Homework 1

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1 Problem 4.2

Use separation of variable in *cartesian* coordinates to solve the infinite *cubical* well (or particle in a box):

$$V(x,y,z) = \begin{cases} 0, & \forall x,y,z \in [0,a] \\ \infty, & \forall x,y,z \notin [0,a] \end{cases}$$

- Find the stationary states, and the corresponding energies.
- Call the distinct energies $E_1, E_2, \dots$ in order of increasing energy. Find E_1, E_2, E_3, E_4, E_5 and E_6. Determine their degeneracies (that is, the number of different states that share the same energy).
- What is the degeneracy of E_{14}, and why is this case interesting?

2 Problem 4.3

Use

$$P_l^m(x) \equiv \left(1 - x^2\right)^{|m|/2} \left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^{|m|} P_l(x)$$
$$P_l(x) \equiv \frac{1}{2l!} \left(\frac{\mathrm{d}}{\mathrm{d}x}\right)^{l} \left(x^2 - 1\right)^{l}$$
$$Y_l^m(\theta, \phi) = \epsilon \sqrt{\frac{(2l+1)}{4\pi} \frac{(l - |m|)!}{(l + |m|)!}} e^{im\phi} P_l^m(\cos[\theta])$$

to construct Y_0^0 and Y_2^l. Check that they are normalized and orthogonal.

3 Problem 4.13

- Find $\langle r \rangle$ and $\langle r^2 \rangle$ for an electron in the ground state of hydrogen. Express your answers in terms of the Bohr radius.
- Find $\langle x \rangle$ and $\langle x^2 \rangle$ for an electron in the ground stat of hydrogen. *Hint:* this requires no noew integration-note that $r^2 = x^2 + y^2 + z^2$, and explot the symmetry of the ground state.
- Find $\langle x^2 \rangle$ in the state $n = 2, l = 1, m = 1$. *Warning:* This state is not symmetrical in x, y, z. Use $x = r \sin\theta \cos\phi$.

4 Problem 4.14

What is the *most probable* value of r, in the ground state of hydrogen? (The answer is not zero!) *Hint:* First ypu must figure out the probability that the electron would be found between r and $r + dr$.

5 Problem 4.23

In problem 4.3 you showed that

$$Y_2^l(\theta, \phi) = -\sqrt{\frac{15}{8\pi}} \sin\theta \cos\theta e^{i\phi}.$$

Apply the raising operator to find $Y_2^2(\theta, \phi)$. Use equation $A_l^m = \hbar\sqrt{l(l+1) - m(m \pm 1)} = \hbar\sqrt{(l \mp m)(l \pm m + 1)}$ to get the normalization.

6 Problem 4.25

If the electron were a classical solid sphere, with radius,

$$r_c = \frac{e^2}{4\pi\epsilon_o mc^2}$$

(the so-called classical electron radius, obtained by assuming the electron's mass is attributable to energy sotred in its electric field, via the Einstein formula $E = mc^2$), and its angular momentum is $\hbar/2$, then how fast (in m/s) would a point on the "equator" be moving? Does this model make sense? (Actually, the radius of the electrin is known experimentally to be much less than r_c but this only makes matters worse.)

7 Problem 4.26

- Check that the spin matrices 4,145 and 4,147 obey the fundamental commutation relations for angular momentum eqn 4.134
- Show that the Pauli spin matrices 4.148 satisfy the product rule

$$\sigma_j \sigma_k = \delta_{jk} + i \sum_l \epsilon_{jkl} \sigma_l,$$

  where the indices stand for x, y, z and ϵ_{jkl} is the Levi-Civita symbol.

8 Problem 4.27

An electron is in the spin state,

$$\Xi = A \begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

- Determine the normalization constant A.
- Find the expectation values of S_x, S_y and S_z.
- Find the "uncertanties" $\sigma_{S_x}, \sigma_{S_y}$ and σ_{S_z}. (Note: These sigmas are standard deviations, not Pauli matrices!)

- Confirm that your results are consistent with all three uncertanty principles 4.100 and its cyclic permutations-only with S in place of L, of course.)

9 Problem 4.32 a)

If ypu measure the component of spin angular momentum along the x direction, at time t, what is the probability that ypu would get $+\hbar/2$?