Algebric and Transcendental functions

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Summary Class sketches for the subject Algebraic and Transcendental Functions. Semester January-May 2025

Circle

A circle is the set of points in a plane that are equidistant from a given point O. The point O is called the center, while the distance r from the center is called the radius. Twice the radius is known as the diameter $d = 2r$ (fig.1).

Derivation of the equation

The main characteristic of the circle is "*equidistant*", that is, that the distance between all those points and the point O are equal. A usefull theorem that translates that characteristic into an equation is the **Pythagorean theorem**. Which describes the relationship between the sides of a right triangle (fig.2) with the equation,

$$a^2 + b^2 = c^2.$$

We can show that any point on the circle forms a right triangle with the horizontal and vertical distances from the point (x, y) to the point O (fig.3).

In that sense, we can relate a with x, b with y and c with the radius. In the special case when the radius is equal to one, we have a *unit circle*,

> **Definition 1: Unit circle**
>
> The unit circle is the circle of radius 1 centered at the origin in the xy-plane. It is represented with the following equation,
>
> $$x^2 + y^2 = 1.$$

General equation

Now, let's give the mathematical representation of the point O as (h, k). Using the knowledge of function translations, we can move the origin of the circle (fig.4) as follows,

$$(x - h)^2 + (y - k)^2 = r^2.$$

Derivation of the equation/function.
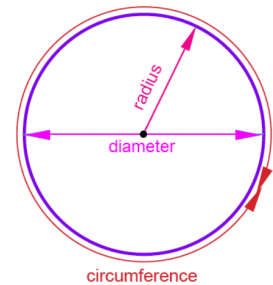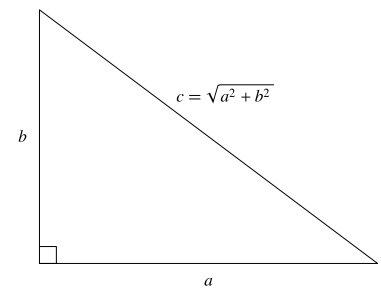Example/Exercises.

Figure 1: Sketch for the board

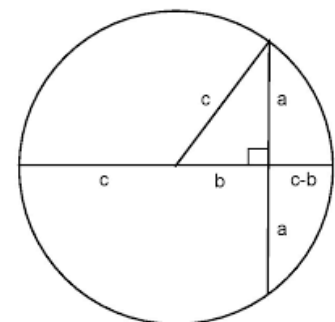Figure 2: Sketch for the board for the pythagoream theorem.

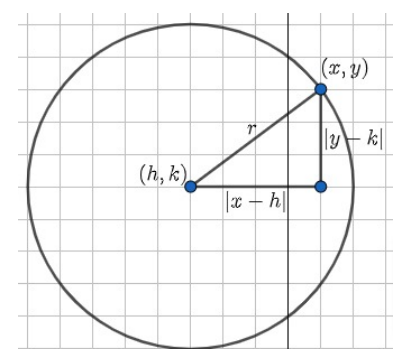Figure 3: Relation between pythagorean theorem and the circle.

Figure 4: Diplaced circle to the point (h, k).

Ellipse

Parabola

Hyperbola

References

James Stewart, L. Redlin, Saleem Watson, and Phyllis Panman. *Precalculus: mathematics for calculus*. Cengage Learning, Boston, MA, seventh edition edition, 2016. ISBN 9781305071759.

Eric W. Weisstein. Pythagorean theorem, a. URL `https://mathworld.wolfram.com/PythagoreanTheorem.html`.

Eric W. Weisstein. Circle, b. URL `https://mathworld.wolfram.com/Circle.html`.