Tecnológico de monterrey

Algebraic and Trascendental Functions

Partial Exam 1

Name _____

ID number _____ **Date** _____ **Group** _____

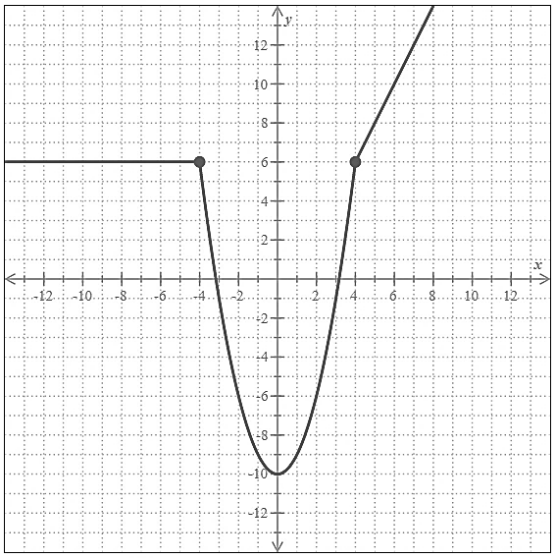
"Adhering to the Code of Ethics for students of the Tec de Monterrey, I pledge to follow the precepts of academic honesty in this evaluation by not using unauthorized or illicit resources to solve it."

_____
Signature

I. Read the following questions and identify the option that best answers each, then write its letter on the line. (4 points each)

1. _____ Given the function $f(x) \coloneqq 5(x-3)^2$, which is the correct translation?
a) 3 units to the right b) 3 units to the left c) 3 units upwards) 3 units downwards

2. _____ Which of the following functions corresponds to the piecewise graph displayed on the cartesian plane.



a) $f(x) = \begin{cases} 6, & x < -4 \\ x^2 - 10, & -4 < x < 4 \\ 2x - 2, & x > 4 \end{cases}$

b) $f(x) = \begin{cases} 6, & x \leq -4 \\ x^2 - 10, & -4 < x \leq 4 \\ 2x - 2, & x > 4 \end{cases}$

c) $f(x) = \begin{cases} -6, & x < -4 \\ -x^2 - 10, & -4 \leq x < 4 \\ -2x + 2, & x > 4 \end{cases}$

d) $f(x) = \begin{cases} 6, & x \leq -4 \\ x^2 - 10, & -4 \leq x \leq 4 \\ 2x - 2, & x \geq 4 \end{cases}$

3. _____ Suppose that $f(x) \coloneqq \frac{1}{x} + a$ and $g(x) \coloneqq \frac{1}{x+a}$. Which of the following options has the correct answer for $(f \circ g)(x)$?

a) x b) 1 c) 0 d) $\frac{1}{x}$

4. _____ Given $f(x) \coloneqq 5x^2$, which of the following functions the inverse function?

a) $f^{-1}(x) = x$ b) $f^{-1}(x) = x^{\frac{1}{2}}$ c) $f^{-1}(x) = \sqrt{5x}$ d) $f^{-1}(x) = \left(\frac{x}{5}\right)^{\frac{1}{2}}$

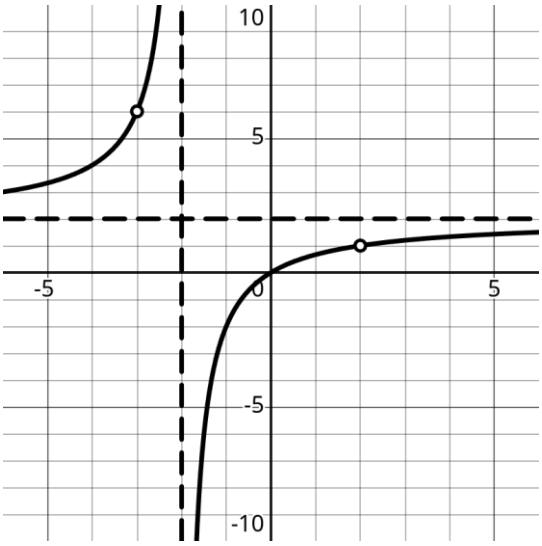
5. _____ A school fund-raising group sells chocolate bars to help finance a swimming pool party. Each chocolate bar costs $10 and they want to sell each bar at $15. What is the correct expression to estimate their profit and the final profit after selling 100 chocolate bars?

a) $f(x) \coloneqq -5x,\ f(100) = -500$ b) $f(x) \coloneqq x + 5,\ f(100) = 105$

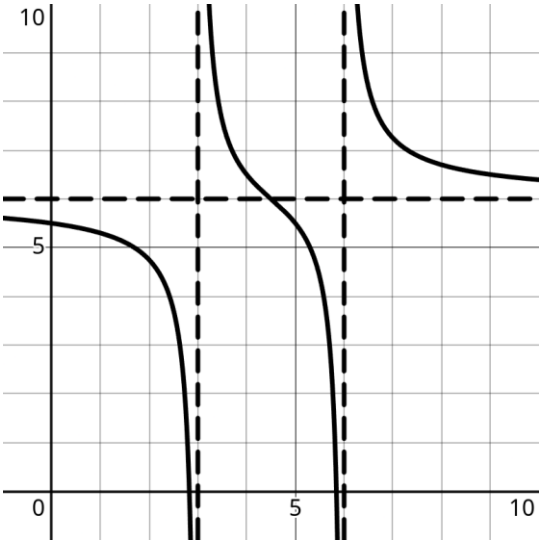
c) $f(x) \coloneqq 5x,\ f(100) = 500$ d) $f(x) \coloneqq 15x,\ f(100) = 1500$

II. Answer the following exercises.

1. Determine the domain and range of the rational function depicted in the following graph. (7 points)



2. For the following graph, find its function in factored form. (7 points)



III. Solve the following exercises in an orderly and clear manner. **Answers without procedure will not be considered.** Underline or frame your final answer.

1. Determine the equations of the vertical and horizontal asymptotes of the following function (10 points)

$$f(x) \coloneqq \frac{7x-15}{x-5}$$

2. Determine the intersections with x and y. Write the final answers as coordinates. (10 points)

$$f(x) \coloneqq \frac{3x}{(x-5)^2} - 5$$

3. Define the coordinate of the empty hole. (10 points)

$$f(x) \coloneqq \frac{x^2(x-1)}{2(x-1)}$$

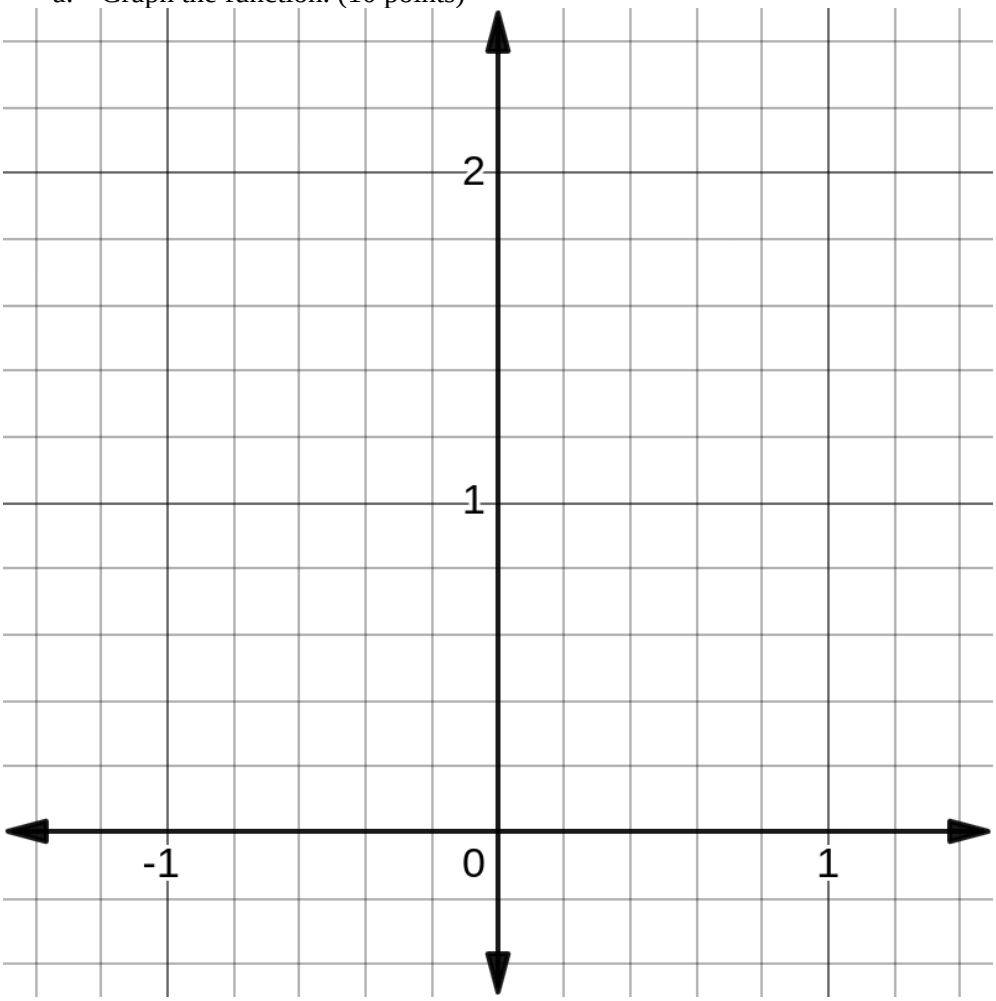
4. Find $h^{-1}(x)$, given the following function $h(x) = 5(x-1)^{-2} + 2$. (10 points)

5. Determine whether $f(x) \coloneqq \frac{1}{4x^2+5}$ and $g(x) \coloneqq \sqrt{4\left(\frac{1}{x} - 5\right)}$ are inverses of each other or not. Simplify your answer as much as possible. Justify your answer using both compositions: $(f \circ g) = (g \circ f) = x$. (10 points)

6. Find the following for the piecewise function,

$$f(x) := \begin{cases} \frac{7}{4} & x \in [-3/2, -1/2] \\ 1/4(x+1) & x \in [-1, 1] \\ -2(x-1)^2 + 2 & x \in [1/2, 3/2] \end{cases}$$

   a. Graph the function. (10 points)



   b. Determine the domain and range of $f(x)$. (6 points)