

CSE 331: Automata & Computability  
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Practice Sheet (Regular Expression)

1. Extended definition of Regular Language.

2. Derived the strings from the following expressions:

- a)**  $(0+1)^*$     **b)**  $(00)^*$     **c)**  $0^*1^*$     **d)**  $(0^*1^*)^*$     **e)**  $0^*1^* + (ab)^*$     **f)**  $(0+1)$     **g)**  $a$   
 $(0+1)b$     **h)**  $a(0+1)^*b$   
**i)**  $1^*01^* + ab$

3. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : w \text{ contains "101" as a substring.} \}$

4. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : w \text{ starts with "101".} \}$

5. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : w \text{ ends with "101".} \}$

6. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : w \text{ contains "00" or "11".} \}$

7. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : w \text{ contains exactly two 1s.} \}$

8. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : w \text{ contains at least two 1s.} \}$

9. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : w \text{ contains at most two 1s.} \}$
10. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : \text{The length of } w \text{ is even or multiple of 2.} \}$
11. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : \text{The length of } w \text{ is odd.} \}$
12. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : \text{The length of } w \text{ is multiple of 3.} \}$
13. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : \text{The length of } w \text{ is **not** multiple of 3.} \}$
14. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : \text{The number of 1's in } w \text{ is a multiple of 3.} \}$
15. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : w \text{ starts and ends with different symbols.} \}$
16. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : w \text{ starts and ends with the same symbols.} \}$

17. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : w \text{ doesn't end with } 01. \}$

18. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : 0\text{'s and } 1\text{'s alternates in } w. \}$

19. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : w \text{ doesn't contain } 00. \}$

20. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : w \text{ doesn't contain } 11. \}$

21. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : w \text{ doesn't contain } 111. \}$  [Practice]

22. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : w \text{ doesn't contain } 10. \}$

23. Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : w \text{ doesn't contain } 00 \text{ and } 11. \}$  Or, **Similar to Question: 18**

24. a) Construct a Regular Expression that generates the language  $L = \{ w \in \{0,1\}^* : w \text{ contains } 0 \text{ in every third position.} \}$

b) Regular language for  $\overline{L}$

25.

Consider the following languages over  $\Sigma = \{0, 1\}$ .

$$L_1 = \{w : w \text{ does not contain } 11\}$$

$$L_2 = \{w : \text{every } 1 \text{ in } w \text{ is followed by at least one } 0\}$$

$$L_3 = \{w : \text{the number of times } 1 \text{ appears in } w \text{ is even}\}$$

Now solve the following problems.

- (a) Give a regular expression for the language  $L_1$ . (2 points)
- (b) Your friend claims that  $L_1 = L_3$ . Prove her wrong by writing down a five-letter string in  $L_1 \setminus L_3$ . Recall  $L_1 \setminus L_3$  contains all strings that are in  $L_1$  but not in  $L_3$ . (2 points)
- (c) Give a regular expression for the language  $L_1 \setminus L_2$ . (2 points)
- (d) Give a regular expression for the language  $L_2$ . (2 points)
- (e) Give a regular expression for the language  $L_2 \setminus L_3$ . (2 points)

26.

Let  $\Sigma = \{0, 1\}$ . Consider the following languages over  $\Sigma$ .

$$L_1 = \{w : w = 1^m 0^n, \text{ where } m, n \geq 0\}$$

$$L_2 = \{w : 1 \text{ does not appear at any even position in } w\}$$

$$L_3 = L_1 \cap L_2$$

Now solve the following problems.

- (a) Give the state diagram for a DFA that recognizes  $L_1$ . (3 points)
- (b) Give the state diagram for a DFA that recognizes  $L_2$ . (3 points)
- (c) If you were to use the "cross product" construction shown in class to obtain a DFA for the language  $L_3$ , how many states would it have? (1 point)
- (d) Find all four-letter strings in  $L_3$ . (1 point)
- (e) Give the state diagram for a DFA that recognizes  $L_3$  using only three states. (2 points)