Chapter 4

Digital Transmission

4-2 ANALOG-TO-DIGITAL CONVERSION

A digital signal is superior to an analog signal because it is more robust to noise and can easily be recovered, corrected and amplified. For this reason, the tendency today is to change an analog signal to digital data. In this section we describe two techniques, pulse code modulation and delta modulation.

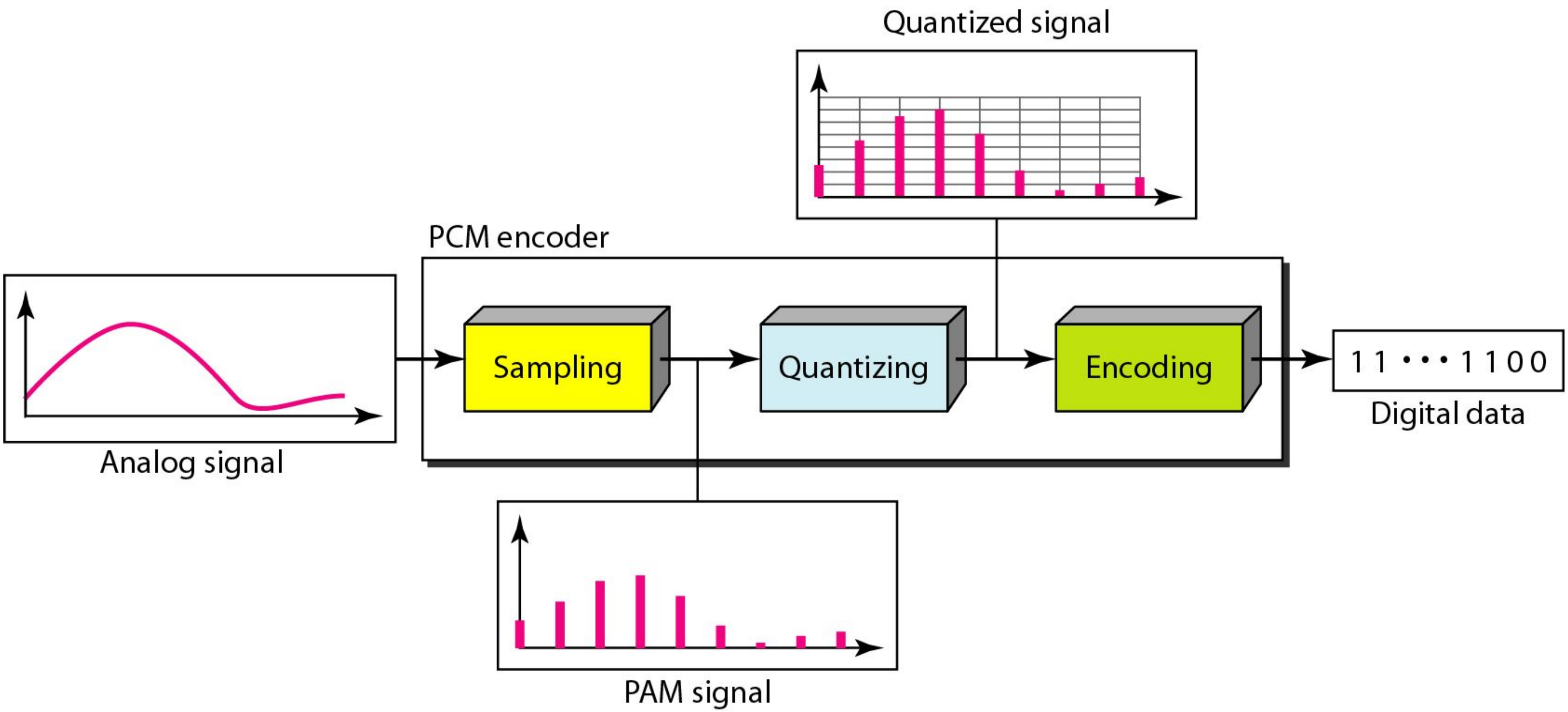
Topics discussed in this section:

- **Pulse Code Modulation (PCM)**
- **Delta Modulation (DM)**

PCM

- PCM consists of three steps to digitize an analog signal:
  1. Sampling
  2. Quantization
  3. Binary encoding
- Before we sample, we have to filter the signal to limit the maximum frequency of the signal as it affects the sampling rate.
- Filtering should ensure that we do not distort the signal, ie remove high frequency components that affect the signal shape.

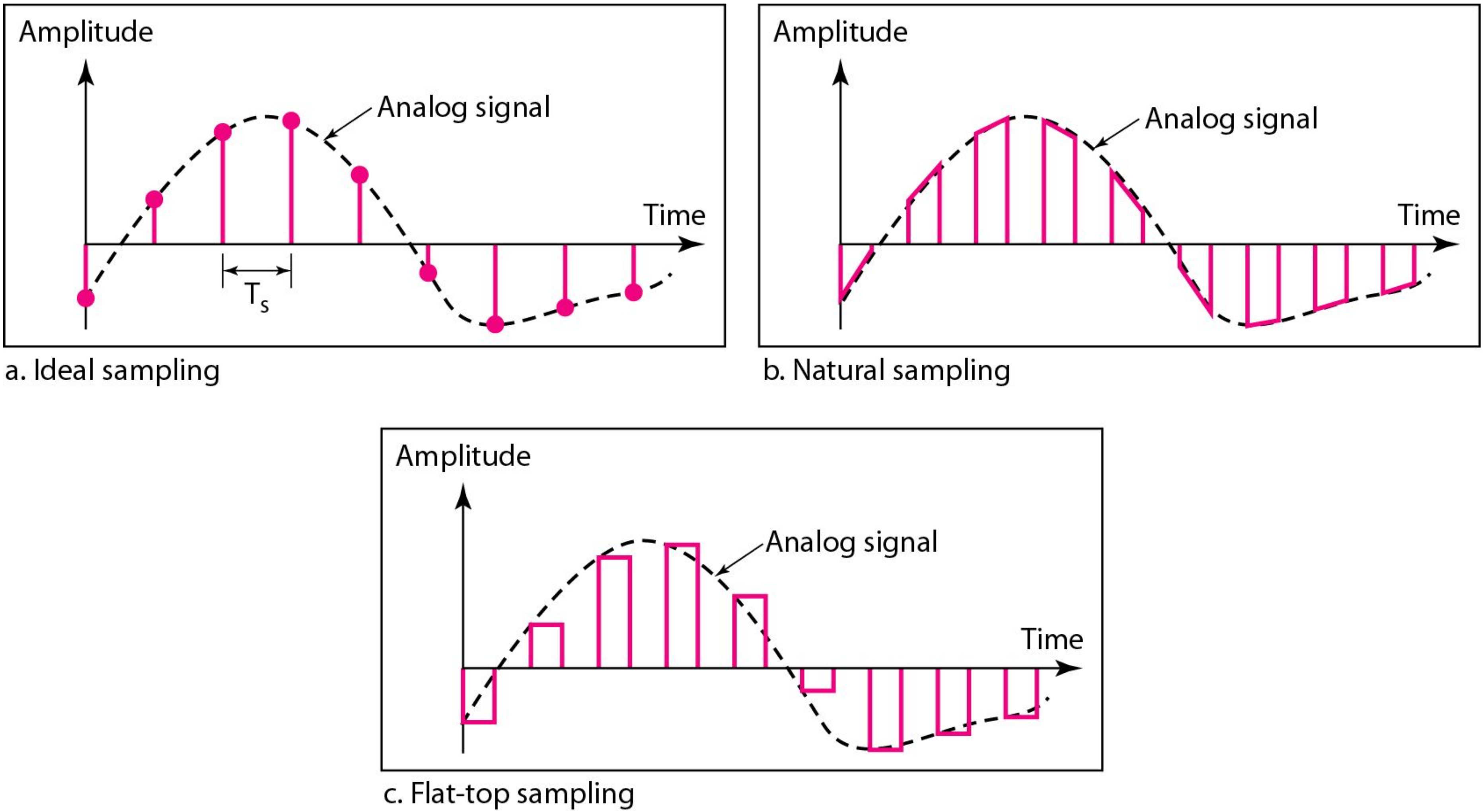
Figure 4.21 *Components of PCM encoder*



Sampling

- Analog signal is sampled every T_S secs.
- T_s is referred to as the sampling interval.
- $f_s = 1/T_s$ is called the sampling rate or sampling frequency.
- There are 3 sampling methods:
  - Ideal - an impulse at each sampling instant
  - Natural - a pulse of short width with varying amplitude
  - Flattop - sample and hold, like natural but with single amplitude value
- The process is referred to as pulse amplitude modulation PAM and the outcome is a signal with analog (non integer) values

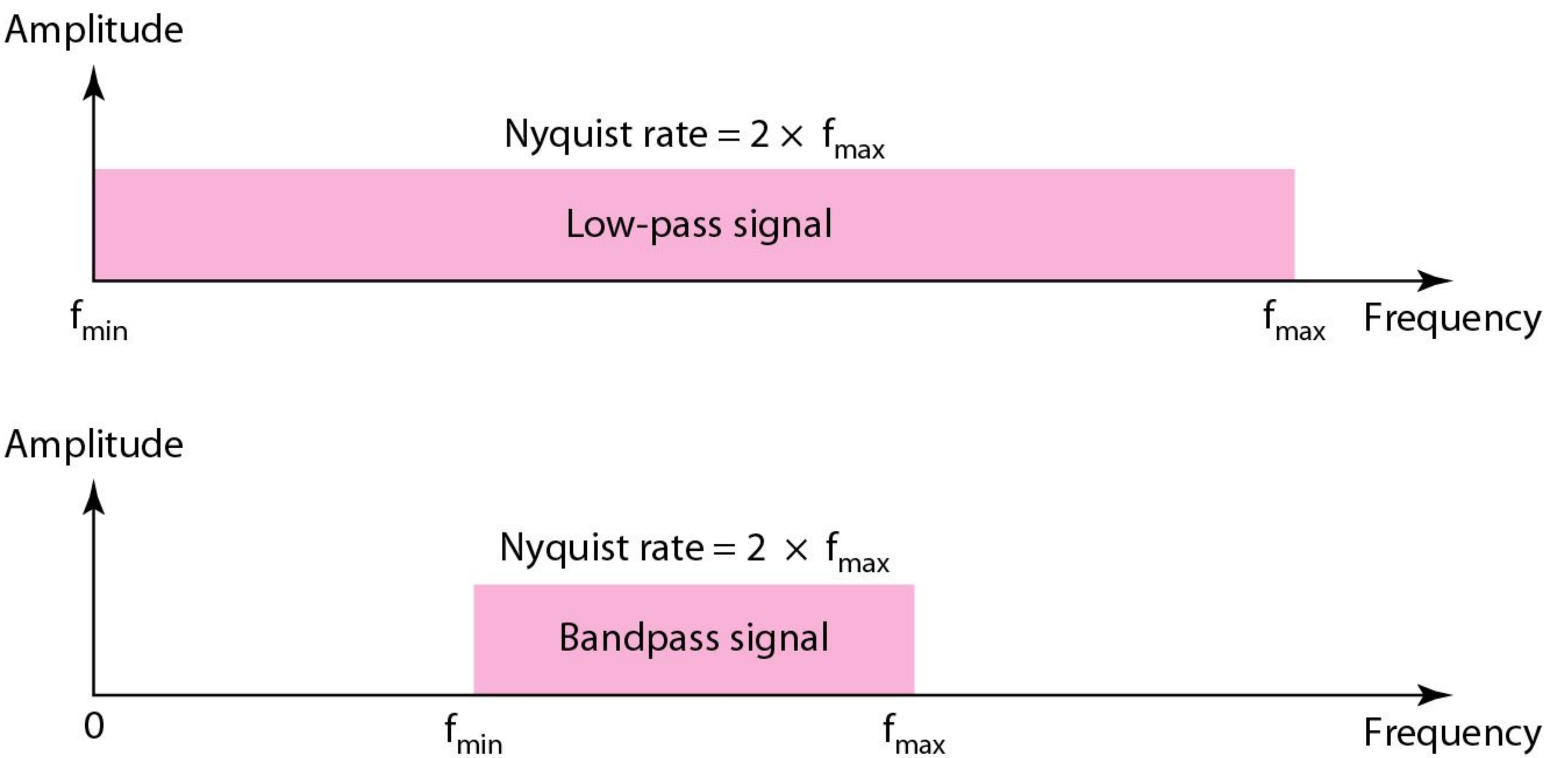
Figure 4.22 *Three different sampling methods for PCM*



Note

According to the Nyquist theorem, the sampling rate must be at least 2 times the highest frequency contained in the signal.

Figure 4.23 *Nyquist sampling rate for low-pass and bandpass signals*

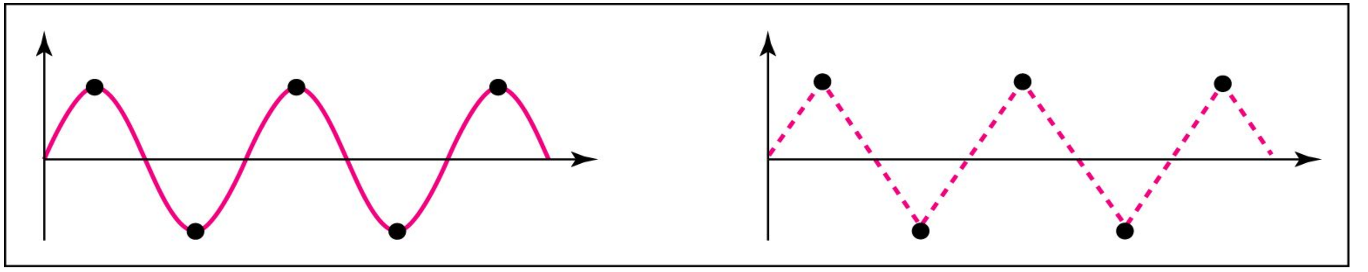


Example 4.6

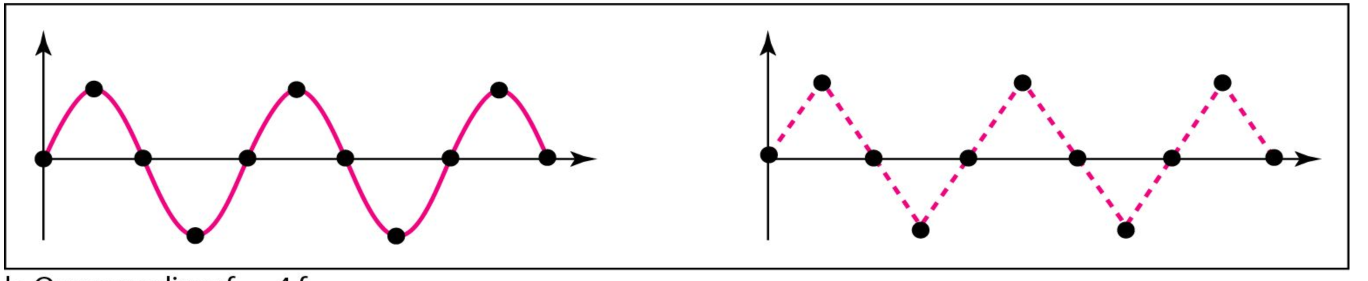
For an intuitive example of the Nyquist theorem, let us sample a simple sine wave at three sampling rates: $f_s = 4f$ (2 times the Nyquist rate), $f_s = 2f$ (Nyquist rate), and $f_s = f$ (one-half the Nyquist rate). Figure 4.24 shows the sampling and the subsequent recovery of the signal.

It can be seen that sampling at the Nyquist rate can create a good approximation of the original sine wave (part a). Oversampling in part b can also create the same approximation, but it is redundant and unnecessary. Sampling below the Nyquist rate (part c) does not produce a signal that looks like the original sine wave.

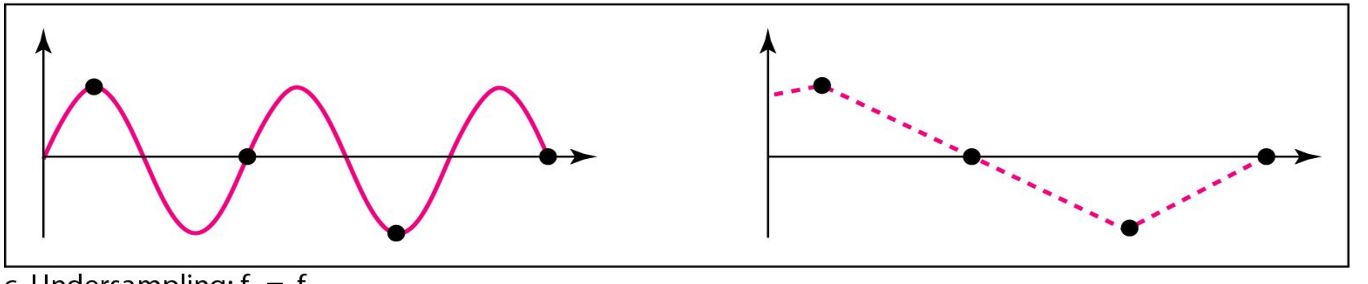
Figure 4.24 *Recovery of a sampled sine wave for different sampling rates*



a. Nyquist rate sampling: $f_s = 2f$



b. Oversampling: $f_s = 4f$

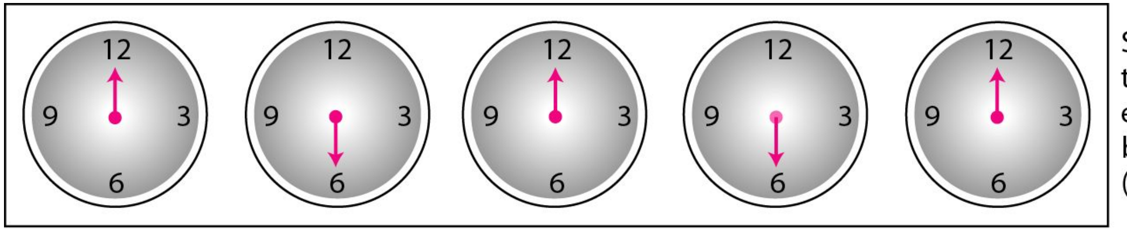


c. Undersampling: $f_s = f$

Example 4.7

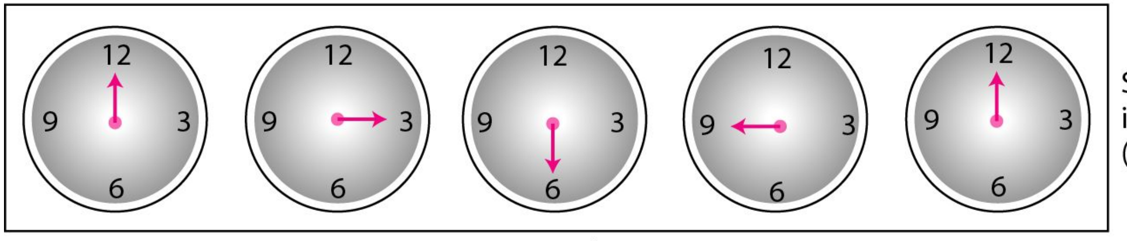
Consider the revolution of a hand of a clock. The second hand of a clock has a period of 60 s. According to the Nyquist theorem, we need to sample the hand every 30 s ($T_s = T$ or $f_s = 2f$). In Figure 4.25a, the sample points, in order, are 12, 6, 12, 6, 12, and 6. The receiver of the samples cannot tell if the clock is moving forward or backward. In part b, we sample at double the Nyquist rate (every 15 s). The sample points are 12, 3, 6, 9, and 12. The clock is moving forward. In part c, we sample below the Nyquist rate ($T_s = T$ or $f_s = f$). The sample points are 12, 9, 6, 3, and 12. Although the clock is moving forward, the receiver thinks that the clock is moving backward.

Figure 4.25 *Sampling of a clock with only one hand*



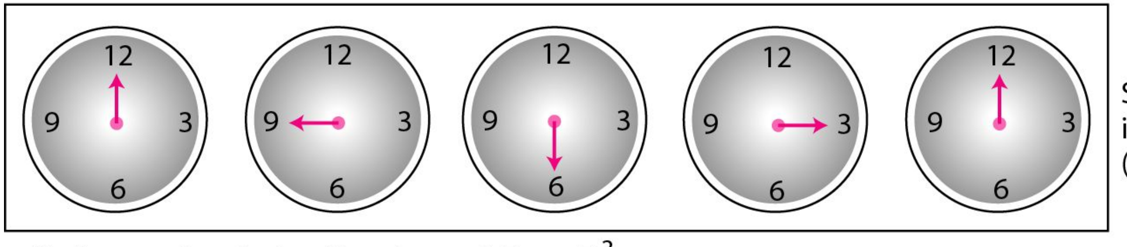
Samples can mean that the clock is moving either forward or backward. (12-6-12-6-12)

a. Sampling at Nyquist rate: $T_s = T\frac{1}{2}$



Samples show clock is moving forward. (12-3-6-9-12)

b. Oversampling (above Nyquist rate): $T_s = T\frac{1}{4}$



Samples show clock is moving backward. (12-9-6-3-12)

c. Undersampling (below Nyquist rate): $T_s = T\frac{3}{4}$

Example 4.8

An example related to Example 4.7 is the seemingly backward rotation of the wheels of a forward-moving car in a movie. This can be explained by under-sampling. A movie is filmed at 24 frames per second. If a wheel is rotating more than 12 times per second, the under-sampling creates the impression of a backward rotation.

Example 4.9

Telephone companies digitize voice by assuming a maximum frequency of 4000 Hz. The sampling rate therefore is 8000 samples per second.

Example 4.10

A complex low-pass signal has a bandwidth of 200 kHz. What is the minimum sampling rate for this signal?

Solution

The bandwidth of a low-pass signal is between 0 and f, where f is the maximum frequency in the signal. Therefore, we can sample this signal at 2 times the highest frequency (200 kHz). The sampling rate is therefore 400,000 samples per second.

Example 4.11

A complex bandpass signal has a bandwidth of 200 kHz. What is the minimum sampling rate for this signal?

Solution

We cannot find the minimum sampling rate in this case because we do not know where the bandwidth starts or ends. We do not know the maximum frequency in the signal.

Quantization

- Sampling results in a series of pulses of varying amplitude values ranging between two limits: a min and a max.
- The amplitude values are infinite between the two limits.
- We need to map the *infinite* amplitude values onto a finite set of known values.
- This is achieved by dividing the distance between min and max into L zones, each of height Δ.

$$\Delta = (\max - \min)/L$$

Quantization Levels

- The midpoint of each zone is assigned a value from 0 to L-1 (resulting in L values)
- Each sample falling in a zone is then approximated to the value of the midpoint.

Quantization Zones

- Assume we have a voltage signal with amplitutes V_{min}=-20V and V_{max}=+20V.
- We want to use L=8 quantization levels.
- Zone width $\Delta = (20 - -20)/8 = 5$
- The 8 zones are: -20 to -15, -15 to -10, -10 to -5, -5 to 0, 0 to +5, +5 to +10, +10 to +15, +15 to +20
- The midpoints are: -17.5, -12.5, -7.5, -2.5, 2.5, 7.5, 12.5, 17.5

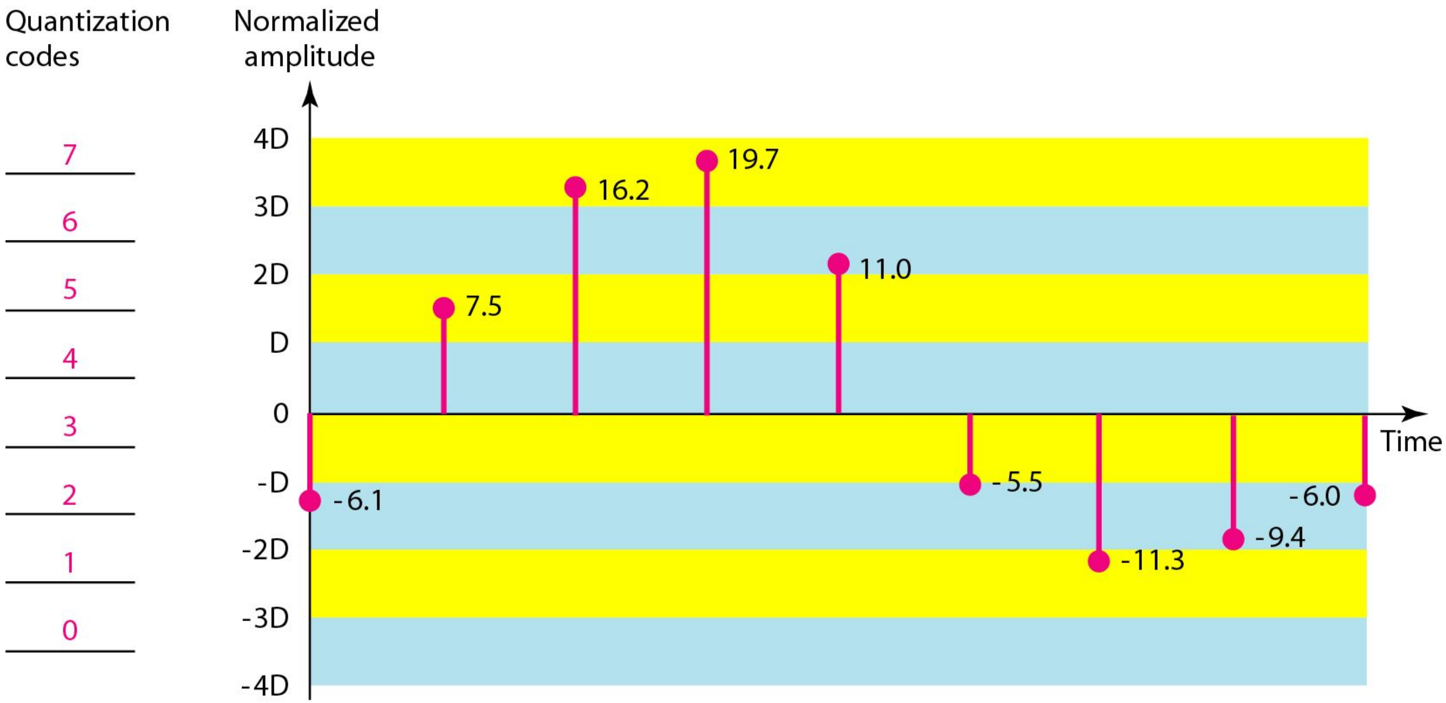
Assigning Codes to Zones

- Each zone is then assigned a binary code.
- The number of bits required to encode the zones, or the number of bits per sample as it is commonly referred to, is obtained as follows:

$$n_b = \log_2 L$$

- Given our example, $n_b = 3$
- The 8 zone (or level) codes are therefore: 000, 001, 010, 011, 100, 101, 110, and 111
- Assigning codes to zones:
  - 000 will refer to zone -20 to -15
  - 001 to zone -15 to -10, etc.

Figure 4.26 *Quantization and encoding of a sampled signal*



Normalized PAM values	-1.22	1.50	3.24	3.94	2.20	-1.10	-2.26	-1.88	-1.20
Normalized quantized values	-1.50	1.50	3.50	3.50	2.50	-1.50	-2.50	-1.50	-1.50
Normalized error	-0.38	0	+0.26	-0.44	+0.30	-0.40	-0.24	+0.38	-0.30
Quantization code	2	5	7	7	6	2	1	2	2
Encoded words	010	101	111	111	110	010	001	010	010

Quantization

- Normalized PAM value is the ratio of
  - Sampled Pulse value/zone width
  - 16.2/5 =3.24

- Normalized Quantized value is the ratio of
  - Midpoint/zone width
  - 17.5/5 = 3.5
- Normalized error = 3.5 - 3.24 = 0.26

Quantization Error

- When a signal is quantized, we introduce an error - the coded signal is an approximation of the actual amplitude value.
- The difference between actual and coded value (midpoint) is referred to as the quantization error.
- The more zones, the smaller Δ which results in smaller errors.
- BUT, the more zones the more bits required to encode the samples -> higher bit rate

Quantization Error and SN_QR

- Signals with lower amplitude values will suffer more from quantization error as the error range: $\Delta/2$, is fixed for all signal levels.
- Non linear quantization is used to alleviate this problem. Goal is to keep SN_QR fixed for all sample values.
- Two approaches:
  - The quantization levels follow a logarithmic curve. Smaller Δ's at lower amplitudes and larger Δ's at higher amplitudes.
  - Companding: The sample values are compressed at the sender into logarithmic zones, and then expanded at the receiver. The zones are fixed in height.

Bit rate and bandwidth requirements of PCM

- The bit rate of a PCM signal can be calculated form the number of bits per sample x the sampling rate

$$\text{Bit rate} = n_b \times f_s$$

- The bandwidth required to transmit this signal depends on the type of line encoding used. Refer to previous section for discussion and formulas.
- A digitized signal will always need more bandwidth than the original analog signal. Price we pay for robustness and other features of digital transmission.

Example 4.14

We want to digitize the human voice. What is the bit rate, assuming 8 bits per sample?

Solution

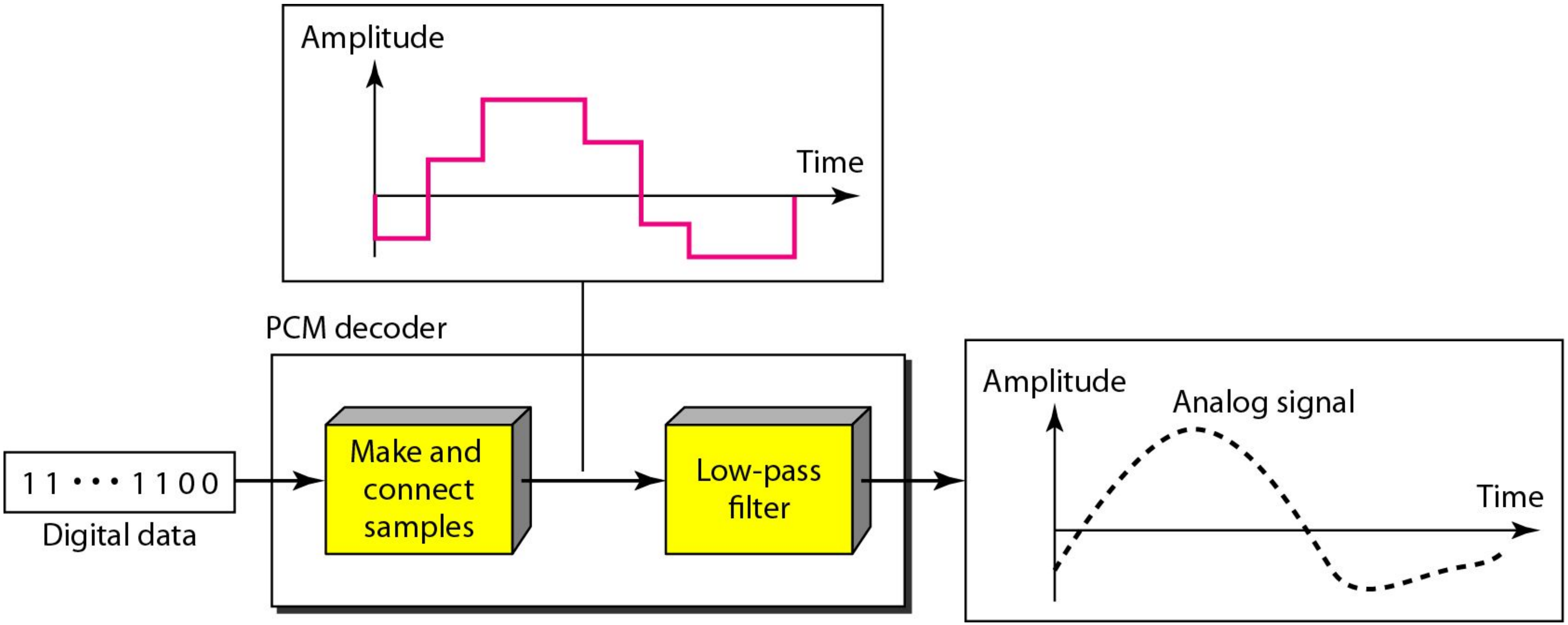
The human voice normally contains frequencies from 0 to 4000 Hz. So the sampling rate and bit rate are calculated as follows:

$$\text{Sampling rate} = 4000 \times 2 = 8000 \text{ samples/s}$$
$$\text{Bit rate} = 8000 \times 8 = 64{,}000 \text{ bps} = 64 \text{ kbps}$$

PCM Decoder

- To recover an analog signal from a digitized signal we follow the following steps:
  - We use a hold circuit that holds the amplitude value of a pulse till the next pulse arrives.
  - We pass this signal through a low pass filter with a cutoff frequency that is equal to the highest frequency in the pre-sampled signal.
- The higher the value of L, the less distorted a signal is recovered.

Figure 4.27 *Components of a PCM decoder*



Example 4.15

We have a low-pass analog signal of 4 kHz. If we send the analog signal, we need a channel with a minimum bandwidth of 4 kHz. If we digitize the signal and send 8 bits per sample, we need a channel with a minimum bandwidth of 8 × 4 kHz = 32 kHz.