Tema 4: Máquinas de vectores soporte con márgenes blandos

$$\frac{\partial \Lambda(\boldsymbol{\theta}, \theta_0, \boldsymbol{\zeta}, \boldsymbol{\alpha}, \boldsymbol{\beta})}{\partial \theta_i} = \theta_i - \sum_{n=1}^{N} c_n \alpha_n x_{ni} = 0 \text{ para } 1 \leq i \leq d \Rightarrow \boldsymbol{\theta}^*(\boldsymbol{\alpha}) = \sum_{n=1}^{N} c_n \alpha_n \boldsymbol{x}_n \quad (1)$$

$$\frac{\partial \Lambda(\boldsymbol{\theta}, \theta_0, \boldsymbol{\zeta}, \boldsymbol{\alpha}, \boldsymbol{\beta})}{\partial \theta_0} = \sum_{n=1}^{n} c_n \alpha_n = 0 \quad (2)$$

$$\frac{\partial \Lambda(\boldsymbol{\theta}, \theta_0, \boldsymbol{\zeta}, \boldsymbol{\alpha}, \boldsymbol{\beta})}{\partial \zeta_n} = \mathcal{C} - \alpha_n - \beta_n = 0 \Rightarrow \alpha_n + \beta_n = \mathcal{C} \quad (3)$$

Tema 4: Máquinas de vectores soporte con márgenes blandos

$$
\begin{aligned}
\Lambda_D(\boldsymbol{\alpha},\boldsymbol{\beta}) \quad &= \quad \Lambda(\boldsymbol{\theta}^*, \theta_0^*, \boldsymbol{\zeta}^*, \boldsymbol{\alpha}, \boldsymbol{\beta}) \\
&= \quad \frac{1}{2}{\boldsymbol{\theta}^*}^t\boldsymbol{\theta}^* + \mathcal{C}\sum_{n=1}^{N}\zeta_n^* - \sum_{n=1}^{N}\alpha_n\left(c_n\left({\boldsymbol{\theta}^*}^t\boldsymbol{x}_n + \theta_0^*\right) + \zeta_n^* - 1\right) - \sum_{n=1}^{N}\beta_n\zeta_n^* \\
\text{Por (1)} \quad &= \quad \frac{1}{2}\sum_{n,n'=1}^{N} c_n c_{n'}\alpha_n\alpha_{n'}{\boldsymbol{x}_n}^t\boldsymbol{x}_{n'} + \mathcal{C}\sum_{n=1}^{N}\zeta_n^* \\
&\qquad\qquad - \sum_{n=1}^{N}\alpha_n\left(c_n\left(\sum_{n'=1}^{N} c_{n'}\alpha_{n'}{\boldsymbol{x}_{n'}}^t\boldsymbol{x}_n + \theta_0^*\right) + \zeta_n^* - 1\right) - \sum_{n=1}^{N}\beta_n\zeta_n^* \\
&= \quad -\frac{1}{2}\sum_{n,n'=1}^{N} c_n c_{n'}\alpha_n\alpha_{n'}{\boldsymbol{x}_n}^t\boldsymbol{x}_{n'} + \sum_{n=1}^{N}\zeta_n^*(\mathcal{C} - \beta_n) - \sum_{n=1}^{N}\alpha_n(c_n\,\theta_0^* + \zeta_n^* - 1) \\
\text{Por (2) y (3)} \quad &= \quad -\frac{1}{2}\sum_{n,n'=1}^{N} c_n c_{n'}\alpha_n\alpha_{n'}{\boldsymbol{x}_n}^t\boldsymbol{x}_{n'} + \cancel{\sum_{n=1}^{N}\zeta_n^*\alpha_n} - \cancel{\theta_0^*\sum_{n=1}^{N}\alpha_n\,c_n} - \cancel{\sum_{n=1}^{N}\alpha_n\,\zeta_n^*} + \sum_{n=1}^{N}\alpha_n \\
&= \quad \sum_{n=1}^{N}\alpha_n - \frac{1}{2}\sum_{n,n'=1}^{N} c_n c_{n'}\alpha_n\alpha_{n'}{\boldsymbol{x}_n}^t\boldsymbol{x}_{n'} \;=\; \Lambda_D(\boldsymbol{\alpha})
\end{aligned}
$$