

This is the title of the Thesis

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Chapter 1

Introduction

Curiosity/phenomenology Paragraph that will tell the reader that hydrogels are cool.

Applications/Market size of the applications sectors If the previous paragraph does not convince the reader, well my last hope is that money does.

Besides, because of such a wide variety of response triggers, hydrogels can serve as sensors or actuators or can be utilized in controlled drug delivery systems, biosensors, tissue engineering scaffolds, and others [20], because of their biomimetic properties and multi functionalities [21](Bustamante-Torres u.a., 2021).

In particular, biomedical applications are very popular and include cell culture [5], wound dressing and healing [2,6], drug delivery [2,7,8], tissue engineering scaffolds [9], bone repair [10], and cartilage regeneration [11](Picchioni u.a., 2018).

Description of the Thesis What the reader will find in each chapter and section.

Why computers and not rheometers? Explain how in silico experiments can help to understand the relation between the network and the mechanical response.

1.1 State of the art: Hydrogels

- Characteristics

- Descriptions
- Synthesis techniques
- Cross-linking (Bond breaking)

General description of a hydrogel A hydrogel is commonly describe as a material composed by a network of polymers chains that exhibits the abilitiy to swell and retain a significant fraction of water within its structure, but will not dissolve in water(Ahmed, 2015; Ahmed u.a., 2025; Priya u.a., 2024; Bustamante-Torres u.a., 2021).¹ The water absorption capacity, network stability of hydrogels, and the conformation of the network with polymer chains are attributable to crosslinking mechanisms(Priya u.a., 2024; Ahmed, 2015). Meanwhile, the polymer chains are predominantly composed with hydrophilic functional groups and can be modified to suit a variety of applications(Ahmed, 2015; Priya u.a., 2024; Bustamante-Torres u.a., 2021).

While the analysis of the impact of functional groups is important, the present project prioritizes the examination of mechanisms that are more pertinent to the mechanical response. The crosslinking mechanisms², in particular, are of particular interest, as they are responsible for resisting dissolution. This suggests that crosslinking mechanisms enable the network to undergo modification by external stimuli.

The subsequent sections will present the essential information to facilitate a comprehensive understanding of the crosslinking mechanisms, their relationship to the mechanical response, the reported mechanical response of hydrogels, and the correlation between rheology experiments and stress curves.

1.1.1 Crosslinking mechanisms

What a low hysteresis means in a hydrogel?

Intro to cross linking A crosslinker is a molecule that functions as a bridge between polymer chains, thereby facilitating the formation of an interconnected network. As previously suggested,

¹the main difference with the microgels, is the size. Hydrogel is bulk, and microgelgel is particle.

²The hydrogels are prepared using different methods like chemical cross-linking of monomers, physical cross-linking using temperature or pH changes, and blending of natural or synthetic polymers.

it is pertinent to understand the mechanisms of crosslinking in order to gain insight into the correlation between these mechanisms and mechanical properties, such as elasticity, viscosity, solubility, glass transition temperature, strength, toughness, and melting point stiffness, swelling capacity, viscosity, and so forth (Priya u.a., 2024). The elements under consideration form stable bonds, which are commonly categorized into two main types: covalent (permanent) and physical (reversible) (Bustamante-Torres u.a., 2021). However, recent mechanisms, such as mechanical crosslinker mechanics, have been demonstrated to form bridges due to the topology of the constituents of the hydrogel.

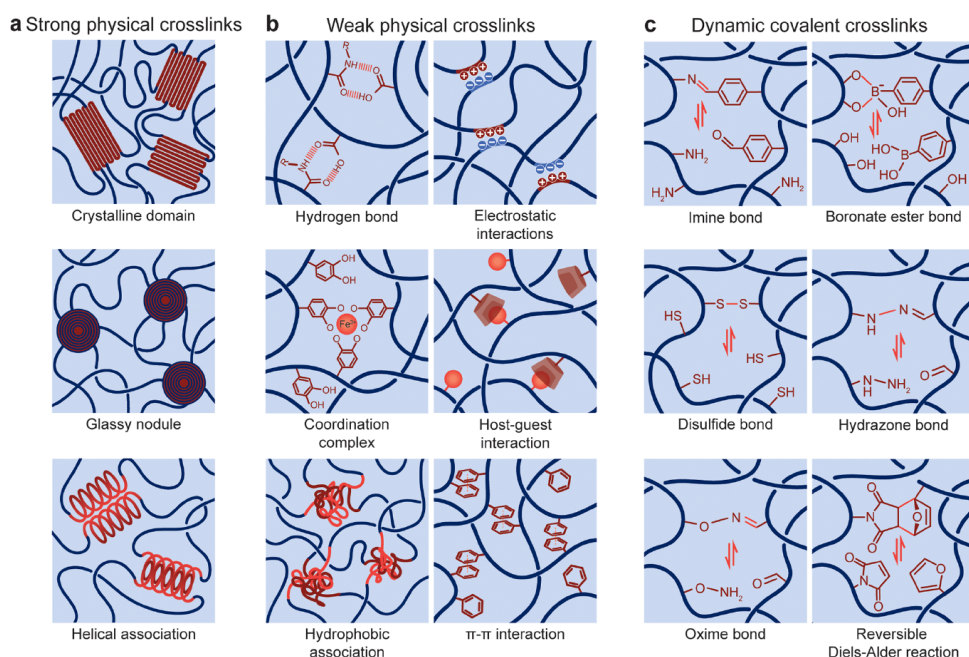


Figure 1.1: Image with the three different crosslinker mechanisms

Difference between physical and chemical bonds Although the concept of bonding is central to comprehending chemical structures and reactions. The criteria employed to characterize a chemical bond, its physical origin, and its nature remain subjects of debate (Kumar u.a., 2021). Consequently, establishing a precise distinction between “covalent” and “non-covalent” bonds remains challenging. Therefore, the description of crosslinker mechanisms is limited to

the principal interactions reported in articles and the synthesis process, rather than focusing on the classification of interactions as “covalent” or “non-covalent”, but rather as “reversible” or “irreversible”. Also in the recent work (Picchioni u.a., 2018) it is shown a “covalent” reversible network. Nonetheless, a general consensus exists that non-covalent bonds are, as a rule, recognized as being weaker than covalent bonds and it is widely accepted that a distinguishing characteristic between covalent and noncovalent bonds is the energy of interaction and equilibrium bond distance (Kumar u.a., 2021; Novikov, 2023).

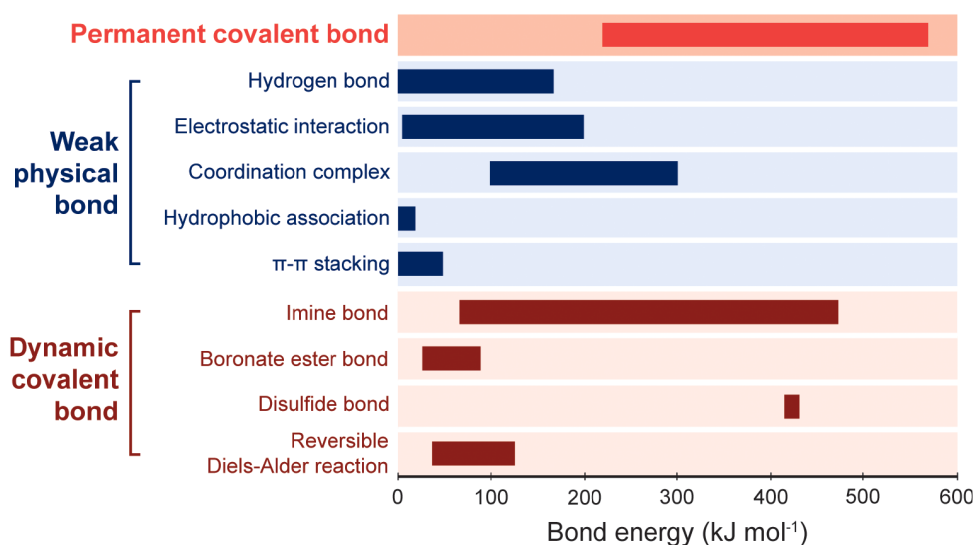


Figure 1.2: Bond energies of various types of permanent covalent crosslinks, weak physical cross-links, and dynamic covalent crosslinks.

Irreversible Cross-linking In irreversible cross-linked hydrogels, polymer chains are synthesized by chain growth polymerization, graft copolymerization, addition and condensation polymerization, enzymatic method, reactive functions groups and gamma and electron beam polymerization (Maitra u.a., /26/2014; Bustamante-Torres u.a., 2021). This types of crosslinking mechanisms exhibit a high degree of strength and stability, leading to a structural arrangement of interconnected polymer chains that is more robust and resistant to alterations in environmental conditions, such as temperature and pH (Maitra u.a., /26/2014).

Reversible Cross-linking In reversible cross-linked hydrogels, polymer chains are held together by molecular entanglements or physicochemical interactions, including van der Waals forces, hydrogen bonds, hydrophobic interactions, charge condensation, crystallite formation, and supramolecular chemistry (Bustamante-Torres u.a., 2021; Maitra u.a., /26/2014). Some of the synthesis methods for reversible crosslinking mechanisms are ionic interaction, crystallization, stereocomplex formation, hydrophobized polysaccharides, protein interaction, amphiphilic copolymers and hydrogen bond (Maitra u.a., /26/2014; Bustamante-Torres u.a., 2021). Furthermore, molecular reversibility can be actually achieved in two different ways: either by making use of equilibrium reactions (e.g., the Diels-Alder one) or through dynamic exchange reactions (e.g., reaction of an excess amino groups with epoxide ones) (Picchioni u.a., 2018).

Mechanical bonds As previously mentioned, a novel class of polymer architecture has recently emerged within the field of polymer science known as mechanically interlocked polymers (MIPs). These polymers are distinguished by the presence of a mechanical bond, that is, a constraint of two (or more) molecular components in space without the formation of covalent bonds (Hart u.a., 2021). While these types of hydrogels exhibit substantial conformational flexibility while preserving a persistent spatial correlation between their components, their synthesis remains challenging.

1.1.2 Mechanical response of hydrogels

General mechanical properties of the crosslinking Even though, physical crosslinking mechanisms are weaker than chemical ones, there numerous interactions contribute to complex behaviors. Meanwhile chemical crosslinking mechanisms are easier to control than physical crosslinking mechanisms because their preparation is independent of pH (Bustamante-Torres u.a., 2021) and they are very brittle due to structural inhomogeneity and lack of energy dissipation (Xu u.a., 2018).

Reversible crosslinking The aforementioned interactions enable hydrogels to undergo structural changes without the rupture of any covalent bonds. Consequently, these materials exhibit enhanced responsiveness to external stimuli, such as temperature, pH, or ionic strength. Additionally, hydrogels demonstrate high water sensitivity and thermal reversibility (Bustamante-Torres

u.a., 2021; Priya u.a., 2024). These materials are known to exhibit distinctive properties, including “self-healing” behavior, where the gel can reform after being broken. The lifespan of these hydrogels is brief, ranging from a few days to a maximum of a month, when maintained within physiological media.

Ir-reversible crosslinking Consequently, chemically cross-linked hydrogels generally exhibit greater mechanical strength and long-term stability. Furthermore, it generally contains regions of the high cross-linking density and low degree of swelling (clusters), dispersed in the regions of the low cross-linking density and high swelling index due to the hydrophobic aggregation of the cross-linking agent (Bustamante-Torres u.a., 2021)..

Network-mechanical response relation Introduce the idea of how by understanding the network we can manipulate/control the mechanical response.

The research of hydrophilic polymers has been complex because the physical properties of solubility or swellability depend on different factors, such as the type of polymer, molecular weight, the ratio of polar groups, and degree of cross-linking (Bustamante-Torres u.a., 2021). High molecular weight and a high degree of cross-linking will reduce the hydrophilicity of the molecule [18,19] (Bustamante-Torres u.a., 2021).

Tunable mechanical response with applications Review of articles of applications

Just describe the phenomena and say that it depends on the structure and so on.

Viscoelasticity

Yield stress

Shear thinning

Transition to talk about printing hydrogels (Correa u.a., 2021) Subsequent work developed safer cross-linking mechanisms, which began a trend toward triggering gelation in situ after

injection, providing a minimally invasive way of administering hydrogels to practically any organ or tissue.^{29,30³}

Unlike earlier hydrogels that relied on covalent cross-links, some of these hydrogels have self-healing properties and possess mechanical properties akin to native tissue, capable of countering natural forces and stresses of a body in motion. More recently, shear-thinning hydrogels were developed that are formed through dynamic and reversible cross-linking.³⁴ For example, physical hydrogels use noncovalent interactions (e.g., supramolecular chemistries) between soluble building blocks in order to self-assemble into a dynamic, reversibly cross-linked network.^{35,36} These “dynamic hydrogels” assembled through reversible cross-links afford the unique property of being injectable even after having formed a gel, due to their shear-thinning and selfhealing behaviors. Current research on dynamic hydrogels has revealed novel and useful capabilities that have opened new frontiers for this technology.

For example, they can stabilize delicate protein and cellular cargoes to combat pharmaceutical cold-chain limitations,³⁹ they can adhere strongly to tissues to form protective barriers and bandages,⁴⁰ and they can be delivered through spray applications to coat complex biological geometries.⁴¹

static covalent cross-links ultimately introduced translational challenges for clinical implementation, since traditional covalent gels require invasive surgical implantation to access non-superficial tissues. Additionally, new manufacturing processes, such as 3D printing, require dynamic rheological properties during processing, disqualifying the use of traditional covalent hydrogels.⁵⁷ Interest in further developing the translational potential of hydrogels led to innovative methods to implant them through minimally invasive means, of which the most clinically relevant is injection through a needle or catheter (Figure 3).

static covalent cross-links ultimately introduced translational challenges for clinical implementation, since traditional covalent gels require invasive surgical implantation to access non-superficial tissues. Additionally, new manufacturing processes, such as 3D printing, require dynamic rheological properties during processing, disqualifying the use of traditional covalent hydrogels.⁵⁷

Hybrid crosslinking hydrogels consist of covalent and noncovalent crosslinking(Xu u.a., 2018). The dynamic physical crosslinking could effectively dissipate energy via destruction and

³Why the gelation in situ after injection is important?

reorganization, and the chemical crosslinking could sustain skeleton construction [19,20](Xu u.a., 2018).

The molecular reversibility can be actually achieved in two different ways: either by making use of equilibrium reactions (e.g., the Diels-Alder one) or through dynamic exchange reactions (e.g., reaction of an excess amino groups with epoxide ones)(Picchioni u.a., 2018).

The general idea is that the use of dynamic covalent bonds allows the polymeric network to adjust itself as a result of an external stimulus(Picchioni u.a., 2018). This can be achieved in principle through other weaker interactions, e.g., hydrogen bonding(Picchioni u.a., 2018).

the use of covalent bonds displays two distinct and clear advantages [25]. In first instance, the network is still covalently linked, which renders it quite robust against small random fluctuations in environmental conditions such as temperature. Furthermore, exchange reactions such as the one of an amine with an imine are often kinetically controlled by the use of catalysts. In turn, this allows the possibility to freeze the network conformation (by slowing the kinetics) when desired(Picchioni u.a., 2018). The general concept behind the use of reversible interactions for the hydrogel polymeric chains is the (reversible) network disruption with immediate release of any loading (Figure 3).

Reversible bonds can be incorporated along the backbone (red circles) or at the crosslinking point (green triangles). The network, when subjected to an appropriate external stimulus, can then break at the crosslinking point (route A) or along the backbone (route B). This generates network fragments that can be quite different in terms of chemical structure even if in both cases the loading (blue circles) will be released. As a result of the network disruption, the load is released as the polymeric chains become soluble and not able anymore to entrap the load(Picchioni u.a., 2018).

1.1.3 Rheology/stress

Main review:(Gu u.a., 2020; Sheiko u.a., 2019)

Bridge of the experiments and interpretation Hysteresis curves to get the sotred energy and the dissipated energy.

Name some network structures The correlation between the structure with the hysteresis loops

Link to mechanical response Same as before.

What if we can change the structure on command and in real time? Bridge to crosslinkers.

How crosslinking affects the mechanical response Crosslinking is another essential process that can be controlled and intentionally modified using ionizing radiations(Priya u.a., 2024).

Chapter 2

Theoretical framework

2.1 Soft colloids

What is a colloid? and type of colloids.

2.1.1 Gels

What is a gel? And examples and so on.

Argument Why we can use a simulation protocol for microgels to modeled hydrogels?

- Why we can model hydrogels as Soft colloids?
- Idea of patchy particles and insterpretaion of interaction rules
- teaser of simulation experiments

Hydrophilic gels that are usually referred to as hydrogels are networks of polymer chains that are sometimes found as colloidal gels in which water is the dispersion medium [1](Ahmed, 2015).

2.2 Molecular dynamics

- Langevin equation
- Velocity Verlet
- Periodic Boundary Conditions

2.2.1 Langevin dynamics

From a general point of view there are two types of methods to make a qualitative description of systems: one focused on simulating dynamics at the microscale, and the other dedicated to deriving or establishing evolutionary equations at the macroscale (Wang u.a., 2025). Since the assumption is made that the mechanical response of a hydrogel is predominantly derived from its internal structure¹ we choose to simulate the dynamics at the microscale. Additionally, by treating the hydrogel as a colloid, permits applying molecular dynamics to model its response under shear deformation. Finally, there are two commonly used mathematical frameworks to model the molecular dynamics, the continuous time random walk (CTRW) model and the Langevin equation (Wang u.a., 2025), in this work we decided² to use the Langevin dynamics mathematical framework.

This is because, the solid phase of the colloid has a large mass and will change their momenta after many collisions with the solvent molecules and the picture which emerges is that of the heavy particles forming a system with a much longer time scale than the solvent molecules (Thijssen, 2007) and Langevin theory takes advantage of this difference in time scale to eliminate the details of the degrees of freedom of the solvent particles and represent their effect by stochastic and dissipative forces allowing longer simulations that would be impossible if the solvent were explicitly included (Pastor, 1994). However, the representation of the solvent by a stochastic and dissipative force, introduces the problem of characterizing two very different timescales, one associated with the slow relaxation of the initial velocity of the Brownian particle and another linked to the frequent collisions that the Brownian particle suffers with particles of the bath (Hansen u.a.,

¹Poner citas que demuestran que no es hipótesis, si no que se sabe

²Supongo que eventualmente justificaré la decisión.

2006)³. Therefore, two terms are used to create a mathematical representation of the solvent: a frictional force proportional to the velocity of the particle and a fluctuating force. Hence,

$$m \frac{d\vec{v}(t)}{dt} = \vec{F}(t) - m\gamma\vec{v}(t) + \vec{R}(t). \quad (2.1)$$

The friction constant γ ⁴ parametrises the effect of solvent damping and activation and is commonly referred to as the collision frequency in the simulation literature, even though formally a Langevin description implies that the solute suffers an infinite number of collisions with infinitesimally small momentum transfer. Also, the fact that the second term is not a function of the position of any of the particles involves the neglect of hydrodynamic interaction or spatial correlation in the friction kernel spatial correlation in the friction kernel (Pastor, 1994). On the other hand, $\vec{R}(t)$ ⁵ is a “random force” subject to the following conditions

$$\begin{aligned} \langle \vec{R}(t) \rangle &= 0 \\ \langle \vec{R}(t) \vec{R}(t') \rangle &= 2k_B T \gamma \delta(t - t') \end{aligned}$$

The no time correlation is equivalent to assuming that the viscoelastic relaxation of the solvent is very rapid with respect to solute motions⁶.

In comparing the results of Langevin dynamics with those of other stochastic methods [28-31], the relevant variable is the velocity relaxation time, τ_v which equals γ^{-1} (Pastor, 1994). The Langevin equation improves conformational sampling over standard molecular dynamics (Paquet u.a., 2015).

- Hablar acerca de que la fuerza aleatoria puede tener distribución gaussiana, pero no necesariamente.
- hablar de la ecuación de Green-Kubo:

$$\eta = \frac{V}{k_B T} \int_0^\infty \langle \sigma_{xy}(t) \sigma_{xy}(0) \rangle dt$$

³Para traer a colación la sensibilidad de la respuesta mecánica al parámetro de damp.

⁴Cuidado con las unidades. Hacer análisis dimensional, porque por la condición de correlación en R , γ ocupa tener unidades de masa entre tiempo, pero en la ecuación, solo ocupa unidades de $1/s$.

⁵No me acuerdo en donde está que se puede asumir que tiene distribución gaussiana.

⁶Grote and Hynes [26] have investigated this assumption for motions involving barrier crossing and have found that while it is seriously in error for passage over sharp barriers (such as 12 recombination); it is quite adequate for conformational transitions such as might be found in polymer motions. (Pastor, 1994)

- No se que tanto hablar de la idea de correlación y su aplicación en estos temas.

2.2.2 Velocity Verlet

Overview of the method

Characteristics of the method

2.2.3 Periodic Boundary Conditions

2.3 Mechanical response

- Macroscopic Stress (Cauchy)
- Microscopic Stress (PhD Thesis of pointwise fields)

2.3.1 Strain and Stress

2.3.2 Pressure and Stress

Introductory paragraph To characterize the behaviour of materials, constitutive relations serve as an input to the continuum theory...⁷

This derivation can be found in the appendix of (Admal u.a., 2010)^{8,9} Consider a system of N interacting particles with each particle position given by

$$\vec{r}_\alpha = \vec{r} + \vec{s}_\alpha, \quad (2.2)$$

where \vec{r} is the position of the center of mass of the system and \vec{s}_α is the position of each point relative to the center of mass. Hence, we can express the momentum of each particle as

$$\vec{p}_\alpha = m_\alpha (\dot{\vec{r}} + \dot{\vec{s}}_\alpha) = m_\alpha (\dot{\vec{r}} + \vec{v}_\alpha^{\text{rel}}). \quad (2.3)$$

⁷Capaz e ir introduciendo ideas del Clausius(Clausius, 1870)

⁸Describe more if what is done in this article

⁹(Eventualmente pondré esto en párrafo) Notation: σ Tensor, $\vec{\sigma}$ vector, $\sigma_{i,j}$ tensor, $\bar{\sigma}$ time average,

Before starting the procedure, let's take into account that the center of mass of the system is given by

$$\vec{r} = \frac{\sum_{\alpha} m_{\alpha} \vec{s}_{\alpha}}{\sum_{\alpha} m_{\alpha}}, \quad (2.4)$$

and by replacing (2.2) in (2.3) we get the following relations, which will be used later,

$$\sum_{\alpha} m_{\alpha} \vec{r}_{\alpha} = \vec{0}, \quad \sum_{\alpha} m_{\alpha} \vec{v}_{\alpha}^{\text{rel}} = \vec{0}. \quad (2.5)$$

Now we can start by computing the time derivative of tensorial product $\vec{r}_{\alpha} \otimes \vec{p}_{\alpha}$ ¹⁰,

$$\frac{d}{dt}(\vec{r}_{\alpha} \otimes \vec{p}_{\alpha}) = \underbrace{\vec{v}_{\alpha}^{\text{rel}} \otimes \vec{p}_{\alpha}}_{\text{Kinetic term}} + \underbrace{\vec{r}_{\alpha} \otimes \vec{f}_{\alpha}}_{\text{Virial term}}, \quad (2.6)$$

which is known as the *dynamical tensor virial theorem* and it is simply an alternative form to express the balance of linear momentum. This theorem becomes useful after making the assumption that there exists a time scale τ , which is short relative to macroscopic processes but long relative to the characteristic time of the particles in the system, over which the particles remain close to their original positions with bounded positions and velocities. Taking advantage of this property we can compute the time average of (2.6),

$$\frac{1}{\tau}(\vec{r}_{\alpha} \otimes \vec{p}_{\alpha}) \Big|_0^{\tau} = \overline{\vec{v}_{\alpha}^{\text{rel}} \otimes \vec{p}_{\alpha}} + \overline{\vec{r}_{\alpha} \otimes \vec{f}_{\alpha}}. \quad (2.7)$$

Assuming that $\vec{r}_{\alpha} \otimes \vec{p}_{\alpha}$ is bounded, and the time scales between microscopic and continuum processes are large enough, the term on the left-hand side can be as small as desired by taking τ sufficiently large and by summing over all particles we achieve the *tensor virial theorem*:

$$\overline{\mathbf{W}} = -2\overline{\mathbf{T}}, \quad (2.8)$$

where

$$\overline{\mathbf{W}} = \sum_{\alpha} \overline{\vec{r}_{\alpha} \otimes \vec{f}_{\alpha}} \quad (2.9)$$

is the time-average virial tensor and

$$\overline{\mathbf{T}} = \frac{1}{2} \sum_{\alpha} \overline{\vec{v}_{\alpha}^{\text{rel}} \otimes \vec{p}_{\alpha}} \quad (2.10)$$

¹⁰It is interesting to note that the tensorial product $\vec{r}_{\alpha} \otimes \vec{p}_{\alpha}$ has units of action and by taking the time derivative we are dealing with terms that has units of energy.

is the time-average kinetic tensor. This expression for the tensor virial theorem applies equally to continuum systems that are not in macroscopic equilibrium as well as those that are at rest.

The assumption of the difference between the time scales allow us to simplify the relation by replacing (2.3) in (2.10), so that,

$$\bar{\mathbf{T}} = \frac{1}{2} \sum_{\alpha} m_{\alpha} \overline{\vec{v}_{\alpha}^{\text{rel}} \otimes \vec{v}_{\alpha}^{\text{rel}}} + \frac{1}{2} \left[\overline{\sum_{\alpha} m_{\alpha} \vec{v}_{\alpha}^{\text{rel}}} \right] \otimes \dot{\vec{r}}, \quad (2.11)$$

which is not the simplification we expected, however, by the relations from (2.5), equation (2.11) simplifies to¹¹

$$\bar{\mathbf{T}} = \frac{1}{2} \sum_{\alpha} m_{\alpha} \overline{\vec{v}_{\alpha}^{\text{rel}} \otimes \vec{v}_{\alpha}^{\text{rel}}}. \quad (2.12)$$

On the other hand, instead of reducing the expression, we start to create the connection with the Cauchy stress tensor by distributing (2.9) into an internal and external contributions,

$$\bar{\mathbf{W}} = \underbrace{\sum_{\alpha} \overline{\vec{r}_{\alpha} \otimes \vec{f}_{\alpha}^{\text{int}}}}_{\bar{\mathbf{W}}_{\text{int}}} + \underbrace{\sum_{\alpha} \overline{\vec{r}_{\alpha} \otimes \vec{f}_{\alpha}^{\text{ext}}}}_{\bar{\mathbf{W}}_{\text{ext}}}. \quad (2.13)$$

The time-average internal virial tensor takes into account the interaction between particle α with the other particles in the system, meanwhile, the time-average external virial tensor considers the interaction with atoms outside the system, via a traction vector \vec{t} and external fields acting on the system represented by $\rho \vec{b}$, where ρ is the mass density of it and \vec{b} is the body force per unit mass applied by the external field. Therefore we can express the following,

$$\sum_{\alpha} \overline{\vec{r}_{\alpha} \otimes \vec{f}_{\alpha}^{\text{ext}}} := \int_{\delta\Omega} \vec{\xi} \otimes \vec{t} dA + \int_{\Omega} \vec{\xi} \otimes \rho \vec{b} dV. \quad (2.14)$$

Where $\vec{\xi}$ is a position vector within the domain Ω occupied by the system of particles with a continuous closed surface $\delta\Omega$. Assuming that Ω is large enough to express the external forces acting on it in the form of the continuum traction vector \vec{t} .

¹¹No estoy muy seguro si incluir una discusión acerca del término cinético en la expresión del virial. Posiblemente un párrafo... posiblemente lo ponga en la interpretación del teorema. También, no se si ir metiendo interpretación durante la derivación o no, pero bueno.

With this we can substitute the traction vector with $\vec{t} = \boldsymbol{\sigma} \vec{n}$, where $\boldsymbol{\sigma}$ represent the Cauchy stress tensor and applying the divergence theorem in (2.14), we have

$$\overline{\mathbf{W}}_{\text{ext}} = \int_{\Omega} \left[\vec{\xi} \otimes \rho \vec{b} + \text{div}_{\vec{\xi}} \left(\vec{\xi} \otimes \boldsymbol{\sigma} \right) \right] dV = \int_{\Omega} \left[\boldsymbol{\sigma}^T + \vec{\xi} \otimes \left(\text{div}_{\vec{\xi}} \boldsymbol{\sigma} + \rho \vec{b} \right) \right] dV \quad (2.15)$$

Since we assume that we are under equilibrium conditions, the term $\text{div}_{\vec{\xi}} \boldsymbol{\sigma} + \rho \vec{b}$ is zero (2.15) it simplifies to

$$\overline{\mathbf{W}}_{\text{ext}} = V \boldsymbol{\sigma}^T. \quad (2.16)$$

By tacking into account that we integrate over the domain Ω we can say that we compute the spatial average of the Cauchy stress tensor,

$$\boldsymbol{\sigma}_{\text{av}} = \frac{1}{V} \int_{\Omega} \boldsymbol{\sigma} dV, \quad (2.17)$$

in which V is the volume of the domain Ω . Replacing (2.16) into (2.13), the tensor virial theorem (2.8) can be expressed as,

$$\sum_{\alpha} \overline{\vec{r}_{\alpha} \otimes \vec{f}_{\alpha}^{\text{int}}} + V \boldsymbol{\sigma}_{\text{av}}^T = - \sum_{\alpha} m_{\alpha} \overline{\vec{v}_{\alpha}^{\text{rel}} \otimes \vec{v}_{\alpha}^{\text{rel}}}. \quad (2.18)$$

Finally, solving for the Cauchy Stress tensor we get,

$$\boldsymbol{\sigma}_{\text{av}} = -\frac{1}{V} \left[\sum_{\alpha} \overline{\vec{f}_{\alpha}^{\text{int}} \otimes \vec{r}_{\alpha}} + \sum_{\alpha} m_{\alpha} \overline{\vec{v}_{\alpha}^{\text{rel}} \otimes \vec{v}_{\alpha}^{\text{rel}}} \right], \quad (2.19)$$

an expression that describe the macroscopic stress tensor in terms of microscopic variables¹².

To end the section it is important to show that (2.19) is symmetric. Therefore, we rewrite the internal force as the sum of forces between the particles,

$$\vec{f}_{\alpha}^{\text{int}} = \sum_{\beta \neq \alpha} \vec{f}_{\alpha\beta}, \quad (2.20)$$

and substituting (2.20) into (2.19), we have

$$\boldsymbol{\sigma}_{\text{av}} = -\frac{1}{V} \left[\sum_{\alpha, \beta \neq \alpha} \overline{\vec{f}_{\alpha\beta} \otimes \vec{r}_{\alpha}} + \sum_{\alpha} m_{\alpha} \overline{\vec{v}_{\alpha}^{\text{rel}} \otimes \vec{v}_{\alpha}^{\text{rel}}} \right]. \quad (2.21)$$

¹²It is important to acknowledge that several mathematical subtleties were not taken into consideration, however all the mathematical formality is adresssed by Nikhil Chandra Admal and E. B. Tadmor in (Admal u.a., 2010)

Due to the property $\vec{f}_{\alpha\beta} = -\vec{f}_{\beta\alpha}$ we obtain the following identity

$$\sum_{\alpha, \beta, \beta \neq \alpha} \vec{f}_{\alpha\beta} \otimes \vec{r}_\alpha = \frac{1}{2} \sum_{\alpha, \beta, \beta \neq \alpha} \left(\vec{f}_{\alpha\beta} \otimes \vec{r}_\alpha + \vec{f}_{\beta\alpha} \otimes \vec{r}_\beta \right) = \frac{1}{2} \sum_{\alpha, \beta, \beta \neq \alpha} \vec{f}_{\alpha\beta} \otimes (\vec{r}_\alpha - \vec{r}_\beta). \quad (2.22)$$

Therefore, by replacing the identity of (2.22) into (2.21), we have

$$\sigma_{av} = -\frac{1}{V} \left[\frac{1}{2} \sum_{\alpha, \beta, \beta \neq \alpha} \overline{\vec{f}_{\alpha\beta} \otimes (\vec{r}_\alpha - \vec{r}_\beta)} + \sum_{\alpha} m_{\alpha} \overline{\vec{v}_{\alpha}^{\text{rel}} \otimes \vec{v}_{\alpha}^{\text{rel}}} \right], \quad (2.23)$$

expressed with indexical notation and using the einstein summation convention,

$$\sigma_{ij}^{\text{av}} = -\frac{1}{V} \left[\frac{1}{2} \sum_{\alpha, \beta, \beta \neq \alpha} \overline{f_i^{\alpha\beta} r_j^{\alpha} + f_i^{\beta\alpha} r_j^{\beta}} + \sum_{\alpha} m_{\alpha} \overline{v_i^{\alpha} v_j^{\alpha \text{rel}}} \right], \quad (2.24)$$

which is the same expression implemented in LAMMPS(Thompson u.a., 2022).¹³

¹³No se si poner la referencia a la pagina de documentacion https://docs.lammps.org/compute_stress_atom.html

Chapter 3

Numerical Experiments

3.1 Simulation tools

Description of what is in this chapter Becuase yes

3.1.1 LAMMPS

Talk about LAMMPS LAMMPS is a computational engine for modeling interacting particles at any length scale, so long as the interactions are primarily short-range, and particles densities are moderately bounded(Thompson u.a., 2022). LAMMPS implemented MD algorithms to enable parallelism across CPUs via MPI, and some versions of these algorithms for GPUs.

For test simulations we used the mpi paralelization scheme, meanwhile the final results where computed with the serial version, since we had access to a cluster.

A LAMMPS input script (text file) is simply a series of lines each beginning with a command name followed by one or more whitespace separated arguments. Programming-like commands are included which define variables, perform conditional tests, execute loops, or invoke shell commands, eg to launch a program external to LAMMPS. The input script is parsed and executed one line at a time which means a single script can be used to run a simulation in stages, altering one or more parameters between the stages, or to run a series of independent simulations where the entire system is reinitialized multiple times.

- Pair styles - Atom styles - Fix styles - Computes styles - units definitions

Make a resume of partitioning, communication, neighbr lists

Since we had access to the lavis cluster we...

Talk about the Fix styles Fix styles implement operations performed during a dynamics timestep or an energy minimization iteration(Thompson u.a., 2022). The `initial_integrate()` and `final_integrate()` methods of fix styles can be used to implement portions of the velocity-Verlet algorithm for various ensembles at the appropriate points in the timestep.

Units Lennar Jones units. distance = σ . time = reduce LJ τ . mass = ratio to unitless 1.0. temperature = reduced LJ temp. pressure = reduced LJ pressure energy = ϵ . velocity = σ/τ . force = reduce LJ force (σ/τ^2). <https://docs.lammps.org/99/units.html>

3.2 Simulation methodology

Overview of the simulation workflow.

The simulation methodology is based on the work presented in(Gnan u.a., 2017) and(Sorichetti u.a., 2023), with the objective of create a representative polymer structure of a microgel and characterize the mechanical response under shear deformation. This methodology creates the structure by using a mixture of two types of patchy particles. The patchy particles are spheres of identical size and mass decorated by patches to represent interaction sites. One type represent a *Crosslinker* and is define by 1 central particle with 4 patches placed at the vertices of a circumscribed tetrahedron. The other one represent a *Monomer* define by 1 central particle and 2 patches placed at the poles.

The interaction between the central particles is modeled with a Weeks-Chandler-Andersen repulsive potential,

$$U_{WCA}(r_{i,j}) = \begin{cases} 4\epsilon_{i,j} \left[\left(\frac{\sigma}{r_{i,j}} \right)^{12} - \left(\frac{\sigma}{r_{i,j}} \right)^6 \right] + \epsilon_{i,j}, & r_{i,j} \in [0, 2^{1/6}\sigma], \\ 0, & r_{i,j} > 2^{1/6}\sigma \end{cases}, \quad (3.1)$$

where $r_{i,j}$ is the distance between the center of the central particles, σ is the diameter of the particles and $\epsilon_{i,j}$ is the energy of the interacton. The patch-patch interaction is modeled with an

attractive potential,

$$U_{\text{patchy}}(r_{\mu v}) = \begin{cases} 2\varepsilon_{\mu v} \left(\frac{\sigma_p^4}{2r_{\mu v}^4} - 1 \right) \exp \left[\frac{\sigma_p}{(r_{\mu v} - r_c)} + 2 \right], & r_{\mu v} \in [0, r_c], \\ 0, & r_{\mu, v} > r_c, \end{cases} \quad (3.2)$$

where $r_{\mu v}$ is the distance between two patches, σ_p is the diameter of the patches, r_c is the cut distance of interaction set to $1.5\sigma_p$ and $\varepsilon_{\mu, v}$ is the interaction energy between the patches. Moreover, the interaction between patches is complemented by a three-body repulsive potential, defined in terms of (3.2), that provides an efficient bond-swapping mechanism making possible to easily equilibrate the system at extremely low temperatures, while at the same time, retaining the single-bond-per-patch condition (Sciortino, 2017),

$$U_{\text{swap}}(r_{l,m}, r_{l,n}) = w \sum_{l,m,n} \varepsilon_{m,n} U_3(r_{l,m}) U_3(r_{l,n}), \quad r_{l,n} \in [0, r_c], \quad (3.3)$$

where

$$U_3(r) = \begin{cases} 1 & r \in [0, r_{\min}], \\ -U_{\text{patchy}}(r)/\varepsilon_{m,n}, & r \in [r_{\min}, r_c] \end{cases}. \quad (3.4)$$

The sum in (3.3) runs over all triples of bonded patches (patch l bonded both with m and n). $r_{l,m}$ and $r_{l,n}$ are the distances between the reference patch and the other two patches. The parameter $\varepsilon_{m,n}$ is the energy of repulsion and w is used to tune the swapping ($w = 1$) and non-swapping bonds ($w \gg 1$). The cut off distance r_c is the same as in the potential of interaction between patches, meanwhile the minimum distance r_{\min} is the distance at the minimum of (3.2), *i.e.* $\varepsilon_{m,n} \equiv |U_{\text{patchy}}(r_{\min})|$. Finally, the energy of interaction between crosslinker patches ($\varepsilon_{\mu^i, \mu^i}$) are set to 0 to allow only crosslinker-monomer and monomer-monomer bonding (figure 3.2).

3.2.1 Assembly protocol

We perform molecular dynamics (MD) simulations at fixed temperature $T = kBT/\varepsilon = 0.05$, where kB is the Boltzmann constant. Thanks to such a low temperature, the system tends to maximize the number of bonds. In addition, owing to the bondswapping mechanism, the system is able to continuously restructure itself, until the large majority of possible bonds are formed. It is important to say that the main difference between the articles cited and the implementation in this thesis are the absence of FENE bonds and the swelling potential.

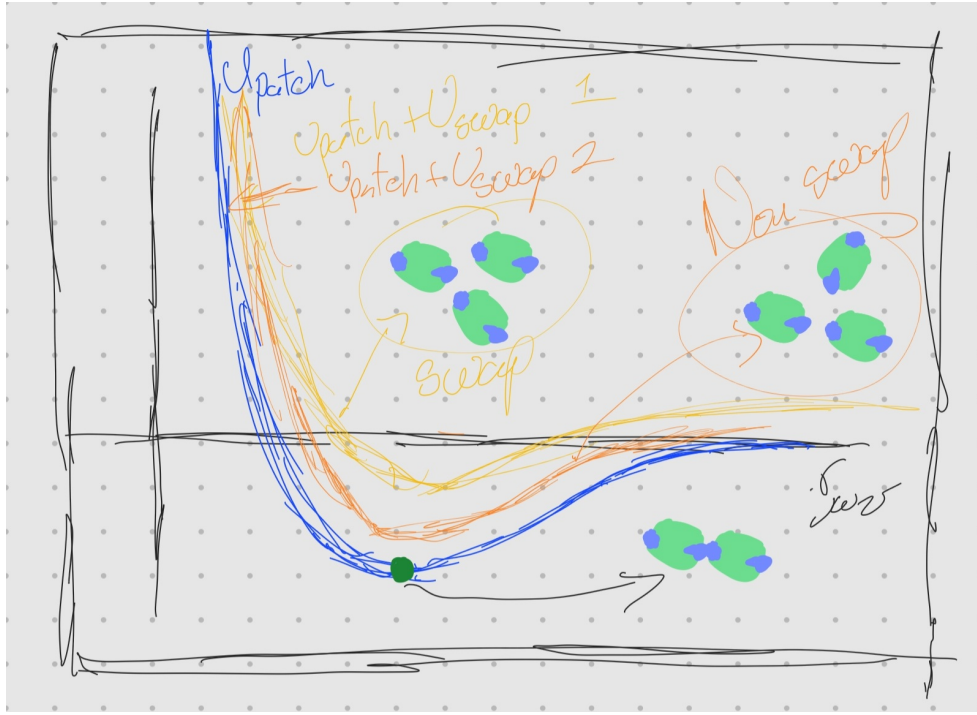


Figure 3.1: La idea de la figura es poner el potencial de interacción entre parches y ver el efecto del potencial de 3 cuerpos cuando $w = 1$ y cuando $w \gg 1$.

3.2.2 Shear protocol

Parameters discussion because yes, why shear rate and that stuff and discussion about the damp

Figures about the deformation?

3.3 Results

3.3.1 Network analysis

After the shear

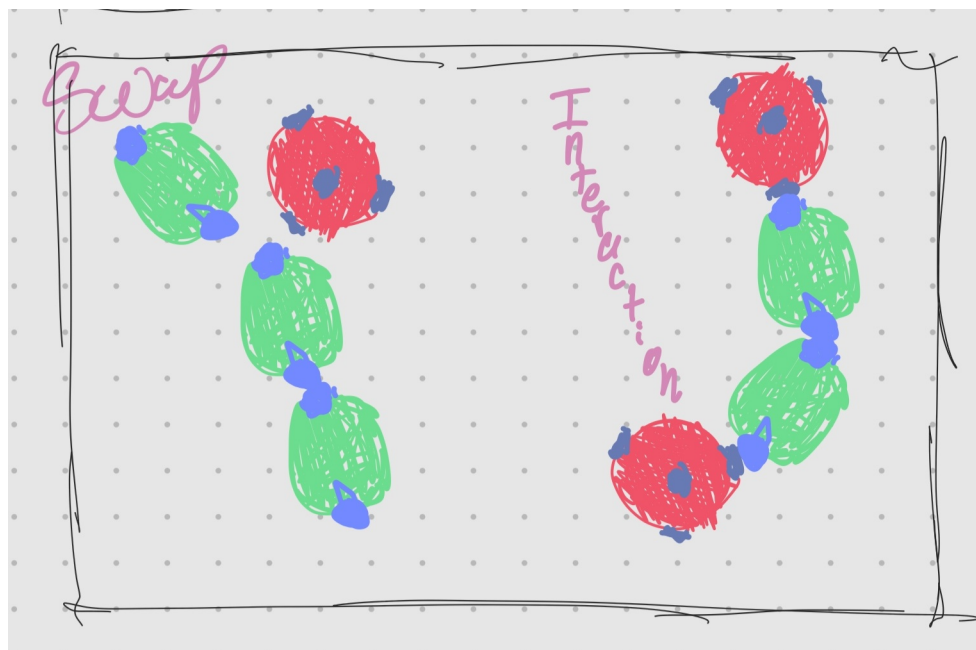


Figure 3.2: La idea de esta es mostrar las posibles configuraciones (monomero-monomero, monomero-crosslinker y un poco de pontecial de 3 cuerpos)

Before the shear

3.3.2 Mechanical response

Varying shear rates and stuff

Chapter 4

Conclusion

- What we achieve
- Future work

4.1 Discussion

About the mechanical response. If it works or not and so on.

About the network characterization. What it work or order arameters and so on.

4.2 Future work

About different crosslinking mechanisms. To use FENE in some crosslinkers and yes.

About parameters Shear rate, temperature, damp

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