

# Homework 2

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## Contents

1	Problem 4.34 . . . . .	1
2	Problem 4.35 . . . . .	3
3	Problem 5.4 . . . . .	3
4	Problem 5.5 . . . . .	3

## 1 Problem 4.34

1. Apply  $\hat{S}_-$  to  $|10\rangle$  and confirm that you get  $\sqrt{2}\hbar|1-1\rangle$
2. Apply  $\hat{S}_+$  to  $|00\rangle$  and confirm that you get zero.
3. Show that  $|11\rangle$  and  $|1-1\rangle$  are eigenstates of  $\hat{S}^2$ , with the appropriate eigenvalue.

### Solution 1: $\hat{S}_- |10\rangle$

Recalling that  $|10\rangle = 1/\sqrt{2}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$  we can compute as follows,

$$\begin{aligned}
 \hat{S}_-^{(T)} |10\rangle &= (\hat{S}_-^{(1)} \oplus \hat{S}_-^{(2)}) \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \\
 &= \frac{1}{\sqrt{2}} \left[ (\hat{S}_-^{(1)} |\uparrow\rangle_{(1)}) |\downarrow\rangle_{(2)} \oplus |\uparrow\rangle_{(1)} (\hat{S}_-^{(2)} |\downarrow\rangle_{(2)}) + (\hat{S}_-^{(1)} |\downarrow\rangle_{(1)}) |\uparrow\rangle_{(2)} \oplus |\downarrow\rangle_{(1)} (\hat{S}_-^{(2)} |\uparrow\rangle_{(2)}) \right] \\
 &= \frac{1}{\sqrt{2}} [\hbar |\downarrow\rangle_{(1)} |\downarrow\rangle_{(2)} + \hbar |\downarrow\rangle_{(1)} |\downarrow\rangle_{(2)}] \\
 &= \frac{\hbar}{\sqrt{2}} (|\downarrow\downarrow\rangle + |\downarrow\downarrow\rangle)
 \end{aligned}$$

$$\hat{S}_-^{(T)} |10\rangle = \sqrt{2}\hbar |1-1\rangle.$$

**Solution 2:**  $\hat{S}_+ |00\rangle$ 

Recalling that  $|00\rangle = 1/\sqrt{2}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$  we can compute as follows,

$$\begin{aligned}
 \hat{S}_+^{(T)} |10\rangle &= (\hat{S}_+^{(1)} \oplus \hat{S}_+^{(2)}) \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \\
 &= \frac{1}{\sqrt{2}} \left[ \left( \hat{S}_+^{(1)} |\uparrow\rangle_{(1)} \right) |\downarrow\rangle_{(2)} \oplus |\uparrow\rangle_{(1)} \left( \hat{S}_+^{(2)} |\downarrow\rangle_{(2)} \right) - \left( \hat{S}_+^{(1)} |\downarrow\rangle_{(1)} \right) |\uparrow\rangle_{(2)} \oplus |\downarrow\rangle_{(1)} \left( \hat{S}_+^{(2)} |\uparrow\rangle_{(2)} \right) \right] \\
 &= \frac{1}{\sqrt{2}} \left[ \hbar |\uparrow\rangle_{(1)} |\uparrow\rangle_{(2)} - \hbar |\uparrow\rangle_{(1)} |\uparrow\rangle_{(2)} \right] \\
 &= \frac{\hbar}{\sqrt{2}} (|\uparrow\uparrow\rangle - |\uparrow\uparrow\rangle)
 \end{aligned}$$

$$\hat{S}_+^{(T)} |00\rangle = 0.$$

**Solution 3: Eigenstates of  $\hat{S}^2$** 

First we compute the expression of  $\hat{S}^2$  for a system with two Hilbert spaces,

$$\begin{aligned}
 (\hat{S}^{(T)})^2 &= (\hat{S}^{(1)} \oplus \hat{S}^{(2)}) \cdot (\hat{S}^{(1)} \oplus \hat{S}^{(2)}) \\
 &= (\hat{S}^{(1)})^2 \oplus (\hat{S}^{(2)})^2 \oplus 2\vec{\hat{S}}^{(1)} \cdot \vec{\hat{S}}^{(2)}
 \end{aligned}$$

Starting with  $|11\rangle = |\uparrow\uparrow\rangle$ ,

$$\begin{aligned}
 (\hat{S}_+^{(T)})^2 |11\rangle &= \left( (\hat{S}^{(1)})^2 \oplus (\hat{S}^{(2)})^2 \oplus 2\vec{\hat{S}}^{(1)} \cdot \vec{\hat{S}}^{(2)} \right) |\uparrow\uparrow\rangle \\
 &= \left[ (\hat{S}^{(1)})^2 |\uparrow\rangle_{(1)} |\uparrow\rangle_{(2)} \right] \oplus \left[ |\uparrow\rangle_{(1)} (\hat{S}^{(2)})^2 |\uparrow\rangle_{(2)} \right] \oplus 2 \left[ \hat{S}_x^{(1)} |\uparrow\rangle_{(1)} \hat{S}_x^{(2)} |\uparrow\rangle_{(2)} \oplus \hat{S}_y^{(1)} |\uparrow\rangle_{(1)} \hat{S}_y^{(2)} |\uparrow\rangle_{(2)} \oplus \hat{S}_z^{(1)} |\uparrow\rangle_{(1)} \hat{S}_z^{(2)} |\uparrow\rangle_{(2)} \right] \\
 &= \frac{3}{4} \hbar^2 |\uparrow\rangle_{(1)} |\uparrow\rangle_{(2)} \oplus \frac{3}{4} \hbar^2 |\uparrow\rangle_{(1)} |\uparrow\rangle_{(2)} \\
 &= \frac{3}{2} \hbar^2 |\uparrow\uparrow\rangle
 \end{aligned}$$

$$\hat{S}_+^{(T)} |00\rangle = 0.$$

$$\begin{aligned}
|11\rangle &= \uparrow\uparrow \\
|10\rangle &= \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow), \quad s = 1 \text{ triplet} \\
|1-1\rangle &= \downarrow\downarrow
\end{aligned}$$

$$|00\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \quad s = 0 \text{ singlet}$$

## 2 Problem 4.35

Quarks carry spin 1/2. Three quarks bind together to make a baryon (such as a proton or neutron): two quarks (or more precisely a quark and an antiquark) bind together to make a meson (such as the pion or the kaon). Assume the quarks are in the ground (so the orbital angular momentum is zero).

- What spins are possible for baryons?
- What spins are possible for mesons?

## 3 Problem 5.4

- If  $\psi_a$  are orthogonal, and both normalized, what is the constant  $A$  in 5.10?
- If  $\psi_a = \psi_b$  (and it is normalized), what is  $A$ ? (This case, of course, occurs only for bosons.)

## 4 Problem 5.5

- Write down the Hamiltonian for two noninteracting identical particles in the infinite square well. Verify that the fermion ground state given in Example 5.1 is an eigenfunction of  $H$ , with the appropriate eigenvalue.
- Find the next two excited states (beyond the ones in Example 5.1)-wave functions and energies-for each of the three cases (distinguishable, identical bosons, identical fermions).