Problem 1

Plot the photon number distribution $p(n) = |\langle n, \psi | n, \psi\rangle|^2$ of a coherent state $|\alpha\rangle$, a Fock state, and a single mode thermal state, for $\langle n\rangle = 1, 10$ and 100 (9 plots in total). For each plot additionally provide Δn and the most probable outcome of the photon number measurement.

Problem 2

Let the operatos $\hat{a}_i$ and $\hat{a}_i^\dagger$, for $i \in \{1, 2\}$, form two sets of creation and annihilation operators that satisfy the commutation relations:

$$\left[\hat{a}_i, \hat{a}_j^\dagger\right] = \delta_{ij}, \quad \left[\hat{a}_i^\dagger, \hat{a}_j^\dagger\right] = 0, \quad \left[\hat{a}_i, \hat{a}_j\right] = 0$$

Let us define two new sets of creation and annihilation operators $\hat{b}_i^\dagger$ and $\hat{b}_i$ for $i = 1, 2$, which also follow the same commutation relations such that

$$\begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix} = \begin{pmatrix} u & \upsilon \\ \upsilon & u \end{pmatrix} \begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix}$$

What relationship must the real numbers u and υ satisfy?

Problem 3

an electric field mode has an equal probability of $1/3$ to be found in each of the states $|0\rangle$, $|2\rangle$ and the uperposition $4|0\rangle + 3|1\rangle$ (before normalization). Find the corresponding density matrix $\hat{\rho}$

Problem 4

Do eigenstates of $\hat{a}^\dagger$ exist? If yes, what is ther decomposition in the Fock state basis?

Problem 5

A bright squeezed state results from the application of a squezzing operator to a coherent state. It is defined as:

$$|r, \alpha\rangle = \hat{S}(r)\,|\alpha\rangle\,,$$

where the squeezing operator is defined as $\hat{S} = \exp\left[r/2(\hat{a}^2 - \hat{a}^{\dagger 2})\right]$. Find the uncertainty of a bright squeezed state in position and momentum quadratures. dos it retain the average photon number asociated with the coherent state? What would be different if we defined the bright squeezed state as a displaced squeezed vacuum states instead?