6. Draw sketches and determine the Fourier Series...

Question

(0)

6. Draw sketches and determine the Fourier Series for the following functions.
   a. $s(x) = \frac{x}{\pi}$, for $-\pi < x < +\pi$
   b. $s(x) = 3|\sin x|$ for $0 \le x < 2\pi$
   c. $s(x) = \begin{cases} 2\sin x & \text{for } 0 \le x < \pi \\ 0 & \text{for } \pi \le x < 2\pi \end{cases}$
   d. $s(x) = \begin{cases} 1 & \text{for } 0 \le x < \pi \\ 0 & \text{for } \pi \le x < \pi \end{cases}$
   e. $s(x) = A - \frac{Ax}{P}$ for $0 \le x < P$

Expert Answer

This solution was written by a subject matter expert. It's designed to help students like you learn core concepts.

Step-by-step

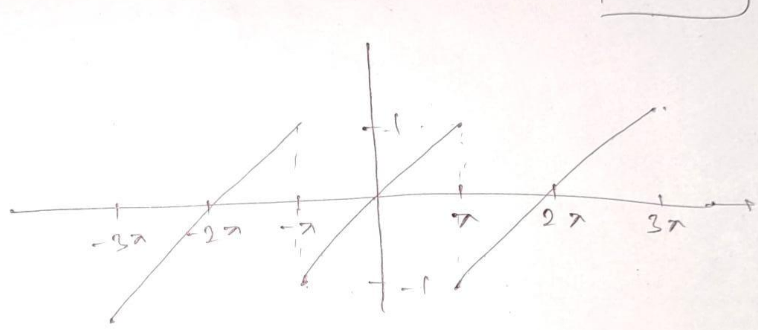
- 1st step
- All steps
- Answer only

Step 1/2

(a) $s(x) = \frac{x}{\pi}$; $-\pi < x < \pi$

time period $(T_0) = 2\pi \Rightarrow \omega_0 = \frac{2\pi}{T_0}$

$\boxed{\omega = 1}$



$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(x)\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{x}{\pi}\,dx$

$= \frac{1}{2\pi^2} \times \frac{1}{2}\left(x^2\right)_{-\pi}^{\pi} \Rightarrow \boxed{a_0 = 0}$

use formula of fourier series coefficient to expand in fourier series.

Explanation:

use formula of fourier series coefficient to expand in fourier series.

Step 2/2

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(x)\cos n\omega_0 x\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\frac{x}{\pi}\cos nx\,dx$$

$$a_n = \frac{1}{\pi^2}\left[\frac{1}{n}(x\sin nx)_{-\pi}^{\pi} + \frac{1}{n^2}(\cos nx)_{-\pi}^{\pi}\right]$$

$$\boxed{a_n = 0}$$

& $b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(x)\sin n\omega_0 x\,dx$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\frac{x}{\pi}\sin nx\,dx$$

$$\therefore \frac{1}{\pi^2}\left[\frac{-1}{n}(x\cos nx)_{-\pi}^{\pi} + \frac{1}{n^2}(\sin nx)_{-\pi}^{\pi}\right]$$

$$= -\frac{1}{n\pi^2}\{\pi(-1)^n + \pi(-1)^n\}$$

$$= -\frac{2(-1)^n}{n\pi} = \frac{2(-1)^{n+1}}{n\pi}$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n\cos n\omega_0 x + \sum_{n=1}^{\infty} b_n\sin n\omega_0 x$$

$$\boxed{f(x) = \sum_{n=1}^{\infty}\frac{2}{n\pi}(-1)^{n+1}\sin nx}$$

use formula of fourier series coefficient to expand in fourier series.

$$A_0 = \frac{1}{T}\int f(x)\,dx$$

$$A_n = \frac{2}{T}\int f(x)\cos(nw_o x)\,dx$$

$$B_n = \frac{2}{T}\int f(x)\sin(nw_o x)\,dx$$

Final answer

use formula of fourier series coefficient to expand in fourier series.

Was this answer helpful?