Review session   Department of Methematics.

Name: _____

1   Examples

Example 1: Factored equation of the circle.

Identify the radius and center of the following circle,

$$(x+3)^2 + (y+1)^2 = 9.$$

Recalling the definition of the circle, $(x-h)^2 + (y-k)^2 = r^2$, and comparing both expressions, we can make the following relations,

$$x+3 \longleftrightarrow x-h, \quad y+1 \longleftrightarrow y-k, \quad 9 \longleftrightarrow r^2.$$

Therefore,

$$3 = -h, \quad 1 = -k, \quad 9 = r^2$$
$$h = -3, \quad k = -1, \quad r = 9^{1/2} = 3.$$

This tell's us that the circle origin is at $(-3, -1)$ with a radius of $r = 3$.

Example 2: Expanded equation of the circle.

Identify the radius and center of the following circle,

$$x^2 + y^2 - 8x + 4y + 4 = 0.$$

Recalling the note, we can make the following comparsion as before,

$$x^2 + y^2 - 8x + 4y + 4 = 0 \quad \longleftrightarrow \quad x^2 + y^2 - 2hx - 2ky + h^2 + k^2 - r^2 = 0.$$

giving us the following relations,

$$-8x = -2hx, \quad +4y = -2ky,$$
$$h = \frac{-8}{-2}, \quad k = \frac{4}{-2},$$
$$h = 4, \quad k = -2.$$

Hence, the origin of the circle is at $(4, -2)$. To find the radius, we start by substituting the values of h and k into the equation and comparing the final terms,

$$x^2 + y^2 - 8x + 4y + 4 = 0 \quad \longleftrightarrow \quad x^2 + y^2 - 2(4)x - 2(-2)y + (4)^2 + (-2)^2 - r^2 = 0.$$
$$x^2 + y^2 - 8x + 4y + 4 = 0 \quad \longleftrightarrow \quad x^2 + y^2 - 8x + 4y + 16 + 4 - r^2 = 0,$$

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therefore

$$4 = 16 + 4 - r^2$$
$$4 - 20 = -r^2$$
$$r = 16^{1/2}$$
$$r = 4.$$

Finally, the radius of the circle is 4.

2 Exercises

- Write the equation of the circle with center at $(6, -9)$ and radius $1/4$.