

## Smart Materials

### Problem set #3: Shape memory effect

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#### Problem 1:

Assuming that the stress required to initiate martensitic transformation in a Nitinol wire increases linearly by 5 MPa for every 1°C increase in temperature, calculate the stress required to initiate martensitic transformation at 50°C. It is given that Nitinol undergoes martensitic transformation at a stress of 300 MPa at 30°C.

Following the assumption of linear relation between the starting martensitic transformation stress with temperature<sup>1</sup>, we can use the following mathematical expression,

$$T(\theta) = C_M\theta + T_o.$$

Where  $T_o$  is the stress required to start martensitic transformation at 0°C. We can compute that value by applying the given data,  $C_M = 5 \text{ MPa } ^\circ\text{C}^{-1}$  and  $T(30^\circ\text{C}) = 300 \text{ MPa}$ ,

$$\begin{aligned} T(\theta) &= C_M\theta + T_o \\ T_o &= T(\theta) - C_M\theta \\ &= 300 \text{ MPa} - 5 \text{ MPa } ^\circ\text{C}^{-1} 30^\circ\text{C} \\ &= 300 \text{ MPa} - 150 \text{ MPa} \\ &= 150 \text{ MPa}. \end{aligned}$$

That that we know this value, we can compute the stress required to initiate martensitic transformation at 50°C as follows:

$$\begin{aligned} T(\theta) &= C_M\theta + T_o \\ T(50^\circ\text{C}) &= 5 \text{ MPa } ^\circ\text{C}^{-1} 50^\circ\text{C} + 150 \text{ MPa} \\ &= 250 \text{ MPa} + 150 \text{ MPa} \\ &= 400 \text{ MPa}. \end{aligned}$$

Hence, martensitic transformation starts by applying 400 MPa of stress at 50°C.

$$T(50^\circ\text{C}) = 400 \text{ MPa}$$

<sup>1</sup> It is important to say that this linear relation is about the starting value of stress to *initiate* the martensitic transformation, we are not analyzing the inial and final points of the transfromation.

#### Problem 2:

A SMA wire shows a stress plateau during martensitic transformation from 350 MPa to 400 MPa. The strain increases from 0.02 to 0.05 during this plateau. Calculate the approximate

work done per unit volume during the martensitic transformation.

**Problem 3:**

A Nitinol wire has a stress-induced martensitic transformation starting at 200 MPa at 30 °C and ending at 500 MPa. The transformation stress increases by 10 MPa/°C. If the wire is heated to 80 °C, determine the new stress required to start and complete the transformation. Calculate the stress required to start and complete the transformation at 80 °C.

From the first sentence, it is understood that the martensitic transformation goes from  $\xi = 0$  at 30 °C with 200 MPa with stress applied to  $\xi = 1$  at 30 °C with 500 MPa. Assuming once more that the stress-induced martensitic transformation is linearly related to temperature, we can use the following mathematical expression:

$$M_s(T) = M_s + \frac{T}{C_M},$$

where  $M_s$  is the martensitic transformation starting temperature at stress 0 and  $C_M$  is the rate of change of the starting point with stress. With the information given in the first two sentences, we can compute  $M_s$ ,

$$\begin{aligned} M_s(T) &= M_s + \frac{T}{C_M} \\ M_s &= M_s(T) - \frac{T}{C_M} \\ M_s &= 30^\circ\text{C} - \frac{200\text{ MPa}}{10\text{ MPa }^\circ\text{C}^{-1}} \\ &= 30^\circ\text{C} - 20^\circ\text{C} \\ &= 10^\circ\text{C}. \end{aligned}$$

Now that we now this value we can compute the starting stress-induced martensitic transformation at 80 °C as follows:

$$\begin{aligned} M_s(T) &= M_s + \frac{T}{C_M} \\ T &= C_M(M_s(T) - M_s) \\ &= 10\text{ MPa }^\circ\text{C}^{-1} (80^\circ\text{C} - 10^\circ\text{C}) \\ &= 700\text{ MPa}. \end{aligned}$$

Therefore, to start stress-induced martensitic transformation at 80 °C we required 700 MPa of stress.

$$T(80^\circ\text{C}) = 700\text{ MPa}$$

**Problem 4:**

The martensite start and finish temperatures ( $M_s$  and  $M_f$ ), the austenite start and finish temperatures ( $A_s$  and  $A_f$ ) and the slopes of the variation of  $M_s$  and  $M_f$  with stress ( $T$ ) i.e.,  $CM$  and the slope of variation of  $A_s$  and  $A_f$  with  $T$ , i.e.,  $CA$ , of a shape memory material are as follows:  $M_s = 25^\circ\text{C}$ ,  $M_f = 5^\circ\text{C}$ ,  $A_s = 29^\circ\text{C}$ ,  $A_f = 51^\circ\text{C}$ ,  $CA = 4.5\text{ MPa}/^\circ\text{C}$ ,  $CM = 11.3\text{ MPa}/^\circ\text{C}$ . The elastic modulus of the material is  $15\text{ GPa}$  and it has a recovery strain of  $8\%$ . For this material compute the following:

- Calculate the martensitic fraction when the material is cooled to  $20^\circ\text{C}$  from  $25^\circ\text{C}$  in a zero-stress state.
- If the temperature is maintained at  $25^\circ\text{C}$ , determine the martensitic fraction when a tensile stress of  $100\text{ MPa}$  is applied to the wire.
- The wire is heated above  $A_f = 51^\circ\text{C}$ , and then cooled under zero stress to  $30^\circ\text{C}$ . Subsequently, it is subjected to loading to activate the shape memory effect, leading to complete phase transformation, starting from point a to point e (as illustrated in the Figure below).

Calculate the stress and strain values at points a, b, c, and d on the stress-strain diagram.

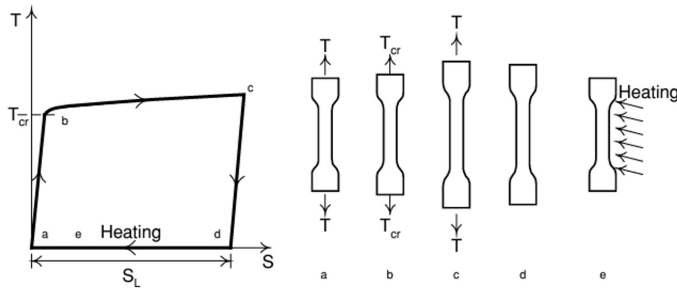


Figure 1: Strain-Stress curve