Propositional Logic Practice Problem Solution

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1. a) $p \to \neg q$
   b) $\neg p \land \neg q$
   c) $q \to \neg p$
   d) $\neg p \to \neg q$

2. a) Sharks have not been spotted near the shore.
   b) Swimming at the New Jersey shore is allowed, and sharks have been spotted near the shore.
   c) Swimming at the New Jersey shore is not allowed, or sharks have been spotted near the shore.
   d) If swimming at the New Jersey shore is allowed, then sharks have not been spotted near the shore.
   e) If sharks have not been spotted near the shore, then swimming at the New Jersey shore is allowed.
   f) If swimming at the New Jersey shore is not allowed, then sharks have not been spotted near the shore.
   g) Swimming at the New Jersey shore is allowed if and only if sharks have not been spotted near the shore.
   h) Swimming at the New Jersey shore is not allowed, and either swimming at the New Jersey shore is allowed or sharks have not been spotted near the shore. Note that we were able to incorporate the parentheses by using the word "either" in the second half of the sentence.
   i) Swimming at the New Jersey shore is allowed or Sharks have been spotted near the shore but not both.

3. a) This is just the negation of p, so we write $\neg p$.
   b) This is a conjunction ("but" means "and"): $p \land \neg q$.
   c) The position of the word "if" tells us which is the antecedent and which is the consequence: $p \to q$.
   d) $\neg p \to \neg q$
   e) The sufficient condition is the antecedent: $p \to q$.
   f) $q \land \neg p$
   g) "Whenever" means "if : $q \to p$.

4. Many forms of the answers for this exercise are possible.
   a) One form of the converse that reads well in English is "I will ski tomorrow only if it snows today." We could state the contrapositive as "If I don't ski tomorrow, then it will not have snowed today." The inverse is "If it does not snow today, then I will not ski tomorrow."
   b) The proposition as stated can be rendered "If there is going to be a quiz, then I will come to class." The converse is "If I come to class, then there will be a quiz." (Or, perhaps even better, "I come to class only if there's going to be a quiz.") The contrapositive is "If I don't come to class, then there won't be a quiz." The inverse is "If there is not going to be a quiz, then I don't come to class."
   c) There is a variable ("a positive integer") in this sentence, so technically it is not a proposition. Nevertheless, we can treat sentences such as this in the same way we treat propositions.

Its converse is "A positive integer is a prime if it has no divisors other than 1 and itself." (Note that this can be false, since the number 1 satisfies the hypothesis but not the conclusion.) The contrapositive of the original proposition is "If a positive integer has a divisor other than 1 and itself, then it is not prime." (We are simplifying a bit here, replacing "does not have no divisors" by "has a divisor." Note that this is always true, assuming that we are talking about positive divisors.) The inverse is "If a positive integer is not prime, then it has a divisor other than 1 and itself."

5.

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

6.

p	q	$q \to p$	$p \wedge (q \to p)$	$p \to (p \wedge (q \to p))$
T	T	T	T	T
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

7.

p	q	$p \to q$	$\neg q \to \neg p$	$(p \to q) \leftrightarrow (\neg q \to \neg p)$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

8.

p	q	$\neg q$	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \to (p \oplus \neg q)$
T	T	F	F	T	T
T	F	T	T	F	F
F	T	F	T	F	F
F	F	T	F	T	T

9. Lets consider p: Germany, q: England, and r: Argentina
We need to demonstrate that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent.

$\boldsymbol{p}$	$\boldsymbol{q}$	$\boldsymbol{r}$	$\boldsymbol{q \wedge r}$	$\boldsymbol{p \vee (q \wedge r)}$	$\boldsymbol{p \vee q}$	$\boldsymbol{p \vee r}$	$\boldsymbol{(p \vee q) \wedge (p \vee r)}$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	T	F	F
F	F	T	F	F	F	T	F
F	F	F	F	F	F	F	F

10. a) There is a student who spends more than five hours every weekday in class.
b) Every student spends more than five hours every weekday in class.
c) There is a student who does not spend more than five hours every weekday in class.
d) No student spends more than five hours every weekday in class. (Or, equivalently, every student spends less than or equal to five hours every weekday in class.)

11. a) This statement is that for every x, if x is a comedian, then x is funny. In English, this is most simply stated, ”Every comedian is funny.”
    b) This statement is that for every x in the domain (universe of discourse), x is a comedian and x is funny. In English, this is most simply stated, ”Every person is a funny comedian.”
    c) This statement is that there exists an x in the domain such that if x is a comedian then x is funny. In English, this might be rendered, ”There exists a person such that ifs/he is a comedian, then s/he is funny.”
    d) This statement is that there exists an x in the domain such that x is a comedian and x is funny. In English, this might be rendered, ”There exists a funny comedian” or ”Some comedians are funny” or ”Some funny people are comedians.

12. a) Every person who is taking STA201 has a Facebook page.
    b) There is a person who is taking STA201 and teaching CSE230.
    c) It is not the case that everyone taking STA201 has a Facebook page or likes to cook.

13. Let P(x) be ”x is perfect”; let F(x) be ”x is your friend”; and let the domain (universe of discourse) be all people.
    a) $\forall x \neg P(x)$
    b) $\neg \forall x P(x)$
    c) $\neg \forall x(F(x) \to P(x))$
    d) $\exists x(F(x) \wedge P(x))$
    e) $\forall x(F(x) \wedge P(x)$
    f) $(\neg \forall x F(x)) \vee (\exists x \neg P(x))$

14. a)$\exists x(P(x) \wedge Q(x))$
    b)$\exists x(P(x) \wedge \neg Q(x))$
    c)$\forall x(P(x) \vee Q(x))$
    d)$\neg \exists x(P(x) \vee Q(x))$

15. a)$\neg \exists(P(x) \wedge Q(x))$
    b) $\forall x(Q(x) \to R(x)$
    c) This is similar to part (a): $\forall x(P(x) \to \neg R(x))$