Algebric and Transcendental functions

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Summary Class sketches for the subject Algebraic and Transcendental Functions. Semester January-May 2025

Circle

A circle is the set of points in a plane that are equidistant from a given point O. The point O is called the center, while the distance r from the center is called the radius. Twice the radius is known as the diameter $d = 2r$ (fig.1).

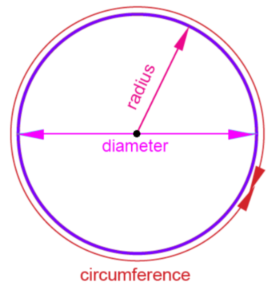

Figure 1: Sketch for the board

Derivation of the equation

The main characteristic of the circle is "*equidistant*", that is, that the distance between all those points and the point O are equal. A usefull theorem that translates that characteristic into an equation is the **Pythagorean theorem**. Which describes the relationship between the sides of a right triangle (fig.2) with the equation,

$$a^2 + b^2 = c^2.$$

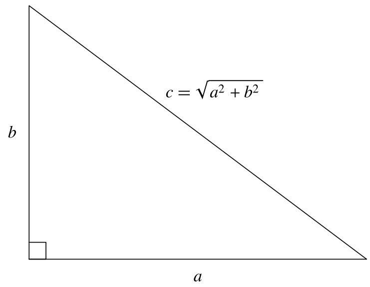

Figure 2: Sketch for the board for the pythagoream theorem.

We can show that any point on the circle forms a right triangle with the horizontal and vertical distances from the point (x, y) to the point O (fig.3).

In that sense, we can relate a with x, b with y and c with the radius. In the special case when the radius is equal to one, we have a *unit circle*,

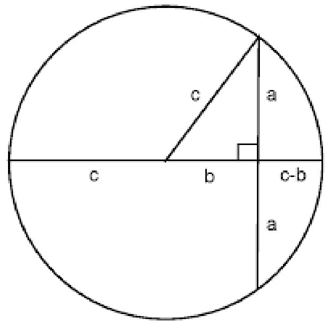

Figure 3: Relation between pythagorean theorem and the circle.

> **Definition 1: Unit circle**
>
> The unit circle is the circle of radius 1 centered at the origin in the xy-plane. It is represented with the following equation,
>
> $$x^2 + y^2 = 1.$$

General equation

Now, let's give the mathematical representation of the point O as (h, k). Using the knowledge of function translations, we can move the origin of the circle (fig.4) as follows,

$$(x-h)^2 + (y-k)^2 = r^2. \tag{1}$$

Now, lets get some mathematical exercises.

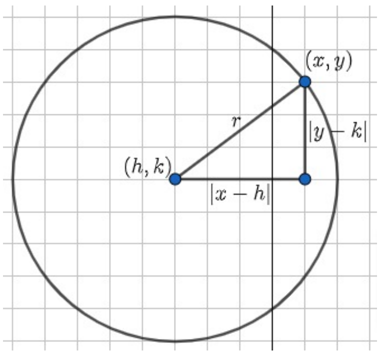

Figure 4: Diplaced circle to the point (h, k).

> **Note 1: Some algebra stuff.**
>
> We are going to expand the equation(1) to get familiarize ourselves

> with different expressions of the same mathematical object.
>
> $$\begin{aligned} (x-h)^2 + (y-k)^2 &= r^2 \\ \left(x^2 - 2hx + h^2\right) + \left(y^2 - 2ky + k^2\right) &= r^2 \\ x^2 + y^2 - 2hx - 2ky + h^2 + k^2 &= r^2 \end{aligned}$$
>
> As we can see, this can be approach as a function of two independent variables x and y ($f(x,y)$) equated to a constant value r^2, $f(x,y) = r^2$. This function is not bijective, indicating that there is no inverse function and the domain and range are bounded.

Examples

> **Example 1: miau**
>
> Identify the radius and center of the following circle,
>
> $$(x+3)^2 + (y+1)^2 = 9.$$

> **Example 2: miau**
>
> Identify the radius and center of the following circle,
>
> $$x^2 + y^2 - 8x + 4y + 4 = 0$$

Exercises

- Write the equation of the circle with center at $(6,-9)$ and radius $1/4$.

  Derivation of the equation/functionnote 1.
  Example/Exercises.

Ellipse

Parabola

Hyperbola

References

James Stewart, L. Redlin, Saleem Watson, and Phyllis Panman. *Precalculus: mathematics for calculus*. Cengage Learning, Boston, MA, seventh edition edition, 2016. ISBN 9781305071759.

Eric W. Weisstein. Pythagorean theorem, a. URL `https://mathworld.wolfram.com/PythagoreanTheorem.html`.

Eric W. Weisstein. Circle, b. URL `https://mathworld.wolfram.com/Circle.html`.