CSE331: Automata and Computability
Practice Sheet
Prepared By: KKP

Question 1: Designing Context-Free Grammars

Give a Context-free Grammar that generates the language

a) L = {w ∈ {0,1}* : w starts with ‘**0**’ and the length of w is even.}
b) L = {w ∈ {a,b}*: every second letter in w is a ‘**b**’.}
c) L = {w ∈ {a,b}*: the length of w is divisible by three}
d) L = {w ∈ {0,1}* : w = 0^n1^{2n+3}, n ≥ 0 }
e) L = {w ∈ {0,1}* : w = 0^n1^n, where n is odd.}
f) L = {w ∈ {a,b}* : the number of ‘**a**’ is at least the number of ‘**b**’ in w.}
[Hint:https://www.youtube.com/watch?v=QF3lTaM56lo&list=PLBENQsMXh3gz85EJ3ZCSa9l9hnUiOer-H&index=35]
g) L = {w ∈ {0,1,2}*: w=$0^i1^j2^k$ where j ≥ 2i + 3k, and i, k ≥ 0}
h) L = {w ∈ {0,1,2,3}*: w=$0^i1^j2^k3^m$ where i=m, j ≥ 3k+2, and m, k ≥ 0}
i) L = {w ∈ {0,1,2}*: w=$0^i1^j2^k$ where i > 2j + 3k, and j, k ≥ 0}
j) If A = {w ∈ {0, 1}* : w contains at least two 0s}, then construct L = {w ∈ {0, 1}∗: w = $0^{3i}v1^{2i}$ where v ∈ A and i ≥ 0}
k) Recall that for a string w, |w| denotes the length of w. Σ = {0,1}
L1 = {w ∈ Σ^*: w contains 11}
L2 = {x#y : y ∈ L1, x ∈ Σ^*, |x| = |y|}
Construct a CFG for L2.
l) L = {w ∈ {0,1}* : w_1#w_2: number of 0s w_1 is equal to number of 1s in w_2.}
m) L = {w ∈ {0,1}* : w_1#w_2: length of w_2 is double of length of w_1.}
n) Recall that for a string w, |w| denotes the length of w. Σ = {0,1}
L1 = {w ∈ Σ^*: w contains exactly two 1s}
L2 = {x#y : x ∈ Σ^*, y ∈ L1, |x| = |y|}
Construct a CFG for L2.
o) L = {$1^i02^j1^k$| i, j, k ≥ 0, 3i ≥ 4k + 2, j is not divisible by three}

Construct a Context-free Grammar for the language expressed by the following regular expression:

a) ((ab)* + (a+b^3) cb)*
b) (a*b* + (ac+b*c) b)*

Question 2: Derivation, Parse tree, Ambiguity

I. S→ASB|SS|SAS|A

A→ASS|BS|B
B→00|11|01|1

Given the above Context-Free Grammar, answer the following questions:

a) Show the Left Most Derivation of 00010111
b) Sketch the parse tree corresponding to the derivation you gave in (a).
c) Show the Right Most Derivation of 00010111
d) Sketch the parse tree corresponding to the derivation you gave in (c).

II. A → A1 | 0A1 | 01

Given the above Context-Free Grammar, answer the following questions:

a) Give a leftmost derivation for the string 001111.
b) Sketch the parse tree corresponding to the derivation you gave in (a).
c) Demonstrate that there are two more parse trees (apart from the one you already found in (b)) for the same string.
d) Find a string w of length six such that w has exactly one parse tree in the grammar above.

III. S → 0S1 | 1S0 | SS | 01 | 10

Given the above Context-Free Grammar, answer the following questions:

a) Show that the grammar above is ambiguous by demonstrating two different parse trees for 011010.
b) Find a string w of length six such that w has exactly one parse tree in the grammar above.

Question 3: Chomsky Normal Form - CNF [Each Question Carries 10 marks]

a) Convert the following grammar into Chomsky Normal Form. You must show work.

S → bBS | ε
B → bYb | bY
C → Ccb | c | A
Y → bcY | ε | YBF

Here b, and c are terminals and the rest are variables.

b) Convert the following grammar into Chomsky Normal Form. You must show work.

S→YaSb
X→aXY | bX | Y
Y→X | b | ϵ

Here a, and b are terminals and the rest are variables.

c) Convert the following grammar into Chomsky Normal Form. You must show work.

S → bXaY |ZXb
X → aY | bY | Y
Y → X | c | ε
Z → ZaX

Here a, b, c are terminals and the rest are variables.

d)

Problem 3 (CO2): Chomsky Normal Form (10 points)

(a) **List** the rules that violate the conditions of Chomsky Normal form in the following grammar. Here a, b, and c are terminals and the rest are variables.

$$A \rightarrow BC \mid bB \mid a$$
$$B \rightarrow bb \mid Cb \mid b \mid C$$
$$C \rightarrow \varepsilon$$

(b) **Write** down the additional rules that need to be added to the following grammar if the production $B \rightarrow \varepsilon$ is removed. Here 0 and 1 are terminals and the rest are variables.

$$S \rightarrow AB \mid 1$$
$$A \rightarrow BAB \mid ABA \mid B \mid 11$$
$$B \rightarrow 00 \mid \varepsilon$$

(c) **Write** down the additional rules that need to be added to the following grammar if the unit productions are removed. Here 0 and 1 are terminals and the rest are variables.

$$S \rightarrow XYX \mid YX \mid X \mid Y$$
$$Y \rightarrow XY \mid X0 \mid 0$$
$$X \rightarrow 1 \mid Y$$

Question 4: CYK Algorithm [Each Question Carries 10 marks]

a) Consider the following grammar in CNF form.

S→BC | CD
B→CB | c
C→DD | c
D→BC | b

Show if string w=cccbb can be derived from the grammar above.

b) Apply the CYK algorithm to determine whether the string “abcaa” can be derived in the following grammar. You must show the entire CYK table. Here a, b, c are terminals and the rest are variables.

S → CA
A → AA | AD | a
B → AB | BC | b
C → CA | BC | c
D → a

Question 5: Pushdown Automata [Each Question Carries 5 marks]

a) L = { w ∈ {0,1}* : w = 0^n1^n, where n is odd.}
b) L = { w ∈ {0,1}* : w = 0^m1^n, where m,n ≥ 1 and m≥n.}
c) L = { w ∈ {0,1}* : w = $0^{n+2}1^n$, where n ≥ 0.}
d) L = { w ∈ {0,1}* : w = 0^n1^{2n}, where n ≥ 0.}
e) L = { w ∈ {0,1}* : w is a palindrome.}
[what if it says even length palindrome or odd length palindrome specifically]
f) L = { w ∈ {0,1}*: w | the length of w is divisible by four}
g) L1 = { w ∈ {0,1}*: the number of 1s in w is multiple of 3.}
L2 = { w ∈ {0,1}* : w contains even numbers of 0.}

Construct a PDA for L = { w ∈ {0,1}*: w = uv, where u ∈ L1,v ∈ L2 and |u| = |v| }

h) L = { w ∈ {0,1}* : w = $0^i1^j0^k$, where j = i+k and i, k ≥ 0.}

i) L = { w ∈ {0,1}* : w_1#w_2: number of 00 substrings in w_1 is equal to number of 11 in w_2.}

i)

Construct a pushdown automaton for the following language.

$$L = \{w\overline{w}^R : w \in \{0,1\}^*\}$$

Here, $\overline{w}^R$ denotes the reverse complement of the string w.

For example, 01001101 ∈ L because the second half of the string, 1101, is the reverse complement of the first half, 0100, i.e. $1101 = \overline{0100}^R$.