

Conic sections: Hyperbola

Department of Mathematics.

Name: _____

1 Examples**Example 1: Factored form of a vertical hyperbola**

We are going to find the center, vertices, foci and asymptotes of the following hyperbola,

$$\frac{(y-8)^2}{4} - \frac{(x+1)^2}{9} = 1.$$

Let's start with the center, which is $C = (-1, 8)$. Now that we know that and also the values of $a = 2$ and $b = 3$, we can compute c using the relation $b^2 \equiv c^2 - a^2$, $\rightarrow c = \sqrt{9+4} = \sqrt{13}$. An important observation is that the term with x is negative, which tells us that is a vertical hyperbola.

With that information we can compute the vertex of the hyperbola as follows,

$$v_1 = (-1, 8-2)$$

$$v_2 = (-1, 8+2)$$

$$v_1 = (-1, 6)$$

$$v_2 = (-1, 10)$$

Then, we do something similar for the foci,

$$F_1 = (-1, 8 - \sqrt{13})$$

$$F_2 = (-1, 8 + \sqrt{13})$$

$$F_1 = (-1, 4.39)$$

$$F_2 = (-1, 11.60)$$

Finally, to compute the asymptotes of the hyperbola, we modify the general equation for hyperbola asymptotes since is a vertical hyperbola, $y - k = \pm \frac{a}{b}(x - h)$,

$$y - k = \pm \frac{a}{b}(x - h) \rightarrow y - 8 = \pm \frac{2}{3}(x + 1)$$

$$y = \pm \frac{2}{3}(x + 1) + 8$$

Example 2: Factored form of a horizontal hyperbola

We are going to find the center, vertices, foci and asymptotes of the following hyperbola,

$$\frac{(x+8)^2}{25} - \frac{(y-9)^2}{4} = 1.$$

Let's start with the center, which is $C = (-8, 9)$. Now that we know that and also the values of $a = 5$ and $b = 2$, we can compute c using the relation $b^2 \equiv c^2 - a^2$, $\rightarrow c = \sqrt{25+4} = \sqrt{29}$. An important observation is that the term with y is negative, which tells us that is a horizontal

hyperbola.

With that information we can compute the vertex of the hyperbola as follows,

$$v_1 = (-8 - 5, 9)$$

$$v_2 = (-8 + 5, 9)$$

$$v_1 = (-13, 9)$$

$$v_2 = (-3, 9)$$

Then, we do something similar for the foci,

$$F_1 = (-8 - \sqrt{29}, 9)$$

$$F_2 = (-8 + \sqrt{29}, 9)$$

$$F_1 = (-13.38, 9)$$

$$F_2 = (-2.61, 9)$$

Finally, to compute the asymptotes of the hyperbola, we use the general equation for hyperbola, $x - h = \pm \frac{a}{b}(y - k)$,

$$\begin{aligned} x - h &= \pm \frac{a}{b}(y - k) \rightarrow x + 8 = \pm \frac{5}{2}(y - 9) \\ x &= \pm \frac{5}{2}(y - 9) - 8 \end{aligned}$$

2 Exercises

2.1 Hyperbola

- Construct the hyperbola equation from it's asymptotes, $y = \pm 1/2(x + 4) - 3$.
- Find the center, vertices, foci and asymptotes of the following hyperbola $(x - 3)^2/9 - (y - 6)^2/4 = 1$.

2.2 Review

- Solve for x , $\log_5(4x) - 3 = -8$
- Determine the horizontal asymptote for the following function $f(x) = 1/3(e^x - 4)^4$
- Determine the vertical and horizontal asymptotes of the following function $f(x) = (7x - 15)/(x - 5)$.
- Find the inverse function of $f(x) = 2x + 4$.