

Team notebook

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1 DP

1.1 1D1D

```
#include <bits/stdc++.h>
using namespace std;

#define ll long long
#define ar array
#define rep(i, n) for(int i = 0; i<(int)n; ++i)
#define repx(i, a ,b) for(int i = (int)a; i<(int)b; ++i)

const int mxN = 2e5+5, M = 1e9+7;
int n, s, dp[mxN], t[mxN], f[mxN];

int w(int j, int i){ //
    if(j >= i) return 1e9; // here is the cost function from (j, i)
    return s*(f[n]-f[j]) + t[i]*(f[i]-f[j]);
}

struct DD{ // index 0 is used as neutral state
    vector<pair<int, int>> v; // (start pos, best k)
    DD() { v.push_back(make_pair(0, 0)); }
    int qry(int i){
        return (--lower_bound(v.begin(), v.end(), make_pair(i+1, 0)))>>second;
    }
    void upd(int x){
        for(int i = (int)v.size()-1; i>=0; --i){
            int y = v[i].first, oldk = v[i].second;
            if(y > x && dp[x] + w(x, y) < dp[oldk] + w(oldk, y))
                v.pop_back();
            else{
                int l = y+1, r = n+1;
                while(l < r){
                    int mid = (l+r)/2;
                    if(dp[x] + w(x, mid) < dp[oldk] + w(oldk, mid))
                        r = mid;
                    else l = mid+1;
                }
                if(r != n+1) v.push_back(make_pair(r, x)); break;
            }
        }
    }
};
```

```
    }
    }
    if(v.size() == 0) v.push_back(make_pair(0, x));
}
};

cin >> n >> s;
DD dd;
t[0] = f[0] = dp[0] = 0;
for(int i = 1; i<=n; ++i){
    cin >> t[i] >> f[i]; t[i] += t[i-1]; f[i] += f[i-1];
}
for(int i = 1; i<=n; ++i){
    int k = dd.qry(i); dp[i] = dp[k] + w(k, i); dd.upd(i);
}
cout << dp[n] << '\n';
// https://vjudge.net/problem/OpenJ\_Bailian-1180
```

1.2 1D2DMaxSum-Multiplication

```
#include "../Header.cpp"

int main(){
    // 1D Max Array Sum
    int n = 9, A[] = { 4, -5, 4, -3, 4, 4, -4, 4, -5 }; //Allow
    all negative numbers
    int sum = A[0], ans = A[0];
    for (int i = 1; i < n; i++){
        sum = max(A[i] + sum, A[i]); // Ignores sum if prev sum is
        worse than A[i]
        ans = max(ans, sum);
    }
    cout << ans << "\n";
    // 2D Max Array Sum
    int B [100][100];
    ans = -INF;
    cin>>n;
    for (int i = 0; i < n; i++) for (int j = 0; j < n; j++){
        cin >> B[i][j];
        if (j > 0) B[i][j] += B[i][j-1]; // Acum sum per Row
    }
```

```

for (int l = 0; l < n; l++) for (int r = l; r < n; r++){
    sum = B[0][r];
    int SubAns = B[0][r];
    if (l > 0){ sum -= B[0][l-1]; SubAns -= B[0][l-1]; }
    for (int row = 1; row < n; row++){
        int aux = B[row][r];
        if(l > 0) aux -= B[row][l-1];
        sum = max(sum + aux, aux);
        SubAns = max (SubAns, sum);
    }
    ans = max(ans, SubAns);
}
cout << ans << "\n";

// Max Array Multiplication
vl c;
bool o = 0;
ans = 1;
ll miniend = 1, maxiend = 1;
for (int i = 0; i < c.size(); i++){
    if(c[i] > 0){
        o = 1;
        if(miniend < 0)miniend *= c[i];
        maxiend *= c[i];
    }
    else if(c[i] == 0){
        miniend = 1;
        maxiend = 1;
    }else{
        int aux = maxiend;
        maxiend = max(1LL, miniend * c[i]);
        miniend = aux*c[i];
    }
    ans = max(ans, maxiend);
}
if(ans == 1 && !o) cout << "0\n";
else{
    cout << ans << "\n";
}

// n dimension accumulative sum

```

```

for dim = 0 to 4
    for a = 0 to na-1
        for b = 0 to nb-1
            for c = 0 to nc - 1
                for d = 0 to nd - 1
                    pa = a - (dim==0); pb = b - (dim==1); pc = c
                        - (dim==2);
                    pd = d - (dim==3);
                    if (pa >= 0 && pb >= 0 && pc >= 0 && pd >= 0)
                        dp(a, b, c, d) += dp (pa, pb, pc, pd)

// o por cada celda
for x in S //(celda de menor a mayor tal que todas las
    anteriores estan procesadas)
}

```

1.3 CoinProblem

```

#include "../Header.cpp"
// Number of ways of reaching a quantity n from a set of coins c
int main(){
    int c[5] = {1, 5, 10, 25, 50};
    int n;
    while(cin >> n){
        int m[n+1];
        m[0] = 1;
        for(int i = 1; i <= n+1; i++) m[i] = 0;
        for(int j = 0; j < 5; j++){
            for(int i = 1; i <= n+1; i++){
                if(i - c[j] >= 0){
                    m[i] += m[i - c[j]];
                    //m[i]=min(m[i],m[i-c[j]]+1);
                    for minimum coins
                }
            }
        }
        cout << m[n] << "\n";
    }
}

```

1.4 ConvexHullTrick

```
#include "../Header.cpp"

typedef ll tc;
struct Line{tc m,h;};
struct CHT { // for minimum (for maximum just change the sign of
    lines)
    vector<Line> c;
    int pos=0;
    tc in(Line a, Line b){
        tc x=b.h-a.h,y=a.m-b.m;
        return x/y+(x%y?!((x>0)^(y>0)):0); // ==ceil(x/y)
    }
    void add(tc m, tc h){ // m's should be non increasing
        Line l=(Line){m,h};
        if(c.size()&&m==c.back().m){
            l.h=min(h,c.back().h);c.pop_back();if(pos)pos--;
        }
        while(c.size()>1&&in(c.back(),l)<=in(c[c.size()-2],c.back())){
            c.pop_back();if(pos)pos--;
        }
        c.pb(l);
    }
    inline bool fbin(tc x, int m){return in(c[m],c[m+1])>x;}
    tc eval(tc x){
        // O(log n) query:
        int s=0,e=c.size();
        while(e-s>1){int m=(s+e)/2;
            if(fbin(x,m-1))e=m;
            else s=m;
        }
        return c[s].m*x+c[s].h;
        // O(1) query (for ordered x's):
        while(pos>0&&fbin(x,pos-1))pos--;
        while(pos<c.size()-1&&!fbin(x,pos))pos++;
        return c[pos].m*x+c[pos].h;
    }
};
```

```
struct Line {
    ll m, c, id;
    ll calc(ll x) {

        return m * x + c;
    }
};
bool obsolete(Line a, Line b, Line c){
    return (c.c - a.c) * (a.m - b.m) < (a.m - c.m) * (b.c - a.c);
}

vector<Line>lines;
void insert(Line l) {
    while(lines.size() > 1) {
        ll sz = lines.size();
        if(obsolete(lines[sz-2], lines[sz-1], l)){
            lines.pop_back();
        } else break;
    }
    lines.push_back(l);
}
```

1.5 ConvexHullTrickDynamic

```
typedef ll tc;
const tc is_query=-(1LL<<62); // special value for query
struct Line {
    tc m,b;
    mutable multiset<Line>::iterator it,end;
    const Line* succ(multiset<Line>::iterator it) const {
        return (++it==end? NULL : &*it);}
    bool operator<(const Line& rhs) const {
        if(rhs.b!=is_query)return m<rhs.m;
        const Line *s=succ(it);
        if(!s)return 0;
        return b-s->b<(s->m-m)*rhs.m;
    }
};
struct HullDynamic : public multiset<Line> { // for maximum
    bool bad(iterator y){
```

```

    iterator z=next(y);
    if(y==begin()){
        if(z==end())return false;
        return y->m==z->m&& y->b<=z->b;
    }
    iterator x=prev(y);
    if(z==end())return y->m==x->m&& y->b<=x->b;
    return
        (x->b-y->b)*(z->m-y->m)>=(y->b-z->b)*(y->m-x->m);
}
iterator next(iterator y){return ++y;}
iterator prev(iterator y){return --y;}
void add(tc m, tc b){
    iterator y=insert((Line){m,b});
    y->it=y;y->end=end();
    if(bad(y)){erase(y);return;}
    while(next(y)!=end()&&bad(next(y)))erase(next(y));
    while(y!=begin()&&bad(prev(y)))erase(prev(y));
}
tc eval(tc x){
    Line l=*lower_bound((Line){x,is_query});
    return l.m*x+l.b;
}
};

```

1.6 Digit_{dp}

```

int dp[12][12][2]; // dp[i][s][f] {i: posicion, s: estado del
                    // problema, f: act < s}
int k, d;

// solve(r) - solve(l-1)
int call(int pos, int cnt, int f){
    if(cnt > k) return 0;
    if(pos == num.size()){
        if(cnt == k) return 1;
        return 0;
    }
    if(dp[pos][cnt][f] != -1) return dp[pos][cnt][f];
    int res = 0, LMT;

```

```

    if(f == 0) LMT = num[pos];
    else LMT = 9;
    /// Try to place all the valid digits such that the number
    /// doesn't exceed b
    for(int dgt = 0; dgt<=LMT; dgt++){
        int nf = f, ncnt = cnt;
        if(f == 0 && dgt < LMT) nf = 1; /// The number is getting
        /// smaller at this position
        if(dgt == d) ncnt++;
        if(ncnt <= k) res += call(pos+1, ncnt, nf);
    }
    return dp[pos][cnt][f] = res;
}
int solve(string s){
    num.clear();
    while(s.size()){
        num.push_back((s.back()-'0')%10);
        s.pop_back();
    }
    reverse(num.begin(), num.end());
    memset(dp, -1, sizeof(dp));
    return call(0, 0, 0);
}

```

1.7 DivideConquerDP

```

#include "../Header.cpp"

// dp(i, j) = min dp(i-1,k-1) + C(k,j) for all k in [0, j]
// C(a,c) + C(b, d) <= C(a,d) + C(b,c) for all a <= b <= c <= d
vp c;
vl acum1, acum2;

ll cost(ll i, ll j){
    return c[j].first * (acum1[j+1] - acum1[i]) - (acum2[j+1] -
    acum2[i]);
}
vector<ll> last, now;

void compute(int l, int r, int optl, int optpr){

```

```

    if (l > r) return;
    int mid = (l + r) / 2;
    pair<ll, int> best = {cost(0, mid), -1};
    for(int k = max(1, optl); k < min(mid, optr) + 1; k++){
        best = min(best, {last[k - 1] + cost(k, mid), k});
    }
    now[mid] = best.first;

    compute(l, mid - 1, optl, best.second);
    compute(mid + 1, r, best.second, optr);
}

int main(){
    ios_base::sync_with_stdio(0); cin.tie(0);
    ll n, k, x, w;
    while(cin >> n >> k){
        c.clear();
        for(int i = 0; i < n; i++){
            cin >> x >> w;
            c.push_back({x, w});
        }
        acum1.clear(); acum2.clear();
        acum1.push_back(0); acum2.push_back(0);

        for(int i = 0; i < n; i++){
            acum1.push_back(c[i].second);
            acum2.push_back(c[i].first * c[i].second);
            acum1.back() += acum1[i];
            acum2.back() += acum2[i];
        }
        last.assign(n, INF);
        now.resize(n);
        for(int i = 0; i < k; i++) { compute(0, n - 1, 0, n - 1);
            swap(last, now); }

        cout<< last[n-1]<<"\n";
    }
}

```

1.8 Knapsack

```

#include "../Header.cpp"

int V[10000], W[10000], M[102][10202];

// index, capacity
int DP(int i, int c){
    if(i==0){
        return 0;
    }
    if(c==0) return 0;
    if(M[i][c] != -1) return M[i][c];
    M[i][c] = DP(i-1, c);
    if(W[i] <= c){
        M[i][c] = max(M[i][c], DP(i-1, c - W[i]) + V[i]);
    }
    return M[i][c];
}

// Variation
int usados=0,espacio_usado;
int knapSack(int W, int wt[], int val[], int n){
    int i, w;
    int K[n+1][W+1][3];
    for (i = 0; i <= n; i++){
        for (w = 0; w <= W; w++){
            if (i==0 || w==0){
                K[i][w][0] = 0;
                K[i][w][1] = 0;
                K[i][w][2] = 0;
            }
            else if (wt[i-1] <= w){
                K[i][w][0] = max(val[i-1] +
                    K[i-1][w-wt[i-1]][0], K[i-1][w][0]);
                if(K[i-1][w][0]>val[i-1] + K[i-1][w-wt[i-1]][0]){
                    K[i][w][1]=K[i-1][w][1];
                    K[i][w][2]=K[i-1][w][2];
                }else{
                    K[i][w][1]=K[i-1][w-wt[i-1]][1]+wt[i-1];
                    K[i][w][2]=K[i-1][w-wt[i-1]][2]+1;
                }
            }
        }
    }
}

```

```

    }
    }else{
        K[i][w][0] = K[i-1][w][0];
        K[i][w][1] = K[i-1][w][1];
        K[i][w][2] = K[i-1][w][2];
    }
}
}
usados=K[n][W][2];
espacio_usado=K[n][W][1];
return K[n][W][0];
}

int main(){
    int v,W,t;
    cin>>t;
    for(int o=0;o<t;o++){
        W=50;
        usados=0;
        cin>>v;
        int val[v];
        int wt[v];
        for(int i=0;i<v;i++){
            cin>>val[i];
            cin>>wt[i];
        }
        int n = sizeof(val)/sizeof(val[0]);
        cout<<knapSack(W, wt, val, n)<<" brinquedos"<<endl;
        cout<<"Peso: " <<espacio_usado<<" kg"<<endl;
        cout<<"sobra(m) " <<v-usados<<" pacote(s)"<<endl<<endl;
    }
}

```

1.9 KnuthOptimization

```

int N; vector<int> A;
vector<vector<int>>> DP, OPT;

int main(){
    DP.assign(N + 1, vi(N + 1));

```

```

    OPT.assign(N + 1, vi(N + 1));
    rep(i, N) DP[i][i + 1] = A[i + 1] - A[i], OPT[i][i + 1] = i;
    repx(d, 2, N + 1) rep(l, N + 1 - d){
        int r = l + d, l_ = OPT[l][r - 1], r_ = OPT[l + 1][r];
        DP[l][r] = 1e9;
        repx(i, l_, r_ + 1){
            int aux = DP[l][i] + DP[i][r] + A[r] - A[l];
            if (aux < DP[l][r]) DP[l][r] = aux, OPT[l][r] = i;
        }
    }
}

```

1.10 LongestIncreasingSubsequence

```

#include "../Header.cpp"

v1 A, p;
void print_LIS(int i) {
    // backtracking
    routine
    if (p[i] == -1) { printf("%d", A[i]); return; } // base case
    print_LIS(p[i]); // backtrack
    printf(" %d", A[i]);
}

//O(nlogn)
int lis(vector<int> const& a) {
    int n = a.size();
    const int INF = 1e9;
    vector<int> d(n+1, INF);
    d[0] = -INF;
    for (int i = 0; i < n; i++) {
        int j = upper_bound(d.begin(), d.end(), a[i]) - d.begin();
        if (d[j-1] < a[i] && a[i] < d[j])
            d[j] = a[i];
    }
    int ans = 0;
    for (int i = 0; i <= n; i++) {
        if (d[i] < INF)
            ans = i;
    }
}

```



```

    return ans;
}
int main(){
    ll t,n; cin>>t;
    while(t--){
        int x; cin>>n;
        for(int i=0;i<n;i++){
            cin>>x;
            A.push_back(x);
        }
        ll LIS[100][100] // LIS for any (i, j)
        for(int z = 0; z < n; z++){
            int k = z, lis_end = z;
            vl L(n, 0), L_id(n, 0);
            p.assign (n, -1)
            for (int i = z; i < n; ++i) {
                int pos = lower_bound(L.begin() + z, L.begin()+k,
                    c[i]) - L.begin();
                if(A[i]==L[pos])pos++;//For non strickly increasing
                    subsequence
                L[pos] = c[i];
                L_id[pos] = i;
                p[i] = pos ? L_id[pos-1] : -1;
                if (pos == k) {
                    k = pos+1;
                    lis_end = i;
                }
            }
            for(int i = z; i < n; i++){
                if(p[i] == -1) LIS[z][i] = 1;
                else LIS[z][i] = 1 + LIS[z][p[i]];
            }
        }
        cout<<"Final LIS is of length: "<< k<<"\n";
        print_LIS(lis_end);cout<<"\n";
        //DP
        vl LI(n, 0), LD(n,0);
        ll in=0,dec=0;
        for(int i=0;i<n;i++){
            LI[i]=1;
            LD[i]=1;

```

```

                for(int j=0;j<i;j++){
                    if(A[j]<A[i])
                        LI[i]=max(LI[i],LI[j]+1);
                    if(A[j]>A[i])
                        LD[i]=max(LD[i],LD[j]+1);
                }
                in=max(in,LI[i]);
                dec=max(dec,LD[i]);
            }
        }
    }
}

```

1.11 connected_component_{dp}

```

int n, k, dp[mxN][mxN][1005][3];
int a[mxN];

int ff(ll i, ll c, ll sum, ll b){ // {i, components, sum,
    borders} 1 indexed
    if(b > 2 || sum > k) return 0; // k = limit sum
    if(c == 0 && i > 1) return 0;
    if(i == n+1) return b == 2 && c == 1; // array completely
        filled
    int &ret = dp[i][c][sum][b];
    if(ret != -1) return ret; // this behind changes
        between problems
    int nsum = sum + (a[i]-a[i-1])*(2*c-b); // all unknown
        positions equals to a[i]
    ll ans = 0;
    if(c >= 2) ans += (c-1)*ff(i+1, c-1, nsum, b); // merge two cc
    if(c >= 1) ans += (2LL*c-b)*ff(i+1, c, nsum, b); // add to a
        component end
    ans += (c+1-b)*(i+1, c+1, nsum, b); // create new component
    if(b < 2) ans += (2LL-b)*ff(i+1, c+1, nsum, b+1); // create
        new end
    if(b < 1) ans += (2LL-b)*ff(i+1, c, nsum, b+1); // extend cc
        to a border
    ans %= M; return ret = ans;
}

```

2 Geometry

2.1 2D Algorithms

```
#include "../Header.cpp"
```

```
double DEG_to_RAD(double d) { return d*PI / 180.0; }
double RAD_to_DEG(double r) { return r*180.0 / PI; }
```

```
struct point { db x, y;
    point() { x = y = 0.0; }
    point(db _x, db _y) : x(_x), y(_y) {}
    bool operator <(const point& p) const { return (x < p.x ? true
        : (x == p.x && y < p.y)); }
    bool operator == (const point& p) const { return abs(p.x - x)
        < EPS && abs(p.y - y) < EPS; }
    point operator + (const point& p) const { return point(x +
        p.x, y + p.y); }
    point operator - (const point& p) const { return point(x -
        p.x, y - p.y); }
    point operator * (db p) const { return point(x * p, y * p); }
    point operator / (db p) const { return point(x / p, y / p); }
    db operator^(const point &p) const {return x * p.y - y * p.x; }
    db operator*(const point &p) const {return x * p.x + y * p.y; }
    db norm_sq() const { return x*x + y*y; }
    point rot() { return point(-y, x); }
    point rot45() { return point(x + y, y - x); }
```

```
// by angles but with cross
```

```
bool half() const { return y > 0 || (y == 0 && x > 0); }
bool operator<(const point &p) const
{
    int h1 = half(), h2 = p.half();
    return h1 != h2 ? h1 > h2 : ((*this) ^ p) > 0;
}
```

```
db ang()
{
    double a = atan2(y, x);
    if (a < 0) a += 2.0 * PI;
    return a;
}
```

```
}
```

```
};
```

```
db dist(const point& p1, const point& p2) {
    return sqrt((p1.x-p2.x)*(p1.x-p2.x)+ (p1.y-p2.y)*(p1.y-p2.y)); }
db dist_sq(point p1, point p2) {
    return (p1.x - p2.x)*(p1.x - p2.x)+(p1.y - p2.y)*(p1.y - p2.y); }
```

```
point rotate(point p, db rad) {
    return point(p.x * cos(rad) - p.y*sin(rad),
        p.x * sin(rad) + p.y*cos(rad)); }
```

```
struct line { db a, b, c; };
```

```
void pointsToLine(point p1, point p2, line &l) {
    if (fabs(p1.x-p2.x) < EPS) // vertical line is fine
        l = {1.0, 0.0, -p1.x}; // default values
    else {
        db a = -(db)(p1.y-p2.y) / (p1.x-p2.x);
        l = {a,
            1.0, // IMPORTANT: we fix the value of b to 1.0
            -(db)(a*p1.x) - p1.y}; }
}
```

```
// for integers, normalized
```

```
void pointsToLine(point& p1, point p2, line &l) {
    l.a = p1.y - p2.y;
    l.b = p2.x - p1.x;
    l.c = p1.x * (p2.y - p1.y) - p1.y * (p2.x - p1.x);
    ll g = __gcd(abs(l.a), __gcd(abs(l.b), abs(l.c)));
    ll sgn = 1;
    if(l.a < 0 || (l.a == 0 && l.b < 0))sgn = -1;
    l.a /= g * sgn; l.b /= g * sgn; l.c /= g * sgn;
}
```

```
// not needed since we will use the more robust form: ax + by + c
= 0
```

```
struct line2 { db m, c; }; // another way to represent a line
```

```

int pointsToLine2(point p1, point p2, line2 &l) {
    if (abs(p1.x-p2.x) < EPS) {        // special case: vertical line
        l.m = INF;                      // l contains m = INF and c = x_value
        l.c = p1.x;                    // to denote vertical line x = x_value
        return 0; // we need this return variable to differentiate
            result
    }
    else {
        l.m = (db)(p1.y-p2.y) / (p1.x-p2.x);
        l.c = p1.y - l.m*p1.x;
        return 1; // l contains m and c of the line equation y = mx
            + c
    } }

bool areParallel(line l1, line l2) { // check coefficients a & b
    return (fabs(l1.a-l2.a) < EPS) && (fabs(l1.b-l2.b) < EPS); }

bool areSame(line l1, line l2) { // also check coefficient c
    return areParallel(l1 ,l2) && (fabs(l1.c-l2.c) < EPS); }

// returns true (+ intersection point) if two lines are intersect
bool areIntersect(line l1, line l2, point &p) {
    if (areParallel(l1, l2)) return false; // no intersection
    // solve system of 2 linear algebraic equations with 2 unknowns
    p.x = (l2.b*l1.c - l1.b*l2.c) / (l2.a*l1.b - l1.a*l2.b);
    // special case: test for vertical line to avoid division by zero
    if (fabs(l1.b) > EPS) p.y = -(l1.a*p.x + l1.c);
    else p.y = -(l2.a*p.x + l2.c);
    return true; }

// Or use pointsToSlope, Revisar, mejor con 2 puntos
void perpendicular_line(point a, line l, line& ans)
{
    point b((-l.b*a.y-l.c)/l.a,a.y+1);
    b.x-=a.x;
    b.y-=a.y;
    b = rotate(b,90);
    b.x+=a.x;
    b.y+=a.y;
    pointsToLine(a,b, ans);
}

```

```

}

//Scalar projection of vector a onto vector b
// if s < -EPS or s > |b| + EPS then the projection is not on the
    segment
db sproject(point a, point b)
{
    return a*b/sqrt(b.norm_sq());
}

bool onSegment(const point& p, const point& p1, const point& p2)
{
    bool x = (abs(p1.x - p2.x) < EPS && abs(p.x - p2.x) < EPS) ||
        (p.x <= max(p1.x, p2.x) && p.x >= min(p1.x, p2.x));
    bool y = (abs(p1.y - p2.y) < EPS && abs(p.y - p2.y) < EPS) ||
        (p.y <= max(p1.y, p2.y) && p.y >= min(p1.y, p2.y));
    return x && y;
}

// convert point and gradient/slope to line, A PARTIR DE UNA
    DIRECCION M
// usar 1/l.a para calcular perpendicular
void pointSlopeToLine(point p, db m, line &l) {
    l.a = -m; // always -m
    l.b = 1; // always 1
    l.c = -((l.a*p.x) + (l.b*p.y)); // compute this
}

void closestPoint(line l, point p, point &ans) {
    line perpendicular; // perpendicular to l and pass through
        p
    if (fabs(l.b) < EPS) { // special case 1: vertical line
        ans.x = -(l.c); ans.y = p.y; return; }

    if (fabs(l.a) < EPS) { // special case 2: horizontal line
        ans.x = p.x; ans.y = -(l.c); return; }

    pointSlopeToLine(p, 1/l.a, perpendicular); // normal line
    // intersect line l with this perpendicular line
    // the intersection point is the closest point
    areIntersect(l, perpendicular, ans); }

```

```

// returns the reflection of point on a line
void reflectionPoint(line l, point p, point &ans) {
    point b;
    closestPoint(l, p, b);           // similar to distToLine
    point v = (b - p);               // create a vector
    ans = p + v + v; }              // translate p twice

// returns the distance from p to the line defined by
// two points a and b (a and b must be different)
// the closest point is stored in the 4th parameter (byref)
db distToLine(point p, point a, point b, point &c) {
    // formula: c = a + u*ab
    point ap = (p - a), ab = (b - a);
    db u = ap * ab / ab.norm_sq();
    c = a + ab * u;                  // translate a to c
    return dist(p, c); }            // Euclidean distance between p and c

// returns the distance from p to the line segment ab defined by
// two points a and b (still OK if a == b)
// the closest point is stored in the 4th parameter (byref)
db distToLineSegment(point p, point a, point b, point &c) {
    point ap = (p - a), ab = (b - a);
    db u = ap * ab / ab.norm_sq();
    if (u < 0.0) { c = point(a.x, a.y); } // closer to a
    return dist(p, a); }             // Euclidean distance between p and a
    if (u > 1.0) { c = point(b.x, b.y); } // closer to b
    return dist(p, b); }             // Euclidean distance between p and b
    return distToLine(p, a, b, c); } // run distToLine as above

bool ccw(point p, point q, point r) {
    return ((q - p)^(r - p)) > -EPS; }

// returns true if point r is on the same line as the line pq
bool collinear(point p, point q, point r) {
    return fabs(((q - p)^(r - p))) < EPS; }

// angle from 0 to 2*PI
db anglet(point a, point o, point b) { // returns angle aob in rad
    point oa = (a - o), ob = (b - o);

```

```

    db ang = acos(oa * ob / sqrt(oa.norm_sq()*ob.norm_sq()));
    if(ang!=0&&!collinear(a,o,b)&&ccw(a,o,b))ang = 2*PI - ang;
    return ang; } // better

db angle(point a, point o, point b) { // returns angle aob in rad
    point oa = (a - o), ob = (b - o);
    return acos(oa * ob / sqrt(oa.norm_sq()*ob.norm_sq())); }

point min(point a, point b)
{
    if(a<b)return a;
    return b;
}
point max(point a, point b)
{
    if(!(a<b))return a;
    return b;
}

// 0 -> No intersection, 1 -> Point intersection, 2 -> segment
// intersection
int SegmentIntersection(point a1, point a2, point b1, point b2,
    point& ans, point& ans2)
{
    line A,B;
    point I;
    pointsToLine(a1,a2,A);
    pointsToLine(b1,b2,B);
    if(areSame(A,B)&&!(a1==a2)&&!(b1==b2))
    {
        ans=max(min(a1,a2),min(b1,b2));
        ans2=min(max(a1,a2),max(b1,b2));
        if(ans2<ans)return 0;
        else if(ans == ans2)return 1;
        return 2;
    }
    if (a1==a2&&b1==b2)
    {
        if(a1==b1)
        {
            ans=a1;

```

```

        return 1;
    }
    return 0;
}
if(a1==a2)
{
    if(fabs(distToLineSegment(a1, b1, b2, ans)-0.0) < EPS)
    {
        ans=a1;
        return 1;
    }
    return 0;
}
if(b1==b2)
{
    if(fabs(distToLineSegment(b1, a1, a2, ans)-0.0) < EPS)
    {
        ans=b1;
        return 1;
    }
    return 0;
}
if (areIntersect(A,B,I) && fabs(distToLineSegment(I, a1, a2,
ans)-0.0) < EPS && fabs(distToLineSegment(I, b1, b2,
ans)-0 < EPS))
{
    return 1;
}
return 0;
}

```

2.2 3DAlgorithms

```
#include "../Header.cpp"
```

```

struct point { db x, y, z;
    point() { x = y = z = 0.0; }
    //point(db r, db u, db v) : x(r*cos(u)*cos(v)),
    y(r*cos(u)*sin(v)), z(r*sin(u)) {}

```

```

    point(db _x, db _y, db _z) : x(_x), y(_y), z(_z) {}
    point operator^(const point &p) const {
        return { y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y *
            p.x};
    }
    db dot(point& p) { return x*p.x + y*p.y + z*p.z; }
    db norm() { return sqrt(x*x + y*y + z*z); }

    bool operator == (const point& p) const
    {
        return abs(p.x - x) < EPS && abs(p.y - y) < EPS && abs(p.z
            - z) < EPS;
    }
    point operator + (const point& p) const
    {
        return point(x + p.x, y + p.y, z + p.z);
    }
    point operator - (const point& p) const
    {
        return point(x - p.x, y - p.y, z - p.z);
    }
    point operator * (db a) const
    {
        return point(x * a, y * a, z * a);
    }
    point operator / (db a) const
    {
        return point(x / a, y / a, z / a);
    }
    point unit() {
        db d = norm();
        return {x/d,y/d,z/d};
    }
};

db angle2(point& x, point& y)
{
    return acos(x.dot(y) / (R*R));
}

bool in_arc(point& p1, point& p2, point& n, point& inter)

```

```

{
    db ab = angle2(p1, p2);
    db ap = angle2(p1, inter);
    point d = (p1 * cos(ap) + (n ^ p1) * sin(ap));
    return ab > ap && inter == d;
}
bool do_intersect_circles()
{
    point a1 = g[j][z], a2 = g[j][0];
    if(z < g[j].size() - 1)
        a2 = g[j][z+1];
    point p1 = route[i], p2 = route[i+1];
    point n1 = (p1^p2).unit(), n2 = (a1^a2).unit();
    point inter = n1^n2;
    if(inter.norm() < EPS)continue;
    inter = inter.unit() * R;

    if(in_arc(p1, p2, n1, inter) && in_arc(a1, a2, n2, inter))
    {
        ag.push_back(angle2(p1, inter));
        continue;
    }
    inter = inter * -1.0;
    if(in_arc(p1, p2, n1, inter) && in_arc(a1, a2, n2, inter))
    {
        ag.push_back(angle2(p1, inter));
        continue;
    }
}

```

2.3 Polygons

```

#include "../Header.cpp"

db DEG_to_RAD(db d) { return d*PI / 180.0; }

db RAD_to_DEG(db r) { return r*180.0 / PI; }

struct point { db x, y;
    point() { x = y = 0.0; }

```

```

    point(db _x, db _y) : x(_x), y(_y) {}
    bool operator <(const point& p) const { return (x < p.x ? true
        : (x == p.x && y < p.y)); }
    bool operator == (const point& p) const { return abs(p.x - x)
        < EPS && abs(p.y - y) < EPS; }
    point operator + (const point& p) const { return point(x +
        p.x, y + p.y); }
    point operator - (const point& p) const { return point(x -
        p.x, y - p.y); }
    point operator * (db p) const { return point(x * p, y * p); }
    point operator / (db p) const { return point(x / p, y / p); }
    db operator^(const point &p) const {return x * p.y - y * p.x; }
    db operator*(const point &p) const {return x * p.x + y * p.y; }
    db norm_sq() const{ return x*x + y*y; }
    point rot(){ return point(-y, x); }

    // by angles but with cross
    bool half() const { return y > 0 || (y == 0 && x > 0); }
    bool operator<(const point &p) const
    {
        int h1 = half(), h2 = p.half();
        return h1 != h2 ? h1 > h2 : ((*this) ^ p) > 0;
    }

    db ang()
    {
        double a = atan2(y, x);
        if (a < 0) a += 2.0 * PI;
        return a;
    }
};

```

```

db dist(point& p1, point& p2) {
    return sqrt((p1.x-p2.x)*(p1.x-p2.x)+ (p1.y-p2.y)*(p1.y-p2.y)); }
db dist_sq(point p1, point p2) {
    return (p1.x - p2.x)*(p1.x - p2.x)+(p1.y - p2.y)*(p1.y - p2.y); }

```

// returns the perimeter, which is the sum of Euclidian distances

```

// of consecutive line segments (polygon edges)
db perimeter(vector<point> &P) {
    db result = 0.0;
    for (ll i = 0; i < (ll)P.size()-1; i++) // remember that P[0] =
        P[n-1]
        result += dist(P[i], P[i+1]);
    return result; }

// returns the area
db area(const vector<point> &P) {
    db result = 0.0;
    for (ll i = 0; i < (ll)P.size()-1; i++) // Shoelace
        formula
        result += P[i]^P[i+1]; // if all points are ll
    return fabs(result)/2.0; } // result can be ll(eger) until last
    step

db seg_integrate(point& a, point& b, db t1, db t2)
{
    // area
    point p1 = a + (b-a) * t1;
    point p2 = a + (b-a) * t2;
    return (p1^p2) / 2.0;
}

db param(point p1, point p2, point a)
{
    if(p1.x != p2.x)
    {
        db sgn = 1;
        if(p1.x > p2.x)sgn = -1;
        return (a.x - p1.x) / abs(p2.x - p1.x) * sgn;
    }
    db sgn = 1;
    if(p1.y > p2.y)sgn = -1;
    return (a.y - p1.y) / abs(p2.y - p1.y) * sgn;
}

// note: to accept collinear points, we have to change the '> 0'
// returns true if point r is on the left side of line pq
bool ccw(point p, point q, point r) {

```

```

    return ((q - p)^(r - p)) > 0; }

int orientation(point p, point q, point r) {
    ll tmp = ((q - p)^(r - p));
    return tmp < 0 ? -1 : tmp == 0 ? 0 : 1; // sign
}

/*bool do_rectangles_intersect(point dl1, point ur1, point dl2,
    point ur2) {
    return max(dl1.x, dl2.x) <= min(ur1.x, ur2.x) && max(dl1.y,
        dl2.y) <= min(ur1.y, ur2.y);
}*/

bool do_segments_intersect(point p1, point q1, point p2, point
    q2) {
    int o11 = orientation(p1, q1, p2);
    int o12 = orientation(p1, q1, q2);
    int o21 = orientation(p2, q2, p1);
    int o22 = orientation(p2, q2, q1);
    // oxx != 0 means cross intersection, no T intersection
    if (o11 != o12 && o21 != o22 && o11 != 0 && o12 != 0 && o21 !=
        0 && o22 != 0) // general case -> non-collinear
        intersection
        return true;
    return false;
}

// returns true if point r is on the same line as the line pq
bool collinear(point p, point q, point r) {
    return fabs(((q - p)^(r - p))) < EPS; }

// angle from 0 to 2*PI
db anglet(point a, point o, point b) { // returns angle aob in rad
    point oa = (a - o), ob = (b - o);
    db ang = acos(oa * ob / sqrt(oa.norm_sq()*ob.norm_sq()));
    if(ang!=0&&!collinear(a,o,b)&&ccw(a,o,b))ang = 2*PI - ang;
    return ang; } // better

db angle(point a, point o, point b) { // returns angle aob in rad
    point oa = (a - o), ob = (b - o);
    return acos(oa * ob / sqrt(oa.norm_sq()*ob.norm_sq())); }
// returns true if we always make the same turn while examining

```

```

// all the edges of the polygon one by one
bool isConvex(const vector<point> &P) {
    ll sz = (ll)P.size();
    if (sz <= 3) return false; // a point/sz=2 or a line/sz=3 is not
        convex
    bool firstTurn = ccw(P[0], P[1], P[2]); // remember one
        result
    for (ll i = 1; i < sz-1; i++) // then compare with the
        others
        if (ccw(P[i], P[i+1], P[(i+2) == sz ? 1 : i+2])) != firstTurn)
            return false; // different sign -> this polygon is
                concave
    return true; } // this polygon is
        convex

// returns true if point p is in either convex/concave polygon P
bool inPolygon(point pt, const vector<point> &P) {
    if ((ll)P.size() < 3) return false; // avoid point or
        line
    db sum = 0; // assume the first vertex is equal to the last
        vertex
    for (ll i = 0; i < (ll)P.size()-1; i++) {
        if (((P[i] - pt)^(P[i+1] - pt)) > 0) //CCW check collinear
            sum += angle(P[i], pt, P[i+1]); // left
                turn/ccw
        else sum -= angle(P[i], pt, P[i+1]); } // right
            turn/cw
    return fabs(sum) > PI; } // 360d -> in, 0d -> out, we have large
        margin

// line segment p-q intersect with line A-B.
point lineIntersectSeg(point p, point q, point A, point B) {
    db a = B.y - A.y;
    db b = A.x - B.x;
    db c = B.x * A.y - A.x * B.y;
    db u = fabs(a * p.x + b * p.y + c);
    db v = fabs(a * q.x + b * q.y + c);
    return point((p.x * v + q.x * u) / (u+v), (p.y * v + q.y * u) /
        (u+v)); }

// cuts polygon Q along the line formed by point a -> point b

```

```

// (note: the last point must be the same as the first point)
// to cut the other side, swap (a,b)
vector<point> cutPolygon(point a, point b, const vector<point>
    &Q) {
    vector<point> P;
    for (ll i = 0; i < (ll)Q.size(); i++) {
        db left1 = (b - a)^(Q[i] - a), left2 = 0;
        if (i != (ll)Q.size()-1) left2 = (b - a)^(Q[i+1] - a);
        if (left1 > -EPS) P.push_back(Q[i]); // Q[i] is on the left
            of ab
        if (left1 * left2 < -EPS) // edge (Q[i], Q[i+1]) crosses
            line ab
            P.push_back(lineIntersectSeg(Q[i], Q[i+1], a, b));
    }
    if (!P.empty() && !(P.back() == P.front()))
        P.push_back(P.front()); // make P's first point = P's last
            point
    return P; }

vector<point> CH_Andrew(vector<point> &Pts) {
    ll n = Pts.size(), k = 0;
    vector<point> H(2*n);
    sort(Pts.begin(), Pts.end()); // sort the points
        lexicographically
    for (ll i = 0; i < n; i++) { // build lower
        hull
        while (k >= 2 && ccw(H[k-2], H[k-1], Pts[i]) <= 0) k--;
        H[k++] = Pts[i];
    }
    for (ll i = n-2, t = k+1; i >= 0; i--) { // build upper
        hull
        while (k >= t && ccw(H[k-2], H[k-1], Pts[i]) <= 0) k--;
        H[k++] = Pts[i];
    }
    H.resize(k);
    return H;
}

point pivot(0, 0);
vector<point> CH_Graham(vector<point> &Pts) {

```



```

vector<point> P(Pts);    // copy all points so that Pts is not
    affected
ll i, j, n = (ll)P.size();
if (n <= 3) {           // corner cases: n=1=point, n=2=line,
    n=3=triangle
    if (!(P[0] == P[n-1])) P.push_back(P[0]); // safeguard from
        corner case
    return P; }          // the CH is P
                          itself

// first, find P0 = point with lowest Y and if tie: rightmost X
ll P0 = 0;
for (i = 1; i < n; i++) //
    O(n)
    if (P[i].y < P[P0].y || (P[i].y == P[P0].y && P[i].x >
        P[P0].x))
        P0 = i;
swap(P[0], P[P0]);      // swap P[P0] with
    P[0]

// second, sort points by angle w.r.t. pivot P0, O(n log n) for
    this sort
pivot = P[0];           // use this global variable as
    reference
sort(++P.begin(), P.end(), [](point a, point b) { // we do not
    sort P[0]
    if (collinear(pivot, a, b)) // special
        case
        return dist(pivot, a) < dist(pivot, b); // check which one
            is closer
    db d1x = a.x-pivot.x, d1y = a.y-pivot.y;
    db d2x = b.x-pivot.x, d2y = b.y-pivot.y;
    return (atan2(d1y, d1x) - atan2(d2y, d2x)) < 0; }); // compare
        2 angles

// third, the ccw tests, although complex, it is just O(n)
vector<point> S;
S.push_back(P[n-1]); S.push_back(P[0]); S.push_back(P[1]); //
    initial S
i = 2;                  // then, we check the
    rest

```

```

while (i < n) {         // note: n must be >= 3 for this method to
    work, O(n)
    j = (ll)S.size()-1;
    if (ccw(S[j-1], S[j], P[i])) S.push_back(P[i++]); // left
        turn, accept
    else S.pop_back(); } // or pop the top of S until we have a
        left turn
return S; } // return the result, overall O(n log n) due to
    angle sorting

point center_of_mass(vector<point>& Q)
{
    point ctr(0,0);
    for (ll i=0; i<Q.size()-1; i++)
    {
        ctr = ctr + Q[i];
    }
    ctr.x/=Q.size()-1;
    ctr.y/=Q.size()-1;
    return ctr;
}

// Pick's theorem
// A = i + b/2 -1
// A: Area polygon with integer coords
// i: Interior points, b: points in the segments

// with vector form of integer segment
// (x0,y0) + t(dx,dy)
ll points_in_segment(point a, point b)
{
    ll absx=abs(a.x-b.x), absy=abs(a.y-b.y);
    return __gcd(absx, absy) + 1;
}

ll memo[101][101][101];

// Dp that pass all possible convex polygons
// from the shortest in(in.y < p.y)
// p si counter cw from in, p
ll all_convex(ll in, ll p, ll q, vector<point>& Q)

```

```

{
    if(memo[in][p][q] != -1) return memo[in][p][q];
    ll ans = 0;

    for(int i = in + 1; i < Q.size(); i++)
    {
        if(i != p && i != q && ccw(Q[in], Q[q], Q[i]) && ccw(Q[p],
            Q[q], Q[i]))
        {
            ans += (all_convex(in, q, i, Q) + 1);
        }
    }
    return memo[in][p][q] = ans;
}

bool comp(point& a, point& b)
{
    return a.y < b.y;
}

//..
vector<point>Q;
sort(ALL(Q), comp);
ll ans = 0;
for(int i = 0; i < n; i++)
{
    for(int p = i + 1; p < n; p++) for(int q = p + 1; q < n; q++)
    {
        if(ccw(Q[i], Q[p], Q[q]))
            ans += all_convex(i, p, q, Q) + 1;
        else
            ans += all_convex(i, q, p, Q) + 1;
        ans %= m;
    }
}

```

2.4 SegmentIntersection

```

#include "../Header.cpp"

struct point { db x, y;
    point() { x = y = 0.0; }

```

```

    point(db _x, db _y) : x(_x), y(_y) {}
    bool operator <(const point& p) const { return (x < p.x ? true
        : (x == p.x && y < p.y)); }
    bool operator == (const point& p) const { return abs(p.x - x)
        < EPS && abs(p.y - y) < EPS; }
    point operator + (const point& p) const { return point(x +
        p.x, y + p.y); }
    point operator - (const point& p) const { return point(x -
        p.x, y - p.y); }
    point operator * (db p) const { return point(x * p, y * p); }
    point operator / (db p) const { return point(x / p, y / p); }
    db operator^(const point &p) const {return x * p.y - y * p.x; }
    db operator*(const point &p) const {return x * p.x + y * p.y; }
    db norm_sq() const{ return x*x + y*y; }
    point rot(){ return point(-y, x); }
    db ang()
    {
        double a = atan2(y, x);
        if (a < 0) a += 2.0 * PI;
        return a;
    }

};

db dist(const point& p1,const point& p2) {
    return sqrt((p1.x-p2.x)*(p1.x-p2.x)+ (p1.y-p2.y)*(p1.y-p2.y)); }

//Constant values to be returned
constexpr int Colinear = -1, NoIntersect = 0, Intersect = 1;
constexpr int CW = 2, CCW = 3;

int orientation(point& p, point& q, point& r) {
    ll tmp = (q - p)^(r - p);
    return tmp < 0 ? CW : tmp == 0 ? Colinear : CCW; // sign
}

struct segment { point p1, p2;
    segment(point _p1, point _p2) : p1(_p1), p2(_p2) {}
};

```

```

//Returns of list of intersection points between segments s1, and
s2
//If they do not intersect, the result is an empty vector
//If they intersect at exactly 1 point, the result contains that
point
//If they overlap for non-0 distance, the left and right points
of that intersection
// are returned
bool onSegment(const point& p, const segment& s)
{
    bool x = (abs(s.p1.x - s.p2.x) < EPS && abs(p.x - s.p2.x) <
        EPS) || (p.x <= max(s.p1.x, s.p2.x) && p.x >= min(s.p1.x,
        s.p2.x));
    bool y = (abs(s.p1.y - s.p2.y) < EPS && abs(p.y - s.p2.y) <
        EPS) || (p.y <= max(s.p1.y, s.p2.y) && p.y >= min(s.p1.y,
        s.p2.y));
    return x && y;
}

```

```

vector<point> intersect(const segment& s1, const segment& s2)
{
    point a = s1.p1, b = s1.p2, c = s2.p1, d = s2.p2;

    if(orientation(a, b, c) == Colinear && orientation(a, b, d) ==
        Colinear &&
        orientation(c, d, a) == Colinear && orientation(c, d, b)
            == Colinear)
    {
        point min_s1 = min(a, b), max_s1 = max(a, b);
        point min_s2 = min(c, d), max_s2 = max(c, d);

        if(max_s1 < min_s2 || max_s2 < min_s1) return {};

        point start = max(min_s1, min_s2), end = min(max_s1,
            max_s2);
        if(start == end)
            return {start};
        else
            return {min(start, end), max(start, end)};
    }
}

```

```

db a1 = b.y - a.y, a2 = d.y - c.y;
db b1 = a.x - b.x, b2 = c.x - d.x;
db c1 = a1*a.x + b1*a.y, c2 = a2*c.x + b2*c.y;
db det = a1*b2 - a2*b1;
if(abs(det) > EPS)
{
    point inter((b2*c1 - b1*c2)/det, (a1*c2 - a2*c1)/det), aux;
    //if(distToLineSegment(inter, s1.p1, s1.p2, aux) <= EPS &&
        distToLineSegment(inter, s2.p1, s2.p2, aux) <= EPS)
        if(onSegment(inter, s1) && onSegment(inter, s2))
            return {inter};
}
return {};
}

```

2.5 TrianglesCircles

```

#include "../Header.cpp"

//define double long long //Para usar enteros

db DEG_to_RAD(db d) { return d * PI / 180.0; }
db RAD_to_DEG(db r) { return r * 180.0 / PI; }

//sweepline rotating a circle around a point
// how many points are in circle radius r
// alpha = atan2(point - center) +- acos(dist/2r)

struct point { db x, y;
    point() { x = y = 0.0; }
    point(db _x, db _y) : x(_x), y(_y) {}
    bool operator <(const point& p) const { return (x < p.x ? true
        : (x == p.x && y < p.y)); }
    bool operator == (const point& p) const { return abs(p.x - x)
        < EPS && abs(p.y - y) < EPS; }
    point operator + (const point& p) const { return point(x +
        p.x, y + p.y); }
    point operator - (const point& p) const { return point(x -
        p.x, y - p.y); }
}

```

```

point operator * (db p) const { return point(x * p, y * p); }
point operator / (db p) const { return point(x / p, y / p); }
db operator^(const point &p) const {return x * p.y - y * p.x; }
db operator*(const point &p) const {return x * p.x + y * p.y; }
db norm_sq() const { return x*x + y*y; }
point rot(){ return point(-y, x); }

// by angles but with cross
bool half() const { return y > 0 || (y == 0 && x > 0); }
bool operator<(const point &p) const
{
    int h1 = half(), h2 = p.half();
    return h1 != h2 ? h1 > h2 : ((*this) ^ p) > 0;
}

db ang()
{
    double a = atan2(y, x);
    if (a < 0) a += 2.0 * PI;
    return a;
}

};

ll insideCircle(point p, point c, ll r) { // all integer version
    ll dx = p.x - c.x, dy = p.y - c.y;
    ll Euc = dx * dx + dy * dy, rSq = r * r; // all integer
    return Euc < rSq ? 0 : Euc == rSq ? 1 : 2; }
    //inside/border/outside

// P1 and P2 intersections of circles and radius r -> pos of
// centers of circles of intersection
bool circle2PtsRad(point p1, point p2, db r, point &c) {
    db d2 = (p1.x - p2.x) * (p1.x - p2.x) +
            (p1.y - p2.y) * (p1.y - p2.y);
    db det = r * r / d2 - 0.25;
    if (det < 0.0) return false;
    db h = sqrt(det);
    c.x = (p1.x + p2.x) * 0.5 + (p1.y - p2.y) * h;
    c.y = (p1.y + p2.y) * 0.5 + (p2.x - p1.x) * h;

```

```

    return true; } // to get the other center, reverse p1
                    and p2

db dist(point& p1, point& p2) { // Euclidean distance
    return sqrt((p1.x-p2.x)*(p1.x-p2.x)+ (p1.y-p2.y)*(p1.y-p2.y));
}

db dist_sq(point p1, point p2) {
    return (p1.x - p2.x)*(p1.x - p2.x)+(p1.y - p2.y)*(p1.y -
    p2.y); }

// a = max x, b = max y from the center, AREA
db A_elipse(db a,db b)
{
    return a*b*PI;
}

// Length of segment with two points on the circumference
// separated by an angle
db chord(db r, db angle)
{
    return sqrt(2*r*r*(1-cos(angle)));
}

//Triangles
db perimeter(db ab, db bc, db ca) {
    return ab + bc + ca; }

db perimeter(point a, point b, point c) {
    return dist(a, b) + dist(b, c) + dist(c, a); }

db area(db ab, db bc, db ca) {
    // Heron's formula, split sqrt(a * b) into sqrt(a) * sqrt(b); in
    // implementation
    db s = 0.5 * perimeter(ab + bc + ca);
    return sqrt(s) * sqrt(s - ab) * sqrt(s - bc) * sqrt(s - ca); }

db area(point a, point b, point c) {
    return area(dist(a, b), dist(b, c), dist(c, a)); }

```

```

// Area of the circle enclosed by an arc and a chord defined by
// an angle
db segment(db r, db angle)
{
    return angle/2.0*r*r-area(chord(r,angle),r,r);
}

// And overlapping rectangle area > 0
bool rectangles_intersect(point a1,point a2,point b1,point
    b2,point& ans1, point& ans2)
{
    if(b1<a1)
    {
        swap(a1,b1);
        swap(a2,b2);
    }
    if(b1.x>=a2.x||b1.y>=a2.y||b2.y<=a1.y)return 0;
    ans1.x=b1.x;
    ans1.y=max(b1.y,a1.y);
    ans2.x=min(b2.x,a2.x);
    ans2.y=min(b2.y,a2.y);
    return 1;
}

struct line { db a, b, c; };

void pointsToLine(point p1, point p2, line &l) {
    if (fabs(p1.x - p2.x) < EPS) {          // vertical line is
        fine
        l.a = 1.0; l.b = 0.0; l.c = -p1.x;    // default
        values
    } else {
        l.a = -(db)(p1.y - p2.y) / (p1.x - p2.x);
        l.b = 1.0;          // IMPORTANT: we fix the value of b
        to 1.0
        l.c = -(db)(l.a * p1.x) - p1.y;
    } }

bool areParallel(line l1, line l2) {      // check coefficient a + b
    return (fabs(l1.a-l2.a) < EPS) && (fabs(l1.b-l2.b) < EPS); }

```

```

bool areSame(line l1, line l2) {          // also check coefficient c
    return areParallel(l1 ,l2) && (fabs(l1.c-l2.c) < EPS); }

// returns true (+ intersection point) if two lines are intersect
bool areIntersect(line l1, line l2, point &p) {
    if (areParallel(l1, l2)) return false;    // no intersection
    // solve system of 2 linear algebraic equations with 2 unknowns
    p.x = (l2.b * l1.c - l1.b * l2.c) / (l2.a * l1.b - l1.a *
        l2.b);
    // special case: test for vertical line to avoid division by
    zero
    if (fabs(l1.b) > EPS) p.y = -(l1.a * p.x + l1.c);
    else p.y = -(l2.a * p.x + l2.c);
    return true; }

db rInCircle(db ab, db bc, db ca) {
    return area(ab, bc, ca) / (0.5 * perimeter(ab, bc, ca)); }

db rInCircle(point a, point b, point c) {
    return rInCircle(dist(a, b), dist(b, c), dist(c, a)); }

// assumption: the required points/lines functions have been
// written
// returns 1 if there is an inCircle center, returns 0 otherwise
// if this function returns 1, ctr will be the inCircle center
// and r is the same as rInCircle
ll inCircle(point p1, point p2, point p3, point &ctr, db &r) {
    r = rInCircle(p1, p2, p3);
    if (fabs(r) < EPS) return 0;            // no inCircle
    center

    line l1, l2;                          // compute these two angle
    bisectors
    db ratio = dist(p1, p2) / dist(p1, p3);
    point p = p2 + (p3 - p2) * (ratio / (1 + ratio));
    pointsToLine(p1, p, l1);

    ratio = dist(p2, p1) / dist(p2, p3);
    p = p1 + (p3 - p1) * (ratio / (1 + ratio));
    pointsToLine(p2, p, l2);
}

```

```

areIntersect(l1, l2, ctr);          // get their intersection
    point
return 1; }

db rCircumCircle(db ab, db bc, db ca) {
    return ab * bc * ca / (4.0 * area(ab, bc, ca)); }

db rCircumCircle(point a, point b, point c) {
    return rCircumCircle(dist(a, b), dist(b, c), dist(c, a)); }

// assumption: the required points/lines functions have been
// written
// returns 1 if there is a circumCenter center, returns 0
// otherwise
// if this function returns 1, ctr will be the circumCircle center
// and r is the same as rCircumCircle
ll circumCircle(point p1, point p2, point p3, point &ctr, db &r){
    db a = p2.x - p1.x, b = p2.y - p1.y;
    db c = p3.x - p1.x, d = p3.y - p1.y;
    db e = a * (p1.x + p2.x) + b * (p1.y + p2.y);
    db f = c * (p1.x + p3.x) + d * (p1.y + p3.y);
    db g = 2.0 * (a * (p3.y - p2.y) - b * (p3.x - p2.x));
    if (fabs(g) < EPS) return 0;

    ctr.x = (d*e - b*f) / g;
    ctr.y = (a*f - c*e) / g;
    r = dist(p1, ctr); // r = distance from center to 1 of the 3
        points
    return 1; }

//
// https://www.nayuki.io/res/smallest-enclosing-circle/computational-geometry-lecture-6.pdf
// O(N) expected time

void smallest_enclosing_circle(vector<point>& pts, point& center,
    db& r) {
    random_shuffle(pts.begin(), pts.end());
    center = pts[0]; r = 0;
    ll N = pts.size();
    for(ll i=1; i<N; i++) {

```

```

        if (dist(pts[i], center) > r + EPS) {
            center = pts[i];
            r = 0;
            for(ll j=0; j<i; j++) {
                if (dist(pts[j], center) > r + EPS) {
                    center = (pts[i] + pts[j]) * 0.5;
                    r = dist(pts[i], center);
                    for(ll k=0; k<j; k++) {
                        if (dist(pts[k], center) > r + EPS) {
                            db rr;
                            circumCircle(pts[i], pts[j],
                                pts[k], center, rr);
                            r = dist(pts[k], center);
                        }
                    }
                }
            }
        }
    }
}

// returns true if point d is inside the circumCircle defined by
// a,b,c
ll inCircumCircle(point a, point b, point c, point d) {
    return (a.x - d.x) * (b.y - d.y) * ((c.x - d.x) * (c.x - d.x)
        + (c.y - d.y) * (c.y - d.y)) +
        (a.y - d.y) * ((b.x - d.x) * (b.x - d.x) + (b.y - d.y) *
            (b.y - d.y)) * (c.x - d.x) +
        ((a.x - d.x) * (a.x - d.x) + (a.y - d.y) * (a.y - d.y)) *
            (b.x - d.x) * (c.y - d.y) -
        ((a.x - d.x) * (a.x - d.x) + (a.y - d.y) * (a.y - d.y)) *
            (b.y - d.y) * (c.x - d.x) -
        (a.y - d.y) * (b.x - d.x) * ((c.x - d.x) * (c.x - d.x) +
            (c.y - d.y) * (c.y - d.y)) -
        (a.x - d.x) * ((b.x - d.x) * (b.x - d.x) + (b.y - d.y) *
            (b.y - d.y)) * (c.y - d.y) > 0 ? 1 : 0;
}

bool canFormTriangle(db a, db b, db c) {
    return (a + b > c) && (a + c > b) && (b + c > a); }

```

```

/*
Si un punto tiene un ngulo >= 60, el lado opuesto no es el menor
del triangulo
*/

// Function to find the circle on
// which the given three points lie
// better CIRCUMCENTER
tuple<db, db, db> findCircle(db x1, db y1, db x2, db y2, db x3,
    db y3)
{
    db x12 = x1 - x2;
    db x13 = x1 - x3;

    db y12 = y1 - y2;
    db y13 = y1 - y3;

    db y31 = y3 - y1;
    db y21 = y2 - y1;

    db x31 = x3 - x1;
    db x21 = x2 - x1;

    db sx13 = x1*x1 - x3*x3;
    db sy13 = y1*y1 - y3*y3;

    db sx21 = x2*x2 - x1*x1;
    db sy21 = y2*y2 - y1*y1;

    db f = ((sx13) * (x12)
        + (sy13) * (x12)
        + (sx21) * (x13)
        + (sy21) * (x13))
        / (2 * ((y31) * (x12) - (y21) * (x13)));
    db g = ((sx13) * (y12)
        + (sy13) * (y12)
        + (sx21) * (y13)
        + (sy21) * (y13))
        / (2 * ((x31) * (y12) - (x21) * (y13)));

    db c = -x1*x1 - y1*y1 - 2 * g * x1 - 2 * f * y1;

```

```

// eqn of circle be  $x^2 + y^2 + 2gx + 2fy + c = 0$ 
// where centre is (h = -g, k = -f) and radius r
// as  $r^2 = h^2 + k^2 - c$ 
db h = -g;
db k = -f;
db sqr_of_r = h * h + k * k - c;

// r is the radius
db r = sqrt(sqr_of_r);

//cout << "Centre = (" << h << ", " << k << ")" << endl;
//cout << "Radius = " << r;
return make_tuple(h, k, r);
}

```

3 Graphs

3.1 BellmanFord

```

#include "../Header.cpp"

int main(){
    ios::sync_with_stdio(false);
    cin.tie(0);
    int v,e,x,y,w,r;
    cin >> v >> e >> r;
    pll h;
    vl d(v, INF);
    d[r] = 0;
    vector<vector<pll>> g(v, vector<pll> (0));
    for(int i = 0 ; i < e; i++){
        cin >> x >> y >> w;
        h.first = y;
        h.second = w;
        g[x].push_back(h);
    }
    /*
inequations solver

```

```

v - u <= p
g[u].push_back({v, p});
g[v].push_back({u, -1});
d[s] = 0
for i in v: g[s].push_back({i, 0})
*/

rep(i, v-1){
    bool mod = 0;
    rep(j, v){
        if(d[j] != INF){
            for(auto it : g[j]){
                d[it.first] = min(d[it.first], d[j] +
                    it.second);
                mod = 1;
            }
        }
    }
    if(mod == 0)break;
}
bool cyc = 0;
rep(j, v)
    for(auto it : g[j])
        if(d[j] < INF && d[it.first] > d[j] + it.second)
            cyc = 1;
// Faster but doesnt support negative cycles
// SPFA from source S
vl dist(v, INF); dist[s] = 0;           // INF = 1e9 here
queue<int> q; q.push(s);                 // like BFS queue
vl in_queue(v, 0); in_queue[s] = 1;     // unique to SPFA
while (!q.empty()) {
    int u = q.front(); q.pop(); in_queue[u] = 0; // pop from
    queue
    for (auto it : g[u]) {                // C++17 style
        if (dist[u]+it.first >= dist[it.second]) continue; //
        not improving, skip
        dist[it.second] = dist[u]+it.first;                //
        relax operation
        if (!in_queue[it.second]) {                        // add to
            the queue

```

```

        q.push(it.second);                // only
        if v is not
        in_queue[it.second] = 1;          //
        already in the queue
    }
}
}
}
}

```

3.2 BiconnectedComponents

```

#include "../Header.cpp"

vector<vl> G;
vl D, L;
// p: -1, L: 0, D: -1
void dfs(int u, int p, int d)
{
    D[u] = L[u] = d;
    for(int v : G[u]) if (v != p)
    {
        if (D[v] == -1)
        {
            dfs(v, u, d + 1);
            if (L[v] > D[u]) {} // (u - v) cut edge
            L[u] = min(L[u], L[v]);
        }
        else L[u] = min(L[u], D[v]);
    }
}

int rc = 0; // Articulation Point
stack<pll> S; // BCC
void dfs(int u, int p, int d)
{
    D[u] = L[u] = d;
    for (int v : G[u]) if (v != p)
    {
        if (D[v] == -1)
        {

```



```

    S.emplace(u, v); dfs(v, u, d + 1);
    if ((p == -1 && ++rc == 2) || (p != -1 && L[v] >= d))
        {} // u is AP
    if (p == -1 or L[v] >= d) while (1) // BCC found
    {
        pll e = S.top(); S.pop();
        if (e == pll(u, v)) break;
    }
    L[u] = min(L[u], L[v]);
}
else if (D[v] < d) { S.emplace(u, v); L[u] = min(L[u],
D[v]); }
}
}

```

3.3 CentroidDecomposition

```

#include "../Header.cpp"

// all tree diameters pass through the centroid
const int MAXN = 1e5 + 5;
vector<int> g[MAXN]; int n;
bool tk[MAXN];
int fat[MAXN]; // father in centroid decomposition
int szt[MAXN]; // size of subtree
int calcsz(int x, int f){
    szt[x]=1;
    for(auto y:g[x]) if(y!=f && !tk[y]) szt[x]+=calcsz(y,x);
    return szt[x];
}
void cdfs(int x=0, int f=-1, int sz=-1){ // O(nlogn)
    if(sz<0) sz=calcsz(x,-1);
    for(auto y:g[x]) if(!tk[y] && szt[y]*2>=sz){
        szt[x]=0; cdfs(y,f,sz); return;
    }
    tk[x]=true; fat[x]=f;
    for(auto y:g[x]) if(!tk[y]) cdfs(y,x);
}
void centroid(){memset(tk,false,sizeof(tk)); cdfs();}
int main(){

```

```

ll t; cin >> t;
for(int T = 1; T <= t; T++) {
    memset(memo, -1, sizeof(memo));
    ll x;
    cin >> N >> K;
    B.clear(); ac.clear(); ac.push_back(0);
    for(int i = 0; i < N; i++){
        cin >> x;
        B.push_back(x); ac.push_back(x);
        ac[i+1] += ac[i];
    }
    ll acum = 0;
    for(int i = 0; i < N; i++){
        acum += B[i];
        if(acum == ac[i+1]) cout<<"1\n";
        else cout <<"0\n";
    }
    ll ans = INF;
    for(int i = 0; i < N; i++){
        ans = min(ans, dp(i, 0, 0));
    }
    if(ans >= INF) ans = -1;
    cout << "Case #" << T << ": ";
    cout << ans << "\n";
}
}

```

3.4 Dijkstra

```

#include "../Header.cpp"

int main(){
    vl d(v, INF);
    priority_queue<pll, vp, greater<pll> > q; //from low to high
    ll s, t;
    q.push({0, s});
    d[s] = 0;
    while(!q.empty()){
        ll w, u;
        tie(w, u) = q.top();

```

```

    q.pop();
    if(w > d[u]) continue;
    for(auto it : g[u]){
        if(d[it.second] > w + it.first){
            d[it.second] = w + it.first;
            q.push({d[it.second], it.second});
        }
    }
}
}
}

```

3.5 D'Esopo-Pape

```

#include "../Header.cpp"

int main(){
    ll v,x,y,e;
    pll h;
    vector<ll>b;
    cin >> v >> e;
    vector<vp > g(v, vp (0));
    vl peso(v, INF);

    int w;
    for(ll i = 0; i < e; i++){
        cin >> x >> y >> w;
        g[x-1].push_back({w, y-1}) ;
        g[y-1].push_back({w, x-1});
    }
    ll s, t;
    vl d(v, INF);
    d[s] = 0;
    vl m(v, 2);
    deque<ll> q;
    q.push_back(s);
    p.assign(v, -1);
    while (!q.empty()) {
        int u = q.front();
        q.pop_front();
        m[u] = 0;

```

```

        for (auto it : g[u]) {
            if (d[it.second] > d[u] + it.first) {
                d[it.second] = d[u] + it.first;
                p[it.second] = u;
                if (m[it.second] == 2) {
                    m[it.second] = 1;
                    q.push_back(it.second);
                } else if (m[it.second] == 0) {
                    m[it.second] = 1;
                    q.push_front(it.second);
                }
            }
        }
    }
    return 0;
}

```

3.6 EulerTour

```

#include "../Header.cpp"

//Euler Tour
vl L, R, d, c;
ll num = -1;
vector<vl>g;
void dfs(ll in, ll p){
    num++;
    L[in] = num;
    d.push_back(c[in]);
    for(auto it : g[in]){
        if(p != it)
            dfs(it, in);
    }
    R[in] = num;
}

```

3.7 FloydWarshall

```

#include "../Header.cpp"

ll p[500][500];
void printPath(int i, int j)
{
    if(i != j) printPath(i, p[i][j]);
    cout << j+1 << " ";
}

int main()
{
    int n, m, q, x, y, w;
    vector<vl> g(n, vl(n, INF));
    rep(i, n)
    {
        g[i][i] = 0;
        //g[i][i] = INF; Detect cheapest positive cycle for each i
    }
    rep(i, m)
    {
        cin >> x >> y >> w;
        g[x][y] = min(g[x][y], w); // handle repeats
    }

    rep(i, n)
        rep(j, n)
            p[i][j] = i;

    rep(k, n)
        rep(i, n)
            rep(j, n)
                //g[i][j] |= (g[i][k] & g[k][j]); to find i is
                //connected with j
                //if at the end g[i][j] & g[j][i], i and j are in
                //the same SCC

                // To find minimal max edge in path from i to j
                //g[i][j] = min(g[i][j], max(g[i][k], g[k][j]));

                if(g[i][k] + g[k][j] < g[i][j])
                {

```

```

                    g[i][j] = g[i][k] + g[k][j];
                    p[i][j] = p[k][j];
                }
            }
        }
    }

    rep(k, n)
        rep(i, n)
            rep(j, n)
                if(g[i][k] != INF && g[k][j] != INF
                    && g[k][k] < 0)
                    g[i][j] = -INF;

    rep(i, q)
    {
        cin >> x >> y;
        if(g[x][y] == INF)
            cout << "Impossible\n";
        else if (g[x][y] == -INF)
            cout << "-Infinity\n";
        else
            cout << g[x][y] << "\n";
    }

    return 0;
}

```

3.8 HeavyLightDecomposition

```

#include "../Header.cpp"

class HLD{
    ST st;
    vi A, H, D, R, P;
    int dfs(vector<vi> &G, int u){
        int ans = 1, M = 0, s;
        for (int v : G[u]) if (v != A[u]){
            A[v] = u, D[v] = D[u] + 1;
            s = dfs(G, v), ans += s;
            if (s > M) H[u] = v, M = s;
        }
        return ans;
    }
}

template <class OP>

```

```

void path(int u, int v, OP op){
    for (; R[u] != R[v]; v = A[R[v]]){
        if (D[R[u]] > D[R[v]]) swap(u, v);
        op(P[R[v]], P[v] + 1);
    }
    if (D[u] > D[v]) swap(u, v);
    op(P[u], P[v] + 1);           // VALUES ON VERTEX
    // op(P[u] + 1, P[v] + 1);    // VALUES ON EDGE
}

public:
HLD(vector<vi> &G, int n) : A(n), D(n), R(n), P(n){
    st = SegmentTree(n);
    H.assign(n, -1); A[0] = -1, D[0] = 0; dfs(G, 0); int p = 0;
    rep(i, n) if (A[i] == -1 || H[A[i]] != i)
        for (int j = i; j != -1; j = H[j]) R[j] = i, P[j] = p++;
}
void set(int v, const node &x) { st.set(P[v], x); } // VALUES
ON VERTEX
// void set(int u, int v, const node &x)           // VALUES ON
EDGE
// {
//     if (D[u] > D[v]) swap(u, v);
//     st.set(P[v], x);
// }
void update(int u, int v, const node &x)           //
OPTIONAL FOR RANGE UPDATES
{ path(u, v, [this, &x](int l, int r) { st.update(l, r, x);
}); }
node query(int u, int v){
    node ans = node();
    path(u, v, [this, &ans](int l, int r) { ans = node(ans,
        st.query(l, r)); });
    return ans;
}
};
// USAGE: HLD<ST<Node>, Node> hld(G, N);
///// NON COMMUTATIVE QUERIES :
class HLD{
    ST st;
    vi A, H, D, R, P;

```

```

int dfs(vector<vi> &G, int u){
    int ans = 1, M = 0, s;
    for (int v : G[u]) if (v != A[u]){
        A[v] = u, D[v] = D[u] + 1;
        s = dfs(G, v), ans += s;
        if (s > M) H[u] = v, M = s;
    }
    return ans;
}

public:
node path(int u, int v){
    node ans1, ans2; bool d = 0;
    for (; R[u] != R[v]; v = A[R[v]]){
        if (D[R[u]] > D[R[v]]) swap(u, v), d = !d;
        if (d) ans1 = node(st.query(P[R[v]], P[v] + 1), ans1);
        else ans2 = node(st.query(P[R[v]], P[v] + 1), ans2);
    }
    if (D[u] > D[v]) swap(u, v), d = !d;
    if (d) ans1 = node(st.query(P[u], P[v] + 1), ans1);
    else ans2 = node(st.query(P[u], P[v] + 1), ans2);
    ans1.sw(); return node(ans1, ans2);
}

HLD(vector<vi> &G, int n) : A(n), st(n), D(n), R(n), P(n){
    st = SegmentTree(n);
    H.assign(n, -1); A[0] = -1, D[0] = 0; dfs(G, 0); int p = 0;
    rep(i, n) if (A[i] == -1 || H[A[i]] != i)
        for (int j = i; j != -1; j = H[j]) R[j] = i, P[j] = p++;
}
void set(int v, const node &x) { st.set(P[v], x); }
};

```

3.9 Hungarian

```

#include "../Header.cpp"

#define rep(i, n) for (int i = 0; i < (int)n; i++)
#define repx(i, a, b) for (int i = (int)a; i < (int)b; i++)

// Minimum/Maximum cost of a perfect matching in complete graph
// O(n^3)

```

```

template<class T>
class Hungarian{
    T inf = numeric_limits<T>::max() / 2;
    bool maxi, swapped = false;
    vector<vector<T>> cost;
    vector<T> u, v;
    vl p, way;
    int l, r;

public:
    // left/right == partition sizes
    Hungarian(int left, int right, bool maximizing){
        l = left, r = right, maxi = maximizing;
        if (swapped = l > r) swap(l, r);
        cost.assign(l + 1, vector<T>(r + 1, 0));
        u.assign(l + 1, 0); v.assign(r + 1, 0);
        p.assign(r + 1, 0); way.assign(r + 1, 0);
    }
    void add_edge(int l, int r, T w){
        assert(l and r); // indices start from 1 !!
        if (swapped) swap(l, r);
        cost[l][r] = maxi ? -w : w;
    }
    // execute after all edges were added
    void calculate(){
        rep(x, i, 1, l + 1){
            vector<bool> used(r+1, false);
            vector<T> minv(r+1, inf);
            int j0 = 0; p[0] = i;
            while (p[j0]){
                int j1, i0 = p[j0]; used[j0] = true;
                T delta = inf;
                rep(x, j, 1, r + 1) if (not used[j])
                {
                    T cur = cost[i0][j] - u[i0] - v[j];
                    if (cur < minv[j]) minv[j] = cur, way[j] = j0;
                    if (minv[j] < delta) delta = minv[j], j1 = j;
                }
                rep(x, j, r + 1){
                    if (used[j]) u[p[j]] += delta, v[j] -= delta;
                    else minv[j] -= delta;
                }
            }
        }
    }
};

```

```

    }
    j0 = j1;
    while (j0) p[j0] = p[way[j0]], j0 = way[j0];
}

// execute after executing calculate()
T answer() { return maxi ? v[0] : -v[0]; }
bool are_matched(int l, int r){
    if (swapped) swap(l, r);
    return p[r] == l;
}
};

int main(){
    ios_base::sync_with_stdio(0); cin.tie(0);
    ll n;
    cin >> n;
    ll d[n][n], pos = (n+1)/2;
    Hungarian<ll> h(pos, pos, 0);
    for(int i = 0; i < n; i++){
        for(int j = 0; j < n; j++){
            cin >> d[i][j];
        }
        for(int i = 0; i < n; i += 2){
            for(int j = 0; j < n; j += 2){
                ll cost = 0;
                if (j > 0) cost += d[i][j-1];
                if (j < n-1) cost += d[i][j+1];
                h.add_edge(i/2+1, j/2+1, cost);
            }
        }
        h.calculate();
        cout << h.answer() << "\n";
    }
}

```

3.10 LCA-SP

```
#include "../Header.cpp"
```

```

ll maxlog2(ll x){
    return (63 - __builtin_clzll(x));
}

// To minimize diameter, connect the center of the diameter of
// two trees
// min(diam1, diam2, dia1+dia2+1) dia1 = diam1/2 (if diameter
// odd)+ 1

struct SparseTableLCA
{
    ll maxlg;
    vector<vl> >SP;
    vector<vl> >MN;
    vl D;
    SparseTableLCA(vector<vl>& g, ll ini=0)
    {
        ll n = g.size();
        vl vis(n,0), parent(n,-1);
        D.resize(n,INF); D[ini]=0;
        queue<ll> q;
        q.emplace(ini);
        while(!q.empty()){
            ll k=q.front(); q.pop();
            if(!vis[k]){
                vis[k]=1;
                for(auto it : g[k])
                    if(!vis[it])
                    {
                        parent[it]=k;
                        D[it]=D[k]+1;
                        q.push(it);
                    }
            }
        }

        SP.clear();
        SP.push_back(parent);
        maxlg = maxlog2(n);

```

```

        repx(i, 1 , maxlg+1)
        {
            vl aux;
            rep(j, n)
            {
                if(SP[i-1][j] != -1)
                    aux.push_back(SP[i-1][SP[i-1][j]]);
                else aux.push_back(-1);
            }
            SP.push_back(aux);
        }
    }

    ll maxL(ll u, ll v) // arista largo maximo
    {
        ll a,b,x=LCA(u, v);
        if(u==x) a = -1;
        else a = query(D[x], u);
        if(v==x) b = -1;
        else b = query(D[x], v);
        return max(a, b);
    }

    ll query(ll a, ll n)
    {
        ll maxi=-1;
        while(D[n] != a)
        {
            maxi = max(maxi, MN[maxlog2(D[n]-a)][n]);
            n=SP[maxlog2(D[n]-a)][n];
        }
        return maxi;
    }

    ll level(ll a, ll n) // up a to depth n
    {
        while(D[n] != a)
            n = SP[maxlog2(D[n]-a)][n];
        return n;
    }

    ll LCA(ll x, ll y)
    {
        if(D[x] <= D[y]) swap(x, y);

```

```

    if(D[x] != D[y])
        x = level(min(D[x], D[y]), x);

    if(x == y) return x;

    for(ll i = maxlg; i>=0; i--)
    {
        if(SP[i][x] != SP[i][y] && SP[i][x] != -1)
        {
            x = SP[i][x];
            y = SP[i][y];
        }
    }
    return SP[0][x];
}

ll Dist(ll u, ll v)
{
    return D[u] + D[v] - 2*D[LCA(u, v)];
}

ll kth_farthest_node(ll u, ll v, ll d)
{
    if(Dist(u, LCA(u, v)) < d)
        return level(D[v] - (Dist(u, v) - d), v);

    else
        return level(D[u] - d, u);
}

// move u k steps in path to v
ll next_path(ll u, ll v, ll k){

    if(D[u] - D[LCA(u, v)] >= k) return level(D[u] - k, u);
    else return level(D[LCA(u, v)] + k - (D[u] - D[LCA(u, v)]),
        v);
}

};

/* edge weight queries
SparseTableLCA(vector<vector<pll>>& g, ll ini)

```

```

{
    ll n = g.size();
    vl vis(n, 0), parent(n, -1), b(n, -1);
    D.resize(n, INF); D[ini] = 0;
    queue<ll> q;
    q.emplace(ini);
    while(!q.empty()){
        ll k = q.front(); q.pop();
        if(!vis[k]){
            vis[k] = 1;
            for(auto it : g[k])
                if(!vis[it.second])
                {
                    b[it.second] = it.first;
                    parent[it.second] = k;
                    D[it.second] = D[k] + 1;
                    q.push(it.second);
                }
        }
    }

    SP.clear();
    SP.push_back(parent);
    maxlg = 63 - __builtin_clzll(n);
    for(ll i = 1; i <= maxlg; i++)
    {
        vl c;
        for(ll j = 0; j < n; j++)
        {
            if(SP[i-1][j] != -1)
                c.push_back(SP[i-1][SP[i-1][j]]);
            else c.push_back(-1);
        }
        SP.push_back(c);
    }
    MN.clear();
    MN.push_back(b);

    for(ll i = 1; i <= maxlg; i++)
    {
        vl c;

```

```

        for(ll j=0;j<n;j++)
        {
            if(MN[i-1][j]!=-1)
                c.push_back(max(MN[i-1][SP[i-1][j]],MN[i-1][j]));
            else c.push_back(-1);
        }
        MN.push_back(c);
    }
}
*/

```

3.11 LaplacianMatrix

```

#include "../Header.cpp"

// Dimension of input square matrix
#define N 4

// Function to get cofactor of mat[p][q] in temp[][]. n is
// current dimension of mat[][]
void getCofactor(int mat[N][N], int temp[N][N], int p,
                 int q, int n){
    int i = 0, j = 0;
    // Looping for each element of the matrix
    for (int row = 0; row < n; row++){
        for (int col = 0; col < n; col++){
            // Copying into temporary matrix only those
            // element which are not in given row and
            // column
            if (row != p && col != q){
                temp[i][j++] = mat[row][col];

                // Row is filled, so increase row index and
                // reset col index
                if (j == n - 1){
                    j = 0;
                    i++;
                }
            }
        }
    }
}

```

```

    }
}

/* Recursive function for finding determinant of matrix.
n is current dimension of mat[][]. */
int determinantOfMatrix(int mat[N][N], int n){
    int D = 0; // Initialize result
    // Base case : if matrix contains single element
    if (n == 1) return mat[0][0];
    int temp[N][N]; // To store cofactors
    int sign = 1; // To store sign multiplier
    // Iterate for each element of first row
    for (int f = 0; f < n; f++){
        // Getting Cofactor of mat[0][f]
        getCofactor(mat, temp, 0, f, n);
        D += sign * mat[0][f]
            * determinantOfMatrix(temp, n - 1);
        // terms are to be added with alternate sign
        sign = -sign;
    }
    return D;
}

/* function for displaying the matrix */
void display(int mat[N][N], int row, int col){
    rep(i, row){
        rep(j, col){
            cout << " " << mat[i][j];
        }
        cout << "\n";
    }
}

// Driver program to test above functions
int main(){
    /* int mat[N][N] = {{6, 1, 1},
                        {4, -2, 5},
                        {2, 8, 7}}; */

    int mat[N][N] = { { 1, 0, 2, -1 },
                      { 3, 0, 0, 5 },
                      { 2, 1, 4, -3 },
                      { 1, 0, 5, 0 } };
}

```



```

int T[N][N]; //Tutte matrix
//range of matrix(number of pivots) is # different matching
//maximum
ll n, m;
for(int i = 0; i < m; i++){
    ll x, y, w;
    cin >> x >> y >> w;
    mat[x][y] = -1;//-w;
    mat[y][x] = -1;//-w;
    mat[x][x] += 1;//w;
    mat[y][y] += 1;//w;

    T[x][x] = 0;
    T[x][y] = rand() % 1e9+7;
    T[y][x] = -T[x][y];
}
// O(N^3)
cout << "Number of spanning trees/weight sum of spanning tree :
    " << determinantOfMatrix(mat, N-1);
cout << "Number of spanning forest with i in one component and
    j in the other : " << determinantOfMatrix(mat, N-i
    -j);//delete i/t row and column
if(determinantOfMatrix(T, N) == 0)
    cout << "Does not exist a maximum matching in the general
        graph (need more verify)";
else
    cout << "Does exist perfect matching";
cout << "Number of spanning trees/weight sum of spanning tree :
    " << determinantOfMatrix(T, N);
return 0;
}

```

3.12 MaxBipartiteMatching

```

#include "../Header.cpp"

// Works with 3-pairing or more (Perfect MCBM)
vl match, vis; // global variables
vector<vl> g;

```

```

int Aug(int L) {
    if (vis[L]) return 0; // L visited, return 0
    vis[L] = 1;
    for (auto it : g[L])
        if ((match[it] == -1) || Aug(match[it])) {
            match[it] = L; // flip status
            return 1; // found 1 matching
        }
    return 0; // no matching
}

int main() {
    ios::sync_with_stdio(false); cin.tie(0); cout.tie(0);
    ll V, Vleft;
    // VLeft and VRight can have common vertices names
    // match[R] -> L
    match.assign(V, -1);
    ll MCBM = 0;
    for(int L = 0; L < Vleft; L++){
        vis.assign(Vleft, 0);
        MCBM += Aug(L);
    }
    cout << "Found " << MCBM << " matchings\n"; // the answer is 2
    // for Figure 4.38
    cerr << "\nTime elapsed: " << 1000 * clock() / CLOCKS_PER_SEC
        << "ms\n";
    return 0;
}

```

3.13 MaxFlowDinic

```

// Time Complexity:
// - general worst case:  $O(|E| * |V|^2)$ 
//  $O(|E| \max\_flow)$  ford fulkenson
// - unit capacities:  $O(\min(V^{2/3}, \sqrt{E}))$ 
// - Bipartite graph (unit capacities) + source & sink (any
    capacities):  $O(E \sqrt{V})$ 

// minimo corte:
// separa en 2 colores el grafo y el minimo corte

```

```
// es la separacin con menos aristas entre distinto color
// mandar infinito a nodos que estan obligados a ser de un colors
// min_cut == max_flow

// max matching = max_flow (grafo bipartito)
// minimo cubrimiento nodos = nodos - max_flow

// maximo matching m*sqrt(n)
```

```
class Dinic{
    struct Edge { int to, rev; ll f, c; };
    int n, t_; vector<vector<Edge>> G;
    vl D; vi q, W;
    bool bfs(int s, int t){
        W.assign(n, 0); D.assign(n, -1); D[s] = 0;
        int f = 0, l = 0; q[l++] = s;
        while (f < l){
            int u = q[f++];
            for (const Edge &e : G[u]) if (D[e.to] == -1 && e.f <
                e.c)
                D[e.to] = D[u] + 1, q[l++] = e.to;
        }
        return D[t] != -1;
    }
    ll dfs(int u, ll f){
        if (u == t_) return f;
        for (int &i = W[u]; i < (int)G[u].size(); ++i){
            Edge &e = G[u][i]; int v = e.to;
            if (e.c <= e.f || D[v] != D[u] + 1) continue;
            ll df = dfs(v, min(f, e.c - e.f));
            if (df > 0) { e.f += df, G[v][e.rev].f -= df; return
                df; }
        }
        return 0;
    }

public:
    Dinic(int N) : n(N), G(N), D(N), q(N) {}
    void addEdge(int u, int v, ll cap){
        G[u].push_back({v, (int)G[v].size(), 0, cap});
```

```
        G[v].push_back({u, (int)G[u].size() - 1, 0, 0}); // cap if
            bidirectional
    }
    ll maxFlow(int s, int t){
        t_ = t; ll ans = 0;
        while (bfs(s, t)) while (ll dl = dfs(s, LLONG_MAX)) ans +=
            dl;
        return ans;
    }
};
```

3.14 MaxFlowFordFulkerson

```
#include "../Header.cpp"
```

```
// minimo cubrimiento aristas = min-cut
// extremos de aristas cortadas que no son s ni t
// max conj, ind = nodos - minimo cubrimiento aristas
```

```
// O(V2)
// O(EF)
```

```
// Number of vertices in given graph
#define V 6
```

```
/* Returns true if there is a path from source 's' to sink 't' in
   residual graph. Also fills parent[] to store the path */
int bfs(int rGraph[V][V], int s, int t, int parent[])
{
    // Create a visited array and mark all vertices as not visited
    bool visited[V];
    memset(visited, 0, sizeof(visited));

    // Create a queue, enqueue source vertex and mark source vertex
    // as visited
    queue<int> q;
    q.push(s);
    visited[s] = true;
    parent[s] = -1;
```

```

// Standard BFS Loop
while (!q.empty())
{
    int u = q.front();
    q.pop();

    for (int v=0; v<V; v++)
    {
        if (visited[v]==false && rGraph[u][v] > 0)
        {
            q.push(v);
            parent[v] = u;
            visited[v] = true;
        }
    }
}

// If we reached sink in BFS starting from source, then return
// true, else false
return (visited[t] == true);
}

// A DFS based function to find all reachable vertices from s.
// The function
// marks visited[i] as true if i is reachable from s. The initial
// values in
// visited[] must be false. We can also use BFS to find reachable
// vertices
void dfs(int rGraph[V][V], int s, bool visited[])
{
    visited[s] = true;
    for (int i = 0; i < V; i++)
        if (rGraph[s][i] && !visited[i])
            dfs(rGraph, i, visited);
}

// Prints the minimum s-t cut
void minCut(int graph[V][V], int s, int t)
{
    int u, v;

```

```

// Create a residual graph and fill the residual graph with
// given capacities in the original graph as residual
// capacities
// in residual graph
int rGraph[V][V]; // rGraph[i][j] indicates residual capacity
// of edge i-j
for (u = 0; u < V; u++)
    for (v = 0; v < V; v++)
        rGraph[u][v] = graph[u][v];

int parent[V]; // This array is filled by BFS and to store path
int max_flow = 0; // There is no flow initially
// Augment the flow while there is a path from source to sink
while (bfs(rGraph, s, t, parent))
{
    // Find minimum residual capacity of the edges along the
    // path filled by BFS. Or we can say find the maximum flow
    // through the path found.
    int path_flow = INT_MAX;
    for (v=t; v!=s; v=parent[v])
    {
        u = parent[v];
        path_flow = min(path_flow, rGraph[u][v]);
    }

    // update residual capacities of the edges and reverse
    // edges
    // along the path
    for (v=t; v!=s; v=parent[v])
    {
        u = parent[v];
        rGraph[u][v] -= path_flow;
        rGraph[v][u] += path_flow;
    }
    max_flow += path_flow;
}

// Flow is maximum now, find vertices reachable from s
bool visited[V];
memset(visited, false, sizeof(visited));

```

```

dfs(rGraph, s, visited);

// Print all edges that are from a reachable vertex to
// non-reachable vertex in the original graph
for (int i = 0; i < V; i++)
    for (int j = 0; j < V; j++)
        if (visited[i] && !visited[j] && graph[i][j])
            cout << i << " - " << j << endl;

return;
}

// Driver program to test above functions
int main()
{
    // Let us create a graph shown in the above example
    int graph[V][V] = { {0, 16, 13, 0, 0, 0},
                        {0, 0, 10, 12, 0, 0},
                        {0, 4, 0, 0, 14, 0},
                        {0, 0, 9, 0, 0, 20},
                        {0, 0, 0, 7, 0, 4},
                        {0, 0, 0, 0, 0, 0}
    };

    minCut(graph, 0, 5);

    return 0;
}

```

3.15 MinCostMaxFlow

```

#include "../Header.cpp"

struct Edge{
    int from, to, capacity, cost;
};
vector<vector<int>> adj, cost, capacity;
const int INF = 1e9;

```

```

void shortest_paths(int n, int v0, vector<int>& d, vector<int>&
p) {
    d.assign(n, INF);
    d[v0] = 0;
    vector<bool> inq(n, false);
    queue<int> q;
    q.push(v0);
    p.assign(n, -1);
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        inq[u] = false;
        for (int v : adj[u]) {
            if (capacity[u][v] > 0 && d[v] > d[u] + cost[u][v]) {
                d[v] = d[u] + cost[u][v];
                p[v] = u;
                if (!inq[v]) {
                    inq[v] = true;
                    q.push(v);
                }
            }
        }
    }
}

// flow, source, to;
int min_cost_flow(int N, vector<Edge> edges, int K, int s, int t)
{
    adj.assign(N, vector<int>());
    cost.assign(N, vector<int>(N, 0));
    capacity.assign(N, vector<int>(N, 0));
    for (Edge e : edges) {
        adj[e.from].push_back(e.to);
        adj[e.to].push_back(e.from);
        cost[e.from][e.to] = e.cost;
        cost[e.to][e.from] = -e.cost;
        capacity[e.from][e.to] = e.capacity;
    }
    int flow = 0;
    int cost = 0;
    vector<int> d, p;
}

```

```

while (flow < K) {
    shortest_paths(N, s, d, p);
    if (d[t] == INF) break;
    // find max flow on that path
    int f = K - flow;
    int cur = t;
    while (cur != s) {
        f = min(f, capacity[p[cur]][cur]);
        cur = p[cur];
    }
    // apply flow
    flow += f;
    cost += f * d[t];
    cur = t;
    while (cur != s) {
        capacity[p[cur]][cur] -= f;
        capacity[cur][p[cur]] += f;
        cur = p[cur];
    }
}
if (flow < K) return -1;
else return cost;
}

int sup[55], inf[55];

int main(){
    int n, q;
    ll ans = 0;
    cin >> n >> q;
    for (int i = 0; i < n; ++i) {
        sup[i] = n;
        inf[i] = 1;
    }
    for (int j = 0; j < q; ++j) {
        int t, l, r, v;
        cin >> t >> l >> r >> v;
        if (t == 1){
            for (int i = l - 1; i <= r - 1; ++i) {
                inf[i] = max(inf[i], v);
            }

```

```

        }else{
            for (int i = l - 1; i <= r - 1; ++i) {
                sup[i] = min(sup[i], v);
            }
        }
    }
    vector<Edge>ee;
    Edge E;
    for(int i = 0; i < n; i++){
        E.from = 0;
        E.to = i + 2;
        E.capacity = 1;
        E.cost = 0;
        ee.push_back(E);
        for(int j = inf[i]; j <= sup[i]; j++){
            E.from = i + 2;
            E.to = j + n + 1;
            E.capacity = 1;
            E.cost = 0;
            ee.push_back(E);
        }
    }
    ll id = n + 2 + n;
    for(int i = n + 2; i <= n + 2 + n - 1; i++){
        for(int j = 0; j < n; j++){
            E.from = i;
            E.to = id;
            E.capacity = 1;
            E.cost = 2 * j + 1;
            ee.push_back(E);
            E.from = id;
            E.to = 1;
            E.capacity = 1;
            E.cost = 0;
            ee.push_back(E);
            id++;
        }
    }
    ans = min_cost_flow(2 * n + n*n + 10, ee, n, 0, 1);
    cout << ans << "\n";
}

```

```

cerr << "\nTime elapsed: " << 1000 * clock() / CLOCKS_PER_SEC
      << "ms\n";
}

```

3.16 MinCostMaxFlow2

```

#include "../Template.cpp"

template <class T>
class MCMF{
    typedef pair<T, T> pTT;
    T INF = numeric_limits<T>::max();
    struct Edge{
        int v; T c, w;
        Edge(int v, T c, T w) : v(v), c(c), w(w) {}
    };
    int n; vvi E;
    vector<Edge> L; vi F; vector<T> D, P; vector<bool> V;
    bool dij(int s, int t){
        D.assign(n, INF); F.assign(n, -1); V.assign(n, false);
        D[s] = 0;
        rep(_, n){
            int best = -1;
            rep(i, n) if (!V[i] && (best == -1 || D[best] > D[i]))
                best = i;
            if (D[best] >= INF) break;
            V[best] = true;
            for (int e : E[best]){
                Edge ed = L[e];
                if (ed.c == 0) continue;
                T toD = D[best] + ed.w + P[best] - P[ed.v];
                if (toD < D[ed.v]) D[ed.v] = toD, F[ed.v] = e;
            }
        }
        return D[t] < INF;
    }
    pTT augment(int s, int t){
        pTT flow(L[F[t]].c, 0);
        for (int v = t; v != s; v = L[F[v] ^ 1].v)
            flow.ff = min(flow.ff, L[F[v]].c), flow.ss += L[F[v]].w;
    }
}

```

```

        for (int v = t; v != s; v = L[F[v] ^ 1].v)
            L[F[v]].c -= flow.ff, L[F[v] ^ 1].c += flow.ff;
        return flow;
    }
public:
    MCMF(int n) : n(n), E(n), D(n), P(n, 0), V(n, 0) {}
    pTT mcmf(int s, int t){
        pTT ans(0, 0);
        if (!dij(s, t)) return ans;
        rep(i, n) if (D[i] < INF) P[i] += D[i];
        while (dij(s, t)){
            auto flow = augment(s, t);
            ans.ff += flow.ff, ans.ss += flow.ff * flow.ss;
            rep(i, n) if (D[i] < INF) P[i] += D[i];
        }
        return ans;
    }
    void addEdge(int u, int v, T c, T w){
        E[u].pb(L.size()); L.eb(v, c, w);
        E[v].pb(L.size()); L.eb(u, 0, -w);
    }
};

```

3.17 SCC

```

#include "../Header.cpp"

vl dfs_num, dfs_low, visited;
vector<vl> g, invg;
void Kosaraju(int u, int pass, vl& S) { // pass = 1 (original), 2
    (transpose)
    dfs_num[u] = 1;
    vl &neighbor = (pass == 1) ? g[u] : invg[u];
    for (auto it : neighbor)
        if (dfs_num[it] == -1)
            Kosaraju(it, pass, S);
    S.push_back(u);
}
// -----
// implementation of Tarjan's SCC algorithm

```

```

stack<int> St;
int cont, numSCC;

void tarjanSCC(int u) {
    dfs_low[u] = dfs_num[u] = cont;
    cont++;
    St.push(u);
    visited[u] = 1;
    for (auto v : g[u]) {
        if (dfs_num[v] == -1)
            tarjanSCC(v);
        if (visited[v])
            dfs_low[u] = min(dfs_low[u], dfs_low[v]);
    }

    if (dfs_low[u] == dfs_num[u]) {
        while (1) {
            int v = St.top(); St.pop(); visited[v] = 0;
            if (u == v) break;
        }
        ++numSCC;
    }
}

int main() {
    int n, m; cin >> n >> m;
    vector<vl> g(n);
    while(m--) {
        int u, v; cin >> u >> v; u--, v--;
        g[u].push_back(v);
    }
    // run Tarjan's SCC
    dfs_num.assign(n, 0); dfs_low.assign(n, 0); visited.assign(n, 0);
    while (!St.empty()) St.pop();
    cont = numSCC = 0;
    for (int u = 0; u < n; ++u)
        if (dfs_num[u] == -1)
            tarjanSCC(u);
    //Kosaraju's SCC
    vl S;
    dfs_num.assign(n, -1);

```

```

    for (int u = 0; u < n; ++u)
        if (dfs_num[u] == -1)
            Kosaraju(u, 1, S);
    int numSCC = 0;
    dfs_num.assign(n, -1);
    for (int i = n-1; i >= 0; --i){
        vl comp;
        if (dfs_num[S[i]] == -1)
            numSCC++, Kosaraju(S[i], 2, comp);           // on
                                                         transposed graph
    }
    return 0;
}

```

3.18 SpecialPathsWithCentroids

```

int n, k, sz[mxN], mxtree[mxN], ans;
vector<pair<int, int>> adj[mxN];
vector<int> new_vis;
bool used[mxN];

int dfs(int u, int p = -1){
    sz[u] = 1;
    for(pair<int, int> v : adj[u]){
        if(v.second != p && !used[v.second]){
            sz[u] += dfs(v.second, u);
        }
    }
    return sz[u];
}

int get_centroid(int u, int ms, int p = -1){ // u = node, ms =
    size of the current tree delimited by previous centroids, p =
    parent
    for(pair<int, int> v : adj[u]){
        if(v.second != p && !used[v.second]){
            if(sz[v.second]*2 > ms){
                return get_centroid(v.second, ms, u);
            }
        }
    }
}

```

```

    }
    return u;
}

int flag[mxN], stamp[mxN];
int timestamp;

void solve(int u){
    ++timestamp;
    int sz_curr_tree = dfs(u);
    int centroid = get_centroid(u, sz_curr_tree);
    used[centroid] = 1;
    flag[0] = 0;
    stamp[0] = timestamp;
    // do something with the centroid
    for(pair<int, int> v : adj[centroid]){

    }
    // end that something with the centroid xdd

    // solve for child trees centroids
    for(pair<int, int> v : adj[centroid]){
        if(used[v.second]) continue;
        solve(v.second);
    }
}

```

3.19 StableMatching

```

#include "../Header.cpp"
// Match the preferences of N clients and M restaurants that no
// side prefer another
// restaurant or client, respectively.

// hospital-residents variation of stable matching
int main(){

    ios_base::sync_with_stdio(0);
    cin.tie(0);
    ll n, m, k, x;

```

```

    cin >> n >> m;

    vl cap(m, 1), match(n, -1);

    vector<queue<ll>>pref(n);
    queue<ll> q;
    for(auto &it : cap) cin >> it; // capacity

    for(int i = 0; i < n; i++)
    {
        q.push(i);
        cin >> k;
        while(k-->
        {
            cin >> x;
            pref[i].push(x-1); //client preference list
        }
    }

    vector<unordered_map<ll, ll>> res_pref(m);
    for(int i = 0; i < m; i++)
    {
        ll id = 0;
        cin >> k;
        while(k-->
        {
            cin >> x;
            res_pref[i][x-1] = -id; // restaurant
            preference list
            id++;
        }
    }

    vector<set<pll>> in_res(m);
    while(!q.empty())
    {
        ll cl = q.front();
        q.pop();

        ll rest = pref[cl].front(); // actual preference
        restaurant

```



```

    in_res[rest].insert({res_pref[rest][cl], cl}); //
        add client to restaurant
    match[cl] = rest;
    pref[cl].pop(); // remove client preference

    if(cap[rest] == 0)
    {
        ll cl_new = (*in_res[rest].begin()).second;
        // erase client with less preference
        in_res[rest].erase(in_res[rest].begin());

        match[cl_new] = -1;
        if(!pref[cl_new].empty()) // add client to
            queue if he has more preferences
            q.push(cl_new);
    }
    else
        cap[rest]--;
}
for(int i = 0; i < n; i++)
    if(match[i] != -1) cout << i+1 << " " << match[i]+1
        << "\n";
}

```

3.20 ToposortDFS

```
#include "../Header.cpp"
```

```

vl s, v;
vector<vl> g;
void dfs(int t)
{
    v[t] = 1;
    for(auto it : g[t])
        if(!v[it])
            dfs(it);

    s.pb(t);
}

```

```

}
int main()
{
    ios::sync_with_stdio(false);
    cin.tie(0);
    ll n;
    g.assign(n, vl());
    v.assign(n, 0);

    rep(i, n)
        if(!v[i])
            dfs(i);

    reverse(ALL(s));
    return 0;
}

```

3.21 ToposortKhan

```
#include "../Header.cpp"
```

```

// All toposort orderings
vector<vl> g;
ll in;
vl indegree, vis;
vl sorted;
vector<char> ans;
bool possible;
void toposort()
{
    bool flag = 0;

    for(int i = 0; i < in; i++)
    {
        if(indegree[i] == 0 && !vis[i])
        {
            sorted.push_back(i);

            for(auto it : g[i])

```

```

        indegree[it]--;
        vis[i] = 1;
        toposort();
        vis[i] = 0;
        sorted.pop_back();

        for(auto it : g[i])
            indegree[it]++;
        flag = 1;
    }
}
if(!flag && sorted.size() == in)
{
    possible = 1;
    for(int i = 0; i < in; i++)
    {
        if(i > 0) cout << " ";
        cout << ans[sorted[i]];
    }
    cout << "\n";
}
}

int main()
{
    ll v,x,y,e,a,b,in=0;
    pll h;
    cin>>v >> e;
    vector<vl > g(v, vl(0));
    for(int i = 0; i < e; i++)
    {
        cin >> x >> y;
        g[x].push_back(y);
    }

    //set indegrees
    vl indegree(v, 0);
    vl sorted;
    for(int i=0; i < v; i++)
    {
        for(auto it : g[i])

```

```

        indegree[it]++;
    }
    queue<ll> q;
    //agregar par a cola para tener los paquetes o niveles de
    //profundidad, sumar
    for(int i = 0; i < v; i++)
        if(indegree[i] == 0)
            q.push(i);

    while(!q.empty()){

        ll t = q.front();
        q.pop();

        for(auto it : g[t]){

            if(--indegree[it] == 0)
                q.push(it);
        }
    }
}

```

4 Header

```

#include<bits/stdc++.h>
#pragma GCC optimize("Ofast")
using namespace std;
typedef long long ll;
typedef unsigned long long ull;
typedef vector<ll> vl;
typedef vector<int> vi;
typedef pair<ll,ll> pll;
typedef vector<pll> vp;
typedef double db;
#define INF 1e17
#define INF32 INT_MAX
#define EPS 1e-7
#define ALL(x) x.begin() , x.end()
#define ALLR(x) x.rbegin() , x.rend()

```

```

#define UNIQUE(c) (c).resize(unique(ALL(c)) - (c).begin())
#define PI acos(-1.0)
#define pb push_back
#define rep(i, n) for (int i = 0; i < (int)n; i++)
#define repx(i, a, b) for (int i = (int)a; i < (int)b; i++)

#define DBG 1

#define cerr \
    if (DBG) cerr

int main() {

    ios::sync_with_stdio(false);
    cin.tie(0);
    cout.tie(0);
    srand((unsigned int) time(0));

    // Code here

    // Compile:
    // g++ Code1.cpp && ./a.out < in > out
    // ulimit -s 1048576 more stack size 1gb
    // g++ -std=c++11 Code1.cpp && a.exe < in > out
    cerr << "\nTime elapsed: " << 1000 * clock() /
        CLOCKS_PER_SEC << "ms\n";
    return 0;
}

```

5 Math

5.1 ArithmeticEval

```

#include "../Header.cpp"

```

```

ll k, n;
string s;

ll eval(ll l, ll r){

```

```

    ll open = 0;
    stack<ll> vals;
    stack<char> ops;

    if(s[l] == '+' || s[l] == '*' || s[r] == '+' || s[r] ==
        '*')return -1;
    if(s[l] == '0' && r-l >= 1 && isdigit(s[l+1]))return -1;

    repx(i, l, r+1){
        if(isdigit(s[i])){
            vals.push((s[i] - '0') % k);
            while(i < r && isdigit(s[i+1])){
                i++;
                vals.top() = (vals.top() * 10 + (s[i] -
                    '0')) % k;
            }
        }
        else if(s[i] == '('){
            open++;
            ops.push(s[i]);
        }
        else if(s[i] == ')'){
            open--;
            if(open < 0) return -1;
            ll in = i-1;
            while(ops.top() != '('){
                ll aux = vals.top();
                vals.pop();
                if(ops.top() == '+'){
                    vals.top() = (vals.top() +
                        aux) % k;
                }
                else{
                    vals.top() = (vals.top() *
                        aux) % k;
                }
                ops.pop();
            }
            ops.pop();
        }
        else{

```

```

        // higher precedence first
        while(!ops.empty() && ops.top() == '*'){
            ll aux = vals.top();
            vals.pop();
            vals.top() = (vals.top() * aux) % k;
            ops.pop();
        }
        ops.push(s[i]);
    }
}
if(open != 0) return -1;
while(!ops.empty()){
    ll aux = vals.top();
    vals.pop();
    if(ops.top() == '+'){
        vals.top() = (vals.top() + aux) % k;
    }
    else{
        vals.top() = (vals.top() * aux) % k;
    }
    ops.pop();
}
return vals.top();
}

int main(){
    ios_base::sync_with_stdio(0); cin.tie(0);
    srand((unsigned int) time(0));
    cin >> k >> n >> s;

    //cout << eval(0, n-1)<<"\n";
    ll ans = 0;
    rep(i, n){
        repx(j, i, n){
            if(eval(i, j) == 0){
                ans++;
                //cout << i << " " << j << "
                //<<eval(i,j)<< "\n";
            }
        }
    }
}

```

```

        cout << ans << "\n";
    }

```

5.2 Combinatory

```

#include "../Header.cpp"

const int M = 1e9+7;

// binary exponent
// Note that  $a^{(b^c) \% p} = a^{(b^c \% (p - 1)) \% p}$  for little
// fermat theorem
ll expmod(ll b, ll e){
    ll ans = 1;
    while(e){
        if(e&1) ans = ans*b %M;
        b = b*b %M; e >>= 1;
    }
    return ans;
}

// Binary exponentiation: a and b relative primes ->  $a^{\phi(m) \% m = 1} \rightarrow a^{\phi(m) - 1} \% mod = a^{-1}$ 
// Binary exponentiation: m prime ->  $a^{\{m - 1\} \% m = 1} \rightarrow a^{\{m - 2\} \% mod = 1}$ 

// When M is prime
ll invmod(ll a){ return expmod(a, M-2); }

//inv modular factoriales
const ll MAXN = 1e5 + 1;
ll F[MAXN], INV[MAXN], FI[MAXN];
// ...

F[0] = 1; repx(i, 1, MAXN) F[i] = F[i-1]*i %M;
INV[1] = 1; repx(i, 2, MAXN) INV[i] = M - (ll)(M/i)*INV[M%i]%M;
FI[0] = 1; repx(i, 1, MAXN) FI[i] = FI[i-1]*INV[i] % M;

// Divide a elements in b segments = choose(a - 1, b - 1)

```

```

// combinatory
ll Comb(ll n, ll k){
    if(n < k) return 0;
    return F[n]*FI[k] %M *FI[n-k] %M;
}

// combinatory precalc
ll C[MAXN][MAXK];
// ...
rep(i, MAXN){
    C[i][0] = 1; if(i < MAXN) C[i][i] = 1;
    repx(j, 1, min(i, (int)MAXK))
        C[i][j] = (C[i-1][j-1] + C[i-1][j])%M;
}

// divide a elements into b segments = C[a-1][b-1]
// each segment has at least 1 element

//Wilson theorem
//(p-1)! mod p = -1 if p prime

//https://cp-algorithms.com/algebra/factorial-modulo.html#multiplicity-of-p

// n! mod p with p prime and ignore multiples of p
int factmod(int n, int p) {
    vector<int> f(p);
    f[0] = 1;
    for (int i = 1; i < p; i++)
        f[i] = f[i-1] * i % p;

    int res = 1;
    while (n > 1) {
        if ((n/p) % 2)
            res = p - res;
        res = res * f[n%p] % p;
        n /= p;
    }
    return res;
}

```

```

// number of times p divides n! = v_p, n! % p*v_p = 0
int multiplicity_factorial(int n, int p) {
    int count = 0;
    do {
        n /= p;
        count += n;
    } while (n);
    return count;
}

// combinatoria n = 1e18, primo chico
// lucas
const int M = 3005;
int C[M][M];
// ...
ll lucas(ll n, ll k, int p){
    ll ans = 1;
    while(n + k){
        ans = (ans * C[n%M][k%M]) % M;
        n /= M; k /= M;
    }
    return ans;
}

// Multinomial Coefficient
ll multinomial(vector<int> K) {
    ll n = 0, ans = 1;
    for (int k : K) n += k, ans = mul(ans, choose(n, k));
    return ans;
}

// Catalan numbers
// Counting problem where the solution is expressed by combining
// two
// identical subproblems with total size = n1 then involves
// Catalan
ll Catalan(ll n) {
    return choose(2 * n, n) * invmod(n + 1) % mod;
}

// C_n = (4n - 2) / (n + 1) * C_{n-1}, C_0 = 1
// Catalan convolution

```

```
// C_n^i = sum_{i1, i2, ..., ik, sum(i_j) = n} C_i1 * C_i2 * ...
// * C_ik
ll CatalanConv(ll n, ll k) {
    return (k + 1) * inv(n + k + 1) % mod * choose(2 * n + k, n) %
        mod;
}
```

5.3 ErathostenesSieve

```
#include "../Header.cpp"

vector<bool> crib;
void criba(ll b){
    crib.assign(b, 1);
    crib[0] = 0; crib[1] = 0;
    repx(k, 2, sqrt(b+1) + 1)
        if(crib[k])
            for(int j=2; (j * k) < b + 1; j++)//optimization j=k
                crib[k*j] = 0;
}

/ Primes in range [L, R] in O((R - L + 1) Log(Log(R)) + sqrt(R) *
  Log(Log(sqrt(R))))
// R ~ 1e12
vector<bool> segmentedSieve(ll L, ll R) {
    // generate all primes up to sqrt(R)
    ll lim = sqrt(R);
    vector<bool> mark(lim + 1, false);
    vl primes;
    repx(i, 2, lim + 1) {
        if (!mark[i]) {
            primes.pb(i);
            for (ll j = i * i; j <= lim; j += i)
                mark[j] = 1;
        }
    }
    vector<bool> isPrime(R - L + 1, true);
    for (ll i : primes)
```

```
        for (ll j = max(i * i, (L + i - 1) / i * i); j <= R; j +=
            i)
            isPrime[j - L] = false;
    if (L == 1)
        isPrime[0] = false;
    return isPrime;
}
```

```
// Linear Sieve: O(n)
// Uses more memory
vl linearSieve(int n = 10000000) {
    vl lp(n + 1, 0), pr;
    repx(i, 2, n + 1) {
        if (lp[i] == 0) {
            lp[i] = i; pr.pb(i);
        }
        for (int j = 0; i * pr[j] <= n; ++j) {
            lp[i * pr[j]] = pr[j];
            if (pr[j] == lp[i]) {
                break;
            }
        }
    }
}

// PRIMALITY TEST
// Trial Division O(sqrt(n))
bool isPrime(int x) {
    for (int d = 2; d * d <= x; d++) {
        if (x % d == 0)
            return false;
    }
    return true;
}

// Fermat theorem: a ^ (p - 1) % p = 1 for all a
// 646 of 10^9 fails
// O(1)
bool probablyPrimeFermat(int n, int iter=5) {
    if (n < 4) return n == 2 || n == 3;
    for (int i = 0; i < iter; i++) {
        int a = 2 + rand() % (n - 3);
```

```

        if (binpower(a, n - 1, n) != 1) return false;
    }
    return true;
}

// Miller Rabin
// Probabilistic check for prime numbers
// 88% of the numbers have a prime factor under 100, optimize the
// checker checking these cases
using u64 = uint64_t;
using u128 = __uint128_t;

u64 binpower(u64 base, u64 e, u64 mod) {
    u64 result = 1;
    base %= mod;
    while (e) {
        if (e & 1) result = (u128)result * base % mod;
        base = (u128)base * base % mod;
        e >>= 1;
    }
    return result;
}

bool check_composite(u64 n, u64 a, u64 d, int s) {
    u64 x = binpower(a, d, n);
    if (x == 1 || x == n - 1) return false;
    for (int r = 1; r < s; r++) {
        x = (u128)x * x % n;
        if (x == n - 1) return false;
    }
    return true;
};

bool MillerRabin(u64 n, int iter=5) { // returns true if n is
    probably prime, else returns false.
    if (n < 4) return n == 2 || n == 3;
    int s = 0;
    u64 d = n - 1;
    while ((d & 1) == 0) {
        d >>= 1;
        s++;
    }

```

```

    }
    for (int i = 0; i < iter; i++) {
        int a = 2 + rand() % (n - 3);
        if (check_composite(n, a, d, s))
            return false;
    }
    return true;
}

// Deterministic MillerRabin
bool MillerRabin(u64 n) { // returns true if n is prime, else
    returns false.
    if (n < 2) return false;
    int r = 0;
    u64 d = n - 1;
    while ((d & 1) == 0) {
        d >>= 1;
        r++;
    }
    for (int a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}) {
        if (n == a) return true;
        if (check_composite(n, a, d, r)) return false;
    }
    return true;
}

```

5.4 Euclid

```

#include "../Header.cpp"

// find (x, y) such that Ax + By = gcd(A, B), and |Ax|, |By| <=
// AB/gcd(A, B)
pll euclid(ll A, ll B) {
    if (!B) return {1, 0};
    pll p = euclid(B, A % B);
    return {p.ss, p.ff - (A / B) * p.ss};
}

// find x in [0, M) such that Ax = 1 mod M
ll minv(ll A, ll M)
{

```

```

    pll p = euclid(A, M);
    assert(p.ff * A + p.ss * M == 1);
    return p.ff + (p.ff < 0) * M;
}

// find (x, y)'s such that Ax + By = R where R is multiplle of
gcd(A, B);
pair<pll, pll> diophantine(ll A, ll B, ll R) {
    ll g = __gcd(A, B), x, y; A /= g, B /= g, R /= g;
    tie(x, y) = euclid(A, B); x *= R, y *= R;
    assert(A * x + B * y == R);
    return {{x, y}, {-B, A}}; // solutions: p+t*ans.snd
}

```

5.5 FFT

```

#include "../Header.cpp"

#define M_PI1 3.141592653589793238462643383279502884L

typedef complex<double> C;
typedef vector<double> vd;

void fft(vector<C> &a)
{
    int n = a.size(), L = 31 - __builtin_clz(n);
    static vector<complex<long double>> R(2, 1);
    static vector<C> rt(2, 1);
    for (static int k = 2; k < n; k *= 2)
    {
        R.resize(n);
        rt.resize(n);
        auto x = polar(1.0L, M_PI1 / k);
        repx(i, k, 2 * k) rt[i] = R[i] = i & 1 ? R[i / 2] * x :
            R[i / 2];
    }
    vector<int> rev(n);
    rep(i, n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    rep(i, n) if (i < rev[i]) swap(a[i], a[rev[i]]);
}

```

```

for (int k = 1; k < n; k *= 2) for (int i = 0; i < n; i += 2 *
    k) rep(j, k)
{
    auto x = (double *)&rt[j + k], y = (double *)&a[i + j + k];
    C z(x[0] * y[0] - x[1] * y[1], x[0] * y[1] + x[1] * y[0]);
    a[i + j + k] = a[i + j] - z;
    a[i + j] += z;
}
}

vd conv(const vl &a, const vl &b)
{
    if (a.empty() || b.empty()) return {};
    vd res(a.size() + b.size() - 1);
    int L = 32 - __builtin_clz(res.size()), n = 1 << L;
    vector<C> in(n), out(n);
    copy(a.begin(), a.end(), in.begin());
    rep(i, b.size()) in[i].imag(b[i]);
    fft(in);
    for (auto &x : in) x *= x;
    rep(i, n) out[i] = in[-i & (n - 1)] - conj(in[i]);
    fft(out);
    rep(i, res.size()) res[i] = imag(out[i]) / (4 * n);
    return res;
}

//slower
vl convMod(const vl &a, const vl &b, int M)
{
    if (a.empty() || b.empty()) return {};
    vl res(a.size() + b.size() - 1);
    int B = 32 - __builtin_clz(res.size()), n = 1 << B, cut =
        int(sqrt(M));
    vector<C> L(n), R(n), outs(n), outl(n);
    rep(i, a.size()) L[i] = C((int)a[i] / cut, (int)a[i] % cut);
    rep(i, b.size()) R[i] = C((int)b[i] / cut, (int)b[i] % cut);
    fft(L), fft(R);
    rep(i, n)
    {
        int j = -i & (n - 1);
        outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
    }
}

```



```

        outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
    }
    fft(outl), fft(outs);
    rep(i, res.size())
    {
        ll av = ll(real(outl[i]) + .5), cv = ll(imag(outs[i]) +
            .5);
        ll bv = ll(imag(outl[i]) + .5) + ll(real(outs[i]) + .5);
        res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
    }
    return res;
}

int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    cout.tie(0);
    int n, m;
    cin >> n >> m;
    string s, t;
    cin >> s >> t;
    // SAME SIZE
    vl an(n, 0), am(n, 0);
    vl bn(n, 0), bm(n, 0);
    for (int k = 0; k < n; ++k) {
        if (s[k] == 'a') an[k] = 1;
        else bn[k] = 1;
    }
    for (int k = 0; k < m; ++k) {
        if (t[k] == 'a') am[k] = 1;
        else bm[k] = 1;
    }
    reverse(am.begin(), am.end());
    reverse(bm.begin(), bm.end());
    vd resA = conv(an, am);
    vd resB = conv(bn, bm);
    vector<vector<int>> ans;
    ans.assign(m+1, vector<int>());

    //n > m
    // All complete count mathces

```

```

    for (int i = n-1; i < 2*n - m; ++i) {
        ans[m - round(resA[i]) - round(resB[i])].push_back(i-n+2);
    }
    // or these ranges for an and bm with original legnth
    for (int j = m-1; j < n; ++j) {
        for (int j = 0; j <= m; ++j) {
            cout << j << " ";
            for (int u: ans[j]) cout << " " << u;
            cout << "\n";
        }
        cerr << "\nTime elapsed: " << 1000 * clock() / CLOCKS_PER_SEC
            << "ms\n";
    }

    // CP-Algorithms FFT
    vector<int> multiply(vector<int> const& a, vector<int> const& b) {
        vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
        int n = 1;
        while (n < a.size() + b.size())
            n <<= 1;
        fa.resize(n);
        fb.resize(n);

        fft(fa, false);
        fft(fb, false);
        for (int i = 0; i < n; i++)
            fa[i] *= fb[i];
        fft(fa, true);

        vector<int> result(n);
        for (int i = 0; i < n; i++)
            result[i] = round(fa[i].real());
        return result;
    }

    using cd = complex<double>;
    const double PI = acos(-1);

    void fft(vector<cd> & a, bool invert) {
        int n = a.size();

```

```

if (n == 1)
    return;

vector<cd> a0(n / 2), a1(n / 2);
for (int i = 0; 2 * i < n; i++) {
    a0[i] = a[2*i];
    a1[i] = a[2*i+1];
}
fft(a0, invert);
fft(a1, invert);

double ang = 2 * PI / n * (invert ? -1 : 1);
cd w(1), wn(cos(ang), sin(ang));
for (int i = 0; 2 * i < n; i++) {
    a[i] = a0[i] + w * a1[i];
    a[i + n/2] = a0[i] - w * a1[i];
    if (invert) {
        a[i] /= 2;
        a[i + n/2] /= 2;
    }
    w *= wn; // Trie to use w = pow(w, i) or something like
             // that
}
}

```

```

// NTT
const int mod = 7340033;
const int root = 5;
const int root_1 = 4404020;
const int root_pw = 1 << 20;

```

```

void fft(vector<int> & a, bool invert) {
    int n = a.size();

    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >>= 1)
            j ^= bit;
        j ^= bit;
    }
}

```

```

if (i < j)
    swap(a[i], a[j]);
}

for (int len = 2; len <= n; len <= 1) {
    int wlen = invert ? root_1 : root;
    for (int i = len; i < root_pw; i <= 1)
        wlen = (int)(1LL * wlen * wlen % mod);

    for (int i = 0; i < n; i += len) {
        int w = 1;
        for (int j = 0; j < len / 2; j++) {
            int u = a[i+j], v = (int)(1LL * a[i+j+len/2] * w %
                mod);
            a[i+j] = u + v < mod ? u + v : u + v - mod;
            a[i+j+len/2] = u - v >= 0 ? u - v : u - v + mod;
            w = (int)(1LL * w * wlen % mod);
        }
    }

    if (invert) {
        int n_1 = inverse(n, mod);
        for (int & x : a)
            x = (int)(1LL * x * n_1 % mod);
    }
}

```

5.6 FFTBits

```

ll c1[MAXN+9], c2[MAXN+9]; // MAXN must be power of 2 !!
void fht(ll* p, int n, bool inv, char t){
    for(int l=1; 2*l<=n; l*=2){
        for(int i=0; i<n; i+=2*l){
            fore(j,0,l){
                ll u=p[i+j], v=p[i+l+j];
                // XOR
                if(t=='x'){
                    if(!inv)p[i+j]=u+v, p[i+l+j]=u-v;
                }
            }
        }
    }
}

```

```

        else
            p[i+j]=(u+v)/2,p[i+1+j]=(u-v)/2;
    }
    // AND
    else if(t=='a'){
        if(!inv)p[i+j]=v,p[i+1+j]=u+v;
        else p[i+j]=-u+v,p[i+1+j]=u;
    }
    // OR
    else if(t=='o'){
        if(!inv)p[i+j]=u+v,p[i+1+j]=u;
        else p[i+j]=v,p[i+1+j]=u-v;
    }
    }
}
}
}
// like polynomial multiplication, but XORing exponents
// instead of adding them (also ANDing, ORing)
vector<ll> multiply(vector<ll>& p1, vector<ll>& p2, char t){
    int n=1<<(32-__builtin_clz(max(SZ(p1),SZ(p2))-1));
    fore(i,0,n)c1[i]=0,c2[i]=0;
    fore(i,0,SZ(p1))c1[i]=p1[i];
    fore(i,0,SZ(p2))c2[i]=p2[i];
    fht(c1,n,false,t);fht(c2,n,false,t);
    fore(i,0,n)c1[i]*=c2[i];
    fht(c1,n,true,t);
    return vector<ll>(c1,c1+n);
}

```

5.7 Fib

```
#include "../Header.cpp"
```

```

// Casini:    F_{n-1} * F_{n+1} - F_n ^ 2 = (-1)^n
// Addition:  F_{n+k} = F_{k} * F_{n+1} + F_{k-1} * F_{n}
// Addition2: F_{2n} = F_n * (F_{n+1} + F_{n-1})
// Addition3: F_n divide F_m ssi n divide a m
// GCD:       gcd(F_n, F_m) = F_{gcd(n,m)}

```

```

// Formula:   F_n = (((1+sqrt(5))/2)^n - ((1-sqrt(5))/2)^n) /
              sqrt(5)
// Approx:    F_n ~ [((1+sqrt(5))/2)^n/sqrt(5)]
// Matrix:    P = {{0, 1}, {1, 1}}

// Fast doubling method
pll fib(n) {
    if(n == 0) return {0, 1};
    auto p = fib(n>>1);
    int c = p.first * (2 * p.second - p.fisrt);
    int d = p.first * p.first + p.second * p.second;
    if(n & 1) return {d, c + d};
    else return {c, d};
}

```

5.8 Functions

```
#include "../Header.cpp"
```

```

using namespace chrono;
auto start1 = high_resolution_clock::now();
auto stop = high_resolution_clock::now();
auto duration = duration_cast<microseconds>(stop - start1);
//cerr << duration.count()/1000 << "ms" << endl;

default_random_engine generator;
uniform_real_distribution<double> distribution(0,LLONG_MAX);
ll num = distribution(generator);

```

```

// files
ifstream input;
input.open("divisibility-tree.in");
input >> n;
ofstream output;
output.open("divisibility-tree.out");
output<<" ";
output.close();

```

```
// suma subconjuntos
```

```

for i = 0 to n-1
for mask = 0 to (1n) - 1
if (mask & (1i))
dp(mask) += dp(mask - (1i))

// suma divisores
for p    P
for x    S (DE MENOR A MAYOR)
if (p divide a x)
dp(x) += dp(x / p)

// hash pairs unorderedmap<pll,ll,hash_pair>
struct hash_pair {
    template <class T1, class T2>
    size_t operator()(const pair<T1, T2>& p) const
    {
        auto hash1 = hash<T1>{}(p.first);
        auto hash2 = hash<T2>{}(p.second);
        return hash1 ^ hash2;
    }
};
int maxlog2(int x) //potencia de 2 mayor que es menor o igual a x
{
    // ll maxlog = 63 - __builtin_clzll(x);
    int maxlog = 31 - __builtin_clz(x);
    return maxlog;
}
int A[10000]; //Set con reset 0(1), Tambien con Map
int t=1;
bool fin(int x)
{
    return A[x]==t;
}
void borrar()
{
    t++;
}
void inse(int x)
{
    A[x]=t;

```

```

}
int res(int a,int b)
{
    int c=1,d;
    for(int i=0;i<b;i++)
    {
        c*=10;
    }
    d=c*10;
    a=a-(a-a%d);
    a=a-(a%c);
    a=a/c;
    return a;
}

//propagate val in mask to all its submask
for (int i = 0; i < p; i++)
{
    for(int mask = 0; mask < (1 << p); mask++)
    {
        if((mask & (1 << i)) == 0)
            f[mask] += f[mask | (1 << i)];
        if(mask & (1 << i)) // to propagate from submasks to mask
            dp[mask] += dp[mask - (1 << i)];
    }
}

// 0(3^n)
rep(m, (1 << n)){
    // 2^k k: number of bits in m
    // iterates over al submasks of m in descending order of value
    for(int s = m; ; s = (s-1) & m){
        cout << s << endl;
        if(s == 0) break;
    }
}

int bit_opst(ll N,ll S)
{

```

```

//Obtain the remainder (modulo) of S when it is divided by N
// (N is a power of 2)
return S &(N -1);
//Determine if S is a power of 2.
return (S &( S -1)) == 0;
//Turn on the last bit in S, e.g.S = (40)10 = (101000)2 S =
// (32)10 = (100000)2.
return S &( S -1);
// Turn on the last zero in S, e.g.S = (41)10 = (101001)2 S =
// (43)10 = (101011)2.
return S|| (S + 1);
// Turn on the last consecutive run of ones in S
return S &( S + 1);
// Turn on the last consecutive run of zeroes in S
return S|| (S -1);
// Turn on all bits
return S = (1 << 62)-1;
// multiply by 2^N
return S<<=N;
// Divide by 2^N
return S>>=N;
// Turn on the N-th bit
return S = S|| (1<<N);
// Check if N-th bit is on
return (S & (1 << N));
// Turn off the N-th bit
return S &= ~(1 << N);
// Alternate status of N-th bit
return S ^= (1 << N);
//Value of the least significant bit on from the right
return N = (S&(-S));
}
//count numbers with i bit set [1, n-1]
ll kol(ll n, ll i)
{
    return (n / (1ll << (i + 1))) * (1ll << i) + max((n % (1ll <<
        (i + 1))) - (1ll << i), 0ll);
}
kol(r+1, i) - kol(l, i);

```

5.9 GaussianElimination

```

#include "../Header.cpp"

const double EPS = 1e-9;
const int INF = 2; // it doesn't actually have to be infinity or
// a big number

int gauss (vector < vector<double> > a, vector<double> & ans) {
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;

    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
        int sel = row;
        for (int i=row; i<n; ++i)
            if (abs (a[i][col]) > abs (a[sel][col]))
                sel = i;
        if (abs (a[sel][col]) < EPS)
            continue;
        for (int i=col; i<=m; ++i)
            swap (a[sel][i], a[row][i]);
        where[col] = row;

        for (int i=0; i<n; ++i)
            if (i != row) {
                double c = a[i][col] / a[row][col];
                for (int j=col; j<=m; ++j)
                    a[i][j] -= a[row][j] * c;
            }
        ++row;
    }

    ans.assign (m, 0);
    for (int i=0; i<m; ++i)
        if (where[i] != -1)
            ans[i] = a[where[i]][m] / a[where[i]][i];
    for (int i=0; i<n; ++i) {
        double sum = 0;
        for (int j=0; j<m; ++j)
            sum += ans[j] * a[i][j];
    }
}

```

```

        if (abs (sum - a[i][m]) > EPS)
            return 0;
    }

    for (int i=0; i<m; ++i)
        if (where[i] == -1)
            return INF;
    return 1;
}

int main(){
    ios_base::sync_with_stdio(0);
    cin.tie(0);

    ll n = 2;

    vector<vl>g (2, vl(3, 0));

    g[0][0] = 1;
    g[1][1] = 1;
    g[0][2] = 1;
    g[1][2] = 2;

    // g: rows: equations, columns: x_1 * p_1 + x_2 * p_2 + x_3 *
    //      p_3 = y
    for(int i = 0; i < n-2; i++)
    {
        for(int z = i+1; z < n-1; z++)
        {
            db mul = g[z][i] / g[i][i];
            for(int j = 0; j < n; j++)
                g[z][j] -= mul * g[i][j];
        }
    }

    vector<db> vals(n, 0);

    for(int i = n-2; i >= 0; i--)
    {

```

```

        db sum = g[i][n-1];
        for(int j = i+1; j < n-1; j++)

            sum -= g[i][j] * vals[j];

        sum /= g[i][i];
        vals[i] = sum;
    }

    for(int i = 0; i < n-1; i++)
        cout << vals[i] << " ";

    cout << endl;

    for(int i = 0; i < n; i++)
    {

        for(int j = 0; j < n+1; j++)
            cout << g[i][j] << " ";
        cout << endl;
    }
}

```

5.10 MathFuncions

```

#include "../Header.cpp"

// pre overflow
ll mul(ll x, ll y) { if (x > MX / y) return MX; return x * y; }
ll sums(ll x, ll y) { if (MX - x < y) return MX; return x + y; }

const int N = 1e5 + 10, LOG_A = 31;
ll basis[LOG_A], sz;

// O(N * LOG), base that produces the maximum
void insertVector(int mask) {
    for (ll i = LOG_A - 1; i >= 0; i--) {
        if ((mask & 1 << i) == 0) continue;

```

```

        if (!basis[i]) {
            basis[i] = mask;
            sz++;
            return;
        }

        mask ^= basis[i];
    }

}

// inclusion, exclusion
ll ans = 0;
forr(bitmask, 1, (1<<n)){
    // bitmask srepresenta la interseccion actual
    bool resta = __builtin_popcount(bitmask)%2;
    ans = (ans + (resta ? 1 : M-1)*cuenta(bitmask) %M) %M;
}

// Catalan number

/*
    Number of ways to place    pairs of parentheses correctly.
    Number of binary trees with    nodes.
    Number of full binary trees with    +    leaves.
    Number of ways to triangulate a convex    +    sided polygon.
*/

ll CAT[MAXN];
// ...
CAT[0] = CAT[1] = 1;
repx(i, 2, MAXN){
    CAT[i] = 0;
    rep(k, i)
        CAT[i]=(CAT[i]+CAT[k]*CAT[i-1-k]%M)%M;
}

ll F[MAXN], INV[MAXN], FI[MAXN];
ll Cat(int n){
    return F[2*n] *FI[n+1]%M *FI[n]%M;
}

// Stirling numbers

```

```

// number of ways to partition a set of n elements into k
// nonempty subsets

ll Stirling[MAXN][MAXN];
// ...
repx(i, 1, MAXN)Stirling[i][1] = 1;
repx(i, 2, MAXN)Stirling[1][i] = 0;
repx(i, 2, MAXN)forr(j, 2, MAXN){
    Stirling[i][j] =
        (Stirling[i-1][j-1] + j*Stirling[i-1][j]%MOD) %MOD;
}

// Bell numbers

// Number of partitions of set of n elements

// a deck of n cards is shuffled by repeatedly removing the top
// card and reinserting it anywhere in the deck (including its
// original position at the top of the deck), with exactly n
// repetitions
// stays the same B_n ways
// Probability B_n / n^n

// nth Bell number equals the number of permutations on n items
// in which no three values that are in sorted order have the
// last two of these three consecutive
ll Stirling[MAXN][MAXN], Bell[MAXN];
// ...
forr(i, MAXN){
    Bell[i] = 0;
    forr(j, MAXN)
        Bell[i] = (Bell[i] + Stirling[i][j]) %MOD;
}

//grundy
int tag[n*n];
int mex(int id)
{
    int ans = 0;
    while(tag[ans] == id) ++ans;
}

```

```

    return ans;
}

ll cn = 0;
for(int i = 0; i < n; i++)
{
    for(int j = 0; j < n; j++)
    {
        ll id = ++cn;
        //abajo
        for(int k = i - 1; k >= 0; k--)
            tag[grundy[k][j]] = id;

        //izquierda
        for(int k = j - 1; k >= 0; k--)
            tag[grundy[i][k]] = id;

        // diagonal
        for(int k = 1; k <= min(i, j); k++)
            tag[grundy[i-k][j-k]] = id;
        grundy[i][j] = mex(id);
    }
}

// fibonacci numbers
f_i = 1 / sqrt(5 * ((1 + sqrt(5)) / 2) ^ n - ((1 - sqrt(5)) / 2)
    ^ n);

// catalan numbers
a_n = 1 / (n+1) * comb(2n, n);

```

5.11 Matrices

```

#include "../Header.cpp"

/*
matrix A: transitions Axb
vector b(rows, 1): base case of dp
Represents last |b| states of dp

```

```

F_n
...
F2
F1
*/

// a^p = a*p mod P
// if a % p == 0 return 0

// to calculate p, can use p mod (P-1)
struct Mat {
    vector<vl> vec;
    Mat(): vec(1, vl(1, 0)) {}
    Mat(int n): vec(n, vl(n) ) {}
    Mat(int n, int m): vec(n, vl(m, 0) ) {}
    vl &operator[](int f){ return vec[f]; }
    const vl &operator[](int f) const { return vec[f]; }
    int size() const { return vec.size(); }
};

Mat operator *(Mat A, Mat B) {
    int n = A.size(), m = A[0].size(), t = B[0].size();
    Mat ans(n, t);
    rep(i, n) rep(j, t) rep(k, m)
        ans[i][j] = (ans[i][j] + A[i][k] * B[k][j] % MOD) % MOD;
    return ans;
}

Mat expmat(Mat A, ll e){
    int n = A.size();
    Mat Ans(n); rep(i, n) Ans[i][i] = 1;
    while(e){
        if(e&1) Ans = Ans*A;
        A = A*A; e >>= 1;
    }
    return Ans;
}

ll Fibo(ll n) {
    Mat V0(1, 2), T(2);
    V0[0] = {1, 1};

```



```

T[0] = {0, 1}; T[1] = {1, 1};
Mat V = V0*expmat(T, n);
return V[0][0];
}

```

5.12 Mobius

```

short mu[MAXN] = {0,1};
void mobius(){
    fore(i,1,MAXN)if(mu[i])for(int
        j=i+i;j<MAXN;j+=i)mu[j]-=mu[i];
}

ll fun(ll N) {
    ll ans=0;
    for(ll i=1;i*i<=N;i++) ans+=mu[i]*(N/(i*i));
    return ans;
}

```

5.13 Modular

```

#include "../Header.cpp"

// Modular inverse of an array
//  $x_{\{i\}}^{-1} = \text{prefix}_{\{i-1\}} * \text{suffix}_{\{i+1\}} * (x_1 * x_2 * \dots * x_n)^{-1}$ 
#include "Euclid.cpp"
vl invs(vl &a, int m) {
    int n = a.size();
    if (n == 0) return {};
    vl b(n);
    ll v = 1;
    for (int i = 0; i != n; ++i) {
        b[i] = v;
        v = (v * a[i]) % m;
    }
    pll p = euclid(v, m);
    ll x = p.ff, y = p.ss;
}

```

```

x = (x % m + m) % m;
for (int i = n - 1; i >= 0; --i) {
    b[i] = (x * b[i]) % m;
    x = (x * a[i]) % m;
}
return b;
}

// LINEAR CONGRUENCE EQUATION
// find x such that  $a * x \% m = b$ 
ll linear_congruence(ll a, ll b, ll m) {
    a %= m, b %= m;
    int g = gcd(a, m);
    if(g % b) return -1;
    return (minv(a, m) * b) % m;
    // more solutions:  $x_i = (x + i * m) \% m$  for all i in [0, g - 1]
}

// CRT for coprime modules
// Use Garner for non coprimes modules
#include "Modular.cpp"
struct Congruence { ll a, m; };

ll chinese_remainder_theorem(vector<Congruence> const&
    congruences) {
    ll M = 1, solution = 0;
    for (auto const& congruence : congruences) M *= congruence.m;
    for (auto const& congruence : congruences) {
        ll a_i = congruence.a;
        ll M_i = M / congruence.m;
        ll N_i = inv(M_i, congruence.m);
        solution = (solution + a_i * M_i % M * N_i) % M;
    }
    return solution;
}

// Benja CRT
#include "Euclid.cpp"
pll CRT(pll a, pll b)
{
}

```

```

    if (a.ss < b.ss) swap(a, b);
    ll x, y; tie(x, y) = euclid(a.ss, b.ss);
    ll g = a.ss * x + b.ss * y, l = a.ss / g * b.ss;
    if ((b.ff - a.ff) % g) return {-1, -1}; // no solution
    x = (b.ff - a.ff) % b.ss * x % b.ss / g * a.ss + a.ff;
    return {x + (x < 0) * l, l};
}

pll CRT(vector<pll> &v)
{
    int N = v.size(); pll ans = v[0];
    rep(i, N) if (i) ans = CRT(ans, v[i]);
}

// DISCRETE LOGARITHM
// Returns minimum x for which  $a^x \% m = b \% m$  in  $O(\sqrt{m})$ 
// Includes case when a and m are not coprime dividing by the gcd
int solve(int a, int b, int m) {
    a %= m, b %= m;
    int k = 1, add = 0, g;
    while ((g = __gcd(a, m)) > 1) {
        if (b == k)
            return add;
        if (b % g)
            return -1;
        b /= g, m /= g, ++add;
        k = (k * 1ll * a / g) % m;
    }

    int n = sqrt(m) + 1;
    int an = 1;
    rep(i, n) an = (an * 1ll * a) % m;

    unordered_map<int, int> vals;
    for (int q = 0, cur = b; q <= n; ++q) {
        vals[cur] = q;
        cur = (cur * 1ll * a) % m;
    }

    for (int p = 1, cur = k; p <= n; ++p) {

```

```

        cur = (cur * 1ll * an) % m;
        if (vals.count(cur)) {
            int ans = n * p - vals[cur] + add;
            return ans;
        }
    }
    return -1;
}

```

5.14 PrimeFactorization

```

#include "../Header.cpp"

// stores smallest prime factor for every number
int spf[MAXN];

// Calculating SPF (Smallest Prime Factor) for every
// number till MAXN.
// Time Complexity :  $O(n \log \log n)$ 
void sieve()
{
    spf[1] = 1;
    for (int i=2; i<MAXN; i++)

        // marking smallest prime factor for every
        // number to be itself.
        spf[i] = i;

    // separately marking spf for every even
    // number as 2
    for (int i=4; i<MAXN; i+=2)
        spf[i] = 2;

    for (int i=3; i*i<MAXN; i++)
    {
        // checking if i is prime
        if (spf[i] == i)
        {
            // marking SPF for all numbers divisible by i
            for (int j=i*i; j<MAXN; j+=i)

```

```

        // marking spf[j] if it is not
        // previously marked
        if (spf[j]==j)
            spf[j] = i;
    }
}

// A O(log n) function returning primefactorization
// by dividing by smallest prime factor at every step
vector<int> getFactorization(int x)
{
    vector<int> ret;
    while (x != 1)
    {
        ret.push_back(spf[x]);
        x = x / spf[x];
    }
    return ret;
}

void primeFactors(ll n) {
    while (n % 2 == 0) {
        cout << 2 << " ";
        n = n/2;
    }

    for (int i = 3; i <= sqrt(n); i = i + 2) {
        while (n % i == 0) {
            cout << i << " ";
            n = n/i;
        }
    }
    if (n > 2)
        cout << n << " ";
}

// FACTORIZATION
// 33.3% of factors O(sqrt(n) / 3)

```

```

vl trial_division3(ll n) {
    vl factorization;
    for (int d : {2, 3, 5}) {
        while (n % d == 0) {
            factorization.pb(d);
            n /= d;
        }
    }
    int increments[] = {4, 2, 4, 2, 4, 6, 2, 6};
    int i = 0;
    for (ll d = 7; d * d <= n; d += increments[i++]) {
        while (n % d == 0) {
            factorization.pb(d);
            n /= d;
        }
        if (i == 8) i = 0;
    }
    if (n > 1) factorization.pb(n);
    return factorization;
}

// Pollard p - 1
// Probabilistic to find divisors
// O(B log(B) log^2(n))
ll pollards_p_minus_1(ll n) {
    int B = 10;
    ll g = 1;
    while (B <= 1000000 && g < n) {
        ll a = 2 + rand() % (n - 3);
        g = gcd(a, n);
        if (g > 1)
            return g;

        // compute a^M
        for (int p : primes) {
            if (p >= B) continue;
            ll p_power = 1;
            while (p_power * p <= B) p_power *= p;
            a = power(a, p_power, n);
            g = gcd(a - 1, n);
            if (g > 1 && g < n) return g;
        }
    }
}

```

```

    }
    B *= 2;
}
return 1;
}
// PRIMALITY DETERMINISTIC UNTIL 10^18
// 2, 3, 5, 13, 19, 73, 193, 407521, 299210837

// POLLARD RHO
// Pollard Rho
ll mult(ll a, ll b, ll mod) { return (__int128)a * b % mod; }

// For a, b <= 10 ^ 18
ll mult2(ll a, ll b, ll mod) {
    ll result = 0;
    while (b) {
        if (b & 1) result = (result + a) % mod;
        a = (a + a) % mod;
        b >>= 1;
    }
    return result;
}

ll f(ll x, ll c, ll mod) {
    return (mult(x, x, mod) + c) % mod;
}

ll rho(ll n, ll x0=2, ll c=1) {
    ll x = x0, y = x0, g = 1;
    while (g == 1) {
        x = f(x, c, n);
        y = f(y, c, n);
        y = f(y, c, n);
        g = gcd(abs(x - y), n);
    }
    return g;
}

// Brent Factorization

```

```

ll brent(ll n, ll x0=2, ll c=1) {
    ll x = x0, g = 1, q = 1, xs, y;
    int m = 128;
    int l = 1;
    while (g == 1) {
        y = x;
        repx(i, 1, l) x = f(x, c, n);
        int k = 0;
        while (k < l && g == 1) {
            xs = x;
            for (int i = 0; i < m && i < l - k; i++) {
                x = f(x, c, n);
                q = mult(q, abs(y - x), n);
            }
            g = gcd(q, n);
            k += m;
        }
        l *= 2;
    }
    if (g == n) {
        do {
            xs = f(xs, c, n);
            g = gcd(abs(xs - y), n);
        } while (g == 1);
    }
    return g;
}

// Eulers Totient Function (PHI function)

// phi(n) := number of integers in [1, n] coprime to n
// phi(p) = p - 1,
// phi(p^k) = p^k - p^{k-1}
// phi(a * b) = phi(a) * phi(b) * g / phi(g) with g = gcd(a, b)
// for n 0(sqrt(n)) using factorization

// Eulers theorem: a and m relatively prime -> a^phi(m) % m = 1
// m is prime -> Fermat little theorem: a^{m-1} % m = 1
// a^n % m = a^{n % phi(m)}
// Generalization: x and m not coprime and n >= log2(n) -> x^n %
    m = x^{phi(m) + [n % phi(m)]}

```

```

int phi(int n) {
    int result = n;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0)
                n /= i;
            result -= result / i;
        }
    }
    if (n > 1) result -= result / n;
    return result;
}

// Phi function for [1, n] in O(n log(log(n)))
void phi_1_to_n(int n) {
    vector<int> phi(n + 1);
    rep(i, n + 1) phi[i] = i;
    repx(i, 2, n + 1) {
        if (phi[i] == i) {
            for (int j = i; j <= n; j += i)
                phi[j] -= phi[j] / i;
        }
    }
}

// Divisor sum property:  $\sum_{d|n} \phi(d) = n$ 
// phi [1, n] with the divisor sum property in O(n log(n))
void phi_1_to_n(int n) {
    vector<int> phi(n + 1);
    for (int i = 0; i <= n; i++)
        phi[i] = i;

    for (int i = 2; i <= n; i++) {
        if (phi[i] == i) {
            for (int j = i; j <= n; j += i)
                phi[j] -= phi[j] / i;
        }
    }
}

```

```

// NUMBER OF DIVISORS AND SUM OF DIVISORS
//  $n = p_1^{e_1} * p_2^{e_2} * \dots * p_k^{e_k}$ 
// d(n) := number of divisors
//  $d(n) = (e_1 + 1) * (e_2 + 1) * \dots * (e_k + 1)$ 
// sigma(n) := sum of divisors
//  $1 + p_1 + p_1^2 + \dots + p_1^{e_1} = (p_1^{e_1+1} - 1) / (p_1 - 1)$ 
//  $\sigma(n) = (p_1^{e_1+1} - 1) / (p_1 - 1) * (p_2^{e_2+1} - 1) / (p_2 - 1) * \dots * (p_k^{e_k+1} - 1) / (p_k - 1)$ 
// Multiplicative functions:  $f(a * b) = f(a) * f(b)$  if a and b
// are coprimes

```

5.15 Simplex

```

#include "../Header.cpp"

#define fore(i,a,b) for(int i=a,ThxDem=b;i<ThxDem;++i)

namespace Simplex {
    vector<int> X,Y;
    vector<vector<db>> > A;
    vector<db> b,c;
    db z;
    int n,m;
    void pivot(int x,int y){
        swap(X[Y],Y[X]);
        b[X]/=A[X][Y];
        fore(i,0,m)if(i!=y)A[X][i]/=A[X][Y];
        A[X][Y]=1/A[X][Y];
        fore(i,0,n)if(i!=x&&abs(A[i][Y])>EPS){
            b[i]-=A[i][Y]*b[X];
            fore(j,0,m)if(j!=y)A[i][j]-=A[i][Y]*A[X][j];
            A[i][Y]=-A[i][Y]*A[X][Y];
        }
        z+=c[Y]*b[X];
        fore(i,0,m)if(i!=y)c[i]-=c[Y]*A[X][i];
        c[Y]=-c[Y]*A[X][Y];
    }
}

pair<db,vector<db>> simplex( // maximize  $c^T x$  s.t.  $Ax \leq b, x \geq 0$ 

```

```

        vector<vector<db> > _A, vector<db> _b, vector<db>
            _c){
// returns pair (maximum value, solution vector)
A=_A;b=_b;c=_c;
n=b.size();m=c.size();z=0.;
X=vector<int>(m);Y=vector<int>(n);
fore(i,0,m)X[i]=i;
fore(i,0,n)Y[i]=i+m;
while(1){
    int x=-1,y=-1;
    db mn=-EPS;
    fore(i,0,n)if(b[i]<mn)mn=b[i],x=i;
    if(x<0)break;
    fore(i,0,m)if(A[x][i]<-EPS){y=i;break;}
    if(y<0) return(make_pair(-1, b));
    assert(y>=0); // no solution to Ax<=b
    pivot(x,y);
}
while(1){
    db mx=EPS;
    int x=-1,y=-1;
    fore(i,0,m)if(c[i]>mx)mx=c[i],y=i;
    if(y<0)break;
    db mn=INF;
    fore(i,0,n)if(A[i][y]>EPS&& b[i]/A[i][y]<mn)mn=b[i]/A[i][y],x=i;
    assert(x>=0); // c^T x is unbounded
    pivot(x,y);
}
vector<db> r(m);
fore(i,0,n)if(Y[i]<m)r[Y[i]]=b[i];
return {z,r};
}
}

int main(){
    ios_base::sync_with_stdio(0);
    cin.tie(0);

    ll n, k, x;
    db y;
    cin >> n >> k >> x;

```

```

vector<db> b, c;
vector<vector<db> > A;

for(int i = 0; i < n; i++)
{
    cin >> y;
    c.push_back(y);
}

vector<db>aux(n, 0);
for(int i = 0; i < k; i++)aux[i] = -1;
A.push_back(aux);
b.push_back(-1.);

for(int i = k; i < n; i++)
{
    aux[i - k] = 0;
    aux[i] = -1;
    A.push_back(aux);
    b.push_back(-1);
}
aux.assign(n, 0);
for(int i = 0; i < n; i++){
    aux[i] = 1;
    A.push_back(aux);
    b.push_back(1);
    aux[i] = 0;
}
aux.assign(n, 1);
A.push_back(aux);
b.push_back(x);
ll in = 0;
/*for(auto it : A)
{
    for(auto it2: it)cout<<it2<<" ";
    cout<<" "<<b[in];
    in++;
    cout<<"*\n";
}*/

```

```

db mx = Simplex::simplex(A,b,c).first;
cout<<(long long)mx<<"\n";

cerr << "\nTime elapsed: " << 1000 * clock() / CLOCKS_PER_SEC
    << "ms\n";
return 0;
}

```

5.16 Simpson

```

const int N = 1000 * 1000; // number of steps (already multiplied
    by 2)

double simpson_integration(double a, double b){
    double h = (b - a) / N;
    double s = f(a) + f(b); // a = x_0 and b = x_2n
    for (int i = 1; i <= N - 1; ++i) { // Refer to final Simpson's
        formula
        double x = a + h * i;
        s += f(x) * ((i & 1) ? 4 : 2);
    }
    s *= h / 3;
    return s;
}

```

6 Misc

6.1 AdHoc

```

#include "../Header.cpp"

int joseph(int n,int m){
    int Result=0;
    for(int i=1;i<=n;i++){
        Result=(Result+m-1)%i+1;
    }
}

```

```

    return(Result);
}

int joseph(int n,int m){
    vl a(n+1, 0);
    //see eliminated
    bool o = 1;
    for(int i = 0; i < n/2; i++){
        a[i+1] = (a[i] + m-1)%(n-i);
        if(a[i+1] < n/2){
            o = 0;
            break;
        }
    }
}

// if k = 2
// move first significant bit to right
int joseph(ll n){
    ll bit = 62;
    while(!(n & (1 << bit))){
        bit--;
    }
    n &= ~(1 << bit);
    return 1 + (n << 1);
}

// matching in DAG gives min number of paths to cover all nodes
// Dilword
// matching in transitive DAG gives max independent set

// primes in a n size range n / log(n)

// nim game
// a_1 ^ a_2 ^ ... ^ a_n = 0: player 1 lose

// nim variation: remove stones from [0, k] piles
// a_1 ^_{k+1} a_2 ^_{k+1} ... ^_{k+1} a_n = 0: player 1 lose
// ^_{k+1} = xor mod (k+1) k+1 bits = 0 mod (k+1)
// sum of pairs
// a*b + b*c + c*a
// (a + b + c + d)^2 - (a^2 + b^2 + c^2)

```

```
// valor{x} = (x, 0)
//combinar((s_1, p_1), (s_2, p_2)) = ((s_1 + s_2),(p_1 + p_2 +
    s_1 * s_2))

// sum of subconj
// 1 + a + b + c + a*b + b*c + c*a + a*b*c
//(1 + a)*(1 + b)*(1 + c)

// valor{x} = 1 + x
//combinar(a, b) = a*b

// x >= y -> x mod y < x/2 counting decimals

/*Para un arbol de tamao N, solo hay un arbol para cada divisor(N)
de tamao divisor(N) que lo puede armar solo consigo mismo

Para hashear un arbol se usan parentesis, el hash es distinto
para cada root,
hay que ordenar los hijos antes de hashear

everyone loses their hats all at once, and each person puts on a
random hat;
in expectation, how many people get their own hats back?
The probability that the each person gets their own hat is 1/N,
and then by linearity of expectation,
the total number of instances of someone getting their own hat is
1/N*N=1.

expeted value to two people will get their original hat : 1/2
for 3: 1/3

*/

// Modular sum optimization
if (R >= MOD) R -= MOD;

/*
euler cycle
```

```
all vertex with even degree
```

```
hamiltonian cycle
d(v) >= n/2 vertex degree
```

```
exact partition  $O(3^{(m/3)})$   $O(2^{(m/2)})$ 
m(4) sets and n(3) objects
101 -
010 -
110
011
```

```
for each i in n:
    choose a row with bit i on
    erase all rows with bit i on
    continue
*/
```

```
// convex hull of max max(X, Y)(2/3) points in recatangle (0, 0)
(X, Y)
```

6.2 Line input

```
#include "Header.cpp"

int main()
{
    // save strings separated by space in a line
    string line, token;
    getline(cin, line);
    stringstream ss(line);
    while(ss >> token){
        cout << token << "\n";
    }
    return 0;
}
```

6.3 NextGreaterLower

```
#include "../Header.cpp"
int main(){

    ios_base::sync_with_stdio(0);
    cin.tie(0);

    ll n;
    cin >> n;

    vl c(n);
    // next value with lower/grater value
    // right greater, left greater, right lower, left lower
    vl Rg(n, n), Lg(n, -1), Rl(n, n), Ll(n, -1);

    rep(i, n){
        cin >> c[i];
    }

    stack<ll> Sg, Sl;
    rep(i, n){
        while(!Sg.empty() && c[Sg.top()] < c[i]){
            Rg[Sg.top()] = i;
            Sg.pop();
        }
        Sg.push(i);

        while(!Sl.empty() && c[Sl.top()] > c[i]){
            Rl[Sl.top()] = i;
            Sl.pop();
        }
        Sl.push(i);
    }
    while(!Sg.empty()) Sg.pop();
    while(!Sl.empty()) Sl.pop();

    for(int i = n-1; i >= 0; i--){
        while(!Sg.empty() && c[Sg.top()] <= c[i]){
            Lg[Sg.top()] = i;
            Sg.pop();
        }
    }
}
```

```
Sg.push(i);

while(!Sl.empty() && c[Sl.top()] > c[i]){
    Ll[Sl.top()] = i;
    Sl.pop();
}
Sl.push(i);
}
cerr << "\nTime elapsed: " << 1000 * clock() / CLOCKS_PER_SEC <<
"ms\n";
return 0;
}
```

7 Pdf antiguos

8 Strings

8.1 AhoCorasick

```
#include "../Header.cpp"
struct AC
{
    ll c = 0, ec = 0, M, A, af = -1;
    vector<vl> N, G; vl L, E;
    vl val;
    // L -> suffix link G -> anti L
    // E -> string finish
    AC (ll M, ll A) : M(M), A(A), N(M, vl(A, 0)), G(M, vl()), E(M,
        0), L(M, 0), val(M, 0) {}
    ll add(string s){ // return endpoint
        af++;
        ll p = 0;
        for (char l : s){
            int t = l - 'a';
            if (!N[p][t]) N[p][t] = ++c;
            p = N[p][t];
        }
        val[p] = 1;
        return p;
    }
}
```

```

}
void init(){
    queue<int> q; q.push(0); L[0] = -1;
    while (!q.empty()){
        int p = q.front(); q.pop();
        for(int c = 0; c < A; c++){
            int u = N[p][c]; if (!u) continue;
            L[u] = L[p] == -1 ? 0 : N[L[p]][c], q.push(u);
            G[L[u]].push_back(u);
        }
        if (p) for(int c = 0; c < A; c++) if (!N[p][c]) N[p][c]
            = N[L[p]][c];
    }
}
};

```

8.2 Hashing

```

#include "../Header.cpp"
struct RH
{
    // choose base B random to avoid hacks 33 37 41
    // randomize V(s[i])
    int B = 1777771, M[2] = {999727999, 1070777777}, P[2] =
        {325255434, 10018302};
    vl H[2], I[2];
    RH(string &s){
        int N = s.size(); rep(k, 2){
            H[k].resize(N + 1), I[k].resize(N + 1);
            H[k][0] = 0, I[k][0] = 1; ll b = 1;
            rep(i, N + 1) if (i){
                H[k][i] = (H[k][i - 1] + b * s[i - 1]) % M[k];
                I[k][i] = (1LL * I[k][i - 1] * P[k]) % M[k];
                b = (b * B) % M[k];
            }
        }
    }
    ll get(int l, int r){ // inclusive - exclusive
        ll h0 = (H[0][r] - H[0][l] + M[0]) % M[0];
        h0 = (1LL * h0 * I[0][l]) % M[0];
    }
};

```

```

        ll h1 = (H[1][r] - H[1][l] + M[1]) % M[1];
        h1 = (1LL * h1 * I[1][l]) % M[1];
        return (h0 << 32) | h1;
    }
};
bool compare(int a, int b){
    ll l = 0, r = n-1, p, res = -1;
    while(l <= r){
        p = (l + r) / 2;
        if(rhs[a].get(0, p) == rhs[b].get(0, p)) l = p+1;
        else {
            res = p;
            r = p-1;
        }
    }
    if(res == -1) return a < b;
    return s[a][res] < s[b][res];
}

```

```

//Suffix Array O(N log^2 N)
rep(n) sa[i] = i;
sort(ALL(sa), compare)

```

8.3 Hashing2D

```

struct Hashing2D {
    vector<vector<int>> hs;
    vector<int> PWX, PWY;
    int n, m;
    static const int PX = 3731, PY = 2999, mod = 998244353;
    Hashing() {}
    Hashing(vector<string>& s) {
        n = (int)s.size(), m = (int)s[0].size();
        hs.assign(n + 1, vector<int>(m + 1, 0));
        PWX.assign(n + 1, 1);
        PWY.assign(m + 1, 1);
        for (int i = 0; i < n; i++) PWX[i + 1] = 1LL * PWX[i] * PX
            % mod;
        for (int i = 0; i < m; i++) PWY[i + 1] = 1LL * PWY[i] * PY
            % mod;
    }
};

```

```

for (int i = 0; i < n; i++) {
    for (int j = 0; j < m; j++) {
        hs[i + 1][j + 1] = s[i][j] - 'a' + 1;
    }
}
for (int i = 0; i <= n; i++) {
    for (int j = 0; j < m; j++) {
        hs[i][j + 1] = (hs[i][j + 1] + 1LL * hs[i][j] * PY
            % mod) % mod;
    }
}
for (int i = 0; i < n; i++) {
    for (int j = 0; j <= m; j++) {
        hs[i + 1][j] = (hs[i + 1][j] + 1LL * hs[i][j] * PX
            % mod) % mod;
    }
}
}
int get_hash(int x1, int y1, int x2, int y2) { // 1-indexed
    assert(1 <= x1 && x1 <= x2 && x2 <= n);
    assert(1 <= y1 && y1 <= y2 && y2 <= m);
    x1--;
    y1--;
    int dx = x2 - x1, dy = y2 - y1;
    return (1LL * (hs[x2][y2] - 1LL * hs[x2][y1] * PWY[dy] %
        mod + mod) % mod -
        1LL * (hs[x1][y2] - 1LL * hs[x1][y1] * PWY[dy] % mod +
        mod) % mod * PWX[dx] % mod + mod) % mod;
}
int get_hash() {
    return get_hash(1, 1, n, m);
}
};

```

8.4 KMP

```
#include "../Header.cpp"
```

```
// Build longest proper prefix/suffix array (lps) for pattern
```

```

// lps[i] = length of the longest proper prefix which is also
// suffix in pattern[0 .. i]
void init_lps(string& s, int lps[]) {
    int n = s.size();
    lps[0] = 0; // base case: no proper prefix/suffix for
    pattern[0 .. 0] (length 1)
    rep(j, 1, n) { // for each s[0 .. j]
        int i = lps[j-1]; // i points to the char next to lps of
        previous iteration
        while (s[i] != s[j] and i > 0) i = lps[i-1];
        lps[j] = s[i] == s[j] ? i+1 : 0;

        //optimization to reutilice the lps in O(n)
        if(i > 0 && s[i] == s[lps[i-1]] && lps[i-1] != 0) lps[i-1]
            = lps[lps[i-1]-1];
    }
}

// Count number of matches of pattern string in target string
// using KMP algorithm
int kmp(string& s, string& t) {
    int n = s.size(), m = t.size();
    int lps[n];
    init_lps(s, lps); // build lps array
    int matches = 0;
    int i = 0; // i tracks current char in pattern to compare
    rep(j, m) { // j tracks each char in target to compare
        // try to keep prefix before i as long as possible while
        ensuring i matches j
        while (s[i] != t[j] && i > 0) i = lps[i-1];
        if (s[i] == t[j]) {
            if (++i == n) { // we matched the whole pattern
                i = lps[n-1]; // shift the pattern so that the
                longest proper prefix/suffix pair is aligned
                matches++;
            }
        }
    }
    return matches;
}

```

```
int main() { // usage
    string target, pattern;
    while (true) {
        cin >> target >> pattern;
        cout << kmp(pattern, target) << " matches\n";
    }
    return 0;
}
```

8.5 KMPMarceloL

```
vector<int> kmppre(string& t){ // r[i]: longest border of t[0,i]
    vector<int> r(t.size()+1);r[0]=-1;
    int j=-1;
    fore(i,0,t.size()){
        while(j>=0&&t[i]!=t[j])j=r[j];
        r[i+1]=++j;
    }
    return r;
}

void kmp(string& s, string& t){ // find t in s
    int j=0;vector<int> b=kmppre(t);
    fore(i,0,s.size()){
        while(j>=0&&s[i]!=t[j])j=b[j];
        if(++j==t.size())printf("Match at
            %d\n",i-j+1),j=b[j];
    }
}
```

8.6 LongestCommonSubstring

```
#include "../Header.cpp"

int LCSuff(string& X, string& Y, int m, int n){
    // Create a table to store lengths of longest common suffixes
    // of
    // substrings. Notethat LCSuff[i][j] contains length of
    // longest
```

```
// common suffix of X[0..i-1] and Y[0..j-1]. The first row and
// first column entries have no logical meaning, they are used
// only
// for simplicity of program
int LCSuff[m+1][n+1];
int result = 0; // To store length of the longest common
// substring
/* Following steps build LCSuff[m+1][n+1] in bottom up
// fashion. */
for (int i=0; i<=m; i++){
    for (int j=0; j<=n; j++){
        if (i == 0 || j == 0)
            LCSuff[i][j] = 0;
        else if (X[i-1] == Y[j-1]){
            LCSuff[i][j] = LCSuff[i-1][j-1] + 1;
            result = max(result, LCSuff[i][j]);
        }
        else LCSuff[i][j] = 0;
    }
}

return result;
}

int main(){
    string X = "OldSite:GeeksforGeeks.org", Y =
        "NewSite:GeeksQuiz.com";
    int m = X.size(), n = Y.size();
    cout << "Length of Longest Common Substring is " <<
        LCSuff(X, Y, m, n) << "\n";
    return 0;
}
```

8.7 Manacher

```
int n;
string s;

int main(){
    vi d1(n); // odd sized palindromes
    for (int i = 0, l = 0, r = -1; i < n; i++) {
```

```

    int k = (i > r) ? 1 : min(d1[l + r - i], r - i + 1);
    while (0 <= i - k && i + k < n && s[i - k] == s[i + k])
        k++;
    d1[i] = k--;
    if (i + k > r) l = i - k, r = i + k;
}
vi d2(n); // even sized palindromes (center to the right)
for (int i = 0, l = 0, r = -1; i < n; i++) {
    int k = (i > r) ? 0 : min(d2[l + r - i + 1], r - i + 1);
    while (0 <= i - k - 1 && i + k < n && s[i - k - 1] == s[i
        + k]) k++;
    d2[i] = k--;
    if (i + k > r) l = i - k - 1, r = i + k;
}
}

```

8.8 PalindromicTree

```

#include "../Header.cpp"

#define MAXN 2000005
ll M = 51123987;
struct Node
{
    // store start and end indexes of current
    // Node inclusively
    // only for first occurrence
    ll start, end;

    // stores length of substring
    ll length;

    // stores insertion Node for all characters a-z
    ll insertEdg[26];

    // stores the Maximum Palindromic Suffix Node for
    // the current Node
    ll suffixEdg;

    ll depht;

```

```

};

// two special dummy Nodes as explained above
Node root1, root2;

// stores Node information for constant time access
Node tree[MAXN];

// Keeps track the current Node while insertion
ll currNode;
string s;
ll ptr;

void insert(ll idx)
{
    //STEP 1//

    /* search for Node X such that s[idx] X S[idx]
       is maximum palindrome ending at position idx
       iterate down the suffix link of currNode to
       find X */
    ll tmp = currNode;
    while (true)
    {
        ll curLength = tree[tmp].length;
        if (idx - curLength >= 1 and s[idx] == s[idx-curLength-1])
            break;
        tmp = tree[tmp].suffixEdg;
    }

    /* Now we have found X ....
       * X = string at Node tmp
       * Check : if s[idx] X s[idx] already exists or not*/
    if (tree[tmp].insertEdg[s[idx]-'a'] != 0)
    {
        // s[idx] X s[idx] already exists in the tree
        currNode = tree[tmp].insertEdg[s[idx]-'a'];
        return;
    }

    // creating new Node

```

```

ptr++;

// making new Node as child of X with
// weight as s[idx]
tree[tmp].insertEdg[s[idx]-'a'] = ptr;

// calculating length of new Node
tree[ptr].length = tree[tmp].length + 2;

// updating end point for new Node
tree[ptr].end = idx;

// updating the start for new Node
tree[ptr].start = idx - tree[ptr].length + 1;

//STEP 2//

/* Setting the suffix edge for the newly created
Node tree[ptr]. Finding some String Y such that
s[idx] + Y + s[idx] is longest possible
palindromic suffix for newly created Node.*/

tmp = tree[tmp].suffixEdg;

// making new Node as current Node
currNode = ptr;
if (tree[currNode].length == 1)
{
    // if new palindrome's length is 1
    // making its suffix link to be null string
    tree[currNode].suffixEdg = 2;
    tree[currNode].depht = 1;
    return;
}
while (true)
{
    ll curLength = tree[tmp].length;
    if (idx-curLength >= 1 and s[idx] == s[idx-curLength-1])
        break;

```

```

        tmp = tree[tmp].suffixEdg;
    }

    // Now we have found string Y
    // linking current Nodes suffix link with s[idx]+Y+s[idx]
    tree[currNode].suffixEdg = tree[tmp].insertEdg[s[idx]-'a'];
    tree[currNode].depht =
        tree[tree[tmp].insertEdg[s[idx]-'a']].depht + 1;
}

// para ir al revez currNode = 1, reverse(s)
// para un string nuevo, devolver currNode o currNode = 1

// driver program
int main()
{
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    // initializing the tree
    root1.length = -1;
    root1.suffixEdg = 1;
    root2.length = 0;
    root2.suffixEdg = 1;
    root1.depht = 0;
    root2.depht = 0;

    tree[1] = root1;
    tree[2] = root2;
    ptr = 2;
    currNode = 1;
    ll l;
    cin >> l;
    cin >> s;
    l = s.length();
    cout<<l<<endl;
    vl sums(l+1, 0);
    ll ans = 0;

    for (ll i=0; i<l; i++){
        insert(i);
        ll nod = currNode, depht = 0;
    }
}

```

```

while(tree[nod].length > 0)
{
    nod = tree[nod].suffixEdg;
    depht++;
}
nod = currNode;
sums[i+1]=(depht + sums[i])%M;
depht--;
while(tree[nod].length > 1)
{
    ans += (depht + sums[i] - sums[i - tree[nod].length +
        1])%M;
    ans %= M;

    nod = tree[nod].suffixEdg;
    depht--;
}

}
cout<<ans<<"\n";

// printing all of its distinct palindromic
// substring
cout << "All distinct palindromic substring for "
    << s << " : \n";
for (int i=3; i<=ptr; i++)
{
    cout << i-2 << ") ";
    for (int j=tree[i].start; j<=tree[i].end; j++)
        cout << s[j];
    cout << endl;
}

cerr << "\nTime elapsed: " << 1000 * clock() / CLOCKS_PER_SEC <<
    "ms\n";
return 0;
}

```

8.9 PalindromicTreeMarceloL

```

struct palindromic_tree{
    static const int SIGMA=26;
    struct Node{
        int len, link, to[SIGMA];
        ll cnt;
        Node(int len, int link=0, ll
            cnt=1):len(len),link(link),cnt(cnt){
            memset(to,0,sizeof(to));
        }
    };
    vector<Node> ns;
    int last;
    palindromic_tree():last(0){ns.pb(Node(-1));ns.pb(Node(0));}
    void reset(){ns.clear();last=0;ns.pb(Node(-1));ns.pb(Node(0));}
    void add(int i, string &s){
        int p=last, c=s[i]-'a';
        while(s[i-ns[p].len-1]!=s[i])p=ns[p].link;
        if(ns[p].to[c]){
            last=ns[p].to[c];
            ns[last].cnt++;
        }else{
            int q=ns[p].link;
            while(s[i-ns[q].len-1]!=s[i])q=ns[q].link;
            q=max(1,ns[q].to[c]);
            last=ns[p].to[c]=SZ(ns);
            ns.pb(Node(ns[p].len+2,q,1));
        }
    }
};

```

8.10 SuffixArray

```

// =====
// Suffix Array Construction : Prefix Doubling + Radix Sort
// =====
// Complexity: O(N*log(N))

```

```
// references:
// https://www.cs.helsinki.fi/u/tpkarkka/opetus/10s/spa/lecture11.pdf
// https://youtu.be/_TUEadu-U_k
#include "../Header.cpp"
#define invrep(i,b,a) for(int i = b; i >= a; --i)

struct SA {
    int n; vl counts, rank, rank_, sa, sa_, lcp; // lcp is optional
    inline int gr(int i) { return i < n ? rank[i]: 0; }
    void csort(int maxv, int k) {
        counts.assign(maxv+1, 0);
        repx(i,0,n) counts[gr(i+k)]++;
        repx(i,1,maxv+1) counts[i] += counts[i-1];
        invrep(i,n-1,0) sa_[--counts[gr(sa[i]+k)]] = sa[i];
        sa.swap(sa_);
    }
    void get_sa(vl& s) {
        repx(i,0,n) sa[i] = i;
        sort(sa.begin(), sa.end(), [&s](int i, int j) { return
            s[i] < s[j]; });
        int r = rank[sa[0]] = 1;
        repx(i,1,n) rank[sa[i]] = (s[sa[i]] != s[sa[i-1]]) ? ++r :
            r;
        for (int h=1; h < n and r < n; h <= 1) {
            csort(r, h); csort(r, 0); r = rank_[sa[0]] = 1;
            repx(i,1,n) {
                if (rank[sa[i]] != rank[sa[i-1]] or
                    gr(sa[i]+h) != gr(sa[i-1]+h)) ++r;
                rank_[sa[i]] = r;
            } rank.swap(rank_);
        }
    }
    // LCP construction in O(N) using Kasai's algorithm
    // reference:
    // https://codeforces.com/blog/entry/12796?#comment-175287
    void get_lcp(vl& s) { // lcp is optional
        lcp.assign(n, 0); int k = 0;
        repx(i,0,n) {
            int r = rank[i]-1;
            if (r == n-1) { k = 0; continue; }

```

```
            int j = sa[r+1];
            while (i+k<n and j+k<n and s[i+k] == s[j+k]) k++;
            lcp[r] = k;
            if (k) k--;
        }
    }
    SA(vl& s) {
        n = s.size();
        rank.resize(n); rank_.resize(n);
        sa.resize(n); sa_.resize(n);
        get_sa(s); get_lcp(s); // lcp is optional
    }
};

int main() { // how to use
    string test; cin >> test;
    vl s;
    for (char c : test) s.push_back(c);
    SA sa(s);
    for (int i : sa.sa) cout << i << ":\t" << test.substr(i) <<
        '\n';
    repx(i,0,s.size()) {
        printf("LCP between %d and %d is %d\n", i, i+1, sa.lcp[i]);
    }
}
```

8.11 SuffixAutomaton

```
struct state {int len,link;map<char,int> next;}; //clear next!!
state st[100005];
int sz,last;
void sa_init(){
    last=st[0].len=0;sz=1;
    st[0].link=-1;
}
void sa_extend(char c){
    int k=sz++,p;
    st[k].len=st[last].len+1;
    for(p=last;p!=-1&&!st[p].next.count(c);p=st[p].link)st[p].next[c]
        if(p==-1)st[k].link=0;
```



```

else {
    int q=st[p].next[c];
    if(st[p].len+1==st[q].len)st[k].link=q;
    else {
        int w=sz++;
        st[w].len=st[p].len+1;
        st[w].next=st[q].next;st[w].link=st[q].link;
        for(;p!=-1&&st[p].next[c]==q;p=st[p].link)st[p].next[c]=w;
        st[q].link=st[k].link=w;
    }
}
last=k;
}

```

8.12 Trie

```
#include "../Header.cpp"
```

```

struct Trie{
    static const int MAX = 1e6;
    int N[MAX][26] = {0}, S[MAX] = {0}, c = 0;
    void add(string s, int a = 1){
        int p = 0; S[p] += a;
        for (char l : s){
            int t = l - 'a';
            if (!N[p][t]) N[p][t] = ++c;
            S[p = N[p][t]] += a;
        }
    }
};

```

```

struct TrieXOR{
    static const int MAX = 1e6;
    int N[MAX][2] = {0}, S[MAX] = {0}, c = 0;
    void add(int x, int a = 1){
        int p = 0; S[p] += a;
        rep(i, 31){
            int t = (x >> (30 - i)) & 1;
            if (!N[p][t]) N[p][t] = ++c;
            S[p = N[p][t]] += a;
        }
    }
};

```

```

}
}
int get(int x){
    if (!S[0]) return -1;
    int p = 0; rep(i, 31){
        int t = ((x >> (30 - i)) & 1) ^ 1;
        if (!N[p][t] || !S[N[p][t]]) t ^= 1;
        p = N[p][t]; if (t) x ^= (1 << (30 - i));
    }
    return x;
}
};

```

```

struct Trie {
    vector<vector<int>> g;
    vector<int> count;
    int vocab;
    Trie(int vocab, int maxdepth = 10000) : vocab(vocab) {
        g.reserve(maxdepth);
        g.emplace_back(vocab, -1);
        count.reserve(maxdepth);
        count.push_back(0);
    }
    int move_to(int u, int c) {
        assert (0 <= c and c < vocab);
        int& v = g[u][c];
        if (v == -1) {
            v = g.size();
            g.emplace_back(vocab, -1);
            count.push_back(0);
        }
        count[v]++;
        return v;
    }
    void insert(const string& s, char ref = 'a') { // insert string
        int u = 0; for (char c : s) u = move_to(u, c - ref);
    }
    void insert(vector<int>& s) { // insert vector<int>
        int u = 0; for (int c : s) u = move_to(u, c);
    }
    db query(const string& s, char ref = 'a'){

```

```

int u = 0;
db cost = 0;
for (char c : s){
    ll co = 0;
    for(auto it : g[u]) if(it != -1)co++;

    ll nex = move_to(u, c - ref);
    if(u == 0 || co > 1 || count[u] != count[nex]) cost++;
    u = nex;
}
return cost;
}
ll dfs(int u, int depht){
    ll ans = INF;
    if(count1[u] == 1 && count2[u] == 1)ans = depht;
    for(int i = 0; i < 26; i++){
        if(g[u][i] != -1) ans = min(ans, dfs(g[u][i], depht +
            1));
    }
    return ans;
}
int size() { return g.size(); }
};

int main(){
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    ll n;
    while(cin >> n){
        string s;
        vector<string > c;
        Trie trie(26);
        for(int i = 0; i < n; i++){
            cin >> s;
            c.push_back(s);
            trie.insert(s);
        }
        db sum = 0;
        for(int i = 0; i < n; i++){
            sum += trie.query(c[i]);

```

```

    }
    cout<<fixed<<setprecision(2)<< sum / db(n) << "\n";
}
}

```

9 Structures

9.1 BinarySearch-Ternary

```

#include "../Header.cpp"
// limit
log(valor maximo/precision)/log(2)
                        /log(3/2) //para ternaria

ll l = 0, r = n-1, res = -1;
while(l <= r){
    ll p = (l + r) / 2;
    if(c[p] <= m)l = p+1;
    else r = p-1;
}
db l = 0, r = PI/2, mini = 1e10;
rep(i, 101){
    db d=(r-l)/3.0,m1=l+d,m2=r-d;
    db c1 = value(m1), c2 = value(m2);
    // Para el maximo cambiar r-m2 con l-m1
    if (c1 < c2) r = m2;
    else l = m1;
    mini = min(mini, (c1+c2)/2.0);
}

// non continuous
ll l = 1,r = m, mini = 1e17;
while(l <= r){
    ll d = (r-l)/3;
    ll m1 = l+d, m2 = r-d;
    ll c1 = value(m1), c2 = value(m2);
    // Para el maximo cambiar r-m2 con l-m1
    if (c1 < c2) r = m2-1;
    else l = m1+1;

```

```

    mini = min(mini, min(c1,c2));
}
ans += mini;

```

9.2 CDQDivideConquer

```

#include "../Header.cpp"

struct Pt {
    int x, y, z, i;
};

const int N = (int)1e5 + 5;

struct BIT {
    int b[N], n;

    void init(int _n) {
        n = _n;
        for(int i = 0 ; i <= n ; ++i) b[i] = 0;
    }
    inline int lowbit(int x) { return x & (-x); }
    void update(int x, int v) {
        for(int i = x ; i <= n ; i += lowbit(i)) b[i] += v;
    }
    int query(int x) {
        int ans = 0;
        for(int i = x ; i > 0 ; i -= lowbit(i)) ans += b[i];
        return ans;
    }
} bit;

vector<Pt> v;
int n, ans[N];

void cdq(int l, int r) {
    if(l + 1 == r) return;
    int m = (l + r) >> 1;
    cdq(l, m); cdq(m, r);

```

```

    int a = l, b = m, sum = 0;
    // need to record the modifications on BIT in order to reset it.
    // The complexity will be  $O(N^2)$  if we resetting it
    // brute-forcelly.
    vector<int> record;
    // temporary array for merge sort
    vector<Pt> tmp;
    while(a < m && b < r) {
        if(v[a].y > v[b].y) bit.update(v[a].z, 1), sum++,
            record.push_back(v[a].z), tmp.push_back(v[a++]);
        else ans[v[b].i] += sum - bit.query(v[b].z),
            tmp.push_back(v[b++]);
    }
    while(a < m) tmp.push_back(v[a++]);
    while(b < r) ans[v[b].i] += sum - bit.query(v[b].z),
        tmp.push_back(v[b++]);
    for(int i = 1 ; i < r ; ++i) v[i] = tmp[i - 1];
    // reset BIT
    for(auto i : record) bit.update(i, -1);
    // release used memory
    vector<int> ().swap(record);
    vector<Pt> ().swap(tmp);
}

void init() {
    cin >> n;
    for(int i = 0 ; i < n ; ++i) {
        int a, b, c; cin >> a >> b >> c;
        v.push_back({a, b, c, i});
    }
    bit.init(n);
}

void solve() {
    // As we require > not >=
    sort(v.begin(), v.end(), [&](auto a, auto b) {
        return (a.x == b.x ? (a.y == b.y ? a.z < b.z : a.y < b.y) :
            a.x > b.x);
    });
    cdq(0, n);
    for(int i = 0 ; i < n ; ++i) cout << ans[i] << '\n';
}

```

9.3 DinamicConnectivity

```
#include "../Header.cpp"
#include "UnionFind.cpp"

// M log M (log M from UF)
struct DinC{
    vector<vp> DC;
    ll T; UF uf;
    DinC(ll times, ll n){
        T = times;
        DC.assign(4*T, vp());
        uf = UF(n);
    }
    void qry(){
        qry(1, 0, T-1);
    }
    void qry(ll n, ll l, ll r){
        for(auto it : DC[n]) uf.join(it.first, it.second);
        if(l == r){ // process time l
            for(auto it : ord[l]){
                ll x1 = uf.find(it.first);
                ll x2 = uf.find(it.second);
                ans += uf.sz[x1] * uf.sz[x2];
            }
        }
        else{
            qry(2*n, l, (l+r)/2);
            qry(2*n+1, (l+r)/2+1, r);
        }
        for(auto it : DC[n]) uf.rollback();
    }
    void upd(ll i, ll j, pll v){
        return upd(1, 0, T-1, i, j, v);
    }
    void upd(ll n, ll l, ll r, ll i, ll j, pll v){
        if(r < i || j < l) return;
        if(i <= l && r <= j){
            DC[n].pb(v);
            return;
        }
        upd(2*n, l, (l+r)/2, i, j, v);
        upd(2*n+1, (l+r)/2+1, r, i, j, v);
    }
};
```

9.4 DisjointIntervals

```
// stores disjoint intervals as [first, second)
struct disjoint_intervals {
    set<pair<int,int> > s;
    void insert(pair<int,int> v){
        if(v.fst>=v.snd) return;
        auto at=s.lower_bound(v); auto it=at;
        if(at!=s.begin()&&(--at)->snd>=v.fst)v.fst=at->fst,--it;
        for(;it!=s.end()&&it->fst<=v.snd;s.erase(it++))
            v.snd=max(v.snd,it->snd);
        segs.insert(v);
    }
};
```

9.5 DominatorTree

```
//idom[i]=parent of i in dominator tree with root=rt, or -1 if
//not exists
int
n, rnk[MAXN], pre[MAXN], anc[MAXN], idom[MAXN], semi[MAXN], low[MAXN];
vector<int> g[MAXN], rev[MAXN], dom[MAXN], ord;
void dfspre(int pos){
    rnk[pos]=SZ(ord); ord.pb(pos);
    for(auto x:g[pos]){
        rev[x].pb(pos);
        if(rnk[x]==n) pre[x]=pos, dfspre(x);
    }
}
int eval(int v){
    if(anc[v]<n&&anc[anc[v]]<n){
```

```

        int x=eval(anc[v]);
        if(rnk[semi[low[v]]]>rnk[semi[x]]) low[v]=x;
        anc[v]=anc[anc[v]];
    }
    return low[v];
}
void dominators(int rt){
    fore(i,0,n){
        dom[i].clear(); rev[i].clear();
        rnk[i]=pre[i]=anc[i]=idom[i]=n;
        semi[i]=low[i]=i;
    }
    ord.clear(); dfspre(rt);
    for(int i=SZ(ord)-1;i;i--){
        int w=ord[i];
        for(int v:rev[w]){
            int u=eval(v);
            if(rnk[semi[w]]>rnk[semi[u]])semi[w]=semi[u];
        }
        dom[semi[w]].pb(w); anc[w]=pre[w];
        for(int v:dom[pre[w]]){
            int u=eval(v);
            idom[v]=(rnk[pre[w]]>rnk[semi[u]]?u:pre[w]);
        }
        dom[pre[w]].clear();
    }
    for(int w:ord) if(w!=rt&&idom[w]!=semi[w])
        idom[w]=idom[idom[w]];
    fore(i,0,n) if(idom[i]==n)idom[i]=-1;
}

```

9.6 FenwickTree

```

#include "../Header.cpp"

struct BIT {
    /*****All Index starts at 1*****/
    vl bit;
    BIT(ll n) { bit.assign(n+1, 0); }
    ll psq(ll k) {

```

```

        ll sum = 0;
        for (; k; k -= (k & -k)) sum += bit[k];
        return sum;
    }
    ll rsq(ll a, ll b) {
        return psq(b) - psq(a-1);
    }
    // increment k'th value by v (and propagate)
    void add(ll k, ll v) {
        for (; k < bit.size(); k += (k & -k)) bit[k] += v;
    }
    void set(ll k, ll v){
        ll aux = rsq(k,k);
        for (; k < bit.size(); k += (k & -k)) bit[k] += v-aux;
        //bit[idx] = min(bit[idx], val);
    }
    // (1, r)
    ll getmin(ll r) {
        ll ret = INF;
        for (; r >= 0; r = (r & (r + 1)) - 1)
            ret = min(ret, bit[r]);
        return ret;
    }
};

```

9.7 FenwickTree2D

```

#include "../Header.cpp"

struct BIT2D { // BIT = binary indexed tree (a.k.a. Fenwick Tree)

    // *****All Index starts at 1*****
    vector<vector<int>> bit;
    BIT2D(int n,int m) { bit.assign(n+1, vector<int>(m+1,0)); }
    // prefix sum query (sum in range 1 .. k)
    int psq(int k, int y) {
        int sum = 0;
        for (; k; k -= (k & -k))for (int ty=y; ty; ty -= (ty & -ty)) sum += bit[k][ty];
        return sum;
    }

```

```

}
// range sum query (sum in range a .. b)
int rsq(int x1, int y1, int x2, int y2) {
    return psq(x2,y2) - psq(x1-1,y2) - psq(x2,y1-1) +
        psq(x1-1,y1-1);
}
// increment k'th value by v (and propagate)
void add(int k, int y, int v) {
    for (; k < bit.size(); k += (k & -k)) for (int ty=y; ty <
        bit[k].size(); ty += (ty & -ty)) bit[k][ty]+=v;
}
void set(int k, int y, int v){
    int aux=rsq(k,y,k,y);
    for (; k < bit.size(); k += (k & -k)) for (int ty=y; ty <
        bit[k].size(); ty += (ty & -ty)) bit[k][ty]+=v-aux;
}
};

```

9.8 Implicit_segment_tree

```

struct Vertex {
    int left, right;
    int sum = 0;
    Vertex *left_child = nullptr, *right_child = nullptr;
    Vertex(int lb, int rb) {
        left = lb;
        right = rb;
    }
    void extend() {
        if (!left_child && left + 1 < right) {
            int t = (left + right) / 2;
            left_child = new Vertex(left, t);
            right_child = new Vertex(t, right);
        }
    }
    void add(int k, int x) {
        extend();
        sum += x;
        if (left_child) {
            if (k < left_child->right)

```

```

        left_child->add(k, x);
    else
        right_child->add(k, x);
}
}
int get_sum(int lq, int rq) {
    if (lq <= left && right <= rq)
        return sum;
    if (max(left, lq) >= min(right, rq))
        return 0;
    extend();
    return left_child->get_sum(lq, rq) +
        right_child->get_sum(lq, rq);
}
};

```

9.9 LinkCutTree

```

struct node {
    int p = 0, c[2] = {0, 0}, pp = 0;
    bool flip = 0;
    int sz = 0, ssz = 0, vsz = 0; // sz -> aux tree size, ssz =
        subtree size in rep tree, vsz = virtual tree size
    long long val = 0, sum = 0, lazy = 0, subsum = 0, vsum = 0;
    node() {}
    node(int x) {
        val = x; sum = x;
        sz = 1; lazy = 0;
        ssz = 1; vsz = 0;
        subsum = x; vsum = 0;
    }
};
struct LCT {
    vector<node> t;
    LCT() {}
    LCT(int n) : t(n + 1) {}

    // <independant splay tree code>
    int dir(int x, int y) { return t[x].c[1] == y; }
    void set(int x, int d, int y) {

```

```

    if (x) t[x].c[d] = y, pull(x);
    if (y) t[y].p = x;
}
void pull(int x) {
    if (!x) return;
    int &l = t[x].c[0], &r = t[x].c[1];
    t[x].sum = t[l].sum + t[r].sum + t[x].val;
    t[x].sz = t[l].sz + t[r].sz + 1;
    t[x].ssz = t[l].ssz + t[r].ssz + t[x].vsz + 1;
    t[x].subsum = t[l].subsum + t[r].subsum + t[x].vsum +
        t[x].val;
}
void push(int x) {
    if (!x) return;
    int &l = t[x].c[0], &r = t[x].c[1];
    if (t[x].flip) {
        swap(l, r);
        if (l) t[l].flip ^= 1;
        if (r) t[r].flip ^= 1;
        t[x].flip = 0;
    }
    if (t[x].lazy) {
        t[x].val += t[x].lazy;
        t[x].sum += t[x].lazy * t[x].sz;
        t[x].subsum += t[x].lazy * t[x].ssz;
        t[x].vsum += t[x].lazy * t[x].vsz;
        if (l) t[l].lazy += t[x].lazy;
        if (r) t[r].lazy += t[x].lazy;
        t[x].lazy = 0;
    }
    // delete later, just testing
    if (!t[x].lazy) {
        int flag = rand()%10000;
        if (l) t[l].lazy = 0;
        if (r) t[r].lazy = 0;
        t[x].lazy = flag;
    }
}
void rotate(int x, int d) {
    int y = t[x].p, z = t[y].p, w = t[x].c[d];
    swap(t[x].pp, t[y].pp);

```

```

    set(y, !d, w);
    set(x, d, y);
    set(z, dir(z, y), x);
}
void splay(int x) {
    for (push(x); t[x].p;) {
        int y = t[x].p, z = t[y].p;
        push(z); push(y); push(x);
        int dx = dir(y, x), dy = dir(z, y);
        if (!z) rotate(x, !dx);
        else if (dx == dy) rotate(y, !dx), rotate(x, !dx);
        else rotate(x, dy), rotate(x, dx);
    }
}
// </independant splay tree code>

// making it a root in the rep. tree
void make_root(int u) {
    access(u);
    int l = t[u].c[0];
    t[l].flip ^= 1;
    swap(t[l].p, t[l].pp);
    t[u].vsz += t[l].ssz;
    t[u].vsum += t[l].subsum;
    set(u, 0, 0);
}
// make the path from root to u a preferred path
// returns last path-parent of a node as it moves up the tree
int access(int _u) {
    int last = _u;
    for (int v = 0, u = _u; u; u = t[v = u].pp) {
        splay(u); splay(v);
        t[u].vsz -= t[v].ssz;
        t[u].vsum -= t[v].subsum;
        int r = t[u].c[1];
        t[u].vsz += t[r].ssz;
        t[u].vsum += t[r].subsum;
        t[v].pp = 0;
        swap(t[r].p, t[r].pp);
        set(u, 1, v);
        last = u;
    }
}

```

```

    }
    splay(_u);
    return last;
}

void link(int u, int v) { // u -> v
    // assert(!connected(u, v));
    make_root(v);
    access(u); splay(u);
    t[v].pp = u;
    t[u].vsz += t[v].ssz;
    t[u].vsum += t[v].subsum;
}

void cut(int u) { // cut par[u] -> u, u is non root vertex
    // assert(connected(u, v));
    access(u);
    assert(t[u].c[0] != 0);
    t[t[u].c[0]].p = 0;
    t[u].c[0] = 0;
    pull(u);
}

// parent of u in the rep. tree
int get_parent(int u) {
    access(u); splay(u); push(u);
    u = t[u].c[0]; push(u);
    while (t[u].c[1]) {
        u = t[u].c[1]; push(u);
    }
    splay(u);
    return u;
}

// root of the rep. tree containing this node
int find_root(int u) {
    access(u); splay(u); push(u);
    while (t[u].c[0]) {
        u = t[u].c[0]; push(u);
    }
    splay(u);
    return u;
}

bool connected(int u, int v) {
    return find_root(u) == find_root(v);
}

```

```

    }
    // depth in the rep. tree
    int depth(int u) {
        access(u); splay(u);
        return t[u].sz;
    }

    int lca(int u, int v) {
        // assert(connected(u, v));
        if (u == v) return u;
        if (depth(u) > depth(v)) swap(u, v);
        access(v);
        return access(u);
    }

    int is_root(int u) {
        return get_parent(u) == 0;
    }

    int component_size(int u) {
        return t[find_root(u)].ssz;
    }

    int subtree_size(int u) {
        int p = get_parent(u);
        if (p == 0) {
            return component_size(u);
        }
        cut(u);
        int ans = component_size(u);
        link(p, u);
        return ans;
    }

    long long component_sum(int u) {
        return t[find_root(u)].subsum;
    }

    long long subtree_sum(int u) {
        int p = get_parent(u);
        if (p == 0) {
            return component_sum(u);
        }
        cut(u);
        long long ans = component_sum(u);
        link(p, u);
        return ans;
    }
}

```



```

    }
    // sum of the subtree of u when root is specified
    long long subtree_query(int u, int root) {
        int cur = find_root(u);
        make_root(root);
        long long ans = subtree_sum(u);
        make_root(cur);
        return ans;
    }
    // path sum
    long long query(int u, int v) {
        int cur = find_root(u);
        make_root(u); access(v);
        long long ans = t[v].sum;
        make_root(cur);
        return ans;
    }
    void upd(int u, int x) {
        access(u); splay(u);
        t[u].val += x;
    }
    // add x to the nodes on the path from u to v
    void upd(int u, int v, int x) {
        int cur = find_root(u);
        make_root(u); access(v);
        t[v].lazy += x;
        make_root(cur);
    }
};

```

9.10 Mo

```

#include "../Header.cpp"

/*
a ancestor of b
[start[a], start[b]]
else
[end[a], start[b]] + lca(a, b)
*/

```

```

ll block;
vl ans;
vl el(1e6+2, 0);
// F = add-remove
// O((N+Q) FN )
struct Query {
    int L, R, id;
};
bool cmp(Query a, Query b){
    if(a.L / block != b.L / block)
        return a.L < b.L;
    return a.L / block % 2 ? a.R < b.R : a.R > b.R;
}
ll add(vl& a, int id){
    //cout<<id<<endl;
    return (2 * el[a[id]] + 1) * a[id];
}
ll remove(vl& a, int id){
    return (-2 * el[a[id]] + 1) * a[id];
}
void Mo(vl& a, vector<Query>& q) {
    block = (int)sqrt(a.size());
    ans.assign(q.size(), 0);
    sort(ALL(q), cmp);
    ll cL = 0, cR = 0, cAns = 0;
    for (int i=0; i<q.size(); i++){
        // L and R values of current range
        int L = q[i].L, R = q[i].R;
        while (cR <= R) {
            cAns += add(a, cR);
            el[a[cR]]++;
            cR++;
        }
        while (cL > L) {
            cAns += add(a, cL-1);
            el[a[cL-1]]++;
            cL--;
        }
        while (cR > R+1) {
            cAns += remove(a, cR-1);
            el[a[cR-1]]--;
        }
    }
}

```

```

        cR--;
    }
    while (cL < L) {
        cAns += remove(a, cL);
        el[a[cL]]--;
        cL++;
    }
    ans[q[i].id] = cAns;
}

int main()
{
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    int n, t, x, y;
    vl c;
    cin >> n >> t;
    for(int i = 0; i < n; i++){
        cin >> x;
        c.push_back(x);
    }
    Query q;
    vector<Query>Q;
    for(int i = 0; i < t; i++){
        cin >> q.L >> q.R;
        q.L--; q.R--;
        q.id = i;
        Q.push_back(q);
    }
    Mo(c, Q);
    for(int i = 0; i < ans.size(); i++){
        cout << ans[i] << "\n";
    }
    return 0;
}

```

9.11 Partially *persistent* DSU

```
struct PPDSU {
```

```

vector<vector<pair<int, int>>> par;
int time = 0; //initial time
PPDSU(int n) : par(n + 1, {{-1, 0}}) {}
bool merge(int u, int v) {
    ++time;
    if ((u = root(u, time)) == (v = root(v, time))) return 0;
    if (par[u].back().first > par[v].back().first) swap(u, v);
    par[u].push_back({par[u].back().first + par[v].back().first,
        time});
    par[v].push_back({u, time}); //par[v] = u
    return 1;
}
bool same(int u, int v, int t) {
    return root(u, t) == root(v, t);
}
int root(int u, int t) { //root of u at time t
    if (par[u].back().first >= 0 && par[u].back().second <= t)
        return root(par[u].back().first, t);
    return u;
}
int size(int u, int t) { //size of the component of u at time t
    u = root(u, t);
    int l = 0, r = (int) par[u].size() - 1, ans = 0;
    while (l <= r) {
        int mid = l + r >> 1;
        if (par[u][mid].second <= t) ans = mid, l = mid + 1;
        else r = mid - 1;
    }
    return -par[u][ans].first;
}
};

```

9.12 PolicyBasedEDD

```

#include "../Header.cpp"

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

```

```

// use less_equal for multiset
template <typename T, typename Comparator = less<T>>
using ordered_set = tree<T, null_type, Comparator, rb_tree_tag,
                        tree_order_statistics_node_update>;

// order_of_key(T x)
// -> returns the number of elements strictly smaller than x
// find_by_order(size_t i)
// -> returns iterator to i-th largest element (counting from 0)
int main(){
    ios::sync_with_stdio(0); cin.tie(0);
    int t;
    cin >> t;
    while(t--){
        int n, k, x, y;
        ordered_set<int> m;
        vector<pll> c;
        cin >> n >> k;
        for(int i = 0; i < n; i++){
            cin >> x >> y;
            c.push_back({x, y});
        }
        sort(ALL(c));
        ll ans = 0;
        for(int i = 0; i < n; i++){
            ll num = m.order_of_key(c[i].second + 1);
            if(abs(num - (n-1 - num)) >= k) ans++;
            m.insert(c[i].second);
        }
        cout<<ans<<"\n";
    }
}

```

9.13 SegmentTree

```

#include "../Header.cpp"
struct SegmentTree{
    vl ST; int N;
    SegmentTree(vl &A){
        N = A.size();
        ST.assign(4*N, 0);
    }
};

```

```

        bd(1,0,N-1,A);
    }
    ll op(ll x, ll y) { return min(x,y); }
    void bd(int n, int l, int r, vl &A){
        if(l == r){
            ST[n] = A[r];
            return;
        }
        bd(2*n,l,(l+r)/2,A);
        bd(2*n+1,(l+r)/2+1,r,A);
        ST[n] = op(ST[2*n], ST[2*n+1]);
    }
    ll qry(int i, int j){
        return qry(1,0,N-1,i,j);
    }
    ll qry(int n, int l, int r, int i, ll j){
        if(r < i || j < l) return 0;
        if(i <= l && r <= j) return ST[n];
        return op(qry(2*n,l,(l+r)/2,i,j),
                  qry(2*n+1,(l+r)/2+1,r,i,j));
    }
    void upd(int i, ll v){
        return upd(1,0,N-1,i,v);
    }
    void upd(int n, int l, int r, int i, ll v){
        if(i < l || r < i) return;
        if(l == r){
            ST[n] = v;
            return;
        }
        upd(2*n,l,(l+r)/2,i,v);
        upd(2*n+1,(l+r)/2+1,r,i,v);
        ST[n] = op(ST[2*n], ST[2*n+1]);
    }
};

```

9.14 SegmentTree2D

```

#include "../Header.cpp"

```

```

struct SegmentTree{
    vector<ll>ST; int N;
    SegmentTree(int Size){
        N = Size; ST.assign(4*N,0);
    }
    void upd(int i, ll v){
        return upd(1,0,N-1,i,v);
    }
    void upd(int n, int l, int r, int i, ll v){
        if(i < l || r < i) return;
        if(l == r){
            ST[n] += v; return;
        }
        upd(2*n,l,(l+r)/2,i,v);
        upd(2*n+1,(l+r)/2+1,r,i,v);
        ST[n] = ST[2*n]+ST[2*n+1];
    }
    ll qry(int i,int j){
        return qry(1,0,N-1,i,j);
    }
    ll qry(int n, int l, int r, int i, int j){
        if(r < i || j < l) return 0;
        if(i <= l && r <= j) return ST[n];
        return
            query(2*n,l,(l+r)/2,i,j)+query(2*n+1,(l+r)/2+1,r,i,j);
    }
};

struct SegmenTree2D{
    vector<SegmentTree>ST;
    int N;
    SegmenTree2D(int Size){
        N = Size; ST.resize(4*N,Size);
    }
    void update(int i, int j, int v){
        return update(1,0,N-1,i,j,v);
    }
    void update(int n, int l, int r, int i, int j, ll v){
        if(i < l || r < i) return;
        if(l == r){
            ST[n].update(j,v);
            return;
        }

```

```

        }
        update(2*n,l,(l+r)/2,i,j,v);
        update(2*n+1,(l+r)/2+1,r,i,j,v);
        ST[n].update(j,v);
    }
    ll query(int i1, int i2, int j1, int j2){
        return query(1,0,N-1,i1,i2,j1,j2);
    }
    ll query(int n, int l, int r, int i1, int i2, int j1, int j2){
        if(l > j1 || i1 > r) return 0;
        if(i1 <= l && r <= j1){
            return ST[n].query(i2,j2);
        }
        return
            query(2*n,l,(l+r)/2,i1,i2,j1,j2)+query(2*n+1,(l+r)/2+1,r,i1,i2,j1,j2);
    }
};

int main()
{
    ll q, n, x, y, l, b, r, t, a;
    SegmenTree2D ST(1025);
    while(cin >> q){
        if(q == 0){
            cin >> n;
        }
        else if(q == 1){
            cin >> x >> y >> a;
            ST.update(x, y, a);
        }
        else if(q == 2){
            cin >> l >> b >> r >> t;
            cout << ST.query(l, b, r, t) << "\n";
        }
        else
            break;
    }
    /*for(int i=0;i<n;i++)
    {
        for(int j=0;j<n;j++)
        {
            cout<<ST.query(i,j,i,j)<<" ";
        }cout<<endl;
    }
    */
}

```

```

    }*/
}
return 0;
}

```

9.15 SegmentTreeBeats

```
#include "../Template.cpp"
```

```

struct Node{
    ll s, mx1, mx2, mxc, mn1, mn2, mnc, lz = 0;
    Node() : s(0), mx1(LLONG_MIN), mx2(LLONG_MIN), mxc(0),
            mn1(LLONG_MAX), mn2(LLONG_MAX), mnc(0) {}
    Node(ll x) : s(x), mx1(x), mx2(LLONG_MIN), mxc(1), mn1(x),
            mn2(LLONG_MAX), mnc(1) {}
    Node(const Node &a, const Node &b){
        // add
        s = a.s + b.s;
        // min
        if (a.mx1 > b.mx1) mx1 = a.mx1, mxc = a.mxc, mx2 =
            max(b.mx1, a.mx2);
        if (a.mx1 < b.mx1) mx1 = b.mx1, mxc = b.mxc, mx2 =
            max(a.mx1, b.mx2);
        if (a.mx1 == b.mx1) mx1 = a.mx1, mxc = a.mxc + b.mxc, mx2
            = max(a.mx2, b.mx2);
        // max
        if (a.mn1 < b.mn1) mn1 = a.mn1, mnc = a.mnc, mn2 =
            min(b.mn1, a.mn2);
        if (a.mn1 > b.mn1) mn1 = b.mn1, mnc = b.mnc, mn2 =
            min(a.mn1, b.mn2);
        if (a.mn1 == b.mn1) mn1 = a.mn1, mnc = a.mnc + b.mnc, mn2
            = min(a.mn2, b.mn2);
    }
};

// 0 - indexed / inclusive - inclusive
template <class node>
struct STB{
    vector<node> st; int n;

```

```

void build(int u, int i, int j, vector<node> &arr){
    if (i == j) { st[u] = arr[i]; return; }
    int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
    build(l, i, m, arr), build(r, m + 1, j, arr);
    st[u] = node(st[l], st[r]);
}

void push_add(int u, int i, int j, ll v){
    st[u].s += (j - i + 1) * v;
    st[u].mx1 += v, st[u].mn1 += v, st[u].lz += v;
    if (st[u].mx2 != LLONG_MIN) st[u].mx2 += v;
    if (st[u].mn2 != LLONG_MAX) st[u].mn2 += v;
}

void push_max(int u, ll v, bool l){ // for min op
    if (v >= st[u].mx1) return;
    st[u].s -= st[u].mx1 * st[u].mxc;
    st[u].mx1 = v;
    st[u].s += st[u].mx1 * st[u].mxc;
    if (l) st[u].mn1 = st[u].mx1;
    else if (v <= st[u].mn1) st[u].mn1 = v;
    else if (v < st[u].mn2) st[u].mn2 = v;
}

void push_min(int u, ll v, bool l){ // for max op
    if (v <= st[u].mn1) return;
    st[u].s -= st[u].mn1 * st[u].mnc;
    st[u].mn1 = v;
    st[u].s += st[u].mn1 * st[u].mnc;
    if (l) st[u].mx1 = st[u].mn1;
    else if (v >= st[u].mx1) st[u].mx1 = v;
    else if (v > st[u].mx2) st[u].mx2 = v;
}

void push(int u, int i, int j){
    if (i == j) return;
    // add
    int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
    push_add(l, i, m, st[u].lz);
    push_add(r, m + 1, j, st[u].lz);
    st[u].lz = 0;
    // min
    push_max(l, st[u].mx1, i == m);
    push_max(r, st[u].mx1, m + 1 == j);
    // max

```

```

    push_min(l, st[u].mn1, i == m);
    push_min(r, st[u].mn1, m + 1 == r);
}
node query(int a, int b, int u, int i, int j){
    if (b < i || j < a) return node();
    if (a <= i && j <= b) return st[u];
    push(u, i, j);
    int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
    return node(query(a, b, l, i, m), query(a, b, r, m + 1,
        j));
}
void update_add(int a, int b, ll v, int u, int i, int j){
    if (b < i || j < a) return;
    if (a <= i && j <= b) { push_add(u, i, j, v); return; }
    push(u, i, j);
    int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
    update_add(a, b, v, l, i, m); update_add(a, b, v, r, m +
        1, j);
    st[u] = node(st[l], st[r]);
}
void update_min(int a, int b, ll v, int u, int i, int j){
    if (b < i || j < a || v >= st[u].mx1) return;
    if (a <= i && j <= b && v > st[u].mx2) { push_max(u, v, i
        == j); return; }
    push(u, i, j);
    int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
    update_min(a, b, v, l, i, m); update_min(a, b, v, r, m +
        1, j);
    st[u] = node(st[l], st[r]);
}
void update_max(int a, int b, ll v, int u, int i, int j){
    if (b < i || j < a || v <= st[u].mn1) return;
    if (a <= i && j <= b && v < st[u].mn2) { push_min(u, v, i
        == j); return; }
    push(u, i, j);
    int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
    update_max(a, b, v, l, i, m); update_max(a, b, v, r, m +
        1, j);
    st[u] = node(st[l], st[r]);
}
}

```

```

STB(vector<node> &v, int N) : n(N), st(N * 4 + 5) { build(0,
    0, n - 1, v); }
node query(int a, int b) { return query(a, b, 0, 0, n - 1); }
void update_add(int a, int b, ll v) { update_add(a, b, v, 0,
    0, n - 1); }
void update_min(int a, int b, ll v) { update_min(a, b, v, 0,
    0, n - 1); }
void update_max(int a, int b, ll v) { update_max(a, b, v, 0,
    0, n - 1); }
};

```

9.16 SegmentTreeIterative

```

#include "../Header.cpp"

struct Node{
    int v;
    Node() { v = 0; } // neutro
    Node(int v) : v(v) {}
    Node(const Node &a, const Node &b) { v = a.v + b.v; }
};

// 0 - indexed / inclusive - exclusive
template <class node>
struct ST{
    vector<node> t; int n;
    ST(vector<node> &arr, int N) : n(N), t(N * 2){
        copy(arr.begin(), arr.end(), t.begin() + n);
        for (int i = n - 1; i > 0; --i) t[i] = node(t[i << 1], t[i
            << 1 | 1]);
    }
    void set(int p, const node &value){
        for (t[p += n] = value; p >>= 1;){
            t[p] = node(t[p << 1], t[p << 1 | 1]);
        }
    }
    node query(int l, int r){
        node ans1, ansr;
        for (l += n, r += n; l < r; l >>= 1, r >>= 1){
            if (l & 1) ans1 = node(ans1, t[l++]);
            if (r & 1) ansr = node(t[--r], ansr);
        }
    }
};

```

```

    }
    return node(ansl, ansr);
}
};

```

9.17 SegmentTreeLazy

```

#include "../Header.cpp"

struct Node
{
    ll mini, cont;
    Node(ll val, ll cont)
    {
        this->mini = val;
        this->cont = cont;
    }
    Node()
    {
        mini = 1e18;
        cont = 0;
    }

    Node operator + (const Node& p) const {
        return Node(min(mini, p.mini), cont + p.cont);
    }
};

ll ans = 0;

struct SegmentTree
{
    vector<Node> ST;
    vl Lazy;
    int N; ll Nul = 0;
    SegmentTree(vector<ll> &A)
    {
        N = A.size();
        ST.resize(4*N+5, Node());
        Lazy.resize(4*N+5, Nul);
    }

```

```

        bd(1,0,N-1,A);
    }
    void up(int n, int l, int r)
    {
        ST[n].mini += Lazy[n];
        if(l != r)
        {
            Lazy[n*2] += Lazy[n];
            Lazy[n*2+1] += Lazy[n];
        }
        Lazy[n] = Nul;
    }
    void bd(int n, int l, int r, vl &A)
    {
        if(l == r)
        {
            ST[n] = Node(A[r], 1);
            return;
        }

        bd(2*n,l,(l+r)/2,A);
        bd(2*n+1,(l+r)/2+1,r,A);
        ST[n] = ST[2*n] + ST[2*n+1];
    }

    Node qry(int n, int l, int r, int i, int j)
    {
        if(r < i || j < l ) return Node();
        if(Lazy[n] != Nul) up(n,l,r);
        if(i <= l && r <= j) return ST[n];
        return qry(2*n,l,(l+r)/2,i,j) +
            qry(2*n+1,(l+r)/2+1,r,i,j);
    }

    void upd(int n, int l, int r, int i, int j, ll v)
    {
        if(Lazy[n] != Nul) up(n,l,r);
        if(l > j || r < i) return;

        if(i <= l && r <= j)
        {

```

```

        Lazy[n] += v;
        if(Lazy[n] != Nul) up(n, l, r);
        return;
    }

    upd(2*n,l,(l+r)/2,i,j,v);
    upd(2*n+1,(l+r)/2+1,r,i,j,v);
    ST[n] = ST[2*n] + ST[2*n+1];
}

void search(int n, int l, int r, int j, ll v)
{
    if(Lazy[n] != Nul) up(n,l,r);
    if(l > j || ST[n].mini > v) return;

    if(l == r)
    {
        ans += ST[n].mini;
        ST[n] = Node();
        if(Lazy[n] != Nul) up(n, l, r);
        return;
    }

    search(2*n,l,(l+r)/2,j,v);
    search(2*n+1,(l+r)/2+1,r,j,v);

    ST[n] = ST[2*n] + ST[2*n+1];
}

Node qry(int i, int j)
{return qry(1,0,N-1,i,j);}
void upd(int i, int j, ll v)
{return upd(1,0,N-1,i,j,v);}
void search(int j, ll v)
{return search(1,0,N-1,j,v);}
};

```

9.18 SegmentTreePersistent

```
#include "../Header.cpp"
```

```

struct SegmentTree
{
    vector<vl> ST; vector<vi> Leftv, Rightv;
    int N;
    SegmentTree(vl &A){
        N = A.size();
        ST.resize(4*N, vl());
        // ST.resize(4*N, vl(1, 0)); whitout build, init
        // with 0s
        Leftv.resize(4*N, vi(1, 0)); // all conected to
        // version 0
        Rightv.resize(4*N, vi(1, 0));
        bd(1,0,N-1,A);
    }
    ll op(ll x, ll y) { return min(x,y); }
    void bd(int n, int l, int r, vl &A){
        if(l==r){
            ST[n].push_back(A[r]);
            return;
        }
        bd(2*n,l,(l+r)/2,A);
        bd(2*n+1,(l+r)/2+1,r,A);
        ST[n].push_back(op(ST[2*n][0], ST[2*n+1][0]));
    }
    ll qry(int i, int j, int vs){
        return qry(1,0,N-1,i,j, vs);
    }
    ll qry(int n, int l, int r, int i, int j, int vs){
        if(r < i || j < l) return 0;
        if(i <= l && r <= j) return ST[n][vs];
        return op(qry(2*n,l,(l+r)/2,i,j, Leftv[n][vs]),
            qry(2*n+1,(l+r)/2+1,r,i,j, Rightv[n][vs]));
    }
    void upd(int i, ll v){
        return upd(1,0,N-1,i,v);
    }
    void upd(int n, int l, int r, int i, ll v){
        if(i < l || r < i) return;
        if(l == r){

```



```

        ST[n].push_back(v); // ST[n].push_back(v +
                           // ST[n].back()) add
        return;
    }
    upd(2*n,1,(l+r)/2,i,v);
    upd(2*n+1,(l+r)/2+1,r,i,v);

    ST[n].push_back(op(ST[2*n].back(),
                      ST[2*n+1].back()));
    Leftv[n].push_back(ST[2*n].size()-1);
    Rightv[n].push_back(ST[2*n+1].size()-1);
}
};

```

9.19 SegmentTreeSubarraySum

```

#include "../Header.cpp"

// Max subarray sum
struct Node{
    //maxPrefixSum, maxSuffixSum, Totalsum, maxSubarraySum
    ll mxP, mxS, sum, subSum;
    Node(){
        mxP = mxS = sum = subSum = -INF;
    }
    Node merge(Node r){
        Node p;
        p.mxP = max(mxP, sum + r.mxP);
        p.mxS = max(r.mxS, r.sum + mxS);
        p.sum = sum + r.sum;
        p.subSum = max({subSum, r.subSum, mxS + r.mxP});
        return p;
    }
    void upd(ll v){
        mxP = mxS = sum = subSum = v;
    }
    void nul(){
        mxP = mxS = subSum = -INF;
        sum = 0;
    }
}

```

```

};
// Node version
struct SegmentTree{
    vector<Node> ST;
    int N;
    SegmentTree(vl &A){
        N = A.size();
        ST.assign(4*N, Node());
        bd(1,0,N-1,A);
    }
    void bd(int n, int l, int r, vl &A){
        if(l == r){
            ST[n].upd(A[l]);
            return;
        }
        bd(2*n,l,(l+r)/2,A);
        bd(2*n+1,(l+r)/2+1,r,A);
        ST[n] = ST[2*n].merge(ST[2*n+1]);
    }
    Node qry(int i, int j){
        return qry(1,0,N-1,i,j);
    }
    Node qry(int n, int l, int r, int i, int j){
        if(r < i || j < l) {
            Node p;
            p.nul();
            return p;
        }
        if(i <= l && r <= j) return ST[n];
        return
            qry(2*n,l,(l+r)/2,i,j).merge(qry(2*n+1,(l+r)/2+1,r,i,j)
    }
    void upd(int i, ll v){
        return upd(1,0,N-1,i,v);
    }
    void upd(int n, int l, int r, int i, ll v){
        if(i < l || r < i) return;
        if(l == r){
            ST[n].upd(v);
            return;
        }
    }
}

```

```

        upd(2*n, l, (l+r)/2, i, v);
        upd(2*n+1, (l+r)/2+1, r, i, v);
        ST[n] = ST[2*n].merge(ST[2*n+1]);
    }
};

```

9.20 SparseTable

```

#include "../Header.cpp"

struct SparseTable{
    vector<vl >SP;
    SparseTable(vl&A){
        int n = A.size();
        SP.push_back(A);
        ll maxlog = 31 - __builtin_clz(n);
        rep(x, 1, maxlog+1){
            vl aux;
            rep(j, n-(1<<i)+1){
                aux.push_back(max(SP[i-1][j], SP[i-1][j+(1<<(i-1))]]));
            }
            SP.push_back(aux);
        }
    }
    ll op(int l, int r){
        ll maxlog = 31 - __builtin_clz(r-l+1);
        return max(SP[maxlog][l], SP[maxlog][r-(1<<maxlog)+1]);
    }
    ll find(int l, int r, ll m){ // maxi
        ll maxlog = 31 - __builtin_clz(r-l+1);
        for(int i = maxlog; i >= 0; i--){
            if(l + (1<<i) <= r && SP[i][l] < m){
                l += (1<<i);
            }
        }
        return l;
    }
};

```

9.21 Treap

```

#include "../Header.cpp"
typedef struct item *pitem;
struct item {
    int pr, key, cnt;
    pitem l, r;
    item(int key):key(key), pr(rand()), cnt(1), l(0), r(0) {}
};

int cnt(pitem t){return t?t->cnt:0;}
void upd_cnt(pitem t){if(t)t->cnt=cnt(t->l)+cnt(t->r)+1;}
void split(pitem t, int key, pitem& l, pitem& r){ // l: < key, r:
    >= key
    if(!t)l=r=0;
    else if(key<t->key)split(t->l, key, l, t->l), r=t;
    else split(t->r, key, t->r, r), l=t;
    upd_cnt(t);
}

void insert(pitem& t, pitem it){
    if(!t)t=it;
    else if(it->pr>t->pr)split(t, it->key, it->l, it->r), t=it;
    else insert(it->key<t->key?t->l:t->r, it);
    upd_cnt(t);
}

void merge(pitem& t, pitem l, pitem r){
    if(!l||!r)t=l?l:r;
    else if(l->pr>r->pr)merge(l->r, l->r, r), t=l;
    else merge(r->l, l, r->l), t=r;
    upd_cnt(t);
}

void erase(pitem& t, int key){
    if(t->key==key)merge(t, t->l, t->r);
    else erase(key<t->key?t->l:t->r, key);
    upd_cnt(t);
}

void unite(pitem& t, pitem l, pitem r){
    if(!l||!r){t=l?l:r;return;}
    if(l->pr<r->pr)swap(l, r);
    pitem p1, p2; split(r, l->key, p1, p2);
    unite(l->l, l->l, p1); unite(l->r, l->r, p2);
    t=l; upd_cnt(t);
}

```

```

}
pitem kth(pitem t, int k){
    if(!t)return 0;
    if(k==cnt(t->l))return t;
    return k<cnt(t->l)?kth(t->l,k):kth(t->r,k-cnt(t->l)-1);
}
pair<int,int> lb(pitem t, int key){ // position and value of
    lower_bound
    if(!t)return {0,1<<30}; // (special value)
    if(key>t->key){
        auto w=lb(t->r,key);w.fst+=cnt(t->l)+1;return w;
    }
    auto w=lb(t->l,key);
    if(w.fst==cnt(t->l))w.snd=t->key;
    return w;
}

pitem ss;
int n,q;

int find(int x){
    int s=1,e=n+1;
    while(e-s>1){
        int m=(s+e)/2;
        if(m-lb(ss,m).fst>x)e=m;
        else s=m;
    }
    return s;
}

int main(){
    scanf("%d%d",&n,&q);
    while(q--){
        char c[4];int k;
        scanf("%s%d",c,&k);
        if(c[0]=='D')insert(ss,new item(find(k)));
        else printf("%d\n",find(k));
    }
    fore(i,0,cnt(ss))assert(lb(ss,kth(ss,i)->key).fst==i);
    return 0;
}

```

9.22 TreapImplicit

```

// example that supports range reverse and addition updates, and
// range sum query
// (commented parts are specific to this problem)
#include "../Header.cpp"
typedef struct item *pitem;
struct item {
    int pr,cnt,val;
    // int sum; // (parameters for range query)
    // bool rev;int add; // (parameters for lazy prop)
    pitem l,r;
    item(int val):
        pr(rand()),cnt(1),val(val),l(0),r(0),/*sum(val),rev(0),add(0)*/
        {}
};

void push(pitem it){
    if(it){
        /*if(it->rev){
            swap(it->l,it->r);
            if(it->l)it->l->rev^=true;
            if(it->r)it->r->rev^=true;
            it->rev=false;
        }
        it->val+=it->add;it->sum+=it->cnt*it->add;
        if(it->l)it->l->add+=it->add;
        if(it->r)it->r->add+=it->add;
        it->add=0;*/
    }
}

int cnt(pitem t){return t?t->cnt:0;}
// int sum(pitem t){return t?push(t),t->sum:0;}
void upd_cnt(pitem t){
    if(t){
        t->cnt=cnt(t->l)+cnt(t->r)+1;
        // t->sum=t->val+sum(t->l)+sum(t->r);
    }
}

void merge(pitem& t, pitem l, pitem r){
    push(l);push(r);
    if(!l||!r)t=l?l:r;
}

```

```

        else if(l->pr>r->pr)merge(l->r,l->r,r),t=l;
        else merge(r->l,l,r->l),t=r;
        upd_cnt(t);
    }
    void split(pitem t, pitem& l, pitem& r, int sz){ // sz:desired
        size of l
        if(!t){l=r=0;return;}
        push(t);
        if(sz<=cnt(t->l))split(t->l,l,t->l,sz),r=t;
        else split(t->r,t->r,r,sz-1-cnt(t->l)),l=t;
        upd_cnt(t);
    }
    void output(pitem t){ // useful for debugging
        if(!t)return;
        push(t);
        output(t->l);printf(" %d",t->val);output(t->r);
    }
    // use merge and split for range updates and queries

    int n,q;
    char s[100005];

    int main(){
        string s;
        cin >> s;
        pitem t=0;
        rep(i, s.size())merge(t,t,new item(s[i]-'a'));

        ll q, x;
        cin >> q;
        while(q--){
            cin >> x;
            pitem r;
            split(t, t, r, x);
            if(t) {t->rev^=true; t->add+=1} if(r) r->rev^=true;
            merge(t, t, r);
        }
        ans(t);
    }

```

9.23 TreapImplicitFather

```

// node father is useful to keep track of the chain of each node
// alternative: splay tree
// IMPORTANT: add pointer f in struct item
#include "../Header.cpp"
#define fore(i,a,b) for(int i=a,ThxDem=b;i<ThxDem;++i)
void merge(pitem& t, pitem l, pitem r){
    push(l);push(r);
    if(!l||!r)t=l?l:r;
    else if(l->pr>r->pr)merge(l->r,l->r,r),l->r->f=t=l;
    else merge(r->l,l,r->l),r->l->f=t=r;
    upd_cnt(t);
}
void split(pitem t, pitem& l, pitem& r, int sz){
    if(!t){l=r=0;return;}
    push(t);
    if(sz<=cnt(t->l)){
        split(t->l,l,t->l,sz);r=t;
        if(l)l->f=0;
        if(t->l)t->l->f=t;
    }
    else {
        split(t->r,t->r,r,sz-1-cnt(t->l));l=t;
        if(r)r->f=0;
        if(t->r)t->r->f=t;
    }
    upd_cnt(t);
}
void push_all(pitem t){
    if(t->f)push_all(t->f);
    push(t);
}
pitem root(pitem t, int& pos){ // get root and position for node t
    push_all(t);
    pos=cnt(t->l);
    while(t->f){
        pitem f=t->f;
        if(t==f->r)pos+=cnt(f->l)+1;
        t=f;
    }
}

```

```

        return t;
    }

    int n,m,c,q;
    map<pair<int,int>,int> w; // owner

    pitem t[105][8005];

    bool join(int k, int x, int y){
        int a,b;
        pitem r0=root(t[k][x],a),r1=root(t[k][y],b);
        if(a&&a<cnt(r0)-1||b&&b<cnt(r1)-1){puts("Forbidden:
            monopoly.");return false;}
        if(r0==r1){puts("Forbidden: redundant.");return false;}
        if(a==0)r0->rev^=1;
        if(b!=0)r1->rev^=1;
        //printf(" %d %d %d %d\n",a,b,cnt(r0),cnt(r1));
        pitem _;
        merge(_,r0,r1);
        return true;
    }

    void disjoint(int k, int x, int y){
        int a,b;
        pitem r0=root(t[k][x],a),r1=root(t[k][y],b);
        assert(r0==r1);assert(abs(a-b)==1);
        pitem _,_;
        //split(a<b?t[k][y]:t[k][x],_,_);
        split(r0,_,_,max(a,b));
    }

    int main(){
        fore(i,0,105)fore(j,0,8005)t[i][j]=new item();
        while(scanf("%d%d%d%d",&n,&m,&c,&q),n){
            fore(i,0,c)fore(j,0,n)t[i][j]->l=t[i][j]->r=t[i][j]->f=0;
            w.clear();
            fore(i,0,m){
                int x,y,k;
                scanf("%d%d%d",&x,&y,&k);x--;y--;k--;
                w[{x,y}]=k;

```

```

                assert(join(k,x,y));
            }
            while(q--){
                int x,y,k;
                scanf("%d%d%d",&x,&y,&k);x--;y--;k--;
                if(!w.count({x,y})){puts("No such
                    cable.");continue;}
                int kk=w[{x,y}];
                if(kk==k){puts("Already owned.");continue;}
                if(join(k,x,y)){
                    w[{x,y}]=k;
                    disjoint(kk,x,y);
                    puts("Sold.");
                }
            }
            puts("");
        }
        return 0;
    }
}

```

9.24 UnionFind

```

#include "../Header.cpp"

struct UF{
    vl p, r, sz;
    UF uf();
    UF(ll n){
        r.assign(n, 0);
        sz.assign(n, 1);
        rep(i, n)
            p.push_back(i);
    }
    ll find(ll x) {return p[x] = p[x] == x ? x : find(p[x]);}
    void join(ll x, ll y){
        if ((x = find(x)) == (y = find(y))) return;
        if(r[x] < r[y]) swap(x, y);
        if(r[x] == r[y]) r[x]++;
        p[y] = x;
        sz[x] += sz[y];
    }
}

```

```

    }
};

// With rollback
struct UF{
    vl p, r, sz;
    stack<vl> S;
    UF () {}
    UF(ll n){
        r.assign(n, 0);
        sz.assign(n, 1);
        for(ll i = 0; i < n; i++)
            p.push_back(i);
    }
    ll find(ll x) {return p[x] == x ? x : find(p[x]);}
    void join(ll x, ll y){
        if ((x = find(x)) == (y = find(y))) return;
        if(r[x] < r[y]) swap(x, y);
        if(r[x] == r[y]) r[x]++;
        S.push({x, y, p[x], p[y]});
        p[y] = x;
        sz[x] += sz[y];
    }
    void rollback(){
        auto a = S.top(); S.pop();
        p[a[0]] = a[2];
        p[a[1]] = a[3];
        sz[a[0]] -= sz[a[1]];
    }
};

```

9.25 WaveletTree

```

#include "../Header.cpp"
typedef vector<int>::iterator iter;

//Wavelet tree with succinct representation of bitmaps
struct WaveTreeSucc {
    vector<vector<int> > C; int s;

```

```

    // sigma = size of the alphabet, ie., one more than the maximum
    // element
    // in S.
    WaveTreeSucc(vector<int> &A, int sigma) : C(sigma*2), s(sigma) {
        build(A.begin(), A.end(), 0, s-1, 1);
    }

    void build(iter b, iter e, int L, int U, int u) {
        if (L == U)
            return;
        int M = (L+U)/2;

        // C[u][i] contains number of zeros until position i-1: [0,i)
        C[u].reserve(e-b+1); C[u].push_back(0);
        for (iter it = b; it != e; ++it)
            C[u].push_back(C[u].back() + (*it<=M));

        iter p = stable_partition(b, e, [=](int i){return i<=M;});

        build(b, p, L, M, u*2);
        build(p, e, M+1, U, u*2+1);
    }

    // Count occurrences of number c until position i.
    // ie, occurrences of c in positions [i,j]
    int rank(int c, int i) const {
        // Internally we consider an interval open on the left: [0, i)
        i++;
        int L = 0, U = s-1, u = 1, M, r;
        while (L != U) {
            M = (L+U)/2;
            r = C[u][i]; u*=2;
            if (c <= M)
                i = r, U = M;
            else
                i -= r, L = M+1, ++u;
        }
        return i;
    }
}

```

```

// Find the k-th smallest element in positions [i,j].
// The smallest element is k=1
int quantile(int k, int i, int j) const {
    // internally we consider an interval open on the left: [i,
    // j)
    j++;
    int L = 0, U = s-1, u = 1, M, ri, rj;
    while (L != U) {
        M = (L+U)/2;
        ri = C[u][i]; rj = C[u][j]; u*=2;
        if (k <= rj-ri)
            i = ri, j = rj, U = M;
        else
            k -= rj-ri, i -= ri, j -= rj,
            L = M+1, ++u;
    }
    return U;
}

// Count number of occurrences of numbers in the range [a, b]
// present in the sequence in positions [i, j], ie, if
// representing a grid it

```

```

// counts number of points in the specified rectangle.
mutable int L, U;
int range(int i, int j, int a, int b) const {
    if (b < a or j < i)
        return 0;
    L = a; U = b;
    return range(i, j+1, 0, s-1, 1);
}

int range(int i, int j, int a, int b, int u) const {
    if (b < L or U < a)
        return 0;
    if (L <= a and b <= U)
        return j-i;
    int M = (a+b)/2, ri = C[u][i], rj = C[u][j];
    return range(ri, rj, a, M, u*2) +
        range(i-ri, j-rj, M+1, b, u*2+1);
}
};

```
