Quantum Optics, Exercise

March 19, 2025

Problem 1

Consider a Mach-Zender interferometer, as shown in Fig. 1. The interferometer consists of two perfect mirrors and two beamsplitters. Both beamsplitters have an intensity reflection coefficient of $\varepsilon_1 = \varepsilon_4 = 1/2$. The matrix describing the beamsplitter transformation is

$$\boldsymbol{M} = \begin{pmatrix} \sqrt{\varepsilon} & i\sqrt{1-\varepsilon} \\ i\sqrt{1-\varepsilon} & \sqrt{\varepsilon} \end{pmatrix}. \tag{1}$$

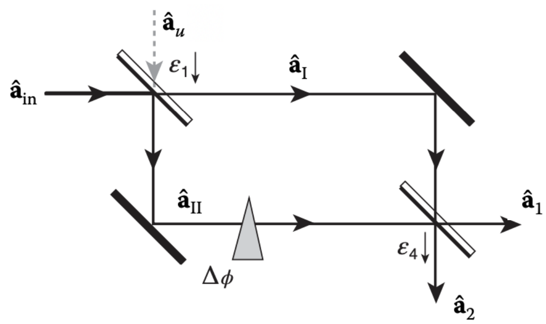

Figure 1: A Mach-Zender interferometer. The input modes correspond to anihilation operators $\hat{a}_{\mathrm{in}}$ and $\hat{a}_{\mathrm{u}}$, the modes propagating inside the interferometer correspond to $\hat{a}_{\mathrm{I}}$ and $\hat{a}_{\mathrm{II}}$, and finally the output modes are $\hat{a}_1$ and $\hat{a}_2$.

(a) Express the anihilation operators corresponding to the modes inside the interferometer $\hat{a}_{\mathrm{I}}$ and $\hat{a}_{\mathrm{II}}$ in terms of the input operators $\hat{a}_{\mathrm{in}}$ and $\hat{a}_{\mathrm{u}}$.

(b) Express the anihilation operators corresponding to the output modes of the interferometer $\hat{a}_1$ and $\hat{a}_2$ in terms of the input operators $\hat{a}_{\mathrm{in}}$ and $\hat{a}_{\mathrm{u}}$ and the phase difference between the interferometer arms $\Delta\phi$.

(c) Show that the operator describing the number of photons entering the interferometer and the operator describing the number of photons exiting the interferometer are identical, in other words, prove that $\hat{n}_{\mathrm{in}} + \hat{n}_{\mathrm{u}} = \hat{n}_1 + \hat{n}_2$.

Let us now consider the following input state $\frac{1}{\sqrt{2}}(|2\rangle_{\mathrm{in}}|0\rangle_{\mathrm{u}} - |0\rangle_{\mathrm{in}}|2\rangle_{\mathrm{u}})$.

(d) Find the quantum state of the electric field inside the interferometer. Comment on the result.

(e) Find the quantum state of the electric field after the second beam splitter (at the output) as a function of phase difference between the interferometer arms $\Delta\phi$.

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