homework3_Ortu

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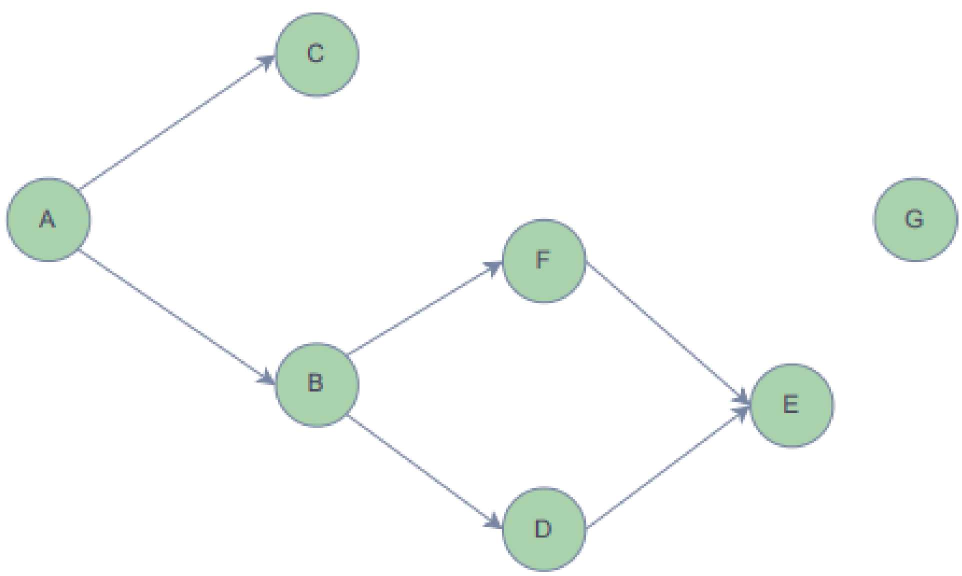
1 Homework 3 - Francesco Ortu

1.1 Ex 1

1. Draw the Bayesian Network representing the joint distribution

$$P(A, B, C, D, E, F, G) = P(A)P(B|A)P(F|B)P(C|A)P(D|B)P(E|D, F)P(G).$$



2. Indicate whether the following statements on (conditional) independence are True or False and motivate your answer.

a. $A \perp\!\!\!\perp D$: FALSE

D is a descendant of A

b. $F \perp\!\!\!\perp D$: FALSE

F and D have the same parents

c. $A \perp\!\!\!\perp B|C$: FALSE

$$P(A,B|C) = \frac{P(A,B,C)}{P(C)} = \frac{P(A)P(C|A)P(B|A)}{P(C)} = P(B|A)P(A|C) \neq P(A|C)P(B|C)$$

d. $A \perp\!\!\!\perp D|B$: TRUE

$$P(A,D|B) = \frac{P(A,D,B)}{P(B)} = \frac{P(A)P(B|A)P(D|B)}{P(B)} = P(A|B)P(D|B)$$

e. $D \perp\!\!\!\perp F|E$: FALSE

head to head

$$P(D,F|E) = \frac{P(D,F,E)}{P(E)} = \frac{P(D,F)P(E|F,D)}{P(E)} \neq P(D|E)P(F|E)$$

f. $B \perp\!\!\!\perp F|E$: FALSE

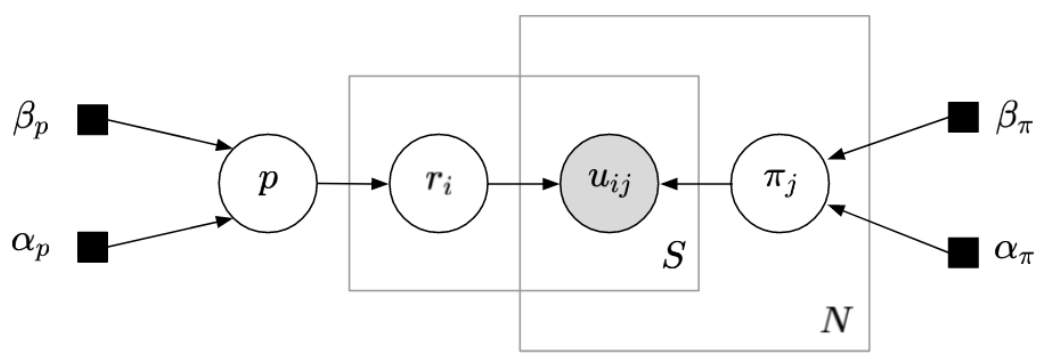
$$P(B,F|E) = \frac{P(B,F,E)}{P(E)} = \frac{P(B)P(F|B)P(E|F)}{P(E)} \neq P(B|E)P(F|E)$$

g. $A \perp\!\!\!\perp D|\{B,F\}$: TRUE

$$\begin{aligned} P(A,D|B,F) &= \frac{P(A,D,B,F)}{P(B,F)} = \frac{P(A)P(B|A)P(F|B)P(D|B)}{P(B)P(F|B)} \\ &= P(A|B)P(D|B) = \frac{P(B,A)P(A)P(F|B)}{P(A)P(B)P(F|B)} \cdot P(D|B) \\ &= \frac{P(B|A)P(A)P(F|B)}{P(B)P(F|B)} \cdot \frac{P(D|B)P(F|B)P(B)}{P(B)P(F|B)} \\ &= \frac{P(A,B,F)}{P(B,F)} \cdot \frac{P(D,B,F)}{P(B,F)} = P(A|B,F)P(D|B,F) \end{aligned}$$

1.2 EX 2

1. Write the generative model represented by the following directed graph, knowing that:
   - p and π_j are sampled from Beta distributions;
   - r_i is sampled from a Bernoulli distribution;
   - u_{ij} is sampled from a Bernoulli distribution with parameter $r_i(1-\pi_j)+(1-r_i)\pi_j$.



2. Implement the generative model using `pyro`.

   Set the hyperparameters to $\alpha_p = 1, \beta_p = 1, \alpha_\pi = 1, \beta_\pi = 5$ and evaluate your model on the observations `data = dist.Bernoulli(0.6).sample((12,6))`.

   Remember to use plate notation and to condition on the observed data!

The joint distribution is:

$$P(p, r, \mu, \pi | \alpha_p, \alpha_\pi, \beta_p, \beta_\pi) = P(p|\alpha_p, \beta_p) \prod_{j=1}^{N} P(\pi_j|\beta_\pi, \alpha_\pi) \prod_{i=1}^{S} P(r_i|p) \prod_{\substack{i=1...N \\ j=1...S}} P(\mu_{ij}|r_i, p_j)$$

Given the observations, we can compute the joint distribution:

$$P(p, r, \pi|\mu, \alpha_p, \alpha_\pi, \beta_p, \beta_\pi) = \frac{P(p, r, \mu, \pi|\alpha_p, \alpha_\pi, \beta_p, \beta_\pi)}{P(\mu|\alpha_p, \alpha_\pi, \beta_p, \beta_\pi)}$$

[ ]:
```python
import pyro
import torch
import pyro.distributions as dist
```

[ ]:
```python
def model(data):
    N = data.size()[0]
    S = data.size()[1]
    #set hyperparameters
    alpha_p = 1
    beta_p = 1
    alpha_pi= 1
    beta_pi = 5


    p = pyro.sample('p', dist.Beta(alpha_p, beta_p))

    r_plate = pyro.plate('r_plate', S, dim = -1)
    pi_plate = pyro.plate("pi_plate", N, dim = -2)

    with pi_plate:
        pi = pyro.sample('pi', dist.Beta(alpha_pi, beta_pi))

    with r_plate:
        r = pyro.sample('r', dist.Bernoulli(probs=p))

    with  pi_plate, r_plate:
        u = pyro.sample( "u", dist.Bernoulli( r * ( 1 - pi) + ( 1 - r ) * pi),␣
 ↪obs=data )

    return p, r,pi,u
```

[ ]:
```
data = dist.Bernoulli(0.6).sample((12,6))

p,r,pi ,u = model(data)

print(p,r,pi,u)

pyro.render_model(model, model_args=(data,),render_distributions=True) #plates
  ↪that are overlapped are rendered as two separate plates
```

```
tensor(0.6268) tensor([1., 1., 0., 0., 1., 1.]) tensor([[0.1436],
        [0.0748],
        [0.0337],
        [0.0497],
        [0.0330],
        [0.2033],
        [0.0222],
        [0.0875],
        [0.1386],
        [0.0146],
        [0.0477],
        [0.2430]]) tensor([[0., 0., 0., 1., 0., 1.],
        [0., 1., 1., 1., 1., 1.],
        [0., 1., 1., 0., 1., 1.],
        [1., 1., 0., 0., 0., 0.],
        [0., 1., 0., 1., 0., 1.],
        [0., 0., 1., 1., 1., 0.],
        [1., 1., 1., 0., 1., 0.],
        [1., 1., 1., 1., 1., 1.],
        [1., 0., 1., 0., 0., 1.],
        [0., 1., 0., 0., 0., 0.],
        [0., 0., 0., 1., 0., 1.],
        [1., 0., 0., 1., 0., 1.]])
```

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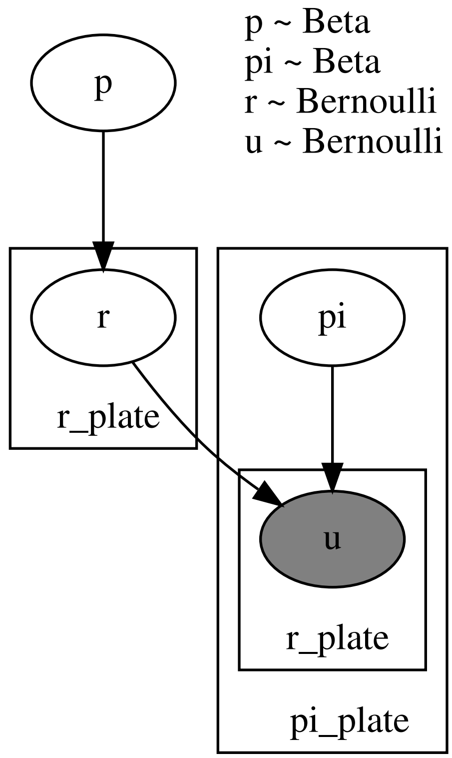
p ~ Beta
pi ~ Beta
r ~ Bernoulli
u ~ Bernoulli
p
r
r_plate
pi
u
r_plate
pi_plate