Practice sheet of Induction & Recursion

1. Using the principle of mathematical induction, prove that

$1^2 + 2^2 + 3^2 + ... + n^2 = (1/6)\{n(n + 1)(2n + 1\}$ for all $n \in N$.

2. Using the principle of mathematical induction, prove that

$1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + ... + (2n - 1)(2n + 1) = (1/3)\{n(4n^2 + 6n - 1)$.

3. By induction prove that $3^n - 1$ is divisible by 2 and is valid for all positive integers.

4. By induction prove that $n^2 - 3n + 4$ is even and valid for all positive integers.

5. Show that $n! > 3^n$ for $n \geq 7$.

6. Use the Principle of Mathematical Induction to verify that, for n any positive integer, $6^n - 1$ is divisible by 5.

7. Check that $a_n = 2^n + 1$ is a solution to the recurrence relation $a_n = 2a_{n-1} - 1$ with $a_1 = 3$.

8. Determine the recursive formula for the sequence 4,8,16,32,64, 128,….?

9. Use iteration to solve the recurrence relation $a_n = a_{n-1} + n$ with $a_n = 4$.

10. Solve the recurrence relation $a_n=7a_{n-1} - 10a_{n-2}$ With $a_0 =2$ and $a_1= 3$.

11. Solve the recurrence relation $a_n= 6a_{n-1}- 9a_{n-2}$ with initial conditions $a_0= 1$ and $a_1 = 4$.

12. Solve the recurrence relation Fn=10Fn-1−25Fn-2 where F0=3 and F1=17

13. Provide a recursive definition for $f(n) = n!$

14. Determine T(2), T(3), T(4), and T(5), if T(n) is recursively defined by T(0) = 2, T(1) = 2, and $T(n) = T(n - 1) + 3T(n - 2) + 4$.

15. One of the most famous recursive definitions is for the Fibonacci sequence f0, f1, f2, ...

Base Case f0 = 0, f1 = 1.

Recursive Case ($n \geq 2$) $f_n = f_{n-1} + f_{n-2}$.

Compute the $f_2, f_3, \ldots, f_{10}$.