Team notebook

Teorema: no me acuerdo

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Contents

1 DP

1.1 1D1D

```
#include <bits/stdc++.h>
using namespace std;

#define ll long long
#define ar array
#define rep(i, n) for(int i = 0; i<(int)n; ++i)
#define repx(i, a ,b) for(int i = (int)a; i<(int)b; ++i)

const int mxN = 2e5+5, M = 1e9+7;
int n, s, dp[mxN], t[mxN], f[mxN];

int w(int j, int i){ //
    if(j >= i) return 1e9; // here is the cost function from (j, i]
    return s*(f[n]-f[j]) + t[i]*(f[i]-f[j]);
}

struct DD{ // index 0 is used as neutral state
    vector<pair<int, int>> v; // (start pos, best k)
    DD() { v.push_back(make_pair(0, 0)); }
    int qry(int i){
        return (--lower_bound(v.begin(), v.end(), make_pair(i+1,
            0)))->second;
    }
    void upd(int x){
        for(int i = (int)v.size()-1; i>=0; --i){
            int y = v[i].first, oldk = v[i].second;
            if(y > x && dp[x] + w(x, y) < dp[oldk] + w(oldk, y))
                v.pop_back();
            else{
                int l = y+1, r = n+1;
                while(l < r){
                    int mid = (l+r)/2;
                    if(dp[x] + w(x, mid) < dp[oldk] + w(oldk, mid))
                        r = mid;
                    else l = mid+1;
                }
                if(r != n+1) v.push_back(make_pair(r, x)); break;
            }
        }
        if(v.size() == 0) v.push_back(make_pair(0, x));
    }
};
    cin >> n >> s;
    DD dd;
    t[0] = f[0] = dp[0] = 0;
    for(int i = 1; i<=n; ++i){
        cin >> t[i] >> f[i]; t[i] += t[i-1]; f[i] += f[i-1];
    }
    for(int i = 1; i<=n; ++i){
        int k = dd.qry(i); dp[i] = dp[k] + w(k, i);dd.upd(i);
    }
    cout << dp[n] << '\n';
// https://vjudge.net/problem/OpenJ_Bailian-1180
```

1.2 1D2DMaxSum-Multiplication

```
#include "../Header.cpp"

int main(){
    // 1D Max Array Sum
    int n = 9, A[] = { 4, -5, 4, -3, 4, 4, -4, 4, -5 }; //Allow
        all negative numbers
    int sum = A[0], ans = A[0];
    for (int i = 1; i < n; i++){
        sum = max(A[i] + sum, A[i]); // Ignores sum if prev sum is
            worse than A[i]
        ans = max(ans, sum);
    }
    cout << ans << "\n";
    // 2D Max Array Sum
    int B [100][100];
    ans = -INF;
    cin>>n;
    for (int i = 0; i < n; i++) for (int j = 0; j < n; j++){
        cin >> B[i][j];
        if (j > 0) B[i][j] += B[i][j-1]; // Acum sum per Row
    }
```

```
for (int l = 0; l < n; l++) for (int r = l; r < n; r++){
    sum = B[0][r];
    int SubAns = B[0][r];
    if (l > 0){ sum -= B[0][l-1]; SubAns -= B[0][l-1]; }
    for (int row = 1; row < n; row++){
        int aux = B[row][r];
        if(l > 0) aux -= B[row][l-1];
        sum = max(sum + aux, aux);
        SubAns = max (SubAns, sum);
    }
    ans = max(ans, SubAns);
}
cout << ans << "\n";

// Max Array Multiplication
vl c;
bool o = 0;
ans = 1;
ll miniend = 1, maxiend = 1;
for (int i = 0; i < c.size(); i++){
    if(c[i] > 0){
        o = 1;
        if(miniend < 0)miniend *= c[i];
        maxiend *= c[i];
    }
    else if(c[i] == 0){
        miniend = 1;
        maxiend = 1;
    }else{
        int aux = maxiend;
        maxiend = max(1LL, miniend * c[i]);
        miniend = aux*c[i];
    }
    ans = max(ans, maxiend);
}
if(ans == 1 && !o) cout << "0\n";
else{
    cout << ans << "\n";
}

// n dimension acumulative sum
    for dim = 0 to 4
        for a = 0 to na-1
            for b = 0 to nb-1
                for c = 0 to nc - 1
                    for d = 0 to nd - 1
                        pa = a - (dim==0); pb = b - (dim==1); pc = c
                            - (dim==2);
                        pd = d - (dim==3);
                        if (pa >= 0 && pb >= 0 && pc >= 0 && pd >= 0)
                            dp(a, b, c, d) += dp (pa, pb, pc, pd)
    // o por cada celda
    for x in S //(celda de menor a mayor tal que todas las
        anteriores estan procesadas)
}
```

1.3 CoinProblem

```
#include "../Header.cpp"
// Number of ways of reaching a quantity n from a set of coins c
int main(){
        int c[5] = {1, 5, 10, 25, 50};
        int n;
        while(cin >> n){
                int m[n+1];
                m[0] = 1;
                for(int i = 1; i <= n+1; i++) m[i] = 0;
                for(int j = 0; j < 5; j++){
                        for(int i = 1; i <= n+1; i++){
                                if(i - c[j] >= 0){
                                        m[i] += m[i - c[j]];
                                        //m[i]=min(m[i],m[i-c[j]]+1);
                                            for minimum coins
                                }
                        }

                }
                cout << m[n] << "\n";
        }
}
```

1.4 ConvexHullTrick

```
#include "../Header.cpp"

typedef ll tc;
struct Line{tc m,h;};
struct CHT { // for minimum (for maximum just change the sign of
    lines)
        vector<Line> c;
        int pos=0;
        tc in(Line a, Line b){
                tc x=b.h-a.h,y=a.m-b.m;
                return x/y+(x%y?!((x>0)^(y>0)):0); // ==ceil(x/y)
        }
        void add(tc m, tc h){ // m's should be non increasing
                Line l=(Line){m,h};
                if(c.size()&&m==c.back().m){
                        l.h=min(h,c.back().h);c.pop_back();if(pos)pos--;
                }
                while(c.size()>1&&in(c.back(),l)<=in(c[c.size()-2],c.back())){
                        c.pop_back();if(pos)pos--;
                }
                c.pb(l);
        }
        inline bool fbin(tc x, int m){return in(c[m],c[m+1])>x;}
        tc eval(tc x){
                // O(log n) query:
                int s=0,e=c.size();
                while(e-s>1){int m=(s+e)/2;
                        if(fbin(x,m-1))e=m;
                        else s=m;
                }
                return c[s].m*x+c[s].h;
                // O(1) query (for ordered x's):
                while(pos>0&&fbin(x,pos-1))pos--;
                while(pos<c.size()-1&&!fbin(x,pos))pos++;
                return c[pos].m*x+c[pos].h;
        }
};
```

```
struct Line {
    ll m, c, id;
    ll calc(ll x) {

        return m * x + c;
    }
};
bool obsolete(Line a, Line b, Line c){
    return (c.c - a.c) * (a.m - b.m) < (a.m - c.m) * (b.c - a.c);
}

vector<Line>lines;
void insert(Line l) {
    while(lines.size() > 1) {
        ll sz = lines.size();
        if(obsolete(lines[sz-2], lines[sz-1], l)){
            lines.pop_back();
        } else break;
    }
    lines.push_back(l);
}
```

1.5 ConvexHullTrickDynamic

```
typedef ll tc;
const tc is_query=-(1LL<<62); // special value for query
struct Line {
        tc m,b;
        mutable multiset<Line>::iterator it,end;
        const Line* succ(multiset<Line>::iterator it) const {
                return (++it==end? NULL : &*it);}
        bool operator<(const Line& rhs) const {
                if(rhs.b!=is_query)return m<rhs.m;
                const Line *s=succ(it);
                if(!s)return 0;
                return b-s->b<(s->m-m)*rhs.m;
        }
};
struct HullDynamic : public multiset<Line> { // for maximum
        bool bad(iterator y){
```

```
            iterator z=next(y);
            if(y==begin()){
                    if(z==end())return false;
                    return y->m==z->m&&y->b<=z->b;
            }
            iterator x=prev(y);
            if(z==end())return y->m==x->m&&y->b<=x->b;
            return
                (x->b-y->b)*(z->m-y->m)>=(y->b-z->b)*(y->m-x->m);
        }
        iterator next(iterator y){return ++y;}
        iterator prev(iterator y){return --y;}
        void add(tc m, tc b){
            iterator y=insert((Line){m,b});
            y->it=y;y->end=end();
            if(bad(y)){erase(y);return;}
            while(next(y)!=end()&&bad(next(y)))erase(next(y));
            while(y!=begin()&&bad(prev(y)))erase(prev(y));
        }
        tc eval(tc x){
            Line l=*lower_bound((Line){x,is_query});
            return l.m*x+l.b;
        }
};
```

1.6 Digit$_{dp}$

```
int dp[12][12][2]; // dp[i][s][f] {i: posicion, s: estado del
    problema, f: act < s}
int k, d;

// solve(r) - solve(l-1)
int call(int pos, int cnt, int f){
    if(cnt > k) return 0;
    if(pos == num.size()){
        if(cnt == k) return 1;
        return 0;
    }
    if(dp[pos][cnt][f] != -1) return dp[pos][cnt][f];
    int res = 0, LMT;
    if(f == 0) LMT = num[pos];
    else LMT = 9;
    /// Try to place all the valid digits such that the number
        doesn't exceed b
    for(int dgt = 0; dgt<=LMT; dgt++){
        int nf = f, ncnt = cnt;
        if(f == 0 && dgt < LMT) nf = 1; /// The number is getting
            smaller at this position
        if(dgt == d) ncnt++;
        if(ncnt <= k) res += call(pos+1, ncnt, nf);
    }
    return dp[pos][cnt][f] = res;
}
int solve(string s){
    num.clear();
    while(s.size()){
        num.push_back((s.back()-'0')%10);
        s.pop_back();
    }
    reverse(num.begin(), num.end());
    memset(dp, -1, sizeof(dp));
    return call(0, 0, 0);
}
```

1.7 DivideConquerDP

```
#include "../Header.cpp"

// dp(i, j) = min dp(i-1,k-1) + C(k,j) for all k in [0, j]
// C(a,c) + C(b, d) <= C(a,d) + C(b,c) for all a <= b <= c <= d
vp c;
vl acum1, acum2;

ll cost(ll i, ll j){
  return c[j].first * (acum1[j+1] - acum1[i]) - (acum2[j+1] -
      acum2[i]);
}
vector<ll> last, now;

void compute(int l, int r, int optl, int optr){
```

```
    if (l > r) return;
    int mid = (l + r) / 2;
    pair<ll, int> best = {cost(0, mid), -1};
    for(int k = max(1, optl); k < min(mid, optr) + 1; k++)
        best = min(best, {last[k - 1] + cost(k, mid), k});
    now[mid] = best.first;

    compute(l, mid - 1, optl, best.second);
    compute(mid + 1, r, best.second, optr);
}

int main(){
  ios_base::sync_with_stdio(0); cin.tie(0);
  ll n, k, x, w;
  while(cin >> n >> k){
    c.clear();
    for(int i = 0; i < n; i++){
      cin >> x >> w;
      c.push_back({x, w});
    }
    acum1.clear(); acum2.clear();
    acum1.push_back(0); acum2.push_back(0);

    for(int i = 0; i < n; i++){
      acum1.push_back(c[i].second);
      acum2.push_back(c[i].first * c[i].second);
      acum1.back() += acum1[i];
      acum2.back() += acum2[i];
    }
    last.assign(n, INF);
    now.resize(n);
    for(int i = 0; i < k; i++) { compute(0, n - 1, 0, n - 1);
        swap(last, now); }

    cout<< last[n-1]<<"\n";

  }
}
```

1.8 Knapsack

```
#include "../Header.cpp"

int V[10000], W[10000], M[102][10202];

// index, capacity
int DP(int i, int c){
    if(i==-1){
        return 0;
    }
    if(c==0) return 0;
    if(M[i][c] != -1) return M[i][c];
    M[i][c] = DP(i-1, c);
    if(W[i] <= c){
        M[i][c] = max(M[i][c], DP(i-1, c - W[i]) + V[i]);
    }
    return M[i][c];
}

// Variation
int usados=0,espacio_usado;
int knapSack(int W, int wt[], int val[], int n){
   int i, w;
   int K[n+1][W+1][3];
   for (i = 0; i <= n; i++){
       for (w = 0; w <= W; w++){
           if (i==0 || w==0){
               K[i][w][0]= 0;
               K[i][w][1]= 0;
               K[i][w][2]= 0;
           }
           else if (wt[i-1] <= w){
                K[i][w][0] = max(val[i-1] +
                    K[i-1][w-wt[i-1]][0],K[i-1][w][0]);
                if(K[i-1][w][0]>val[i-1] + K[i-1][w-wt[i-1]][0]){
                    K[i][w][1]=K[i-1][w][1];
                    K[i][w][2]=K[i-1][w][2];
                }else{
                   K[i][w][1]=K[i-1][w-wt[i-1]][1]+wt[i-1];
                   K[i][w][2]=K[i-1][w-wt[i-1]][2]+1;
```

```
                }
            }else{
                K[i][w][0] = K[i-1][w][0];
                K[i][w][1] = K[i-1][w][1];
                K[i][w][2] = K[i-1][w][2];
            }
        }
    }
    usados=K[n][W][2];
    espacio_usado=K[n][W][1];
    return K[n][W][0];
}

int main(){
    int v,W,t;
    cin>>t;
    for(int o=0;o<t;o++){
        W=50;
        usados=0;
        cin>>v;
        int val[v];
        int wt[v];
        for(int i=0;i<v;i++){
            cin>>val[i];
            cin>>wt[i];
        }
        int n = sizeof(val)/sizeof(val[0]);
        cout<<knapSack(W, wt, val, n)<<" brinquedos"<<endl;
        cout<<"Peso: "<<espacio_usado<<" kg"<<endl;
        cout<<"sobra(m) "<<v-usados<<" pacote(s)"<<endl<<endl;
    }
}
```

1.9 KnuthOptimization

```
int N; vector<int> A;
vector<vector<int>> DP, OPT;

int main(){
    DP.assign(N + 1, vi(N + 1));
    OPT.assign(N + 1, vi(N + 1));
    rep(i, N) DP[i][i + 1] = A[i + 1] - A[i], OPT[i][i + 1] = i;
    repx(d, 2, N + 1) rep(l, N + 1 - d){
        int r = l + d, l_ = OPT[l][r - 1], r_ = OPT[l + 1][r];
        DP[l][r] = 1e9;
        repx(i, l_, r_ + 1){
            int aux = DP[l][i] + DP[i][r] + A[r] - A[l];
            if (aux < DP[l][r]) DP[l][r] = aux, OPT[l][r] = i;
        }
    }
}
```

1.10 LongestIncreasingSubsequence

```
#include "../Header.cpp"

vl A, p;
void print_LIS(int i) {                    // backtracking
    routine
  if (p[i] == -1) { printf("%d", A[i]); return; }// base case
  print_LIS(p[i]);                         // backtrack
  printf(" %d", A[i]);
}

//O(nlogn)
int lis(vector<int> const& a) {
    int n = a.size();
    const int INF = 1e9;
    vector<int> d(n+1, INF);
    d[0] = -INF;
    for (int i = 0; i < n; i++) {
        int j = upper_bound(d.begin(), d.end(), a[i]) - d.begin();
        if (d[j-1] < a[i] && a[i] < d[j])
            d[j] = a[i];
    }
    int ans = 0;
    for (int i = 0; i <= n; i++) {
        if (d[i] < INF)
            ans = i;
    }
```

```
    return ans;
}
int main(){
    ll t,n; cin>>t;
    while(t--){
        int x; cin>>n;
        for(int i=0;i<n;i++){
            cin>>x;
            A.push_back(x);
        }
        ll LIS[100][100] // LIS for any (i, j)
        for(int z = 0; z < n; z++){
            int k = z, lis_end = z;
            vl L(n, 0), L_id(n, 0);
            p.assign (n, -1)
            for (int i = z; i < n; ++i) {
                int pos = lower_bound(L.begin() + z, L.begin()+k,
                    c[i]) - L.begin();
                if(A[i]==L[pos])pos++;//For non strickly increasing
                    subsequence
                L[pos] = c[i];
                L_id[pos] = i;
                p[i] = pos ? L_id[pos-1] : -1;
                if (pos == k) {
                k = pos+1;
                lis_end = i;
                }
            }
            for(int i = z; i < n; i++){
                if(p[i] == -1) LIS[z][i] = 1;
                else LIS[z][i] = 1 + LIS[z][p[i]];
            }
        }
        cout<<"Final LIS is of length: "<< k<<"\n";
        print_LIS(lis_end);cout<<"\n";
        //DP
        vl LI(n, 0), LD(n,0);
        ll in=0,dec=0;
        for(int i=0;i<n;i++){
            LI[i]=1;
            LD[i]=1;
            for(int j=0;j<i;j++){
                if(A[j]<A[i])
                    LI[i]=max(LI[i],LI[j]+1);
                if(A[j]>A[i])
                    LD[i]=max(LD[i],LD[j]+1);
            }
            in=max(in,LI[i]);
            dec=max(dec,LD[i]);
        }
    }
}
```

1.11 connected$_{c}$omponent$_{d}$p

```
int n, k, dp[mxN][mxN][1005][3];
int a[mxN];

int ff(ll i, ll c, ll sum, ll b){ // {i, components, sum,
    borders} 1 indexed
    if(b > 2 || sum > k) return 0; // k = limit sum
    if(c == 0 && i > 1) return 0;
    if(i == n+1) return b == 2 && c == 1; // array completely
        filled
    int &ret = dp[i][c][sum][b];
    if(ret != -1) return ret;          // this behind changes
        between problems
    int nsum = sum + (a[i]-a[i-1])*(2*c-b); // all unknown
        positions equals to a[i]
    ll ans = 0;
    if(c >= 2) ans += (c-1)*ff(i+1, c-1, nsum, b); // merge two cc
    if(c >= 1) ans += (2LL*c-b)*ff(i+1, c, nsum, b); // add to a
        component end
    ans += (c+1-b)*(i+1, c+1, nsum, b); // create new component
    if(b < 2) ans += (2LL-b)*ff(i+1, c+1, nsum, b+1); // create
        new end
    if(b < 1) ans += (2LL-b)*ff(i+1, c, nsum, b+1); // extend cc
        to a border
    ans %= M; return ret = ans;
}
```

2 Geometry

2.1 2DAlgorithms

```
#include "../Header.cpp"

double DEG_to_RAD(double d) { return d*PI / 180.0; }
double RAD_to_DEG(double r) { return r*180.0 / PI; }

struct point { db x, y;
    point() { x = y = 0.0; }
    point(db _x, db _y) : x(_x), y(_y) {}
    bool operator <(const point& p) const { return (x < p.x ? true
        : (x == p.x && y < p.y)); }
    bool operator == (const point& p) const { return abs(p.x - x)
        < EPS && abs(p.y - y) < EPS; }
    point operator + (const point& p) const { return point(x +
        p.x, y + p.y); }
    point operator - (const point& p) const { return point(x -
        p.x, y - p.y); }
    point operator * (db p) const { return point(x * p, y * p); }
    point operator / (db p) const { return point(x / p, y / p); }
    db operator^(const point &p) const {return x * p.y - y * p.x; }
    db operator*(const point &p) const {return x * p.x + y * p.y; }
    db norm_sq() const{ return x*x + y*y; }
    point rot(){ return point(-y, x); }
    point rot45(){ return point(x + y, y - x); }

    // by angles but with cross
    bool half() const { return y > 0 || (y == 0 && x > 0); }
    bool operator<(const point &p) const
    {
        int h1 = half(), h2 = p.half();
        return h1 != h2 ? h1 > h2 : ((*this) ^ p) > 0;
    }

    db ang()
    {
        double a = atan2(y, x);
        if (a < 0) a += 2.0 * PI;
        return a;
    }

};

db dist(const point& p1,const point& p2) {
  return sqrt((p1.x-p2.x)*(p1.x-p2.x)+ (p1.y-p2.y)*(p1.y-p2.y)); }
db dist_sq(point p1, point p2) {
  return (p1.x - p2.x)*(p1.x - p2.x)+(p1.y - p2.y)*(p1.y - p2.y); }


point rotate(point p, db rad) {
  return point(p.x * cos(rad) - p.y*sin(rad),
               p.x * sin(rad) + p.y*cos(rad)); }


struct line { db a, b, c; };


void pointsToLine(point p1, point p2, line &l) {
  if (fabs(p1.x-p2.x) < EPS)              // vertical line is fine
    l = {1.0, 0.0, -p1.x};                      // default values
  else {
    db a = -(db)(p1.y-p2.y) / (p1.x-p2.x);
    l = {a,
         1.0,             // IMPORTANT: we fix the value of b to 1.0
         -(db)(a*p1.x) - p1.y}; }
  }

// for integers, normalized
void pointsToLine(point& p1, point p2, line &l) {
  l.a = p1.y - p2.y;
  l.b = p2.x - p1.x;
  l.c = p1.x * (p2.y - p1.y) - p1.y * (p2.x - p1.x);
  ll g = __gcd(abs(l.a), __gcd(abs(l.b), abs(l.c)));
  ll sgn = 1;
  if(l.a < 0 || (l.a == 0 && l.b < 0))sgn = -1;
  l.a /= g * sgn; l.b /= g * sgn; l.c /= g * sgn;
}
// not needed since we will use the more robust form: ax + by + c
    = 0
struct line2 { db m, c; };   // another way to represent a line
```

```
int pointsToLine2(point p1, point p2, line2 &l) {
 if (abs(p1.x-p2.x) < EPS) {        // special case: vertical line
   l.m = INF;                  // l contains m = INF and c = x_value
   l.c = p1.x;                // to denote vertical line x = x_value
   return 0;  // we need this return variable to differentiate
       result
 }
 else {
   l.m = (db)(p1.y-p2.y) / (p1.x-p2.x);
   l.c = p1.y - l.m*p1.x;
   return 1;    // l contains m and c of the line equation y = mx
       + c
} }

bool areParallel(line l1, line l2) {   // check coefficients a & b
  return (fabs(l1.a-l2.a) < EPS) && (fabs(l1.b-l2.b) < EPS); }

bool areSame(line l1, line l2) {       // also check coefficient c
  return areParallel(l1 ,l2) && (fabs(l1.c-l2.c) < EPS); }

// returns true (+ intersection point) if two lines are intersect
bool areIntersect(line l1, line l2, point &p) {
  if (areParallel(l1, l2)) return false;       // no intersection
  // solve system of 2 linear algebraic equations with 2 unknowns
  p.x = (l2.b*l1.c - l1.b*l2.c) / (l2.a*l1.b - l1.a*l2.b);
  // special case: test for vertical line to avoid division by zero
  if (fabs(l1.b) > EPS) p.y = -(l1.a*p.x + l1.c);
  else                  p.y = -(l2.a*p.x + l2.c);
  return true; }

// Or use pointsToSlope, Revisar, mejor con 2 puntos
void perpendicular_line(point a, line l, line& ans)
{
    point b((-l.b*a.y-l.c)/l.a,a.y+1);
    b.x-=a.x;
    b.y-=a.y;
    b = rotate(b,90);
    b.x+=a.x;
    b.y+=a.y;
    pointsToLine(a,b, ans);
}


//Scalar projection of vector a onto vector b
// if s < -EPS or s > |b| + EPS then the projection is not on the
    segment
db sproject(point a, point b)
{
  return a*b/sqrt(b.norm_sq());
}

bool onSegment(const point& p, const point& p1, const point& p2)
{
    bool x = (abs(p1.x - p2.x) < EPS && abs(p.x - p2.x) < EPS) ||
        (p.x <= max(p1.x, p2.x) && p.x >= min(p1.x, p2.x));
    bool y = (abs(p1.y - p2.y) < EPS && abs(p.y - p2.y) < EPS) ||
        (p.y <= max(p1.y, p2.y) && p.y >= min(p1.y, p2.y));
    return x && y;
}

// convert point and gradient/slope to line, A PARTIR DE UNA
    DIRECCION M
// usar 1/l.a para calcular perpendicular
void pointSlopeToLine(point p, db m, line &l) {
  l.a = -m;                                          // always -m
  l.b = 1;                                            // always 1
  l.c = -((l.a*p.x) + (l.b*p.y)); }              // compute this

void closestPoint(line l, point p, point &ans) {
  line perpendicular;      // perpendicular to l and pass through
      p
  if (fabs(l.b) < EPS) {          // special case 1: vertical line
    ans.x = -(l.c); ans.y = p.y;    return; }

  if (fabs(l.a) < EPS) {        // special case 2: horizontal line
    ans.x = p.x;    ans.y = -(l.c); return; }

  pointSlopeToLine(p, 1/l.a, perpendicular);      // normal line
  // intersect line l with this perpendicular line
  // the intersection point is the closest point
  areIntersect(l, perpendicular, ans); }
```

```
// returns the reflection of point on a line
void reflectionPoint(line l, point p, point &ans) {
  point b;
  closestPoint(l, p, b);                  // similar to distToLine
  point v = (b - p);                         // create a vector
  ans = p + v + v; }       // translate p twice

// returns the distance from p to the line defined by
// two points a and b (a and b must be different)
// the closest point is stored in the 4th parameter (byref)
db distToLine(point p, point a, point b, point &c) {
  // formula: c = a + u*ab
  point ap = (p - a), ab = (b - a);
  db u = ap * ab / ab.norm_sq();
  c = a + ab * u;                // translate a to c
  return dist(p, c); }         // Euclidean distance between p and c

// returns the distance from p to the line segment ab defined by
// two points a and b (still OK if a == b)
// the closest point is stored in the 4th parameter (byref)
db distToLineSegment(point p, point a, point b, point &c) {
  point ap = (p - a), ab = (b - a);
  db u = ap * ab / ab.norm_sq();
  if (u < 0.0) { c = point(a.x, a.y);              // closer to a
    return dist(p, a); }       // Euclidean distance between p and a
  if (u > 1.0) { c = point(b.x, b.y);              // closer to b
    return dist(p, b); }       // Euclidean distance between p and b
  return distToLine(p, a, b, c); }      // run distToLine as above

bool ccw(point p, point q, point r) {
  return ((q - p)^(r - p)) > -EPS; }

// returns true if point r is on the same line as the line pq
bool collinear(point p, point q, point r) {
  return fabs(((q - p)^(r - p))) < EPS; }


// angle from 0 to 2*PI
db anglet(point a, point o, point b) { // returns angle aob in rad
  point oa = (a - o), ob = (b - o);
  db ang = acos(oa * ob / sqrt(oa.norm_sq()*ob.norm_sq()));
  if(ang!=0&&!collinear(a,o,b)&&ccw(a,o,b))ang = 2*PI - ang;
  return ang; } // better

db angle(point a, point o, point b) { // returns angle aob in rad
  point oa = (a - o), ob = (b - o);
  return acos(oa * ob / sqrt(oa.norm_sq()*ob.norm_sq())); }

point min(point a,point b)
{
    if(a<b)return a;
    return b;
}
point max(point a, point b)
{
    if(!(a<b))return a;
    return b;
}

// 0 -> No intersection, 1 -> Point intersection, 2 -> segment
    intersection
int SegmentIntersection(point a1, point a2, point b1, point b2,
    point& ans, point& ans2)
{
    line A,B;
    point I;
    pointsToLine(a1,a2,A);
    pointsToLine(b1,b2,B);
    if(areSame(A,B)&&!(a1==a2)&&!(b1==b2))
    {
        ans=max(min(a1,a2),min(b1,b2));
        ans2=min(max(a1,a2),max(b1,b2));
        if(ans2<ans)return 0;
        else if(ans == ans2)return 1;
        return 2;
    }
    if (a1==a2&&b1==b2)
    {
        if(a1==b1)
        {
            ans=a1;
```

```
            return 1;
        }
        return 0;
    }
    if(a1==a2)
    {
        if(fabs(distToLineSegment(a1, b1, b2, ans)-0.0) < EPS)
        {
            ans=a1;
            return 1;
        }
        return 0;
    }
    if(b1==b2)
    {
        if(fabs(distToLineSegment(b1, a1, a2, ans)-0.0) < EPS)
        {
            ans=b1;
            return 1;
        }
        return 0;
    }
    if (areIntersect(A,B,I) && fabs(distToLineSegment(I, a1, a2,
        ans)-0.0) < EPS && fabs(distToLineSegment(I, b1, b2,
        ans)-0 < EPS))
    {
        return 1;
    }
    return 0;
}
```

2.2 3DAlgorithms

```
#include "../Header.cpp"


struct point { db x, y, z;
    point() { x = y = z = 0.0; }
    //point(db r, db u, db v) : x(r*cos(u)*cos(v)),
        y(r*cos(u)*sin(v)), z(r*sin(u)) {}
        point(db _x, db _y, db _z) : x(_x), y(_y), z(_z) {}
        point operator^(const point &p) const {
               return { y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y *
                   p.x};
        }
        db dot(point& p) { return x*p.x + y*p.y + z*p.z; }
    db norm() { return sqrt(x*x + y*y + z*z); }

        bool operator == (const point& p) const
    {
        return abs(p.x - x) < EPS && abs(p.y - y) < EPS && abs(p.z
            - z) < EPS;
    }
    point operator + (const point& p) const
    {
        return point(x + p.x, y + p.y, z + p.z);
    }
    point operator - (const point& p) const
    {
        return point(x - p.x, y - p.y, z - p.z);
    }
    point operator * (db a) const
    {
        return point(x * a, y * a, z * a);
    }
    point operator / (db a) const
    {
        return point(x / a, y / a, z / a);
    }
        point unit() {
        db d = norm();
        return {x/d,y/d,z/d};
    }

};
db angle2(point& x, point& y)
{
        return acos(x.dot(y) / (R*R));
}

bool in_arc(point& p1, point& p2, point& n, point& inter)
```

```
{
        db ab = angle2(p1, p2);
        db ap = angle2(p1, inter);
        point d = (p1 * cos(ap) + (n ^ p1) * sin(ap));
        return ab > ap && inter == d;
}
bool do_intersect_circles()
{
    point a1 = g[j][z], a2 = g[j][0];
    if(z < g[j].size() - 1)
        a2 = g[j][z+1];
    point p1 = route[i], p2 = route[i+1];
    point n1 = (p1^p2).unit(), n2 = (a1^a2).unit();
    point inter = n1^n2;
    if(inter.norm() < EPS)continue;
    inter = inter.unit() * R;

    if(in_arc(p1, p2, n1, inter) && in_arc(a1, a2, n2, inter))
    {
        ag.push_back(angle2(p1, inter));
        continue;
    }
    inter = inter * -1.0;
    if(in_arc(p1, p2, n1, inter) && in_arc(a1, a2, n2, inter))
    {
        ag.push_back(angle2(p1, inter));
        continue;
    }
}
```

2.3 Polygons

```
#include "../Header.cpp"

db DEG_to_RAD(db d) { return d*PI / 180.0; }

db RAD_to_DEG(db r) { return r*180.0 / PI; }

struct point { db x, y;
    point() { x = y = 0.0; }
    point(db _x, db _y) : x(_x), y(_y) {}
    bool operator <(const point& p) const { return (x < p.x ? true
        : (x == p.x && y < p.y)); }
    bool operator == (const point& p) const { return abs(p.x - x)
        < EPS && abs(p.y - y) < EPS; }
    point operator + (const point& p) const { return point(x +
        p.x, y + p.y); }
    point operator - (const point& p) const { return point(x -
        p.x, y - p.y); }
    point operator * (db p) const { return point(x * p, y * p); }
    point operator / (db p) const { return point(x / p, y / p); }
    db operator^(const point &p) const {return x * p.y - y * p.x; }
    db operator*(const point &p) const {return x * p.x + y * p.y; }
    db norm_sq() const{ return x*x + y*y; }
    point rot(){ return point(-y, x); }

    // by angles but with cross
    bool half() const { return y > 0 || (y == 0 && x > 0); }
    bool operator<(const point &p) const
    {
        int h1 = half(), h2 = p.half();
        return h1 != h2 ? h1 > h2 : ((*this) ^ p) > 0;
    }

    db ang()
    {
        double a = atan2(y, x);
        if (a < 0) a += 2.0 * PI;
        return a;
    }

  };


db dist(point& p1, point& p2) {
  return sqrt((p1.x-p2.x)*(p1.x-p2.x)+ (p1.y-p2.y)*(p1.y-p2.y)); }
db dist_sq(point p1, point p2) {
  return (p1.x - p2.x)*(p1.x - p2.x)+(p1.y - p2.y)*(p1.y - p2.y); }


// returns the perimeter, which is the sum of Euclidian distances
```

```
// of consecutive line segments (polygon edges)
db perimeter(vector<point> &P) {
  db result = 0.0;
  for (ll i = 0; i < (ll)P.size()-1; i++) // remember that P[0] =
      P[n-1]
    result += dist(P[i], P[i+1]);
  return result; }

// returns the area
db area(const vector<point> &P) {
  db result = 0.0;
  for (ll i = 0; i < (ll)P.size()-1; i++)        // Shoelace
      formula
    result += P[i]^P[i+1]; // if all points are ll
  return fabs(result)/2.0; }  // result can be ll(eger) until last
      step

db seg_integrate(point& a, point& b, db t1, db t2)
{
  // area
  point p1 = a + (b-a) * t1;
  point p2 = a + (b-a) * t2;
  return (p1^p2) / 2.0;
}

db param(point p1, point p2, point a)
{
  if(p1.x != p2.x)
  {
    db sgn = 1;
    if(p1.x > p2.x)sgn = -1;
    return (a.x - p1.x) / abs(p2.x - p1.x) * sgn;
  }
  db sgn = 1;
  if(p1.y > p2.y)sgn = -1;
  return (a.y - p1.y) / abs(p2.y - p1.y) * sgn;
}

// note: to accept collinear points, we have to change the '> 0'
// returns true if point r is on the left side of line pq
bool ccw(point p, point q, point r) {
  return ((q - p)^(r - p)) > 0; }

int orientation(point p, point q, point r) {
    ll tmp = ((q - p)^(r - p));
    return tmp < 0 ? -1 : tmp == 0 ? 0 : 1; // sign
}
/*bool do_rectangles_intersect(point dl1, point ur1, point dl2,
    point ur2) {
    return max(dl1.x, dl2.x) <= min(ur1.x, ur2.x) && max(dl1.y,
        dl2.y) <= min(ur1.y, ur2.y);
}*/

bool do_segments_intersect(point p1, point q1, point p2, point
    q2) {
    int o11 = orientation(p1, q1, p2);
    int o12 = orientation(p1, q1, q2);
    int o21 = orientation(p2, q2, p1);
    int o22 = orientation(p2, q2, q1);
    // oxx != 0 means cross intersection, no T intersection
    if (o11 != o12 && o21 != o22 && o11 != 0 && o12 != 0 && o21 !=
        0 && o22 != 0) // general case -> non-collinear
        intersection
        return true;
    return false;
}

// returns true if point r is on the same line as the line pq
bool collinear(point p, point q, point r) {
  return fabs(((q - p)^(r - p))) < EPS; }

// angle from 0 to 2*PI
db anglet(point a, point o, point b) { // returns angle aob in rad
  point oa = (a - o), ob = (b - o);
  db ang = acos(oa * ob / sqrt(oa.norm_sq()*ob.norm_sq()));
  if(ang!=0&&!collinear(a,o,b)&&ccw(a,o,b))ang = 2*PI - ang;
  return ang; } // better

db angle(point a, point o, point b) { // returns angle aob in rad
  point oa = (a - o), ob = (b - o);
  return acos(oa * ob / sqrt(oa.norm_sq()*ob.norm_sq())); }
// returns true if we always make the same turn while examining
```

```
// all the edges of the polygon one by one
bool isConvex(const vector<point> &P) {
  ll sz = (ll)P.size();
  if (sz <= 3) return false; // a point/sz=2 or a line/sz=3 is not
      convex
  bool firstTurn = ccw(P[0], P[1], P[2]);       // remember one
      result
  for (ll i = 1; i < sz-1; i++)        // then compare with the
      others
    if (ccw(P[i], P[i+1], P[(i+2) == sz ? 1 : i+2]) != firstTurn)
      return false;          // different sign -> this polygon is
          concave
  return true; }                                // this polygon is
      convex

// returns true if point p is in either convex/concave polygon P
bool inPolygon(point pt, const vector<point> &P) {
  if ((ll)P.size() < 3) return false;          // avoid point or
      line
  db sum = 0;  // assume the first vertex is equal to the last
      vertex
  for (ll i = 0; i < (ll)P.size()-1; i++) {
    if (((P[i] - pt)^(P[i+1] - pt)) > 0) //CCW check collinear
        sum += angle(P[i], pt, P[i+1]);             // left
            turn/ccw
    else sum -= angle(P[i], pt, P[i+1]); }           // right
        turn/cw
  return fabs(sum) > PI; } // 360d -> in, 0d -> out, we have large
      margin

// line segment p-q intersect with line A-B.
point lineIntersectSeg(point p, point q, point A, point B) {
  db a = B.y - A.y;
  db b = A.x - B.x;
  db c = B.x * A.y - A.x * B.y;
  db u = fabs(a * p.x + b * p.y + c);
  db v = fabs(a * q.x + b * q.y + c);
  return point((p.x * v + q.x * u) / (u+v), (p.y * v + q.y * u) /
      (u+v)); }

// cuts polygon Q along the line formed by point a -> point b
// (note: the last point must be the same as the first point)
// to cut the other side, swap (a,b)
vector<point> cutPolygon(point a, point b, const vector<point>
    &Q) {
  vector<point> P;
  for (ll i = 0; i < (ll)Q.size(); i++) {
    db left1 = (b - a)^(Q[i] - a), left2 = 0;
    if (i != (ll)Q.size()-1) left2 = (b - a)^(Q[i+1] - a);
    if (left1 > -EPS) P.push_back(Q[i]);   // Q[i] is on the left
        of ab
    if (left1 * left2 < -EPS)     // edge (Q[i], Q[i+1]) crosses
        line ab
      P.push_back(lineIntersectSeg(Q[i], Q[i+1], a, b));
  }
  if (!P.empty() && !(P.back() == P.front()))
    P.push_back(P.front());     // make P's first point = P's last
        point
  return P; }

vector<point> CH_Andrew(vector<point> &Pts) {
  ll n = Pts.size(), k = 0;
  vector<point> H(2*n);
  sort(Pts.begin(), Pts.end());     // sort the points
      lexicographically
  for (ll i = 0; i < n; i++) {                     // build lower
      hull
    while (k >= 2 && ccw(H[k-2], H[k-1], Pts[i]) <= 0) k--;
    H[k++] = Pts[i];
  }
  for (ll i = n-2, t = k+1; i >= 0; i--) {         // build upper
      hull
    while (k >= t && ccw(H[k-2], H[k-1], Pts[i]) <= 0) k--;
    H[k++] = Pts[i];
  }
  H.resize(k);
  return H;
}

point pivot(0, 0);
vector<point> CH_Graham(vector<point> &Pts) {
```

```
vector<point> P(Pts);   // copy all points so that Pts is not
    affected
ll i, j, n = (ll)P.size();
if (n <= 3) {       // corner cases: n=1=point, n=2=line,
    n=3=triangle
  if (!(P[0] == P[n-1])) P.push_back(P[0]); // safeguard from
      corner case
  return P; }                                  // the CH is P
      itself

// first, find P0 = point with lowest Y and if tie: rightmost X
ll P0 = 0;
for (i = 1; i < n; i++)                                    //
    O(n)
  if (P[i].y < P[P0].y || (P[i].y == P[P0].y && P[i].x >
      P[P0].x))
    P0 = i;
swap(P[0], P[P0]);                           // swap P[P0] with
    P[0]

// second, sort points by angle w.r.t. pivot P0, O(n log n) for
    this sort
pivot = P[0];                 // use this global variable as
    reference
sort(++P.begin(), P.end(), [](point a, point b) { // we do not
    sort P[0]
  if (collinear(pivot, a, b))                        // special
      case
    return dist(pivot, a) < dist(pivot, b); // check which one
        is closer
  db d1x = a.x-pivot.x, d1y = a.y-pivot.y;
  db d2x = b.x-pivot.x, d2y = b.y-pivot.y;
  return (atan2(d1y, d1x) - atan2(d2y, d2x)) < 0; }); // compare
      2 angles

// third, the ccw tests, although complex, it is just O(n)
vector<point> S;
S.push_back(P[n-1]); S.push_back(P[0]); S.push_back(P[1]); //
    initial S
i = 2;                                   // then, we check the
    rest
  while (i < n) {   // note: n must be >= 3 for this method to
      work, O(n)
    j = (ll)S.size()-1;
    if (ccw(S[j-1], S[j], P[i])) S.push_back(P[i++]); // left
        turn, accept
    else S.pop_back(); } // or pop the top of S until we have a
        left turn
  return S; } // return the result, overall O(n log n) due to
      angle sorting

point center_of_mass(vector <point>& Q)
{
    point ctr(0,0);
    for (ll i=0;i<Q.size()-1;i++)
    {
        ctr = ctr + Q[i];
    }
    ctr.x/=Q.size()-1;
    ctr.y/=Q.size()-1;
    return ctr;
}
// Pick's theorem
// A = i + b/2 -1
// A: Area poligon with integer coords
// i: Interior points, b: points in the segments

// with vector form of integer segment
// (x0,y0) + t(dx,dy)
ll points_in_segment(point a, point b)
{
  ll absx=abs(a.x-b.x),absy=abs(a.y-b.y);
  return __gcd(absx,absy) + 1;

}

ll memo[101][101][101];

// Dp that pass all possible convex poligons
// from the shortest in(in.y < p.y)
// p si counter cw from in, p
ll all_convex(ll in, ll p, ll q, vector<point>& Q)
```

```
{
    if(memo[in][p][q] != -1) return memo[in][p][q];
    ll ans = 0;

    for(int i = in + 1; i < Q.size(); i++)
    {
        if(i != p && i != q && ccw(Q[in], Q[q], Q[i]) && ccw(Q[p],
            Q[q], Q[i]))
        {
            ans += (all_convex(in, q, i, Q) + 1);
        }
    }
    return memo[in][p][q] = ans;
}
bool comp(point& a, point& b)
{
    return a.y < b.y;
}
//..
vector<point>Q;
sort(ALL(Q), comp);
ll ans = 0;
for(int i = 0; i < n; i++)
{
    for(int p = i + 1;p < n; p++) for(int q = p + 1; q < n; q++)
    {
        if(ccw(Q[i], Q[p], Q[q]))
            ans += all_convex(i, p, q, Q) + 1;
        else
            ans += all_convex(i, q, p, Q) + 1;
        ans %= m;
    }
}
```

2.4 SegmentIntersection

```
#include "../Header.cpp"

struct point { db x, y;
    point() { x = y = 0.0; }
    point(db _x, db _y) : x(_x), y(_y) {}
    bool operator <(const point& p) const { return (x < p.x ? true
        : (x == p.x && y < p.y)); }
    bool operator == (const point& p) const { return abs(p.x - x)
        < EPS && abs(p.y - y) < EPS; }
    point operator + (const point& p) const { return point(x +
        p.x, y + p.y); }
    point operator - (const point& p) const { return point(x -
        p.x, y - p.y); }
    point operator * (db p) const { return point(x * p, y * p); }
    point operator / (db p) const { return point(x / p, y / p); }
    db operator^(const point &p) const {return x * p.y - y * p.x; }
    db operator*(const point &p) const {return x * p.x + y * p.y; }
    db norm_sq() const{ return x*x + y*y; }
    point rot(){ return point(-y, x); }
    db ang()
    {
        double a = atan2(y, x);
        if (a < 0) a += 2.0 * PI;
        return a;
    }

  };

db dist(const point& p1,const point& p2) {
  return sqrt((p1.x-p2.x)*(p1.x-p2.x)+ (p1.y-p2.y)*(p1.y-p2.y)); }


//Constant values to be returned
constexpr int Colinear = -1, NoIntersect = 0, Intersect = 1;
constexpr int CW = 2, CCW = 3;

int orientation(point& p, point& q, point& r) {
    ll tmp = (q - p)^(r - p);
    return tmp < 0 ? CW : tmp == 0 ? Colinear : CCW; // sign
}

struct segment { point p1, p2;
  segment(point _p1, point _p2) : p1(_p1), p2(_p2) {}
};
```

```
//Returns of list of intersection points between segments s1, and
    s2
//If they do not intersect, the result is an empty vector
//If they intersect at exactly 1 point, the result contains that
    point
//If they overlap for non-0 distance, the left and right points
    of that intersection
//  are returned
bool onSegment(const point& p, const segment& s)
{
    bool x = (abs(s.p1.x - s.p2.x) < EPS && abs(p.x - s.p2.x) <
        EPS) || (p.x <= max(s.p1.x, s.p2.x) && p.x >= min(s.p1.x,
        s.p2.x));
    bool y = (abs(s.p1.y - s.p2.y) < EPS && abs(p.y - s.p2.y) <
        EPS) || (p.y <= max(s.p1.y, s.p2.y) && p.y >= min(s.p1.y,
        s.p2.y));
    return x && y;
}

vector<point> intersect(const segment& s1, const segment& s2)
{

    point a = s1.p1, b = s1.p2, c = s2.p1, d = s2.p2;

    if(orientation(a, b, c) == Colinear && orientation(a, b, d) ==
        Colinear &&
        orientation(c, d, a) == Colinear && orientation(c, d, b)
            == Colinear)
    {
        point min_s1 = min(a, b), max_s1 = max(a, b);
        point min_s2 = min(c, d), max_s2 = max(c, d);

        if(max_s1 < min_s2 || max_s2 < min_s1) return {};

        point start = max(min_s1, min_s2), end = min(max_s1,
            max_s2);
        if(start == end)
            return {start};
        else
            return {min(start, end), max(start, end)};
    }
    db a1 = b.y - a.y, a2 = d.y - c.y;
    db b1 = a.x - b.x, b2 = c.x - d.x;
    db c1 = a1*a.x + b1*a.y, c2 = a2*c.x + b2*c.y;
    db det = a1*b2 - a2*b1;
    if(abs(det) > EPS)
    {
        point inter((b2*c1 - b1*c2)/det, (a1*c2 - a2*c1)/det), aux;
        //if(distToLineSegment(inter, s1.p1, s1.p2, aux) <= EPS &&
            distToLineSegment(inter, s2.p1, s2.p2, aux) <= EPS)
        if(onSegment(inter, s1) && onSegment(inter, s2))
            return {inter};
    }
    return {};
}
```

2.5 TrianglesCircles

```
#include "../Header.cpp"

//#define double long long //Para usar enteros

db DEG_to_RAD(db d) { return d * PI / 180.0; }
db RAD_to_DEG(db r) { return r * 180.0 / PI; }

//sweepline rotating a circle around a point
// how many points are in circle radius r
// alpha = atan2(point - center) +- acos(dist/2r)

struct point { db x, y;
    point() { x = y = 0.0; }
    point(db _x, db _y) : x(_x), y(_y) {}
    bool operator <(const point& p) const { return (x < p.x ? true
        : (x == p.x && y < p.y)); }
    bool operator == (const point& p) const { return abs(p.x - x)
        < EPS && abs(p.y - y) < EPS; }
    point operator + (const point& p) const { return point(x +
        p.x, y + p.y); }
    point operator - (const point& p) const { return point(x -
        p.x, y - p.y); }
```

```
    point operator * (db p) const { return point(x * p, y * p); }
    point operator / (db p) const { return point(x / p, y / p); }
    db operator^(const point &p) const {return x * p.y - y * p.x; }
    db operator*(const point &p) const {return x * p.x + y * p.y; }
    db norm_sq() const{ return x*x + y*y; }
    point rot(){ return point(-y, x); }

    // by angles but with cross
    bool half() const { return y > 0 || (y == 0 && x > 0); }
    bool operator<(const point &p) const
    {
        int h1 = half(), h2 = p.half();
        return h1 != h2 ? h1 > h2 : ((*this) ^ p) > 0;
    }

    db ang()
    {
        double a = atan2(y, x);
        if (a < 0) a += 2.0 * PI;
        return a;
    }

};

ll insideCircle(point p, point c, ll r) { // all integer version
    ll dx = p.x - c.x, dy = p.y - c.y;
    ll Euc = dx * dx + dy * dy, rSq = r * r;        // all integer
    return Euc < rSq ? 0 : Euc == rSq ? 1 : 2; }
        //inside/border/outside

// P1 an d P2 intersections of circles and radius r -> pos of
    centers of circles of intersection
bool circle2PtsRad(point p1, point p2, db r, point &c) {
    db d2 = (p1.x - p2.x) * (p1.x - p2.x) +
            (p1.y - p2.y) * (p1.y - p2.y);
    db det = r * r / d2 - 0.25;
    if (det < 0.0) return false;
    db h = sqrt(det);
    c.x = (p1.x + p2.x) * 0.5 + (p1.y - p2.y) * h;
    c.y = (p1.y + p2.y) * 0.5 + (p2.x - p1.x) * h;
    return true; }      // to get the other center, reverse p1
        and p2

db dist(point& p1, point& p2) {           // Euclidean distance
    return sqrt((p1.x-p2.x)*(p1.x-p2.x)+ (p1.y-p2.y)*(p1.y-p2.y));
        }
db dist_sq(point p1, point p2) {
    return (p1.x - p2.x)*(p1.x - p2.x)+(p1.y - p2.y)*(p1.y -
        p2.y); }


// a = max x, b = max y from the center, AREA
db A_elipse(db a,db b)
{
    return a*b*PI;
}

// Length of segment with two points on the circunference
// separated by an angle
db chord(db r, db angle)
{
    return sqrt(2*r*r*(1-cos(angle)));
}

//Triangles
db perimeter(db ab, db bc, db ca) {
    return ab + bc + ca; }

db perimeter(point a, point b, point c) {
    return dist(a, b) + dist(b, c) + dist(c, a); }

db area(db ab, db bc, db ca) {
  // Heron's formula, split sqrt(a * b) into sqrt(a) * sqrt(b); in
      implementation
    db s = 0.5 * perimeter(ab + bc + ca);
    return sqrt(s) * sqrt(s - ab) * sqrt(s - bc) * sqrt(s - ca); }

db area(point a, point b, point c) {
    return area(dist(a, b), dist(b, c), dist(c, a)); }
```

```
// Area of the circle enclosed by an arc and a chord defined by
    an angle
db segment(db r, db angle)
{
    return angle/2.0*r*r-area(chord(r,angle),r,r);
}

// And overlaping rectangle area > 0
bool rectangles_intersect(point a1,point a2,point b1,point
    b2,point& ans1, point& ans2)
{
    if(b1<a1)
    {
        swap(a1,b1);
        swap(a2,b2);
    }
    if(b1.x>=a2.x||b1.y>=a2.y||b2.y<=a1.y)return 0;
    ans1.x=b1.x;
    ans1.y=max(b1.y,a1.y);
    ans2.x=min(b2.x,a2.x);
    ans2.y=min(b2.y,a2.y);
    return 1;

}

struct line { db a, b, c; };

void pointsToLine(point p1, point p2, line &l) {
    if (fabs(p1.x - p2.x) < EPS) {          // vertical line is
        fine
        l.a = 1.0;  l.b = 0.0;  l.c = -p1.x;         // default
            values
    } else {
        l.a = -(db)(p1.y - p2.y) / (p1.x - p2.x);
        l.b = 1.0;            // IMPORTANT: we fix the value of b
            to 1.0
        l.c = -(db)(l.a * p1.x) - p1.y;
} }

bool areParallel(line l1, line l2) {    // check coefficient a + b
    return (fabs(l1.a-l2.a) < EPS) && (fabs(l1.b-l2.b) < EPS); }

bool areSame(line l1, line l2) {       // also check coefficient c
    return areParallel(l1 ,l2) && (fabs(l1.c-l2.c) < EPS); }

// returns true (+ intersection point) if two lines are intersect
bool areIntersect(line l1, line l2, point &p) {
    if (areParallel(l1, l2)) return false;       // no intersection
    // solve system of 2 linear algebraic equations with 2 unknowns
    p.x = (l2.b * l1.c - l1.b * l2.c) / (l2.a * l1.b - l1.a *
        l2.b);
    // special case: test for vertical line to avoid division by
        zero
    if (fabs(l1.b) > EPS) p.y = -(l1.a * p.x + l1.c);
    else                  p.y = -(l2.a * p.x + l2.c);
    return true; }

db rInCircle(db ab, db bc, db ca) {
    return area(ab, bc, ca) / (0.5 * perimeter(ab, bc, ca)); }

db rInCircle(point a, point b, point c) {
    return rInCircle(dist(a, b), dist(b, c), dist(c, a)); }

// assumption: the required points/lines functions have been
    written
// returns 1 if there is an inCircle center, returns 0 otherwise
// if this function returns 1, ctr will be the inCircle center
// and r is the same as rInCircle
ll inCircle(point p1, point p2, point p3, point &ctr, db &r) {
    r = rInCircle(p1, p2, p3);
    if (fabs(r) < EPS) return 0;                // no inCircle
        center

    line l1, l2;                  // compute these two angle
        bisectors
    db ratio = dist(p1, p2) / dist(p1, p3);
    point p = p2 + (p3 - p2) * (ratio / (1 + ratio));
    pointsToLine(p1, p, l1);

    ratio = dist(p2, p1) / dist(p2, p3);
    p = p1 + (p3 - p1) * (ratio / (1 + ratio));
    pointsToLine(p2, p, l2);
```

```
    areIntersect(l1, l2, ctr);       // get their intersection
        point
    return 1; }

db rCircumCircle(db ab, db bc, db ca) {
    return ab * bc * ca / (4.0 * area(ab, bc, ca)); }

db rCircumCircle(point a, point b, point c) {
    return rCircumCircle(dist(a, b), dist(b, c), dist(c, a)); }

// assumption: the required points/lines functions have been
    written
// returns 1 if there is a circumCenter center, returns 0
    otherwise
// if this function returns 1, ctr will be the circumCircle center
// and r is the same as rCircumCircle
ll circumCircle(point p1, point p2, point p3, point &ctr, db &r){
    db a = p2.x - p1.x, b = p2.y - p1.y;
    db c = p3.x - p1.x, d = p3.y - p1.y;
    db e = a * (p1.x + p2.x) + b * (p1.y + p2.y);
    db f = c * (p1.x + p3.x) + d * (p1.y + p3.y);
    db g = 2.0 * (a * (p3.y - p2.y) - b * (p3.x - p2.x));
    if (fabs(g) < EPS) return 0;

    ctr.x = (d*e - b*f) / g;
    ctr.y = (a*f - c*e) / g;
    r = dist(p1, ctr); // r = distance from center to 1 of the 3
        points
    return 1; }

//
    https://www.nayuki.io/res/smallest-enclosing-circle/computational-geometry-lecture-6.pdf
// O(N) expected time

void smallest_enclosing_circle(vector<point>& pts, point& center,
    db& r) {
    random_shuffle(pts.begin(), pts.end());
    center = pts[0]; r = 0;
    ll N = pts.size();
    for(ll i=1;i<N;i++) {
        if (dist(pts[i] , center) > r + EPS) {
            center = pts[i];
            r = 0;
            for(ll j=0;j<i;j++) {
                if (dist(pts[j] , center) > r + EPS) {
                    center = (pts[i] + pts[j]) * 0.5;
                    r = dist(pts[i] , center);
                    for(ll k=0;k<j;k++) {
                        if (dist(pts[k] , center) > r + EPS) {
                            db rr;
                            circumCircle(pts[i], pts[j],
                                pts[k],center,rr);
                            r = dist(pts[k] , center);
                        }
                    }
                }
            }
        }
    }
}

// returns true if point d is inside the circumCircle defined by
    a,b,c
ll inCircumCircle(point a, point b, point c, point d) {
    return (a.x - d.x) * (b.y - d.y) * ((c.x - d.x) * (c.x - d.x)
        + (c.y - d.y) * (c.y - d.y)) +
        (a.y - d.y) * ((b.x - d.x) * (b.x - d.x) + (b.y - d.y) *
            (b.y - d.y)) * (c.x - d.x) +
        ((a.x - d.x) * (a.x - d.x) + (a.y - d.y) * (a.y - d.y)) *
            (b.x - d.x) * (c.y - d.y) -
        ((a.x - d.x) * (a.x - d.x) + (a.y - d.y) * (a.y - d.y)) *
            (b.y - d.y) * (c.x - d.x) -
        ((a.x - d.x) * (a.x - d.x) + (a.y - d.y) * (a.y - d.y)) *
            (b.x - d.x) * ((c.x - d.x) * (c.x - d.x) +
            (c.y - d.y) * (c.y - d.y)) -
        (a.x - d.x) * ((b.x - d.x) * (b.x - d.x) + (b.y - d.y) *
            (b.y - d.y)) * (c.y - d.y) > 0 ? 1 : 0;
}

bool canFormTriangle(db a, db b, db c) {
    return (a + b > c) && (a + c > b) && (b + c > a); }
```

```
/*
Si un punto tiene un ngulo >= 60, el lado opuesto no es el menor
    del triangulo
*/

// Function to find the circle on
// which the given three points lie
// better CIRCUMCENTER
tuple<db, db, db> findCircle(db x1, db y1, db x2, db y2, db x3,
    db y3)
{
    db x12 = x1 - x2;
    db x13 = x1 - x3;

    db y12 = y1 - y2;
    db y13 = y1 - y3;

    db y31 = y3 - y1;
    db y21 = y2 - y1;

    db x31 = x3 - x1;
    db x21 = x2 - x1;

    db sx13 = x1*x1 - x3*x3;
    db sy13 = y1*y1 - y3*y3;

    db sx21 = x2*x2 - x1*x1;
    db sy21 = y2*y2 - y1*y1;

    db f = ((sx13) * (x12)
            + (sy13) * (x12)
            + (sx21) * (x13)
            + (sy21) * (x13))
           / (2 * ((y31) * (x12) - (y21) * (x13)));
    db g = ((sx13) * (y12)
            + (sy13) * (y12)
            + (sx21) * (y13)
            + (sy21) * (y13))
           / (2 * ((x31) * (y12) - (x21) * (y13)));

    db c = -x1*x1 - y1*y1 - 2 * g * x1 - 2 * f * y1;

    // eqn of circle be x^2 + y^2 + 2*g*x + 2*f*y + c = 0
    // where centre is (h = -g, k = -f) and radius r
    // as r^2 = h^2 + k^2 - c
    db h = -g;
    db k = -f;
    db sqr_of_r = h * h + k * k - c;

    // r is the radius
    db r = sqrt(sqr_of_r);

    //cout << "Centre = (" << h << ", " << k << ")" << endl;
    //cout << "Radius = " << r;
    return make_tuple(h, k, r);
}
```

3 Graphs

3.1 AATeoremas

```
//Teoremas:

//In any bipartite graph, the number of edges in a maximum
    matching equals the number of vertices in a minimum vertex
    cover.

//Maxflow == Min cut cost

// Min cut s-t en el grafo residual (de FordFulkerson) es elegir
    las aristas de nodos reachable (de s) a non reachable.
```

3.2 BellmanFord

```
#include "../Header.cpp"

int main(){
    ios::sync_with_stdio(false);
```

```
cin.tie(0);
int v,e,x,y,w,r;
cin >> v >> e >> r;
pll h;
vl d(v, INF);
d[r] = 0;
vector<vector<pll> > g(v, vector<pll> (0));
for(int i = 0 ; i < e; i++){
    cin >> x >> y >> w;
    h.first = y;
    h.second = w;
    g[x].push_back(h);
}
/*
inequations solver
v - u <= p
g[u].push_back({v, p});
g[v].push_back({u, -1});
d[s] = 0
for i in v: g[s].push_back({i, 0})
*/

rep(i, v-1){
    bool mod = 0;
    rep(j, v){
        if(d[j] != INF){
            for(auto it : g[j]){
                d[it.first] = min(d[it.first], d[j] +
                    it.second);
                mod = 1;
            }
        }
    }
    if(mod == 0)break;
}
bool cyc = 0;
rep(j, v)
    for(auto it : g[j])
        if(d[j] < INF && d[it.first] > d[j] + it.second)
            cyc = 1;
// Faster but doesnt support negative cycles
    // SPFA from source S
    vl dist(v, INF); dist[s] = 0;              // INF = 1e9 here
    queue<int> q; q.push(s);                   // like BFS queue
    vl in_queue(v, 0); in_queue[s] = 1;        // unique to SPFA
    while (!q.empty()) {
        int u = q.front(); q.pop(); in_queue[u] = 0; // pop from
            queue
        for (auto it : g[u]) {              // C++17 style
            if (dist[u]+it.first >= dist[it.second]) continue;  //
                not improving, skip
            dist[it.second] = dist[u]+it.first;                 //
                relax operation
            if (!in_queue[it.second]) {                    // add to
                the queue
                q.push(it.second);                          // only
                    if v is not
                in_queue[it.second] = 1;                    //
                    already in the queue
            }
        }
    }
}
```

3.3 BiconnectedComponents

```
#include "../Header.cpp"

vector<vl> G;
vl D, L;
// p: -1, L: 0, D: -1
void dfs(int u, int p, int d)
{
    D[u] = L[u] = d;
    for(int v : G[u]) if (v != p)
    {
        if (D[v] == -1)
        {
            dfs(v, u, d + 1);
            if (L[v] > D[u]) {} // (u - v) cut edge
            L[u] = min(L[u], L[v]);
```

```
        }
        else L[u] = min(L[u], D[v]);
    }
}

int rc = 0; // Articulation Point
stack<pll> S; // BCC
void dfs(int u, int p, int d)
{
    D[u] = L[u] = d;
    for (int v : G[u]) if (v != p)
    {
        if (D[v] == -1)
        {
            S.emplace(u, v); dfs(v, u, d + 1);
            if ((p == -1 && ++rc == 2) || (p != -1 && L[v] >= d))
                {} // u is AP
            if (p == -1 or L[v] >= d) while (1) // BCC found
            {
                pll e = S.top(); S.pop();
                if (e == pll(u, v)) break;
            }
            L[u] = min(L[u], L[v]);
        }
        else if (D[v] < d) { S.emplace(u, v); L[u] = min(L[u],
            D[v]); }
    }
}
```

3.4 CentroidDecomposition

```
#include "../Header.cpp"

// all tree diameters pass through the centroid
const int MAXN = 1e5 + 5;
vector<int> g[MAXN];int n;
bool tk[MAXN];
int fat[MAXN]; // father in centroid decomposition
int szt[MAXN]; // size of subtree
int calcsz(int x, int f){
        szt[x]=1;
        for(auto y:g[x])if(y!=f&&!tk[y])szt[x]+=calcsz(y,x);
        return szt[x];
}
void cdfs(int x=0, int f=-1, int sz=-1){ // O(nlogn)
        if(sz<0)sz=calcsz(x,-1);
        for(auto y:g[x])if(!tk[y]&&szt[y]*2>=sz){
                szt[x]=0;cdfs(y,f,sz);return;
        }
        tk[x]=true;fat[x]=f;
        for(auto y:g[x])if(!tk[y])cdfs(y,x);
}
void centroid(){memset(tk,false,sizeof(tk));cdfs();}
int main(){
  ll t; cin >> t;
  for(int T = 1; T <= t; T++) {
    memset(memo, -1, sizeof(memo));
    ll x;
    cin >> N >> K;
    B.clear(); ac.clear(); ac.push_back(0);
    for(int i = 0; i < N; i++){
      cin >> x;
      B.push_back(x); ac.push_back(x);
      ac[i+1] += ac[i];
    }
    ll acum = 0;
    for(int i = 0; i < N; i++){
        acum += B[i];
        if(acum == ac[i+1])cout<<"1\n";
        else cout <<"0\n";
    }
    ll ans = INF;
    for(int i = 0; i < N; i++){
      ans = min(ans, dp(i, 0, 0));
    }
    if(ans >= INF) ans = -1;
    cout << "Case #" << T << ": ";
    cout << ans << "\n";
  }
}
```

3.5 Dijkstra

```
#include "../Header.cpp"

int main(){
    vl d(v, INF);
    priority_queue<pll, vp, greater<pll> > q; //from low to high
    ll s, t;
    q.push({0, s});
    d[s] = 0;
    while(!q.empty()){
        ll w, u;
        tie(w, u) = q.top();
        q.pop();
        if(w > d[u]) continue;
        for(auto it : g[u]){
            if(d[it.second] > w + it.first){
                d[it.second] = w + it.first;
                q.push({d[it.second], it.second});
            }
        }
    }
}
```

3.6 D´Esopo-Pape

```
#include "../Header.cpp"

int main(){
    ll v,x,y,e;
    pll h;
    vector<ll>b;
    cin >> v >> e;
    vector<vp > g(v, vp (0));
    vl peso(v, INF);

    int w;
    for(ll i = 0; i < e; i++){
        cin >> x >> y >> w;
        g[x-1].push_back({w, y-1}) ;
        g[y-1].push_back({w, x-1});
    }
    ll s, t;
    vl d(v, INF);
    d[s] = 0;
    vl m(v, 2);
    deque<ll> q;
    q.push_back(s);
    p.assign(v, -1);
    while (!q.empty()) {
        int u = q.front();
        q.pop_front();
        m[u] = 0;
        for (auto it : g[u]) {
            if (d[it.second] > d[u] + it.first) {
                d[it.second] = d[u] + it.first;
                p[it.second] = u;
                if (m[it.second] == 2) {
                    m[it.second] = 1;
                    q.push_back(it.second);
                } else if (m[it.second] == 0) {
                    m[it.second] = 1;
                    q.push_front(it.second);
                }
            }
        }
    }
    return 0;
}
```

3.7 EulerTour

```
#include "../Header.cpp"

//Euler Tour
vl L, R, d, c;
ll num = -1;
vector<vl>g;
void dfs(ll in, ll p){
```

```
    num++;
    L[in] = num;
    d.push_back(c[in]);
    for(auto it : g[in]){
        if(p != it)
            dfs(it, in);
    }
    R[in] = num;
}
```

3.8 FloydWarshall

```
#include "../Header.cpp"

ll p[500][500];
void printPath(int i, int j)
{
    if(i != j) printPath(i, p[i][j]);
    cout << j+1 << " ";
}

int main()
{
    int n, m, q, x ,y ,w;
    vector<vl > g(n, vl(n, INF));
    rep(i, n)
    {
        g[i][i] = 0;
        //g[i][i] = INF; Detect cheapest positive cycle for each i
    }
    rep(i, m)
    {
        cin >> x >> y >> w;
        g[x][y] = min(g[x][y], w); // handle repeats
    }

    rep(i, n)
        rep(j, n)
            p[i][j] = i;

    rep(k, n)
        rep(i, n)
            rep(j, n)
                //g[i][j] |= (g[i][k] & g[k][j]); to find i is
                    connected with j
                // if at the end g[i][j] & g[j][i], i and j are in
                    the same SCC

                // To find minimal max edge in path from i to j
                //g[i][j] = min(g[i][j], max(g[i][k], g[k][j]));

                if(g[i][k] + g[k][j] < g[i][j])
                {
                    g[i][j] = g[i][k] + g[k][j];
                    p[i][j] = p[k][j];

                }
    rep(k, n)
        rep(i, n)
            rep(j, n)
                if(g[i][k] != INF && g[k][j] !=INF
                    && g[k][k] < 0)
                        g[i][j] = -INF;
   rep(i, q)
    {
        cin >> x >> y;
        if(g[x][y] == INF)
            cout << "Impossible\n";
        else if (g[x][y] == -INF)
            cout << "-Infinity\n";
        else
            cout << g[x][y] << "\n";
    }
        return 0;
}
```

3.9 HeavyLightDecomposition

```
#include "../Header.cpp"
```

```
class HLD{
    ST st;
    vi A, H, D, R, P;
    int dfs(vector<vi> &G, int u){
        int ans = 1, M = 0, s;
        for (int v : G[u]) if (v != A[u]){
            A[v] = u, D[v] = D[u] + 1;
            s = dfs(G, v), ans += s;
            if (s > M) H[u] = v, M = s;
        }
        return ans;
    }
    template <class OP>
    void path(int u, int v, OP op){
        for (; R[u] != R[v]; v = A[R[v]]){
            if (D[R[u]] > D[R[v]]) swap(u, v);
            op(P[R[v]], P[v] + 1);
        }
        if (D[u] > D[v]) swap(u, v);
        op(P[u], P[v] + 1);             // VALUES ON VERTEX
        // op(P[u] + 1, P[v] + 1);      // VALUES ON EDGE
    }

public:
    HLD(vector<vi> &G, int n) : A(n), D(n), R(n), P(n){
        st = SegmentTree(n);
        H.assign(n, -1); A[0] = -1, D[0] = 0; dfs(G, 0); int p = 0;
        rep(i, n) if (A[i] == -1 || H[A[i]] != i)
            for (int j = i; j != -1; j = H[j]) R[j] = i, P[j] = p++;
    }
    void set(int v, const node &x) { st.set(P[v], x); } // VALUES
        ON VERTEX
    // void set(int u, int v, const node &x)       // VALUES ON
        EDGE
    // {
    //     if (D[u] > D[v]) swap(u, v);
    //     st.set(P[v], x);
    // }
    void update(int u, int v, const node &x)            //
        OPTIONAL FOR RANGE UPDATES
    { path(u, v, [this, &x](int l, int r) { st.update(l, r, x);
        }); }
    node query(int u, int v){
        node ans = node();
        path(u, v, [this, &ans](int l, int r) { ans = node(ans,
            st.query(l, r)); });
        return ans;
    }
};
// USAGE: HLD<ST<Node>, Node> hld(G, N);
//// NON COMMUTATIVE QUERIES :
class HLD{
    ST st;
    vi A, H, D, R, P;
    int dfs(vector<vi> &G, int u){
        int ans = 1, M = 0, s;
        for (int v : G[u]) if (v != A[u]){
            A[v] = u, D[v] = D[u] + 1;
            s = dfs(G, v), ans += s;
            if (s > M) H[u] = v, M = s;
        }
        return ans;
    }
public:
    node path(int u, int v){
        node ans1, ans2; bool d = 0;
        for (; R[u] != R[v]; v = A[R[v]]){
            if (D[R[u]] > D[R[v]]) swap(u, v), d = !d;
            if (d) ans1 = node(st.query(P[R[v]], P[v] + 1), ans1);
            else ans2 = node(st.query(P[R[v]], P[v] + 1), ans2);
        }
        if (D[u] > D[v]) swap(u, v), d = !d;
        if (d) ans1 = node(st.query(P[u], P[v] + 1), ans1);
        else ans2 = node(st.query(P[u], P[v] + 1), ans2);
        ans1.sw(); return node(ans1, ans2);
    }
    HLD(vector<vi> &G, int n) : A(n), st(n), D(n), R(n), P(n){
        st = SegmentTree(n);
        H.assign(n, -1); A[0] = -1, D[0] = 0; dfs(G, 0); int p = 0;
        rep(i, n) if (A[i] == -1 || H[A[i]] != i)
            for (int j = i; j != -1; j = H[j]) R[j] = i, P[j] = p++;
```

```
    }
    void set(int v, const node &x) { st.set(P[v], x); }
};
```

3.10 Hungarian

```
#include "../Header.cpp"

#define rep(i, n) for (int i = 0; i < (int)n; i++)
#define repx(i, a, b) for (int i = (int)a; i < (int)b; i++)

// Minimum/Maximum cost of a perfect matching in complete graph
// O(n^3)
template<class T>
class Hungarian{
    T inf = numeric_limits<T>::max() / 2;
    bool maxi, swapped = false;
    vector<vector<T>> cost;
    vector<T> u, v;
    vl p, way;
    int l, r;

public:
    // left/right == partition sizes
    Hungarian(int left, int right, bool maximizing){
        l = left, r = right, maxi = maximizing;
        if (swapped = l > r) swap(l, r);
        cost.assign(l + 1, vector<T>(r + 1, 0));
        u.assign(l + 1, 0); v.assign(r + 1, 0);
        p.assign(r + 1, 0); way.assign(r + 1, 0);
    }
    void add_edge(int l, int r, T w){
        assert(l and r); // indices start from 1 !!
        if (swapped) swap(l, r);
        cost[l][r] = maxi ? -w : w;
    }
    // execute after all edges were added
    void calculate(){
        repx(i, 1, l + 1){
            vector<bool> used(r+1, false);
            vector<T> minv(r+1, inf);
            int j0 = 0; p[0] = i;
            while (p[j0]){
                int j1, i0 = p[j0]; used[j0] = true;
                T delta = inf;
                repx(j, 1, r + 1) if (not used[j])
                {
                    T cur = cost[i0][j] - u[i0] - v[j];
                    if (cur < minv[j]) minv[j] = cur, way[j] = j0;
                    if(minv[j] < delta) delta = minv[j], j1 = j;
                }
                rep(j, r + 1){
                    if (used[j]) u[p[j]] += delta, v[j] -= delta;
                    else minv[j] -= delta;
                }
                j0 = j1;
            }
            while (j0) p[j0] = p[way[j0]], j0 = way[j0];
        }
    }

    // execute after executing calculate()
    T answer() { return maxi ? v[0] : -v[0]; }
    bool are_matched(int l, int r){
        if (swapped) swap(l, r);
        return p[r] == l;
    }
};

int main(){
    ios_base::sync_with_stdio(0); cin.tie(0);
    ll n;
    cin >> n;
    ll d[n][n], pos = (n+1)/2;
    Hungarian<ll> h(pos, pos, 0);
    for(int i = 0; i < n; i++)
        for(int j = 0; j < n; j++)
            cin >> d[i][j];
    for(int i = 0; i < n; i += 2){
        for(int j = 0; j < n; j+= 2){
            ll cost = 0;
```

```
            if(j > 0) cost += d[i][j-1];
            if(j < n-1) cost += d[i][j+1];
            h.add_edge(i/2+1, j/2+1, cost);
        }
    }
    h.calculate();
    cout << h.answer() << "\n";
}
```

3.11 LCA-SP

```
#include "../Header.cpp"

ll maxlog2(ll x){
    return (63 - __builtin_clzll(x));
}

// To minimize diameter, connect the center of the diameter of
    two trees
// min(diam1, diam2, dia1+dia2+1) dia1 = diam1/2 (if diameter
    odd)+ 1

struct SparseTableLCA
{
    ll maxlg;
    vector<vl >SP;
    vector<vl >MN;
    vl D;
    SparseTableLCA(vector<vl>& g, ll ini=0)
    {
        ll n = g.size();
        vl vis(n,0), parent(n,-1);
        D.resize(n,INF);D[ini]=0;
        queue<ll> q;
        q.emplace(ini);
        while(!q.empty()){
            ll k=q.front();q.pop();
            if(!vis[k]){
                vis[k]=1;
                for(auto it : g[k])
                if(!vis[it])
                {
                    parent[it]=k;
                    D[it]=D[k]+1;
                    q.push(it);
                }

            }
        }

        SP.clear();
        SP.push_back(parent);
        maxlg = maxlog2(n);

        repx(i, 1 , maxlg+1)
        {
            vl aux;
            rep(j, n)
            {
                if(SP[i-1][j]!=-1)
                aux.push_back(SP[i-1][SP[i-1][j]]);
                else aux.push_back(-1);
            }
            SP.push_back(aux);
        }
    }

    ll maxL(ll u,ll v)//arista largo maximo
    {
        ll a,b,x=LCA(u, v);
        if(u==x)a =- 1;
        else a = query(D[x], u);
        if(v==x)b =- 1;
        else b = query(D[x], v);
        return max(a, b);
    }
    ll query(ll a,ll n)
    {
        ll maxi=-1;
        while(D[n]!=a)
        {
```

```
        maxi = max(maxi, MN[maxlog2(D[n]-a)][n]);
        n=SP[maxlog2(D[n]-a)][n];
    }
    return maxi;
}
ll level(ll a, ll n) // up a to depth n
{
    while(D[n] != a)
        n = SP[maxlog2(D[n]-a)][n];
    return n;
}
ll LCA(ll x,ll y)
{
    if(D[x] <= D[y]) swap(x, y);

    if(D[x] != D[y])
        x = level(min(D[x], D[y]), x);

    if(x == y) return x;

    for(ll i = maxlg; i>=0; i--)
    {
        if(SP[i][x] != SP[i][y] && SP[i][x] != -1)
        {
            x = SP[i][x];
            y = SP[i][y];
        }
    }
    return SP[0][x];
}
ll Dist(ll u,ll v)
{
    return D[u] + D[v] - 2*D[LCA(u, v)];
}
ll kth_fartest_node(ll u, ll v, ll d)
{
    if(Dist(u, LCA(u, v)) < d)
        return level(D[v] - (Dist(u, v) - d), v);

    else
        return level(D[u] - d, u);
    }

    // move u k steps in path to v
    ll next_path(ll u, ll v, ll k){

      if(D[u] - D[LCA(u, v)] >= k) return level(D[u] - k, u);
      else return level(D[LCA(u, v)] + k - (D[u] - D[LCA(u, v)]),
          v);
    }
};


/* edge weight queries
SparseTableLCA(vector<vector<pll>>& g, ll ini)
    {
        ll n = g.size();
        vl vis(n,0), parent(n,-1), b(n,-1);
        D.resize(n,INF);D[ini]=0;
        queue<ll> q;
        q.emplace(ini);
        while(!q.empty()){
            ll k=q.front();q.pop();
            if(!vis[k]){
                vis[k]=1;
                for(auto it : g[k])
                    if(!vis[it.second])
                    {
                        b[it.second]=it.first;
                        parent[it.second]=k;
                        D[it.second]=D[k]+1;
                        q.push(it.second);
                    }
            }
        }

        SP.clear();
        SP.push_back(parent);
        maxlg= 63 - __builtin_clzll(n);
        for(ll i = 1; i <= maxlg; i++)
        {
            vl c;
```

```
        for(ll j=0;j<n;j++)
        {
            if(SP[i-1][j]!=-1)
            c.push_back(SP[i-1][SP[i-1][j]]);
            else c.push_back(-1);
        }
        SP.push_back(c);
    }
    MN.clear();
    MN.push_back(b);

    for(ll i=1;i<=maxlg;i++)
    {
        vl c;
        for(ll j=0;j<n;j++)
        {
            if(MN[i-1][j]!=-1)
            c.push_back(max(MN[i-1][SP[i-1][j]],MN[i-1][j]));
            else c.push_back(-1);
        }
        MN.push_back(c);
    }
}
*/
```

3.12 LaplacianMatrix

```
#include "../Header.cpp"

// Dimension of input square matrix
#define N 4

// Function to get cofactor of mat[p][q] in temp[][]. n is
// current dimension of mat[][]
void getCofactor(int mat[N][N], int temp[N][N], int p,
                 int q, int n){
    int i = 0, j = 0;
    // Looping for each element of the matrix
    for (int row = 0; row < n; row++){
        for (int col = 0; col < n; col++){
            // Copying into temporary matrix only those
            // element which are not in given row and
            // column
            if (row != p && col != q){
                temp[i][j++] = mat[row][col];

                // Row is filled, so increase row index and
                // reset col index
                if (j == n - 1){
                    j = 0;
                    i++;
                }
            }
        }
    }
}

/* Recursive function for finding determinant of matrix.
   n is current dimension of mat[][]. */
int determinantOfMatrix(int mat[N][N], int n){
    int D = 0; // Initialize result
    // Base case : if matrix contains single element
    if (n == 1) return mat[0][0];
    int temp[N][N]; // To store cofactors
    int sign = 1; // To store sign multiplier
    // Iterate for each element of first row
    for (int f = 0; f < n; f++){
        // Getting Cofactor of mat[0][f]
        getCofactor(mat, temp, 0, f, n);
        D += sign * mat[0][f]
             * determinantOfMatrix(temp, n - 1);
        // terms are to be added with alternate sign
        sign = -sign;
    }
    return D;
}
/* function for displaying the matrix */
void display(int mat[N][N], int row, int col){
    rep(i, row){
        rep(j, col){
            cout << " " << mat[i][j];
```

```
        }
        cout << "\n";
    }
}
// Driver program to test above functions
int main(){
    /* int mat[N][N] = {{6, 1, 1},
                       {4, -2, 5},
                       {2, 8, 7}}; */

    int mat[N][N] = { { 1, 0, 2, -1 },
                      { 3, 0, 0, 5 },
                      { 2, 1, 4, -3 },
                      { 1, 0, 5, 0 } };
    int T[N][N]; //Tutte matrix
    //range of matrix(number of pivots) is # different matching
        maximum
    ll n, m;
    for(int i = 0; i < m; i++){
        ll x, y, w;
        cin >> x >> y >> w;
        mat[x][y] = -1;//-w;
        mat[y][x] = -1;//-w;
        mat[x][x] += 1;//w;
        mat[y][y] += 1;//w;

        T[x][x] = 0;
        T[x][y] = rand() % 1e9+7;
        T[y][x] = -T[x][y];
    }
    // O(N^3)
    cout <<"Number of spanning trees/weight sum of spanning tree :
        " << determinantOfMatrix(mat, N-1);
    cout <<"Number of spanning forest with i in one component and
        j in the other : " << determinantOfMatrix(mat, N-i
        -j);//delete i/t row and column
    if(determinantOfMatrix(T, N) == 0)
        cout <<"Does not exist a maximum matching in the general
            graph (need more verify)";
    else
        cout <<"Does exist perfect matching";
    cout <<"Number of spanning trees/weight sum of spanning tree :
        " << determinantOfMatrix(T, N);
    return 0;
}
```

3.13 MaxBipartiteMatching

```
#include "../Header.cpp"

// Works with 3-pairing or more (Perfect MCBM)
vl match, vis;                                  // global variables
vector<vl> g;

int Aug(int L) {
  if (vis[L]) return 0;                         // L visited, return 0
  vis[L] = 1;
  for (auto it : g[L])
    if ((match[it] == -1) || Aug(match[it])) {
      match[it] = L;                             // flip status
      return 1;                                 // found 1 matching
    }
  return 0;                                     // no matching
}

int main() {
    ios::sync_with_stdio(false); cin.tie(0); cout.tie(0);
    ll V, Vleft;
    // VLeft and VRight can have common vertices names
    // match[R] -> L
    match.assign(V, -1);
    ll MCBM = 0;
    for(int L = 0; L < Vleft; L++){
        vis.assign(Vleft, 0);
        MCBM += Aug(L);
    }
    cout << "Found " << MCBM << " matchings\n"; // the answer is 2
        for Figure 4.38
    cerr << "\nTime elapsed: " << 1000 * clock() / CLOCKS_PER_SEC
        << "ms\n";
    return 0;
```

```
}
```

3.14 MaxFlowDinic

```
// Time Complexity:
// - general worst case: O (|E| * |V|^2)
// O (|E||max_flow|) ford fulkenson
// - unit capacities: O( min( V^(2/3), sqrt(E) ) )
// - Bipartite graph (unit capacities) + source & sink (any
    capacities): O(E sqrt V)

// minimo corte:
// separa en 2 colores el grafo y el minimo corte
// es la separacin con menos aristas entre distinto color
// mandar infinito a nodos que estan obligados a ser de un colors
// min_cut == max_flow

// max matching = max_flow (grafo bipartito)
// minimo cubrimiento nodos = nodos - max_flow


// maximo matching m*sqrt(n)

// usados: ll, vl, vi;

class Dinic{
    struct Edge { int to, rev; ll f, c; };
    int n, t_; vector<vector<Edge>> G;
    vl D; vi q, W;

    bool bfs(int s, int t){
        W.assign(n, 0); D.assign(n, -1); D[s] = 0;
        int f = 0, l = 0; q[l++] = s;
        while (f < l){
            int u = q[f++];
            for (const Edge &e : G[u]) if (D[e.to] == -1 && e.f <
                e.c)
                D[e.to] = D[u] + 1, q[l++] = e.to;
        }
        return D[t] != -1;
    }
    ll dfs(int u, ll f){
        if (u == t_) return f;
        for (int &i = W[u]; i < (int)G[u].size(); ++i){
            Edge &e = G[u][i]; int v = e.to;
            if (e.c <= e.f || D[v] != D[u] + 1) continue;
            ll df = dfs(v, min(f, e.c - e.f));
            if (df > 0) { e.f += df, G[v][e.rev].f -= df; return
                df; }
        }
        return 0;
    }

public:
    Dinic(int N) : n(N), G(N), D(N), q(N) {}
    void addEdge(int u, int v, ll cap){
        G[u].push_back({v, (int)G[v].size(), 0, cap});
        G[v].push_back({u, (int)G[u].size() - 1, 0, 0}); // cap if
            bidirectional
    }
    ll maxFlow(int s, int t){
        t_ = t; ll ans = 0;
        while (bfs(s, t)) while (ll dl = dfs(s, LLONG_MAX)) ans +=
            dl;
        return ans;
    }
};

int n,m,a,b;
ll c;

int main()
{
    cin >> n >> m; // n vertices, m aristas
    Dinic Grafo(n+1);

    for (int i = 0; i < m ; i ++)
    {
        cin >> a >> b >> c; // a conecta b con peso c
        Grafo.addEdge(a,b,c);
    }
```

```
    c = Grafo.maxFlow(1,n);
    cout << c << "\n";
}
```

3.15 MaxFlowFordFulkerson

```
#include "../Header.cpp"


// minimo cubrimiento aristas = min-cut
// extremos de aristas cortadas que no son s ni t
// max conj, ind = nodos - minimo cubrimiento aristas

// O(VE^2)
// O(EF)

// Number of vertices in given graph
#define V 6

/* Returns true if there is a path from source 's' to sink 't' in
  residual graph. Also fills parent[] to store the path */
int bfs(int rGraph[V][V], int s, int t, int parent[])
{
    // Create a visited array and mark all vertices as not visited
    bool visited[V];
    memset(visited, 0, sizeof(visited));

    // Create a queue, enqueue source vertex and mark source vertex
    // as visited
    queue <int> q;
    q.push(s);
    visited[s] = true;
    parent[s] = -1;

    // Standard BFS Loop
    while (!q.empty())
    {
        int u = q.front();
        q.pop();

        for (int v=0; v<V; v++)
        {
            if (visited[v]==false && rGraph[u][v] > 0)
            {
                q.push(v);
                parent[v] = u;
                visited[v] = true;
            }
        }
    }

    // If we reached sink in BFS starting from source, then return
    // true, else false
    return (visited[t] == true);
}

// A DFS based function to find all reachable vertices from s.
    The function
// marks visited[i] as true if i is reachable from s. The initial
    values in
// visited[] must be false. We can also use BFS to find reachable
    vertices
void dfs(int rGraph[V][V], int s, bool visited[])
{
    visited[s] = true;
    for (int i = 0; i < V; i++)
       if (rGraph[s][i] && !visited[i])
           dfs(rGraph, i, visited);
}

// Prints the minimum s-t cut
void minCut(int graph[V][V], int s, int t)
{
    int u, v;

    // Create a residual graph and fill the residual graph with
    // given capacities in the original graph as residual
        capacities
    // in residual graph
```

```
int rGraph[V][V]; // rGraph[i][j] indicates residual capacity
    of edge i-j
for (u = 0; u < V; u++)
    for (v = 0; v < V; v++)
         rGraph[u][v] = graph[u][v];

int parent[V]; // This array is filled by BFS and to store path
int max_flow = 0; // There is no flow initially
// Augment the flow while there is a path from source to sink
while (bfs(rGraph, s, t, parent))
{
    // Find minimum residual capacity of the edhes along the
    // path filled by BFS. Or we can say find the maximum flow
    // through the path found.
    int path_flow = INT_MAX;
    for (v=t; v!=s; v=parent[v])
    {
        u = parent[v];
        path_flow = min(path_flow, rGraph[u][v]);
    }

    // update residual capacities of the edges and reverse
        edges
    // along the path
    for (v=t; v != s; v=parent[v])
    {
        u = parent[v];
        rGraph[u][v] -= path_flow;
        rGraph[v][u] += path_flow;
    }
    max_flow += path_flow;
}

// Flow is maximum now, find vertices reachable from s
bool visited[V];
memset(visited, false, sizeof(visited));
dfs(rGraph, s, visited);

// Print all edges that are from a reachable vertex to
// non-reachable vertex in the original graph
for (int i = 0; i < V; i++)
      for (int j = 0; j < V; j++)
         if (visited[i] && !visited[j] && graph[i][j])
              cout << i << " - " << j << endl;

    return;
}

// Driver program to test above functions
int main()
{
    // Let us create a graph shown in the above example
    int graph[V][V] = { {0, 16, 13, 0, 0, 0},
                        {0, 0, 10, 12, 0, 0},
                        {0, 4, 0, 0, 14, 0},
                        {0, 0, 9, 0, 0, 20},
                        {0, 0, 0, 7, 0, 4},
                        {0, 0, 0, 0, 0, 0}
                      };

    minCut(graph, 0, 5);

    return 0;
}
```

3.16 MinCostMaxFlow

```
#include "../Header.cpp"

struct Edge{
    int from, to, capacity, cost;
};
vector<vector<int>> adj, cost, capacity;
const int INF = 1e9;

void shortest_paths(int n, int v0, vector<int>& d, vector<int>&
    p) {
    d.assign(n, INF);
    d[v0] = 0;
    vector<bool> inq(n, false);
    queue<int> q;
```

```
    q.push(v0);
    p.assign(n, -1);
    while (!q.empty()) {
        int u = q.front();
        q.pop();
        inq[u] = false;
        for (int v : adj[u]) {
            if (capacity[u][v] > 0 && d[v] > d[u] + cost[u][v]) {
                d[v] = d[u] + cost[u][v];
                p[v] = u;
                if (!inq[v]) {
                    inq[v] = true;
                    q.push(v);
                }
            }
        }
    }
}

 // flow, source, to;
int min_cost_flow(int N, vector<Edge> edges, int K, int s, int t)
    {
    adj.assign(N, vector<int>());
    cost.assign(N, vector<int>(N, 0));
    capacity.assign(N, vector<int>(N, 0));
    for (Edge e : edges) {
        adj[e.from].push_back(e.to);
        adj[e.to].push_back(e.from);
        cost[e.from][e.to] = e.cost;
        cost[e.to][e.from] = -e.cost;
        capacity[e.from][e.to] = e.capacity;
    }
    int flow = 0;
    int cost = 0;
    vector<int> d, p;
    while (flow < K) {
        shortest_paths(N, s, d, p);
        if (d[t] == INF) break;
        // find max flow on that path
        int f = K - flow;
        int cur = t;
        while (cur != s) {
            f = min(f, capacity[p[cur]][cur]);
            cur = p[cur];
        }
        // apply flow
        flow += f;
        cost += f * d[t];
        cur = t;
        while (cur != s) {
            capacity[p[cur]][cur] -= f;
            capacity[cur][p[cur]] += f;
            cur = p[cur];
        }
    }
    if (flow < K) return -1;
    else return cost;
}

int sup[55], inf[55];

int main(){
    int n, q;
    ll ans = 0;
    cin >> n >> q;
    for (int i = 0; i < n; ++i) {
        sup[i] = n;
        inf[i] = 1;
    }
    for (int j = 0; j < q; ++j) {
        int t, l, r, v;
        cin >> t >> l >> r >> v;
        if (t == 1){
            for (int i = l - 1; i <= r - 1; ++i) {
                inf[i] = max(inf[i], v);
            }
        }else{
            for (int i = l - 1; i <= r - 1; ++i) {
                sup[i] = min(sup[i], v);
            }
        }
    }
```

```
    vector<Edge>ee;
    Edge E;
    for(int i = 0; i < n; i++){
        E.from = 0;
        E.to = i + 2;
        E.capacity = 1;
        E.cost = 0;
        ee.push_back(E);
        for(int j = inf[i]; j <= sup[i]; j++){
            E.from = i + 2;
            E.to = j + n + 1;
            E.capacity = 1;
            E.cost = 0;
            ee.push_back(E);
        }
    }
    ll id = n + 2 + n;
    for(int i = n + 2; i <= n + 2 + n - 1; i++){
        for(int j = 0; j < n; j++){
            E.from = i;
            E.to = id;
            E.capacity = 1;
            E.cost = 2 * j + 1;
            ee.push_back(E);
            E.from = id;
            E.to = 1;
            E.capacity = 1;
            E.cost = 0;
            ee.push_back(E);
            id++;
        }
    }
    ans = min_cost_flow(2 * n + n*n + 10, ee, n, 0, 1);
    cout << ans << "\n";
    cerr << "\nTime elapsed: " << 1000 * clock() / CLOCKS_PER_SEC
        << "ms\n";
}
```

3.17 MinCostMaxFlow2

```
#include "../Template.cpp"

template <class T>
class MCMF{
    typedef pair<T, T> pTT;
    T INF = numeric_limits<T>::max();
    struct Edge{
        int v; T c, w;
        Edge(int v, T c, T w) : v(v), c(c), w(w) {}
    };
    int n; vvi E;
    vector<Edge> L; vi F; vector<T> D, P; vector<bool> V;
    bool dij(int s, int t){
        D.assign(n, INF); F.assign(n, -1); V.assign(n, false);
        D[s] = 0;
        rep(_, n){
            int best = -1;
            rep(i, n) if (!V[i] && (best == -1 || D[best] > D[i]))
                best = i;
            if (D[best] >= INF) break;
            V[best] = true;
            for (int e : E[best]){
                Edge ed = L[e];
                if (ed.c == 0) continue;
                T toD = D[best] + ed.w + P[best] - P[ed.v];
                if (toD < D[ed.v]) D[ed.v] = toD, F[ed.v] = e;
            }
        }
        return D[t] < INF;
    }
    pTT augment(int s, int t){
        pTT flow(L[F[t]].c, 0);
        for (int v = t; v != s; v = L[F[v] ^ 1].v)
            flow.ff = min(flow.ff, L[F[v]].c), flow.ss += L[F[v]].w;
        for (int v = t; v != s; v = L[F[v] ^ 1].v)
            L[F[v]].c -= flow.ff, L[F[v] ^ 1].c += flow.ff;
        return flow;
    }
public:
    MCMF(int n) : n(n), E(n), D(n), P(n, 0), V(n, 0) {}
    pTT mcmf(int s, int t){
```

```
        pTT ans(0, 0);
        if (!dij(s, t)) return ans;
        rep(i, n) if (D[i] < INF) P[i] += D[i];
        while (dij(s, t)){
            auto flow = augment(s, t);
            ans.ff += flow.ff, ans.ss += flow.ff * flow.ss;
            rep(i, n) if (D[i] < INF) P[i] += D[i];
        }
        return ans;
    }
    void addEdge(int u, int v, T c, T w){
        E[u].pb(L.size()); L.eb(v, c, w);
        E[v].pb(L.size()); L.eb(u, 0, -w);
    }
};
```

3.18 MinimumSpanningTree

```
#include "../Header.cpp"
int V,E;
int w,v,u;
// codigo competitive programming 4
class UnionFind { // OOP style
    private: vi p, rank, setSize; // vi p is the key part
    int numSets;
    public:
        UnionFind(int N) {
            p.assign(N, 0); for (int i = 0; i < N; ++i) p[i] = i;
            rank.assign(N, 0); // optional speedup
            setSize.assign(N, 1); // optional feature
            numSets = N; // optional feature
        }
        int findSet(int i) { return (p[i] == i) ? i : (p[i] =
            findSet(p[i])); }
        bool isSameSet(int i, int j) { return findSet(i) ==
            findSet(j); }
        int numDisjointSets() { return numSets; } // optional
        int sizeOfSet(int i) { return setSize[findSet(i)]; } //
            optional
        void unionSet(int i, int j) {
            if (isSameSet(i, j)) return; // i and j are in same set
            int x = findSet(i), y = findSet(j); // find both rep
                items
            if (rank[x] > rank[y]) swap(x, y); // keep x shorter
                than y
            p[x] = y; // set x under y
            if (rank[x] == rank[y]) ++rank[y]; // optional speedup
            setSize[y] += setSize[x]; // combine set sizes at y
            --numSets; // a union reduces numSets
            }
        };

//

int main()
{
    cin >> V >> E;

    vector< pair<int, ii> > Edgelist;

    for(int i = 0; i< E; i++)
    {
        cin >> u >> v >> w;
        Edgelist.push_back(make_pair(w,ii(u,v)));
    }
sort(Edgelist.begin(),Edgelist.end()); // sort de built in de pair

int mst_cost = 0, num_taken = 0; // no edge has been taken
UnionFind UF(V); // all V are disjoint sets

for (auto &[w, e] : Edgelist)
{
    u = e.first -1 ; v = e.second -1;

    if (UF.isSameSet(u, v)) continue; // already in the same CC

    mst_cost += w;
    UF.unionSet(u, v); // link them
    ++num_taken; // 1 more edge is taken
    if (num_taken == V-1) break;
```

```
}

cout << mst_cost << '\n';
}
```

3.19 SCC

```
#include "../Header.cpp"
// Strong Connected Component
vl dfs_num, dfs_low, visited;
vector<vl> g, invg;
void Kosaraju(int u, int pass, vl& S) { // pass = 1 (original), 2
    (transpose)
  dfs_num[u] = 1;
  vl &neighbor = (pass == 1) ? g[u] : invg[u];
  for (auto it : neighbor)
    if (dfs_num[it] == -1)
      Kosaraju(it, pass, S);
  S.push_back(u);
}
// ----------------------------------------
// implementation of Tarjan's SCC algorithm
stack<int> St;
int cont, numSCC;

void tarjanSCC(int u) {
  dfs_low[u] = dfs_num[u] = cont;
  cont++;
  St.push(u);
  visited[u] = 1;
  for (auto v : g[u]) {
    if (dfs_num[v] == -1)
      tarjanSCC(v);
    if (visited[v])
      dfs_low[u] = min(dfs_low[u], dfs_low[v]);
  }

  if (dfs_low[u] == dfs_num[u]) {
    while (1) {
      int v = St.top(); St.pop(); visited[v] = 0;
      if (u == v) break;
    }
    ++numSCC;
  }
}
int main() {
    int n, m; cin >> n >> m;
    vector<vl> g(n);
    while(m--) {
        int u, v; cin >> u >> v; u--, v--;
        g[u].push_back(v);
    }
    // run Tarjan's SCC
    dfs_num.assign(n, 0); dfs_low.assign(n, 0); visited.assign(n,
        0);
    while (!St.empty()) St.pop();
    cont = numSCC = 0;
    for (int u = 0; u < n; ++u)
     if (dfs_num[u] == -1)
       tarjanSCC(u);
    //Kosaraju's SCC
    vl S;
    dfs_num.assign(n, -1);
    for (int u = 0; u < n; ++u)
        if (dfs_num[u] == -1)
            Kosaraju(u, 1, S);
    int numSCC = 0;
    dfs_num.assign(n, -1);
    for (int i = n-1; i >= 0; --i){
        vl comp;
        if (dfs_num[S[i]] == -1)
            numSCC++, Kosaraju(S[i], 2, comp);        // on
                transposed graph
    }
    return 0;
}
```

3.20 SpecialPathsWithCentroids

```
int n, k, sz[mxN], mxtree[mxN], ans;
```

```
vector<pair<int, int>> adj[mxN];
vector<int> new_vis;
bool used[mxN];

int dfs(int u, int p = -1){
    sz[u] = 1;
    for(pair<int, int> v : adj[u]){
        if(v.second != p && !used[v.second]){
            sz[u] += dfs(v.second, u);
        }
    }
    return sz[u];
}

int get_centroid(int u, int ms, int p = -1){ // u = node, ms =
    size of the current tree delimited by previous centroids, p =
    parent
    for(pair<int, int> v : adj[u]){
        if(v.second != p && !used[v.second]){
            if(sz[v.second]*2 > ms){
                return get_centroid(v.second, ms, u);
            }
        }
    }
    return u;
}

int flag[mxN], stamp[mxN];
int timestamp;

void solve(int u){
    ++timestamp;
    int sz_curr_tree = dfs(u);
    int centroid = get_centroid(u, sz_curr_tree);
    used[centroid] = 1;
    flag[0] = 0;
    stamp[0] = timestamp;
    // do something with the centroid
    for(pair<int, int> v : adj[centroid]){

    }
    // end that something with the centroid xdd

    // solve for child trees centroids
    for(pair<int, int> v : adj[centroid]){
        if(used[v.second]) continue;
        solve(v.second);
    }
}
```

3.21 StableMatching

```
#include "../Header.cpp"
// Match the preferences of N clients and M restaurants that no
    side prefer another
// restaurant or client, respectively.

// hospital-residents variation of stable matching
int main(){

        ios_base::sync_with_stdio(0);
        cin.tie(0);
        ll n, m, k, x;
        cin >> n >> m;

        vl cap(m, 1), match(n, -1);

        vector<queue<ll>>pref(n);
        queue<ll> q;
        for(auto &it : cap) cin >> it; // capacity

        for(int i = 0; i < n; i++)
        {
                q.push(i);
                cin >> k;
                while(k--)
                {
                        cin >> x;
                        pref[i].push(x-1); //client preference list
                }
        }
```

```
vector<unordered_map<ll, ll>> res_pref(m);
for(int i = 0; i < m; i++)
{
        ll id = 0;
        cin >> k;
        while(k--)
        {
                cin >> x;
                res_pref[i][x-1] = -id; // restaurant
                    preference list
                id++;
        }
}

vector<set<pll>> in_res(m);
while(!q.empty())
{
        ll cl = q.front();
        q.pop();

        ll rest = pref[cl].front(); // actual preference
            restaurant

        in_res[rest].insert({res_pref[rest][cl], cl});//
            add client to restaurant
        match[cl] = rest;
        pref[cl].pop(); // remove client preference

        if(cap[rest] == 0)
        {
                ll cl_new = (*in_res[rest].begin()).second;
                    // erase client with less preference
                in_res[rest].erase(in_res[rest].begin());

                match[cl_new] = -1;
                if(!pref[cl_new].empty()) // add client to
                    queue if he has more preferences
                        q.push(cl_new);
        }
        else
                        cap[rest]--;
        }
        for(int i = 0; i < n; i++)
                if(match[i] != -1) cout << i+1 <<" "<<match[i]+1
                    <<"\n";

}
```

3.22 ToposortDFS

```
#include "../Header.cpp"

vl s, v;
vector<vl > g;
void dfs(int t)
{
    v[t] = 1;
    for(auto it : g[t])
        if(!v[it])
            dfs(it);

    s.pb(t);
}
int main()
{
    ios::sync_with_stdio(false);
    cin.tie(0);
        ll n;
        g.assign(n, vl());
    v.assign(n, 0);

    rep(i, n)
        if(!v[i])
            dfs(i);

    reverse(ALL(s));
        return 0;
}
```

3.23 ToposortKhan

```cpp
#include "../Header.cpp"

// All toposort orderings
vector<vl> g;
ll in;
vl indegree, vis;
vl sorted;
vector<char> ans;
bool possible;
void toposort()
{
    bool flag = 0;

    for(int i = 0; i < in; i++)
    {
        if(indegree[i] == 0 && !vis[i])
        {
            sorted.push_back(i);


            for(auto it : g[i])
                indegree[it]--;
            vis[i] = 1;
            toposort();
            vis[i] = 0;
            sorted.pop_back();

            for(auto it : g[i])
                indegree[it]++;
            flag = 1;
        }
    }
    if(!flag && sorted.size() == in)
    {
        possible = 1;
        for(int i = 0; i < in; i++)
        {
            if(i > 0) cout << " ";
            cout << ans[sorted[i]];
        }
        cout << "\n";
    }
}

int main()
{
        ll v,x,y,e,a,b,in=0;
        pll h;
        cin>>v >> e;
        vector<vl > g(v, vl(0));
        for(int i = 0; i < e; i++)
    {
        cin >> x >> y;
        g[x].push_back(y);
    }

    //set indegrees
    vl indegree(v, 0);
    vl sorted;
    for(int i=0; i < v; i++)
    {
        for(auto it : g[i])
            indegree[it]++;
    }
    queue<ll> q;
    //agregar par a cola para tener los paquetes o niveles de
        profundidad, sumar
    for(int i = 0; i < v; i++)
        if(indegree[i] == 0)
            q.push(i);

    while(!q.empty()){

        ll t = q.front();
        q.pop();

        for(auto it : g[t]){

            if(--indegree[it] == 0)
                q.push(it);
```

```
        }
    }
}
```

4 Header

```
#include<bits/stdc++.h>
#pragma GCC optimize("Ofast")
using namespace std;
typedef long long ll;
typedef unsigned long long ull;
typedef vector<ll> vl;
typedef vector<int> vi;
typedef pair<ll,ll> pll;
typedef vector<pll> vp;
typedef double db;
#define INF 1e17
#define INF32 INT_MAX
#define EPS 1e-7
#define ALL(x) x.begin() , x.end()
#define ALLR(x) x.rbegin() , x.rend()
#define UNIQUE(c) (c).resize(unique(ALL(c)) - (c).begin())
#define PI acos(-1.0)
#define pb push_back
#define rep(i, n) for (int i = 0; i < (int)n; i++)
#define repx(i, a, b) for (int i = (int)a; i < (int)b; i++)

#define DBG 1

#define cerr \
    if (DBG) cerr

int main() {

        ios::sync_with_stdio(false);
        cin.tie(0);
        cout.tie(0);
        srand((unsigned int) time(0));

        // Code here

        // Compile:
        // g++ Code1.cpp && ./a.out < in > out
        // ulimit -s 1048576 more stack size 1gb
        // g++ -std=c++11 Code1.cpp && a.exe < in > out
        cerr << "\nTime elapsed: " << 1000 * clock() /
            CLOCKS_PER_SEC << "ms\n";

        //fijar precision de double
        double f =3.14159;

        cout << setprecision(2) << f << '\n';

        return 0;
}
```

5 Math

5.1 ArithmeticEval

```
#include "../Header.cpp"

ll k, n;
string s;

ll eval(ll l, ll r){
        ll open = 0;
        stack<ll> vals;
        stack<char> ops;

        if(s[l] == '+' || s[l] == '*' || s[r] == '+' || s[r] ==
            '*')return -1;
        if(s[l] == '0' && r-l >= 1 && isdigit(s[l+1]))return -1;

        repx(i, l, r+1){
                if(isdigit(s[i])){
                        vals.push((s[i] - '0') % k);
                        while(i < r && isdigit(s[i+1])){
```

```
                i++;
                vals.top() = (vals.top() * 10 + (s[i]
                    - '0')) % k;
            }
        }
        else if(s[i] == '('){
            open++;
            ops.push(s[i]);
        }
        else if(s[i] == ')'){
            open--;
            if(open < 0) return -1;
            ll in = i-1;
            while(ops.top() != '('){
                ll aux = vals.top();
                vals.pop();
                if(ops.top() == '+'){
                    vals.top() = (vals.top() +
                        aux) % k;
                }
                else{
                    vals.top() = (vals.top() *
                        aux) % k;
                }
                ops.pop();
            }
            ops.pop();
        }
        else{
            // higher precedence first
            while(!ops.empty() && ops.top() == '*'){
                ll aux = vals.top();
                vals.pop();
                vals.top() = (vals.top() * aux) % k;
                ops.pop();
            }
            ops.push(s[i]);
        }
}
if(open != 0) return -1;
while(!ops.empty()){
            ll aux = vals.top();
            vals.pop();
            if(ops.top() == '+'){
                vals.top() = (vals.top() + aux) % k;
            }
            else{
                vals.top() = (vals.top() * aux) % k;
            }
            ops.pop();
        }
        return vals.top();
}

int main(){
        ios_base::sync_with_stdio(0); cin.tie(0);
        srand((unsigned int) time(0));
        cin >> k >> n >> s;

        //cout << eval(0, n-1)<<"\n";
        ll ans = 0;
        rep(i, n){
            repx(j, i, n){
                if(eval(i, j) == 0){
                    ans++;
                    //cout << i << " " << j <<"
                        "<<eval(i,j)<< "\n";
                }
            }
        }
        cout << ans << "\n";
}
```

5.2 Combinatory

```
#include "../Header.cpp"

const int M = 1e9+7;

// binary exponent
```

```
// Note that a^(b^c) % p = a^(b^c % (p - 1)) % p for little
    fermat therorem
ll expmod(ll b, ll e){
    ll ans = 1;
    while(e){
        if(e&1) ans = ans*b %M;
        b = b*b %M; e >>= 1;
    }
    return ans;
}

// Binary exponentiation: a and b relative primes -> a ^ phi(m) %
    m = 1 -> a ^ {phi(m) - 1} % mod = a^-1
// Binary exponentiation: m prime -> a ^ {m - 1} % m = 1 -> a ^
    {m - 2} % mod = 1

// When M is prime
ll invmod(ll a){ return expmod(a, M-2); }

//inv modular factoriales
const ll MAXN = 1e5 + 1;
ll F[MAXN], INV[MAXN], FI[MAXN];
// ...

F[0] = 1; repx(i, 1, MAXN) F[i] = F[i-1]*i %M;
INV[1] = 1; repx(i, 2, MAXN) INV[i] = M - (ll)(M/i)*INV[M%i]%M;
FI[0] = 1; repx(i, 1, MAXN) FI[i] = FI[i-1]*INV[i] % M;


// Divide a elements in b segments = choose(a - 1, b - 1)
// combinatory
ll Comb(ll n, ll k){
    if(n < k) return 0;
    return F[n]*FI[k] %M *FI[n-k] %M;
}

// combinatury precalc
ll C[MAXN][MAXK];
// ...
rep(i, MAXN){
    C[i][0] = 1; if(i < MAXN) C[i][i] = 1;
    repx(j, 1, min(i, (int)MAXK))
        C[i][j] = (C[i-1][j-1] + C[i-1][j])%M;
}


// divide a elements into b segments = C[a-1][b-1]
// each segment has at least 1 element

//Wilson theorem
//(p-1)! mod p = -1 if p prime

//https://cp-algorithms.com/algebra/factorial-modulo.html#multiplicity-o

// n! mod p with p prime and ignore multiples of p
int factmod(int n, int p) {
    vector<int> f(p);
    f[0] = 1;
    for (int i = 1; i < p; i++)
        f[i] = f[i-1] * i % p;

    int res = 1;
    while (n > 1) {
        if ((n/p) % 2)
            res = p - res;
        res = res * f[n%p] % p;
        n /= p;
    }
    return res;
}

// number of times p divides n! = v_p, n! % p*v_p = 0
int multiplicity_factorial(int n, int p) {
    int count = 0;
    do {
        n /= p;
        count += n;
    } while (n);
    return count;
}

// combinatoria n = 1e18, primo chico
```

```
// lucas
const int M = 3005;
int C[M][M];
// ...
ll lucas(ll n, ll k, int p){
    ll ans = 1;
    while(n + k){
        ans = (ans * C[n%M][k%M]) % M;
        n /= M; k /= M;
    }
    return ans;
}

// Multinomial Coefficient
ll multinomial(vector<int> K) {
    ll n = 0, ans = 1;
    for (int k : K) n += k, ans = mul(ans, choose(n, k));
    return ans;
}

// Catalan numbers
// Counting problem where the solution is expressed by combining
    two
// identical subproblems with total size = n1 then involves
    Catalan
ll Catalan(ll n) {
    return choose(2 * n, n) * invmod(n + 1) % mod;
}
// C_n = (4n - 2) / (n + 1) * C_{n-1}, C_0 = 1
// Catalan convolution
// C_n^i = sum_{i1, i2, ..., ik, sum(i_j) = n} C_i1 * C_i2 * ...
    * C_ik
ll CatalanConv(ll n, ll k) {
    return (k + 1) * inv(n + k + 1) % mod * choose(2 * n + k, n) %
        mod;
}
```

5.3 ErathostenesSieve

```
#include "../Header.cpp"

vector <bool> crib;
void criba(ll b){
    crib.assign(b, 1);
    crib[0] = 0; crib[1] = 0;
    repx(k, 2, sqrt(b+1) + 1)
        if(crib[k])
            for(int j=2; (j * k) < b + 1; j++)//optimization j=k
                crib[k*j] = 0;
}


/ Primes in range [L, R] in O((R - L + 1) Log(Log(R)) + sqrt(R) *
    Log(Log(sqrt(R))))
// R ~ 1e12
vector<bool> segmentedSieve(ll L, ll R) {
    // generate all primes up to sqrt(R)
    ll lim = sqrt(R);
    vector<bool> mark(lim + 1, false);
    vl primes;
    repx(i, 2, lim + 1) {
        if (!mark[i]) {
            primes.pb(i);
            for (ll j = i * i; j <= lim; j += i)
                mark[j] = 1;
        }
    }
    vector<bool> isPrime(R - L + 1, true);
    for (ll i : primes)
        for (ll j = max(i * i, (L + i - 1) / i * i); j <= R; j +=
            i)
            isPrime[j - L] = false;
    if (L == 1)
        isPrime[0] = false;
    return isPrime;
}

// Linear Sieve: O(n)
// Uses more memory
vl linearSieve(int n = 10000000) {
    vl lp(n + 1, 0), pr;
```

```
    repx(i, 2, n + 1) {
        if (lp[i] == 0) {
            lp[i] = i; pr.pb(i);
        }
        for (int j = 0; i * pr[j] <= n; ++j) {
            lp[i * pr[j]] = pr[j];
            if (pr[j] == lp[i]) {
                break;
            }
        }
    }
}

// PRIMALITY TEST
// Trial Division O(sqrt(n))
bool isPrime(int x) {
    for (int d = 2; d * d <= x; d++) {
        if (x % d == 0)
            return false;
    }
    return true;
}
// Fermat theorem: a ^ (p - 1) % p = 1 for all a
// 646 of 10^9 fails
// O(1)
bool probablyPrimeFermat(int n, int iter=5) {
    if (n < 4) return n == 2 || n == 3;
    for (int i = 0; i < iter; i++) {
        int a = 2 + rand() % (n - 3);
        if (binpower(a, n - 1, n) != 1) return false;
    }
    return true;
}

// Miller Rabin
// Probabilistic check for prime numbers
// 88% of the numbers have a prime factor under 100, optimize the
    checker checking these cases
using u64 = uint64_t;
using u128 = __uint128_t;

u64 binpower(u64 base, u64 e, u64 mod) {
    u64 result = 1;
    base %= mod;
    while (e) {
        if (e & 1) result = (u128)result * base % mod;
        base = (u128)base * base % mod;
        e >>= 1;
    }
    return result;
}

bool check_composite(u64 n, u64 a, u64 d, int s) {
    u64 x = binpower(a, d, n);
    if (x == 1 || x == n - 1) return false;
    for (int r = 1; r < s; r++) {
        x = (u128)x * x % n;
        if (x == n - 1) return false;
    }
    return true;
};

bool MillerRabin(u64 n, int iter=5) { // returns true if n is
    probably prime, else returns false.
    if (n < 4) return n == 2 || n == 3;
    int s = 0;
    u64 d = n - 1;
    while ((d & 1) == 0) {
        d >>= 1;
        s++;
    }
    for (int i = 0; i < iter; i++) {
        int a = 2 + rand() % (n - 3);
        if (check_composite(n, a, d, s))
            return false;
    }
    return true;
}

// Deterministic MillerRabin
bool MillerRabin(u64 n) { // returns true if n is prime, else
    returns false.
```

```
    if (n < 2) return false;
    int r = 0;
    u64 d = n - 1;
    while ((d & 1) == 0) {
        d >>= 1;
        r++;
    }
    for (int a : {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}) {
        if (n == a) return true;
        if (check_composite(n, a, d, r)) return false;
    }
    return true;
}
```

5.4 Euclid

```
#include "../Header.cpp"

// find (x, y) such that Ax + By = gcd(A, B), and |Ax|, |By| <=
    AB/gcd(A, B)
pll euclid(ll A, ll B) {
    if (!B) return {1, 0};
    pll p = euclid(B, A % B);
    return {p.ss, p.ff - (A / B) * p.ss};
}
// find x in [0, M) such that Ax = 1 mod M
ll minv(ll A, ll M)
{
    pll p = euclid(A, M);
    assert(p.ff * A + p.ss * M == 1);
    return p.ff + (p.ff < 0) * M;
}

// find (x, y)'s such that Ax + By = R where R is multiplle of
    gcd(A, B);
pair<pll, pll> diophantine(ll A, ll B, ll R) {
    ll g = __gcd(A, B), x, y; A /= g, B /= g, R /= g;
    tie(x, y) = euclid(A, B); x *= R, y *= R;
    assert(A * x + B * y == R);
    return {{x, y}, {-B, A}}; // solutions: p+t*ans.snd
}
```

5.5 FFT

```
#include "../Header.cpp"

#define M_PIl 3.14159265358979323846264338327950288 4L

typedef complex<double> C;
typedef vector<double> vd;

void fft(vector<C> &a)
{
    int n = a.size(), L = 31 - __builtin_clz(n);
    static vector<complex<long double>> R(2, 1);
    static vector<C> rt(2, 1);
    for (static int k = 2; k < n; k *= 2)
    {
        R.resize(n);
        rt.resize(n);
        auto x = polar(1.0L, M_PIl / k);
        repx(i, k, 2 * k) rt[i] = R[i] = i & 1 ? R[i / 2] * x :
            R[i / 2];
    }
    vector<int> rev(n);
    rep(i, n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    rep(i, n) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2) for (int i = 0; i < n; i += 2 *
        k) rep(j, k)
    {
        auto x = (double *)&rt[j + k], y = (double *)&a[i + j + k];
        C z(x[0] * y[0] - x[1] * y[1], x[0] * y[1] + x[1] * y[0]);
        a[i + j + k] = a[i + j] - z;
        a[i + j] += z;
    }
}

vd conv(const vl &a, const vl &b)
{
    if (a.empty() || b.empty()) return {};
```

```
    vd res(a.size() + b.size() - 1);
    int L = 32 - __builtin_clz(res.size()), n = 1 << L;
    vector<C> in(n), out(n);
    copy(a.begin(), a.end(), in.begin());
    rep(i, b.size()) in[i].imag(b[i]);
    fft(in);
    for (auto &x : in) x *= x;
    rep(i, n) out[i] = in[-i & (n - 1)] - conj(in[i]);
    fft(out);
    rep(i, res.size()) res[i] = imag(out[i]) / (4 * n);
    return res;
}

//slower
vl convMod(const vl &a, const vl &b, int M)
{
    if (a.empty() || b.empty()) return {};
    vl res(a.size() + b.size() - 1);
    int B = 32 - __builtin_clz(res.size()), n = 1 << B, cut =
        int(sqrt(M));
    vector<C> L(n), R(n), outs(n), outl(n);
    rep(i, a.size()) L[i] = C((int)a[i] / cut, (int)a[i] % cut);
    rep(i, b.size()) R[i] = C((int)b[i] / cut, (int)b[i] % cut);
    fft(L), fft(R);
    rep(i, n)
    {
        int j = -i & (n - 1);
        outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
        outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / 1i;
    }
    fft(outl), fft(outs);
    rep(i, res.size())
    {
        ll av = ll(real(outl[i]) + .5), cv = ll(imag(outs[i]) +
            .5);
        ll bv = ll(imag(outl[i]) + .5) + ll(real(outs[i]) + .5);
        res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
    }
    return res;
}
```

```
int main() {
    ios::sync_with_stdio(false);
    cin.tie(0);
    cout.tie(0);
    int n, m;
    cin >> n >> m;
    string s, t;
    cin >> s >> t;
    // SAME SIZE
    vl an(n, 0), am(n ,0);
    vl bn(n, 0), bm(n ,0);
    for (int k = 0; k < n; ++k) {
        if (s[k] == 'a') an[k] = 1;
        else bn[k] = 1;
    }
    for (int k = 0; k < m; ++k) {
        if (t[k] == 'a') am[k] = 1;
        else bm[k] = 1;
    }
    reverse(am.begin(), am.end());
    reverse(bm.begin(), bm.end());
    vd resA= conv(an,am);
    vd resB = conv(bn,bm);
    vector<vector<int>> ans;
    ans.assign(m+1, vector<int>());

    //n > m
    // All complete count mathces
    for (int i = n-1; i < 2*n - m; ++i) {
        ans[m - round(resA[i]) - round(resB[i])].push_back(i-n+2);
    }
    // or these ranges for an and bm with original legnths
    for (int j = m-1; j < n; ++j) {
    for (int j = 0; j <= m; ++j) {
        cout << j << ":";
        for (int u: ans[j]) cout << " " << u;
        cout << "\n";
    }
    cerr << "\nTime elapsed: " << 1000 * clock() / CLOCKS_PER_SEC
        << "ms\n";
}
```

```
// CP-Algorithms FFT
vector<int> multiply(vector<int> const& a, vector<int> const& b) {
    vector<cd> fa(a.begin(), a.end()), fb(b.begin(), b.end());
    int n = 1;
    while (n < a.size() + b.size())
        n <<= 1;
    fa.resize(n);
    fb.resize(n);

    fft(fa, false);
    fft(fb, false);
    for (int i = 0; i < n; i++)
        fa[i] *= fb[i];
    fft(fa, true);

    vector<int> result(n);
    for (int i = 0; i < n; i++)
        result[i] = round(fa[i].real());
    return result;
}

using cd = complex<double>;
const double PI = acos(-1);

void fft(vector<cd> & a, bool invert) {
    int n = a.size();
    if (n == 1)
        return;

    vector<cd> a0(n / 2), a1(n / 2);
    for (int i = 0; 2 * i < n; i++) {
        a0[i] = a[2*i];
        a1[i] = a[2*i+1];
    }
    fft(a0, invert);
    fft(a1, invert);

    double ang = 2 * PI / n * (invert ? -1 : 1);
    cd w(1), wn(cos(ang), sin(ang));
    for (int i = 0; 2 * i < n; i++) {
        a[i] = a0[i] + w * a1[i];
        a[i + n/2] = a0[i] - w * a1[i];
        if (invert) {
            a[i] /= 2;
            a[i + n/2] /= 2;
        }
        w *= wn; // Trie to use w = pow(w, i) or something like
            that
    }
}


// NTT
const int mod = 7340033;
const int root = 5;
const int root_1 = 4404020;
const int root_pw = 1 << 20;

void fft(vector<int> & a, bool invert) {
    int n = a.size();

    for (int i = 1, j = 0; i < n; i++) {
        int bit = n >> 1;
        for (; j & bit; bit >>= 1)
            j ^= bit;
        j ^= bit;

        if (i < j)
            swap(a[i], a[j]);
    }

    for (int len = 2; len <= n; len <<= 1) {
        int wlen = invert ? root_1 : root;
        for (int i = len; i < root_pw; i <<= 1)
            wlen = (int)(1LL * wlen * wlen % mod);

        for (int i = 0; i < n; i += len) {
            int w = 1;
            for (int j = 0; j < len / 2; j++) {
```

```
            int u = a[i+j], v = (int)(1LL * a[i+j+len/2] * w %
                mod);
            a[i+j] = u + v < mod ? u + v : u + v - mod;
            a[i+j+len/2] = u - v >= 0 ? u - v : u - v + mod;
            w = (int)(1LL * w * wlen % mod);
        }
    }
}

if (invert) {
    int n_1 = inverse(n, mod);
    for (int & x : a)
        x = (int)(1LL * x * n_1 % mod);
}
}
```

5.6 FFTBits

```
ll c1[MAXN+9],c2[MAXN+9]; // MAXN must be power of 2 !!
void fht(ll* p, int n, bool inv, char t){
    for(int l=1;2*l<=n;l*=2){
        for(int i=0;i<n;i+=2*l){
            fore(j,0,l){
                ll u=p[i+j],v=p[i+l+j];
                // XOR
                if(t=='x'){
                    if(!inv)p[i+j]=u+v,p[i+l+j]=u-v;
                    else
                        p[i+j]=(u+v)/2,p[i+l+j]=(u-v)/2;
                }
                // AND
                else if(t=='a'){
                    if(!inv)p[i+j]=v,p[i+l+j]=u+v;
                    else p[i+j]=-u+v,p[i+l+j]=u;
                }
                // OR
                else if(t=='o'){
                    if(!inv)p[i+j]=u+v,p[i+l+j]=u;
                    else p[i+j]=v,p[i+l+j]=u-v;
                }
            }
        }
    }
}
// like polynomial multiplication, but XORing exponents
// instead of adding them (also ANDing, ORing)
vector<ll> multiply(vector<ll>& p1, vector<ll>& p2, char t){
    int n=1<<(32-__builtin_clz(max(SZ(p1),SZ(p2))-1));
    fore(i,0,n)c1[i]=0,c2[i]=0;
    fore(i,0,SZ(p1))c1[i]=p1[i];
    fore(i,0,SZ(p2))c2[i]=p2[i];
    fht(c1,n,false,t);fht(c2,n,false,t);
    fore(i,0,n)c1[i]*=c2[i];
    fht(c1,n,true,t);
    return vector<ll>(c1,c1+n);
}
```

5.7 Fib

```
#include "../Header.cpp"

// Casini:     F_{n-1} * F_{n+1} - F_n ^ 2 = (-1)^n
// Addition:   F_{n + k} = F_{k} * F_{n + 1} + F_{k - 1} * F_{n}
// Addition2:  F_{2n} = F_n * (F_{n+1} + F_{n-1})
// Addition3:  F_n divide F_m ssi n divide a m
// GCD:        gcd(F_n, F_m) = F_{gcd(n,m)}
// Formula:    F_n = (((1+sqrt(5))/2)^n - ((1-sqrt(5))/2)^n) /
    sqrt(5)
// Approx:     F_n ~ [((1+sqrt(5))/2)^n/sqrt(5)]
// Matrix:     P = {{0, 1}, {1, 1}}

// Fast doubling method
pll fib(n) {
    if(n == 0) return {0, 1};
    auto p = fib(n>>1);
    int c = p.first * (2 * p.second - p.fisrt);
    int d = p.first * p.first + p.second * p.second;
    if(n & 1) return {d, c + d};
    else return {c, d};
}
```

5.8 Functions

```
#include "../Header.cpp"

using namespace chrono;
auto start1 = high_resolution_clock::now();
auto stop = high_resolution_clock::now();
auto duration = duration_cast<microseconds>(stop - start1);
//cerr << duration.count()/1000 << "ms" << endl;

default_random_engine generator;
uniform_real_distribution<double> distribution(0,LLONG_MAX);
ll num = distribution(generator);


// files
ifstream input;
input.open("divisibility-tree.in");
input >> n;
ofstream output;
output.open("divisibility-tree.out");
output<<" ";
output.close();

// suma subconjuntos
for i = 0 to n-1
for mask = 0 to (1n) - 1
if (mask & (1i))
dp(mask) += dp(mask - (1i))

// suma divisores
for p    P
for x    S (DE MENOR A MAYOR)
if (p divide a x)
dp(x) += dp(x / p)


// hash pairs unrderedmap<pll,ll,hash_pair>
```

```
struct hash_pair {
    template <class T1, class T2>
    size_t operator()(const pair<T1, T2>& p) const
    {
        auto hash1 = hash<T1>{}(p.first);
        auto hash2 = hash<T2>{}(p.second);
        return hash1 ^ hash2;
    }
};
int maxlog2(int x) //potecncia de 2 mayor que es menor o igual a x
{
    // ll maxlog = 63 - __builtin_clzll(x);
    int maxlog = 31 - __builtin_clz(x);
    return maxlog;
}
int A[10000];//Set con reset O(1),Tambien con Map
int t=1;
bool fin(int x)
{
    return A[x]==t;
}
void borrar()
{
    t++;
}
void inse(int x)
{
    A[x]=t;
}
int res(int a,int b)
{
    int c=1,d;
    for(int i=0;i<b;i++)
    {
        c*=10;
    }
    d=c*10;
    a=a-(a-a%d);
    a=a-(a%c);
    a=a/c;
    return a;
```

```
}


//propagate val in mask to all its submask
for (int i = 0; i < p; i++)
{
    for(int mask = 0; mask < (1 << p); mask++)
    {
        if((mask & (1 << i)) == 0)
            f[mask] += f[mask | (1 << i)];
        if(mask & (1 << i)) // to propagate from submasks to mask
            dp[mask] += dp[mask - (1 << i)];
    }
}

// O(3^n)
rep(m, (1 << n)){
    // 2^k k: number of bits in m
    // iterates over al submasks of m in descending order of value
    for(int s = m; ; s = (s-1) & m){
        cout << s << endl;
        if(s == 0) break;
    }
}

int bit_opst(ll N,ll S)
{
    //Obtain the remainder (modulo) of S when it is divided by N
        (N is a power of 2)
    return S &(N -1);
    //Determine if S is a power of 2.
    return (S &( S -1)) == 0;
    //Turn  o  the last bit in S, e.g.S = (40)10 = (101000)2 S =
        (32)10 = (100000)2.
    return S &( S -1);
    // Turn on the last zero in S, e.g.S = (41)10 = (101001)2 S =
        (43)10 = (101011)2.
    return S|| (S + 1);
    // Turn  o  the last consecutive run of ones in S
    return S &( S + 1);
    // Turn on the last consecutive run of zeroes in S
    return S|| (S -1);
    // Turn on all bits
    return S = (1 << 62)-1;
    // multiply by 2^N
    return S<<=N;
    // Divide by 2^N
    return S>>=N;
    // Turn on the N-th bit
    return S = S||(1<<N);
    // Check if N-th bit is on
    return (S & (1 << N));
    // Turn off the N-th bit
    return S &= ~(1 << N);
    // Alternate satatus of N-th bir
    return S ^= (1 << N);
    //Value of the least significant bit on from the right
    return N = (S&(-S));
}
//count numbers with i bit set [1, n-1]
ll kol(ll n, ll i)
{
    return (n / (1ll << (i + 1))) * (1ll << i) + max((n % (1ll <<
        (i + 1))) - (1ll << i), 0ll);
}
kol(r+1, i) - kol(l, i);
```

5.9 GaussianElimination

```
#include "../Header.cpp"

const double EPS = 1e-9;
const int INF = 2; // it doesn't actually have to be infinity or
    a big number

int gauss (vector < vector<double> > a, vector<double> & ans) {
    int n = (int) a.size();
    int m = (int) a[0].size() - 1;

    vector<int> where (m, -1);
    for (int col=0, row=0; col<m && row<n; ++col) {
```

```
        int sel = row;
        for (int i=row; i<n; ++i)
            if (abs (a[i][col]) > abs (a[sel][col]))
                sel = i;
        if (abs (a[sel][col]) < EPS)
            continue;
        for (int i=col; i<=m; ++i)
            swap (a[sel][i], a[row][i]);
        where[col] = row;

        for (int i=0; i<n; ++i)
            if (i != row) {
                double c = a[i][col] / a[row][col];
                for (int j=col; j<=m; ++j)
                    a[i][j] -= a[row][j] * c;
            }
        ++row;
    }

    ans.assign (m, 0);
    for (int i=0; i<m; ++i)
        if (where[i] != -1)
            ans[i] = a[where[i]][m] / a[where[i]][i];
    for (int i=0; i<n; ++i) {
        double sum = 0;
        for (int j=0; j<m; ++j)
            sum += ans[j] * a[i][j];
        if (abs (sum - a[i][m]) > EPS)
            return 0;
    }

    for (int i=0; i<m; ++i)
        if (where[i] == -1)
            return INF;
    return 1;
}

int main(){
        ios_base::sync_with_stdio(0);
    cin.tie(0);
    ll n = 2;

    vector<vl>g (2, vl(3, 0));

    g[0][0] = 1;
    g[1][1] = 1;
    g[0][2] = 1;
    g[1][2] = 2;

// g: rows: equations, columns: x_1 * p_1 + x_2 * p_2 + x_3 *
    p_3 = y
    for(int i = 0; i < n-2; i++)
    {

            for(int z = i+1; z < n-1; z++)
            {
                    db mul = g[z][i] / g[i][i];
                    for(int j = 0; j < n; j++)
                            g[z][j] -= mul * g[i][j];

            }

    }
    vector<db> vals(n, 0);

    for(int i = n-2; i >= 0; i--)
    {
            db sum = g[i][n-1];
            for(int j = i+1; j < n-1; j++)

                    sum -= g[i][j] * vals[j];

            sum /= g[i][i];
            vals[i] = sum;
    }

    for(int i = 0; i < n-1; i++)
            cout << vals[i] << " ";

    cout << endl;
```

```
	for(int i = 0; i < n; i++)
	{

		for(int j = 0; j < n+1; j++)
			cout << g[i][j] << " ";
		cout << endl;

	}

}
```

5.10 MathFuncions

```
#include "../Header.cpp"

// pre overflow
ll mul(ll x, ll y) { if (x > MX / y) return MX; return x * y; }
ll sums(ll x, ll y) { if (MX - x < y) return MX; return x + y; }

const int N = 1e5 + 10, LOG_A = 31;
ll basis[LOG_A], sz;

// O(N * LOG), base that produces the maximum
void insertVector(int mask) {
	for (ll i = LOG_A - 1; i >= 0; i--) {
		if ((mask & 1 << i) == 0) continue;

		if (!basis[i]) {
			basis[i] = mask;
	    sz++;
			return;
		}

		mask ^= basis[i];
	}
}

// inclusion, exclusion
ll ans = 0;
forr(bitmask, 1, (1<<n)){
    // bitmask srepresenta la interseccion actual
    bool resta = __builtin_popcount(bitmask)%2;
    ans = (ans + (resta ? 1 : M-1)*cuenta(bitmask) %M) %M;
}

// Catalan number

/*
    Number of ways to place   pairs of parentheses correctly.
    Number of binary trees with   nodes.
    Number of full binary trees with  +     leaves.
    Number of ways to triangulate a convex  +     sided polygon.
*/

ll CAT[MAXN];
// ...
CAT[0] = CAT[1] = 1;
repx(i, 2, MAXN){
    CAT[i] = 0;
    rep(k, i)
        CAT[i]=(CAT[i]+CAT[k]*CAT[i-1-k]%M)%M;
}
ll F[MAXN], INV[MAXN], FI[MAXN];
ll Cat(int n){
    return F[2*n] *FI[n+1]%M *FI[n]%M;
}
// Stirling numbers

// number of ways to partition a set of n elements into k
    nonempty subsets

ll Stirling[MAXN][MAXN];
// ...
repx(i, 1, MAXN)Stirling[i][1] = 1;
repx(i, 2, MAXN)Stirling[1][i] = 0;
repx(i, 2, MAXN)forr(j, 2, MAXN){
    Stirling[i][j] =
    (Stirling[i-1][j-1] + j*Stirling[i-1][j]%MOD) %MOD;
}

// Bell numbers
```

```
// Number of partitions of set of n elements

// a deck of n cards is shuffled by repeatedly removing the top
    card and reinserting it anywhere in the deck (including its
    original position at the top of the deck), with exactly n
    repetitions
// stays the same B_n ways
// Probability B_n / n^n

// nth Bell number equals the number of permutations on n items
    in which no three values that are in sorted order have the
    last two of these three consecutive
ll Stirling[MAXN][MAXN], Bell[MAXN];
// ...
forn(i, MAXN){
    Bell[i] = 0;
    forn(j, MAXN)
        Bell[i] = (Bell[i] + Stirling[i][j]) %MOD;
}

//grundy
int tag[n*n];
int mex(int id)
{
    int ans = 0;
    while(tag[ans] == id) ++ans;
    return ans;
}

ll cn = 0;
for(int i = 0; i < n; i++)
{
    for(int j = 0; j < n; j++)
    {
        ll id = ++cn;
        //abajo
        for(int k = i - 1; k >= 0; k--)
            tag[grundy[k][j]] ] = id;

        //izquierda
        for(int k = j - 1; k >= 0; k--)
            tag[grundy[i][k]] ] = id;

        // diagonal
        for(int k = 1; k <= min(i, j); k++)
            tag[grundy[i-k][j-k]] ] = id;
        grundy[i][j] = mex(id);
    }
}

// fibonacci numbers
f_i = 1 / sqrt(5 * ((1 + sqrt(5)) / 2) ^ n - ((1 - sqrt(5)) / 2)
    ^ n);

// catalan numbers
a_n = 1 / (n+1) * comb(2n, n);
```

5.11 Matrices

```
#include "../Header.cpp"

/*
matrix A: transitions Axb
vector b(rows, 1): base case of dp
Represents last |b| states of dp

F_n
...
F2
F1
*/

// a^p = a*p mod P
// if a % p == 0 return 0

// to calculate p, can use p mod (P-1)
struct Mat {
    vector<vl> vec;
    Mat(): vec(1, vl(1, 0)) {}
    Mat(int n): vec(n, vl(n) ) {}
```

```
    Mat(int n, int m): vec(n, vl(m, 0) ) {}
    vl &operator[](int f){ return vec[f]; }
    const vl &operator[](int f) const { return vec[f]; }
    int size() const { return vec.size(); }
};

Mat operator *(Mat A, Mat B) {
    int n = A.size(), m = A[0].size(), t = B[0].size();
    Mat ans(n, t);
    rep(i, n) rep(j, t) rep(k, m)
        ans[i][j] = (ans[i][j] + A[i][k] * B[k][j] % MOD) % MOD;
    return ans;
}

Mat expmat(Mat A, ll e){
    int n = A.size();
    Mat Ans(n); rep(i, n) Ans[i][i] = 1;
    while(e){
        if(e&1) Ans = Ans*A;
        A = A*A; e >>= 1;
    }
    return Ans;
}

ll Fibo(ll n) {
    Mat V0(1, 2), T(2);
    V0[0] = {1, 1};
    T[0] = {0, 1}; T[1] = {1, 1};
    Mat V = V0*expmat(T, n);
    return V[0][0];
}
```

5.12 Mobius

```
short mu[MAXN] = {0,1};
void mobius(){
       fore(i,1,MAXN)if(mu[i])for(int
           j=i+i;j<MAXN;j+=i)mu[j]-=mu[i];
}
```

```
ll fun(ll N) {
       ll ans=0;
       for(ll i=1;i*i<=N;i++) ans+=mu[i]*(N/(i*i));
       return ans;
}
```

5.13 Modular

```
#include "../Header.cpp"

// Modular inverse of an array
// x_{i} ^ -1 = prefix_{i-1} * suffix{i+1} * (x1 * x2 * ... *
    x_n) ^ -1
#include "Euclid.cpp"
vl invs(vl &a, int m) {
    int n = a.size();
    if (n == 0) return {};
    vl b(n);
    ll v = 1;
    for (int i = 0; i != n; ++i) {
        b[i] = v;
        v = (v * a[i]) % m;
    }
    pll p = euclid(v, m);
    ll x = p.ff, y = p.ss;
    x = (x % m + m) % m;
    for (int i = n - 1; i >= 0; --i) {
        b[i] = (x * b[i]) % m;
        x = (x * a[i]) % m;
    }
    return b;
}

// LINEAR CONGUENCE EQUATION
// find x such that a * x % m = b
ll linear_congruence(ll a, ll b, ll m) {
    a %= m, b %= m;
    int g = gcd(a, n);
    if(g % b) return -1;
    return (minv(a, m) * b) % m;
```

```
    // more solutions: x_i = (x + i * m) % m for all i in [0, g -
        1]
}

// CRT for coprime modules
// Use garner for non coprimes modules
#include "Modular.cpp"
struct Congruence { ll a, m; };

ll chinese_remainder_theorem(vector<Congruence> const&
    congruences) {
    ll M = 1, solution = 0;
    for (auto const& congruence : congruences) M *= congruence.m;
    for (auto const& congruence : congruences) {
        ll a_i = congruence.a;
        ll M_i = M / congruence.m;
        ll N_i = inv(M_i, congruence.m);
        solution = (solution + a_i * M_i % M * N_i) % M;
    }
    return solution;
}

// Benja CRT
#include "Euclid.cpp"
pll CRT(pll a, pll b)
{
    if (a.ss < b.ss) swap(a, b);
    ll x, y; tie(x, y) = euclid(a.ss, b.ss);
    ll g = a.ss * x + b.ss * y, l = a.ss / g * b.ss;
    if ((b.ff - a.ff) % g) return {-1, -1}; // no solution
    x = (b.ff - a.ff) % b.ss * x % b.ss / g * a.ss + a.ff;
    return {x + (x < 0) * l, l};
}

pll CRT(vector<pll> &v)
{
    int N = v.size(); pll ans = v[0];
    rep(i, N) if (i) ans = CRT(ans, v[i]);
}
```

```
// DISCRETE LOGARITHM
// Returns minimum x for which a ^ x % m = b % m in O(sqrt(m))
// Includes case when a and m are not coprime dividing by the gcd
int solve(int a, int b, int m) {
    a %= m, b %= m;
    int k = 1, add = 0, g;
    while ((g = __gcd(a, m)) > 1) {
        if (b == k)
            return add;
        if (b % g)
            return -1;
        b /= g, m /= g, ++add;
        k = (k * 1ll * a / g) % m;
    }

    int n = sqrt(m) + 1;
    int an = 1;
    rep(i, n) an = (an * 1ll * a) % m;

    unordered_map<int, int> vals;
    for (int q = 0, cur = b; q <= n; ++q) {
        vals[cur] = q;
        cur = (cur * 1ll * a) % m;
    }

    for (int p = 1, cur = k; p <= n; ++p) {
        cur = (cur * 1ll * an) % m;
        if (vals.count(cur)) {
            int ans = n * p - vals[cur] + add;
            return ans;
        }
    }
    return -1;
}
```

5.14 PrimeFactorization

```
#include "../Header.cpp"

// stores smallest prime factor for every number
```

```
int spf[MAXN];

// Calculating SPF (Smallest Prime Factor) for every
// number till MAXN.
// Time Complexity : O(nloglogn)
void sieve()
{
    spf[1] = 1;
    for (int i=2; i<MAXN; i++)

        // marking smallest prime factor for every
        // number to be itself.
        spf[i] = i;

    // separately marking spf for every even
    // number as 2
    for (int i=4; i<MAXN; i+=2)
        spf[i] = 2;

    for (int i=3; i*i<MAXN; i++)
    {
        // checking if i is prime
        if (spf[i] == i)
        {
            // marking SPF for all numbers divisible by i
            for (int j=i*i; j<MAXN; j+=i)

                // marking spf[j] if it is not
                // previously marked
                if (spf[j]==j)
                    spf[j] = i;
        }
    }
}

// A O(log n) function returning primefactorization
// by dividing by smallest prime factor at every step
vector<int> getFactorization(int x)
{
    vector<int> ret;
    while (x != 1)
    {
        ret.push_back(spf[x]);
        x = x / spf[x];
    }
    return ret;
}

void primeFactors(ll n) {
    while (n % 2 == 0) {
        cout << 2 << " ";
        n = n/2;
    }

    for (int i = 3; i <= sqrt(n); i = i + 2) {
        while (n % i == 0) {
            cout << i << " ";
            n = n/i;
        }
    }
    if (n > 2)
        cout << n << " ";
}


// FACTORIZATION
// 33.3% of factors O(sqrt(n) / 3)
vl trial_division3(ll n) {
    vl factorization;
    for (int d : {2, 3, 5}) {
        while (n % d == 0) {
            factorization.pb(d);
            n /= d;
        }
    }
    int increments[] = {4, 2, 4, 2, 4, 6, 2, 6};
    int i = 0;
    for (ll d = 7; d * d <= n; d += increments[i++]) {
        while (n % d == 0) {
            factorization.pb(d);
            n /= d;
        }
```

```
        if (i == 8) i = 0;
    }
    if (n > 1) factorization.pb(n);
    return factorization;
}

// Pollard p - 1
// Probabilistic to find divisors
// O(B log(B) log^2(n))
ll pollards_p_minus_1(ll n) {
    int B = 10;
    ll g = 1;
    while (B <= 1000000 && g < n) {
        ll a = 2 + rand() % (n - 3);
        g = gcd(a, n);
        if (g > 1)
            return g;

        // compute a^M
        for (int p : primes) {
            if (p >= B) continue;
            ll p_power = 1;
            while (p_power * p <= B) p_power *= p;
            a = power(a, p_power, n);
            g = gcd(a - 1, n);
            if (g > 1 && g < n) return g;
        }
        B *= 2;
    }
    return 1;
}
// PRIMALITY DETERMINISTIC UNTIL 10^18
// 2, 3, 5, 13, 19, 73, 193, 407521, 299210837


// POLLARD RHO
// Pollard Rho
ll mult(ll a, ll b, ll mod) { return (__int128)a * b % mod; }

// For a, b <= 10 ^ 18
ll mult2(ll a, ll b, ll mod) {
    ll result = 0;
    while (b) {
        if (b & 1) result = (result + a) % mod;
        a = (a + a) % mod;
        b >>= 1;
    }
    return result;
}


ll f(ll x, ll c, ll mod) {
    return (mult(x, x, mod) + c) % mod;
}

ll rho(ll n, ll x0=2, ll c=1) {
    ll x = x0, y = x0, g = 1;
    while (g == 1) {
        x = f(x, c, n);
        y = f(y, c, n);
        y = f(y, c, n);
        g = gcd(abs(x - y), n);
    }
    return g;
}

// Brent Factorization
ll brent(ll n, ll x0=2, ll c=1) {
    ll x = x0, g = 1, q = 1, xs, y;
    int m = 128;
    int l = 1;
    while (g == 1) {
        y = x;
        repx(i, 1, l) x = f(x, c, n);
        int k = 0;
        while (k < l && g == 1) {
            xs = x;
            for (int i = 0; i < m && i < l - k; i++) {
                x = f(x, c, n);
                q = mult(q, abs(y - x), n);
            }
            g = gcd(q, n);
```

```
            k += m;
        }
        l *= 2;
    }
    if (g == n) {
        do {
            xs = f(xs, c, n);
            g = gcd(abs(xs - y), n);
        } while (g == 1);
    }
    return g;
}

// Eulers Totient Function (PHI function)

// phi(n) := number of integers in [1, n] coprime to n
// phi(p) = p - 1,
// phi(p^k) = p^k - p^{k-1}
// phi(a * b) = phi(a) * phi(b) * g / phi(g) with g = gcd(a, b)
// for n O(sqrt(n)) using factorization

// Eulers theorem: a and m relatively prime -> a^phi(m) % m = 1
// m is prime -> Fermat little theorem: a^{m-1} % m = 1
// a^n % m = a^{n % phi(m)}
// Generalization: x and m not coprime and n >= log2(n) -> x^n %
    m = x^{phi(m) + [n % phi(m)]}

int phi(int n) {
    int result = n;
    for (int i = 2; i * i <= n; i++) {
        if (n % i == 0) {
            while (n % i == 0)
                n /= i;
            result -= result / i;
        }
    }
    if (n > 1) result -= result / n;
    return result;
}

// Phi function for [1, n] in O(n log(log(n)))
void phi_1_to_n(int n) {
    vector<int> phi(n + 1);
    rep(i, n + 1) phi[i] = i;
    repx(i, 2, n + 1) {
        if (phi[i] == i) {
            for (int j = i; j <= n; j += i)
                phi[j] -= phi[j] / i;
        }
    }
}


// Divisor sum property: sum_{d|n} phi(d) = n
// phi [1, n] with the divisor sum property in O(n log(n))
void phi_1_to_n(int n) {
    vector<int> phi(n + 1);
    for (int i = 0; i <= n; i++)
        phi[i] = i;

    for (int i = 2; i <= n; i++) {
        if (phi[i] == i) {
            for (int j = i; j <= n; j += i)
                phi[j] -= phi[j] / i;
        }
    }
}

// NUMBER OF DIVISORS AND SUM OF DIVISORS
// n = p1^e1 * p2^e2 * ... * pk^ek
// d(n) := number of divisors
// d(n) = (e1 + 1) * (e2 + 1) * ... * (ek + 1)
// sigma(n) := sum of divisors
// 1 + p1 + p1^2 + ... + p1^e1 = (p1^{e1+1} - 1) / (p1 - 1)
// sigma(n) = (p1^{e1+1} - 1) / (p1 - 1) * (p2^{e2+1} - 1) / (p2
    - 1) * ... * (pk^{ek+1} - 1) / (pk - 1)
// Multiplicative functions: f(a * b) = f(a) * f(b) if a and b
    are coprimes
```

5.15 Simplex

```cpp
#include "../Header.cpp"

#define fore(i,a,b) for(int i=a,ThxDem=b;i<ThxDem;++i)

namespace Simplex {
vector<int> X,Y;
vector<vector<db> > A;
vector<db> b,c;
db z;
int n,m;
void pivot(int x,int y){
	swap(X[y],Y[x]);
	b[x]/=A[x][y];
	fore(i,0,m)if(i!=y)A[x][i]/=A[x][y];
	A[x][y]=1/A[x][y];
	fore(i,0,n)if(i!=x&&abs(A[i][y])>EPS){
		b[i]-=A[i][y]*b[x];
		fore(j,0,m)if(j!=y)A[i][j]-=A[i][y]*A[x][j];
		A[i][y]=-A[i][y]*A[x][y];
	}
	z+=c[y]*b[x];
	fore(i,0,m)if(i!=y)c[i]-=c[y]*A[x][i];
	c[y]=-c[y]*A[x][y];
}
pair<db,vector<db> > simplex( // maximize c^T x s.t. Ax<=b, x>=0
		vector<vector<db> > _A, vector<db> _b, vector<db>
		    _c){
	// returns pair (maximum value, solution vector)
	A=_A;b=_b;c=_c;
	n=b.size();m=c.size();z=0.;
	X=vector<int>(m);Y=vector<int>(n);
	fore(i,0,m)X[i]=i;
	fore(i,0,n)Y[i]=i+m;
	while(1){
		int x=-1,y=-1;
		db mn=-EPS;
		fore(i,0,n)if(b[i]<mn)mn=b[i],x=i;
		if(x<0)break;
		fore(i,0,m)if(A[x][i]<-EPS){y=i;break;}
	if(y<0) return(make_pair(-1, b));
		assert(y>=0); // no solution to Ax<=b
		pivot(x,y);
	}
	while(1){
		db mx=EPS;
		int x=-1,y=-1;
		fore(i,0,m)if(c[i]>mx)mx=c[i],y=i;
		if(y<0)break;
		db mn=INF;
		fore(i,0,n)if(A[i][y]>EPS&&b[i]/A[i][y]<mn)mn=b[i]/A[i][y]
		assert(x>=0); // c^T x is unbounded
		pivot(x,y);
	}
	vector<db> r(m);
	fore(i,0,n)if(Y[i]<m)r[Y[i]]=b[i];
	return {z,r};
}
}

int main(){
	ios_base::sync_with_stdio(0);
    cin.tie(0);

    ll n, k, x;
    db y;
    cin >> n >> k >> x;

    vector<db> b, c;
    vector<vector<db> > A;

    for(int i = 0; i < n; i++)
    {
        cin >> y;
        c.push_back(y);
    }

    vector<db>aux(n, 0);
    for(int i = 0; i < k; i++)aux[i] = -1;
    A.push_back(aux);
    b.push_back(-1.);

    for(int i = k; i < n; i++)
```

```
    {
        aux[i - k] = 0;
        aux[i] = -1;
        A.push_back(aux);
        b.push_back(-1);
    }
    aux.assign(n, 0);
    for(int i = 0; i < n; i++)
    {
        aux[i] = 1;
        A.push_back(aux);
        b.push_back(1);
        aux[i] = 0;
    }
    aux.assign(n, 1);
    A.push_back(aux);
    b.push_back(x);
    ll in = 0;
    /*for(auto it : A)
    {
        for(auto it2: it)cout<<it2<<" ";
        cout<<" "<<b[in];
        in++;
        cout<<"*\n";
    }*/

    db mx = Simplex::simplex(A,b,c).first;
    cout<<(long long)mx<<"\n";

    cerr << "\nTime elapsed: " << 1000 * clock() / CLOCKS_PER_SEC
        << "ms\n";
    return 0;
}
```

5.16 Simpson

```
const int N = 1000 * 1000; // number of steps (already multiplied
    by 2)

double simpson_integration(double a, double b){
    double h = (b - a) / N;
    double s = f(a) + f(b); // a = x_0 and b = x_2n
    for (int i = 1; i <= N - 1; ++i) { // Refer to final Simpson's
        formula
        double x = a + h * i;
        s += f(x) * ((i & 1) ? 4 : 2);
    }
    s *= h / 3;
    return s;
}
```

6 Misc

6.1 AdHoc

```
#include "../Header.cpp"

int joseph(int n,int m){
    int Result=0;
    for(int i=1;i<=n;i++){
        Result=(Result+m-1)%i+1;
    }
    return(Result);
}
int joseph(int n,int m){
    vl a(n+1, 0);
    //see eliminated
    bool o = 1;
    for(int i = 0; i < n/2; i++){
        a[i+1] = (a[i] + m-1)%(n-i);
        if(a[i+1] < n/2){
            o = 0;
            break;
        }
    }
}
// if k = 2
// move first significant bit to right
int joseph(ll n){
```

```
    ll bit = 62;
    while(!(n & (1 << bit))){
        bit--;
    }
    n &= ~(1 << bit);
    return 1 + (n << 1);
}

// matching in DAG gives min number of paths to cover all nodes
// Dilword
// matching in transitive DAG gives max independent set

// primes in a n size range n / log(n)

// nim game
// a_1 ^ a_2 ^ ... ^ a_n = 0: player 1 lose

// nim variation: remove stones from [0, k] piles
// a_1 ^_k+1 a_2 ^_k+1 ... ^_k+1 a_n = 0: player 1 lose
// ^_k+1 = xor mod (k+1) k+1 bits = 0 mod (k+1)
// sum of pairs
// a*b + b*c + c*a
//(a + b + c + d)^2 - (a^2 + b^2 + c^2)

// valor{x} = (x, 0)
//combinar((s_1, p_1), (s_2, p_2)) = ((s_1 + s_2),(p_1 + p_2 +
    s_1 * s_2))

// sum of subconj
// 1 + a + b + c + a*b + b*c + c*a + a*b*c
//(1 + a)*(1 + b)*(1 + c)

// valor{x} = 1 + x
//combinar(a, b) = a*b

// x >= y -> x mod y < x/2 counting decimals

/*Para un arbol de tamao N, solo hay un arbol para cada divisor(N)
de  tamao  divisor(N) que lo puede armar solo consigo mismo

Para hashear un arbol se usan parentesis, el hash es distinto
    para cada root,
hay que ordenar los hijos antes de hashear



everyone loses their hats all at once, and each person puts on a
    random hat;
in expectation, how many people get their own hats back?
The probability that the each person gets their own hat is 1/N,
and then by linearity of expectation,
the total number of instances of someone getting their own hat is
    1/N*N=1.

expeted value to two people will get their original hat : 1/2
for 3: 1/3

*/

// Modular sum optimization
if (R >= MOD) R -= MOD;


/*
euler cycle
all vertex with even degree


hamiltonian cycle
d(v) >= n/2 vertex degree


exact partition O(3^(m/3)) O(2^(m/2))
m(4) sets and n(3) objects
101 -
010 -
110
011

for each i in n:
    choose a row with bit i on
```

```
    erase all rows with bit i on
    continue
*/


// convex hull of max max(X, Y)^(2/3) points in recatangle (0, 0)
    (X, Y)
```

6.2 Line input

```
#include "Header.cpp"

int main()
{
    // save strings separated by space in a line
    string line, token;
    getline(cin, line);
    stringstream ss(line);
    while(ss >> token){
        cout << token << "\n";
    }
    return 0;
}
```

6.3 NextGreaterLower

```
#include "../Header.cpp"
int main(){

        ios_base::sync_with_stdio(0);
        cin.tie(0);
  ll n;
        cin >> n;

  vl c(n);
  // next value with lower/grater value
  // right greater, left greater, right lower, left lower
  vl Rg(n, n), Lg(n, -1), Rl(n, n), Ll(n, -1);

  rep(i, n){
    cin >> c[i];
  }

  stack<ll> Sg, Sl;
  rep(i, n){
    while(!Sg.empty() && c[Sg.top()] < c[i]){
      Rg[Sg.top()] = i;
      Sg.pop();
    }
    Sg.push(i);

    while(!Sl.empty() && c[Sl.top()] > c[i]){
      Rl[Sl.top()] = i;
      Sl.pop();
    }
    Sl.push(i);
  }
  while(!Sg.empty()) Sg.pop();
  while(!Sl.empty()) Sl.pop();

  for(int i = n-1; i >= 0; i--){
    while(!Sg.empty() && c[Sg.top()] <= c[i]){
      Lg[Sg.top()] = i;
      Sg.pop();
    }
    Sg.push(i);

    while(!Sl.empty() && c[Sl.top()] > c[i]){
      Ll[Sl.top()] = i;
      Sl.pop();
    }
    Sl.push(i);
  }
  cerr << "\nTime elapsed: " << 1000 * clock() / CLOCKS_PER_SEC <<
      "ms\n";
  return 0;
}
```

7 Strings

7.1 AhoCorasick

```
#include "../Header.cpp"
struct AC
{
    ll c = 0, ec = 0, M, A, af = -1;
    vector<vl> N, G; vl L, E;
    vl val;
    // L -> suffix link G -> anti L
    // E -> string finish
    AC (ll M, ll A) : M(M), A(A), N(M, vl(A, 0)), G(M, vl()), E(M,
        0), L(M, 0), val(M, 0) {}
    ll add(string s){ // return endpoint
        af++;
        ll p = 0;
        for (char l : s){
            int t = l - 'a';
            if (!N[p][t]) N[p][t] = ++c;
            p = N[p][t];
        }
        val[p] = 1;
        return p;
    }
    void init(){
        queue<int> q; q.push(0); L[0] = -1;
        while (!q.empty()){
            int p = q.front(); q.pop();
            for(int c = 0; c < A; c++){
                int u = N[p][c]; if (!u) continue;
                L[u] = L[p] == -1 ? 0 : N[L[p]][c], q.push(u);
                G[L[u]].push_back(u);
            }
            if (p) for(int c = 0; c < A; c++) if (!N[p][c]) N[p][c]
                = N[L[p]][c];
        }
    }
};
```

7.2 Hashing

```
#include "../Header.cpp"
struct RH
{
    // choose base B random to avoid hacks 33 37 41
    // randomize V(s[i])
    int B = 1777771, M[2] = {999727999, 1070777777}, P[2] =
        {325255434, 10018302};
    vl H[2], I[2];
    RH(string &s){
        int N = s.size(); rep(k, 2){
            H[k].resize(N + 1), I[k].resize(N + 1);
            H[k][0] = 0, I[k][0] = 1; ll b = 1;
            rep(i, N + 1) if (i){
                H[k][i] = (H[k][i - 1] + b * s[i - 1]) % M[k];
                I[k][i] = (1LL * I[k][i - 1] * P[k]) % M[k];
                b = (b * B) % M[k];
            }
        }
    }
    ll get(int l, int r){ // inclusive - exclusive
        ll h0 = (H[0][r] - H[0][l] + M[0]) % M[0];
        h0 = (1LL * h0 * I[0][l]) % M[0];
        ll h1 = (H[1][r] - H[1][r] + M[1]) % M[1];
        h1 = (1LL * h1 * I[1][l]) % M[1];
        return (h0 << 32) | h1;
    }
};
bool compare(int a, int b){
    ll l = 0, r = n-1, p, res = -1;
    while(l <= r){
        p = (l + r) / 2;
        if(rhs[a].get(0, p) == rhs[b].get(0, p))l = p+1;
        else {
            res = p;
            r = p-1;
        }
    }
    if(res == -1)return a < b;
    return s[a][res] < s[b][res];
```

```
}

//Suffix Array O(N log^2 N)
rep(n) sa[i] = i;
sort(ALL(sa), compare)
```

7.3 Hashing2D

```
struct Hashing2D {
    vector<vector<int>> hs;
    vector<int> PWX, PWY;
    int n, m;
    static const int PX = 3731, PY = 2999, mod = 998244353;
    Hashing() {}
    Hashing(vector<string>& s) {
        n = (int)s.size(), m = (int)s[0].size();
        hs.assign(n + 1, vector<int>(m + 1, 0));
        PWX.assign(n + 1, 1);
        PWY.assign(m + 1, 1);
        for (int i = 0; i < n; i++) PWX[i + 1] = 1LL * PWX[i] * PX
            % mod;
        for (int i = 0; i < m; i++) PWY[i + 1] = 1LL * PWY[i] * PY
            % mod;
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < m; j++) {
                hs[i + 1][j + 1] = s[i][j] - 'a' + 1;
            }
        }
        for (int i = 0; i <= n; i++) {
            for (int j = 0; j < m; j++) {
                hs[i][j + 1] = (hs[i][j + 1] + 1LL * hs[i][j] * PY
                    % mod) % mod;
            }
        }
        for (int i = 0; i < n; i++) {
            for (int j = 0; j <= m; j++) {
                hs[i + 1][j] = (hs[i + 1][j] + 1LL * hs[i][j] * PX
                    % mod) % mod;
            }
        }
    }
    int get_hash(int x1, int y1, int x2, int y2) { // 1-indexed
        assert(1 <= x1 && x1 <= x2 && x2 <= n);
        assert(1 <= y1 && y1 <= y2 && y2 <= m);
        x1--;
        y1--;
        int dx = x2 - x1, dy = y2 - y1;
        return (1LL * (hs[x2][y2] - 1LL * hs[x2][y1] * PWY[dy] %
            mod + mod) % mod -
            1LL * (hs[x1][y2] - 1LL * hs[x1][y1] * PWY[dy] % mod +
                mod) % mod * PWX[dx] % mod + mod) % mod;
    }
    int get_hash() {
        return get_hash(1, 1, n, m);
    }
};
```

7.4 KMP

```
#include "../Header.cpp"

// Build longest proper prefix/suffix array (lps) for pattern
// lps[i] = length of the longest proper prefix which is also
    suffix in pattern[0 .. i]
void init_lps(string& s, int lps[]) {
    int n = s.size();
    lps[0] = 0; // base case: no proper prefix/suffix for
        pattern[0 .. 0] (length 1)
    repx(j, 1, n) { // for each s[0 .. j]
        int i = lps[j-1]; // i points to the char next to lps of
            previous iteration
        while (s[i] != s[j] and i > 0) i = lps[i-1];
        lps[j] = s[i] == s[j] ? i+1 : 0;

        //optimization to reutilice the lps in O(n)
        if(i > 0 && s[i] == s[lps[i-1]] && lps[i-1] != 0) lps[i-1]
            = lps[lps[i-1]-1];
    }
}
```

```
// Count number of matches of pattern string in target string
    using KMP algorithm
int kmp(string& s, string& t) {
    int n = s.size(), m = t.size();
    int lps[n];
    init_lps(s, lps); // build lps array
    int matches = 0;
    int i = 0; // i tracks current char in pattern to compare
    rep(j, m) { // j tracks each char in target to compare
        // try to keep prefix before i as long as possible while
            ensuring i matches j
        while (s[i] != t[j] && i > 0) i = lps[i-1];
        if (s[i] == t[j]) {
            if (++i == n) { // we matched the whole pattern
                i = lps[n-1]; // shift the pattern so that the
                    longest proper prefix/suffix pair is aligned
                matches++;
            }
        }
    }
    return matches;
}

int main() { // usage
    string target, pattern;
    while (true) {
        cin >> target >> pattern;
        cout << kmp(pattern, target) << " matches\n";
    }
    return 0;
}
```

7.5 KMPMarceloL

```
vector<int> kmppre(string& t){ // r[i]: longest border of t[0,i)
       vector<int> r(t.size()+1);r[0]=-1;
       int j=-1;
       fore(i,0,t.size()){
              while(j>=0&&t[i]!=t[j])j=r[j];
              r[i+1]=++j;
       }
       return r;
}
void kmp(string& s, string& t){ // find t in s
       int j=0;vector<int> b=kmppre(t);
       fore(i,0,s.size()){
              while(j>=0&&s[i]!=t[j])j=b[j];
              if(++j==t.size())printf("Match at
                  %d\n",i-j+1),j=b[j];
       }
}
```

7.6 LongestCommonSubstring

```
#include "../Header.cpp"

int LCSubStr(string& X, string& Y, int m, int n){
    // Create a table to store lengths of longest common suffixes
        of
    // substrings. Notethat LCSuff[i][j] contains length of
        longest
    // common suffix of X[0..i-1] and Y[0..j-1]. The first row and
    // first column entries have no logical meaning, they are used
        only
    // for simplicity of program
    int LCSuff[m+1][n+1];
    int result = 0; // To store length of the longest common
        substring
    /* Following steps build LCSuff[m+1][n+1] in bottom up
        fashion. */
    for (int i=0; i<=m; i++){
        for (int j=0; j<=n; j++){
            if (i == 0 || j == 0)
                LCSuff[i][j] = 0;
            else if (X[i-1] == Y[j-1]){
                LCSuff[i][j] = LCSuff[i-1][j-1] + 1;
                result = max(result, LCSuff[i][j]);
            }
            else LCSuff[i][j] = 0;
        }
```

```
    }
    return result;
}

int main(){
    string X = "OldSite:GeeksforGeeks.org", Y =
        "NewSite:GeeksQuiz.com";
    int m = X.size(), n = Y.size();
    cout << "Length of Longest Common Substring is " <<
        LCSubStr(X, Y, m, n) << "\n";
    return 0;
}
```

7.7 Manacher

```
int n;
string s;

int main(){
    vi d1(n); // odd sized palindromes
    for (int i = 0, l = 0, r = -1; i < n; i++) {
        int k = (i > r) ? 1 : min(d1[l + r - i], r - i + 1);
        while (0 <= i - k && i + k < n && s[i - k] == s[i + k])
            k++;
        d1[i] = k--;
        if (i + k > r) l = i - k, r = i + k;
    }
    vi d2(n); // even sized palindromes (center to the right)
    for (int i = 0, l = 0, r = -1; i < n; i++) {
        int k = (i > r) ? 0 : min(d2[l + r - i + 1], r - i + 1);
        while (0 <= i - k - 1 && i + k < n && s[i - k - 1] == s[i
            + k]) k++;
        d2[i] = k--;
        if (i + k > r) l = i - k - 1, r = i + k;
    }
}
```

7.8 PalindromicTree

```
#include "../Header.cpp"

#define MAXN 2000005
ll M = 51123987;
struct Node
{
    // store start and end indexes of current
    // Node inclusively
    // only for first ocurrence
    ll start, end;

    // stores length of substring
    ll length;

    // stores insertion Node for all characters a-z
    ll insertEdg[26];

    // stores the Maximum Palindromic Suffix Node for
    // the current Node
    ll suffixEdg;

    ll depht;
};

// two special dummy Nodes as explained above
Node root1, root2;

// stores Node information for constant time access
Node tree[MAXN];

// Keeps track the current Node while insertion
ll currNode;
string s;
ll ptr;

void insert(ll idx)
{
//STEP 1//

    /* search for Node X such that s[idx] X S[idx]
       is maximum palindrome ending at position idx
```

```
    iterate down the suffix link of currNode to
    find X */
    ll tmp = currNode;
    while (true)
    {
        ll curLength = tree[tmp].length;
        if (idx - curLength >= 1 and s[idx] == s[idx-curLength-1])
            break;
        tmp = tree[tmp].suffixEdg;
    }

    /* Now we have found X ....
     * X = string at Node tmp
     * Check : if s[idx] X s[idx] already exists or not*/
    if(tree[tmp].insertEdg[s[idx]-'a'] != 0)
    {
        // s[idx] X s[idx] already exists in the tree
        currNode = tree[tmp].insertEdg[s[idx]-'a'];
        return;
    }

    // creating new Node
    ptr++;

    // making new Node as child of X with
    // weight as s[idx]
    tree[tmp].insertEdg[s[idx]-'a'] = ptr;

    // calculating length of new Node
    tree[ptr].length = tree[tmp].length + 2;

    // updating end point for new Node
    tree[ptr].end = idx;

    // updating the start for new Node
    tree[ptr].start = idx - tree[ptr].length + 1;



//STEP 2//
    /* Setting the suffix edge for the newly created
       Node tree[ptr]. Finding some String Y such that
       s[idx] + Y + s[idx] is longest possible
       palindromic suffix for newly created Node.*/

    tmp = tree[tmp].suffixEdg;

    // making new Node as current Node
    currNode = ptr;
    if (tree[currNode].length == 1)
    {
        // if new palindrome's length is 1
        // making its suffix link to be null string
        tree[currNode].suffixEdg = 2;
        tree[currNode].depht = 1;
        return;
    }
    while (true)
    {
        ll curLength = tree[tmp].length;
        if (idx-curLength >= 1 and s[idx] == s[idx-curLength-1])
            break;
        tmp = tree[tmp].suffixEdg;
    }

    // Now we have found string Y
    // linking current Nodes suffix link with s[idx]+Y+s[idx]
    tree[currNode].suffixEdg = tree[tmp].insertEdg[s[idx]-'a'];
    tree[currNode].depht =
        tree[tree[tmp].insertEdg[s[idx]-'a']].depht + 1;

}
// para ir al revez currNode = 1, reverse(s)
// para un string nuevo, devolver currNode o currNode = 1

// driver program
int main()
{
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    // initializing the tree
```

```
root1.length = -1;
root1.suffixEdg = 1;
root2.length = 0;
root2.suffixEdg = 1;
root1.depht = 0;
root2.depht = 0;

tree[1] = root1;
tree[2] = root2;
ptr = 2;
currNode = 1;
ll l;
cin >> l;
cin >> s;
l = s.length();
cout<<l<<endl;
vl sums(l+1, 0);
ll ans = 0;

for (ll i=0; i<l; i++){
    insert(i);
    ll nod = currNode, depht = 0;
    while(tree[nod].length > 0)
    {
      nod = tree[nod].suffixEdg;
      depht++;
    }
    nod = currNode;
    sums[i+1]=(depht + sums[i])%M;
    depht--;
    while(tree[nod].length > 1)
    {
      ans += (depht + sums[i] - sums[i - tree[nod].length +
          1])%M;
      ans %= M;

      nod = tree[nod].suffixEdg;
      depht--;
    }

}
```

```
    cout<<ans<<"\n";

    // printing all of its distinct palindromic
    // substring
    cout << "All distinct palindromic substring for "
         << s << " : \n";
    for (int i=3; i<=ptr; i++)
    {
        cout << i-2 << ") ";
        for (int j=tree[i].start; j<=tree[i].end; j++)
            cout << s[j];
        cout << endl;
    }


  cerr << "\nTime elapsed: " << 1000 * clock() / CLOCKS_PER_SEC <<
      "ms\n";
  return 0;
}
```

7.9 PalindromicTreeMarceloL

```
struct palindromic_tree{
    static const int SIGMA=26;
    struct Node{
        int len, link, to[SIGMA];
        ll cnt;
        Node(int len, int link=0, ll
            cnt=1):len(len),link(link),cnt(cnt){
            memset(to,0,sizeof(to));
        }
    };
    vector<Node> ns;
    int last;
    palindromic_tree():last(0){ns.pb(Node(-1));ns.pb(Node(0));}
    void reset(){ns.clear();last=0;ns.pb(Node(-1));ns.pb(Node(0));}
    void add(int i, string &s){
        int p=last, c=s[i]-'a';
        while(s[i-ns[p].len-1]!=s[i])p=ns[p].link;
        if(ns[p].to[c]){
```

```
            last=ns[p].to[c];
            ns[last].cnt++;
        }else{
            int q=ns[p].link;
            while(s[i-ns[q].len-1]!=s[i])q=ns[q].link;
            q=max(1,ns[q].to[c]);
            last=ns[p].to[c]=SZ(ns);
            ns.pb(Node(ns[p].len+2,q,1));
        }
    }
};
```

7.10 SuffixArray

```
// ============================================================
// Suffix Array Construction : Prefix Doubling + Radix Sort
// ============================================================
// Complexity: O(N*log(N))
// references:
    https://www.cs.helsinki.fi/u/tpkarkka/opetus/10s/spa/lecture11.pdf
// https://youtu.be/_TUeAdu-U_k
#include "../Header.cpp"
#define invrep(i,b,a) for(int i = b; i >= a; --i)


struct SA {
    int n; vl counts, rank, rank_, sa, sa_, lcp; // lcp is optional
    inline int gr(int i) { return i < n ? rank[i]: 0; }
    void csort(int maxv, int k) {
        counts.assign(maxv+1, 0);
        repx(i,0,n) counts[gr(i+k)]++;
        repx(i,1,maxv+1) counts[i] += counts[i-1];
        invrep(i,n-1,0) sa_[--counts[gr(sa[i]+k)]] = sa[i];
        sa.swap(sa_);
    }
    void get_sa(vl& s) {
        repx(i,0,n) sa[i] = i;
        sort(sa.begin(), sa.end(), [&s](int i, int j) { return
            s[i] < s[j]; });
        int r = rank[sa[0]] = 1;
        repx(i,1,n) rank[sa[i]] = (s[sa[i]] != s[sa[i-1]]) ? ++r :
            r;
        for (int h=1; h < n and r < n; h <<= 1) {
            csort(r, h); csort(r, 0); r = rank_[sa[0]] = 1;
            repx(i,1,n) {
                if (rank[sa[i]] != rank[sa[i-1]] or
                    gr(sa[i]+h) != gr(sa[i-1]+h)) ++r;
                rank_[sa[i]] = r;
            } rank.swap(rank_);
        }
    }
    // LCP construction in O(N) using Kasai's algorithm
    // reference:
        https://codeforces.com/blog/entry/12796?#comment-175287
    void get_lcp(vl& s) { // lcp is optional
        lcp.assign(n, 0); int k = 0;
        repx(i,0,n) {
            int r = rank[i]-1;
            if (r == n-1) { k = 0; continue; }
            int j = sa[r+1];
            while (i+k<n and j+k<n and s[i+k] == s[j+k]) k++;
            lcp[r] = k;
            if (k) k--;
        }
    }
    SA(vl& s) {
        n = s.size();
        rank.resize(n); rank_.resize(n);
        sa.resize(n); sa_.resize(n);
        get_sa(s); get_lcp(s); // lcp is optional
    }
};

int main() { // how to use
    string test; cin >> test;
    vl s;
    for (char c : test) s.push_back(c);
    SA sa(s);
    for (int i : sa.sa) cout << i << ":\t" << test.substr(i) <<
        '\n';
    repx(i,0,s.size()) {
```

```
        printf("LCP between %d and %d is %d\n", i, i+1, sa.lcp[i]);
    }
}
```

7.11 SuffixAutomaton

```
struct state {int len,link;map<char,int> next;}; //clear next!!
state st[100005];
int sz,last;
void sa_init(){
        last=st[0].len=0;sz=1;
        st[0].link=-1;
}
void sa_extend(char c){
        int k=sz++,p;
        st[k].len=st[last].len+1;
        for(p=last;p!=-1&&!st[p].next.count(c);p=st[p].link)st[p].next[c]=k;
        if(p==-1)st[k].link=0;
        else {
                int q=st[p].next[c];
                if(st[p].len+1==st[q].len)st[k].link=q;
                else {
                        int w=sz++;
                        st[w].len=st[p].len+1;
                        st[w].next=st[q].next;st[w].link=st[q].link;
                        for(;p!=-1&&st[p].next[c]==q;p=st[p].link)st[p].next[c]=w;
                        st[q].link=st[k].link=w;
                }
        }
        last=k;
}
```

7.12 Trie

```
#include "../Header.cpp"

struct Trie{
    static const int MAX = 1e6;
    int N[MAX][26] = {0}, S[MAX] = {0}, c = 0;
    void add(string s, int a = 1){
        int p = 0; S[p] += a;
        for (char l : s){
            int t = l - 'a';
            if (!N[p][t]) N[p][t] = ++c;
            S[p = N[p][t]] += a;
        }
    }
};

struct TrieXOR{
    static const int MAX = 1e6;
    int N[MAX][2] = {0}, S[MAX] = {0}, c = 0;
    void add(int x, int a = 1){
        int p = 0; S[p] += a;
        rep(i, 31){
            int t = (x >> (30 - i)) & 1;
            if (!N[p][t]) N[p][t] = ++c;
            S[p = N[p][t]] += a;
        }
    }
    int get(int x){
        if (!S[0]) return -1;
        int p = 0; rep(i, 31){
            int t = ((x >> (30 - i)) & 1) ^ 1;
            if (!N[p][t] || !S[N[p][t]]) t ^= 1;
            p = N[p][t]; if (t) x ^= (1 << (30 - i));
        }
        return x;
    }
};

struct Trie {
    vector<vector<int>> g;
    vector<int> count;
    int vocab;
    Trie(int vocab, int maxdepth = 10000) : vocab(vocab) {
        g.reserve(maxdepth);
        g.emplace_back(vocab, -1);
        count.reserve(maxdepth);
```

```
    count.push_back(0);
}
int move_to(int u, int c) {
    assert (0 <= c and c < vocab);
    int& v = g[u][c];
    if (v == -1) {
        v = g.size();
        g.emplace_back(vocab, -1);
        count.push_back(0);
    }
    count[v]++;
    return v;
}
void insert(const string& s, char ref = 'a') { // insert string
    int u = 0; for (char c : s) u = move_to(u, c - ref);
}
void insert(vector<int>& s) { // insert vector<int>
    int u = 0; for (int c : s) u = move_to(u, c);
}
db query(const string& s, char ref = 'a'){
  int u = 0;
  db cost = 0;
  for (char c : s){
    ll co = 0;
    for(auto it : g[u]) if(it != -1)co++;

    ll nex = move_to(u, c - ref);
    if(u == 0 || co > 1 || count[u] != count[nex]) cost++;
    u = nex;
  }
  return cost;
}
ll dfs(int u, int depht){
    ll ans = INF;
    if(count1[u] == 1 && count2[u] == 1)ans = depht;
    for(int i = 0; i < 26; i++){
        if(g[u][i] != -1) ans = min(ans, dfs(g[u][i], depht +
            1));
    }
    return ans;
}
```

```
    int size() { return g.size(); }
};


int main(){
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    ll n;
    while(cin >> n){
      string s;
      vector<string > c;
      Trie trie(26);
      for(int i = 0; i < n; i++){
        cin >> s;
        c.push_back(s);
        trie.insert(s);
      }
      db sum = 0;
      for(int i = 0; i < n; i++){
        sum += trie.query(c[i]);
      }
      cout<<fixed<<setprecision(2)<< sum / db(n) << "\n";
    }
}
```

8 Structures

8.1 BinarySearch-Ternary

```
#include "../Header.cpp"
// limit
log(valor maximo/precision)/log(2)
                          /log(3/2) //para ternaria

ll l = 0, r = n-1, res = -1;
while(l <= r){
    ll p = (l + r) / 2;
    if(c[p] <= m)l = p+1;
    else r = p-1;
```

```
}
db l = 0, r = PI/2, mini = 1e10;
rep(i, 101){
    db d=(r-l)/3.0,m1=l+d,m2=r-d;
    db c1 = value(m1), c2 = value(m2);
    // Para el maximo cambiar r-m2 con l-m1
    if (c1 < c2) r = m2;
    else l = m1;
    mini = min(mini, (c1+c2)/2.0);
}

// non continuous
ll = 1,r = m, mini = 1e17;
while(l <= r){
    ll d = (r-l)/3;
    ll m1 = l+d, m2 = r-d;
    ll c1 = value(m1), c2 = value(m2);
    // Para el maximo cambiar r-m2 con l-m1
    if (c1 < c2) r = m2-1;
    else l = m1+1;
    mini = min(mini, min(c1,c2));
}
ans += mini;
```

8.2 CDQDivideConquer

```
#include "../Header.cpp"


struct Pt {
  int x, y, z, i;
};

const int N = (int)1e5 + 5;

struct BIT {
  int b[N], n;

  void init(int _n) {
    n = _n;
    for(int i = 0 ; i <= n ; ++i) b[i] = 0;
  }
  inline int lowbit(int x) { return x & (-x); }
  void update(int x, int v) {
    for(int i = x ; i <= n ; i += lowbit(i)) b[i] += v;
  }
  int query(int x) {
    int ans = 0;
    for(int i = x ; i > 0 ; i -= lowbit(i)) ans += b[i];
    return ans;
  }
} bit;

vector<Pt> v;
int n, ans[N];

void cdq(int l, int r) {
  if(l + 1 == r) return;
  int m = (l + r) >> 1;
  cdq(l, m); cdq(m, r);
  int a = l, b = m, sum = 0;
  // need to record the modifications on BIT in order to reset it.
     The complexity will be $O(N^2)$ if we resetting it
     brute-forcely.
  vector<int> record;
  // temporary array for merge sort
  vector<Pt> tmp;
  while(a < m && b < r) {
    if(v[a].y > v[b].y) bit.update(v[a].z, 1), sum++,
        record.push_back(v[a].z), tmp.push_back(v[a++]);
    else ans[v[b].i] += sum - bit.query(v[b].z),
        tmp.push_back(v[b++]);
  }
  while(a < m) tmp.push_back(v[a++]);
  while(b < r) ans[v[b].i] += sum - bit.query(v[b].z),
      tmp.push_back(v[b++]);
  for(int i = l ; i < r ; ++i) v[i] = tmp[i - l];
  // reset BIT
  for(auto i : record) bit.update(i, -1);
  // release used memory
  vector<int> ().swap(record);
```

```
  vector<Pt> ().swap(tmp);
}

void init() {
  cin >> n;
  for(int i = 0 ; i < n ; ++i) {
    int a, b, c; cin >> a >> b >> c;
    v.push_back({a, b, c, i});
  }
  bit.init(n);
}
void solve() {
  // As we require > not >=
  sort(v.begin(), v.end(), [&](auto a, auto b) {
      return (a.x == b.x ? (a.y == b.y ? a.z < b.z : a.y < b.y) :
          a.x > b.x);
  });
  cdq(0, n);
  for(int i = 0 ; i < n ; ++i) cout << ans[i] << '\n';
}
```

8.3 DinamicConnectivity

```
#include "../Header.cpp"
#include "UnionFind.cpp"

// M log M (log M from UF)
struct DinC{
        vector<vp> DC;
        ll T; UF uf;
        DinC(ll times, ll n){
                T = times;
                DC.assign(4*T, vp());
                uf = UF(n);
        }
        void qry(){
                qry(1, 0, T-1);
        }
        void qry(ll n, ll l, ll r){
                for(auto it : DC[n]) uf.join(it.first, it.second);
                if(l == r){ // process time l
                        for(auto it : ord[l]){
                                ll x1 = uf.find(it.first);
                                ll x2 = uf.find(it.second);
                                ans += uf.sz[x1] * uf.sz[x2];
                        }

                }
                else{
                        qry(2*n,l,(l+r)/2);
                        qry(2*n+1,(l+r)/2+1,r);
                }
                for(auto it : DC[n]) uf.rollback();
        }
        void upd(ll i,ll j, pll v){
                return upd(1, 0, T-1, i, j, v);
        }
        void upd(ll n, ll l, ll r, ll i, ll j, pll v){
                if(r < i || j < l) return;
                if(i <= l && r <= j){
                    DC[n].pb(v);
                    return;
                }
                upd(2*n,l,(l+r)/2,i,j,v);
                upd(2*n+1,(l+r)/2+1,r,i,j,v);
        }
};
```

8.4 DisjointIntervals

```
// stores disjoint intervals as [first, second)
struct disjoint_intervals {
        set<pair<int,int> > s;
        void insert(pair<int,int> v){
                if(v.fst>=v.snd) return;
                auto at=s.lower_bound(v);auto it=at;
                if(at!=s.begin()&&(--at)->snd>=v.fst)v.fst=at->fst,--it;
                for(;it!=s.end()&&it->fst<=v.snd;s.erase(it++))
                        v.snd=max(v.snd,it->snd);
```

```
                segs.insert(v);
        }
};
```

8.5 DominatorTree

```
//idom[i]=parent of i in dominator tree with root=rt, or -1 if
    not exists
int
    n,rnk[MAXN],pre[MAXN],anc[MAXN],idom[MAXN],semi[MAXN],low[MAXN];
vector<int> g[MAXN],rev[MAXN],dom[MAXN],ord;
void dfspre(int pos){
        rnk[pos]=SZ(ord); ord.pb(pos);
        for(auto x:g[pos]){
                rev[x].pb(pos);
                if(rnk[x]==n) pre[x]=pos,dfspre(x);
        }
}
int eval(int v){
        if(anc[v]<n&&anc[anc[v]]<n){
                int x=eval(anc[v]);
                if(rnk[semi[low[v]]]>rnk[semi[x]]) low[v]=x;
                anc[v]=anc[anc[v]];
        }
        return low[v];
}
void dominators(int rt){
        fore(i,0,n){
                dom[i].clear(); rev[i].clear();
                rnk[i]=pre[i]=anc[i]=idom[i]=n;
                semi[i]=low[i]=i;
        }
        ord.clear(); dfspre(rt);
        for(int i=SZ(ord)-1;i;i--){
                int w=ord[i];
                for(int v:rev[w]){
                        int u=eval(v);
                        if(rnk[semi[w]]>rnk[semi[u]])semi[w]=semi[u];
                }
                dom[semi[w]].pb(w); anc[w]=pre[w];
                for(int v:dom[pre[w]]){
                        int u=eval(v);
                        idom[v]=(rnk[pre[w]]>rnk[semi[u]]?u:pre[w]);
                }
                dom[pre[w]].clear();
        }
        for(int w:ord) if(w!=rt&&idom[w]!=semi[w])
            idom[w]=idom[idom[w]];
        fore(i,0,n) if(idom[i]==n)idom[i]=-1;
}
```

8.6 FenwickTree

```
#include "../Header.cpp"

struct BIT {
    /**********All Index starts at 1***********/
    vl bit;
    BIT(ll n) { bit.assign(n+1, 0); }
    ll psq(ll k) {
        ll sum = 0;
        for (; k; k -= (k & -k)) sum += bit[k];
        return sum;
    }
    ll rsq(ll a, ll b) {
        return psq(b) - psq(a-1);
    }
    // increment k'th value by v (and propagate)
    void add(ll k, ll v) {
        for (; k < bit.size(); k += (k & -k)) bit[k] += v;
    }
    void set(ll k, ll v){
        ll aux = rsq(k,k);
        for (; k < bit.size(); k += (k & -k)) bit[k] += v-aux;
        //bit[idx] = min(bit[idx], val);
    }
    // (1, r)
    ll getmin(ll r) {
        ll ret = INF;
        for (; r >= 0; r = (r & (r + 1)) - 1)
```

```
            ret = min(ret, bit[r]);
        return ret;
    }
};
```

8.7 FenwickTree2D

```
#include "../Header.cpp"

struct BIT2D { // BIT = binary indexed tree (a.k.a. Fenwick Tree)

    //  ********All Index starts at 1**********
    vector<vector<int> > bit;
    BIT2D(int n,int m) { bit.assign(n+1, vector<int>(m+1,0)); }
    // prefix sum query (sum in range 1 .. k)
    int psq(int k, int y) {
        int sum = 0;
        for (; k; k -= (k & -k))for (int ty=y; ty; ty -= (ty &
            -ty)) sum += bit[k][ty];
        return sum;
    }
    // range sum query (sum in range a .. b)
    int rsq(int x1, int y1, int x2, int y2) {
        return psq(x2,y2) - psq(x1-1,y2)- psq(x2,y1-1) +
            psq(x1-1,y1-1);
    }
    // increment k'th value by v (and propagate)
    void add(int k, int y, int v) {
        for (; k < bit.size(); k += (k & -k)) for (int ty=y; ty <
            bit[k].size(); ty += (ty & -ty)) bit[k][ty]+=v;
    }
    void set(int k, int y, int v){
        int aux=rsq(k,y,k,y);
        for (; k < bit.size(); k += (k & -k)) for (int ty=y; ty <
            bit[k].size(); ty += (ty & -ty)) bit[k][ty]+=v-aux;
    }
};
```

8.8 Implicit$_s egment_t ree$

```
struct Vertex {
    int left, right;
    int sum = 0;
    Vertex *left_child = nullptr, *right_child = nullptr;
    Vertex(int lb, int rb) {
        left = lb;
        right = rb;
    }
    void extend() {
        if (!left_child && left + 1 < right) {
            int t = (left + right) / 2;
            left_child = new Vertex(left, t);
            right_child = new Vertex(t, right);
        }
    }
    void add(int k, int x) {
        extend();
        sum += x;
        if (left_child) {
            if (k < left_child->right)
                left_child->add(k, x);
            else
                right_child->add(k, x);
        }
    }
    int get_sum(int lq, int rq) {
        if (lq <= left && right <= rq)
            return sum;
        if (max(left, lq) >= min(right, rq))
            return 0;
        extend();
        return left_child->get_sum(lq, rq) +
            right_child->get_sum(lq, rq);
    }
};
```

8.9 LinkCutTree

```
struct node {
    int p = 0, c[2] = {0, 0}, pp = 0;
    bool flip = 0;
    int sz = 0, ssz = 0, vsz = 0; // sz -> aux tree size, ssz =
        subtree size in rep tree, vsz = virtual tree size
    long long val = 0, sum = 0, lazy = 0, subsum = 0, vsum = 0;
    node() {}
    node(int x) {
        val = x; sum = x;
        sz = 1; lazy = 0;
        ssz = 1; vsz = 0;
        subsum = x; vsum = 0;
    }
};
struct LCT {
    vector<node> t;
    LCT() {}
    LCT(int n) : t(n + 1) {}

    // <independant splay tree code>
    int dir(int x, int y) { return t[x].c[1] == y; }
    void set(int x, int d, int y) {
        if (x) t[x].c[d] = y, pull(x);
        if (y) t[y].p = x;
    }
    void pull(int x) {
        if (!x) return;
        int &l = t[x].c[0], &r = t[x].c[1];
        t[x].sum = t[l].sum + t[r].sum + t[x].val;
        t[x].sz = t[l].sz + t[r].sz + 1;
        t[x].ssz = t[l].ssz + t[r].ssz + t[x].vsz + 1;
        t[x].subsum = t[l].subsum + t[r].subsum + t[x].vsum +
            t[x].val;
    }
    void push(int x) {
        if (!x) return;
        int &l = t[x].c[0], &r = t[x].c[1];
        if (t[x].flip) {
            swap(l, r);
            if (l) t[l].flip ^= 1;
            if (r) t[r].flip ^= 1;
            t[x].flip = 0;
        }
        if (t[x].lazy) {
            t[x].val += t[x].lazy;
            t[x].sum += t[x].lazy * t[x].sz;
            t[x].subsum += t[x].lazy * t[x].ssz;
            t[x].vsum += t[x].lazy * t[x].vsz;
            if (l) t[l].lazy += t[x].lazy;
            if (r) t[r].lazy += t[x].lazy;
            t[x].lazy = 0;
        }
        // delete later, just testing
        if(!t[x].lazy){
            int flag = rand()%10000;
            if(l) t[l].lazy = 0;
            if(r) t[r].lazy = 0;
            t[x].lazy = flag;
        }
    }
    void rotate(int x, int d) {
        int y = t[x].p, z = t[y].p, w = t[x].c[d];
        swap(t[x].pp, t[y].pp);
        set(y, !d, w);
        set(x, d, y);
        set(z, dir(z, y), x);
    }
    void splay(int x) {
        for (push(x); t[x].p;) {
            int y = t[x].p, z = t[y].p;
            push(z); push(y); push(x);
            int dx = dir(y, x), dy = dir(z, y);
            if (!z) rotate(x, !dx);
            else if (dx == dy) rotate(y, !dx), rotate(x, !dx);
            else rotate(x, dy), rotate(x, dx);
        }
    }
    // </independant splay tree code>

    // making it a root in the rep. tree
    void make_root(int u) {
        access(u);
```

```
    int l = t[u].c[0];
    t[l].flip ^= 1;
    swap(t[l].p, t[l].pp);
    t[u].vsz += t[l].ssz;
    t[u].vsum += t[l].subsum;
    set(u, 0, 0);
  }
// make the path from root to u a preferred path
// returns last path-parent of a node as it moves up the tree
  int access(int _u) {
    int last = _u;
    for (int v = 0, u = _u; u; u = t[v = u].pp) {
      splay(u); splay(v);
      t[u].vsz -= t[v].ssz;
      t[u].vsum -= t[v].subsum;
      int r = t[u].c[1];
      t[u].vsz += t[r].ssz;
      t[u].vsum += t[r].subsum;
      t[v].pp = 0;
      swap(t[r].p, t[r].pp);
      set(u, 1, v);
      last = u;
    }
    splay(_u);
    return last;
  }
  void link(int u, int v) { // u -> v
    // assert(!connected(u, v));
    make_root(v);
    access(u); splay(u);
    t[v].pp = u;
    t[u].vsz += t[v].ssz;
    t[u].vsum += t[v].subsum;
  }
  void cut(int u) { // cut par[u] -> u, u is non root vertex
    // assert(connected(u, v));
    access(u);
    assert(t[u].c[0] != 0);
    t[t[u].c[0]].p = 0;
    t[u].c[0] = 0;
    pull(u);
  }
// parent of u in the rep. tree
  int get_parent(int u) {
    access(u); splay(u); push(u);
    u = t[u].c[0]; push(u);
    while (t[u].c[1]) {
      u = t[u].c[1]; push(u);
    }
    splay(u);
    return u;
  }
// root of the rep. tree containing this node
  int find_root(int u) {
    access(u); splay(u); push(u);
    while (t[u].c[0]) {
      u = t[u].c[0]; push(u);
    }
    splay(u);
    return u;
  }
  bool connected(int u, int v) {
    return find_root(u) == find_root(v);
  }
// depth in the rep. tree
  int depth(int u) {
    access(u); splay(u);
    return t[u].sz;
  }
  int lca(int u, int v) {
    // assert(connected(u, v));
    if (u == v) return u;
    if (depth(u) > depth(v)) swap(u, v);
    access(v);
    return access(u);
  }
  int is_root(int u) {
    return get_parent(u) == 0;
  }
  int component_size(int u) {
    return t[find_root(u)].ssz;
  }
```

```
  int subtree_size(int u) {
      int p = get_parent(u);
      if (p == 0) {
          return component_size(u);
      }
      cut(u);
      int ans = component_size(u);
      link(p, u);
      return ans;
  }
  long long component_sum(int u) {
      return t[find_root(u)].subsum;
  }
  long long subtree_sum(int u) {
      int p = get_parent(u);
      if (p == 0) {
          return component_sum(u);
      }
      cut(u);
      long long ans = component_sum(u);
      link(p, u);
      return ans;
  }
// sum of the subtree of u when root is specified
  long long subtree_query(int u, int root) {
      int cur = find_root(u);
      make_root(root);
      long long ans = subtree_sum(u);
      make_root(cur);
      return ans;
  }
// path sum
  long long query(int u, int v) {
      int cur = find_root(u);
      make_root(u); access(v);
      long long ans = t[v].sum;
      make_root(cur);
      return ans;
  }
  void upd(int u, int x) {
      access(u); splay(u);
      t[u].val += x;
  }
// add x to the nodes on the path from u to v
  void upd(int u, int v, int x) {
      int cur = find_root(u);
      make_root(u); access(v);
      t[v].lazy += x;
      make_root(cur);
  }
};
```

8.10 Mo

```
#include "../Header.cpp"

/*
a ancestor of b
[start[a], start[b]]
else
[end[a], start[b]] + lca(a, b)
*/
ll block;
vl ans;
vl el(1e6+2, 0);
// F = add-remove
// O((N+Q) FN )
struct Query {
    int L, R, id;
};
bool cmp(Query a, Query b){
    if(a.L / block != b.L / block)
        return a.L < b.L;
    return a.L / block % 2 ? a.R < b.R : a.R > b.R;
}
ll add(vl& a, int id){
    //cout<<id<<endl;
    return (2 * el[a[id]] + 1) * a[id];
}
ll remove(vl& a, int id){
    return (-2 * el[a[id]] + 1) * a[id];
```

```
}
void Mo(vl& a, vector<Query>& q) {
    block = (int)sqrt(a.size());
    ans.assign(q.size(),0);
    sort(ALL(q), cmp);
    ll cL = 0, cR = 0, cAns = 0;
    for (int i=0; i<q.size(); i++){
        // L and R values of current range
        int L = q[i].L, R = q[i].R;
        while (cR <= R) {
            cAns += add(a, cR);
            el[a[cR]]++;
            cR++;
        }
        while (cL > L) {
            cAns += add(a, cL-1);
            el[a[cL-1]]++;
            cL--;
        }
        while (cR > R+1) {
            cAns += remove(a, cR-1);
            el[a[cR-1]]--;
            cR--;
        }
        while (cL < L) {
            cAns += remove(a, cL);
            el[a[cL]]--;
            cL++;
        }
        ans[q[i].id] = cAns;
    }
}

int main()
{
    ios_base::sync_with_stdio(0);
    cin.tie(0);
    int n, t, x, y;
    vl c;
    cin >> n >> t;
    for(int i = 0; i < n; i++){
        cin >> x;
        c.push_back(x);
    }
    Query q;
    vector<Query>Q;
    for(int i = 0; i < t; i++){
        cin >> q.L >> q.R;
        q.L--; q.R--;
        q.id = i;
        Q.push_back(q);
    }
    Mo(c, Q);
    for(int i = 0; i < ans.size(); i++){
        cout << ans[i] << "\n";
    }
    return 0;
}
```

8.11 Partially$_{persistent_{DSU}}$

```
struct PPDSU {
  vector<vector<pair<int, int>>> par;
  int time = 0; //initial time
  PPDSU(int n) : par(n + 1, {{-1, 0}}) {}
  bool merge(int u, int v) {
    ++time;
    if ((u = root(u, time)) == (v = root(v, time))) return 0;
    if (par[u].back().first > par[v].back().first) swap(u, v);
    par[u].push_back({par[u].back().first + par[v].back().first,
        time});
    par[v].push_back({u, time}); //par[v] = u
    return 1;
  }
  bool same(int u, int v, int t) {
    return root(u, t) == root(v, t);
  }
  int root(int u, int t) { //root of u at time t
    if (par[u].back().first >= 0 && par[u].back().second <= t)
        return root(par[u].back().first, t);
    return u;
```

```
  }
  int size(int u, int t) { //size of the component of u at time t
    u = root(u, t);
    int l = 0, r = (int) par[u].size() - 1, ans = 0;
    while (l <= r) {
      int mid = l + r >> 1;
      if (par[u][mid].second <= t) ans = mid, l = mid + 1;
      else r = mid - 1;
    }
    return -par[u][ans].first;
  }
};
```

8.12 PolicyBasedEDD

```
#include "../Header.cpp"

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

// use less_equal for multiset
template <typename T, typename Comparator = less<T>>
using ordered_set = tree<T, null_type, Comparator, rb_tree_tag,
                         tree_order_statistics_node_update>;
// order_of_key(T x)
// -> returns the number of elements strictly smaller than x
// find_by_order(size_t i)
// -> returns iterator to i-th largest element (counting from 0)
int main(){
  ios::sync_with_stdio(0); cin.tie(0);
  int t;
  cin >> t;
  while(t--){
    int n, k, x, y;
    ordered_set<int> m;
    vector<pll> c;
    cin >> n >> k;
    for(int i = 0; i < n; i ++){
      cin >> x >> y;
      c.push_back({x, y});
    }
    sort(ALL(c));
    ll ans = 0;
    for(int i = 0; i < n; i++){
      ll num =m.order_of_key(c[i].second + 1);
      if(abs(num - (n-1 - num)) >= k)ans++;
      m.insert(c[i].second);
    }
    cout<<ans<<"\n";
  }
}
```

8.13 SegmentTree

```
#include "../Header.cpp"
struct SegmentTree{
        vl ST; int N;
        SegmentTree(vl &A){
                N = A.size();
                ST.assign(4*N, 0);
                bd(1,0,N-1,A);
        }
        ll op(ll x, ll y) { return min(x,y); }
        void bd(int n, int l, int r, vl &A){
                if(l == r){
                        ST[n] = A[r];
                        return;
                }
                bd(2*n,l,(l+r)/2,A);
                bd(2*n+1,(l+r)/2+1,r,A);
                ST[n] = op(ST[2*n], ST[2*n+1]);
        }
        ll qry(int i, int j){
                return qry(1,0,N-1,i,j);
        }
        ll qry(int n, int l, int r, int i, ll j){
                if(r < i || j < l) return 0;
                if(i <= l && r <= j) return ST[n];
```

```
            return op(qry(2*n,l,(l+r)/2,i,j),
                qry(2*n+1,(l+r)/2+1,r,i,j));
        }
        void upd(int i, ll v){
            return upd(1,0,N-1,i,v);
        }
        void upd(int n, int l, int r, int i, ll v){
            if(i < l || r < i) return;
            if(l == r){
                ST[n] = v;
                return;
            }
            upd(2*n,l,(l+r)/2,i,v);
            upd(2*n+1,(l+r)/2+1,r,i,v);
            ST[n] = op(ST[2*n], ST[2*n+1]);
        }
};
```

8.14 SegmentTree2D

```
#include "../Header.cpp"

struct SegmentTree{
    vector<ll>ST; int N;
    SegmentTree(int Size){
        N = Size; ST.assign(4*N,0);
    }
    void upd(int i, ll v){
        return upd(1,0,N-1,i,v);
    }
    void upd(int n, int l, int r, int i, ll v){
        if(i < l || r < i) return;
        if(l == r){
            ST[n] += v; return;
        }
        upd(2*n,l,(l+r)/2,i,v);
        upd(2*n+1,(l+r)/2+1,r,i,v);
        ST[n] = ST[2*n]+ST[2*n+1];
    }
    ll qry(int i,int j){
        return qry(1,0,N-1,i,j);
    }
    ll qry(int n, int l, int r, int i, int j){
        if(r < i || j < l) return 0;
        if(i <= l && r <= j) return ST[n];
        return
            query(2*n,l,(l+r)/2,i,j)+query(2*n+1,(l+r)/2+1,r,i,j);
    }
};
struct SegmenTree2D{
    vector<SegmentTree>ST;
    int N;
    SegmenTree2D(int Size){
        N = Size; ST.resize(4*N,Size);
    }
    void update(int i, int j, int v){
        return update(1,0,N-1,i,j,v);
    }
    void update(int n, int l, int r, int i, int j, ll v){
        if(i < l || r < i) return;
        if(l == r){
            ST[n].update(j,v);
            return;
        }
        update(2*n,l,(l+r)/2,i,j,v);
        update(2*n+1,(l+r)/2+1,r,i,j,v);
        ST[n].update(j,v);
    }
    ll query(int i1, int i2, int j1, int j2){
        return query(1,0,N-1,i1,i2,j1,j2);
    }
    ll query(int n, int l, int r, int i1, int i2, int j1, int j2){
        if(l > j1 || i1 > r) return 0;
        if(i1 <= l && r <= j1){
            return ST[n].query(i2,j2);
        }
        return
            query(2*n,l,(l+r)/2,i1,i2,j1,j2)+query(2*n+1,(l+r)/2+1,r,i1,i
    }
};
int main()
```

```
{
    ll q, n, x, y, l, b, r, t, a;
    SegmenTree2D ST(1025);
    while(cin >> q){
        if(q == 0){
            cin >> n;
        }
        else if(q == 1){
            cin >> x >> y >> a;
            ST.update(x, y, a);
        }
        else if(q == 2){
            cin >> l >> b >> r >> t;
            cout << ST.query(l, b, r, t) <<"\n";
        }
        else
            break;
        /*for(int i=0;i<n;i++)
        {
            for(int j=0;j<n;j++)
            {
                cout<<ST.query(i,j,i,j)<<" ";
            }cout<<endl;
        }*/
    }
    return 0;
}
```

8.15 SegmentTreeBeats

```
#include "../Template.cpp"

struct Node{
    ll s, mx1, mx2, mxc, mn1, mn2, mnc, lz = 0;
    Node() : s(0), mx1(LLONG_MIN), mx2(LLONG_MIN), mxc(0),
        mn1(LLONG_MAX), mn2(LLONG_MAX), mnc(0) {}
    Node(ll x) : s(x), mx1(x), mx2(LLONG_MIN), mxc(1), mn1(x),
        mn2(LLONG_MAX), mnc(1) {}
    Node(const Node &a, const Node &b){
        // add
        s = a.s + b.s;
        // min
        if (a.mx1 > b.mx1) mx1 = a.mx1, mxc = a.mxc, mx2 =
            max(b.mx1, a.mx2);
        if (a.mx1 < b.mx1) mx1 = b.mx1, mxc = b.mxc, mx2 =
            max(a.mx1, b.mx2);
        if (a.mx1 == b.mx1) mx1 = a.mx1, mxc = a.mxc + b.mxc, mx2
            = max(a.mx2, b.mx2);
        // max
        if (a.mn1 < b.mn1) mn1 = a.mn1, mnc = a.mnc, mn2 =
            min(b.mn1, a.mn2);
        if (a.mn1 > b.mn1) mn1 = b.mn1, mnc = b.mnc, mn2 =
            min(a.mn1, b.mn2);
        if (a.mn1 == b.mn1) mn1 = a.mn1, mnc = a.mnc + b.mnc, mn2
            = min(a.mn2, b.mn2);
    }
};

// 0 - indexed / inclusive - inclusive
template <class node>
struct STB{
    vector<node> st; int n;

    void build(int u, int i, int j, vector<node> &arr){
        if (i == j) { st[u] = arr[i]; return; }
        int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
        build(l, i, m, arr), build(r, m + 1, j, arr);
        st[u] = node(st[l], st[r]);
    }
    void push_add(int u, int i, int j, ll v){
        st[u].s += (j - i + 1) * v;
        st[u].mx1 += v, st[u].mn1 += v, st[u].lz += v;
        if (st[u].mx2 != LLONG_MIN) st[u].mx2 += v;
        if (st[u].mn2 != LLONG_MAX) st[u].mn2 += v;
    }
    void push_max(int u, ll v, bool l){ // for min op
        if (v >= st[u].mx1) return;
        st[u].s -= st[u].mx1 * st[u].mxc;
        st[u].mx1 = v;
        st[u].s += st[u].mx1 * st[u].mxc;
        if (l) st[u].mn1 = st[u].mx1;
```

```
    else if (v <= st[u].mn1) st[u].mn1 = v;
    else if (v < st[u].mn2) st[u].mn2 = v;
}
void push_min(int u, ll v, bool l){ // for max op
    if (v <= st[u].mn1) return;
    st[u].s -= st[u].mn1 * st[u].mnc;
    st[u].mn1 = v;
    st[u].s += st[u].mn1 * st[u].mnc;
    if (l) st[u].mx1 = st[u].mn1;
    else if (v >= st[u].mx1) st[u].mx1 = v;
    else if (v > st[u].mx2) st[u].mx2 = v;
}
void push(int u, int i, int j){
    if (i == j) return;
    // add
    int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
    push_add(l, i, m, st[u].lz);
    push_add(r, m + 1, j, st[u].lz);
    st[u].lz = 0;
    // min
    push_max(l, st[u].mx1, i == m);
    push_max(r, st[u].mx1, m + 1 == j);
    // max
    push_min(l, st[u].mn1, i == m);
    push_min(r, st[u].mn1, m + 1 == r);
}
node query(int a, int b, int u, int i, int j){
    if (b < i || j < a) return node();
    if (a <= i && j <= b) return st[u];
    push(u, i, j);
    int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
    return node(query(a, b, l, i, m), query(a, b, r, m + 1,
        j));
}
void update_add(int a, int b, ll v, int u, int i, int j){
    if (b < i || j < a) return;
    if (a <= i && j <= b) { push_add(u, i, j, v); return; }
    push(u, i, j);
    int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
    update_add(a, b, v, l, i, m); update_add(a, b, v, r, m +
        1, j);
    st[u] = node(st[l], st[r]);
}
void update_min(int a, int b, ll v, int u, int i, int j){
    if (b < i || j < a || v >= st[u].mx1) return;
    if (a <= i && j <= b && v > st[u].mx2) { push_max(u, v, i
        == j); return; }
    push(u, i, j);
    int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
    update_min(a, b, v, l, i, m); update_min(a, b, v, r, m +
        1, j);
    st[u] = node(st[l], st[r]);
}
void update_max(int a, int b, ll v, int u, int i, int j){
    if (b < i || j < a || v <= st[u].mn1) return;
    if (a <= i && j <= b && v < st[u].mn2) { push_min(u, v, i
        == j); return; }
    push(u, i, j);
    int m = (i + j) / 2, l = u * 2 + 1, r = u * 2 + 2;
    update_max(a, b, v, l, i, m); update_max(a, b, v, r, m +
        1, j);
    st[u] = node(st[l], st[r]);
}

STB(vector<node> &v, int N) : n(N), st(N * 4 + 5) { build(0,
    0, n - 1, v); }
node query(int a, int b) { return query(a, b, 0, 0, n - 1); }
void update_add(int a, int b, ll v) { update_add(a, b, v, 0,
    0, n - 1); }
void update_min(int a, int b, ll v) { update_min(a, b, v, 0,
    0, n - 1); }
void update_max(int a, int b, ll v) { update_max(a, b, v, 0,
    0, n - 1); }
};
```

8.16 SegmentTreeIterative

```
#include "../Header.cpp"

struct Node{
    int v;
```

```
    Node() { v = 0; } // neutro
    Node(int v) : v(v) {}
    Node(const Node &a, const Node &b) { v = a.v + b.v; }
};

// 0 - indexed / inclusive - exclusive
template <class node>
struct ST{
    vector<node> t; int n;
    ST(vector<node> &arr, int N) : n(N), t(N * 2){
        copy(arr.begin(), arr.end(), t.begin() + n);
        for (int i = n - 1; i > 0; --i) t[i] = node(t[i << 1], t[i
            << 1 | 1]);
    }
    void set(int p, const node &value){
        for (t[p += n] = value; p >>= 1;)
            t[p] = node(t[p << 1], t[p << 1 | 1]);
    }
    node query(int l, int r){
        node ansl, ansr;
        for (l += n, r += n; l < r; l >>= 1, r >>= 1){
            if (l & 1) ansl = node(ansl, t[l++]);
            if (r & 1) ansr = node(t[--r], ansr);
        }
        return node(ansl, ansr);
    }
};
```

8.17 SegmentTreeLazy

```
#include "../Header.cpp"

struct Node
{
  ll mini, cont;
  Node(ll val, ll cont)
  {
    this->mini = val;
    this->cont = cont;
  }
  Node()
  {
    mini = 1e18;
    cont = 0;
  }

  Node operator + (const Node& p) const {
    return Node(min(mini, p.mini), cont + p.cont);
  }
};

ll ans = 0;

struct SegmentTree
{
        vector<Node> ST;
  vl Lazy;
        int N; ll Nul = 0;
        SegmentTree(vector<ll> &A)
        {
                N = A.size();
                ST.resize(4*N+5, Node());
                Lazy.resize(4*N+5,Nul);
                bd(1,0,N-1,A);
        }
        void up(int n, int l, int r)
        {
                        ST[n].mini += Lazy[n];
                        if(l != r)
                        {
                                        Lazy[n*2] += Lazy[n];
                                        Lazy[n*2+1] += Lazy[n];
                        }
                        Lazy[n] = Nul;
        }
        void bd(int n, int l, int r, vl &A)
        {
                if(l == r)
                {
                        ST[n] = Node(A[r], 1);
                        return;
```

```
        }

        bd(2*n,l,(l+r)/2,A);
        bd(2*n+1,(l+r)/2+1,r,A);
        ST[n] = ST[2*n] + ST[2*n+1];
    }

    Node qry(int n, int l, int r, int i, int j)
    {
        if(r < i || j < l ) return Node();
  if(Lazy[n] != Nul) up(n,l,r);
        if(i <= l && r <= j) return ST[n];
        return qry(2*n,l,(l+r)/2,i,j) +
            qry(2*n+1,(l+r)/2+1,r,i,j);
    }

    void upd(int n, int l, int r, int i, int j, ll v)
    {
        if(Lazy[n] != Nul) up(n,l,r);
        if(l > j || r < i) return;

        if(i <= l && r <= j)
        {
            Lazy[n] += v;
            if(Lazy[n] != Nul) up(n, l, r);
            return;
        }

        upd(2*n,l,(l+r)/2,i,j,v);
        upd(2*n+1,(l+r)/2+1,r,i,j,v);
        ST[n] = ST[2*n] + ST[2*n+1];
    }

void search(int n, int l, int r, int j, ll v)
    {
        if(Lazy[n] != Nul) up(n,l,r);
  if(l > j || ST[n].mini > v) return;

  if(l == r)
  {
    ans += ST[n].mini;
    ST[n] = Node();
    if(Lazy[n] != Nul) up(n, l, r);
                return;
    }

        search(2*n,l,(l+r)/2,j,v);
            search(2*n+1,(l+r)/2+1,r,j,v);

            ST[n] = ST[2*n] + ST[2*n+1];
        }

  Node qry(int i, int j)
        {return qry(1,0,N-1,i,j);}
  void upd(int i, int j, ll v)
        {return upd(1,0,N-1,i,j,v);}
  void search(int j, ll v)
        {return search(1,0,N-1,j,v);}
};
```

8.18 SegmentTreePersistent

```
#include "../Header.cpp"

struct SegmentTree
{
    vector<vl> ST; vector<vi> Leftv, Rightv;
    int N;
    SegmentTree(vl &A){
        N = A.size();
        ST.resize(4*N, vl());
        // ST.resize(4*N, vl(1, 0)); whitout build, init
            with 0s
        Leftv.resize(4*N, vi(1, 0)); // all conected to
            version 0
        Rightv.resize(4*N, vi(1, 0));
        bd(1,0,N-1,A);
    }
    ll op(ll x, ll y) { return min(x,y); }
    void bd(int n, int l, int r, vl &A){
        if(l==r){
```

```
                ST[n].push_back(A[r]);
                return;
        }
        bd(2*n,l,(l+r)/2,A);
        bd(2*n+1,(l+r)/2+1,r,A);
        ST[n].push_back(op(ST[2*n][0], ST[2*n+1][0]));
    }
    ll qry(int i, int j, int vs){
        return qry(1,0,N-1,i,j, vs);
    }
    ll qry(int n, int l, int r, int i, int j, int vs){
        if(r < i || j < l) return 0;
        if(i <= l && r <= j) return ST[n][vs];
        return op(qry(2*n,l,(l+r)/2,i,j, Leftv[n][vs]),
            qry(2*n+1,(l+r)/2+1,r,i,j, Rightv[n][vs]));
    }
    void upd(int i, ll v){
        return upd(1,0,N-1,i,v);
    }
    void upd(int n, int l, int r, int i, ll v){
        if(i < l || r < i) return;
        if(l == r){
            ST[n].push_back(v); // ST[n].push_back(v +
                ST[n].back()) add
            return;
        }
        upd(2*n,l,(l+r)/2,i,v);
        upd(2*n+1,(l+r)/2+1,r,i,v);

        ST[n].push_back(op(ST[2*n].back(),
            ST[2*n+1].back()));
        Leftv[n].push_back(ST[2*n].size()-1);
        Rightv[n].push_back(ST[2*n+1].size()-1);
    }
};
```

8.19 SegmentTreeSubarraySum

```
#include "../Header.cpp"
```

```
// Max subarray sum
struct Node{
    //maxPrefixSum, maxSuffixSum, Totalsum, maxSubarraySum
    ll mxP, mxS, sum, subSum;
    Node(){
        mxP = mxS = sum = subSum = -INF;
    }
    Node merge(Node r){
        Node p;
        p.mxP = max(mxP, sum + r.mxP);
        p.mxS = max(r.mxS, r.sum + mxS);
        p.sum = sum + r.sum;
        p.subSum = max({subSum, r.subSum, mxS + r.mxP});
        return p;
    }
    void upd(ll v){
        mxP = mxS = sum = subSum = v;
    }
    void nul(){
        mxP = mxS = subSum = -INF;
        sum = 0;
    }
};
// Node version
struct SegmentTree{
    vector<Node> ST;
    int N;
    SegmentTree(vl &A){
        N = A.size();
        ST.assign(4*N, Node());
        bd(1,0,N-1,A);
    }
    void bd(int n, int l, int r, vl &A){
        if(l == r){
            ST[n].upd(A[l]);
            return;
        }
        bd(2*n,l,(l+r)/2,A);
        bd(2*n+1,(l+r)/2+1,r,A);
        ST[n] = ST[2*n].merge(ST[2*n+1]);
    }
```

```
        Node qry(int i, int j){
                return qry(1,0,N-1,i,j);
        }
        Node qry(int n, int l, int r, int i, int j){
                if(r < i || j < l) {
                        Node p;
                        p.nul();
                        return p;
                }
                if(i <= l && r <= j) return ST[n];
                return
                    qry(2*n,l,(l+r)/2,i,j).merge(qry(2*n+1,(l+r)/2+1,r,i,j));
        }
        void upd(int i, ll v){
                return upd(1,0,N-1,i,v);
        }
        void upd(int n, int l, int r, int i, ll v){
                if(i < l || r < i) return;
                if(l == r){
                        ST[n].upd(v);
                        return;
                }
                upd(2*n,l,(l+r)/2,i,v);
                upd(2*n+1,(l+r)/2+1,r,i,v);
                ST[n] = ST[2*n].merge(ST[2*n+1]);
        }
};
```

8.20 SparseTable

```
#include "../Header.cpp"

struct SparseTable{
    vector<vl >SP;
    SparseTable(vl&A){
        int n = A.size();
        SP.push_back(A);
        ll maxlog = 31 - __builtin_clz(n);
        repx(i, 1 ,maxlog+1){
            vl aux;
            rep(j, n-(1<<i)+1){
                aux.push_back(max(SP[i-1][j],SP[i-1][j+(1<<(i-1))]));
            }
            SP.push_back(aux);
        }
    }
    ll op(int l, int r){
        ll maxlog = 31 - __builtin_clz(r-l+1);
        return max(SP[maxlog][l],SP[maxlog][r-(1<<maxlog)+1]);
    }
    ll find(int l, int r, ll m){ // maxi
      ll maxlog = 31 - __builtin_clz(r-l+1);
      for(int i = maxlog; i >= 0; i--){
        if(l + (1<<i) <= r && SP[i][l] < m){
          l += (1<<i);
        }
      }
      return l;
    }
};
```

8.21 Treap

```
#include "../Header.cpp"
typedef struct item *pitem;
struct item {
        int pr,key,cnt;
        pitem l,r;
        item(int key):key(key),pr(rand()),cnt(1),l(0),r(0) {}
};
int cnt(pitem t){return t?t->cnt:0;}
void upd_cnt(pitem t){if(t)t->cnt=cnt(t->l)+cnt(t->r)+1;}
void split(pitem t, int key, pitem& l, pitem& r){ // l: < key, r:
    >= key
        if(!t)l=r=0;
        else if(key<t->key)split(t->l,key,l,t->l),r=t;
        else split(t->r,key,t->r,r),l=t;
        upd_cnt(t);
}
void insert(pitem& t, pitem it){
```

```
        if(!t)t=it;
        else if(it->pr>t->pr)split(t,it->key,it->l,it->r),t=it;
        else insert(it->key<t->key?t->l:t->r,it);
        upd_cnt(t);
}
void merge(pitem& t, pitem l, pitem r){
        if(!l||!r)t=l?l:r;
        else if(l->pr>r->pr)merge(l->r,l->r,r),t=l;
        else merge(r->l,l,r->l),t=r;
        upd_cnt(t);
}
void erase(pitem& t, int key){
        if(t->key==key)merge(t,t->l,t->r);
        else erase(key<t->key?t->l:t->r,key);
        upd_cnt(t);
}
void unite(pitem &t, pitem l, pitem r){
        if(!l||!r){t=l?l:r;return;}
        if(l->pr<r->pr)swap(l,r);
        pitem p1,p2;split(r,l->key,p1,p2);
        unite(l->l,l->l,p1);unite(l->r,l->r,p2);
        t=l;upd_cnt(t);
}
pitem kth(pitem t, int k){
        if(!t)return 0;
        if(k==cnt(t->l))return t;
        return k<cnt(t->l)?kth(t->l,k):kth(t->r,k-cnt(t->l)-1);
}
pair<int,int> lb(pitem t, int key){ // position and value of
    lower_bound
        if(!t)return {0,1<<30}; // (special value)
        if(key>t->key){
                auto w=lb(t->r,key);w.fst+=cnt(t->l)+1;return w;
        }
        auto w=lb(t->l,key);
        if(w.fst==cnt(t->l))w.snd=t->key;
        return w;
}

pitem ss;
int n,q;
```

```
int find(int x){
        int s=1,e=n+1;
        while(e-s>1){
                int m=(s+e)/2;
                if(m-lb(ss,m).fst>x)e=m;
                else s=m;
        }
        return s;
}

int main(){
        scanf("%d%d",&n,&q);
        while(q--){
                char c[4];int k;
                scanf("%s%d",c,&k);
                if(c[0]=='D')insert(ss,new item(find(k)));
                else printf("%d\n",find(k));
        }
        fore(i,0,cnt(ss))assert(lb(ss,kth(ss,i)->key).fst==i);
        return 0;
}
```

8.22 TreapImplicit

```
// example that supports range reverse and addition updates, and
    range sum query
// (commented parts are specific to this problem)
#include "../Header.cpp"
typedef struct item *pitem;
struct item {
        int pr,cnt,val;
//      int sum; // (paramters for range query)
//      bool rev;int add; // (parameters for lazy prop)
        pitem l,r;
        item(int val):
            pr(rand()),cnt(1),val(val),l(0),r(0)/*,sum(val),rev(0),add(0)
            {}
};
void push(pitem it){
```

```
        if(it){
                /*if(it->rev){
                        swap(it->l,it->r);
                        if(it->l)it->l->rev^=true;
                        if(it->r)it->r->rev^=true;
                        it->rev=false;
                }
                it->val+=it->add;it->sum+=it->cnt*it->add;
                if(it->l)it->l->add+=it->add;
                if(it->r)it->r->add+=it->add;
                it->add=0;*/
        }
}
int cnt(pitem t){return t?t->cnt:0;}
// int sum(pitem t){return t?push(t),t->sum:0;}
void upd_cnt(pitem t){
        if(t){
                t->cnt=cnt(t->l)+cnt(t->r)+1;
                // t->sum=t->val+sum(t->l)+sum(t->r);
        }
}
void merge(pitem& t, pitem l, pitem r){
        push(l);push(r);
        if(!l||!r)t=l?l:r;
        else if(l->pr>r->pr)merge(l->r,l->r,r),t=l;
        else merge(r->l,l,r->l),t=r;
        upd_cnt(t);
}
void split(pitem t, pitem& l, pitem& r, int sz){ // sz:desired
    size of l
        if(!t){l=r=0;return;}
        push(t);
        if(sz<=cnt(t->l))split(t->l,l,t->l,sz),r=t;
        else split(t->r,t->r,r,sz-1-cnt(t->l)),l=t;
        upd_cnt(t);
}
void output(pitem t){ // useful for debugging
        if(!t)return;
        push(t);
        output(t->l);printf(" %d",t->val);output(t->r);
}
```

```
// use merge and split for range updates and queries

int n,q;
char s[100005];

int main(){
        string s;
        cin >> s;
        pitem t=0;
        rep(i, s.size())merge(t,t,new item(s[i]-'a'));

        ll q, x;
        cin >> q;
        while(q--)
        {
                cin >> x;
                pitem r;
                split(t, t, r, x);
                if(t) {t->rev^=true; t->add+=1} if(r) r->rev^=true;
                merge(t, t, r);
        }
        ans(t);
}
```

8.23 TreapImplictFather

```
// node father is useful to keep track of the chain of each node
// alternative: splay tree
// IMPORTANT: add pointer f in struct item
#include "../Header.cpp"
#define fore(i,a,b) for(int i=a,ThxDem=b;i<ThxDem;++i)
void merge(pitem& t, pitem l, pitem r){
        push(l);push(r);
        if(!l||!r)t=l?l:r;
        else if(l->pr>r->pr)merge(l->r,l->r,r),l->r->f=t=l;
        else merge(r->l,l,r->l),r->l->f=t=r;
        upd_cnt(t);
}
void split(pitem t, pitem& l, pitem& r, int sz){
        if(!t){l=r=0;return;}
```

```
        push(t);
        if(sz<=cnt(t->l)){
                split(t->l,l,t->l,sz);r=t;
                if(l)l->f=0;
                if(t->l)t->l->f=t;
        }
        else {
                split(t->r,t->r,r,sz-1-cnt(t->l));l=t;
                if(r)r->f=0;
                if(t->r)t->r->f=t;
        }
        upd_cnt(t);
}
void push_all(pitem t){
        if(t->f)push_all(t->f);
        push(t);
}
pitem root(pitem t, int& pos){ // get root and position for node t
        push_all(t);
        pos=cnt(t->l);
        while(t->f){
                pitem f=t->f;
                if(t==f->r)pos+=cnt(f->l)+1;
                t=f;
        }
        return t;
}


int n,m,c,q;
map<pair<int,int>,int> w; // owner

pitem t[105][8005];

bool join(int k, int x, int y){
        int a,b;
        pitem r0=root(t[k][x],a),r1=root(t[k][y],b);
        if(a&&a<cnt(r0)-1||b&&b<cnt(r1)-1){puts("Forbidden:
            monopoly.");return false;}
        if(r0==r1){puts("Forbidden: redundant.");return false;}
        if(a==0)r0->rev^=1;
        if(b!=0)r1->rev^=1;
        //printf(" %d %d %d %d\n",a,b,cnt(r0),cnt(r1));
        pitem _;
        merge(_,r0,r1);
        return true;
}

void disjoin(int k, int x, int y){
        int a,b;
        pitem r0=root(t[k][x],a),r1=root(t[k][y],b);
        assert(r0==r1);assert(abs(a-b)==1);
        pitem _,__;
        //split(a<b?t[k][y]:t[k][x],_,__);
        split(r0,_,__,max(a,b));
}

int main(){
        fore(i,0,105)fore(j,0,8005)t[i][j]=new item();
        while(scanf("%d%d%d%d",&n,&m,&c,&q),n){
                fore(i,0,c)fore(j,0,n)t[i][j]->l=t[i][j]->r=t[i][j]->f=0;
                w.clear();
                fore(i,0,m){
                        int x,y,k;
                        scanf("%d%d%d",&x,&y,&k);x--;y--;k--;
                        w[{x,y}]=k;
                        assert(join(k,x,y));
                }
                while(q--){
                        int x,y,k;
                        scanf("%d%d%d",&x,&y,&k);x--;y--;k--;
                        if(!w.count({x,y})){puts("No such
                            cable.");continue;}
                        int kk=w[{x,y}];
                        if(kk==k){puts("Already owned.");continue;}
                        if(join(k,x,y)){
                                w[{x,y}]=k;
                                disjoin(kk,x,y);
                                puts("Sold.");
                        }
                }
                puts("");
```

```
    }
    return 0;
}
```

8.24 UnionFind

```
#include "../Header.cpp"

struct UF{
    vl p, r, sz;
    UF uf();
    UF(ll n){
        r.assign(n, 0);
        sz.assign(n, 1);
        rep(i, n)
            p.push_back(i);
    }
    ll find(ll x) {return p[x] = p[x] == x ? x : find(p[x]);}
    void join(ll x, ll y){
        if ((x = find(x)) == (y = find(y))) return;
        if(r[x] < r[y]) swap(x, y);
        if(r[x] == r[y]) r[x]++;
        p[y] = x;
        sz[x] += sz[y];
    }
};


// With rollback
struct UF{
    vl p, r, sz;
    stack<vl> S;
    UF () {}
    UF(ll n){
        r.assign(n, 0);
        sz.assign(n, 1);
        for(ll i = 0; i < n; i++)
            p.push_back(i);
    }
    ll find(ll x) {return p[x] == x ? x : find(p[x]);}
    void join(ll x, ll y){
        if ((x = find(x)) == (y = find(y))) return;
        if(r[x] < r[y]) swap(x, y);
        if(r[x] == r[y]) r[x]++;
        S.push({x, y, p[x], p[y]});
        p[y] = x;
        sz[x] += sz[y];
    }
    void rollback(){
        auto a = S.top(); S.pop();
        p[a[0]] = a[2];
        p[a[1]] = a[3];
        sz[a[0]] -= sz[a[1]];
    }
};
```

8.25 WaveletTree

```
#include "../Header.cpp"
typedef vector<int>::iterator iter;

//Wavelet tree with succinct representation of bitmaps
struct WaveTreeSucc {
  vector<vector<int> > C; int s;

  // sigma = size of the alphabet, ie., one more than the maximum
      element
  // in S.
  WaveTreeSucc(vector<int> &A, int sigma) : C(sigma*2), s(sigma) {
    build(A.begin(), A.end(), 0, s-1, 1);
  }

  void build(iter b, iter e, int L, int U, int u) {
    if (L == U)
      return;
    int M = (L+U)/2;

    // C[u][i] contains number of zeros until position i-1: [0,i)
    C[u].reserve(e-b+1); C[u].push_back(0);
    for (iter it = b; it != e; ++it)
```

```
    C[u].push_back(C[u].back() + (*it<=M));

  iter p = stable_partition(b, e, [=](int i){return i<=M;});

  build(b, p, L, M, u*2);
  build(p, e, M+1, U, u*2+1);
}

// Count occurrences of number c until position i.
// ie, occurrences of c in positions [i,j]
int rank(int c, int i) const {
  // Internally we consider an interval open on the left: [0, i)
  i++;
  int L = 0, U = s-1, u = 1, M, r;
  while (L != U) {
    M = (L+U)/2;
    r = C[u][i]; u*=2;
    if (c <= M)
      i = r, U = M;
    else
      i -= r, L = M+1, ++u;
  }
  return i;
}

// Find the k-th smallest element in positions [i,j].
// The smallest element is k=1
int quantile(int k, int i, int j) const {
  // internally we we consider an interval open on the left: [i,
      j)
  j++;
  int L = 0, U = s-1, u = 1, M, ri, rj;
  while (L != U) {
    M = (L+U)/2;
    ri = C[u][i]; rj = C[u][j]; u*=2;
    if (k <= rj-ri)
      i = ri, j = rj, U = M;
    else
      k -= rj-ri, i -= ri, j -= rj,
        L = M+1, ++u;
  }
  return U;
}

// Count number of occurrences of numbers in the range [a, b]
// present in the sequence in positions [i, j], ie, if
    representing a grid it
// counts number of points in the specified rectangle.
mutable int L, U;
int range(int i, int j, int a, int b) const {
  if (b < a or j < i)
    return 0;
  L = a; U = b;
  return range(i, j+1, 0, s-1, 1);
}

int range(int i, int j, int a, int b, int u) const {
  if (b < L or U < a)
    return 0;
  if (L <= a and b <= U)
    return j-i;
  int M = (a+b)/2, ri = C[u][i], rj = C[u][j];
  return range(ri, rj, a, M, u*2) +
    range(i-ri, j-rj, M+1, b, u*2+1);
}
};
```