Algebraic and Transcendental functions

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April 8, 2025

Summary Class sketches for the subject Algebraic and Transcendental Functions. Semester January-May 2025

Hyperbola

A hyperbola is a curve defined by all points that satifies that the difference of the distance from a point P to a point F_1 with the distance from a point P to a point F_2 remains as a constant value of $2a$, as shown in figure1. The distance between point F_1 and F_2 is defined to be $2c$.

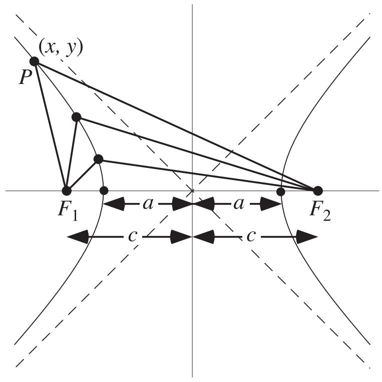

Figure 1: Skecth of an hyperbola

Derivation of the equation

We are going to call the distance from point P to point F_1 as r_1, and the distance from point P to point F_2 as r_2. By definition, the substraction of those distances needs to be equal to $2a$, therefore we have the following equation,

$$\sqrt{(x-c)^2+y^2} - \sqrt{(x+c)^2+y^2} = 2a.$$

As before, we are going to modify that equation to express it in a more friendly way, taking into account that $b^2 = c^2 - a^2$,

$$\begin{aligned}
\left[\sqrt{(x-c)^2+y^2}\right]^2 &= \left[2a - \sqrt{(x+c)^2+y^2}\right]^2 \\
(x-c)^2+y^2 &= 4a^2 - 4a\sqrt{(x+c)^2+y^2} + (x+c)^2+y^2 \\
\cancelto{0}{x^2} - 2cx + \cancelto{0}{c^2} + \cancelto{0}{y^2} &= 4a^2 - 4a\sqrt{(x+c)^2+y^2} + \cancelto{0}{x^2} + 2cx + \cancelto{0}{c^2} + \cancelto{0}{y^2} \\
\frac{1}{4a}\left[4a\sqrt{(x+c)^2+y^2}\right] &= \frac{1}{4a}\left[4a^2+4cx\right] \\
\left[\sqrt{(x+c)^2+y^2}\right]^2 &= \left[a + \frac{cx}{a}\right]^2 \\
(x+c)^2+y^2 &= a^2 + 2cx + \frac{c^2x^2}{a^2} \\
x^2 + \cancelto{0}{2cx} + c^2 + y^2 &= a^2 + \cancelto{0}{2cx} + \frac{c^2x^2}{a^2} \\
x^2 + c^2 + y^2 &= a^2 + \frac{c^2x^2}{a^2} \\
x^2\cancelto{-b^2/a^2}{\left(1-\frac{c^2}{a^2}\right)} + y^2 &= \cancelto{-b^2}{a^2-c^2} \\
\frac{1}{-b^2}\left[\frac{-b^2}{a^2}x^2+y^2\right] &= \cancelto{1}{\frac{-b^2}{-b^2}} \\
\frac{x^2}{a^2} - \frac{y^2}{b^2} &= 1
\end{aligned}$$

References

James Stewart, L. Redlin, Saleem Watson, and Phyllis Panman. *Precalculus: mathematics for calculus*. Cengage Learning, Boston, MA, seventh edition edition, 2016. ISBN 9781305071759.

Eric W. Weisstein. Ellipse, a. URL `https://mathworld.wolfram.com/Ellipse.html`.

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