Challenge:
Quantum Teleportation Protocol Modification

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Partial Entangled State

For a general two qubits system

$$|\varphi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

The system is entangled if the two qubits are **not separable** [1,2]. Meaning the state cannot be written as the tensor product of the two individual q-bits:

$$|\varphi\rangle \neq |\varphi_1\rangle|\varphi_2\rangle$$

A state is maximally entangled when each non-zero term in the superposition has the same probability and is partially entangled when that isn't the case. The level of entanglement can be measured with the concurrence [2]:

$$C = 2|ad - bc|$$

In our circuit, an entangled state is obtained after applying a Y-Rotation gate and a c-not gate to qubits two and three ($|\varphi_1\rangle$)

$$|\varphi_1\rangle = |\psi\rangle(-\sin(\theta/2)\,|00\rangle + \cos(\theta/2)\,|11\rangle)$$

For this state, its entanglement is:

$$C = |\sin\theta|$$

Quantum Gates

Quantum gates are linear transformations using unitary matrices ($UU^\dagger = I$). The following gates are examples of them. They were used herein.

Steps to teleport: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$|\psi_1\rangle$: Quantum entanglement between Alice second qubit and receiver (Bob).

$|\psi_2\rangle$: Alice first qubit contains the state to be teleported. Alice interacts this first qubit with the partially entangled state.

$|\psi_3\rangle$: Alice helps to codify the information into a easier way to measure.

$|\psi_4\rangle$: Alice measures the generated state to collapse the system. Now it is possible to translate to classical bits.

Bob decides which gates he needs to construct the teleported state.
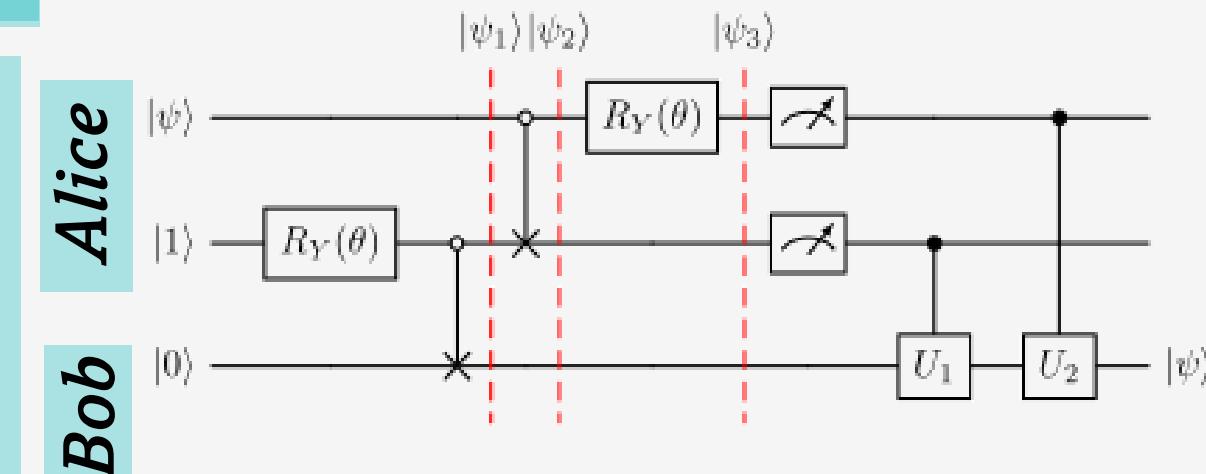
Figure 1. Modified QTP circuit. Rotational Y gate was implemented instead of Hadamard.

If Alice Measures:	then Bob's state is:	Gate to be applied
$|00\rangle$	$\cos(\theta/2)\sin(\theta/2)\,(\alpha|0\rangle - \beta|1\rangle)$	Z
$|01\rangle$	$\alpha\cos^2(\theta/2)\,|1\rangle - \beta\sin^2(\theta/2)\,|0\rangle$	—
$|10\rangle$	$\alpha\sin^2(\theta/2)\,|0\rangle + \beta\cos^2(\theta/2)\,|1\rangle$	—
$|11\rangle$	$\cos(\theta/2)\sin(\theta/2)\,(\alpha|1\rangle + \beta|0\rangle)$	X

Results

For 2-qubit system, Alice will have 4 possible cases to teleport $|\psi\rangle$. Entanglement will depend on θ:

1. Strong entanglement, in which probability is uniformly distributed.
2. Smooth entanglement, when amplitude of a case is greater than the others.
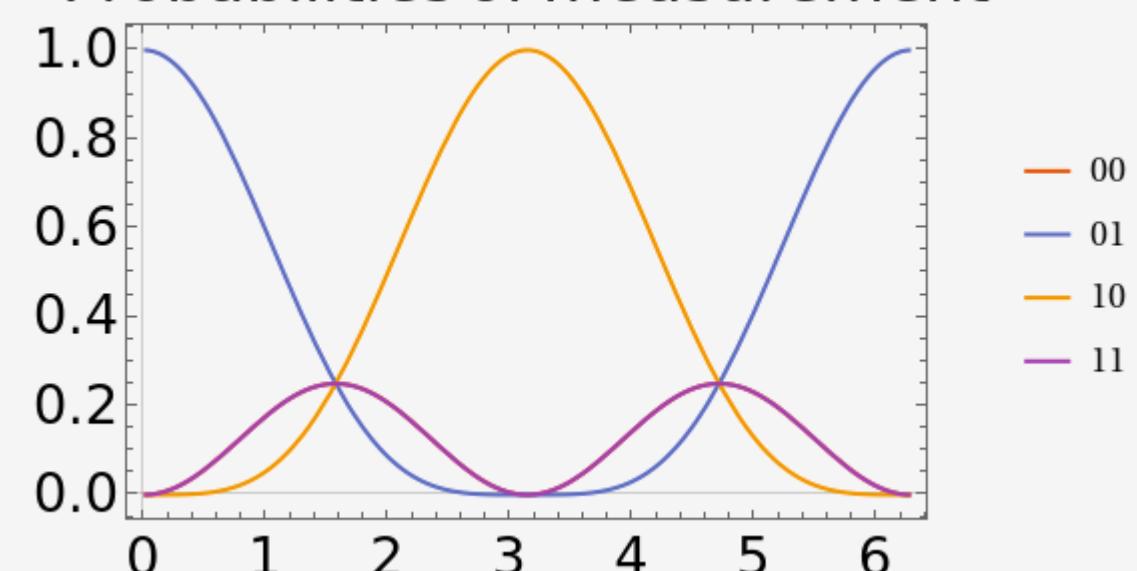
Figure 2. Probability of measuring each state acording to the rotational gate's angle.
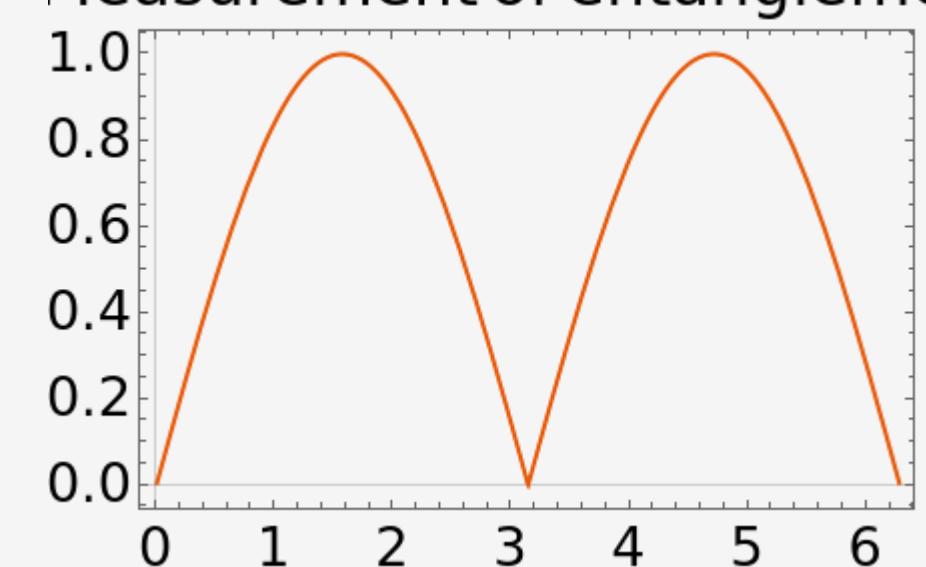
Figure 3. Degree of entanglement in terms of the rotation angle.

Conclusions

If $|\varphi_1\rangle$ is not maximally entangled, then it's not always possible for Bob to reconstruct Alice's initial state $|\psi\rangle$. Teleportation will be possible if states $|00\rangle$ or $|11\rangle$ are measured, but not if $|01\rangle$ or $|10\rangle$ are the ones detected.

Reference:
[1] Bolokian, M., Houshmand, M., Sadeghizadeh, MS. et al. Multi-Party Quantum Teleportation with Selective Receiver. Int J Theor Phys 60, 828–837 (2021). https://doi.org/10.1007/s10773-020-04702-y
[2] Wootters, William K. Entanglement of formation of an arbitrary state of two qubits. *Physical Review Letters* 80.10 (1998): 2245.