1. Show that: 0 + 1 + 2 + ….. + n = $\frac{n(n+1)}{2}$

2. For any natural number n such that $n \geq 2$, show that n + 1 < n^2.
   $[We\ must\ choose\ n_0 = 2]$

3. For any natural number n such $n \geq 4$, $prove\ that\ n! > n^2$

4. $Fibonacci\ Sequence$: $1, 1, 2, 3, 5, 8, 13, 21, \ldots$.
   $Show\ that\ the\ identity$ $F_1 + F_3 + \ldots + F_{2n-1} = F_{2n} - 1$ $is\ true\ for\ all\ n \geq 1$.

5. $For\ any\ natural\ number\ n\ and\ for\ any\ real\ number\ x,\ prove\ that$
   $(1 - x)(1 + x + x^2 + x^3 + \ldots + x^n) = 1 - x^{n+1}$

6. $Use\ the\ corollary$ $\sum_{i=0}^{n} a \cdot r^i = a \cdot \frac{1-r^{n+1}}{1-r}$ $to\ find\ the\ sum\ of$
   $3 \cdot 2 + 3 \cdot 2^2 + 3 \cdot 2^3 + \ldots + 3 \cdot 2^n$

7. $Use\ the\ corollary$ $\sum_{i=0}^{n} a \cdot r^i = a \cdot \frac{1-r^{n+1}}{1-r}$ $to\ find\ the\ sum\ of$
   $2 + 10 + 50 + \ldots + 1250$

8. $The\ terms\ of\ a\ sequence\ are\ given\ recursively\ as$
   $a_0 = 0$, $a_1 = 2$, and $a_n = 4(a_{n-1} - a_{n-2})$ $for\ n \geq 2$
   $Prove\ by\ induction\ that\ b_n = n \cdot 2^n\ is\ a\ closed\ form\ for\ the\ sequence.$
   $That\ is,\ prove\ that\ a_n = b_n\ for\ every\ n \in N$

9. $The\ terms\ of\ a\ sequence\ are\ given\ recursively\ as$
   $a_0 = 1,\ a_1 = 1,\quad and\quad a_n = 2 \cdot a_{n-1} + 3 \cdot a_{n-2}\quad for\ n \geq 2$
   $Prove\ by\ induction\ that\ b_n = \frac{1}{2} \cdot 3^n + \frac{1}{2} \cdot (-1)^n\ is\ a\ closed\ form\ for\ the\ sequence$
   $[a_n = b_n]$

10. $Find\ the\ first\ six\ values\ [from\ F(0)\ to\ F(5)]\ of\ the\ function\ defined\ on\ N\ given\ by$
    $F(0) = 2,\ F(1) = 3,\ F(2) = 5,\ and\ F(n) = 2F(n-1) + 3F(n-2) + F(n-3)$
    $for\ n \geq 3$.

11. $Tower\ of\ Hanoi\ is\ a\ mathematical\ puzzle\ where\ we\ have\ three\ rods\ and.$
$n\ disks.\ The\ objective\ of\ the\ puzzle\ is\ to\ move\ the\ entire\ stack\ to\ another\ rod,$
$obeying\ the\ following\ simple\ rules:$
$a.\ Only\ one\ disk\ can\ be\ moved\ at\ a\ time.$
$b.\ Each\ move\ consists\ of\ taking\ the\ upper\ disk\ from\ one\ of\ the\ stacks$
$and\ placing\ it\ on\ top\ of\ another\ stack\ i.e.\ a\ disk\ can\ only\ be\ moved\ if\ it\ is\ the$
$uppermost\ disk\ on\ a\ stack.$
$c.\ No\ disk\ may\ be\ placed\ on\ top\ of\ a\ smaller\ disk.$
$Solve\ the\ problem\ using\ recursion:\ T(n) = 2T(n-1) + 1\ for\ n > 1$
$[T(1) = 1]$

12. $Solve\ the\ following\ first - order\ recurrence\ using\ back\ substitution:$

$$T(n) = \begin{cases} cT(n-1) + f(n) & \text{for } n \geq k \\ f(k) & \text{for } n = k \end{cases}$$

13. $Solve\ the\ following\ first - order\ recurrence\ using\ back\ substitution:$
$T(n) = 2T(\frac{n}{2}) + n\ ,\ [T(1) = 1]$

14. $Solve\ the\ following\ first - order\ recurrence\ using\ back\ substitution:$
$T(n) = 2T(\frac{n}{2}) + 1\ ,\ [T(1) = 1]$

15. $Solve\ the\ following:$
$T(n) = T(n-1) + n\ ,\ [T(1) = 1]$