

Thermodynamic Functions ΔH , ΔS , ΔG and Reaction Equilibrium

Dr. Fernando Rodríguez-Macías

Francisco Javier Vázquez Tavares A00827546

Contents

Problem 1

Plot the photon number distribution $p(n) = |\langle n, \psi | n, \psi \rangle|^2$ of a coherent state $|\alpha\rangle$, a Fock state, and a single mode thermal state, for $\langle n \rangle = 1, 10$ and 100 (9 plots in total). For each plot additionally provide Δn and the most probable outcome of the photon number measurement.

Problem 2

Let the operators \hat{a}_i and \hat{a}_i^\dagger , for $i \in \{1, 2\}$, form two sets of creation and annihilation operators that satisfy the commutation relations:

$$[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}, \quad [\hat{a}_i^\dagger, \hat{a}_j^\dagger] = 0, \quad [\hat{a}_i, \hat{a}_j] = 0$$

Let us define two new sets of creation and annihilation operators \hat{b}_i^\dagger and \hat{b}_i for $i = 1, 2$, which also follow the same commutation relations such that

$$\begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix} = \begin{pmatrix} u & v \\ v & u \end{pmatrix} \begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix}$$

What relationship must the real numbers u and v satisfy?

Problem 3

an electric field mode has an equal probability of $1/3$ to be found in each of the states $|0\rangle$, $|2\rangle$ and the superposition $4|0\rangle + 3|1\rangle$ (before normalization). Find the corresponding density matrix $\hat{\rho}$

Problem 4

Do eigenstates of \hat{a}^\dagger exist? If yes, what is their decomposition in the Fock state basis?

Problem 5

A bright squeezed state results from the application of a squeezing operator to a coherent state. It is defined as:

$$|r, \alpha\rangle = \hat{S}(r) |\alpha\rangle,$$

where the squeezing operator is defined as $\hat{S} = \exp [r/2(\hat{a}^2 - \hat{a}^{\dagger 2})]$. Find the uncertainty of a bright squeezed state in position and momentum quadratures. Does it retain the average photon number associated with the coherent state? What would be different if we defined the bright squeezed state as a displaced squeezed vacuum states instead?