Quantumm Optics Class-Notes and others

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Summary Class notes, post-class notes and others for the course of Quantum Optics. Semester February-June 2025

Harmonic oscilator

February 12

First Quantization

February 17

Properties of Quantumm electric field

Februrary 19

Fock States

February 19

Coherent states

February 24

According to the professor the sections **??**, where the hard part of the course. The next sections we are going to describe different states and analize there properties.

Lets remember the operator of an enlectric field with x component and one mode,

$$E_x = i\left(\frac{\hbar\omega}{2\epsilon_o V}\right)^{1/2}\left(\hat{a}\exp\left[-i\omega t\right] - \hat{a}^\dagger\exp\left[-i\omega t\right]\right),$$

which can be expressed in terms of quadratures as,

$$E_x = 2\left(\frac{\hbar\omega}{2\epsilon_o V}\right)^{1/2}\left(\hat{X}_1\sin\left(\omega t\right) + \hat{X}_2\cos\left(\omega t\right)\right)$$

So, when we want to get the expeted value of the electric field of a Fock state $|n\rangle$ we get $\langle n|E|n\rangle = 0$. So we need other states to model a laser. A useful observation is that $\langle n|\hat{X}_1^2|n\rangle = 1/4(2n+1)$ and $\Delta\hat{X}_1\Delta\hat{X}_2 = 1/4(2n+1)$.

We are going to study the "*Gleuber states*". Which are the states that can describe the laser. For that we have 4 definitions,

Definition 1 Eigenstates of $\hat{a}$

$$\hat{a}\left|\alpha\right\rangle = \alpha\left|\alpha\right\rangle, \quad \alpha \in \mathbb{C}.$$

Reminders of some properties.

$$\begin{aligned}
\left[\hat{a}, \hat{a}^\dagger\right] &= 1 \\
\hat{X}_1 &= \frac{\hat{a}+\hat{a}^\dagger}{2} \\
\hat{X}_2 &= \frac{\hat{a}-\hat{a}^\dagger}{2i} \\
\hat{a} &= \hat{X}_1 + i\hat{X}_2 \\
\hat{a}^\dagger &= \hat{X}_1 - i\hat{X}_2 \\
\left[\hat{X}_1, \hat{X}_2\right] &= \frac{i}{2} \\
\Delta\hat{X}_1\Delta\hat{X}_2 &\geq \frac{1}{4}
\end{aligned}$$

Definition 2 Displaced vacuum[1].

$$\hat{D}(\alpha) = \exp\left[\alpha\hat{a}^{\dagger} - \alpha^{*}\hat{a}\right], \quad |\alpha\rangle = \hat{D}(\alpha)\,|0\rangle$$

[1] It is important to use the following definition of e^{x}, becuase the argument are matrices and vectors,

$$\exp(x) = \sum \frac{x^{n}}{n!}.$$

Definition 3 Fock States

$$|\alpha\rangle = \exp\left[-\frac{|\alpha|^{2}}{2}\right] \sum \frac{\alpha^{n}}{\sqrt{n!}}\,|n\rangle$$

Definition 4

$$\Delta\hat{X}_{1}\Delta\hat{X}_{2} = \frac{1}{2}$$

Squeezed States

February 26