Measures of Central Tendency:

Ungrouped Data:

A manufacturer of electronic components is interested in determining the lifetime of certain type of battery. A sample in hours of life , is as follows:

123, 116, 122, 110, 175, 125, 125, 111, 118, 117

i)Find the sample mean, median and mode.
ii) Also Show that the sum of deviation of each value from their mean is zero.

Solution:

i)Mean, $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$

$$= \frac{123+116+.....+117}{10}$$

=124.2 hours

So the average lifetime of all the batteries is 124.2 hours

ii) The sum of deviation of each value from their mean is zero

Calculation Table

x_i	$\bar{x}$	$(x_i - \bar{x})$
123	**124.2**	**-1.2**
116		**-8.2**
122		**-2.2**
110		**-14.2**
175		**50.8**
125		**0.8**
125		**0.8**
111		**-13.2**
118		**-6.2**
117		**-7.2**
		$\sum_{i=1}^{n}(x_i - \bar{x}) = 0$

So The sum of deviation of each value from their mean is zero, $\sum_{i=1}^{n}(x_i - \bar{x}) = 0$

Median: Since n=10 is even

Arrange the data in ascending order:

110,111, 116, 117, 118, 122, 123, 125, 125, 175

Median=Average of $\frac{n}{2}$ **and** $(\frac{n}{2}+1)\ thobservation$

=Average of 5th and 6th observation

$= \frac{118+122}{2} = 120$

So the median lifetime is 120 hours i.e. The middle most battery has lifetime of 120 hours.

Mode:

Mode= The value in the series x_i **which has maximum frequency.**

x	Tally marks	fi
110	/	1
111	/	1
116	/	1
117	/	1
118	/	1
122	/	1
123	/	1
125	//	2
175	/	1
Total		10

Since 125 occurs most frequently. So, mode is 125.

Example of Grouped data:

The Lifetime of 40 similar car batteries is as follows:

Lifetime in years	Number of batteries
1.5-2.0	2
2.0-2.5	1
2.5-3.0	4
3.0-3.5	15
3.5-4.0	10
4.0-4.5	5
4.5-5.0	3

i)Find the mean, median, and mode

ii)Also show the median and mode graphically

Solution:

Calculation table:

Lifetime in years	**Number of batteries** f_i	x_i **=class midvalue**	$f_i x_i$
1.5-2.0	2	**1.75**	**3.5**
2.0-2.5	1	**2.25**	**2.25**
2.5-3.0	4	**2.75**	**11**
3.0-3.5	15	**3.25**	**48.75**
3.5-4.0	10	**3.75**	**37.5**
4.0-4.5	5	**4.25**	**21.25**
4.5-5.0	3	**4.75**	**14.25**

	N=40		$\sum_{i=1}^{n} f_i x_i = 138.5$

Sample Mean, $\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{N}$

$$= \frac{138.5}{40}$$

=3.4625 year.

So, the average lifetime of all the batteries is 3.4625 year.

Median group= $\frac{N}{2} = \frac{40}{2} = 20th\ observation$

Lifetime in years	**Number of batteries** f_i	x_i **=class midvalue**	**Cumulative Frequency**
1.5-2.0	2	**1.75**	**2**
2.0-2.5	1	**2.25**	**3**
2.5-3.0	4	**2.75**	**7**
3.0-3.5	15	**3.25**	**22 Median group**
3.5-4.0	10	**3.75**	**32**
4.0-4.5	5	**4.25**	**37**
4.5-5.0	3	**4.75**	**40**
	N=40		

4th group is the **Median group**

Median $= L + \dfrac{\dfrac{N}{2} - P.c.f}{f_m} \times c$

$= 3.0 + \dfrac{\dfrac{40}{2} - 7}{15} \times 0.5$

$= 3.43$years

So, the middle most batteries has lifetime of 3.43 years

Mode: 4th group is the modal group.

Here, $\Delta_1 = 15 - 4 = 11$

$\Delta_2 = 15 - 10 = 5$

Mode= $L + \dfrac{\Delta_1}{\Delta_1 + \Delta_2} \times c$

$= 3.0 + \dfrac{11}{11+5} \times 0.5$

=3.34 year

So the maximum batteries has a lifetime of 3.34 year.

Examples of geometric mean for ungrouped data:

Example: The annual rates of growth of output of a factory in 5 years are

5.0%, 2.5%, 7.5%, 10.0%, 5.0%.

Find the average rates of growth of output for the period.

Solution: So will calculate geometric mean because data are given in percentages.

Let the initial value is 100.

x_i	$\log x_i$
105.0	2.021

102.5	2.011
107.5	2.031
110.0	2.041
105.0	2.021
Total	$\sum \log x_i = 10.125$

We know log GM= $\frac{\sum_{i=1}^{n} \log x_i}{n} = \frac{10.125}{5}$

=2.025

GM=Antilog (2.025)=105.93

So the average rate of growth of output for the period is =105.93-100=5.93%

Example: In a certain factory a unit of work is completed by A in 4 minute, by B in 5 Minute, C in 6 minute, D in 10 minute, and E in 12 minute. What is the average number of unit of work completed per minute? Which average do you consider here and why?

Solution:

Since the information are given in unit of time. So we will use harmonic mean.

Calculation Table

	x_i	$\frac{1}{x_i}$

A	4	0.25
B	5	0.20
C	6	0.167
D	10	0.100
E	12	0.083
		$\sum_{i=1}^{n}\frac{1}{x_i}=0.8$

We know, HM= $\frac{n}{\sum_{i=1}^{n}\frac{1}{x_i}}$ =5/0.8=6.25 minute

The average number of unit of work completed per minute is 1/6.25=0.16unit.

Relation between AM, GM and HM:

For same data $AM \geq GM \geq HM$

i)Check: 10, 20, 30, 50, 60

Solution:

x_i	$\frac{1}{x_i}$	$\log x_i$
10	0.10	1.00
20	0.05	1.30
30	0.033	1.47
50	0.02	1.69
60	0.016	1.77

$\sum_{i=1}^{n} x_i = 170$ $\sum_{i=1}^{n}\frac{1}{x_i}=0.22$ $\sum_{i=1}^{n}\log x_i$ =7.23

We know AM= $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{170}{5} = 34$

$$\log GM = \frac{\sum_{i=1}^{n} \log x_i}{n} = \frac{7.23}{5} = 1.446$$

$$GM = Anti\log(1.446) = 27.93$$

$$HM = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}} = \frac{5}{.22} = 22.72$$

Since the *AM*=34, *GM*=27.93, and *HM*=22.72, So it is clear that $AM \geq GM \geq HM$.

ii)For same data $AM \geq GM \geq HM$

Profit in Lac Taka	**Number of companies**
25-35	4
35-45	7
45-55	15
55-65	14
65-75	8
75-85	2

Calculation Table

Class interval	x_i	f_i	$f_i x_i$	$f_i \log x_i$	$\frac{f_i}{x_i}$

25-35	30	4	120	5.908	.133
35-45	40	7	280	11.214	.175
45-55	50	15	750	25.485	.3
55-65	60	14	840	24.894	.233
65-75	70	8	560	14.761	.114
75-85	80	2	160	3.806	.025
		N=50	$\sum f_i x_i = 2710$	$\sum f_i log x_i = 86.068$	$\sum \frac{f_i}{x_i} = 0.98$

We know,

$$AM = \frac{\sum f_i x_i}{N} = \frac{2710}{50} = 54.2$$

We know,

$$logGM = \frac{\sum f_i log\, x_i}{N} = \frac{86.068}{50} = 1.72136$$

$$GM = Antilog(1.72136) = 52.65$$

We know,

$$HM = \frac{N}{\sum \frac{f_i}{x_i}} = \frac{50}{.98} = 51.020$$

Since the *AM*=54.2, *GM*=52.65, and *HM*=51.020, So it is clear that $AM \geq GM \geq HM$

Weighted average:

Ex : In a company, there are 50 skilled workers with daily wage 500 TK,25 semi-skilled workers with daily wage 400 Tk and 15 unskilled workers with daily wage 300 TK. What is the average daily wages of the workers?

Solution: Let , W_1 = no.of Skilled worker=50

W2= no. semi-skilled worker=25

W3= no. of unskilled worker=15

Weighted Average = $\frac{W_1X_1+W_2X_2+W_3X_3}{W_1+W_2+W_3}$

$$= \frac{50\times 500+25\times 400+15\times 300}{50+25+15}$$

=438.89TK

So the workers earn on average 438.89 Taka daily.