L}=\frac{100}{10}=10\ A$$

Phase lag on inductor is 90º, and then $\varphi = \frac{\pi}{4} - \varphi_i = \frac{\pi}{2} \Rightarrow \varphi_i = \frac{\pi}{4} - \frac{\pi}{2} = -\frac{\pi}{4}\, rad$

As all the dipoles are connected in series, the intensity of current on inductor is the same for all the dipoles, being:

$$i(t) = i_L(t) = i_R(t) = i_C(t) = 10\cos(1000t - \frac{\pi}{4}) \quad A$$

From intensity, voltages on terminals of resistor and capacitor can be computed:

on resistor $U_{Rm} = I_m R = 10 \cdot 5 = 50\, V$

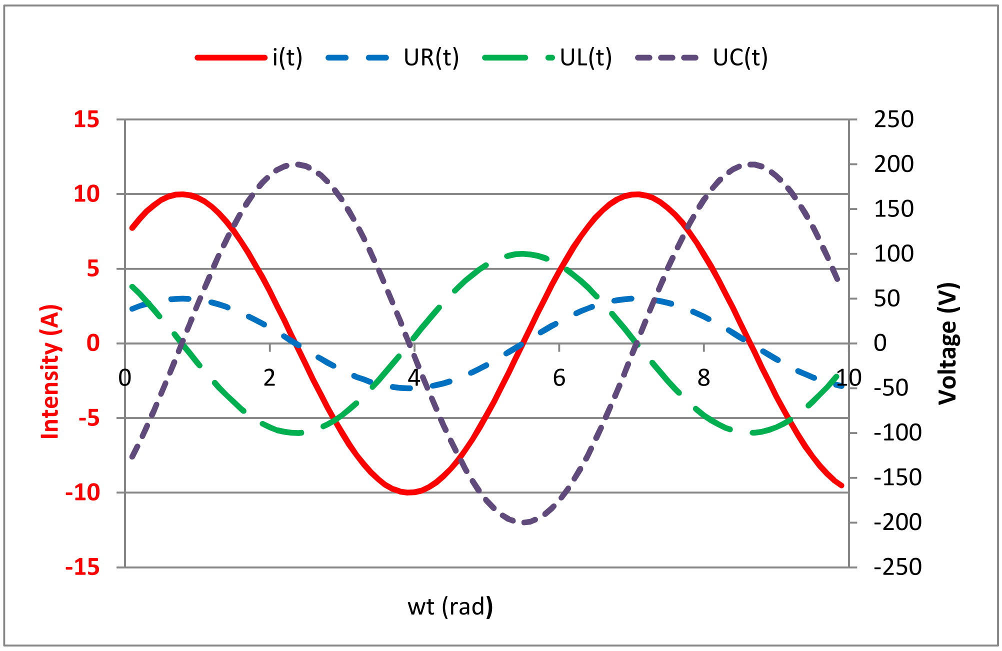
and as phase lag on resistor is zero: $u_R(t) = 50\cos(1000t - \frac{\pi}{4}) \quad V$

On capacitor: $X_C = \frac{1}{C\omega} = \frac{1}{50 \cdot 10^{-6} \cdot 1000} = 20\,\Omega \Rightarrow U_{Cm} = I_m X_C = 10 \cdot 20 = 200\, V$

And phase lag $\varphi = \varphi_u - (-\frac{\pi}{4}) = -\frac{\pi}{2} \Rightarrow \varphi_u = -\frac{\pi}{2} - \frac{\pi}{4} = -\frac{3\pi}{4} rad$

$$u_C(t) = 200\cos(1000t - \frac{3\pi}{4}) \quad V$$

The graph with voltages and intensity:



It can be noted that, obviously, intensity and voltage on resistor go in phase, phase lag between voltages on inductor and capacitor is 180º (or -180º), voltage on inductor goes ahead intensity, and voltage on capacitor goes behind intensity.

9.4 RLC series circuit. Impedance and phase lag.

A *RLC* series circuit is a circuit having a resistor, an inductor and a capacitor connected in series (*RLC* dipole). As we have above seen, the intensity of current is the same for all the basic dipoles on circuit, and the voltage on each device can be computed. In order to simplify computations, we have taken initial phase of intensity zero ($\varphi_i=0$).

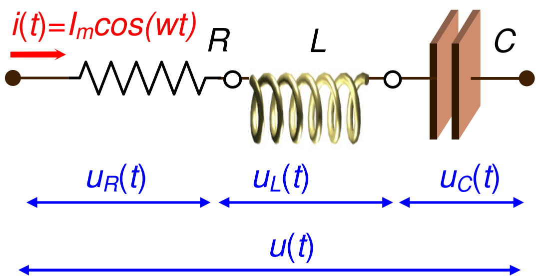


Figure 9-5. *RLC* series dipole

But the main problem is now to relate the intensity of current *i(t)* to the voltage on terminals of *RLC* circuit, *u(t)*. This voltage could be computed by only adding the voltages on the devices on circuit:

$$u(t)=u_R(t)+u_L(t)+u_C(t)=RI_m cos(\omega t)+L\omega I_m cos(\omega t+\frac{\pi}{2})+\frac{1}{C\omega}I_m cos(\omega t-\frac{\pi}{2}) \quad (1)$$

The addition of sinusoidal functions is a new sinusoidal function, and *u(t)* is a sinusoidal function, but from this equation is difficult to assess its amplitude and its initial phase. Nevertheless, the amplitude and phase of *u(t)* can be computed using two new features of a *RLC* circuit, its **Impedance** and its **phase lag**. Let's suppose that the voltage on terminals of *RLC* dipole, *u(t)* is:

$$u(t)=U_m cos(\omega t+\varphi) \quad (2)$$

By expanding (1) remembering that $cos(A+B)=cos A cos B-sin A sin B$ and taking in account that $sin(\pi/2)=1$ and $cos(\pi/2)=0$:

$$u(t)=RI_m cos(\omega t)+L\omega I_m(cos(\omega t)cos(\frac{\pi}{2})-sin(\omega t)sin(\frac{\pi}{2}))+\frac{I_m}{C\omega}(cos(\omega t)cos(\frac{\pi}{2})+sin(\omega t)sin(\frac{\pi}{2}))=$$
$$=RI_m cos(\omega t)-L\omega I_m sin(\omega t)+\frac{I_m}{C\omega}sin(\omega t) \quad (3)$$

And expanding (2): $$u(t)=U_m(cos(\omega t)cos(\varphi)-sin(\omega t)sin(\varphi)) \quad (4)$$

Identifying coefficients of $sin(\omega t)$ and $cos(\omega t)$ between (3) and (4):

$$sin(\omega t): \quad -L\omega I_m+\frac{I_m}{C\omega}=-U_m sin(\varphi) \quad (5)$$

$$cos(\omega t): \quad RI_m=U_m cos(\varphi) \quad (6)$$

By dividing both equations (5)/(6) we get the phase lag between voltage on terminals of *RLC* dipole and intensity of current:

$$\boxed{\frac{Lw-\frac{1}{Cw}}{R}=tg\varphi}$$

φ is the phase lag between voltage and intensity.

Squaring (5) and (6) and adding them, taking in account that $cos^2(\varphi)+sin^2(\varphi)=1$:

$$(R^2+(L\omega-\frac{1}{C\omega})^2)I_m^2=U_m^2(cos^2(\varphi)+sin^2(\varphi))\Rightarrow U_m=I_m\sqrt{R^2+(L\omega-\frac{1}{C\omega})^2}$$

The rate between U_m and I_m is the **Impedance** (***Z***) of RLC series circuit:

$$\boxed{\frac{U_m}{I_m}=Z=\sqrt{R^2+(L\omega-\frac{1}{C\omega})^2}=\sqrt{R^2+(X_L-X_C)^2}}$$

Obviously, the impedance of a resistor equals *R*, for an inductor equals X_L and for a capacitor equals X_C.

Example 9-2

On circuit of example 9-1, find the voltage on terminals of circuit.

Solution:

From the impedance of this circuit:

$$Z=\sqrt{5^2+(10-20)^2}=11{,}18\,\Omega\Rightarrow U_m=I_mZ=10*11{,}18=111{,}8\,V$$

Phase lag

$$tg\varphi=\frac{10-20}{5}=-2\Rightarrow\varphi=-63{,}4º\Rightarrow\varphi_u=\varphi+\varphi_i=-63{,}4-45=-108{,}4º$$

And voltage on terminals of circuit: $u(t)=111{,}8\cos(1000t-108{,}4º)\,V$

The difference between the inductive reactance (X_L) and the capacitive reactance (X_C) is the reactance (X):

$X=X_L-X_C$ also measured in Ohms

The main magnitudes of an *RLC* series circuit can be easily remembered drawing the **Impedance triangle**, a triangle being the horizontal cathetus the resistance of circuit, the vertical cathetus the reactance of circuit, the hypotenuse the impedance of circuit, and the angle between Z and R, the phase lag of circuit.

Depending on magnitudes of *L* and *C*, X can be positive (inductive circuit), negative (capacitive circuit), or zero (resonant circuit).

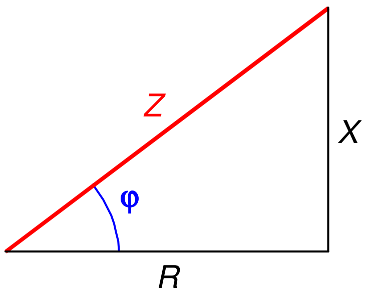


Figure 9-6. Impedance triangle for an inductive circuit

Review about phase lag and Impedance for some dipoles

Phase lag and impedance for basic dipoles (R, L and C) and for dipole RLC (RLC series circuit) can be summarized on next table:

Dipole	Phase lag $\phi=\phi_u-\phi_i$	Impedance Z
Resistor	0	R
Inductor	90º	$X_L = L\omega$
Capacitor	-90º	$X_C = 1/C\omega$
RLC series	$\varphi = arctg\left(\dfrac{L\omega-\dfrac{1}{C\omega}}{R}\right)$	$Z=\dfrac{U_m}{I_m}=\sqrt{R^2+\left(L\omega-\dfrac{1}{C\omega}\right)^2}$

Table 9-1 Summary of phase lag and impedance for basic dipoles and RLC series circuit.

9.5 Power on A.C.

Let's imagine a dipole (both a resistor, an inductor, a capacitor, or a *RLC* series dipole) flowed by a intensity of current $i(t)=I_m sin(\omega t)$; we'll use the sine function instead cosine and initial phase zero for intensity in order to simplify the computations. Voltage on terminals of dipole will be $u(t)=U_m sin(\omega t+\varphi)$. The amplitude U_m is depending on impedance of dipole, and φ is 0 for a resistor, 90º for an inductor, -90º for a capacitor, and any magnitude between -90º and 90º for a *RLC* series dipole.

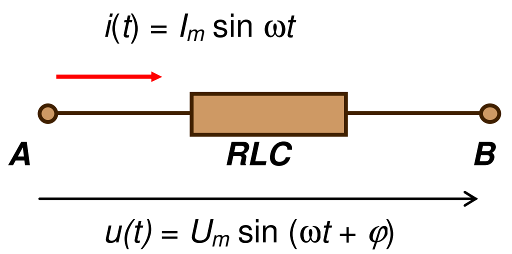


Figure 9-7. Voltage and intensity on a *RLC* series dipole

At any time, the consumed power by the dipole, **Instantaneous power**, is the product of intensity and voltage:

$$p(t)=i(t)u(t)=I_m sin(\omega t)U_m sin(\omega t+\varphi)$$

Remembering that $sin(A+B)=sin A cos B+cos A sin B$ and that $sin 2A = 2 sin A cos A$ expanding the above equation

$$p(t)=i(t)u(t)=I_m sin(\omega t)U_m sin(\omega t+\varphi)=I_mU_m[sin(\omega t)cos\varphi+cos(\omega t)sin\varphi]sin(\omega t)=$$
$$=U_mI_m cos\varphi sin^2 \omega t+\frac{U_mI_m}{2}sin\varphi sin 2\omega t$$

The instantaneous power is the addition of two terms: the first one is always positive, because cosφ is always positive (φ is in the range -90º to 90º) and sin^2wt is also always positive. This term is known as **Active power** or **Real power**:

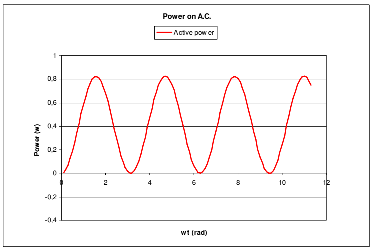
$$P_a(t)=U_mI_m cos\varphi sin^2 \omega t$$

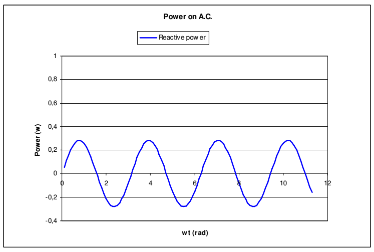
The second term can be positive or negative, according the magnitudes of φ and t; this term is known as **Reactive power**:

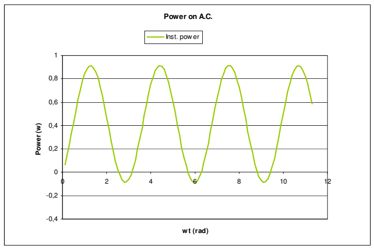
$$P_r(t) = \frac{U_m I_m}{2} \sin\varphi \sin 2\omega t$$

Active power (P_a) + Reactive power (P_R) = Instantaneous power (P_i)

As an example, giving values I_m=1 A, U_m=1 V, ω=1 rad/s, φ=0,6 rad and drawing the functions corresponding to active, reactive and instantaneous power, we can see:







Active power, as we have said is always positive, meaning that this power is consumed at any time by the dipole; reactive power is a function with a frequency 2 times that of current, and sometimes is consumed by the dipole (when it's positive) and sometimes is generated by the dipole (when it's negative). Along a whole cycle, the average value of reactive power is zero ($\frac{1}{T}\int_0^T \frac{U_m I_m}{2} \sin\varphi \sin 2\omega t dt = 0$), but the average value of active power equals the average value of instant power, not being zero:

$$P_{i_{av}} = P_{a_{av}} = \frac{1}{T}\int_0^T U_m I_m \cos\varphi \sin^2 \omega t dt = \frac{U_m I_m \cos\varphi}{T}\int_0^T \sin^2 \omega t dt = \frac{U_m I_m}{2}\cos\varphi = U_{rms} I_{rms} \cos\varphi$$

Along a cycle, the average power (both the instantaneous power as the active power) is always positive, meaning that the power is consumed by the dipole. **Cosφ** is known as **Power factor**, because for a given intensity and voltage, it determines the consumed power; it depends on the inductive or capacitive feature of circuit. The maximum consumed power is reached for a circuit with φ=0 (voltage and intensity on phase).

Applying these results to basic dipoles:

Resistor:

For a resistor φ=0 (cosφ=1) and $U_m = I_m R$

The reactive power is zero at any time, $P_r(t) = 0$

From above equation, the average values $P_{i_{av}} = P_{a_{av}} = \frac{U_m^2}{2R} = \frac{U_{rms}^2}{R}$

On a resistor there isn't reactive power at any moment (cosφ=0), and all the consumed power is active power, lost by Joule heating. Besides, from this equation can be easily understood the physical meaning of rms (random mean square) magnitudes:

An alternating current consumes on a resistor the same power than a direct current (D.C.) having the rms magnitudes (intensity and voltage).

Inductor:

For an inductor φ=90º (cosφ=0) and $U_m = L\omega I_m$

The active power is zero at any time, $P_a(t) = 0$

And the reactive power on a time t is $P_r(t) = \frac{U_m I_m}{2} sin 2\omega t = \frac{U_m^2}{2L\omega} sin 2\omega t$

An inductor takes energy from circuit along a half of cycle, storing this energy as magnetic field, and it returns this energy to the circuit in the another half of cycle. But an inductor doesn't consume net energy; it neither produces nor consumes power. It happens at a frequency two times that of electric current (2ωt).

Capacitor:

For a capacitor φ=-90º (cosφ=0) and $U_m = \frac{I_m}{C\omega}$

The active power is zero at any time, $P_a(t) = 0$

And the reactive power on a time t is $P_r(t) = -\frac{U_m I_m}{2} sin 2\omega t = -\frac{U_m^2 C\omega}{2} sin 2\omega t$

As an inductor, a capacitor takes energy from circuit along a half of cycle, storing this energy as electric field, and it returns this energy to the circuit in the another half of cycle. But a capacitor doesn't consume net energy; it neither produces nor consumes power. It happens at a frequency two times that of electric current (2ωt).

It's important to note the negative sign of reactive power for a capacitor. It only means that the power is negative (capacitor consuming energy) when sin2ωt is positive, and then the reactive power on a capacitor is in opposition of phase (phase lag 180º) against the reactive power on an inductor. It means that when inductor is taking energy from circuit, the capacitor is giving it, and the other way round on the other half of cycle.

If the amplitudes of reactive power on inductor and capacitor are equal (it happens only when $X_L=X_C$), then they are cancelled, and the reactive power on circuit is always zero; but if they aren't cancelled, the reactive power isn't zero and the intensity of current necessary to carry this energy produces Joule heating on wires of circuit, losing energy, even if it can't be taken advantage of the reactive power.

PROPOSED PROBLEM:

A *RLC* series circuit made up by L=2 H, C=2 μF and R=20 Ohm is connected to an adjustable frequency generator, giving an amplitude U_m=100 V.

If generator is adjusted to 60 Hz:

a) Find the maximum intensity of current (amplitude of intensity).

b) Find the amplitudes of voltage on resistor, inductor and capacitor.

c) Find the phase lag.

d) Write instantaneous magnitudes of intensity *i(t)*, voltage on generator *u(t)*, resistor $u_R(t)$, inductor $u_L(t)$, and capacitor $u_C(t)$.

e) Compute the power factor

f) Find the average power given by generator to the circuit, verifying that this power is that consumed on resistor.

9.6 Questions and problems

1. Consider a capacitive circuit. ¿How does the capacitive reactance change if the angular frequency increases two times?

2. A capacitor C = 0,5μF is connected to an A.C. generator; the amplitude of voltage on terminals of generator is U_m = 300 V ¿Which is the amplitude of intensity of current along the capacitor, I_m, if angular frequency is

a) ω= 100 rad/s? b) ω= 1000 rad/s?

Sol: a) 0,015 A b) 0,15 A

3. An inductor L = 45 mH is connected to an A.C. generator; the amplitude of voltage on terminals of generator is U_m = 300 V and the inductive reactance of inductor is X_L = 1300 Ω. ¿Which is

a) the angular frequency ω and the frequency f of generator?
b) ¿The amplitude of intensity, I_m?
c) Find that angular frequency that equals the inductive reactance of inductor to the capacitive reactance of problem 2.

Sol:
a) ω = $2{,}9.10^4$ rad/s, f= 181514,67 Hz b) 0,23 A c) 21081,85 rad/s

4. A circuit is made up by two basic dipoles in series. The terminals of this circuit are connected to an A.C. generator giving a voltage $u(t)$ = 150 cos(500 t+ 10º) V, and flowing along the circuit an intensity of current $i(t)$ = 13,42cos(500t - 53,4º) A. Determine the two basic dipoles and their magnitudes.

Sol: R = 5 Ω L = 0,02 H

5. Along a circuit made up by two basic dipoles in series and a generator giving a voltage $u(t)$ = 200sen(2000t + 50º) V flows an intensity $i(t)$ = 4cos(2000t + 13,2º) A. Determine such dipoles and their magnitudes.

Sol: R = 29,7 Ω C = 12,4 μF

6. On a RL series circuit, R = 5 Ω and L = 0,06 H, drop of potential on terminals of inductor is $u_L(t)$ = 15cos200t V. Compute:

a) the intensity of current b) phase lag and impedance of circuit
c) voltage on terminals of circuit

Sol:
a) $i(t)$ = 1,25 cos(200t - 90º) A; b) φ = 67,4º Z = 13 Ω
c) $u(t)$ = 16,3cos(200t - 22,6º) V

7. Along a RLC series circuit, R = 2 Ω, L = 1,6 mH and C = 20 μF, flows an intensity of current $i(t)$ = 3cos(5000t - 60º) A.

Compute and draw on a graph the sinusoidal functions corresponding to the drop of potential on each device and the drop of potential on terminals of *RLC* circuit.

Sol:

$u_R(t)$ = 6cos(5000t - 60º) V, $u_L(t)$ = 24cos(5000t + 30º) V,

$u_C(t)$ = 30cos(5000t - 150º) V, $u(t) = 6(2)^{1/2}$cos(5000t - 105º) V

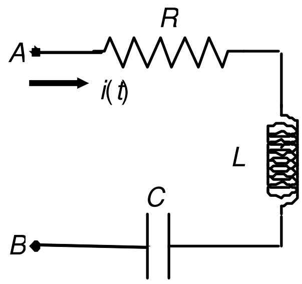
8. A resistor 5 Ω sized and a capacitor are connected in series. Voltage on resistor is $u_R(t)$ = 25 cos(2000t + 30º) V. If voltage on terminals of *RC* dipole goes 60º behind current, ¿which is the capacitance *C* of capacitor?

Sol: *C* = 57,7 µF

9. The voltage applied on terminals of a *RLC* series circuit goes 30º ahead the current. Amplitude of voltage on inductor is two times the amplitude of voltage on capacitor, and $u_L(t)$ = 10cos1000t V. If *R* = 20 Ω find values of *L* and *C*.

Sol: *L* = 23,1 mH, *C* = 86,6 µF

10. Along circuit on picture flows an intensity of current $i(t) = 10(2)^{1/2}$cos(100t+90º) A. If *R*=10 Ω, *L*=0,5 H and *C*=20 µF



a) Compute power factor of *RLC* dipole.
b) Compute the average power along a cycle on each basic dipole.

Sol: a) cos φ = 0,022 b) P_{Rav}= 1000 W, P_{Lav}= P_{Cav}=0

11. ¿How does the power factor depend on resistance *R*, on inductance *L* and on capacitance *C* in a *RLC* series circuit?

13. Along a *RL* series circuit, having *L* = 0,05 H, flows an intensity of current $i(t) = 2(2)^{1/2}$cos500t A. Voltage measured with a voltmeter on terminals of resistor is V_R = 50 V. Determine:

a) Magnitude of *R*
b) *u*(*t*) on terminals of generator
c) if a capacitor is added in series with *R* and *L*, compute the capacitance of this capacitor in order to get 30º of phase lag between voltage on generator and intensity of current.
d) the new *i(t)* after the capacitor has been added.

Sol:

a) *R* = 25 Ω b) *v(t)* = 100 cos(500t + 45º) V,

c) *C* = 189 µF, d) $I_1(t) = 2,45(2)^{1/2}$ cos(500t + 15º) A

GLOSSARY

Frequency is the number of cycles by second of sinusoidal function.

Initial phase is the phase of sinusoidal function on time t = 0.

Phase lag φ is the difference of phases between the initial phases of two sinusoidal functions, usually voltaje and intensity $\varphi = \varphi_u - \varphi_i$

Root mean square (rms): square root of average value of square of sinusoidal function along a cycle:

$$U_{rms} = \frac{U_m}{\sqrt{2}} \qquad I_{rms} = \frac{I_m}{\sqrt{2}}$$

Impedance: Rate between amplitudes of drop of potential and intensity in a *RLC* dipole

$$Z = \frac{U_m}{I_m}$$

Inductive reactance: Rate between amplitudes of difference of potential and intensity on an inductor.

$$X_L = L\omega$$

Capacitive reactance: Rate between amplitudes of difference of potential and intensity on a capacitor.

$$X_C = \frac{1}{C\omega}$$

Active power: Is the consumed power by Joule heating in a *RLC* dipole

$$P_a = IU\cos\varphi$$

Power factor: cosφ is the cosinus of phase lag. It only depends on devices of circuit and the angular frequency of current.