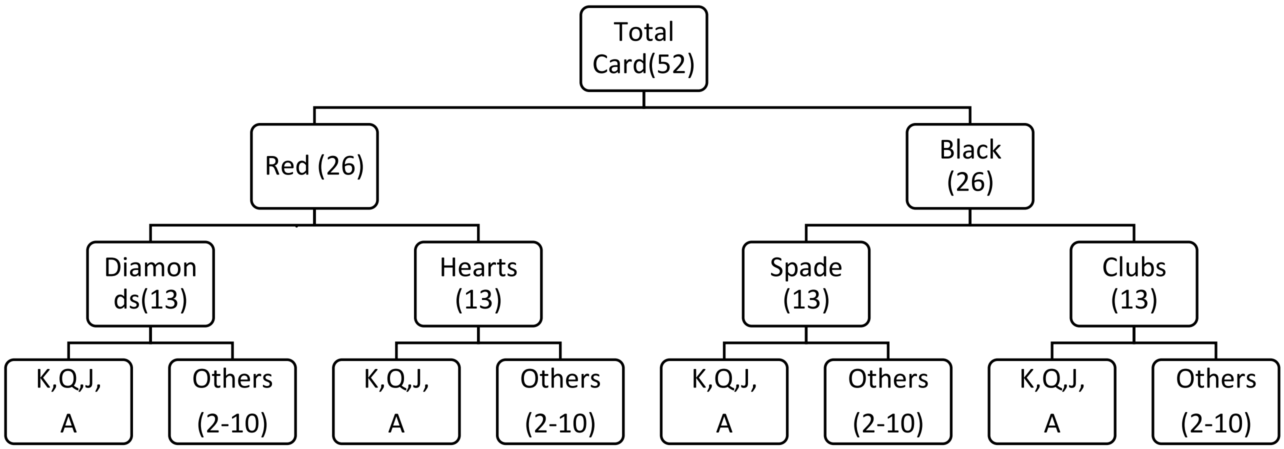
Problems about playing cards:



Problem 1:

A card is drawn from a pack of 52 cards. Estimate the probability of getting –

i. The card is red
ii. The card is a diamond
iii. The card is an ace
iv. The card is a spade
v. The card is hearts or king

Solution:

i. Let, event A :The card is a red

There are 26 red cards out of 52 cards.

So, the required probability, P(A) = $\frac{Number\ of\ favourable\ \ outcomes}{Total\ number\ of\ possible\ outcomes}$

$= \frac{26}{52}$

$= \frac{1}{2} = 0.5$

ii. Let, event B : The card is a diamond

There are 13 diamond cards out of 52 cards.

So, the required probability, P(B) = $\frac{Number\ of\ favourable\ \ outcomes}{Total\ number\ of\ possible\ outcomes}$

$= \frac{13}{52}$

$= \frac{1}{4} \quad = \quad 0.25$

iii.

Let, event C : The card is an ace.

There are 4 ace cards out of 52 cards.

So, the required probability, P(C) = $\frac{Number\ of\ favourable\ \ outcomes}{Total\ number\ of\ possible\ outcomes}$

$= \frac{4}{52}$

$= \frac{1}{13}$

iv.

Let, event D : The card is an spade.

There are 13 spade cards out of 52 cards.

So, the required probability, P(D) = $\frac{Number\ of\ favourable\ \ outcomes}{Total\ number\ of\ possible\ outcomes}$

$= \frac{13}{52}$

$= \frac{1}{4}$

v.

Let, E : the card is hearts

There are 13 heart cards out of 52 cards.

F : the card is king

There are 4 king cards out of 52 cards.

Normally, event E and F are not mutually exclusive events. Since the set of hearts contain a king.

There are 1 card which is king and heart.

So, the probability of getting hearts or king is,

$P(E \cup F) = P(E) + P(F) – P(E \cap F)$

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$

$= \frac{4}{13}$

Problem 2:

A bag contains 4 white, 3 black and 5 red balls. Estimate the probability of getting a white or red ball at random at a single draw.

Solution:

The bag contains 4 white, 3 black and 5 red balls.

A : The ball is white

B: The ball is red

So, the probability of getting a white or red ball, P (A U B) is-

P(AUB) = P(A) + P(B)   [as they are mutually exclusive]

$$= \frac{4}{12} + \frac{5}{12}$$

$$= \frac{9}{12}$$

$$= \frac{3}{4}$$

Problem 3:

Two bags contained 12 white , 7 red and 11 black ; 7 white, 10 red and 13 black respectively. One ball is drawn at random from each bag. Find the probability that-

i. Both balls are of white color.
ii. Both balls are of same color.

Solution:

In the first bag, 12 white, 7 red and 11 black balls. Total number of balls = 30

In the second bag, 7 white, 10 red and 13 black balls. Total number of balls = 30

(i) Event A: both balls are of white color. i.e., one white ball is drawn from the first bag and one white ball is drawn from the second bag.

The required probability, P(A)

= P(Probability of drawing white ball from first bag) . P(Probability of drawing white ball from second bag)

$$= \frac{12}{30} . \frac{7}{30}$$

$$= \frac{84}{900}$$

= 0.0933

(ii)

Event B : One red ball is drawn from the first bag and one red ball is drawn from the second bag.

Event C : One black ball is drawn from the first bag and one black ball is drawn from the second bag.

The required probability,

P(A or B or C) = P(AUBUC) = P(A) + P(B) + P(C)

P(B) = P(Probability of drawing red ball from first bag) . P(Probability of drawing red ball from second bag)

$= \frac{7}{30} \cdot \frac{10}{30}$

$= \frac{70}{900}$

= 0.0778

P(C) = P(Probability of drawing black ball from first bag) . P(Probability of drawing black ball from second bag)

$= \frac{11}{30} \cdot \frac{13}{30}$

$= \frac{143}{900}$

= .1589

So, P(AUBUC) = P(A) + P(B) + P(C)

= 0.0933 + .0778 + 0.1589

= 0.33

Problem 4:

One fair coin is tossed two times. Construct the sample space of the experiment. Find the probability of getting i) all head ii) at least one head iii) at best one head iv) a head and a tail.

Solution:

A fair coin is tossed for two times. The sample space of the experiment,

S = { HH, HT, TH, TT}

Total number of equally likely cases of sample points is, n(S) = 4

(i) Let , the event A : All head

The set of favourable cases of event A : {HH}, n(A) = 1

So, the required probability, P(A) = $\frac{n\,(A)}{n\,(S)}$

$= \frac{1}{4}$

(ii) Let , the event B : at least one head

The set of favourable cases of event A : {HH, HT, TH}, n(B) = 3

So, the required probability, P(B) = $\frac{n\,(B)}{n\,(S)}$

$= \frac{3}{4}$

(iii) Let , the event C : at best one head

The set of favourable cases of event C : {HT, TH, TT}, n(C) = 3

So, the required probability, P(C) = $\frac{n\,(C)}{n\,(S)}$

$= \frac{3}{4}$

(iv) Let , the event D : a head and a tail

The set of favourable cases of event D : {HT, TH}, n(A) = 2

So, the required probability, P(D) = $\frac{n\,(D)}{n\,(S)}$

$= \frac{2}{4}$

$= \frac{1}{2}$

Problem 5:

Two dice are thrown at random. Write down the sample space and estimate the probability of-

i. The total numbers on the dice is 8
ii. The first dice show 6
iii. Both the dice show the same number.

Solution:

When two dice are thrown, the sample space S are listed below-

S	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Total number of possible outcome n (S) = 36

(i) Let, event A : The total of the numbers on the dice is 8.

The set of favourable cases of event A : { (2,6), (6, 2), (4, 4), (3,5), (5, 3) }

The number of favourable cases of event A, n(A) = 5

Therefore, the probability, P(A) = $\frac{n\ (A)}{n\ (S)}$

$= \frac{5}{36}$

(ii) Let, event B : The first die show 6.

The set of favourable cases of event B : { (6,1), (6, 2), (6, 3), (6,4), (6, 5), (6, 6) }

The number of favourable cases of event B, n(B) = 6

Therefore, the probability, P(B) = $\frac{n\ (B)}{n\ (S)}$

$= \frac{6}{36}$

$= \frac{1}{6}$

(iii) Let, event C : Both dice show the same number.

The set of favourable cases of event C: { (1,1), (2, 2), (3, 3), (4, 4), (5,5), (6, 6) }

The number of favourable cases of event C, n(C) = 6

Therefore, the probability, P(C) = $\frac{n\,(C)}{n\,(S)}$

$$= \frac{6}{36}$$

$$= \frac{1}{6}$$

<u>Problem 6:</u>

Suppose 35% of the students failed in English, 25% of the students failed in statistics and 15% of the students failed in both English and statistics. A student is selected at random.

(i) If he failed in statistics, estimate the probability that he failed in English?
(ii) If he failed English, estimate is the probability that he failed in statistics?
(iii) Calculate the probability that he failed in English or statistics?

<u>Solution:</u>

Let us define two events E and S as follows :

E = The students who failed in English

S = The students who failed in statistics

It is given that,

P(E) = 35% =0.35

P(S) = 25% = 0.25

P(E∩S) = 15% = 0.15

(i)

The probability that a student failed in English, given that he was failed in statistics is,

$P(E|S) = \frac{P(E \cap S)}{P(S)}$

$= \frac{0.15}{0.25}$

$= 0.6$

(ii)

The probability that a student failed in statistics, given that he was failed in English is

$P(S|E) = \frac{P(S \cap E)}{P(E)}$

$= \frac{0.15}{0.35}$

$= 0.4286$

(iv) The probability that a student failed in English or statistics is –

$P(E \cup S) = P(E) + P(S) - P(E \cap S)$

$= 0.35 + .25 - 0.\ 15$

$= 0.45$

Problem 7:

Of 1000 assembled components, 10 have a working defect and 20 have a structural defect. There is a good reason to assume that no component has both defects. What is the probability that randomly chosen component will have either type of defect?

Solution:

Let us define the events A and B as follows :

A : The component has working defect

B : The component has structural defect.

From the information we have,

P(A) = $\frac{10}{1000} = 0.01,$

P(B) = $\frac{20}{1000} = 0.02,$

P(A∩B) $= 0$

Then the required probability,

P(A U B) = P(A) + P(B)

= 0.01 + 0.02

= 0.03

Problem 8:

Two events such that , P(A) = $\frac{1}{2}$ and P(B) = $\frac{1}{3}$ and P(A|B) = $\frac{1}{4}$. Find P(A U B) and P(A ∩ B).

Solution:

Here, P(A) = $\frac{1}{2}$ and P(B) = $\frac{1}{3}$ and P(A|B) = $\frac{1}{4}$

We know, P(A|B) $= \frac{P(A\cap B)}{P(B)}$

Then, P(A∩B) = P(A|B) P(B)

= $\frac{1}{4} \cdot \frac{1}{3}$

= $\frac{1}{12}$

Again we know,

P(AUB) = P(A) +P(B) – P(A∩B)

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{12}$

$= \frac{3}{4}$

Problem 9:

A candidate is selected for interview of management trainees for 3 companies. For the first companies there are 12 candidates, for the second there are 15 candidates, and for the third there are 10 candidates. What are the chances of his getting job at least one of thecompany?

Solution:

P(The candidate will get job in at least one of the company)=1-P(he will not get job in any company)

P(1)+P(2)+P(3)=1-P(0)

P(the candidate will not get job in the 1st company) = $1-\frac{1}{12} = \frac{11}{12}$

P(the candidate will not get job in the 2nd company) = $1-\frac{1}{15} = \frac{14}{15}$

P(the candidate will not get job in the 3rd company) = $1-\frac{1}{10} = \frac{9}{10}$

Since the events are independent. So the probability of not getting job is also independent.

P (the candidate will not get job in any company) = $\frac{11}{12} \cdot \frac{14}{15} \cdot \frac{9}{10} = 0.77$

Again, P(The candidate will get job in at least one of the company) =1- P(he will not get job in any company)=1-0.77=0.23

So, there is 23% chance that the candidate will get job in at least one of the companies.

Problem 10 :

A study showed that 65% of managers had some business education and 50% had some engineering education. Furthermore, 20 percent of the managers had some business education but no engineering education. What is the probability that a manager has some business education, given that he has some engineering education?

Solution:

Let, A= a manager has some business education

B= a manager has some engineering education

P(A)=0.65 and P(B)=0.50

$P(A \cap B) = 0.65 - 0.20 = 0.45$

We know,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.45}{0.50} = 0.9$$

There is 90% chance that a manager has business education given that he has some engineering education.

or

So, the probability that a manager has some business education, given that he has some engineering education is 0.90.

Problem 11:

Two factories manufacture the same machine part. Each part is classified having either 0,1,2 or 3 manufacturing defects. The joint probability distribution for this is given below:

Number of defects

	0	1	2	3
Manufacturer A	0.1250	0.0625	0.1875	0.1250
Manufacturer B	0.0625	0.0625	0.1250	0.2500

i) A part is observed to have no defects. What is the probability that it wasproduced by manufacturer A?

ii) A part is known to have been produced by manufacturer A. What is probability that the part has no defects?

iii) A part is known to have two or more defects. What is probability that it was manufactured by A?

iv) A part is known to have one or more defects. What is the probability that it was manufactured by B?

Solution:

Number of defects

	0	1	2	3	Total
Manufacturer A	0.1250	0.0625	0.1875	0.1250	.5
Manufacturer B	0.0625	0.0625	0.1250	0.2500	.5
Total	0.1875	0.1250	0.3125	0.3750	1.00

(i) P(A| No defect) = $\frac{P(\,A \text{ and no defects})}{P(\,No\ defect)} = \frac{0.125}{.1875} = 0.6667$

(ii) P(No defect |A) = $\frac{P(\,A \text{ and no defects})}{P(\,A)} = \frac{0.125}{.5} = 0.2500$

(iii) P(A | Two or more defects) = $\frac{P(\,A \text{ and two or more defects})}{P(\,\text{Two or more defects})} = \frac{.1875+.1250}{0.6875} = 0.4545$

(iv) P(B | one or more defects) = $\frac{P(\,B\ and\ one\ or\ more\ \ defects)}{P(\,one\ or\ more\ defects)} = \frac{.0625+.1250+.250}{0.8125} = 0.5385$

Problem 12:

An electrical system consists of four components. The system works if the components A and B work and either of the components C or D work. The reliability (Probability of working) of each component is also shown in the figure. Find the probability that i) The entire system works ii) the components C does not work given that the entire system works. Assume that all four components work independently.

P(A works) = 0.9

P(B works) = 0.9, P(C works) = 0.8, P(D works) = 0.8

Solution:

P(entire system work)=P(A ∩ B ∩ (C ∪ D))

$=P(A) \times P(B) \times P(C \cup D)$

$= P(A) \times P(B) \times \{P(C) + P(D) - P(C \cap D)\}$

$= P(A) \times P(B) \times \{P(C) + P(D) - P(C) \times P(D)\}$

$=0.9\times0.9\times\{0.8+0.8-0.8\times0.8\}$

$=0.7776$

ii)

P(System C does not work |The entire system work)

$= \frac{p(\,System\ C\ does\ not\ work\ and\ the\ entire\ system\ works)}{P(\,The\ entire\ sustem\ works)}$

$= \frac{P(A \cap B \cap C' \cap D)}{0.7776}$

$= \frac{P(A)P(B)\big(1-P(C)\big)P(D)}{0.7776}$

$= \frac{0.9\times0.9\times(1-0.8)\times0.8}{0.7776} = 0.1667$