# Conic sections: Ellipse

Department of Mathematics.

Name: ________________________________

## 1 Examples

**Example 1: Factored equation of the ellipse**

Find the center, axis and foci of the following ellipse,

$$\frac{(x-2)^2}{100}+\frac{(y-5)^2}{64}=1.$$

Comparing the given equation with the general equation for the ellipse we can get the following insight,

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \quad \longleftrightarrow \quad \frac{(x-2)^2}{100}+\frac{(y-5)^2}{64}=1$$
$$-h=-2, \quad -k=-5, \quad a^2=100, \quad b^2=64$$
$$h=2, \quad k=5, \quad a=10, \quad b=8.$$

Therefore, we know that the orign of the ellipse is at $(2,5)$, also that the semiminor axis is equal to 8 and the semimajor axis is equal to 10. Lastly, to compute the foci of the ellipse we need to add and substract the $x$ component of the origin to the semimajor axis,

$$F_1=(2,5)-(10,0)=(-8,5), \qquad F_2=(2,5)+(10,0)=(12,5)$$

**Example 2: Expanded equation of the ellipse**

Find the center, axis and foci of the following ellipse,

$$x^2+4y^2+6x+16y+3=0.$$

As before, we compare this expression with the expanded form of the ellipse,

$$\frac{1}{a^2}\left(x^2-2xh+h^2\right)+\frac{1}{b^2}\left(y^2-2yk+k^2\right)=1 \longleftrightarrow \left(x^2+6x\right)+4\left(y^2+4y\right)=-3.$$

As we can see, we are missing terms. If we just say that $a=1$ and $b=1/2$, there is no solution for the equation. Hence, we need to complete the square ($(x+\alpha)^2=x^2+2\alpha x+\alpha^2$),

$$\Longrightarrow\ 6x=2\alpha x \to \alpha=3, \qquad \Longrightarrow\ 4y=2\alpha y \to \alpha=2$$
$$(x+3)^2=x^2+6x+9, \quad (y+2)^2=y^2+4y+4$$

To keep the same expression we need to re-write those terms as follows,

$$\left(x^2+6x\right)=(x+3)^2-9, \quad \left(y^2+4y\right)=(y+2)^2-4,$$

therefore,

$$\begin{aligned}(x+3)^2 - 9 + 4\left[(y+2)^2 - 4\right] &= -3 \\ (x+3)^2 + 4(y+2)^2 &= 22,\end{aligned}$$

finally, we divide by 22 the equation,

$$\begin{gathered}\frac{1}{22}(x+3)^2 + \frac{4}{22}(y+2)^2 = 1 \longleftrightarrow \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \\ 3 = -h, \quad 2 = -k, \qquad \frac{1}{22} = a^2, \quad \frac{2}{11} = b^2 \\ h = -3, \quad k = -2, \qquad a = \sqrt{22}, \quad b = \sqrt{\frac{2}{11}}.\end{gathered}$$

Hence, the ellipse is center at $O = (-3, -2)$ with focal points at $F_1 = (-3 - \sqrt{22}, -2)$ and $F_2 = (-3 + \sqrt{22}, -2)$.

# 2 Exercises

## 2.1 Ellipse

- Find the center, vertices and foci of the following ellipse

$$\frac{(x-1)^2}{16} + \frac{(y+5)^2}{25} = 1$$

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- Write the compacted equation of the ellipse $16x^2 - 48x + 9y^2 + 12y + 28 = 0$.

## 2.2 Review

- The hydrogen ion concentration of a sample of a mystery liquid is $\mathrm{H}^+ = 6.88 \times 10^{-3}$. Calculate the pH of the substance considering that $\mathrm{pH} = -\log[\mathrm{H}^+]$.
- Determine if the following functions are inverses by computing both compositions, $f(x) = (x-5)/2$ and $g(x) = 2x+5$.
- Express the domain and rage of the functions $f(x) = (x-5)^{1/2}$ and $g(x) = 1/(x-2)^2$.
- Find the critical point of the function $f(x) = \exp[(x-4)(x+4)]$.