Course Title: Linear Algebra & Fourier Analysis

Course Code: MAT216

Assignment 01

Question 01: Determine the values of λ and μ, so that the following system of linear equations has

i) no solution ii) more than one solution iii) unique solution

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

Question 02: Solve the following system

(Use reduced row echelon form)

$$\frac{1}{x} + \frac{2}{y} - \frac{4}{z} = 1$$
$$\frac{2}{x} + \frac{3}{y} + \frac{8}{z} = 0$$
$$-\frac{1}{x} + \frac{9}{y} + \frac{10}{z} = 5$$

Question 03: Use the LU decomposition to find the solution of the following system

$$3x_1 - 6x_2 - 3x_3 = -3$$
$$2x_1 + 6x_3 = -22$$
$$-4x_1 + 7x_2 + 4x_3 = 3$$

Question 04: Suppose that a certain diet calls for 7 units of fat, 9 units of protein, and 16 units of carbohydrates for the main meal, and suppose that an individual has three possible foods to choose from to meet these requirements:

Food 1: Each ounce contains 2 units of fat, 2 units of protein, and 4 units of carbohydrates.
Food 2: Each ounce contains 3 units of fat, 1 unit of protein, and 2 units of carbohydrates.
Food 3: Each ounce contains 1 unit of fat, 3 units of protein, and 5 units of carbohydrates

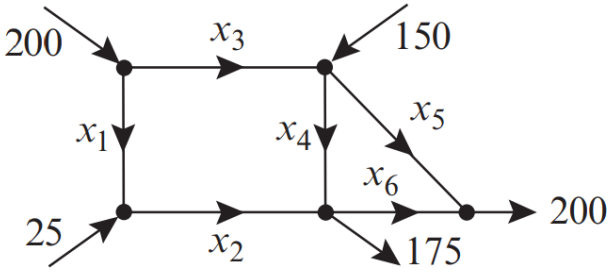
Let *x, y,* and *z* denote the number of ounces of the first, second, and third foods that the dieter will consume at the main meal. Find a linear system in *x, y,* and *z* whose solution tells how many ounces of each food must be consumed to meet the diet requirements.

Question 05: The accompanying figure shows known flow rates of hydro-carbons into and out of a network of pipes at an oil refinery.

(Use reduced row echelon form)

a) Set up a linear system whose solution provides the unknown flow rates.

b) Solve the system for the unknown flow rates.

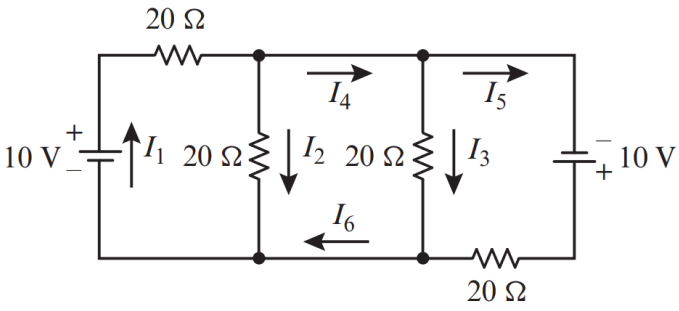


Question 06: Alka-Seltzer contains sodium bicarbonate $(NaHCO_3)$ and citric acid $(H_3C_6H_5O_7)$. When a tablet is dissolved in water, the following reaction produces sodium citrate, water, and carbon dioxide (gas):

$$NaHCO_3 + H_3C_6H_5O_7 \rightarrow Na_3C_6H_5O_7 + H_2O + CO_2$$

Hints: The Law of conservation of mass says that "Atoms are neither created, nor destroyed, during any chemical reaction." Thus, the same collection of atoms is present after a reaction as before the reaction.

Question 07: Find I_n, where $n = 1, 2, 3, 4, 5, 6$



Hints: Kirchhoff's law:

i) The total current entering a junction or a node is equal to the charge leaving the node as no charge is lost.

ii) The voltage around a loop equals the sum of every voltage drop in the same loop for any closed network and equals zero.

Question 08: Find the cubic polynomial whose graph passes through the points $(0, 1), (-1, -1), (1, 3), (4, -1)$

Question 09: Use the following Correspondence to decode the message.

A=1	D=4	G=7	J=10	M=13	P=16	S=19	V=22	Y=25
B=2	E=5	H=8	K=11	N=14	Q=17	T=20	W=23	Z=26
C=3	F=6	I=9	L=12	O=15	R=18	U=21	X=24	Space=0/27

Decode the message using the key $K = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{pmatrix}$ and find the inverse matrix using $(K|I) \rightarrow (I|K^{-1})$

11	20	43	25	10	41	22	14	41

OR,

$$\begin{bmatrix} 11 \\ 20 \\ 43 \end{bmatrix}, \begin{bmatrix} 25 \\ 10 \\ 41 \end{bmatrix}, \begin{bmatrix} 22 \\ 14 \\ 41 \end{bmatrix}$$

Question 10: Use the following Correspondence to decode the message.

A=1	D=4	G=7	J=10	M=13	P=16	S=19	V=22	Y=25
B=2	E=5	H=8	K=11	N=14	Q=17	T=20	W=23	Z=26
C=3	F=6	I=9	L=12	O=15	R=18	U=21	X=24	Space=0/27

Decode the message using the key $K = \begin{pmatrix} -3 & -3 & -4 \\ 0 & 1 & 1 \\ 4 & 3 & 4 \end{pmatrix}$

$$\begin{bmatrix} -122 \\ 23 \\ 138 \end{bmatrix}, \begin{bmatrix} -123 \\ 19 \\ 139 \end{bmatrix}, \begin{bmatrix} -176 \\ 47 \\ 181 \end{bmatrix}, \begin{bmatrix} -182 \\ 41 \\ 197 \end{bmatrix}, \begin{bmatrix} -96 \\ 22 \\ 101 \end{bmatrix}, \begin{bmatrix} -91 \\ 10 \\ 111 \end{bmatrix}, \begin{bmatrix} -183 \\ 32 \\ 203 \end{bmatrix}$$

*** *Please do not copy from others. Your entire effort might go in vain for unfair play.*

The more you explain, the more you score.

GOOD LUCK!!!