Quantum Computation (Group 550)

Exam 1 (Second part)

Date: September 11 2025

Instructions: In this part, you must write and test Python code inside a Jupyter notebook. Submit the notebook as part of your exam. Each problem must include:

1. Your Python code (well-documented).
2. The output of the code execution.
3. A short explanation (1–2 sentences) of what the result shows.
4. Goal: 100 (pts)

1. **Eigenvalues of Pauli Matrices. (20)**

   Write a Python program that:

   (a) Defines the three Pauli matrices $\sigma_x, \sigma_y, \sigma_z$.

   (b) Uses `numpy.linalg.eig` (or equivalent) to compute their eigenvalues and eigenvectors.

   (c) Prints the results in a readable way.

   Task: Verify that the eigenvalues are ± 1.

2. **Gram–Schmidt Orthonormalization. (30 pts)**

   (a) Write a Python function that performs the Gram–Schmidt process on a set of input vectors.

   (b) Test your function with:
   - Two vectors in $\mathbb{C}^2$.
   - Three vectors in $\mathbb{C}^3$.

   (c) Print the resulting orthonormal basis.

   Task: Verify that the resulting vectors are orthonormal (check inner products).

3. **Idempotence of Pauli Matrices. (30 pts)**

   (a) Write a Python script that checks whether a matrix A satisfies $A^2 = I$.

   (b) Apply it to each Pauli matrix.

   (c) Print the results with an explanation.

   Task: Confirm whether Pauli matrices are idempotent (i.e., test if $A^2 = A$ or $A^2 = I$).

4. **Normal Operators. (30 pts)**

   A matrix A is called **normal** if it commutes with its adjoint:

   $$AA^\dagger = A^\dagger A.$$

   (a) Write a Python function that checks whether a given matrix is normal.

(b) Test your function with:

- A Hermitian matrix (e.g., $\begin{bmatrix} 2 & i \\ -i & 3 \end{bmatrix}$).
- A unitary matrix (e.g., one of the Pauli matrices).
- A non-normal matrix (e.g., $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$).

(c) Print and explain the results.

Task: Verify that Hermitian and unitary operators are normal, but not all matrices are.