Conic sections: Circle

Department of Mathematics.

Name: _____

1 Examples

Example 1: Factored equation of the circle.

Identify the radius and center of the following circle,

$$(x+3)^2+(y+1)^2=9.$$

Recalling the definition of the circle, $(x-h)^2+(y-k)^2=r^2$, and comparing both expressions, we can make the following relations,

$$x+3 \longleftrightarrow x-h, \quad y+1 \longleftrightarrow y-k, \quad 9 \longleftrightarrow r^2.$$

Therefore,

$$\begin{aligned} 3=-h, \quad 1=-k, \quad 9=r^2 \\ h=-3, \quad k=-1, \quad r=9^{1/2}=3. \end{aligned}$$

This tell's us that the circle origin is at $(-3,-1)$ with a radius of $r=3$.

Example 2: Expanded equation of the circle.

Identify the radius and center of the following circle,

$$x^2+y^2-8x+4y+4=0.$$

Recalling the note, we can make the following comparsion as before,

$$x^2+y^2-8x+4y+4=0 \quad \longleftrightarrow \quad x^2+y^2-2hx-2ky+h^2+k^2-r^2=0.$$

giving us the following relations,

$$\begin{aligned} -8x=-2hx, \quad +4y=-2ky, \\ h=\frac{-8}{-2}, \quad k=\frac{4}{-2}, \\ h=4, \quad k=-2. \end{aligned}$$

Hence, the origin of the circle is at $(4,-2)$. To find the radius, we start by substituting the values of h and k into the equation and comparing the final terms,

$$\begin{aligned} x^2+y^2-8x+4y+4=0 \quad &\longleftrightarrow \quad x^2+y^2-2(4)x-2(-2)y+(4)^2+(-2)^2-r^2=0. \\ x^2+y^2-8x+4y+4=0 \quad &\longleftrightarrow \quad x^2+y^2-8x+4y+16+4-r^2=0, \end{aligned}$$

therefore

$$\begin{aligned} 4 &= 16+4-r^2 \\ 4-20 &= -r^2 \\ r &= 16^{1/2} \\ r &= 4. \end{aligned}$$

Finally, the radius of the circle is 4.

2 Exercises

2.1 Circle

1. With help of the pythagorean theorem find the function of a circle with origin at $O = (3/2, 2)$ and the point $(2, 3/2)$.
2. Write the equation of the circle with center at $(6, -9)$ and radius $1/4$.
3. Find the origing, radius of the following circle equation $x^2 - 8x + y^2 - 10y + 5 = 0$, then write the factored equation.

2.2 Review

1. Taking into account the following functions $f(x) = x + c$ and $g(x) = x + h$, compute $(f \circ g)(x) - f(x)$.
2. Determine the vertical and horizontal asymptotes of the following function $f(x) = (7x - 15)/(x - 5)$.
3. Use the properties of logarithms to condes the following expression $\log\left[\ln\left(x^6\right)/6\ln(x)\right] - \log[8\ln(x)]$.
4. Determine the critical point, asymptote, domain and range of the following function $f(x) = \ln(x+6) - 4$.