Dynamic Mode Decomposition: Recognition of Spatial-Temporal patterns in large-scale Neural Recordings for imagination and movement tasks

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1 Introduction

Neural recordings play a crucial role in studying and understanding the nervous system, particularly in the field of neuroscience. It involves monitoring the electrical activity of nerve cells, called neurons, using electrodes or other sensing devices. These recordings allow scientists to examine in detail how neurons communicate with each other to perform cognitive, motor, and sensory functions. Despite the vast amount of data generated by these recordings, the activity of complex neural networks can often be described by a relatively low number of distinct patterns. Thus, recognizing patterns in neural recordings holds fundamental importance. This process relies on analyzing and identifying patterns of electrical activity within the brain, recorded through techniques such as electroencephalography (**EEG**), electrocorticography (**ECoG**), or neural activity recorded by micro-electrodes. In addition, the identification of these spatial-temporal patterns allows the construction of dynamic models and the application of machine learning tools for pattern analysis.

Through patterns recognition in neural recordings, we are able to:

1. **Understand Brain Function:** The brain is a complex network of interconnected neurons that generate electrical signals in response to internal and external stimuli. Recognizing patterns of neural activity allows us to analyze how it processes information, stores memories, generates emotions, and controls movement, contributing to the basic understanding of cognitive and neural processes.

2. **Diagnose and Treat Brain Disorders:** Recognizing anomalous patterns in neural activity can aid in early diagnosis and understanding of neurological and psychiatric diseases such as epilepsy, Parkinson's disease, depression, and other conditions. Identifying such patterns can lead to new targeted therapeutic approaches and better assessment of treatment effectiveness.

3. Develop Brain-Computer Interfaces (BCIs): Neural pattern recognition is essential in the development of brain-computer interfaces (BCIs), which enable individuals to communicate or control external devices directly through brain activity. This technology holds promise for people with motor or communication disabilities, opening new possibilities for interaction with the surrounding world.

4. Develop Neuroprosthetics: Pattern recognition can be used to control neural prostheses or implants that replace or restore compromised functions, such as limb movement. Precise identification of neural activity patterns allows the translation of patient intentions into commands for prostheses, thereby improving their utility and functionality.

5. Enhance Cognitive Performance and Learning: Through pattern recognition, it's possible to observe brain activity during learning and cognitive effort. This can lead to more personalized teaching strategies and interventions to improve learning and attention.
Therefore, pattern recognition in neural recordings is essential for gaining a better understanding of the human brain's functioning, developing new medical and communication technologies, and advancing neuroscience and artificial intelligence.

Measurements of neural activity from tens to hundreds of simultaneously recorded channels constitute of time traces that explore a network with complex dynamics and generate, as mentioned before, a vast amount of date. Thus, exploiting these data sources is not straightforward because such sources tend to be high-dimensional and because these data sources are not directly compatible with traditional modeling and control techniques. To deal with this issue, a model reduction technique can be used in order to construct a low-dimensional representation of the physical model.
A possible reduction technique is the modal decomposition[1], also known as dynamic mode decomposition **(DMD)**. DMD exploits the fact that high-dimensional dynamical systems tend to reside in low-dimensional spaces and thus seeks to construct reduced order models in such spaces. In the context of spatio-temporal systems, the low-dimensional space is composed of modes that capture dominant spatial features of the system.

The paper is structured as follows:

- ***Section 2***: Introduction of concepts and algorithm of DMD models;
- ***Section 3***: Explanation of Data-acquisition and Data-processing;
- ***Section 4***: Illustration of obtained results;
- ***Section 5***: Conclusion.

[1] Modal decomposition has been successfully applied in every scientific and engineering discipline.

2 Dynamic Mode Decomposition

The dynamic mode decomposition (**DMD**) is a new approach to explore spatial-temporal patterns in large-scale neural recordings. It combines well-characterized advantages from two of the most powerful data analysis tools used today: time-domain power spectral analysis and principal component analysis (**PCA**) in space.

2.1 Why do we use it?

The DMD is a modal decomposition algorithm that describes dynamical data at high dimensionality using coupled spatial and temporal modes. It is robust to noise variations, at sub-sampling velocity and fits easily to a very high number of measurements acquired simultaneously.(1)

2.2 Theoretical Concept

In this framework we present the *exact* DMD developed by Tu et al., although several algorithms have been proposed, since earlier formulations required uniform sampling of the dynamics in time, while the approach presented here works with irregularly sampled data and with concatenated data from several different experiments or numerical simulations. Moreover, the exact formulation provides a precise mathematical definition of DMD that allows for rigorous theoretical results.

Exact DMD is based on the efficient and numerically well-conditioned SVD. It is inherently data-driven which first step is to collect a number of pairs of snapshots of the state of a system. These last ones may be denoted by $\{x(t_k), x(t'_k)\}_{k=1}^{m}$ where $t'_k = t_k + \Delta t$, and the time step Δt is sufficiently small to solve the highest frequencies in the dynamics(1). As before, a snapshot may be the state of a system, that is reshaped into an high-dimensional column vector:

$$X = \begin{pmatrix} | & | & | \\ | & | & | \\ x(t_1) & \dots & x(t_m) \\ | & | & | \\ | & | & | \end{pmatrix}$$

$$X' = \begin{pmatrix} | & | & | \\ | & | & | \\ x(t'_1) & \dots & x(t'_m) \\ | & | & | \\ | & | & | \end{pmatrix}$$

We assume uniform sampling in time, so that $t'_k = t_{k+1}$ and $t_k = k\Delta t$. In the case of **EEG** recordings, these measurements may be voltages from n channels of an electrode array sampled every Δt.

The DMD algorithm seeks the leading spectral decomposition of the **best-fit linear operator** A that relates the two snapshot matrices in time:

$$X' \approx AX$$

Then, A establishes a linear dynamical system that best approximates the snapshot measurements forward in time. With uniform sampling:

$$x_{k+1} \approx Ax_k$$

where A comes from:

$$A = \arg\min_{A} ||X' - AX||_F = X'X^{\dagger}$$

In the following section, we see the *Algorithm*.

2.3 Algorithm

Given X data matrix:

- **Step 1.** Computation of the singular value decomposition of X:

  $$X \approx \tilde{U}\tilde{\Sigma}\tilde{V}^*$$

  where $\tilde{U} \in \mathbb{C}^{n\times r}$, $\tilde{\Sigma} \in \mathbb{C}^{r\times r}$, $\tilde{V} \in \mathbb{C}^{m\times r}$, and $r \leq m$ denotes either the exact or approximate rank of the data matrix X. In practice, choosing the approximate rank r is one of the most important and subjective steps in DMD, and in dimensionality reduction in general.

- **Step 2.** Computation of matrix A:

  $$A = X'X^\dagger$$

  However, we are only interested in the leading r eigenvalues and eigenvectors of A, so pre-multiply by $\tilde{U}^*$ (complex conjugate) and post-multiply by $\tilde{U}$, we obtain:

  $$\tilde{A} = \tilde{U}^*\tilde{A}\tilde{U} = \tilde{U}^*X'\tilde{V}\tilde{\Sigma}^{-1}.$$

  So that $\tilde{A}$, reduced version, has the same eigenvalue of the full matrix A, we need only to use $\tilde{A}$: $\tilde{x_{k+1}} = \tilde{A}\tilde{x_k}$

- **Step 3.** Computation of spectral decomposition of $\tilde{A}$;

  $$\tilde{A}W = W\Lambda$$

  where the elements of the diagonal matrix are the DMD eigenvalues, which correspond to eigenvalues of the full A matrix.

- **Step 4.** Computation of DMD modes: The high-dimensional DMD modes Φ are reconstructing using the eigenvectors W of the reduced system and the time-shifted snapshot matrix X' according to:

  $$\Phi = X'\tilde{V}\tilde{\Sigma}^{-1}W$$

  where Φ represent the modes of the DMD. These DMD modes are eigenvectors of the high-dimensional A matrix corresponding to the eigenvalues in Λ, i.e.:

  $$A\Phi = \Phi\Lambda$$

2.4 DMD's Application

In this framework we have validated the DMD approach to provide sensorimotor maps on simple tasks. In particular it is interesting to observe that these sensorimotor maps show statistically changes in the sensorimotor cortex in the specific frequency range.

The tasks are related to:

- *Imagination*
- *Movement*

2.5 DMD's Properties

The data X may be real or complex valued, however in the case of recordings from electrode arrays, we will proceed assuming X are real valued measurements of voltage. Further, the decomposition is unique, and it is also possible to compute the DMD of non-uniformly sampled data.

2.6 Spatial Domain Limitation

DMD spatial modes are based on PCA modes, therefore, relatively local spatial correlations (affecting only a very small number of measurement locations) that do not contain a lot of energy may be less likely to emerge as coherent modes in both PCA and DMD.

2.7 Temporal Domain Limitation

In the temporal domain, the DMD converts time series information into a sum of complex sinusoidal. This implies that the non-linear dynamics, which are not well approximated by the $e^{\omega t}$, will not be well approximated by the DMD, it is not suitable for capturing transient dynamics, in general it is used as window technique.

3 Data modelling

Our work is based on an article (2) which work consists in studying, through DMD, the sensorimotor map to analyze spindle networks.

3.1 Data acquisition

We acquired the data from the following link: `https://physionet.org/content/eegmmidb/1.0.0/`.
The experiment consists in: a set of 64-channels EEG from subjects who performed a series of motor/imaginary tasks. The complete set of data consists of over 1500 one/two minutes EEG recordings, obtained from 109 volunteers. Among all tasks, we have chosen two kind of them:

- *Move fists*
- *Imagine to move fists*

and we saw the difference in brain part actions and frequency content between movement/imagination.
We pre-announce that all the analysis has been done with MATLAB considering only one patient.

3.2 Data processing

The data is acquired from '.edf' files and read with **edfread** MATLAB function. Firstly, since **edfread** returns the data into a *timetable* structure, both for imagination and movement, we created a function which extract the data in order to put them in a matrix. This MATLAB function is called **DATASET**.
In particular, it takes in input the data from the file, the time of simulation and gives in output 1 column vector for each part of the brain. As last step, it puts together all based on the application (imagination/movement). Moreover, in the **DATASET** function, we have used another function that we created, called "dataextraction(data.nameofelectrodes, time of simulation)" with which we extract the single electrode data for each brain part, and then we put all together in a single matrix that is specific of that lobe.

3.3 Implementing DMD with an Augmented Data Matrix

DMD was originally used in the study of large fluid flow fields, where typically $n \gg m$. In contrast, in neuroscience we are often interested in electrode arrays that have tens of channels sampled at hundreds of samples per second, therefore $n < m$. This implies that the SVD of X produces v non-zero singular values, where v is the smaller of n and $m-1$. This property restricts the maximum number of DMD modes and eigenvalues to n, which is often too few to fully capture the dynamics over m snapshots in time. The solution to this rank mismatch is to construct augmented versions of the data matrices, appending to the snapshot measurements with $h-1$ time-shifted versions of themselves, thus augmenting the number of measurements to be h_n. Specifically, we construct a new augmented data matrix:

$$X_{aug} = \begin{pmatrix} | & | & | & | \\ x_1 & x_2 & \dots & x_{m-h} \\ | & | & | & | \\ | & | & | & | \\ x_2 & x_3 & \dots & x_{m-h+1} \\ | & | & | & | \\ | & | & | & | \\ x_h & x_{h+1} & \dots & x_{m-1} \\ | & | & | & | \end{pmatrix}$$

and similarly for X'_{aug}. DMD is then applied using X_{aug} and X'_{aug} instead of X and X'.

3.4 How to choose h?

To estimate h, it's suggested the smallest integer h such that $h_n > 2m$, where n is the number of recording channel and m is the number of matrix patterns.

4 Data-Analysis

In this section we will show the Dataset Initialization for our work and then, we will see the reconstructed signal from recording data. Once we have information about the spectrum, it is straightforward to derive patterns, about Movement and Imagination, from the DMD spectrum Plot.

4.1 Initialization

For both tasks, it is necessary to give in input the simulation-time and the number of electrodes. In particular for our study, we have chosen 23 electrodes to analyze, divided as follows:

- *Imagination/Movement*: 7 electrodes for Central part, 9 electrodes for Frontal part, 4 electrode for Temporal part, 3 electrodes for Occipital part.

Moreover, for each task we have considered 3 runs, hence we have three Dataset Matrix for each task:

$$X_1 = \begin{pmatrix} | & | & | & | \\ | & | & | & | \\ x_{C1} & x_{F1} & x_{T1} & x_{O1} \\ | & | & | & | \\ | & | & | & | \end{pmatrix}$$

$$X_2 = \begin{pmatrix} | & | & | & | \\ | & | & | & | \\ x_{C2} & x_{F2} & x_{T2} & x_{O2} \\ | & | & | & | \\ | & | & | & | \end{pmatrix}$$

$$X_3 = \begin{pmatrix} | & | & | & | \\ | & | & | & | \\ x_{C3} & x_{F3} & x_{T3} & x_{O3} \\ | & | & | & | \\ | & | & | & | \end{pmatrix}$$

With these, using an appropriate temporal axis, we are able to show the signals captured from the experiment (see *Figure 1* and *Figure 2*).

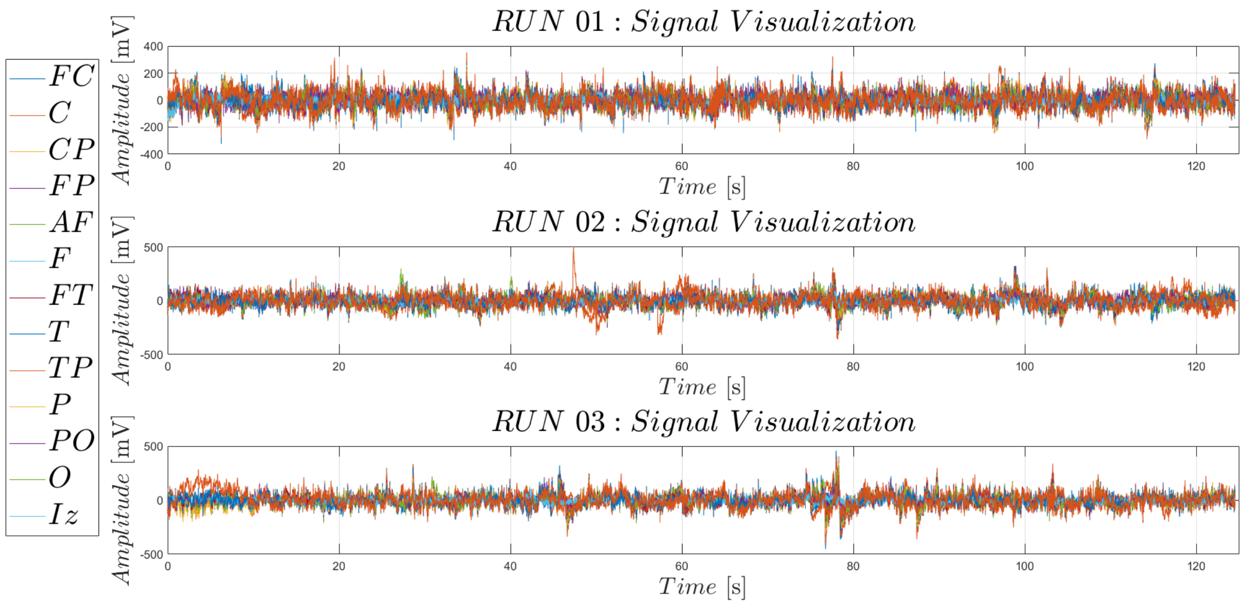

Figure 1: *3 run for imagination task*

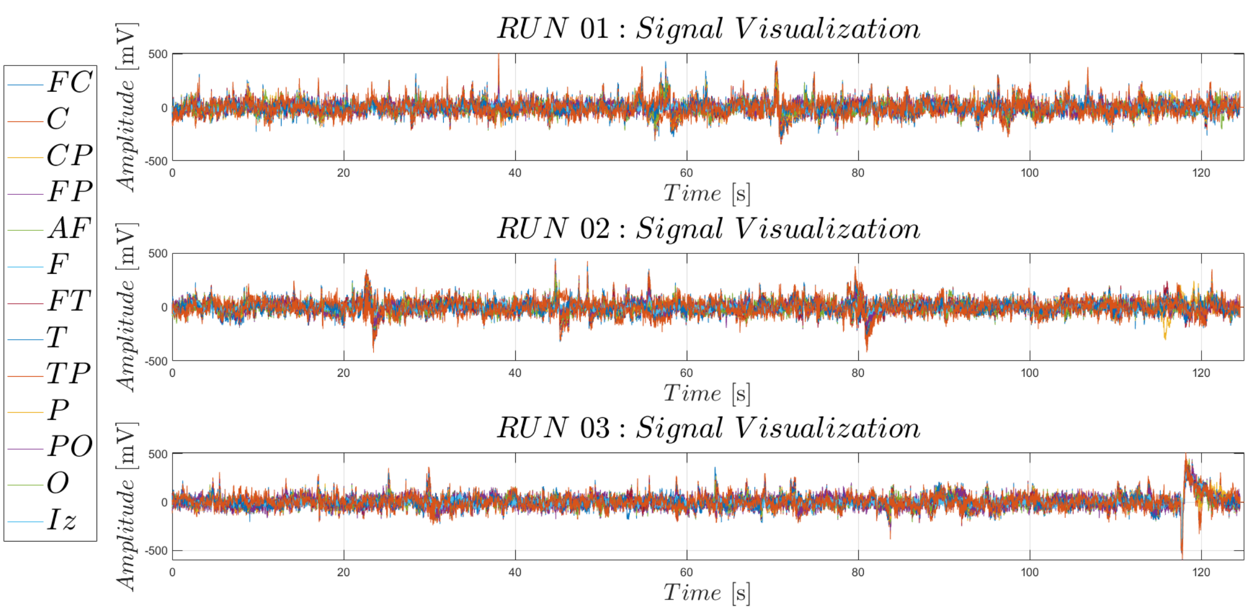

Figure 2: *3 run for movement task*

4.2 Spectral analysis

The DMD function used in this work, created by the researchers (2), has the following syntax:

$$[\Phi, \mu, \lambda, diagS, x_o] = DMD(X_i)$$

where the subscript i, referred to the data matrix, indicate the run for each task.
In particular:

- Φ is a matrix where each column of its is a DMD mode ϕ_i(*each spatial DMD mode* ϕ_i *describes its temporal dynamics*);
- μ is the Fourier spectrum of modes;
- λ is the DMD spectrum of modes;
- $diagS$ is the singular value of the data matrix;
- x_o is the initial condition vector.

Remark: Since the EEG data are strictly real valued, the decomposition produces complex conjugate pairs of eigenvalues and modes.

By this function, the useful terms for our analysis are Φ and μ. Through the DMD spectrum function, defined as follows:

$$[frequencies, PSD] = DMD\_spectrum(\Phi, \mu)$$

we obtain the power spectral densities PSD and the *frequencies* information that we need to recognize the characteristics patterns of each task.
In the following subsections, we state the information for each task.

4.2.1 Frequency bands

From the Neurophisiology, we know that each task corresponds to a given frequency band of interest:

Task	Frequency Band
imagine open/close fists	4-8 Hz
open/close fists	20 Hz
close eyes	8-14 Hz
open eyes	8-14 Hz
muscular artifacts	30-300 Hz

Table 1: Tasks Frequency Band

To make things more specific, we report a table with the name of each frequency band:

name	Frequency Band
δ band	4-8 Hz
α band	8-14 Hz
β band	14-30 Hz
γ band	30-300 Hz

Table 2: Frequency Band Table

4.3 Imagination

In *Figure 3*, we see the frequency content given by the analysis stated in the introduction of this chapter, analyzing the $PSD(f)$. The questions now are: *Can we recognize the pattern of the imagination task? Can we see something else?* We would like to see an activity in the δ band since it is responsible for the imagination.

In *Figure 3* we see an activity in the frequency band between 2-4 Hz, as in the proximity of the δ band, so we can suppose that the patient is doing quite well the task. An interesting aspect is that, there are some peaks around 20 Hz, corresponding to the opening movement of the fist, this means that the patient thought to do that, but at the same he was doing it.

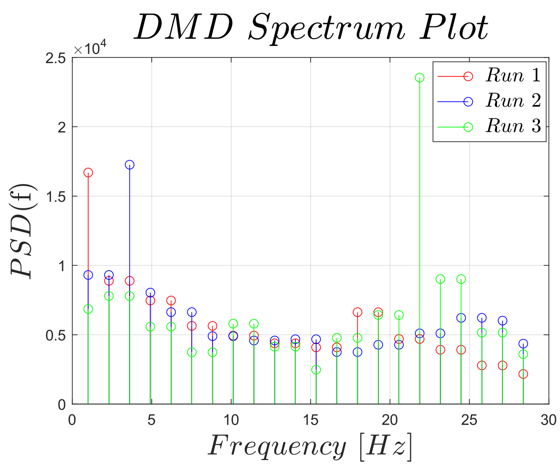

Figure 3: *Frequency content for 3 runs of Imagination task*

It's important also to note that, we have changes at the third run, this could mean that the patient lose concentration in doing tasks, during the experiment. We can conclude that, for this first task act the frontal part, responsible for the imagination, the occipital part, responsible for the visualization activity and the central for the movement task(see *Figure 4*).

4.4 Movement

Same analysis for the movement. The questions again are: *Can we recognize the pattern of the movement task? Can we see something else?* We would like to see an activity in the β band.

In *Figure 5*, we see an activity in the frequency band around 20 Hz, exactly in the beta band, this means that the patient was opening/closing fists, but pay attention, because we have the similar situation as the previous case. In particular, we have a peak corresponding to the imagination task the δ band. We can conclude that, for this first task act the frontal part, responsible for the imagination, the occipital part, responsible for the visualization activity and the central

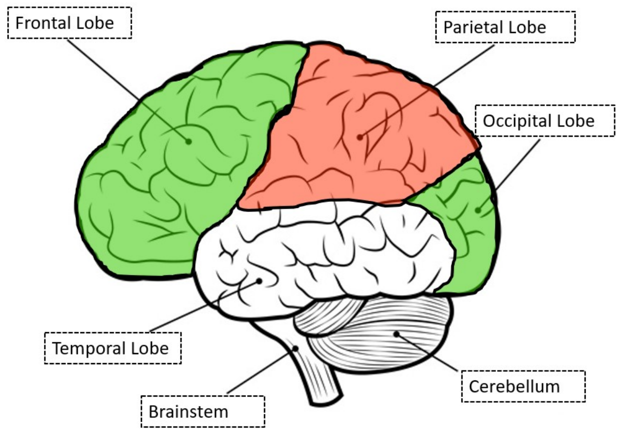

Figure 4: **Lobe Activity**: *Green regions highlight the lobes with more activity, while the red region highlights the undesirable lobe activity.*

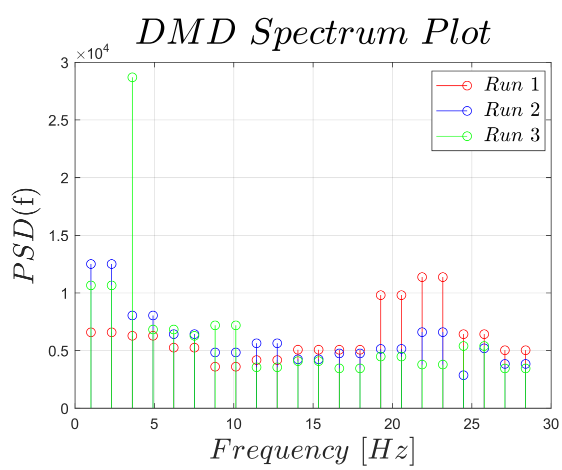

Figure 5: *Frequency content for 3 runs of Movement task*

for the movement task(see *Figure 6*). If we note the difference, between the *Figure 4* and the *Figure 6*, the red areas are in the opposite *"direction"* with respect the corresponding task, in other words we would expect that the Parietal Lobe was red for Movement, as for the Frontal Lobe for the Imagination; but as we have just stated, it happens the converse.(Note that both Figures refers to the activity on the long run, as the end of the experiment).

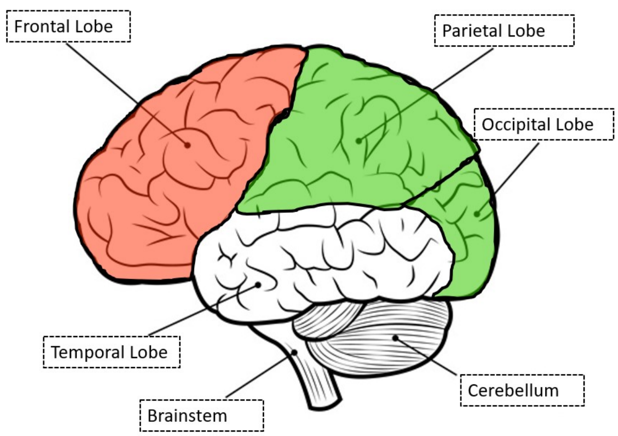

Figure 6: **Lobe Activity**: *Green regions highlight the lobes with more activity, while the red region highlights the undesirable lobe activity.*

5 Conclusions

In this work, we have seen the dynamic mode decomposition **(DMD)** in order to analyse and visualize large-scale neural recordings. In particular, we saw how DMD can be used to extract the coherent patterns by decomposing the data into a low-dimensional representation. This extraction was validated on two simple motor tasks where selective and separable regions of sensorimotor cortex were identified. Although DMD has previously been described in studying high-dimensional dynamical systems in a different discipline, here we described in detail its adaption to use with EEG recordings.
Therefore we can conclude saying that dimensionality reduction is a useful concept in building meaningful models based on dynamics of neuronal networks because there exists low-dimensional structures in large-scale data. Moreover, in contrast to other static modal decomposition techniques, DMD provides not only modes, but also a relatively low-dimensional, efficient linear model for how the most dynamically important modes evolve in time. The interesting thing is that such dynamic models of high-dimensional dynamic data are ***equation-free***, in the sense that they are entirely **data-driven** and do not rely on a set of governing equations.

Instead, what can we say about the obtained results?
Analyzing the plot of the DMD Spectrum we realized that the patient, on the long run, confused the tasks, in particular:

- *In the plot of the imagination data we have a spike around the 20hz*;
- *In the plot of the movement we have a spike around the 5hz.*

Due to this information we can affirm that at the third run, the patient inverted the tasks. We can therefore say that through the DMD Spectral analysis it is possible to extract important information about the activity of the brain.

References

[1] S. L. Brunton and J. Kutz, *Data Driven Science and Engineering*, 2017.

[2] B. W.Brunton, L. A.Johnson, J. G.Ojenam, and J. Kutz, "Extracting spatial-temporal coeherent patterns in large-scale neural recordings using dynamic mode decomposition," 2014. [Online]. Available: https://arxiv.org/abs/1409.5496