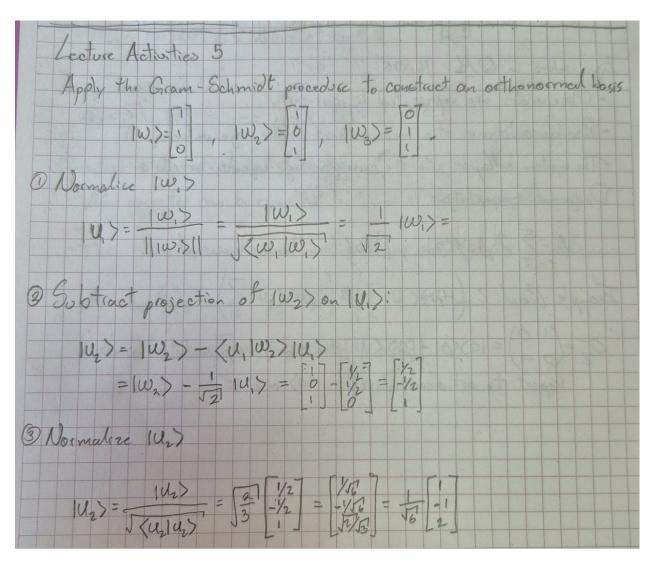
Lecture Activity 5

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1 Page 14 by hand



(a) Subtract projections
$$|u_{3}\rangle = |u_{3}\rangle - (\langle u_{1}|u_{2}\rangle |u_{1}\rangle + \langle v_{2}|u_{3}\rangle |u_{2}\rangle)$$

$$= |u_{3}\rangle - (\langle u_{1}|u_{2}\rangle |u_{1}\rangle + \langle v_{2}|u_{3}\rangle |u_{2}\rangle)$$

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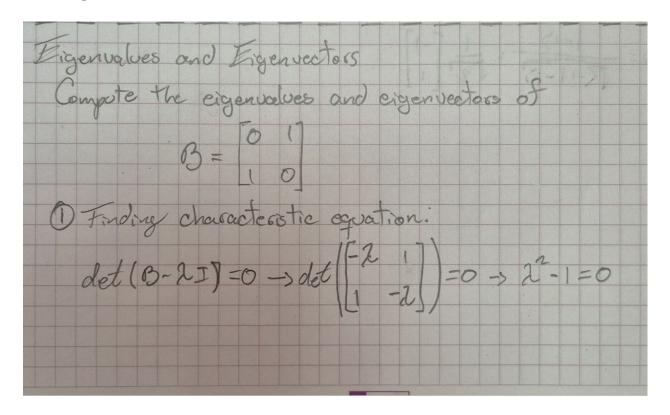
$$= |u_{3}\rangle - (\langle u_{1}|u_{1}\rangle |u_{2}\rangle + \langle v_{2}|u_{2}\rangle |u_{2}\rangle)$$

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$$= |u_{3}\rangle - |v_{2}\rangle - |v_{3}\rangle |u_{2}\rangle + |v_{3}\rangle |u_{2}\rangle + |v_{3}\rangle |u_{2}\rangle + |v_{3}\rangle |u_{3}\rangle + |v_{3}\rangle |u_{3}\rangle + |v_{3}\rangle |u_{2}\rangle + |v_{3}\rangle |u_{3}\rangle + |v_{3}\rangle +$$

2 Page 28



| 23 slue f | The agenvalues 2 =0 > 2= 1 > 2 = ±1 -> 2 = {- | 1, 13 |
|-----------|--|------------|
| 3 Solving | (B-ZI) (V) =0 to find eigenvector | |
| | [a] $[a]$ $[a+b=0]$ $[a-b=0]$ $[a-b=0]$ | a b |
| 2= | | |
| 3.6) | 1][a] | 11,8 |
| 2= | 1 -> 1-1>= [-1] | |
| | ring eigenvectors | |
| (+1) = | $\frac{1+1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ | |
| -1>=- | 1-15 - 12 [-1] | |

3 Jypter notebook

Notebook with the procedures/answers of the lecture activity.

3.1 2 Finding eigenvalues and eigenvectors (Page 28)

Considering the following matrix

$$B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

We are going to use python to find the eigenvalues and eigen vectors.

```
[29]: # Declare the matrix
B = np.array([[0, 1],[1, 0]]);

# Print the matrix
print("Matrix\n")
matprint(B, fmt="g")
print("\n")

# Use the command to find the eigenvalues and eigenvectors
eigenvalues, eigenvectors = np.linalg.eig(B)

# Print the answers
for i in range(len(eigenvalues)):
    print(f"Eigenvalue {i+1}: {eigenvalues[i]}")
    print(f"Corresponding eigenvector:\n{eigenvectors[:, i]}\n")
```

Matrix

0 1 1 0

Eigenvalue 1: 1.0 Corresponding eigenvector:

```
[0.70710678 0.70710678]

Eigenvalue 2: -1.0

Corresponding eigenvector:
[-0.70710678 0.70710678]
```

3.2 For each eigenvalue, write down its corresponding eigenvector (coding)

We are going to use the same command to find eigenvalues and eigenvectors of the following matrix

$$B = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & 9 \end{pmatrix}.$$

```
Matrix
```

2 0 0 0 3 4 0 4 9

[0.

Eigenvalue 1: 11.0

Corresponding eigenvector:

[0. 0.4472136 0.89442719]

Eigenvalue 2: 1.0

Corresponding eigenvector:

0.89442719 -0.4472136]

Eigenvalue 3: 2.0 Corresponding eigenvector:

[1. 0. 0.]