Homework 4

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Contents

1 Problem 4.16

A hydrogenic atom consists of a single electron orbiting a nucleus with Z protons ($Z = 1$ would be itself, $Z = 2$ is ionized helium, $Z = 3$ is doubly ionized lithium, and so on). Determine

1. Bohr energies $E_n(Z)$
2. Binding energy $E_1(z)$
3. Bohr radius $a(Z)$
4. Rydberg constant $R(Z)$

for a hydrogenic atom. (Express your answers as appropriate multiples of the hydrogen values.) Where in the electromagnetic spectrum would the Lyman series fall, for $Z = 2$ and $Z = 3$? Hint: There's nothing much to calculate here-in the potential

$$V(r) = -\frac{e^2}{4\pi\epsilon_0}\frac{1}{r}$$

$e^2 \to Ze^2$, so all you have to do is make the same substitution in all the final results.

Solution 1: Possible spins for baryons

$$\boxed{s = \left\{\frac{3}{2}, \frac{1}{2}\right\}.}$$

Solution 2: Possible spins for mesons

$$\boxed{s = \{1, 0\}.}$$

2 Problem 5.6

Imagine teo noninteracting particles, each of mass m, in the infinite square well. If one is in the state ψ_n (eqn 2.28), and the other in state ψ_l ($l \neq n$), calculate $\langle (x_1 - x_2)^2 | (x_1 - x_2)^2 \rangle$, assuming

2.28

1. they are distinguishable particles
2. they are identical bosons
3. they are identical fermions

Solution 3: Constant of normalization A

$$\boxed{A = \frac{1}{\sqrt{2}}.}$$

Solution 4: Constant of normalization A with $\psi_a = \psi_b$

$$\boxed{A = \frac{1}{2}.}$$

3 Problem 5.9

1. Suppose you put both electrons in a helium atom into the $n = 2$ state; what would the energy of the emitted electron be?
2. Describe (quantitatively) the spectrum of the helium ion, He^{+}.

Solution 5: Hamiltonian of non-interacting identical particles

$$\boxed{\hat{H}\psi(x_1, x_2) = 5K\psi(x_1, x_2), \quad K = \frac{\pi^2\hbar^2}{2a^2m}.}$$

Solution 6: Energies and states of the next two excited states

$$\boxed{\begin{aligned} \psi_{1,3} &= \frac{\sqrt{2}}{a}\left[\sin\left(\pi\frac{x_1}{a}\right)\sin\left(3\pi\frac{x_2}{a}\right) - \sin\left(3\pi\frac{x_2}{a}\right)\sin\left(\pi\frac{x_1}{a}\right)\right], \quad E = 10K \\ \psi_{2,3} &= \frac{\sqrt{2}}{a}\left[\sin\left(2\pi\frac{x_1}{a}\right)\sin\left(3\pi\frac{x_2}{a}\right) - \sin\left(3\pi\frac{x_2}{a}\right)\sin\left(2\pi\frac{x_1}{a}\right)\right], \quad E = 13K \end{aligned}}$$

4 Problem 5.10

Discuss (qualitatively) the energy level scheme for thelium if

1. electrons were identical bosons
2. if electrons were distinguishable particles (but with the same mass and charge). Pretend these "electrons" still have spin 1/2, so the spin configuration are the singlet and the triplet.

5 Problem 5.12

1. Figure out the electron configurations (in the notation of eqn 5.33) for the first two rows of the periodic table (up to neon), and check your results against table 5.1
2. Figure out the corresponding total angular momenta, in the notation of eqn 5.34, for the first four elements. List all possibilities for boron, carbon and nitrogen.

6 Problem 5.14

The ground state of dysprosium (element 66, in the 6th row of the Periodic Table) is listed as $^5I_8$. What are the total spin, total arbital and grand total angular momentum quantum numbers? Suggest a likely electron configuration for dysprosium.