Detailed study of Jacobian for candidates to bielliptic curves of the quotient curves with square-free level

CHAPTER 1

The modular curves $X_0(p_1p_2)/W$ with $|W| = 2$.

We only list the cases where $X_0(pq)/w_p$ has genus ≥ 2. (The case $X_0(pq)/w_{pq}$ concerning bielliptic is done in the paper $X_0^+(N)$ and we not consider here).

1. N=85

genus 4, fix w_{17}:$X_0(N)/w_{17}$

```
 [*
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 + q^5 + (a - 3)*q^6 + (a - 1)*q^7 - a*q^8 +
        (-2*a + 1)*q^9 + a*q^10 + (-a + 3)*q^11 + (-a + 1)*q^12 - 4*q^13 + (-a +
        3)*q^14 + (-a + 1)*q^15 - 5*q^16 - q^17 + (a - 6)*q^18 + (2*a + 2)*q^19
        + O(q^20)
*]
[*
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field,
    Number Field with defining polynomial $.1^2 - 3 over the Rational Field
*]
[* 85, 85 *]
```

genus 4, fix w_5:$X_0(N)/w_5$

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + q^2 + 2*q^3 - q^4 - q^5 + 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^11 -
        2*q^12 + 2*q^13 - 2*q^14 - 2*q^15 - q^16 + q^17 + q^18 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field
*]
[* 17, 85, 85 *]
```

The elliptic curves in the $\mathbb{Q}$-decomposition are E17a, E85a, over $\mathbb{Q}$ we have $n(E85a, 2) = 5-4$, $n(E17a, 8) = 11 - 8$, thus is not bielliptic over $\mathbb{Q}$.

2. N=93

: $N = 93 = 3 \cdot 31$:

genus 4, fix w_3**:** $X_0(N)/w_3$

```
[*
    q + a*q^2 - 2*a*q^3 + (a - 1)*q^4 + q^5 + (-2*a - 2)*q^6 + (2*a - 3)*q^7 +
        (-2*a + 1)*q^8 + (4*a + 1)*q^9 + a*q^10 + 2*q^11 - 2*q^12 - 2*a*q^13 +
        (-a + 2)*q^14 - 2*a*q^15 - 3*a*q^16 + (-2*a + 4)*q^17 + (5*a + 4)*q^18 +
        (-2*a + 1)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 + (-2*a - 5)*q^5 - a*q^6 + (2*a + 1)*q^7 +
        (4*a + 3)*q^8 + q^9 + (a + 2)*q^10 + 2*a*q^11 + (3*a + 3)*q^12 + (2*a +
        2)*q^13 + (-5*a - 2)*q^14 + (2*a + 5)*q^15 + (-3*a + 2)*q^16 + (-4*a -
        8)*q^17 + a*q^18 + (-2*a - 7)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field
*]
[* 31, 93 *]
```

genus 5, fix w_31**:** $X_0(N)/w_{31}$

```
[*
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 + (-2*a - 5)*q^5 - a*q^6 + (2*a + 1)*q^7 +
        (4*a + 3)*q^8 + q^9 + (a + 2)*q^10 + 2*a*q^11 + (3*a + 3)*q^12 + (2*a +
        2)*q^13 + (-5*a - 2)*q^14 + (2*a + 5)*q^15 + (-3*a + 2)*q^16 + (-4*a -
        8)*q^17 + a*q^18 + (-2*a - 7)*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a^2 - a + 2)*q^5 + a*q^6 + (-a^2 - a +
        4)*q^7 - q^8 + q^9 + (-a^2 - 2*a + 1)*q^10 + (2*a^2 - 6)*q^11 + (a^2 -
        2)*q^12 + (2*a^2 - 4)*q^13 + (-a^2 + 1)*q^14 + (-a^2 - a + 2)*q^15 +
        (-2*a^2 - a + 4)*q^16 + (2*a^2 + 2*a - 6)*q^17 + a*q^18 + (-a^2 + 3*a +
        4)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 4*x + 1 over the Rational Field
*]
[* 93, 93 *]
```

3. N=106

genus 6: $X_0(N)/w_2$

```
[*
    q - q^2 - 3*q^3 - q^4 + 3*q^6 - 4*q^7 + 3*q^8 + 6*q^9 + 3*q^12 - 3*q^13 +
        4*q^14 - q^16 - 3*q^17 - 6*q^18 - 5*q^19 + O(q^20),
    q + a*q^2 + (-a^2 - a + 3)*q^3 + (a^2 - 2)*q^4 + (a^2 - 3)*q^5 - q^6 + (a^2
        - 1)*q^7 + (-a^2 - a + 1)*q^8 + (-3*a^2 - 2*a + 7)*q^9 + (-a^2 + 1)*q^10
        + (a^2 + 2*a - 3)*q^11 + (2*a^2 + a - 6)*q^12 + q^13 + (-a^2 + 2*a +
        1)*q^14 + (3*a^2 + 2*a - 9)*q^15 + (-2*a^2 - 2*a + 3)*q^16 + (2*a -
        1)*q^17 + (a^2 - 2*a - 3)*q^18 + (a + 4)*q^19 + O(q^20),
```

```
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - q^8 - 2*q^9 + 4*q^10 - 4*q^11 - q^12 +
        q^13 + 4*q^15 + q^16 + 5*q^17 + 2*q^18 - 7*q^19 + O(q^20),
    q - q^2 + 2*q^3 + q^4 + q^5 - 2*q^6 - 2*q^7 - q^8 + q^9 - q^10 + 5*q^11 +
        2*q^12 - 4*q^13 + 2*q^14 + 2*q^15 + q^16 + 3*q^17 - q^18 - 4*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^3 + x^2 - 3*x - 1 over the Rational
    Field,
    Rational Field,
    Rational Field
*]
[* 53, 53, 106, 106 *] 53a,..,106b,106d
```

We have $n(53a;9) = 21 - 14$, $n(106b;25) = 57 - 40$, $n(106d;5) = 11 - 10$. Therefore $X_0(106)/w_2$ is not bielliptic over $\mathbb{Q}$ and because $g \geq 6$ is not bielliptic.

genus 5: $X_0(N)/w53$

```
[*
    q - q^2 - 3*q^3 - q^4 + 3*q^6 - 4*q^7 + 3*q^8 + 6*q^9 + 3*q^12 - 3*q^13 +
        4*q^14 - q^16 - 3*q^17 - 6*q^18 - 5*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - q^8 - 2*q^9 + 4*q^10 - 4*q^11 - q^12 +
        q^13 + 4*q^15 + q^16 + 5*q^17 + 2*q^18 - 7*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + q^6 - 4*q^7 + q^8 - 2*q^9 + q^12 + 5*q^13 - 4*q^14 +
        q^16 - 3*q^17 - 2*q^18 - q^19 + O(q^20),
    q + q^2 - 2*q^3 + q^4 + 3*q^5 - 2*q^6 + 2*q^7 + q^8 + q^9 + 3*q^10 - 3*q^11
        - 2*q^12 - 4*q^13 + 2*q^14 - 6*q^15 + q^16 + 3*q^17 + q^18 - 4*q^19 +
        O(q^20),
    q - q^2 - 3*q^3 - q^4 + 3*q^6 - 4*q^7 + 3*q^8 + 6*q^9 + 3*q^12 - 3*q^13 +
        4*q^14 - q^16 - 3*q^17 - 6*q^18 - 5*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 53, 106, 106, 106, 106 *]
```

The Jacobian is $53a, 106b, 106c, 106a, 53a$,thus the factor $53a$ repeated.

$n(53a;9) = 16 - 14$, $n(106b;3) = 12 - 10$, $n(106c;3) = 12 - 6$, $n(106a;5) = 7 - 6$. Thus is not bielliptic over $\mathbb{Q}$.

4. N=115

genus 5, $X_0(115)/w_5$**:**

```
[*
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
```

```
    q + 2*q^2 + 2*q^4 - q^5 + q^7 - 3*q^9 - 2*q^10 + 2*q^11 - 2*q^13 + 2*q^14 -
        4*q^16 + 3*q^17 - 6*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (4*a +
        3)*q^8 - 2*q^9 - a*q^10 + (2*a + 2)*q^11 + (3*a + 3)*q^12 + (2*a -
        1)*q^13 + (2*a + 2)*q^14 + q^15 + (-3*a + 2)*q^16 + (-4*a - 8)*q^17 -
        2*a*q^18 + (6*a + 10)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field
*]
[* 23, 115, 115 *]
```

Not bielliptic over $\mathbb{Q}$: $n(115a; 2) = 5 - 2$.

genus 6, $X_0(115)/w_{23}$**:**

```
[*
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (4*a +
        3)*q^8 - 2*q^9 - a*q^10 + (2*a + 2)*q^11 + (3*a + 3)*q^12 + (2*a -
        1)*q^13 + (2*a + 2)*q^14 + q^15 + (-3*a + 2)*q^16 + (-4*a - 8)*q^17 -
        2*a*q^18 + (6*a + 10)*q^19 + O(q^20),
    q + a*q^2 + (-a^2 + a + 2)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^3 + a^2 +
        2*a)*q^6 + (a^3 - 2*a^2 - 4*a + 3)*q^7 + (a^3 - 4*a)*q^8 + (a^2 - a -
        1)*q^9 + a*q^10 + (-2*a + 2)*q^11 + (-a^3 + 3*a - 2)*q^12 + (-2*a^3 +
        3*a^2 + 7*a - 4)*q^13 + (-2*a - 2)*q^14 + (-a^2 + a + 2)*q^15 + (2*a^3 -
        2*a^2 - 5*a + 2)*q^16 + (-a^3 + 2*a^2 + 2*a - 3)*q^17 + (a^3 - a^2 -
        a)*q^18 + (2*a - 2)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^4 - 2*x^3 - 4*x^2 + 5*x + 2 over the
    Rational Field
*]
[* 115, 115 *]
```

Thus is not bielliptic (has genus ≥ 6).

5. N=122

genus 7, $X_0(122)/w_2$**:**

```
[*
    q - q^2 - 2*q^3 - q^4 - 3*q^5 + 2*q^6 + q^7 + 3*q^8 + q^9 + 3*q^10 - 5*q^11
        + 2*q^12 + q^13 - q^14 + 6*q^15 - q^16 + 4*q^17 - q^18 - 4*q^19 +
        O(q^20),
    q + a*q^2 + (-a^2 + 3)*q^3 + (a^2 - 2)*q^4 + (a^2 - 2*a - 2)*q^5 + (-a^2 +
        1)*q^6 + (a^2 - a - 3)*q^7 + (a^2 - a - 1)*q^8 + (-2*a^2 + 2*a + 5)*q^9
        + (-a^2 + a - 1)*q^10 + (a + 4)*q^11 + (a^2 - 2*a - 5)*q^12 + (-2*a^2 +
        2*a + 1)*q^13 - q^14 + (3*a^2 - 2*a - 7)*q^15 + (-2*a^2 + 2*a + 3)*q^16
        + (-a^2 + 2*a + 1)*q^17 + (-a + 2)*q^18 + (3*a^2 - 7)*q^19 + O(q^20),
    q - q^2 - 2*q^3 + q^4 + q^5 + 2*q^6 - 5*q^7 - q^8 + q^9 - q^10 - 3*q^11 -
        2*q^12 - 3*q^13 + 5*q^14 - 2*q^15 + q^16 - q^18 + O(q^20),
    q - q^2 + a*q^3 + q^4 - a*q^6 + (-a + 3)*q^7 - q^8 + a*q^9 + (-2*a + 2)*q^11
```

```
        + a*q^12 + (-2*a + 4)*q^13 + (a - 3)*q^14 + q^16 + (2*a - 2)*q^17 -
        a*q^18 + (3*a - 1)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 3*x + 1 over the Rational
    Field,
    Rational Field,
    Number Field with defining polynomial x^2 - x - 3 over the Rational Field
*]
[* 61, 61, 122, 122 *]: 61a,...,122a,...
```

$n(61a, 122a; 9) = 25 - 24$, thus is not bielliptic.

genus 6, $X_0(122)/w_{61}$**:**

```
[*
    q - q^2 - 2*q^3 - q^4 - 3*q^5 + 2*q^6 + q^7 + 3*q^8 + q^9 + 3*q^10 - 5*q^11
        + 2*q^12 + q^13 - q^14 + 6*q^15 - q^16 + 4*q^17 - q^18 - 4*q^19 +
        O(q^20),
    q - q^2 - 2*q^3 + q^4 + q^5 + 2*q^6 - 5*q^7 - q^8 + q^9 - q^10 - 3*q^11 -
        2*q^12 - 3*q^13 + 5*q^14 - 2*q^15 + q^16 - q^18 + O(q^20),
    q + q^2 + a*q^3 + q^4 + (-a^2 - 3*a + 3)*q^5 + a*q^6 + (2*a^2 + 3*a - 5)*q^7
        + q^8 + (a^2 - 3)*q^9 + (-a^2 - 3*a + 3)*q^10 + (-a^2 - a + 1)*q^11 +
        a*q^12 + (-a^2 - a + 3)*q^13 + (2*a^2 + 3*a - 5)*q^14 + (-2*a^2 - 2*a +
        2)*q^15 + q^16 + (-2*a^2 - 4*a + 4)*q^17 + (a^2 - 3)*q^18 + (a^2 + 2*a -
        4)*q^19 + O(q^20),
    q - q^2 - 2*q^3 - q^4 - 3*q^5 + 2*q^6 + q^7 + 3*q^8 + q^9 + 3*q^10 - 5*q^11
        + 2*q^12 + q^13 - q^14 + 6*q^15 - q^16 + 4*q^17 - q^18 - 4*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 + x^2 - 5*x + 2 over the Rational
    Field,
    Rational Field
*]
[* 61, 122, 122, 122 *] 61a,122a,...,61a.
```

(There is repetition in tha Jacobian decomposition over $\mathbb{Q}$)

$n(122a; 11) = 32 - 30$. With 61a we can not say nothing.

6. 129

genus 6, $X_0(129)/w_3$**:**

```
[*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (-a + 2)*q^5 - 2*q^6 + (a - 2)*q^7 - 2*a*q^8 - q^9 +
        (2*a - 2)*q^10 + (2*a - 1)*q^11 + (2*a + 1)*q^13 + (-2*a + 2)*q^14 +
        (-2*a + 2)*q^15 - 4*q^16 + (2*a + 5)*q^17 - a*q^18 + (-2*a - 2)*q^19 +
        O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 2*q^7 + q^9 - 5*q^11 + 2*q^12 + 3*q^13 + 2*q^15 +
```

```
        4*q^16 - 3*q^17 + 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + (2*a - 1)*q^4 + (-a + 2)*q^5 - a*q^6 + (-2*a + 3)*q^7 + (a
        + 2)*q^8 + q^9 - q^10 + (-a + 4)*q^11 + (-2*a + 1)*q^12 - 5*q^13 + (-a -
        2)*q^14 + (a - 2)*q^15 + 3*q^16 - 2*a*q^17 + a*q^18 + (4*a - 5)*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 1 over the Rational Field
*]
[* 43, 43, 129, 129 *]
```

$n(43a; 4) = 15 - 10$, and $129a$ nothing.

genus 7, $X_0(129)/w_{43}$**:**

```
[*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 2*q^7 + q^9 - 5*q^11 + 2*q^12 + 3*q^13 + 2*q^15 +
        4*q^16 - 3*q^17 + 2*q^19 + O(q^20),
    q + q^2 + q^3 - q^4 + 2*q^5 + q^6 - 3*q^8 + q^9 + 2*q^10 - q^12 - 2*q^13 +
        2*q^15 - q^16 - 6*q^17 + q^18 + 4*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a - 2)*q^5 + a*q^6 + (-a^2 + 6)*q^7 +
        (-2*a^2 + a + 8)*q^8 + q^9 + (-a^2 - 2*a)*q^10 + (a^2 - a - 5)*q^11 +
        (a^2 - 2)*q^12 + 3*q^13 + (2*a^2 + a - 8)*q^14 + (-a - 2)*q^15 + (3*a^2
        - 2*a - 12)*q^16 + (-a^2 + 5)*q^17 + a*q^18 + (-a^2 - 2*a + 2)*q^19 +
        O(q^20),
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^3 + 2*$.1^2 - 5*$.1 - 8 over the
    Rational Field,
    Rational Field
*]
[* 43, 129, 129, 129, 129 *]
```

Corresponds to $43a, 129a, 129b, A_f, 43a$.

$n(129a, ; 2) = 8 - 6$, $n(129b; 2) = 8 - 4$. Nothing on $43a$.

7. 133

genus 6, $X_0(133)/w_7$:

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q + a*q^2 + a*q^3 + (-3*a - 3)*q^4 + (-2*a - 3)*q^5 + (-3*a - 1)*q^6 - q^7 +
        (4*a + 3)*q^8 + (-3*a - 4)*q^9 + (3*a + 2)*q^10 + (a - 3)*q^11 + (6*a +
```

```
        3)*q^12 + q^13 - a*q^14 + (3*a + 2)*q^15 + (-3*a + 2)*q^16 + (3*a +
        3)*q^17 + (5*a + 3)*q^18 - q^19 + O(q^20),
    q + a*q^2 + (-a^2 + 5)*q^3 + (a^2 - 2)*q^4 + (a^2 - a - 4)*q^5 + (-2*a^2 + a
        + 7)*q^6 - q^7 + (2*a^2 - 7)*q^8 + (-2*a^2 + a + 8)*q^9 + (a^2 - 7)*q^10
        + (-a + 3)*q^11 + (-a^2 - a + 4)*q^12 + (a^2 - a - 4)*q^13 - a*q^14 +
        (3*a^2 - 2*a - 13)*q^15 + (2*a^2 + a - 10)*q^16 + (-2*a^2 - a + 11)*q^17
        + (-3*a^2 + 14)*q^18 + q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial $.1^2 + 3*$.1 + 1 over the Rational
    Field,
    Number Field with defining polynomial $.1^3 - 2*$.1^2 - 4*$.1 + 7 over the
    Rational Field
*]
[* 19, 133, 133 *]
```

Only $19a$ in the Jacobian decomposition to become bielliptic over $\mathbb{Q}$, and we cannot discard on $\mathbb{F}_{p^n}$-points with $p \nmid N$.

genus 4, $X_0(133)/w_{19}$

```
[*
    q + a*q^2 + a*q^3 + (-3*a - 3)*q^4 + (-2*a - 3)*q^5 + (-3*a - 1)*q^6 - q^7 +
        (4*a + 3)*q^8 + (-3*a - 4)*q^9 + (3*a + 2)*q^10 + (a - 3)*q^11 + (6*a +
        3)*q^12 + q^13 - a*q^14 + (3*a + 2)*q^15 + (-3*a + 2)*q^16 + (3*a +
        3)*q^17 + (5*a + 3)*q^18 - q^19 + O(q^20),
    q + a*q^2 + (-a + 2)*q^3 + (a - 1)*q^4 + q^5 + (a - 1)*q^6 + q^7 + (-2*a +
        1)*q^8 + (-3*a + 2)*q^9 + a*q^10 + (a - 1)*q^11 + (2*a - 3)*q^12 - q^13
        + a*q^14 + (-a + 2)*q^15 - 3*a*q^16 + (3*a - 1)*q^17 + (-a - 3)*q^18 -
        q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial $.1^2 + 3*$.1 + 1 over the Rational
    Field,
    Number Field with defining polynomial $.1^2 - $.1 - 1 over the Rational
    Field
*]
[* 133, 133 *]
```

Not over $\mathbb{Q}$, may study in $\overline{\mathbb{Q}}$.

8. 134

genus 8, $X_0(134)/w_2$:

```
[*
    q + 2*q^2 - 2*q^3 + 2*q^4 + 2*q^5 - 4*q^6 - 2*q^7 + q^9 + 4*q^10 - 4*q^11 -
        4*q^12 + 2*q^13 - 4*q^14 - 4*q^15 - 4*q^16 + 3*q^17 + 2*q^18 + 7*q^19 +
        O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-3*a - 3)*q^4 - 3*q^5 + q^6 + (3*a + 4)*q^7 +
        (4*a + 3)*q^8 + (3*a + 5)*q^9 - 3*a*q^10 + (-2*a - 3)*q^11 + (3*a +
        6)*q^12 + (-3*a - 8)*q^13 + (-5*a - 3)*q^14 + (3*a + 9)*q^15 + (-3*a +
        2)*q^16 + (-2*a - 6)*q^17 + (-4*a - 3)*q^18 + (3*a + 5)*q^19 + O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (-a - 1)*q^4 + (-2*a + 1)*q^5 + q^6 - a*q^7 +
```

```
        (-2*a - 1)*q^8 + (a - 1)*q^9 + (3*a - 2)*q^10 + q^11 + (-a - 2)*q^12 +
        a*q^13 + (a - 1)*q^14 + (a - 1)*q^15 + 3*a*q^16 + (-2*a + 2)*q^17 +
        (-2*a + 1)*q^18 + (a - 5)*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + (a^2 + a - 5)*q^5 - a*q^6 + (-2*a^2 - 2*a + 12)*q^7
        - q^8 + (a^2 - 3)*q^9 + (-a^2 - a + 5)*q^10 + (-a^2 - 2*a + 6)*q^11 +
        a*q^12 + (a^2 - 2)*q^13 + (2*a^2 + 2*a - 12)*q^14 + (2*a^2 + 3*a -
        11)*q^15 + q^16 + (-a^2 - a + 5)*q^17 + (-a^2 + 3)*q^18 + 2*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 8*x + 11 over the Rational
    Field
*]
[* 67, 67, 67, 134 *]
```

Only $67a$ appears in the Jacobian for an e.c. over $\mathbb{Q}$, and $n(67a; 9) = 27 - 24$, therefore is not bielliptic.

genus 7, $X_0(134)/w_{67}$

```
[*
    q + a*q^2 + (-a - 3)*q^3 + (-3*a - 3)*q^4 - 3*q^5 + q^6 + (3*a + 4)*q^7 +
        (4*a + 3)*q^8 + (3*a + 5)*q^9 - 3*a*q^10 + (-2*a - 3)*q^11 + (3*a +
        6)*q^12 + (-3*a - 8)*q^13 + (-5*a - 3)*q^14 + (3*a + 9)*q^15 + (-3*a +
        2)*q^16 + (-2*a - 6)*q^17 + (-4*a - 3)*q^18 + (3*a + 5)*q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 + (-a^2 + a + 1)*q^5 + a*q^6 + (2*a^2 - 6*a)*q^7 + q^8
        + (a^2 - 3)*q^9 + (-a^2 + a + 1)*q^10 + (-3*a^2 + 6*a + 2)*q^11 + a*q^12
        + (3*a^2 - 8*a - 2)*q^13 + (2*a^2 - 6*a)*q^14 + (-2*a^2 + a + 1)*q^15 +
        q^16 + (-a^2 + 5*a - 3)*q^17 + (a^2 - 3)*q^18 + (-4*a^2 + 12*a + 2)*q^19
        + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-3*a - 3)*q^4 - 3*q^5 + q^6 + (3*a + 4)*q^7 +
        (4*a + 3)*q^8 + (3*a + 5)*q^9 - 3*a*q^10 + (-2*a - 3)*q^11 + (3*a +
        6)*q^12 + (-3*a - 8)*q^13 + (-5*a - 3)*q^14 + (3*a + 9)*q^15 + (-3*a +
        2)*q^16 + (-2*a - 6)*q^17 + (-4*a - 3)*q^18 + (3*a + 5)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 3*x^2 + 1 over the Rational
    Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field
*]
[* 67, 134, 134 *]
```

It is not bielliptic.

9. 146

genus 8, $X_0(146)/w_2$:

```
[*
    q + q^2 - q^4 + 2*q^5 + 2*q^7 - 3*q^8 - 3*q^9 + 2*q^10 - 2*q^11 - 6*q^13 +
        2*q^14 - q^16 + 2*q^17 - 3*q^18 + 8*q^19 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-3*a - 3)*q^4 + a*q^5 + q^6 - 3*q^7 + (4*a +
```

```
        3)*q^8 + (3*a + 5)*q^9 + (-3*a - 1)*q^10 + (-a - 3)*q^11 + (3*a +
        6)*q^12 + (3*a + 5)*q^13 - 3*a*q^14 + q^15 + (-3*a + 2)*q^16 + (-6*a -
        9)*q^17 + (-4*a - 3)*q^18 + q^19 + O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + (a + 1)*q^4 - a*q^5 - 3*q^6 - q^7 + 3*q^8 + (-a +
        1)*q^9 + (-a - 3)*q^10 + (a + 3)*q^11 + (-a - 2)*q^12 + (a - 1)*q^13 -
        a*q^14 + 3*q^15 + (a - 2)*q^16 + (2*a - 3)*q^17 - 3*q^18 - 7*q^19 +
        O(q^20),
    q - q^2 + a*q^3 + q^4 + 1/2*(-a^2 + 4)*q^5 - a*q^6 + 1/2*a^2*q^7 - q^8 +
        (a^2 - 3)*q^9 + 1/2*(a^2 - 4)*q^10 + (-a^2 - 2*a + 6)*q^11 + a*q^12 +
        1/2*(-a^2 + 8)*q^13 - 1/2*a^2*q^14 + (-2*a + 2)*q^15 + q^16 + (a^2 + 2*a
        - 6)*q^17 + (-a^2 + 3)*q^18 + (-a^2 - 2*a + 8)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^2 - x - 3 over the Rational Field,
    Number Field with defining polynomial x^3 - 8*x + 4 over the Rational Field
*]
[* 73, 73, 73, 146 *]
```

Only $73a$ appears in the $\mathbb{Q}$-decomposition Jacobian, $n(73a;5) = 10 - 8$, therefore is not bielliptic.

genus 8,, $X_0(146)/w_{73}$

```
[*
    q + a*q^2 + (-a - 3)*q^3 + (-3*a - 3)*q^4 + a*q^5 + q^6 - 3*q^7 + (4*a +
        3)*q^8 + (3*a + 5)*q^9 + (-3*a - 1)*q^10 + (-a - 3)*q^11 + (3*a +
        6)*q^12 + (3*a + 5)*q^13 - 3*a*q^14 + q^15 + (-3*a + 2)*q^16 + (-6*a -
        9)*q^17 + (-4*a - 3)*q^18 + q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 + 1/2*(-a^3 - a^2 + 4*a + 2)*q^5 + a*q^6 + 1/2*(2*a^3
        + a^2 - 14*a + 2)*q^7 + q^8 + (a^2 - 3)*q^9 + 1/2*(-a^3 - a^2 + 4*a +
        2)*q^10 + (a^2 - 4)*q^11 + a*q^12 + 1/2*(-3*a^2 - 2*a + 10)*q^13 +
        1/2*(2*a^3 + a^2 - 14*a + 2)*q^14 + 1/2*(-a^3 - 4*a^2 + 6*a + 4)*q^15 +
        q^16 + (-a^3 - a^2 + 6*a)*q^17 + (a^2 - 3)*q^18 + (a^2 + 2*a - 4)*q^19 +
        O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-3*a - 3)*q^4 + a*q^5 + q^6 - 3*q^7 + (4*a +
        3)*q^8 + (3*a + 5)*q^9 + (-3*a - 1)*q^10 + (-a - 3)*q^11 + (3*a +
        6)*q^12 + (3*a + 5)*q^13 - 3*a*q^14 + q^15 + (-3*a + 2)*q^16 + (-6*a -
        9)*q^17 + (-4*a - 3)*q^18 + q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^4 - 8*x^2 + 4*x + 4 over the
    Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field
*]
[* 73, 146, 146 *]
```

It is not bielliptic.

10. 158

genus 10, $X_0(158)/w_2$:

```
[*
```

```
    q - q^2 - q^3 - q^4 - 3*q^5 + q^6 - q^7 + 3*q^8 - 2*q^9 + 3*q^10 - 2*q^11 +
        q^12 + 3*q^13 + q^14 + 3*q^15 - q^16 - 6*q^17 + 2*q^18 + 4*q^19 +
        O(q^20),
    q + a*q^2 + (-a^4 + a^3 + 3*a^2 - 3*a + 1)*q^3 + (a^2 - 2)*q^4 + (a^4 -
        4*a^2 - a + 3)*q^5 + (a^4 - 3*a^3 - 3*a^2 + 9*a - 1)*q^6 + (a^4 - a^3 -
        5*a^2 + 3*a + 3)*q^7 + (a^3 - 4*a)*q^8 + (-a^4 + a^3 + 5*a^2 - 5*a -
        2)*q^9 + (2*a^3 - a^2 - 5*a + 1)*q^10 + (-a^4 - 2*a^3 + 6*a^2 + 7*a -
        6)*q^11 + (-a^4 + a^3 + 3*a^2 - 3*a - 1)*q^12 + (a^3 + a^2 - 2*a -
        3)*q^13 + (-a^4 + a^3 + 3*a^2 - 5*a + 1)*q^14 + (-a^4 + 3*a^3 + a^2 -
        9*a + 3)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + (-2*a^3 + 6*a + 2)*q^17 + (a^4
        - a^3 - 5*a^2 + 6*a - 1)*q^18 + (-3*a^3 + 3*a^2 + 10*a - 8)*q^19 +
        O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - 3*q^7 - q^8 - 2*q^9 + q^10 + 4*q^11 - q^12
        - 7*q^13 + 3*q^14 + q^15 + q^16 - 4*q^17 + 2*q^18 - 6*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 + 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 - 3*q^10 + q^12 +
        5*q^13 + q^14 + 3*q^15 + q^16 + 2*q^18 + 2*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 - 2*q^5 - a*q^6 + 4*q^7 - q^8 + 3*q^9 + 2*q^10 +
        a*q^12 + (-2*a + 2)*q^13 - 4*q^14 - 2*a*q^15 + q^16 + (-2*a + 2)*q^17 -
        3*q^18 + 2*a*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^5 - 6*x^3 + 8*x - 1 over the
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 6 over the Rational Field
*]
[* 79, 79, 158, 158, 158 *]
```

over $\mathbb{Q}$-decomposition Jacobian appears: $79a, 158b, 158d$, and $n(79a, 158d; 25) = 68 - 54$. It remains $158b$????

genus 5, $X_0(158)/w_{79}$:

```
[*
    q - q^2 - q^3 - q^4 - 3*q^5 + q^6 - q^7 + 3*q^8 - 2*q^9 + 3*q^10 - 2*q^11 +
        q^12 + 3*q^13 + q^14 + 3*q^15 - q^16 - 6*q^17 + 2*q^18 + 4*q^19 +
        O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - 3*q^7 - q^8 - 2*q^9 + q^10 + 4*q^11 - q^12
        - 7*q^13 + 3*q^14 + q^15 + q^16 - 4*q^17 + 2*q^18 - 6*q^19 + O(q^20),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 + q^10 + 2*q^11 - q^12
        - q^13 + 3*q^14 - q^15 + q^16 - 2*q^17 - 2*q^18 + O(q^20),
    q + q^2 + 2*q^3 + q^4 - 2*q^5 + 2*q^6 + q^8 + q^9 - 2*q^10 - 4*q^11 + 2*q^12
        + 2*q^13 - 4*q^15 + q^16 - 2*q^17 + q^18 + O(q^20),
    q - q^2 - q^3 - q^4 - 3*q^5 + q^6 - q^7 + 3*q^8 - 2*q^9 + 3*q^10 - 2*q^11 +
        q^12 + 3*q^13 + q^14 + 3*q^15 - q^16 - 6*q^17 + 2*q^18 + 4*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
```

```
    Rational Field,
    Rational Field
*]
[* 79, 158, 158, 158, 158 *]
```

The $\mathbb{Q}$-decomposition Jacobian is: $79a, 158b, 158c, 158e, 79a$. Over bielliptic over $\mathbb{Q}$ we have is not bielliptic over $\mathbb{Q}$: $n(158e; 3) = 6 - 4$, $n(69a, 158b, 158c; 9) = 32 - 30$.

11. 161

genus 7, $X_0(161)/w_7$:

```
[*
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a - 1)*q^4 + (-2*a - 2)*q^5 - a*q^6 - q^7 + (-2*a -
        1)*q^8 - 2*q^9 - 2*q^10 + (4*a + 2)*q^11 + (a + 1)*q^12 + (2*a - 1)*q^13
        - a*q^14 + (2*a + 2)*q^15 + 3*a*q^16 - 2*a*q^18 + (-2*a - 6)*q^19 +
        O(q^20),
    q + a*q^2 + 1/2*(-a^2 + 5)*q^3 + (a^2 - 2)*q^4 + 1/2*(-a^2 + 5)*q^5 +
        1/2*(a^2 - 1)*q^6 - q^7 + (-a^2 + a + 1)*q^8 + (-a^2 - a + 3)*q^9 +
        1/2*(a^2 - 1)*q^10 + (-a + 1)*q^11 + 1/2*(a^2 + 4*a - 9)*q^12 + (a^2 -
        3)*q^13 - a*q^14 + (-a^2 - a + 6)*q^15 + (-4*a + 3)*q^16 + 1/2*(a^2 -
        1)*q^17 + (-2*a - 1)*q^18 + (2*a^2 + 2*a - 4)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 + x^2 - 5*x - 1 over the Rational
    Field
*]
[* 23, 161, 161 *]
```

Not bielliptic.

genus 8, $X_0(161)/w_{23}$:

```
[*
    q - q^2 - q^4 + 2*q^5 + q^7 + 3*q^8 - 3*q^9 - 2*q^10 + 4*q^11 + 6*q^13 -
        q^14 - q^16 - 2*q^17 + 3*q^18 + 4*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a - 1)*q^4 + (-2*a - 2)*q^5 - a*q^6 - q^7 + (-2*a -
        1)*q^8 - 2*q^9 - 2*q^10 + (4*a + 2)*q^11 + (a + 1)*q^12 + (2*a - 1)*q^13
        - a*q^14 + (2*a + 2)*q^15 + 3*a*q^16 - 2*a*q^18 + (-2*a - 6)*q^19 +
        O(q^20),
    q + a*q^2 + 1/2*(a^4 - a^3 - 8*a^2 + 5*a + 11)*q^3 + (a^2 - 2)*q^4 +
        1/2*(-a^4 - a^3 + 10*a^2 + 5*a - 21)*q^5 + 1/2*(a^4 + a^3 - 12*a^2 - 5*a
        + 27)*q^6 + q^7 + (a^3 - 4*a)*q^8 + (-a^2 - a + 7)*q^9 + 1/2*(-3*a^4 +
        a^3 + 22*a^2 - 5*a - 27)*q^10 + (-a^4 + 8*a^2 + a - 12)*q^11 + 1/2*(a^4
        - a^3 - 6*a^2 + a + 5)*q^12 + (a^4 - 9*a^2 + 14)*q^13 + a*q^14 + (a^3 -
        8*a + 3)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + 1/2*(a^4 + a^3 - 6*a^2 - 5*a -
        3)*q^17 + (-a^3 - a^2 + 7*a)*q^18 + (-2*a + 2)*q^19 + O(q^20)
*]
[*
```

```
    Rational Field,
    Number Field with defining polynomial $.1^2 + $.1 - 1 over the Rational
    Field,
    Number Field with defining polynomial $.1^5 - 2*$.1^4 - 9*$.1^3 + 17*$.1^2 +
        16*$.1 - 27 over the Rational Field
*]
[* 161, 161, 161 *]
```

Only over $\mathbb{Q}$ appears: $161a$, $n(161a, 5) = 10 - 8$, therefore is not bielliptic.

12. 166

genus 10, $X_0(166)/w_2$:

```
[*
    q - q^2 - q^3 - q^4 - 2*q^5 + q^6 - 3*q^7 + 3*q^8 - 2*q^9 + 2*q^10 + 3*q^11
        + q^12 - 6*q^13 + 3*q^14 + 2*q^15 - q^16 + 5*q^17 + 2*q^18 + 2*q^19 +
        O(q^20),
    q + a*q^2 + 1/2*(a^4 - a^3 - 7*a^2 + 3*a + 8)*q^3 + (a^2 - 2)*q^4 +
        1/2*(-a^5 - a^4 + 9*a^3 + 7*a^2 - 16*a - 4)*q^5 + 1/2*(a^5 - a^4 - 7*a^3
        + 3*a^2 + 8*a)*q^6 + 1/4*(3*a^5 - a^4 - 25*a^3 + 3*a^2 + 38*a)*q^7 +
        (a^3 - 4*a)*q^8 + 1/4*(-a^5 + a^4 + 9*a^3 - 7*a^2 - 20*a + 12)*q^9 +
        (-a^5 + 7*a^3 + 2*a^2 - 8*a - 4)*q^10 + 1/4*(-a^5 + a^4 + 5*a^3 + a^2 -
        16)*q^11 + (-a^3 + a^2 + 3*a - 4)*q^12 + (a^3 - 5*a + 2)*q^13 + 1/2*(a^5
        + a^4 - 9*a^3 - 11*a^2 + 18*a + 12)*q^14 + (a^4 - 7*a^2 + 6)*q^15 + (a^4
        - 6*a^2 + 4)*q^16 + 1/4*(a^5 - 3*a^4 - 7*a^3 + 17*a^2 + 14*a - 16)*q^17
        - 2*q^18 + 1/2*(3*a^5 - a^4 - 23*a^3 - a^2 + 32*a + 8)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 + q^7 - q^8 - 2*q^9 + 2*q^10 - 5*q^11 -
        q^12 - 2*q^13 - q^14 + 2*q^15 + q^16 - 3*q^17 + 2*q^18 - 2*q^19 +
        O(q^20),
    q - q^2 + a*q^3 + q^4 + 1/2*(a + 4)*q^5 - a*q^6 + 1/2*(a - 2)*q^7 - q^8 +
        (-2*a + 1)*q^9 + 1/2*(-a - 4)*q^10 + (-a + 2)*q^11 + a*q^12 + 1/2*(-a +
        2)*q^13 + 1/2*(-a + 2)*q^14 + (a + 2)*q^15 + q^16 + 1/2*(a + 8)*q^17 +
        (2*a - 1)*q^18 + 1/2*(-a - 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^6 - x^5 - 9*x^4 + 7*x^3 + 20*x^2 -
        12*x - 8 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 4 over the Rational Field
*]
[* 83, 83, 166, 166 *]
```

Over $\mathbb{Q}$: $83a, 166a$, $n(83a, 166a; 25) = 67 - 64$. It is not bielliptic.

genus 6, $X_0(166)/w_{83}$

```
[*
    q - q^2 - q^3 - q^4 - 2*q^5 + q^6 - 3*q^7 + 3*q^8 - 2*q^9 + 2*q^10 + 3*q^11
        + q^12 - 6*q^13 + 3*q^14 + 2*q^15 - q^16 + 5*q^17 + 2*q^18 + 2*q^19 +
        O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 + q^7 - q^8 - 2*q^9 + 2*q^10 - 5*q^11 -
        q^12 - 2*q^13 - q^14 + 2*q^15 + q^16 - 3*q^17 + 2*q^18 - 2*q^19 +
        O(q^20),
```

```
    q + q^2 + a*q^3 + q^4 + 1/2*(-a^2 - a + 4)*q^5 + a*q^6 + 1/2*(a^2 - 3*a -
        2)*q^7 + q^8 + (a^2 - 3)*q^9 + 1/2*(-a^2 - a + 4)*q^10 + (-a + 2)*q^11 +
        a*q^12 + 1/2*(-a^2 + a - 2)*q^13 + 1/2*(a^2 - 3*a - 2)*q^14 + (-a^2 - a
        + 2)*q^15 + q^16 + 1/2*(3*a^2 + a - 16)*q^17 + (a^2 - 3)*q^18 +
        1/2*(-5*a^2 + a + 18)*q^19 + O(q^20),
    q - q^2 - q^3 - q^4 - 2*q^5 + q^6 - 3*q^7 + 3*q^8 - 2*q^9 + 2*q^10 + 3*q^11
        + q^12 - 6*q^13 + 3*q^14 + 2*q^15 - q^16 + 5*q^17 + 2*q^18 + 2*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^3 - $.1^2 - 6*$.1 + 4 over the
    Rational Field,
    Rational Field
*]
[* 83, 166, 166, 166 *]
```

Over $\mathbb{Q}$: $83a, 166a, 83a$. $n(166a; 13) = 37 - 32$. We can not say nothing???

13. 177

genus 10, $X_0(177)/w_3$:

```
[*
    q + a*q^2 + 1/4*(-a^4 + 5*a^2 - 2*a)*q^3 + (a^2 - 2)*q^4 + 1/4*(3*a^4 +
        2*a^3 - 23*a^2 - 12*a + 28)*q^5 + (-a^3 + 4*a - 2)*q^6 + 1/2*(-a^4 - a^3
        + 7*a^2 + 3*a - 6)*q^7 + (a^3 - 4*a)*q^8 + 1/2*(a^3 + 2*a^2 - 5*a -
        4)*q^9 + 1/2*(a^4 + 2*a^3 - 9*a^2 - 10*a + 12)*q^10 + 1/2*(-a^4 - 2*a^3
        + 9*a^2 + 12*a - 16)*q^11 + 1/2*(-a^4 + 3*a^2 - 2*a)*q^12 + 1/2*(-a^4 -
        2*a^3 + 9*a^2 + 12*a - 12)*q^13 + 1/2*(-a^4 - 2*a^3 + 5*a^2 + 10*a -
        8)*q^14 + 1/4*(a^4 + 2*a^3 - 9*a^2 - 8*a + 8)*q^15 + (a^4 - 6*a^2 +
        4)*q^16 + (a^4 - 8*a^2 + 9)*q^17 + 1/2*(a^4 + 2*a^3 - 5*a^2 - 4*a)*q^18
        + 1/4*(3*a^4 + 6*a^3 - 23*a^2 - 32*a + 36)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a - 1)*q^4 + (-2*a - 1)*q^5 - a*q^6 + (a - 3)*q^7 +
        (-2*a - 1)*q^8 + q^9 + (a - 2)*q^10 + (2*a + 1)*q^11 + (a + 1)*q^12 +
        (-2*a - 5)*q^13 + (-4*a + 1)*q^14 + (2*a + 1)*q^15 + 3*a*q^16 + 3*a*q^17
        + a*q^18 + 5*a*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-a^2 + a + 2)*q^5 - a*q^6 + (a + 3)*q^7 +
        q^8 + q^9 + (a^2 - 2*a - 1)*q^10 + (-a^2 - a + 2)*q^11 + (-a^2 + 2)*q^12
        + (-a^2 - a + 4)*q^13 + (a^2 + 3*a)*q^14 + (a^2 - a - 2)*q^15 + (-2*a^2
        + a + 4)*q^16 + (3*a^2 - 2*a - 7)*q^17 + a*q^18 + (-a^2 + 5)*q^19 +
        O(q^20)
*]
[*
    Number Field with defining polynomial x^5 - 9*x^3 + 2*x^2 + 16*x - 8 over
    the Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 4*x - 1 over the Rational Field
*]
[* 59, 177, 177 *]
```

It is not bielliptic.

genus 4, $X_0(177)/w_{59}$:

```
[*
    q + a*q^2 - q^3 + (-a - 1)*q^4 + (-2*a - 1)*q^5 - a*q^6 + (a - 3)*q^7 +
        (-2*a - 1)*q^8 + q^9 + (a - 2)*q^10 + (2*a + 1)*q^11 + (a + 1)*q^12 +
        (-2*a - 5)*q^13 + (-4*a + 1)*q^14 + (2*a + 1)*q^15 + 3*a*q^16 + 3*a*q^17
        + a*q^18 + 5*a*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a - 1)*q^4 + q^5 + a*q^6 + (-a + 1)*q^7 + (-2*a + 1)*q^8
        + q^9 + a*q^10 + (-2*a + 3)*q^11 + (a - 1)*q^12 - q^13 - q^14 + q^15 -
        3*a*q^16 + (-3*a + 2)*q^17 + a*q^18 + (3*a - 4)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field
*]
[* 177, 177 *]
```

It is not bielliptic over $\mathbb{Q}$. Over $\overline{\mathbb{Q}}$?

14. 205

genus 9, $X_0(205)/w_5$:

```
[*
    q + a*q^2 + 1/2*(-a^2 - 2*a + 3)*q^3 + (a^2 - 2)*q^4 + (-a - 1)*q^5 +
        1/2*(-a^2 - 2*a - 1)*q^6 + 1/2*(a^2 + 2*a + 1)*q^7 + (-a^2 + a + 1)*q^8
        + a*q^9 + (-a^2 - a)*q^10 + 1/2*(3*a^2 + 2*a - 9)*q^11 + 1/2*(a^2 - 2*a
        - 7)*q^12 + (-a^2 + 3)*q^13 + 1/2*(a^2 + 6*a + 1)*q^14 + (a^2 + 2*a -
        1)*q^15 + (-4*a + 3)*q^16 - 2*q^17 + a^2*q^18 + 1/2*(-3*a^2 - 2*a +
        13)*q^19 + O(q^20),
    q - q^2 + 2*q^3 - q^4 - q^5 - 2*q^6 + 2*q^7 + 3*q^8 + q^9 + q^10 + 6*q^11 -
        2*q^12 + 2*q^13 - 2*q^14 - 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 - q^3 + (-a - 1)*q^4 - q^5 - a*q^6 - 3*a*q^7 + (-2*a - 1)*q^8 -
        2*q^9 - a*q^10 + (2*a - 3)*q^11 + (a + 1)*q^12 + 3*a*q^13 + (3*a -
        3)*q^14 + q^15 + 3*a*q^16 + (2*a + 1)*q^17 - 2*a*q^18 + (-3*a - 4)*q^19
        + O(q^20),
    q + a*q^2 + (-a^2 + a + 4)*q^3 + (a^2 - 2)*q^4 - q^5 + (-a^2 + 7)*q^6 + (a^2
        - 7)*q^7 + (2*a^2 - 7)*q^8 + (-3*a^2 + a + 13)*q^9 - a*q^10 + (-a^2 - a
        + 6)*q^11 + (a - 1)*q^12 + (-a^2 + 3)*q^13 + (2*a^2 - 3*a - 7)*q^14 +
        (a^2 - a - 4)*q^15 + (2*a^2 + a - 10)*q^16 + (3*a^2 - a - 10)*q^17 +
        (-5*a^2 + a + 21)*q^18 + (a^2 + 1)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^3 + x^2 - 5*x - 1 over the Rational
    Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 2*x^2 - 4*x + 7 over the
    Rational Field
*]
[* 41, 205, 205, 205 *]
```

Over $\mathbb{Q}$ only $205b$: $n(205b; 9) = 36 - 24$. Not bielliptic.

genus 6, $X_0(205)/w_{41}$:

```
[*
    q + q^2 + 2*q^3 - q^4 + q^5 + 2*q^6 + 2*q^7 - 3*q^8 + q^9 + q^10 - 2*q^12 -
        4*q^13 + 2*q^14 + 2*q^15 - q^16 + 4*q^17 + q^18 + O(q^20),
    q + a*q^2 - q^3 + (-a - 1)*q^4 - q^5 - a*q^6 - 3*a*q^7 + (-2*a - 1)*q^8 -
        2*q^9 - a*q^10 + (2*a - 3)*q^11 + (a + 1)*q^12 + 3*a*q^13 + (3*a -
        3)*q^14 + q^15 + 3*a*q^16 + (2*a + 1)*q^17 - 2*a*q^18 + (-3*a - 4)*q^19
        + O(q^20),
    q + a*q^2 + (a^2 - a - 2)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^2 + 2*a + 1)*q^6 +
        (-a^2 + 3)*q^7 + q^8 + (a^2 - 3*a - 1)*q^9 + a*q^10 + (-a^2 + a +
        4)*q^11 + (-a + 3)*q^12 + (-a^2 + 2*a + 3)*q^13 + (-a - 1)*q^14 + (a^2 -
        a - 2)*q^15 + (-2*a^2 + a + 4)*q^16 + (-a^2 - a + 2)*q^17 + (-3*a^2 +
        3*a + 1)*q^18 + (-a^2 + 1)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 4*x - 1 over the Rational Field
*]
[* 205, 205, 205 *]
```

Over $\mathbb{Q}$ only $205c$: $n(205c;4) = 17 - 16$. It is not bielliptic.

15. 206

genus 13, $X_0(206)/w_2$:

```
[*
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 + (-a - 3)*q^5 - a*q^6 - q^7 + (4*a +
        3)*q^8 - 2*q^9 + q^10 + a*q^11 + (3*a + 3)*q^12 + (3*a + 3)*q^13 -
        a*q^14 + (a + 3)*q^15 + (-3*a + 2)*q^16 + (a - 3)*q^17 - 2*a*q^18 +
        (-3*a - 2)*q^19 + O(q^20),
    q + a*q^2 + (-a^5 + 3*a^4 + 3*a^3 - 11*a^2 - a + 8)*q^3 + (a^2 - 2)*q^4 +
        (2*a^5 - 5*a^4 - 9*a^3 + 19*a^2 + 9*a - 13)*q^5 + (-a^5 + 2*a^4 + 6*a^3
        - 10*a^2 - 8*a + 11)*q^6 + (-a^4 + 2*a^3 + 4*a^2 - 5*a - 3)*q^7 + (a^3 -
        4*a)*q^8 + (-a^5 + 3*a^4 + 5*a^3 - 15*a^2 - 7*a + 17)*q^9 + (3*a^5 -
        7*a^4 - 15*a^3 + 27*a^2 + 19*a - 22)*q^10 + (-a^5 + 2*a^4 + 4*a^3 -
        4*a^2 - 4*a - 1)*q^11 + (-a^4 + a^3 + 5*a^2 - 3*a - 5)*q^12 + (2*a^5 -
        4*a^4 - 11*a^3 + 15*a^2 + 14*a - 11)*q^13 + (-a^5 + 2*a^4 + 4*a^3 -
        5*a^2 - 3*a)*q^14 + (a^4 - 3*a^3 - a^2 + 7*a - 5)*q^15 + (a^4 - 6*a^2 +
        4)*q^16 + (-3*a^5 + 7*a^4 + 16*a^3 - 30*a^2 - 21*a + 30)*q^17 + (-a^5 +
        4*a^4 + 2*a^3 - 16*a^2 + a + 11)*q^18 + (-a^5 + 3*a^4 + 4*a^3 - 14*a^2 -
        3*a + 13)*q^19 + O(q^20),
    q - q^2 + 2*q^3 + q^4 + 4*q^5 - 2*q^6 - q^8 + q^9 - 4*q^10 - 6*q^11 + 2*q^12
        - 2*q^13 + 8*q^15 + q^16 + 2*q^17 - q^18 - 4*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + (-a + 1)*q^5 - a*q^6 + (a - 2)*q^7 - q^8 + (a +
        4)*q^9 + (a - 1)*q^10 + 4*q^11 + a*q^12 + (-2*a + 2)*q^13 + (-a +
        2)*q^14 - 7*q^15 + q^16 + (-a - 1)*q^17 + (-a - 4)*q^18 + 6*q^19 +
        O(q^20),
    q - q^2 + a*q^3 + q^4 + (a - 1)*q^5 - a*q^6 + (a + 4)*q^7 - q^8 + (-3*a -
        2)*q^9 + (-a + 1)*q^10 + a*q^12 + (2*a + 6)*q^13 + (-a - 4)*q^14 + (-4*a
        + 1)*q^15 + q^16 + (-a + 1)*q^17 + (3*a + 2)*q^18 + 2*q^19 + O(q^20)
```

```
*]
[*
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^6 - 4*x^5 - x^4 + 17*x^3 - 9*x^2 -
        16*x + 11 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - x - 7 over the Rational Field,
    Number Field with defining polynomial x^2 + 3*x - 1 over the Rational Field
*]
[* 103, 103, 206, 206, 206 *]
```

Over $\mathbb{Q}$ only $206a$: $n(206a, 3) = 6 - 4$ is not bielliptic.

genus 8: $X_0(206)/w_{103}$:

```
[*
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 + (-a - 3)*q^5 - a*q^6 - q^7 + (4*a +
        3)*q^8 - 2*q^9 + q^10 + a*q^11 + (3*a + 3)*q^12 + (3*a + 3)*q^13 -
        a*q^14 + (a + 3)*q^15 + (-3*a + 2)*q^16 + (a - 3)*q^17 - 2*a*q^18 +
        (-3*a - 2)*q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 + (-a^3 + 5*a - 2)*q^5 + a*q^6 + (2*a^3 - a^2 - 12*a +
        9)*q^7 + q^8 + (a^2 - 3)*q^9 + (-a^3 + 5*a - 2)*q^10 + (-2*a^3 + 2*a^2 +
        10*a - 10)*q^11 + a*q^12 + (2*a^3 - 10*a + 4)*q^13 + (2*a^3 - a^2 - 12*a
        + 9)*q^14 + (-2*a^3 + 10*a - 5)*q^15 + q^16 + (2*a^3 - 3*a^2 - 12*a +
        12)*q^17 + (a^2 - 3)*q^18 + (-2*a^2 - 2*a + 8)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 + (-a - 3)*q^5 - a*q^6 - q^7 + (4*a +
        3)*q^8 - 2*q^9 + q^10 + a*q^11 + (3*a + 3)*q^12 + (3*a + 3)*q^13 -
        a*q^14 + (a + 3)*q^15 + (-3*a + 2)*q^16 + (a - 3)*q^17 - 2*a*q^18 +
        (-3*a - 2)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^4 - 2*x^3 - 5*x^2 + 12*x - 5 over
    the Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field
*]
[* 103, 206, 206 *]
```

Not bielliptic.

16. 209

genus 10: $X_0(209)/w_{11}$

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q + a*q^2 + (-a - 1)*q^3 - q^5 + (-a - 2)*q^6 + (-a - 2)*q^7 - 2*a*q^8 +
        2*a*q^9 - a*q^10 - q^11 + (3*a - 2)*q^13 + (-2*a - 2)*q^14 + (a +
        1)*q^15 - 4*q^16 + (a + 2)*q^17 + 4*q^18 - q^19 + O(q^20),
    q + a*q^2 + 1/2*(-a^4 + 7*a^2 - 2*a - 4)*q^3 + (a^2 - 2)*q^4 + 1/2*(a^5 -
        9*a^3 + 14*a + 6)*q^5 + 1/2*(-a^5 + 7*a^3 - 2*a^2 - 4*a)*q^6 + 1/4*(-a^6
        + 12*a^4 - 37*a^2 + 26)*q^7 + (a^3 - 4*a)*q^8 + 1/4*(a^6 - 12*a^4 +
        4*a^3 + 41*a^2 - 20*a - 26)*q^9 + 1/2*(a^6 - 9*a^4 + 14*a^2 + 6*a)*q^10
        - q^11 + 1/2*(-a^6 + 9*a^4 - 2*a^3 - 18*a^2 + 4*a + 8)*q^12 + 1/4*(-a^6
```

```
        - 2*a^5 + 10*a^4 + 18*a^3 - 27*a^2 - 36*a + 14)*q^13 + 1/4*(a^6 - 2*a^5
        - 10*a^4 + 22*a^3 + 27*a^2 - 40*a - 30)*q^14 + 1/4*(a^6 + 2*a^5 - 10*a^4
        - 18*a^3 + 23*a^2 + 28*a + 6)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + (a^4 - a^3
        - 9*a^2 + 7*a + 12)*q^17 + 1/4*(-a^6 + 2*a^5 + 14*a^4 - 18*a^3 - 47*a^2
        + 40*a + 30)*q^18 + q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial $.1^2 - 2 over the Rational Field,
    Number Field with defining polynomial $.1^7 + $.1^6 - 14*$.1^5 - 10*$.1^4 +
        59*$.1^3 + 27*$.1^2 - 66*$.1 - 30 over the Rational Field
*]
[* 19, 209, 209 *]
```

Over $\mathbb{Q}$ only $19a$: $n(19a;16) = 20 - 18$. It is not bielliptic.

genus 8, $X_0(209)/w_{19}$:

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q + a*q^2 + (-a - 1)*q^3 - q^5 + (-a - 2)*q^6 + (-a - 2)*q^7 - 2*a*q^8 +
        2*a*q^9 - a*q^10 - q^11 + (3*a - 2)*q^13 + (-2*a - 2)*q^14 + (a +
        1)*q^15 - 4*q^16 + (a + 2)*q^17 + 4*q^18 - q^19 + O(q^20),
    q + a*q^2 + 1/2*(a^4 - 2*a^3 - 5*a^2 + 8*a + 2)*q^3 + (a^2 - 2)*q^4 +
        1/2*(-a^3 + 7*a - 2)*q^5 + 1/2*(a^3 - 2*a^2 - 3*a + 4)*q^6 + 1/2*(-a^3 +
        3*a + 4)*q^7 + (a^3 - 4*a)*q^8 + 1/2*(a^3 - 2*a^2 - 7*a + 8)*q^9 +
        1/2*(-a^4 + 7*a^2 - 2*a)*q^10 + q^11 + 1/2*(-a^4 + 2*a^3 + 7*a^2 - 12*a
        - 4)*q^12 + 1/2*(-a^4 + 7*a^2 - 4)*q^13 + 1/2*(-a^4 + 3*a^2 + 4*a)*q^14
        + 1/2*(-a^4 + 4*a^3 + a^2 - 16*a + 10)*q^15 + (a^4 - 6*a^2 + 4)*q^16 +
        (a^4 - a^3 - 5*a^2 + 3*a)*q^17 + 1/2*(a^4 - 2*a^3 - 7*a^2 + 8*a)*q^18 -
        q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial $.1^2 - 2 over the Rational Field,
    Number Field with defining polynomial $.1^5 - 2*$.1^4 - 6*$.1^3 + 10*$.1^2 +
        5*$.1 - 4 over the Rational Field
*]
[* 11, 209, 209 *]
```

Over $\mathbb{Q}$ only $11a$: $n(11a;4) = 13 - 10$. It is not bielliptic.

17. 213

genus 12, $X_0(213)/w_3$:

```
[*
    q + a*q^2 + (-a^2 + 3)*q^3 + (a^2 - 2)*q^4 + (-a - 1)*q^5 + (-2*a + 3)*q^6 +
        (2*a^2 + 2*a - 6)*q^7 + (a - 3)*q^8 + (-a^2 - 3*a + 6)*q^9 + (-a^2 -
        a)*q^10 + (-2*a^2 - 2*a + 6)*q^11 + (3*a - 6)*q^12 + 4*q^13 + (2*a^2 +
        4*a - 6)*q^14 + (a^2 + 2*a - 6)*q^15 + (-a^2 - 3*a + 4)*q^16 + (2*a^2 +
        2*a - 6)*q^17 + (-3*a^2 + a + 3)*q^18 + (-a^2 - a + 7)*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (a^2 - 2)*q^4 + (-a^2 + a + 5)*q^5 - a^2*q^6 - 2*a*q^7 +
        (-a^2 + 3)*q^8 + (a^2 - 3)*q^9 + (2*a^2 + a - 3)*q^10 + (2*a^2 - 6)*q^11
```

```
        + (a^2 - 2*a - 3)*q^12 + (-2*a^2 + 4)*q^13 - 2*a^2*q^14 + (-2*a^2 - a +
        3)*q^15 + (-a^2 - a + 1)*q^16 + (2*a^2 + 2*a - 6)*q^17 + (-a^2 + a +
        3)*q^18 + (a^2 + 2*a - 2)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a - 1)*q^4 - a*q^5 - a*q^6 - 3*q^7 + (-2*a - 1)*q^8 +
        q^9 + (a - 1)*q^10 + (-2*a - 3)*q^11 + (a + 1)*q^12 + (3*a - 1)*q^13 -
        3*a*q^14 + a*q^15 + 3*a*q^16 + (2*a + 1)*q^17 + a*q^18 + (-2*a - 5)*q^19
        + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-a^2 + 2*a + 1)*q^5 - a*q^6 + (-a^2 + a +
        4)*q^7 + (a^3 - 4*a)*q^8 + q^9 + (-a^3 + 2*a^2 + a)*q^10 + (-a^3 + a^2 +
        3*a + 1)*q^11 + (-a^2 + 2)*q^12 + (-a^3 + 2*a^2 + a)*q^13 + (-a^3 + a^2
        + 4*a)*q^14 + (a^2 - 2*a - 1)*q^15 + (3*a^3 - 4*a^2 - 7*a + 3)*q^16 +
        (2*a^3 - 5*a^2 - 5*a + 6)*q^17 + a*q^18 + (3*a^3 - 5*a^2 - 9*a + 7)*q^19
        + O(q^20)
*]
[*
    Number Field with defining polynomial x^3 - 5*x + 3 over the Rational Field,
    Number Field with defining polynomial x^3 + x^2 - 4*x - 3 over the Rational
    Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Number Field with defining polynomial x^4 - 3*x^3 - 2*x^2 + 7*x + 1 over the
    Rational Field
*]
[* 71, 71, 213, 213 *]
```

It is not bielliptic.

genus 5, $X_0(213)/w_{71}$:

```
[*
    q + q^2 + q^3 - q^4 + 2*q^5 + q^6 + 2*q^7 - 3*q^8 + q^9 + 2*q^10 - q^12 -
        2*q^13 + 2*q^14 + 2*q^15 - q^16 + q^18 + O(q^20),
    q + a*q^2 - q^3 + (-a - 1)*q^4 - a*q^5 - a*q^6 - 3*q^7 + (-2*a - 1)*q^8 +
        q^9 + (a - 1)*q^10 + (-2*a - 3)*q^11 + (a + 1)*q^12 + (3*a - 1)*q^13 -
        3*a*q^14 + a*q^15 + 3*a*q^16 + (2*a + 1)*q^17 + a*q^18 + (-2*a - 5)*q^19
        + O(q^20),
    q + a*q^2 + q^3 + (a + 1)*q^4 - a*q^5 + a*q^6 - q^7 + 3*q^8 + q^9 + (-a -
        3)*q^10 + 3*q^11 + (a + 1)*q^12 + (-a - 1)*q^13 - a*q^14 - a*q^15 + (a -
        2)*q^16 + 3*q^17 + a*q^18 + (-2*a - 1)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - x - 3 over the Rational Field
*]
[* 213, 213, 213 *]
```

Over $\mathbb{Q}$ only $213a$: $n(213a; 7) = 14 - 12$. Over $\overline{\mathbb{Q}}$???

18. 215

genus 10: $X_0(215)/w_5$:

```
[*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
```

```
    q + a*q^2 - a*q^3 + (-a + 2)*q^5 - 2*q^6 + (a - 2)*q^7 - 2*a*q^8 - q^9 +
        (2*a - 2)*q^10 + (2*a - 1)*q^11 + (2*a + 1)*q^13 + (-2*a + 2)*q^14 +
        (-2*a + 2)*q^15 - 4*q^16 + (2*a + 5)*q^17 - a*q^18 + (-2*a - 2)*q^19 +
        O(q^20),
    q - 2*q^4 - q^5 - 2*q^7 - 3*q^9 - q^11 - q^13 + 4*q^16 - 3*q^17 - 2*q^19 +
        O(q^20),
    q + a*q^2 + (a^5 - 2*a^4 - 6*a^3 + 9*a^2 + 6*a - 2)*q^3 + (a^2 - 2)*q^4 -
        q^5 + (a^5 - a^4 - 8*a^3 + 3*a^2 + 15*a + 3)*q^6 + (-2*a^5 + 3*a^4 +
        13*a^3 - 12*a^2 - 16*a + 2)*q^7 + (a^3 - 4*a)*q^8 + (2*a^5 - 3*a^4 -
        13*a^3 + 10*a^2 + 16*a + 7)*q^9 - a*q^10 + (-3*a^5 + 3*a^4 + 23*a^3 -
        9*a^2 - 38*a - 9)*q^11 + (a^4 - 2*a^3 - 6*a^2 + 8*a + 7)*q^12 + (-2*a +
        2)*q^13 + (-3*a^5 + 3*a^4 + 22*a^3 - 10*a^2 - 32*a - 6)*q^14 + (-a^5 +
        2*a^4 + 6*a^3 - 9*a^2 - 6*a + 2)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + (4*a^5
        - 4*a^4 - 30*a^3 + 12*a^2 + 48*a + 12)*q^17 + (3*a^5 - 3*a^4 - 24*a^3 +
        10*a^2 + 41*a + 6)*q^18 + (2*a^5 - 2*a^4 - 16*a^3 + 6*a^2 + 28*a +
        8)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^6 - 3*x^5 - 5*x^4 + 17*x^3 + 3*x^2 -
        17*x - 3 over the Rational Field
*]
[* 43, 43, 215, 215 *]
```

Over $\mathbb{Q}$ only $43a, 215a$: $n(43a; 4) = 18 - 10, n(215a; 16) = 34 - 18$. Not bielliptic.

genus 11, $X_0(215)/w_{43}$:

```
[*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q - 2*q^4 - q^5 - 2*q^7 - 3*q^9 - q^11 - q^13 + 4*q^16 - 3*q^17 - 2*q^19 +
        O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (a^2 - 2)*q^4 + q^5 + (a^2 + a)*q^6 + (-a^2 - 2*a
        + 1)*q^7 + (-2*a^2 - a + 3)*q^8 + (a^2 + 2*a - 2)*q^9 + a*q^10 + (-a^2 +
        a + 7)*q^11 + (-a^2 + a + 1)*q^12 + (-2*a - 2)*q^13 + (-2*a - 3)*q^14 +
        (a + 1)*q^15 + (a^2 - 3*a - 2)*q^16 + (-2*a + 2)*q^17 + (a + 3)*q^18 +
        (-2*a^2 - 4*a + 6)*q^19 + O(q^20),
    q + a*q^2 + (-a^3 + 5*a)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^4 + 5*a^2)*q^6 +
        (a^4 - a^3 - 6*a^2 + 6*a + 2)*q^7 + (a^3 - 4*a)*q^8 + (a^4 + a^3 - 6*a^2
        - 6*a + 5)*q^9 + a*q^10 + (a^3 - 6*a - 1)*q^11 + (-2*a^4 + 13*a^2 - 5*a
        - 4)*q^12 + (-a^4 + 5*a^2 + a + 3)*q^13 + (a^4 + a^3 - 7*a^2 - 3*a +
        4)*q^14 + (-a^3 + 5*a)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + (a^4 - 7*a^2 + a
        + 1)*q^17 + (3*a^4 + a^3 - 19*a^2 + 4)*q^18 + (-2*a^4 + 14*a^2 - 2*a -
        10)*q^19 + O(q^20),
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20)
*]
[*
```

```
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^3 + 2*$.1^2 - 3*$.1 - 3 over the
    Rational Field,
    Number Field with defining polynomial $.1^5 - 2*$.1^4 - 7*$.1^3 + 13*$.1^2 +
        5*$.1 - 4 over the Rational Field,
    Rational Field
*]
[* 43, 215, 215, 215, 215 *]
```

Over $\mathbb{Q}$ we have $43a, 215a, 43a$: $n(215a; 2) = 7 - 6$, $n(43a; 4) = 13 - 10$. Not bielliptic.

19. 221

genus 9, $X_0(221)/w_{13}$:

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + q^2 + 2*q^3 - q^4 + 2*q^5 + 2*q^6 + 2*q^7 - 3*q^8 + q^9 + 2*q^10 -
        6*q^11 - 2*q^12 - q^13 + 2*q^14 + 4*q^15 - q^16 + q^17 + q^18 + 4*q^19 +
        O(q^20),
    q - q^2 - q^4 + 4*q^5 - 2*q^7 + 3*q^8 - 3*q^9 - 4*q^10 + 6*q^11 - q^13 +
        2*q^14 - q^16 + q^17 + 3*q^18 + 8*q^19 + O(q^20),
    q + a*q^2 + (a - 1)*q^3 + (-a - 1)*q^4 + (-2*a - 1)*q^5 + (-2*a + 1)*q^6 +
        (-a - 1)*q^7 + (-2*a - 1)*q^8 + (-3*a - 1)*q^9 + (a - 2)*q^10 + 3*a*q^11
        + a*q^12 - q^13 - q^14 + (3*a - 1)*q^15 + 3*a*q^16 - q^17 + (2*a -
        3)*q^18 + (3*a - 2)*q^19 + O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + 3*q^4 + (a - 1)*q^5 + (a - 5)*q^6 + 2*q^7 + a*q^8
        + (-2*a + 3)*q^9 + (-a + 5)*q^10 + 2*q^11 + (-3*a + 3)*q^12 - q^13 +
        2*a*q^14 + (2*a - 6)*q^15 - q^16 + q^17 + (3*a - 10)*q^18 + (-2*a +
        2)*q^19 + O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (-a + 3)*q^4 - q^5 + 5*q^6 + (-a - 3)*q^7 + (2*a -
        5)*q^8 + (a + 3)*q^9 - a*q^10 + (a + 2)*q^11 + (3*a - 2)*q^12 - q^13 +
        (-2*a - 5)*q^14 + (-a - 1)*q^15 + (-5*a + 4)*q^16 + q^17 + (2*a +
        5)*q^18 + (-a + 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 + $.1 - 1 over the Rational
    Field,
    Number Field with defining polynomial $.1^2 - 5 over the Rational Field,
    Number Field with defining polynomial $.1^2 + $.1 - 5 over the Rational
    Field
*]
[* 17, 221, 221, 221, 221, 221 *]
```

Over $\mathbb{Q}$, $17a, 221a, 221b$:$n(17a, 221a; 8) = 12 - 8, n(221b; 2) = 6 - 4$ Not bielliptic.

genus 8: $X_0(221)/w_{17}$

```
[*
```

```
    q + a*q^2 + (a - 1)*q^3 + (-a - 1)*q^4 + (-2*a - 1)*q^5 + (-2*a + 1)*q^6 +
        (-a - 1)*q^7 + (-2*a - 1)*q^8 + (-3*a - 1)*q^9 + (a - 2)*q^10 + 3*a*q^11
        + a*q^12 - q^13 - q^14 + (3*a - 1)*q^15 + 3*a*q^16 - q^17 + (2*a -
        3)*q^18 + (3*a - 2)*q^19 + O(q^20),
    q + a*q^2 + 1/2*(-a^5 + a^4 + 8*a^3 - 5*a^2 - 13*a + 2)*q^3 + (a^2 - 2)*q^4
        + 1/2*(a^4 - a^3 - 6*a^2 + 3*a + 3)*q^5 + 1/2*(-a^4 + a^3 + 6*a^2 - 3*a
        - 3)*q^6 + (-a^3 + 5*a + 2)*q^7 + (a^3 - 4*a)*q^8 + (-a^2 + 4)*q^9 +
        1/2*(a^5 - a^4 - 6*a^3 + 3*a^2 + 3*a)*q^10 + (-a^2 + 3)*q^11 + 1/2*(a^5
        - a^4 - 10*a^3 + 7*a^2 + 23*a - 4)*q^12 + q^13 + (-a^4 + 5*a^2 +
        2*a)*q^14 + (a^3 - 7*a)*q^15 + (a^4 - 6*a^2 + 4)*q^16 - q^17 + (-a^3 +
        4*a)*q^18 + (a^5 - a^4 - 8*a^3 + 6*a^2 + 13*a - 1)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial $.1^2 + $.1 - 1 over the Rational
    Field,
    Number Field with defining polynomial $.1^6 - $.1^5 - 9*$.1^4 + 6*$.1^3 +
        19*$.1^2 - 5*$.1 - 3 over the Rational Field
*]
[* 221, 221 *]
```

Not bielliptic.

20. 287

genus 14, $X_0(287)/w_7$

```
[*
    q + a*q^2 + 1/2*(-a^2 - 2*a + 3)*q^3 + (a^2 - 2)*q^4 + (-a - 1)*q^5 +
        1/2*(-a^2 - 2*a - 1)*q^6 + 1/2*(a^2 + 2*a + 1)*q^7 + (-a^2 + a + 1)*q^8
        + a*q^9 + (-a^2 - a)*q^10 + 1/2*(3*a^2 + 2*a - 9)*q^11 + 1/2*(a^2 - 2*a
        - 7)*q^12 + (-a^2 + 3)*q^13 + 1/2*(a^2 + 6*a + 1)*q^14 + (a^2 + 2*a -
        1)*q^15 + (-4*a + 3)*q^16 - 2*q^17 + a^2*q^18 + 1/2*(-3*a^2 - 2*a +
        13)*q^19 + O(q^20),
    q + a*q^2 + (-a - 1)*q^3 + (-a - 1)*q^4 + (a + 1)*q^5 - q^6 - q^7 + (-2*a -
        1)*q^8 + (a - 1)*q^9 + q^10 - q^11 + (a + 2)*q^12 + (-2*a - 5)*q^13 -
        a*q^14 + (-a - 2)*q^15 + 3*a*q^16 + (-2*a - 3)*q^17 + (-2*a + 1)*q^18 +
        (3*a + 1)*q^19 + O(q^20),
    q + a*q^2 + (-a + 3)*q^3 + (a^2 - 2)*q^4 + (-2*a^2 + 4*a + 2)*q^5 + (-a^2 +
        3*a)*q^6 - q^7 + (4*a^2 - 7*a - 1)*q^8 + (a^2 - 6*a + 6)*q^9 + (-4*a^2 +
        8*a + 2)*q^10 + (2*a^2 - 6*a)*q^11 + (-a^2 + 5*a - 5)*q^12 + (-a^2 + 5*a
        - 1)*q^13 - a*q^14 + (-2*a^2 + 4*a + 4)*q^15 + (7*a^2 - 13*a)*q^16 +
        (-a^2 - 2*a + 7)*q^17 + (-2*a^2 + 3*a - 1)*q^18 + (3*a^2 - 8*a - 3)*q^19
        + O(q^20),
    q + a*q^2 + (-a^3 + 5*a)*q^3 + (a^2 - 2)*q^4 + (a^5 - 9*a^3 - a^2 + 19*a +
        6)*q^5 + (-a^4 + 5*a^2)*q^6 - q^7 + (a^3 - 4*a)*q^8 + (-a^5 + 10*a^3 +
        2*a^2 - 24*a - 8)*q^9 + (-a^5 + a^4 + 9*a^3 - 4*a^2 - 18*a - 5)*q^10 +
        (a^5 + a^4 - 11*a^3 - 8*a^2 + 30*a + 15)*q^11 + (-a^5 + 7*a^3 -
        10*a)*q^12 + (a^5 + a^4 - 10*a^3 - 8*a^2 + 22*a + 14)*q^13 - a*q^14 +
        (-2*a^5 - a^4 + 20*a^3 + 7*a^2 - 47*a - 15)*q^15 + (a^4 - 6*a^2 +
        4)*q^16 + (a^3 + a^2 - 5*a - 3)*q^17 + (a^5 - 8*a^3 - a^2 + 16*a +
        5)*q^18 + (-a^4 - a^3 + 6*a^2 + 4*a - 2)*q^19 + O(q^20)
*]
```

```
[*
    Number Field with defining polynomial x^3 + x^2 - 5*x - 1 over the Rational
    Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 4*x^2 + 3*x + 1 over the
    Rational Field,
    Number Field with defining polynomial x^6 + x^5 - 10*x^4 - 10*x^3 + 23*x^2 +
        24*x + 5 over the Rational Field
*]
[* 41, 287, 287, 287 *]
```

Not bielliptic.

genus 10, $X_0(287)/w_{41}$:

```
[*
    q + a*q^2 + (-a - 1)*q^3 + (-a - 1)*q^4 + (a + 1)*q^5 - q^6 - q^7 + (-2*a -
        1)*q^8 + (a - 1)*q^9 + q^10 - q^11 + (a + 2)*q^12 + (-2*a - 5)*q^13 -
        a*q^14 + (-a - 2)*q^15 + 3*a*q^16 + (-2*a - 3)*q^17 + (-2*a + 1)*q^18 +
        (3*a + 1)*q^19 + O(q^20),
    q + a*q^2 + (a^2 - a - 3)*q^3 + (a^2 - 2)*q^4 + 2*q^5 + (a - 3)*q^6 + q^7 +
        (a^2 - 3)*q^8 + (-2*a^2 - a + 9)*q^9 + 2*a*q^10 - 2*q^11 + (-a^2 - a +
        6)*q^12 + (-a^2 + 6)*q^13 + a*q^14 + (2*a^2 - 2*a - 6)*q^15 + (-a^2 + a
        + 1)*q^16 + (-2*a^2 + a + 6)*q^17 + (-3*a^2 + a + 6)*q^18 + (a + 4)*q^19
        + O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (a^2 - 2)*q^4 + (a^4 - 7*a^2 + a + 6)*q^5 + (a^2 +
        a)*q^6 + q^7 + (a^3 - 4*a)*q^8 + (a^2 + 2*a - 2)*q^9 + (-a^4 - a^3 +
        5*a^2 - 3)*q^10 + (-a^4 - a^3 + 3*a^2 + 2*a + 3)*q^11 + (a^3 + a^2 - 2*a
        - 2)*q^12 + (-a^4 - a^3 + 6*a^2 + 3*a - 4)*q^13 + a*q^14 + (-a^3 - 2*a^2
        + a + 3)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + (a^4 + 2*a^3 - 4*a^2 - 7*a +
        3)*q^17 + (a^3 + 2*a^2 - 2*a)*q^18 + (-a^4 + a^3 + 6*a^2 - 6*a - 4)*q^19
        + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 4*x + 3 over the Rational
    Field,
    Number Field with defining polynomial x^5 + x^4 - 6*x^3 - 4*x^2 + 6*x + 3
    over the Rational Field
*]
[* 287, 287, 287 *]
```

Not bielliptic.

21. 299

genus 14, $X_0(299)/w_{13}$

```
[*
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (a - 1)*q^4 + (-a - 1)*q^5 + (-a - 1)*q^6 - q^7 + (-2*a
        + 1)*q^8 + (a - 2)*q^9 + (-2*a - 1)*q^10 + (a - 2)*q^11 - q^12 - q^13 -
```

```
        a*q^14 + (2*a + 1)*q^15 - 3*a*q^16 + (3*a - 2)*q^17 + (-a + 1)*q^18 +
        (2*a - 3)*q^19 + O(q^20),
    q + a*q^2 + 1/16*(-3*a^9 - 3*a^8 + 47*a^7 + 44*a^6 - 233*a^5 - 195*a^4 +
        397*a^3 + 282*a^2 - 184*a - 112)*q^3 + (a^2 - 2)*q^4 + 1/32*(7*a^9 +
        9*a^8 - 117*a^7 - 130*a^6 + 649*a^5 + 565*a^4 - 1379*a^3 - 796*a^2 +
        976*a + 352)*q^5 + 1/8*(-3*a^9 - 5*a^8 + 49*a^7 + 74*a^6 - 261*a^5 -
        337*a^4 + 519*a^3 + 508*a^2 - 344*a - 192)*q^6 + 1/16*(-3*a^9 + a^8 +
        51*a^7 - 16*a^6 - 289*a^5 + 73*a^4 + 617*a^3 - 58*a^2 - 400*a - 64)*q^7
        + (a^3 - 4*a)*q^8 + 1/16*(5*a^9 + 3*a^8 - 87*a^7 - 46*a^6 + 507*a^5 +
        231*a^4 - 1137*a^3 - 460*a^2 + 832*a + 320)*q^9 + 1/2*(a^9 + a^8 -
        16*a^7 - 15*a^6 + 83*a^5 + 70*a^4 - 160*a^3 - 114*a^2 + 106*a + 56)*q^10
        + 1/32*(7*a^9 + 13*a^8 - 105*a^7 - 190*a^6 + 481*a^5 + 849*a^4 - 711*a^3
        - 1248*a^2 + 336*a + 480)*q^11 + 1/8*(-5*a^9 - 5*a^8 + 81*a^7 + 76*a^6 -
        431*a^5 - 357*a^4 + 867*a^3 + 574*a^2 - 584*a - 272)*q^12 - q^13 +
        1/8*(-a^9 - 3*a^8 + 19*a^7 + 46*a^6 - 127*a^5 - 227*a^4 + 349*a^3 +
        400*a^2 - 320*a - 192)*q^14 + 1/32*(-3*a^9 - a^8 + 45*a^7 + 6*a^6 -
        181*a^5 + 43*a^4 + 51*a^3 - 272*a^2 + 336*a + 288)*q^15 + (a^4 - 6*a^2 +
        4)*q^16 + 1/16*(5*a^9 + 3*a^8 - 87*a^7 - 46*a^6 + 507*a^5 + 231*a^4 -
        1121*a^3 - 476*a^2 + 752*a + 352)*q^17 + 1/2*(a^9 + a^8 - 17*a^7 -
        16*a^6 + 97*a^5 + 81*a^4 - 215*a^3 - 146*a^2 + 160*a + 80)*q^18 +
        1/32*(-13*a^9 - 15*a^8 + 227*a^7 + 234*a^6 - 1355*a^5 - 1179*a^4 +
        3245*a^3 + 2176*a^2 - 2640*a - 1120)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Number Field with defining polynomial x^10 - x^9 - 19*x^8 + 18*x^7 + 127*x^6
        - 109*x^5 - 357*x^4 + 252*x^3 + 400*x^2 - 192*x - 128 over the Rational
    Field
*]
[* 23, 299, 299 *]
```

Not bielliptic.

genus 11, $X_0(299)/w_{23}$:

```
[*
    q + a*q^2 - a*q^3 + (a - 1)*q^4 + (-a - 1)*q^5 + (-a - 1)*q^6 - q^7 + (-2*a
        + 1)*q^8 + (a - 2)*q^9 + (-2*a - 1)*q^10 + (a - 2)*q^11 - q^12 - q^13 -
        a*q^14 + (2*a + 1)*q^15 - 3*a*q^16 + (3*a - 2)*q^17 + (-a + 1)*q^18 +
        (2*a - 3)*q^19 + O(q^20),
    q + a*q^2 + 3*q^4 + (a + 1)*q^5 + (-a + 1)*q^7 + a*q^8 - 3*q^9 + (a +
        5)*q^10 + (-a - 3)*q^11 + q^13 + (a - 5)*q^14 - q^16 + 2*q^17 - 3*a*q^18
        + (-a + 5)*q^19 + O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + (a + 2)*q^4 + (-a + 1)*q^5 - 4*q^6 + 2*a*q^7 + (a
        + 4)*q^8 + (-a + 2)*q^9 - 4*q^10 + (-a + 3)*q^11 + (-2*a - 2)*q^12 +
        q^13 + (2*a + 8)*q^14 + (-a + 5)*q^15 + 3*a*q^16 - 6*q^17 + (a - 4)*q^18
        + (-a + 3)*q^19 + O(q^20),
    q + a*q^2 + a*q^3 + (-a + 3)*q^4 + (-a + 1)*q^5 + (-a + 5)*q^6 + q^7 + (2*a
        - 5)*q^8 + (-a + 2)*q^9 + (2*a - 5)*q^10 + (a + 2)*q^11 + (4*a - 5)*q^12
        + q^13 + a*q^14 + (2*a - 5)*q^15 + (-5*a + 4)*q^16 + (a + 2)*q^17 + (3*a
        - 5)*q^18 + (-2*a - 5)*q^19 + O(q^20),
```

```
    q + a*q^3 - 2*q^4 + 1/2*(-a^2 + 7)*q^5 + (a + 1)*q^7 + (a^2 - 3)*q^9 +
        1/2*(-a^2 - 2*a + 9)*q^11 - 2*a*q^12 + q^13 + 1/2*(a^2 - 2*a - 5)*q^15 +
        4*q^16 + (-a^2 + 7)*q^17 + 1/2*(a^2 + 2*a - 5)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Number Field with defining polynomial x^2 - x - 4 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 5 over the Rational Field,
    Number Field with defining polynomial x^3 + x^2 - 9*x - 5 over the Rational
    Field
*]
[* 299, 299, 299, 299, 299 *]
```

Not bielliptic.

22. 178

genus 10, $X_0(178)/w_2$:

```
[*
    q - q^2 - q^3 - q^4 - q^5 + q^6 - 4*q^7 + 3*q^8 - 2*q^9 + q^10 - 2*q^11 +
        q^12 + 2*q^13 + 4*q^14 + q^15 - q^16 + 3*q^17 + 2*q^18 - 5*q^19 +
        O(q^20),
    q + q^2 + 2*q^3 - q^4 - 2*q^5 + 2*q^6 + 2*q^7 - 3*q^8 + q^9 - 2*q^10 -
        4*q^11 - 2*q^12 + 2*q^13 + 2*q^14 - 4*q^15 - q^16 + 6*q^17 + q^18 -
        2*q^19 + O(q^20),
    q + a*q^2 + 1/2*(-a^4 + a^3 + 7*a^2 - 5*a - 8)*q^3 + (a^2 - 2)*q^4 + (-a^2 +
        4)*q^5 + 1/2*(2*a^4 - 3*a^3 - 15*a^2 + 13*a + 17)*q^6 + 1/2*(a^4 - 8*a^2
        - 2*a + 13)*q^7 + (a^3 - 4*a)*q^8 + (a^2 - a - 4)*q^9 + (-a^3 +
        4*a)*q^10 + (-a^3 + 5*a + 2)*q^11 + 1/2*(-3*a^4 + 3*a^3 + 19*a^2 - 15*a
        - 18)*q^12 + (-a^4 + a^3 + 8*a^2 - 5*a - 11)*q^13 + 1/2*(-a^4 + 2*a^3 +
        8*a^2 - 8*a - 17)*q^14 + 1/2*(a^4 - a^3 - 5*a^2 + 5*a + 2)*q^15 + (a^4 -
        6*a^2 + 4)*q^16 + (a^4 - a^3 - 7*a^2 + 4*a + 4)*q^17 + (a^3 - a^2 -
        4*a)*q^18 + 1/2*(a^3 - a^2 - 3*a + 9)*q^19 + O(q^20),
    q - q^2 + 2*q^3 + q^4 + 2*q^5 - 2*q^6 - q^8 + q^9 - 2*q^10 + 2*q^12 - 4*q^13
        + 4*q^15 + q^16 + 2*q^17 - q^18 - 2*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + (-2*a - 3)*q^5 - a*q^6 - 2*q^7 - q^8 + (-2*a -
        2)*q^9 + (2*a + 3)*q^10 + 2*a*q^11 + a*q^12 - 2*q^13 + 2*q^14 + (a -
        2)*q^15 + q^16 + (2*a - 1)*q^17 + (2*a + 2)*q^18 + (a + 2)*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^5 + x^4 - 10*x^3 - 10*x^2 + 21*x +
        17 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 89, 89, 89, 178, 178 *]
```

Over $\mathbb{Q}$, $89a, 89b, 178b$: $n(89b, 178b; 3) = 6 - 4, n(89a; 9) = 38 - 30$.Not bielliptic.

genus 8, $X_0(178)/w_{89}$:

```
[*
    q - q^2 - q^3 - q^4 - q^5 + q^6 - 4*q^7 + 3*q^8 - 2*q^9 + q^10 - 2*q^11 +
        q^12 + 2*q^13 + 4*q^14 + q^15 - q^16 + 3*q^17 + 2*q^18 - 5*q^19 +
        O(q^20),
    q + q^2 + q^3 + q^4 + 3*q^5 + q^6 - 4*q^7 + q^8 - 2*q^9 + 3*q^10 - 6*q^11 +
        q^12 + 2*q^13 - 4*q^14 + 3*q^15 + q^16 + 3*q^17 - 2*q^18 + 5*q^19 +
        O(q^20),
    q - q^2 + a*q^3 + q^4 + (-2*a - 3)*q^5 - a*q^6 - 2*q^7 - q^8 + (-2*a -
        2)*q^9 + (2*a + 3)*q^10 + 2*a*q^11 + a*q^12 - 2*q^13 + 2*q^14 + (a -
        2)*q^15 + q^16 + (2*a - 1)*q^17 + (2*a + 2)*q^18 + (a + 2)*q^19 +
        O(q^20),
    q + q^2 + a*q^3 + q^4 - a*q^5 + a*q^6 + 1/2*(-a^2 - a + 6)*q^7 + q^8 + (a^2
        - 3)*q^9 - a*q^10 + 2*q^11 + a*q^12 + 1/2*(a^2 - 3*a - 6)*q^13 +
        1/2*(-a^2 - a + 6)*q^14 - a^2*q^15 + q^16 + (-a^2 + 4)*q^17 + (a^2 -
        3)*q^18 + (a - 4)*q^19 + O(q^20),
    q - q^2 - q^3 - q^4 - q^5 + q^6 - 4*q^7 + 3*q^8 - 2*q^9 + q^10 - 2*q^11 +
        q^12 + 2*q^13 + 4*q^14 + q^15 - q^16 + 3*q^17 + 2*q^18 - 5*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 8*x + 4 over the Rational
    Field,
    Rational Field
*]
[* 89, 178, 178, 178, 178 *]
```

Over $\mathbb{Q}$, $89a, 178a, 89a{:}n(89a, 178a; 9) = 32 - 30$. Not bielliptic.

23. 183

genus 9, $X_0(183)/w_3$:

```
[*
    q - q^2 - 2*q^3 - q^4 - 3*q^5 + 2*q^6 + q^7 + 3*q^8 + q^9 + 3*q^10 - 5*q^11
        + 2*q^12 + q^13 - q^14 + 6*q^15 - q^16 + 4*q^17 - q^18 - 4*q^19 +
        O(q^20),
    q + a*q^2 + (-a^2 + 3)*q^3 + (a^2 - 2)*q^4 + (a^2 - 2*a - 2)*q^5 + (-a^2 +
        1)*q^6 + (a^2 - a - 3)*q^7 + (a^2 - a - 1)*q^8 + (-2*a^2 + 2*a + 5)*q^9
        + (-a^2 + a - 1)*q^10 + (a + 4)*q^11 + (a^2 - 2*a - 5)*q^12 + (-2*a^2 +
        2*a + 1)*q^13 - q^14 + (3*a^2 - 2*a - 7)*q^15 + (-2*a^2 + 2*a + 3)*q^16
        + (-a^2 + 2*a + 1)*q^17 + (-a + 2)*q^18 + (3*a^2 - 7)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-a - 2)*q^7 + (a - 2)*q^8
        + q^9 - a*q^10 + (-a - 2)*q^11 + (2*a + 1)*q^12 - 3*q^13 - q^14 + q^15 +
        3*q^16 - 6*q^17 + a*q^18 + (4*a + 6)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + 2*q^5 - a*q^6 + (-2*a^2 + 2*a + 4)*q^7 +
        (a^2 - a - 1)*q^8 + q^9 + 2*a*q^10 + (-a^2 + 3)*q^11 + (-a^2 + 2)*q^12 +
        (2*a^2 - 2*a - 2)*q^13 + (-2*a + 2)*q^14 - 2*q^15 + (-2*a^2 + 2*a +
        3)*q^16 + (-a^2 - 2*a + 7)*q^17 + a*q^18 + (-2*a - 2)*q^19 + O(q^20)
```

```
*]
[*
    Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 3*x + 1 over the Rational
    Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 3*x + 1 over the Rational
    Field
*]
[* 61, 61, 183, 183 *]
```

Over $\mathbb{Q}$ only $61a$: $n(61a; 4) = 20 - 16$. Not bielliptic.

genus 10, $X_0(183)/w_{61}$:

```
[*
    q - q^2 - 2*q^3 - q^4 - 3*q^5 + 2*q^6 + q^7 + 3*q^8 + q^9 + 3*q^10 - 5*q^11
        + 2*q^12 + q^13 - q^14 + 6*q^15 - q^16 + 4*q^17 - q^18 - 4*q^19 +
        O(q^20),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-a - 2)*q^7 + (a - 2)*q^8
        + q^9 - a*q^10 + (-a - 2)*q^11 + (2*a + 1)*q^12 - 3*q^13 - q^14 + q^15 +
        3*q^16 - 6*q^17 + a*q^18 + (4*a + 6)*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + 1/2*(a^5 + 2*a^4 - 10*a^3 - 16*a^2 + 21*a
        + 20)*q^5 + a*q^6 + 1/2*(-2*a^5 - 3*a^4 + 18*a^3 + 22*a^2 - 34*a -
        23)*q^7 + (a^3 - 4*a)*q^8 + q^9 + 1/2*(2*a^5 + a^4 - 18*a^3 - 10*a^2 +
        30*a + 17)*q^10 + 1/2*(-a^4 + 6*a^2 - 2*a - 5)*q^11 + (a^2 - 2)*q^12 +
        1/2*(-a^5 + 10*a^3 - 21*a + 2)*q^13 + 1/2*(-3*a^5 - 4*a^4 + 26*a^3 +
        28*a^2 - 43*a - 34)*q^14 + 1/2*(a^5 + 2*a^4 - 10*a^3 - 16*a^2 + 21*a +
        20)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + (a^5 + a^4 - 9*a^3 - 6*a^2 + 16*a +
        5)*q^17 + a*q^18 + (a^5 + a^4 - 8*a^3 - 8*a^2 + 11*a + 13)*q^19 +
        O(q^20),
    q - q^2 - 2*q^3 - q^4 - 3*q^5 + 2*q^6 + q^7 + 3*q^8 + q^9 + 3*q^10 - 5*q^11
        + 2*q^12 + q^13 - q^14 + 6*q^15 - q^16 + 4*q^17 - q^18 - 4*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^6 - 11*x^4 + 2*x^3 + 31*x^2 - 10*x -
        17 over the Rational Field,
    Rational Field
*]
[* 61, 183, 183, 183 *]
```

Over $\mathbb{Q}$ only $61a, 61a$: $n(61a; 25) = 56 - 54$.

24. 249

genus 14, $X_0(249)/w_3$:

```
[*
    q - q^2 - q^3 - q^4 - 2*q^5 + q^6 - 3*q^7 + 3*q^8 - 2*q^9 + 2*q^10 + 3*q^11
        + q^12 - 6*q^13 + 3*q^14 + 2*q^15 - q^16 + 5*q^17 + 2*q^18 + 2*q^19 +
        O(q^20),
    q + a*q^2 + 1/2*(a^4 - a^3 - 7*a^2 + 3*a + 8)*q^3 + (a^2 - 2)*q^4 +
```

```
        1/2*(-a^5 - a^4 + 9*a^3 + 7*a^2 - 16*a - 4)*q^5 + 1/2*(a^5 - a^4 - 7*a^3
        + 3*a^2 + 8*a)*q^6 + 1/4*(3*a^5 - a^4 - 25*a^3 + 3*a^2 + 38*a)*q^7 +
        (a^3 - 4*a)*q^8 + 1/4*(-a^5 + a^4 + 9*a^3 - 7*a^2 - 20*a + 12)*q^9 +
        (-a^5 + 7*a^3 + 2*a^2 - 8*a - 4)*q^10 + 1/4*(-a^5 + a^4 + 5*a^3 + a^2 -
        16)*q^11 + (-a^3 + a^2 + 3*a - 4)*q^12 + (a^3 - 5*a + 2)*q^13 + 1/2*(a^5
        + a^4 - 9*a^3 - 11*a^2 + 18*a + 12)*q^14 + (a^4 - 7*a^2 + 6)*q^15 + (a^4
        - 6*a^2 + 4)*q^16 + 1/4*(a^5 - 3*a^4 - 7*a^3 + 17*a^2 + 14*a - 16)*q^17
        - 2*q^18 + 1/2*(3*a^5 - a^4 - 23*a^3 - a^2 + 32*a + 8)*q^19 + O(q^20),
    q + q^2 - q^3 - q^4 - q^5 - q^6 - 4*q^7 - 3*q^8 + q^9 - q^10 - 3*q^11 + q^12
        + 2*q^13 - 4*q^14 + q^15 - q^16 + 4*q^17 + q^18 - q^19 + O(q^20),
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 3*q^11 + q^12 -
        6*q^13 - q^15 - q^16 - 4*q^17 - q^18 - 7*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + 1/2*(-a^4 - 4*a^3 + 4*a^2 + 20*a + 1)*q^5
        - a*q^6 + (a^4 + 2*a^3 - 5*a^2 - 8*a + 2)*q^7 + (a^3 - 4*a)*q^8 + q^9 +
        1/2*(-a^4 + 6*a^2 - 2*a + 1)*q^10 + 1/2*(-a^4 - 4*a^3 + 2*a^2 + 18*a +
        9)*q^11 + (-a^2 + 2)*q^12 + (a^3 - 5*a + 2)*q^13 + (-a^4 - a^3 + 6*a^2 +
        5*a - 1)*q^14 + 1/2*(a^4 + 4*a^3 - 4*a^2 - 20*a - 1)*q^15 + (a^4 - 6*a^2
        + 4)*q^16 + (2*a^4 + 4*a^3 - 10*a^2 - 18*a)*q^17 + a*q^18 + 1/2*(-5*a^4
        - 12*a^3 + 24*a^2 + 52*a + 5)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^6 - x^5 - 9*x^4 + 7*x^3 + 20*x^2 -
        12*x - 8 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^5 + 3*x^4 - 4*x^3 - 14*x^2 - 3*x + 1
    over the Rational Field
*]
[* 83, 83, 249, 249, 249 *]
```

Over $\mathbb{Q}$, $83a, 249a, 249b$:$n(83a, 249a, 249b; 4) = 22 - 16$. Not bielliptic.

genus 8, $X_0(249)/w_{83}$:

```
[*
    q - q^2 - q^3 - q^4 - 2*q^5 + q^6 - 3*q^7 + 3*q^8 - 2*q^9 + 2*q^10 + 3*q^11
        + q^12 - 6*q^13 + 3*q^14 + 2*q^15 - q^16 + 5*q^17 + 2*q^18 + 2*q^19 +
        O(q^20),
    q + q^2 - q^3 - q^4 - q^5 - q^6 - 4*q^7 - 3*q^8 + q^9 - q^10 - 3*q^11 + q^12
        + 2*q^13 - 4*q^14 + q^15 - q^16 + 4*q^17 + q^18 - q^19 + O(q^20),
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 3*q^11 + q^12 -
        6*q^13 - q^15 - q^16 - 4*q^17 - q^18 - 7*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a + 2)*q^5 + a*q^6 + (-a^2 + 3)*q^7 +
        (a^3 - 4*a)*q^8 + q^9 + (-a^2 + 2*a)*q^10 + (-2*a^3 + a^2 + 8*a -
        2)*q^11 + (a^2 - 2)*q^12 + (-a^3 + 5*a - 2)*q^13 + (-a^3 + 3*a)*q^14 +
        (-a + 2)*q^15 + (2*a^3 - 2*a^2 - 8*a + 5)*q^16 + (2*a^3 - 2*a^2 - 8*a +
        6)*q^17 + a*q^18 + (2*a^3 - 9*a)*q^19 + O(q^20),
    q - q^2 - q^3 - q^4 - 2*q^5 + q^6 - 3*q^7 + 3*q^8 - 2*q^9 + 2*q^10 + 3*q^11
        + q^12 - 6*q^13 + 3*q^14 + 2*q^15 - q^16 + 5*q^17 + 2*q^18 + 2*q^19 +
        O(q^20)
*]
```

```
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 - 2*x^3 - 4*x^2 + 8*x - 1 over the
    Rational Field,
    Rational Field
*]
[* 83, 249, 249, 249, 249 *]
```

Over $\mathbb{Q}$: $83a, 249a, 249b, 83a$: $n(83a, 249a, 249b; 4) = 21 - 16$. Not bielliptic.

25. 303

genus 17, $X_0(303)/w_3$

```
[*
    q - 2*q^3 - 2*q^4 - q^5 - 2*q^7 + q^9 - 2*q^11 + 4*q^12 + q^13 + 2*q^15 +
        4*q^16 + 3*q^17 - 5*q^19 + O(q^20),
    q + a*q^2 + 1/4*(a^6 + a^5 - 10*a^4 - 10*a^3 + 19*a^2 + 17*a + 2)*q^3 + (a^2
        - 2)*q^4 + 1/4*(-2*a^6 - 3*a^5 + 22*a^4 + 28*a^3 - 58*a^2 - 45*a +
        30)*q^5 + 1/4*(a^6 + 3*a^5 - 12*a^4 - 28*a^3 + 33*a^2 + 45*a - 14)*q^6 +
        1/4*(-a^5 - 2*a^4 + 10*a^3 + 16*a^2 - 21*a - 14)*q^7 + (a^3 - 4*a)*q^8 +
        1/4*(a^6 + 2*a^5 - 10*a^4 - 20*a^3 + 21*a^2 + 34*a - 4)*q^9 +
        1/4*(-3*a^6 - 4*a^5 + 32*a^4 + 36*a^3 - 77*a^2 - 56*a + 28)*q^10 +
        1/4*(-a^6 + 12*a^4 - 35*a^2 + 20)*q^11 + 1/4*(a^6 - a^5 - 10*a^4 + 6*a^3
        + 23*a^2 - 5*a - 18)*q^12 + 1/4*(3*a^6 + 4*a^5 - 34*a^4 - 36*a^3 +
        91*a^2 + 48*a - 40)*q^13 + 1/4*(-a^6 - 2*a^5 + 10*a^4 + 16*a^3 - 21*a^2
        - 14*a)*q^14 + 1/4*(-3*a^6 - 3*a^5 + 34*a^4 + 30*a^3 - 93*a^2 - 55*a +
        50)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + 1/4*(3*a^6 + 3*a^5 - 32*a^4 - 28*a^3
        + 79*a^2 + 45*a - 42)*q^17 + 1/4*(2*a^6 + 3*a^5 - 22*a^4 - 26*a^3 +
        50*a^2 + 39*a - 14)*q^18 + 1/2*(a^5 - 10*a^3 + 21*a + 4)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a - 1)*q^5 - a*q^6 + (-a - 2)*q^7 - 2*a*q^8 + q^9 + (-a
        - 2)*q^10 + 2*q^11 + (2*a - 3)*q^13 + (-2*a - 2)*q^14 + (a + 1)*q^15 -
        4*q^16 + (-a - 3)*q^17 + a*q^18 - 3*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (a^6 + a^5 - 8*a^4 - 6*a^3 + 14*a^2 + 3*a
        - 3)*q^5 - a*q^6 + (-a^6 - 2*a^5 + 8*a^4 + 12*a^3 - 14*a^2 - 5*a +
        4)*q^7 + (a^3 - 4*a)*q^8 + q^9 + (a^6 + 4*a^5 - 6*a^4 - 26*a^3 + 2*a^2 +
        21*a + 4)*q^10 + (-a^3 - a^2 + 6*a + 2)*q^11 + (-a^2 + 2)*q^12 + (-a^6 -
        3*a^5 + 5*a^4 + 20*a^3 + 4*a^2 - 18*a - 3)*q^13 + (-2*a^6 - 4*a^5 +
        12*a^4 + 26*a^3 - 4*a^2 - 20*a - 4)*q^14 + (-a^6 - a^5 + 8*a^4 + 6*a^3 -
        14*a^2 - 3*a + 3)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + (a^6 + 3*a^5 - 6*a^4 -
        20*a^3 + 2*a^2 + 17*a + 5)*q^17 + a*q^18 + (-a^5 + 7*a^3 - a^2 - 8*a +
        3)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^7 - 13*x^5 + 2*x^4 + 47*x^3 - 16*x^2
        - 43*x + 14 over the Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Number Field with defining polynomial x^7 - 12*x^5 + 40*x^3 + x^2 - 24*x - 4
    over the Rational Field
```

```
*]
[* 101, 101, 303, 303 *]
```

Over $\mathbb{Q}$ only $101a$: $n(101a;4) = 19 - 18$. It is not bielliptic.

genus 10, $X_0(303)/w_{101}$:

```
[*
    q - 2*q^3 - 2*q^4 - q^5 - 2*q^7 + q^9 - 2*q^11 + 4*q^12 + q^13 + 2*q^15 +
        4*q^16 + 3*q^17 - 5*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a - 1)*q^5 - a*q^6 + (-a - 2)*q^7 - 2*a*q^8 + q^9 + (-a
        - 2)*q^10 + 2*q^11 + (2*a - 3)*q^13 + (-2*a - 2)*q^14 + (a + 1)*q^15 -
        4*q^16 + (-a - 3)*q^17 + a*q^18 - 3*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (a^4 - a^3 - 5*a^2 + 3*a + 5)*q^5 + a*q^6
        + (-a^5 + a^4 + 5*a^3 - 5*a^2 - 3*a + 4)*q^7 + (a^3 - 4*a)*q^8 + q^9 +
        (a^5 - a^4 - 5*a^3 + 3*a^2 + 5*a)*q^10 + (-2*a^4 + a^3 + 11*a^2 - 4*a -
        8)*q^11 + (a^2 - 2)*q^12 + (2*a^5 - 2*a^4 - 11*a^3 + 9*a^2 + 10*a -
        5)*q^13 + (-2*a^4 + 10*a^2 - 6)*q^14 + (a^4 - a^3 - 5*a^2 + 3*a +
        5)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + (-a^4 + a^3 + 3*a^2 - 3*a + 3)*q^17 +
        a*q^18 + (-a^5 + 2*a^4 + 7*a^3 - 11*a^2 - 10*a + 7)*q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 - q^5 - 2*q^7 + q^9 - 2*q^11 + 4*q^12 + q^13 + 2*q^15 +
        4*q^16 + 3*q^17 - 5*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Number Field with defining polynomial x^6 - x^5 - 7*x^4 + 5*x^3 + 13*x^2 -
        4*x - 6 over the Rational Field,
    Rational Field
*]
[* 101, 303, 303, 303 *]
```

Over $\mathbb{Q}$, $101a, 101a$: $n(101a;4) = 26 - 18$. It is not bielliptic.

26. Magma code

```
L:=[* *];F:=[* *];Level:=[* *];  N:=129; Factorization(N);N;
Nd:=Divisors(N); NotAtkinLehnerfix:=3; for j in Nd do
MS:=NewformDecomposition(CuspidalSubspace(ModularSymbols(j,2,1)));
     m:=#MS;
     M:=PrimeDivisors(j);
     Nr:=Numerator(N/j);
     divi:=GCD(j,Nr);
     jj:=Numerator(j/divi);
     Mm:=PrimeDivisors(jj);
     mm:=#Mm;
     D:=Factorization(jj);
       for i in [1..m] do
          f:=Eigenform(MS[i],20);
          K:=Parent(Coefficient(f,3)); d:=Dimension(MS[i]);
          X:=IdentityMatrix(Rationals(), d);
          u:=0;
        for kk in  [1..mm] do
            if GCD(divi,D[kk][1]) eq 1 then
```

```
               if D[kk][1] ne NotAtkinLehnerfix then
               Y:=AtkinLehner(MS[i],(D[kk][1])^(D[kk][2]));
               if Y eq X then
               else
               u:=1;
               end if;
               end if;
            end if;
        end for;
        if u eq 0 then
        L:=Append(L,f);
        F:=Append(F,K);
        Level:=Append(Level,j);
              else
               end if;




     end for;
 end for;
 L;
 F; Level;
 felm:=# F;

p:=11;

C:=ComplexField(100); R<x>:=PolynomialRing(C); pj:=0*x+1; Roo:=[*
*]; for j in [1 .. felm] do
  if Degree(F[j]) eq 1 then
    cc:=Roots(x^2-Coefficient(L[j],p)*x+p,C);
    Roo:=Append(Roo,cc);
    pj:=pj*(x^2-Coefficient(L[j],p)*x+p);
  else
   dd:=Degree(F[j]);
   u:=Roots(DefiningPolynomial(F[j]),C); uu:= # u;
      for m in [1 .. uu] do
      f := hom< F[j] -> C | u[m][1]>;
      cc2:=Roots(x^2-f(Coefficient(L[j],p))*x+p,C);
      Roo:=Append(Roo,cc2);
      pj:=pj*(x^2-f(Coefficient(L[j],p))*x+p);
      end for;
  end if;
end for; pj; PR:=[* *];
 d2:=Degree(pj);
long:= # Roo; Roo[1][1][2];


for nn in [1 .. 20] do s:=0;
```

```
  for i in [1 .. long] do
    for j in [1..2] do
     if Roo[i][j][2] gt 0 then
s:=s+(Roo[i][j][2])*(Roo[i][j][1])^(nn) ;
  else
  s:=s;
  end if;
end for; end for;
 a:=Round(1+p^(nn)-s); PR:=Append(PR,a); end for; PR; N;
Jj:=[* *]; for aaa in [1..10] do
 ss:=0;
  adiv:=Divisors(aaa);
    for kk in adiv do
       vv:=aaa/kk;vv:=Numerator(vv);
      ss:=ss+(MoebiusMu(vv))*(PR[kk]);
     end for;

  vvv:=ss/aaa;
  Rr:=Integers(2); bb:=Rr!vvv;
  Jj:=Append(Jj,bb);
end for; Jj; jjel:=# Jj; ssum:=0; var:=0; for t in [1..jjel] do
  if Jj[t] eq 1 then
     tred:=Rr!t;
     if tred eq 1 then
     ssum:=ssum+t;
     var:=t;
     else
     ssum:=ssum;
     end if;
  else
     ssum:=ssum;
  end if;
end for; var; ssum;p;

PR;

PR2:=[* *]; a3:=3; cearrels:=Roots(x^2-a3*x+p,C);

for i in [1..20] do

b:=2*(p^i+1-Round(cearrels[1][1]^i+ p^i/cearrels[1][1]^i));
PR2:=Append(PR2,b); end for; PR2;
```

27. 33

genus 2, $X_0(33)/w_3$

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q + q^2 - q^3 - q^4 - 2*q^5 - q^6 + 4*q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 +
```

```
        q^12 - 2*q^13 + 4*q^14 + 2*q^15 - q^16 - 2*q^17 + q^18 + O(q^20)
*]
[*
    Rational Field,
    Rational Field
*]
```

The jacobian is $11a, 33a$.

28. 35

genus 2, $X_0(35)/w_7$:

```
[*
    q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
        (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
        a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
        - 2)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial $.1^2 + $.1 - 4 over the Rational
    Field
*]
[* 35 *]
```

29. 38

genus 2, $X_0(38)/w_2$

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 - q^7 - q^8 - 2*q^9 - 6*q^11 + q^12 + 5*q^13 +
        q^14 + q^16 + 3*q^17 + 2*q^18 + q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field
*]
[* 19, 38 *]
```

The Jacobian is $19a, 38a$.

30. 39

genus 2, $X_0(39)/w_{13}$:

```
[*
    q + a*q^2 + q^3 + (-2*a - 1)*q^4 + (-2*a - 2)*q^5 + a*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + q^9 + (2*a - 2)*q^10 - 2*q^11 + (-2*a - 1)*q^12 - q^13 +
        (-2*a + 2)*q^14 + (-2*a - 2)*q^15 + 3*q^16 + (4*a + 6)*q^17 + a*q^18 +
        (-2*a - 2)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field
```

```
*]
[* 39 *]
```

31. 46

genus 3, $X_0(46)/w_2$:

```
[*
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q - q^2 + q^4 + 4*q^5 - 4*q^7 - q^8 - 3*q^9 - 4*q^10 + 2*q^11 - 2*q^13 +
        4*q^14 + q^16 - 2*q^17 + 3*q^18 - 2*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field
*]
[* 23, 46 *]
```

32. 51

genus 3, $X_0(51)/w_3$:

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a + 2)*q^4 + (-a + 1)*q^5 - a*q^6 + (a - 4)*q^8 + q^9 +
        (2*a - 4)*q^10 + (-a - 1)*q^11 + (a - 2)*q^12 + (a + 3)*q^13 + (a -
        1)*q^15 - 3*a*q^16 + q^17 + a*q^18 + (3*a + 3)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial $.1^2 + $.1 - 4 over the Rational
    Field
*]
[* 17, 51 *]
```

33. 55

genus 3, $X_0(55)/w_5$:

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q + a*q^2 + (-2*a + 2)*q^3 + (2*a - 1)*q^4 - q^5 + (-2*a - 2)*q^6 - 2*q^7 +
        (a + 2)*q^8 + 5*q^9 - a*q^10 + q^11 + (-2*a - 6)*q^12 + (2*a - 6)*q^13 -
        2*a*q^14 + (2*a - 2)*q^15 + 3*q^16 + (2*a + 2)*q^17 + 5*a*q^18 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial $.1^2 - 2*$.1 - 1 over the Rational
    Field
*]
```

```
[* 11, 55 *]
```

34. 62

genus 4, $X_0(62)/w_2$:

```
[*
    q + a*q^2 - 2*a*q^3 + (a - 1)*q^4 + q^5 + (-2*a - 2)*q^6 + (2*a - 3)*q^7 +
        (-2*a + 1)*q^8 + (4*a + 1)*q^9 + a*q^10 + 2*q^11 - 2*q^12 - 2*a*q^13 +
        (-a + 2)*q^14 - 2*a*q^15 - 3*a*q^16 + (-2*a + 4)*q^17 + (5*a + 4)*q^18 +
        (-2*a + 1)*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + (-2*a + 2)*q^5 - a*q^6 + 2*q^7 - q^8 + (2*a - 1)*q^9
        + (2*a - 2)*q^10 + (a - 4)*q^11 + a*q^12 + (-3*a + 2)*q^13 - 2*q^14 +
        (-2*a - 4)*q^15 + q^16 + (2*a - 2)*q^17 + (-2*a + 1)*q^18 - 4*q^19 +
        O(q^20)
*]
[*
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 2 over the Rational Field
*]
[* 31, 62 *]
```

35. 69

genus 4, $X_0(69)/w_3$:

```
[*
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + 3*q^4 + (-a - 1)*q^5 - a*q^6 + (-a + 1)*q^7 + a*q^8 + q^9
        + (-a - 5)*q^10 + 4*q^11 - 3*q^12 + 2*a*q^13 + (a - 5)*q^14 + (a +
        1)*q^15 - q^16 + (-a - 5)*q^17 + a*q^18 + (-a + 5)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field
*]
[* 23, 69 *]
```

36. 87

genus 5, $X_0(87)/w_3$:

```
[*
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 +
        (2*a - 2)*q^18 + 6*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-2*a^2 + 8)*q^5 - a*q^6 + (a^2 - a -
        2)*q^7 + (2*a^2 - 7)*q^8 + q^9 + (-4*a^2 + 14)*q^10 + (a^2 - a - 6)*q^11
        + (-a^2 + 2)*q^12 + (-a^2 - a + 6)*q^13 + (a^2 + 2*a - 7)*q^14 + (2*a^2
        - 8)*q^15 + (2*a^2 + a - 10)*q^16 + (3*a^2 - a - 10)*q^17 + a*q^18 +
        (2*a - 2)*q^19 + O(q^20)
```

```
*]
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 2*x^2 - 4*x + 7 over the
    Rational Field
*]
[* 29, 87 *]
```

genus 2, $X_0(87)/w_{29}$:

```
[*
    q + a*q^2 + q^3 + (a - 1)*q^4 + (-2*a + 2)*q^5 + a*q^6 + (-2*a - 1)*q^7 +
        (-2*a + 1)*q^8 + q^9 - 2*q^10 + (2*a + 1)*q^11 + (a - 1)*q^12 + (4*a -
        3)*q^13 + (-3*a - 2)*q^14 + (-2*a + 2)*q^15 - 3*a*q^16 + 3*q^17 + a*q^18
        + (2*a - 6)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field
*]
[* 87 *]
```

37. 94

genus 6, $X_0(94)/w_2$:

```
[*
    q + a*q^2 + (a^3 - a^2 - 6*a + 4)*q^3 + (a^2 - 2)*q^4 + (-4*a^3 + 2*a^2 +
        20*a - 10)*q^5 + (-a^2 - a + 1)*q^6 + (3*a^3 - a^2 - 16*a + 7)*q^7 +
        (a^3 - 4*a)*q^8 + (3*a^3 - a^2 - 14*a + 6)*q^9 + (-2*a^3 + 10*a -
        4)*q^10 + (2*a^3 - 2*a^2 - 10*a + 6)*q^11 + (-3*a^3 + a^2 + 13*a -
        8)*q^12 + (-4*a^3 + 2*a^2 + 22*a - 8)*q^13 + (2*a^3 - a^2 - 8*a +
        3)*q^14 + (-4*a^3 + 4*a^2 + 22*a - 16)*q^15 + (a^3 - a^2 - 5*a + 5)*q^16
        + (a^3 + a^2 - 6*a)*q^17 + (2*a^3 + a^2 - 9*a + 3)*q^18 + (-2*a^3 + 10*a
        - 2)*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + 1/2*(-a + 4)*q^5 - a*q^6 + (-a - 2)*q^7 - q^8 +
        5*q^9 + 1/2*(a - 4)*q^10 + 1/2*(-a + 8)*q^11 + a*q^12 + 1/2*(-a -
        4)*q^13 + (a + 2)*q^14 + (2*a - 4)*q^15 + q^16 - 5*q^18 + 1/2*(3*a -
        8)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^4 - x^3 - 5*x^2 + 5*x - 1 over the
    Rational Field,
    Number Field with defining polynomial x^2 - 8 over the Rational Field
*]
```

It is not bielliptic.

38. 95

genus 5, $X_0(95)/w_5$:

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q + a*q^2 + (-a^3 + 5*a - 2)*q^3 + (a^2 - 2)*q^4 - q^5 + (2*a^3 - a^2 - 10*a
```

```
        + 9)*q^6 + (-2*a^2 - 2*a + 8)*q^7 + (a^3 - 4*a)*q^8 + (-2*a + 1)*q^9 -
        a*q^10 + (2*a^2 + 2*a - 6)*q^11 + (-3*a^3 + 2*a^2 + 15*a - 14)*q^12 +
        (a^3 + 2*a^2 - 3*a - 4)*q^13 + (-2*a^3 - 2*a^2 + 8*a)*q^14 + (a^3 - 5*a
        + 2)*q^15 + (-2*a^3 + 8*a - 5)*q^16 + (2*a^3 - 10*a + 6)*q^17 + (-2*a^2
        + a)*q^18 + q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial $.1^4 + 2*$.1^3 - 6*$.1^2 - 8*$.1 + 9
    over the Rational Field
*]
[* 19, 95 *]
```

OVer $\mathbb{Q}$, $19a$: $n(19a; 16) = 21 - 18$. It is not over $\mathbb{Q}$. Over $\overline{\mathbb{Q}}$???

genus 3, $X_0(95)/w_{19}$:

```
[*
    q + a*q^2 + (-a^2 + 3)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^2 + 1)*q^6 + (2*a^2 -
        2*a - 4)*q^7 + (a^2 - a - 1)*q^8 + (-2*a^2 + 2*a + 5)*q^9 + a*q^10 +
        (-2*a - 2)*q^11 + (a^2 - 2*a - 5)*q^12 + (a^2 - 2*a + 1)*q^13 + (2*a -
        2)*q^14 + (-a^2 + 3)*q^15 + (-2*a^2 + 2*a + 3)*q^16 + (-2*a^2 + 4*a +
        4)*q^17 + (-a + 2)*q^18 - q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial $.1^3 - $.1^2 - 3*$.1 + 1 over the
    Rational Field
*]
[* 95 *]
```

39. 119

genus 6, $X_0(119)/w_7$:

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + (-a^4 + 6*a^2 + a - 4)*q^3 + (a^2 - 2)*q^4 + (2*a^4 + a^3 -
        15*a^2 - 6*a + 18)*q^5 + (-2*a^4 - 2*a^3 + 15*a^2 + 10*a - 17)*q^6 - q^7
        + (a^3 - 4*a)*q^8 + (2*a^4 + a^3 - 13*a^2 - 8*a + 13)*q^9 + (5*a^4 + a^3
        - 34*a^2 - 10*a + 34)*q^10 + (-2*a^4 - 2*a^3 + 14*a^2 + 12*a - 14)*q^11
        + (-4*a^4 - a^3 + 26*a^2 + 9*a - 26)*q^12 + (-2*a^4 + 14*a^2 - 14)*q^13
        - a*q^14 + (-a^4 - a^3 + 7*a^2 + 3*a - 4)*q^15 + (a^4 - 6*a^2 + 4)*q^16
        + q^17 + (5*a^4 + 3*a^3 - 36*a^2 - 15*a + 34)*q^18 + (-2*a^4 + 14*a^2 +
        2*a - 14)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial $.1^5 - 2*$.1^4 - 8*$.1^3 + 14*$.1^2 +
        14*$.1 - 17 over the Rational Field
*]
[* 17, 119 *]
```

Only $17a$ over $\mathbb{Q}$: $n(17a; 7) = 9 - 8$. It is not bielliptic.

genus 4, $X_0(119)/w_{17}$:

```
[*
    q + a*q^2 + (-a^3 - a^2 + 4*a + 1)*q^3 + (a^2 - 2)*q^4 + (a^3 + a^2 -
        4*a)*q^5 + (-a^2 + 3)*q^6 + q^7 + (a^3 - 4*a)*q^8 + (-a^3 - 3*a^2 + 2*a
        + 7)*q^9 + (a^2 + a - 3)*q^10 - 2*a*q^11 + (a^3 + 2*a^2 - 5*a - 2)*q^12
        + (2*a^3 + 4*a^2 - 6*a - 4)*q^13 + a*q^14 + (2*a^2 + 2*a - 9)*q^15 +
        (-a^3 - a^2 + a + 1)*q^16 - q^17 + (-2*a^3 - 3*a^2 + 6*a + 3)*q^18 +
        (-2*a^3 - 4*a^2 + 4*a + 8)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial $.1^4 + $.1^3 - 5*$.1^2 - $.1 + 3 over
    the Rational Field
*]
[* 119 *]
```

40. Magma programme to deal Jacobian $X_0(N)/ < w_d, w_{d_2}, .. >$

```
L:=[* *];F:=[* *];Level:=[* *];  N:=370; Factorization(N);N;
Nd:=Divisors(N); AtkinLehnerfix:=[*2,5,10*];
Involutions:=#AtkinLehnerfix;

for j in Nd do
MS:=NewformDecomposition(CuspidalSubspace(ModularSymbols(j,2,1)));
     m:=#MS;
     M:=PrimeDivisors(j);
     Nr:=Numerator(N/j);
     divi:=GCD(j,Nr);
     jj:=Numerator(j/divi);
     Mm:=PrimeDivisors(jj);
     Nn:=Divisors(jj);
     mm:=#Mm;
     mn:=#Nn;
     D:=Factorization(jj);
       for i in [1..m] do
          f:=Eigenform(MS[i],20);
          K:=Parent(Coefficient(f,3)); d:=Dimension(MS[i]);
          X:=IdentityMatrix(Rationals(), d);
          u:=0;
        for kk in  [1..mn] do
            if GCD(divi,Nn[kk]) eq 1 then
              for jo in [1..Involutions] do
                dd:=GCD(Nn[kk],AtkinLehnerfix[jo]);
               if dd ne 1 then
               Y:=AtkinLehner(MS[i],dd);
                 if Y eq X then
                 else
                 u:=1;
                 end if;
               end if;
             end for;
           end if;
```

```
        end for;
        if u eq 0 then
        L:=Append(L,f);
        F:=Append(F,K);
        Level:=Append(Level,j);
              else
               end if;




     end for;
 end for;
 L;
 F; Level;
 felm:=# F;

p:=11;

C:=ComplexField(100); R<x>:=PolynomialRing(C); pj:=0*x+1; Roo:=[*
*]; for j in [1 .. felm] do
  if Degree(F[j]) eq 1 then
    cc:=Roots(x^2-Coefficient(L[j],p)*x+p,C);
    Roo:=Append(Roo,cc);
    pj:=pj*(x^2-Coefficient(L[j],p)*x+p);
  else
   dd:=Degree(F[j]);
   u:=Roots(DefiningPolynomial(F[j]),C); uu:= # u;
      for m in [1 .. uu] do
      f := hom< F[j] -> C | u[m][1]>;
      cc2:=Roots(x^2-f(Coefficient(L[j],p))*x+p,C);
      Roo:=Append(Roo,cc2);
      pj:=pj*(x^2-f(Coefficient(L[j],p))*x+p);
      end for;
  end if;
end for; pj; PR:=[* *];
 d2:=Degree(pj);
long:= # Roo; Roo[1][1][2];


for nn in [1 .. 20] do s:=0;
  for i in [1 .. long] do
    for j in [1..2] do
     if Roo[i][j][2] gt 0 then
s:=s+(Roo[i][j][2])*(Roo[i][j][1])^(nn) ;
  else
  s:=s;
  end if;
end for; end for;
```

```
 a:=Round(1+p^(nn)-s); PR:=Append(PR,a); end for; PR; N;
Jj:=[* *]; for aaa in [1..10] do
 ss:=0;
  adiv:=Divisors(aaa);
    for kk in adiv do
       vv:=aaa/kk;vv:=Numerator(vv);
      ss:=ss+(MoebiusMu(vv))*(PR[kk]);
     end for;

  vvv:=ss/aaa;
  Rr:=Integers(2); bb:=Rr!vvv;
  Jj:=Append(Jj,bb);
end for; Jj; jjel:=# Jj; ssum:=0; var:=0; for t in [1..jjel] do
  if Jj[t] eq 1 then
     tred:=Rr!t;
     if tred eq 1 then
     ssum:=ssum+t;
     var:=t;
     else
     ssum:=ssum;
     end if;
  else
     ssum:=ssum;
  end if;
end for; var; ssum;p;

PR;

PR2:=[* *]; a3:=3; cearrels:=Roots(x^2-a3*x+p,C);

for i in [1..20] do

b:=2*(p^i+1-Round(cearrels[1][1]^i+ p^i/cearrels[1][1]^i));
PR2:=Append(PR2,b); end for; PR2;
```

41. Magma programme

Need to modify, for the previous one, to think..

```
L:=[* *];F:=[* *];Level:=[* *];  N:=370; Factorization(N);N;
Nd:=Divisors(N); AtkinLehnerfix:=[*10,74,185*];
Involutions:=#AtkinLehnerfix;

for j in Nd do
MS:=NewformDecomposition(CuspidalSubspace(ModularSymbols(j,2,1)));
     m:=#MS;
     M:=PrimeDivisors(j);
     Nr:=Numerator(N/j);
     divi:=GCD(j,Nr);
     jj:=Numerator(j/divi);
     Mm:=PrimeDivisors(jj);
```

```
     Nn:=Divisors(jj);
     mm:=#Mm;
     mn:=#Nn;
     D:=Factorization(jj);
       for i in [1..m] do
          f:=Eigenform(MS[i],20);
          K:=Parent(Coefficient(f,3)); d:=Dimension(MS[i]);
          X:=IdentityMatrix(Rationals(), d);
          u:=0;

            if GCD(divi,j) eq 1 then
              for jo in [1..Involutions] do
                dd:=GCD(j,AtkinLehnerfix[jo]);
               if dd eq AtkinLehnerfix[jo] then
                 Y:=AtkinLehner(MS[i],dd);
                  if Y eq X then
                  else
                  u:=1;
                  end if;
               else
                  if dd eq 1 then
                  else
                  end if;
               end if;
             end for;
            end if;

         if u eq 0 then
         L:=Append(L,f);
         F:=Append(F,K);
         Level:=Append(Level,j);
               else
                end if;




     end for;
 end for;
 L;
 F; Level;
 felm:=# F;

p:=3;

C:=ComplexField(100); R<x>:=PolynomialRing(C); pj:=0*x+1; Roo:=[*
*]; for j in [1 .. felm] do
  if Degree(F[j]) eq 1 then
    cc:=Roots(x^2-Coefficient(L[j],p)*x+p,C);
```

```
    Roo:=Append(Roo,cc);
    pj:=pj*(x^2-Coefficient(L[j],p)*x+p);
  else
   dd:=Degree(F[j]);
   u:=Roots(DefiningPolynomial(F[j]),C); uu:= # u;
      for m in [1 .. uu] do
      f := hom< F[j] -> C | u[m][1]>;
      cc2:=Roots(x^2-f(Coefficient(L[j],p))*x+p,C);
      Roo:=Append(Roo,cc2);
      pj:=pj*(x^2-f(Coefficient(L[j],p))*x+p);
      end for;
  end if;
end for; pj; PR:=[* *];
 d2:=Degree(pj);
long:= # Roo; Roo[1][1][2];


for nn in [1 .. 20] do s:=0;
  for i in [1 .. long] do
    for j in [1..2] do
     if Roo[i][j][2] gt 0 then
s:=s+(Roo[i][j][2])*(Roo[i][j][1])^(nn) ;
  else
  s:=s;
  end if;
end for; end for;
 a:=Round(1+p^(nn)-s); PR:=Append(PR,a); end for; PR; N;
Jj:=[* *]; for aaa in [1..10] do
 ss:=0;
  adiv:=Divisors(aaa);
    for kk in adiv do
       vv:=aaa/kk;vv:=Numerator(vv);
      ss:=ss+(MoebiusMu(vv))*(PR[kk]);
     end for;

  vvv:=ss/aaa;
  Rr:=Integers(2); bb:=Rr!vvv;
  Jj:=Append(Jj,bb);
end for; Jj; jjel:=# Jj; ssum:=0; var:=0; for t in [1..jjel] do
  if Jj[t] eq 1 then
     tred:=Rr!t;
     if tred eq 1 then
     ssum:=ssum+t;
     var:=t;
     else
     ssum:=ssum;
     end if;
  else
     ssum:=ssum;
```

```
  end if;
end for; var; ssum;p;

PR;

PR2:=[* *]; a3:=-3; cearrels:=Roots(x^2-a3*x+p,C);

for i in [1..20] do

b:=2*(p^i+1-Round(cearrels[1][1]^i+ p^i/cearrels[1][1]^i));
PR2:=Append(PR2,b); end for; PR2;
```

42. N=57=3*19

42.1. $X_0(57)/ < w_3 >$**, genus 2.** [* $q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 + 4*q^16 - 3*q^17 + q^19 + O(q^20), q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 - 2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 + O(q^20)$ *] [* Rational Field, Rational Field *] [* 19, 57 *] 19a, 57a

42.2. $X_0(57)/ < w_{19} >$**, genus 3.** [* $q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 - 2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 + O(q^20), q + q^2 + q^3 - q^4 - 2*q^5 + q^6 - 3*q^8 + q^9 - 2*q^10 - q^12 + 6*q^13 - 2*q^15 - q^16 - 6*q^17 + q^18 - q^19 + O(q^20), q - 2*q^2 + q^3 + 2*q^4 + q^5 - 2*q^6 + 3*q^7 + q^9 - 2*q^10 - 3*q^11 + 2*q^12 - 6*q^13 - 6*q^14 + q^15 - 4*q^16 + 3*q^17 - 2*q^18 - q^19 + O(q^20)$ *] [* Rational Field, Rational Field, Rational Field *] [* 57, 57, 57 *] 57a,57b,57c, $n(57b; 2) = 6 - 4$ (need equation, from Petri)

43. N=58

43.1. $X_0(58)/w_2$**,genus 3.** [* $q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 + (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 + (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 + (2*a - 2)*q^18 + 6*q^19 + O(q^20), q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11 - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 + O(q^20)$ *] [* Number Field with defining polynomial $.1^2 + 2*.1$ - 1 over the Rational Field, Rational Field *] [* 29, 58 *], 58a, rk=1.

43.2. $X_0(58)/w_{29}$**,genus 2.** Not necessarely for quadratic points, but two bielliptic quotients. That should be 58a and 58b.

44. N=65

44.1. $X_0(65)/w_5$**,genus 3.** [* $q-q^2-2*q^3-q^4-q^5+2*q^6-4*q^7+3*q^8+q^9+q^10+2*q^11+2*q^12-q^13+4*q^14+2*q^15-q^16+2*q^17-q^18-6*q^19+O(q^20), q+a*q^2+(-a+1)*q^3+q^4-q^5+(a-3)*q^6+2*q^7-a*q^8+(-2*a+1)*q^9-a*q^10+(a-3)*q^11+(-a+1)*q^12+q^13+2*a*q^14+(a-1)*q^15-5*q^16+2*a*q^17+(a-6)*q^18+(3*a-1)*q^19+O(q^20)$ *] [* Rational Field, Number Field with defining polynomial $.1^2 - 3$ over the Rational Field *] [* 65, 65 *], 65a, rk=1.

44.2. $X_0(65)/w_{13}$**,genus 3.** [* $q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 + 2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + O(q^20), q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a - 2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a + 3)*q^19 + O(q^20)$ *] [* Rational Field, Number Field with defining polynomial $.1^2 + 2*.1$ - 1 over the Rational Field *] [* 65, 65 *] 65a, rk=1.

45. N=74

45.1. $X_0(74)/w_2$**, genus 4.** [* $q-2*q^2-3*q^3+2*q^4-2*q^5+6*q^6-q^7+6*q^9+4*q^10-5*q^11-6*q^12-2*q^13+2*q^14+6*q^15-4*q^16-12*q^18+O(q^20), q+q^3-2*q^4-q^7-2*q^9+3*q^11-2*q^12-4*q^13+4*q^16+6*q^17+2*q^19+O(q^20), q-q^2+a*q^3+q^4+(-a+1)*q^5-a*q^6+(-2*a+4)*q^7-q^8+(3*a-2)*q^9+(a-1)*q^10+(-a+1)*q^11+a*q^12+(a-2)*q^13+(2*a-4)*q^14+(-2*a-1)*q^15+q^16-6*q^17+(-3*a+2)*q^18+2*q^19+O(q^20)$ *] [* Rational Field, Rational Field, Number Field with defining polynomial $.1^2 - 3*.1 - 1$ over the Rational Field *] [* 37, 37, 74 *]37a,37a, rk=1,rk=0, Need equation Petri.

45.2. $X_0(74)/w_{37}$**,genus 4.** [* $q - 2 * q^2 - 3 * q^3 + 2 * q^4 - 2 * q^5 + 6 * q^6 - q^7 + 6 * q^9 + 4 * q^10 - 5 * q^11 - 6 * q^12 - 2 * q^13 + 2 * q^14 + 6 * q^15 - 4 * q^16 - 12 * q^18 + O(q^20), q + q^2 + a * q^3 + q^4 + (-3 * a - 1) * q^5 + a * q^6 + 2 * a * q^7 + q^8 + (-a - 2) * q^9 + (-3 * a - 1) * q^10 + (-a - 3) * q^11 + a * q^12 + (3 * a + 2) * q^13 + 2 * a * q^14 + (2 * a - 3) * q^15 + q^16 + (4 * a + 2) * q^17 + (-a - 2) * q^18 + (-4 * a - 2) * q^19 + O(q^20), q - 2 * q^2 - 3 * q^3 + 2 * q^4 - 2 * q^5 + 6 * q^6 - q^7 + 6 * q^9 + 4 * q^10 - 5 * q^11 - 6 * q^12 - 2 * q^13 + 2 * q^14 + 6 * q^15 - 4 * q^16 - 12 * q^18 + O(q^20)$ *] [* Rational Field, Number Field with defining polynomial $.1^2+.1 - 1$ over the Rational Field, Rational Field *] [* 37, 74, 74 *] 37a repetida, rk=1.

46. N=77

46.1. $X_0(77)/w_7$**,genus 3.** [* $q - 2 * q^2 - q^3 + 2 * q^4 + q^5 + 2 * q^6 - 2 * q^7 - 2 * q^9 - 2 * q^10 + q^11 - 2 * q^12 + 4 * q^13 + 4 * q^14 - q^15 - 4 * q^16 - 2 * q^17 + 4 * q^18 + O(q^20), q - 3 * q^3 - 2 * q^4 - q^5 - q^7 + 6 * q^9 - q^11 + 6 * q^12 - 4 * q^13 + 3 * q^15 + 4 * q^16 + 2 * q^17 - 6 * q^19 + O(q^20), q + q^2 + 2 * q^3 - q^4 - 2 * q^5 + 2 * q^6 - q^7 - 3 * q^8 + q^9 - 2 * q^10 + q^11 - 2 * q^12 + 4 * q^13 - q^14 - 4 * q^15 - q^16 + 4 * q^17 + q^18 + O(q^20)$ *] [* Rational Field, Rational Field, Rational Field *] [* 11, 77, 77 *] 11a (rk=0),77a(rk=1),77c(rk=0).$n(11a, 4) = 12 - 10$,$n(77c, 3) = 6 - 4$. Thus to 77a.!!!

46.2. $X_0(77)/w_{11}$**, genus 4.** [* $q-3*q^3-2*q^4-q^5-q^7+6*q^9-q^11+6*q^12-4*q^13+3*q^15+4*q^16+2*q^17-6*q^19+O(q^20), q+q^3-2*q^4+3*q^5+q^7-2*q^9-q^11-2*q^12-4*q^13+3*q^15+4*q^16-6*q^17+2*q^19+O(q^20), q+a*q^2+(-a+1)*q^3+3*q^4-2*q^5+(a-5)*q^6+q^7+a*q^8+(-2*a+3)*q^9-2*a*q^10-q^11+(-3*a+3)*q^12+(a+1)*q^13+a*q^14+(2*a-2)*q^15-q^16+(-a-1)*q^17+(3*a-10)*q^18+(2*a+2)*q^19+O(q^20)$ *] [* Rational Field, Rational Field, Number Field with defining polynomial $.1^2 - 5$ over the Rational Field *] [* 77, 77, 77 *] 77a(rk=1),77b(rk=0) $n(77b, 3) = 4 - 2$, only 77a.

47. N=82

47.1. $X_0(82)/w_2$**, genus 4.** [* $q + a * q^2 + 1/2 * (-a^2 - 2 * a + 3) * q^3 + (a^2 - 2) * q^4 + (-a - 1) * q^5 + 1/2 * (-a^2 - 2 * a - 1) * q^6 + 1/2 * (a^2 + 2 * a + 1) * q^7 + (-a^2 + a + 1) * q^8 + a * q^9 + (-a^2 - a) * q^10 + 1/2 * (3 * a^2 + 2 * a - 9) * q^11 + 1/2 * (a^2 - 2 * a - 7) * q^12 + (-a^2 + 3) * q^13 + 1/2 * (a^2 + 6 * a + 1) * q^14 + (a^2 + 2 * a - 1) * q^15 + (-4 * a + 3) * q^16 - 2 * q^17 + a^2 * q^18 + 1/2 * (-3 * a^2 - 2 * a + 13) * q^19 + O(q^20), q - q^2 - 2 * q^3 + q^4 - 2 * q^5 + 2 * q^6 - 4 * q^7 - q^8 + q^9 + 2 * q^10 - 2 * q^11 - 2 * q^12 + 4 * q^13 + 4 * q^14 + 4 * q^15 + q^16 - 2 * q^17 - q^18 + 6 * q^19 + O(q^20)$ *] [* Number Field with defining polynomial $.1^3 + .1^2 - 5 * .1 - 1$ over the Rational Field, Rational Field *] [* 41, 82 *]82a (rk=1).

47.2. $X_0(82)/w_{41}$**, genus 3.** [* $q-q^2-2*q^3+q^4-2*q^5+2*q^6-4*q^7-q^8+q^9+2*q^10-2*q^11-2*q^12+4*q^13+4*q^14+4*q^15+q^16-2*q^17-q^18+6*q^19+O(q^20), q+q^2+a*q^3+q^4-2*a*q^5+a*q^6+(-a-2)*q^7+q^8-q^9-2*a*q^10+3*a*q^11+a*q^12+(-a-2)*q^14-4*q^15+q^16+(4*a+2)*q^17-q^18+(-a-4)*q^19+O(q^20)$ *] [* Rational Field, Number Field with defining polynomial $.1^2 - 2$ over the Rational Field *] [* 82, 82 *] 82a, rk=1.

48. 86

48.1. $X_0(86)/w_2$**, genus 5.** [* $q-2*q^2-2*q^3+2*q^4-4*q^5+4*q^6+q^9+8*q^10+3*q^11-4*q^12-5*q^13+8*q^15-4*q^16-3*q^17-2*q^18-2*q^19+O(q^20), q+a*q^2-a*q^3+(-a+2)*q^5-2*q^6+(a-2)*q^7-2*a*q^8-q^9+(2*a-2)*q^10+(2*a-1)*q^11+(2*a+1)*q^13+(-2*a+2)*q^14+(-2*a+2)*q^15-4*q^16+(2*a+5)*$

$q^17-a*q^18+(-2*a-2)*q^19+O(q^20), q-q^2+a*q^3+q^4+(-a+1)*q^5-a*q^6+2*q^7-q^8+(-a+2)*q^9+(a-1)*q^10+a*q^12+2*q^13-2*q^14+(2*a-5)*q^15+q^16+(a-4)*q^17+(a-2)*q^18+(-3*a-1)*q^19+O(q^20)$ *] [* Rational Field, Number Field with defining polynomial $.1^2-2$ over the Rational Field, Number Field with defining polynomial $.1^2+.1$ - 5 over the Rational Field *] [* 43, 43, 86 *] 43a (rk=1).

48.2. $X_0(86)/w_{43}$**,genus 4.** [* $q-2*q^2-2*q^3+2*q^4-4*q^5+4*q^6+q^9+8*q^10+3*q^11-4*q^12-5*q^13+8*q^15-4*q^16-3*q^17-2*q^18-2*q^19+O(q^20), q+q^2+a*q^3+q^4+(-a-1)*q^5+a*q^6+(-4*a+2)*q^7+q^8+(a-2)*q^9+(-a-1)*q^10+(4*a-4)*q^11+a*q^12+(4*a-2)*q^13+(-4*a+2)*q^14+(-2*a-1)*q^15+q^16-a*q^17+(a-2)*q^18+(a+5)*q^19+O(q^20), q-2*q^2-2*q^3+2*q^4-4*q^5+4*q^6+q^9+8*q^10+3*q^11-4*q^12-5*q^13+8*q^15-4*q^16-3*q^17-2*q^18-2*q^19+O(q^20)$ *] [* Rational Field, Number Field with defining polynomial $.1^2-.1$ - 1 over the Rational Field, Rational Field *] [* 43, 86, 86 *] 43a, repetida (rk=1),

49. $N = 91$

49.1. $X_0(91)/w_7$**,genus 4.** [* $q-2*q^2+2*q^4-3*q^5-q^7-3*q^9+6*q^10-6*q^11-q^13+2*q^14-4*q^16+4*q^17+6*q^18+5*q^19+O(q^20), q+a*q^2+(-a^2+a+2)*q^3+(a^2-2)*q^4+(-a+1)*q^5+(-2*a+2)*q^6-q^7+(a^2-2)*q^8+(-2*a+3)*q^9+(-a^2+a)*q^10+(a^2-a-2)*q^11-4*q^12+q^13-a*q^14+(-a^2+3*a)*q^15+(-a^2+2*a+2)*q^16+(a^2+a-2)*q^17+(-2*a^2+3*a)*q^18+(-a-1)*q^19+O(q^20)$ *] [* Rational Field, Number Field with defining polynomial $.1^3-.1^2-4*.1+2$ over the Rational Field *] [* 91, 91 *] 91a (rk=1)

49.2. $X_0(91)/w_{13}$**,genus 3.** [* $q-2*q^2+2*q^4-3*q^5-q^7-3*q^9+6*q^10-6*q^11-q^13+2*q^14-4*q^16+4*q^17+6*q^18+5*q^19+O(q^20), q+a*q^2-a*q^3+(a+3)*q^5-2*q^6+q^7-2*a*q^8-q^9+(3*a+2)*q^10-3*a*q^11-q^13+a*q^14+(-3*a-2)*q^15-4*q^16-a*q^17-a*q^18+(3*a-3)*q^19+O(q^20)$ *] [* Rational Field, Number Field with defining polynomial $.1^2-2$ over the Rational Field *] [* 91, 91 *] 91a (rk=1).

50. 111

50.1. $X_0(111)/w_3$**,genus 5.** [* $q-2*q^2-3*q^3+2*q^4-2*q^5+6*q^6-q^7+6*q^9+4*q^10-5*q^11-6*q^12-2*q^13+2*q^14+6*q^15-4*q^16-12*q^18+O(q^20), q+q^3-2*q^4-q^7-2*q^9+3*q^11-2*q^12-4*q^13+4*q^16+6*q^17+2*q^19+O(q^20), q+a*q^2-q^3+(a^2-2)*q^4+(-a^2+5)*q^5-a*q^6+(-2*a^2+2*a+4)*q^7+(3*a^2-3*a-5)*q^8+q^9+(-3*a^2+4*a+5)*q^10+(2*a^2-4*a-2)*q^11+(-a^2+2)*q^12+(2*a^2-4*a-4)*q^13+(-4*a^2+2*a+10)*q^14+(a^2-5)*q^15+(4*a^2-2*a-11)*q^16+(-a^2+4*a+1)*q^17+a*q^18+(2*a^2-2*a-8)*q^19+O(q^20)$ *] [* Rational Field, Rational Field, Number Field with defining polynomial $.1^3-3*.1^2-.1+5$ over the Rational Field *] [* 37, 37, 111 *] 37a,(rk=1),37b(rk=0).$n(37b;16) = 22-18$. Sol 37a.

50.2. $X_0(111)/w_{37}$**,genus 6.** [* $q-2*q^2-3*q^3+2*q^4-2*q^5+6*q^6-q^7+6*q^9+4*q^10-5*q^11-6*q^12-2*q^13+2*q^14+6*q^15-4*q^16-12*q^18+O(q^20), q+a*q^2+q^3+(a^2-2)*q^4+(-a^3-2*a^2+3*a+4)*q^5+a*q^6+(2*a^3+2*a^2-8*a-2)*q^7+(a^3-4*a)*q^8+q^9+(-2*a^3-3*a^2+6*a+5)*q^10+(2*a^2-6)*q^11+(a^2-2)*q^12+(-2*a^3-4*a^2+6*a+10)*q^13+(2*a^3+4*a^2-6*a-10)*q^14+(-a^3-2*a^2+3*a+4)*q^15+(-2*a-1)*q^16+(-a^3+3*a-2)*q^17+a*q^18+(2*a^2+2*a-4)*q^19+O(q^20), q-2*q^2-3*q^3+2*q^4-2*q^5+6*q^6-q^7+6*q^9+4*q^10-5*q^11-6*q^12-2*q^13+2*q^14+6*q^15-4*q^16-12*q^18+O(q^20)$ *] [* Rational Field, Number Field with defining polynomial $.1^4-6*.1^2+2*.1+5$ over the Rational Field, Rational Field *] [* 37, 111, 111 *] 37a (repetida), rk=1

51. $N = 118$

51.1. $X_0(118)/w_2$**,genus 7.** [* $q+a*q^2+1/4*(-a^4+5*a^2-2*a)*q^3+(a^2-2)*q^4+1/4*(3*a^4+2*a^3-23*a^2-12*a+28)*q^5+(-a^3+4*a-2)*q^6+1/2*(-a^4-a^3+7*a^2+3*a-6)*q^7+(a^3-4*a)*q^8+1/2*(a^3+2*a^2-5*a-4)*q^9+1/2*(a^4+2*a^3-9*a^2-10*a+12)*q^10+1/2*(-a^4-2*a^3+9*a^2+12*a-16)*q^11+1/2*(-a^4+3*a^2-2*a)*q^12+1/2*(-a^4-2*a^3+9*a^2+12*a-12)*q^13+1/2*(-a^4-2*a^3+5*a^2+10*a-8)*q^14+1/4*(a^4+2*a^3-9*a^2-8*a+8)*q^15+(a^4-6*a^2+4)*q^16+(a^4-8*a^2+9)*q^17+1/2*(a^4+2*a^3-$

$5*a^2-4*a)*q^18+1/4*(3*a^4+6*a^3-23*a^2-32*a+36)*q^19+O(q^20), q-q^2-q^3+q^4-3*q^5+q^6-q^7-q^8-2*q^9+3*q^10-2*q^11-q^12-2*q^13+q^14+3*q^15+q^16-2*q^17+2*q^18+3*q^19+O(q^20), q-q^2+2*q^3+q^4+2*q^5-2*q^6-3*q^7-q^8+q^9-2*q^10+q^11+2*q^12+3*q^13+3*q^14+4*q^15+q^16-q^17-q^18-8*q^19+O(q^20)$ *] [* Number Field with defining polynomial $.1^5-9*.1^3+2*.1^2+16*.1$ - 8 over the Rational Field, Rational Field, Rational Field *] [* 59, 118, 118 *] 118a(rk=1),118d(rk=0),$n(118d;9) = 27-24$, sol 118a. OK.

51.2. $X_0(118)/w_{59}$**,genus 3.** [* $q-q^2-q^3+q^4-3*q^5+q^6-q^7-q^8-2*q^9+3*q^10-2*q^11-q^12-2*q^13+q^14+3*q^15+q^16-2*q^17+2*q^18+3*q^19+O(q^20), q+q^2-q^3+q^4+q^5-q^6+3*q^7+q^8-2*q^9+q^10+2*q^11-q^12-6*q^13+3*q^14-q^15+q^16-2*q^17-2*q^18-5*q^19+O(q^20), q+q^2+2*q^3+q^4-2*q^5+2*q^6-3*q^7+q^8+q^9-2*q^10-q^11+2*q^12-3*q^13-3*q^14-4*q^15+q^16+7*q^17+q^18+4*q^19+O(q^20)$ *] [* Rational Field, Rational Field, Rational Field *] [* 118, 118, 118 *] 118a(rk=1),118b,118c (???) Need Petri equations.

52. $N = 123$

52.1. $X_0(123)/w_3$**,genus 7.** [* $q+a*q^2+1/2*(-a^2-2*a+3)*q^3+(a^2-2)*q^4+(-a-1)*q^5+1/2*(-a^2-2*a-1)*q^6+1/2*(a^2+2*a+1)*q^7+(-a^2+a+1)*q^8+a*q^9+(-a^2-a)*q^10+1/2*(3*a^2+2*a-9)*q^11+1/2*(a^2-2*a-7)*q^12+(-a^2+3)*q^13+1/2*(a^2+6*a+1)*q^14+(a^2+2*a-1)*q^15+(-4*a+3)*q^16-2*q^17+a^2*q^18+1/2*(-3*a^2-2*a+13)*q^19+O(q^20), q-q^3-2*q^4-2*q^5-4*q^7+q^9+5*q^11+2*q^12-4*q^13+2*q^15+4*q^16-5*q^17-2*q^19+O(q^20), q+a*q^2-q^3+(a^2-2)*q^4+(-a^2+a+4)*q^5-a*q^6+(-a^2-a+4)*q^7+(a^2-2)*q^8+q^9+2*q^10+(-a-1)*q^11+(-a^2+2)*q^12+(a^2-a)*q^13+(-2*a^2+2)*q^14+(a^2-a-4)*q^15+(-a^2+2*a+2)*q^16+(2*a^2-a-5)*q^17+a*q^18+(a^2-a-2)*q^19+O(q^20)$ *] [* Number Field with defining polynomial $.1^3+.1^2-5*.1-1$ over the Rational Field, Rational Field, Number Field with defining polynomial $.1^3-.1^2-4*.1+2$ over the Rational Field *][* 41, 123, 123 *] 123b(rk=1).

52.2. $X_0(123)/w_{41}$**,genus 3.** [* $q-q^3-2*q^4-2*q^5-4*q^7+q^9+5*q^11+2*q^12-4*q^13+2*q^15+4*q^16-5*q^17-2*q^19+O(q^20), q+a*q^2+q^3+(-a+2)*q^5+a*q^6+(a-2)*q^7-2*a*q^8+q^9+(2*a-2)*q^10+(-a+1)*q^11+(-3*a+2)*q^13+(-2*a+2)*q^14+(-a+2)*q^15-4*q^16+(a+1)*q^17+a*q^18+(a-4)*q^19+O(q^20)$ *] [* Rational Field, Number Field with defining polynomial $.1^2-2$ over the Rational Field *] [* 123, 123 *]123b(rk=1).

53. $N = 141$

53.1. $X_0(141)/w_3$**,genus 8.** [* $q+a*q^2+(a^3-a^2-6*a+4)*q^3+(a^2-2)*q^4+(-4*a^3+2*a^2+20*a-10)*q^5+(-a^2-a+1)*q^6+(3*a^3-a^2-16*a+7)*q^7+(a^3-4*a)*q^8+(3*a^3-a^2-14*a+6)*q^9+(-2*a^3+10*a-4)*q^10+(2*a^3-2*a^2-10*a+6)*q^11+(-3*a^3+a^2+13*a-8)*q^12+(-4*a^3+2*a^2+22*a-8)*q^13+(2*a^3-a^2-8*a+3)*q^14+(-4*a^3+4*a^2+22*a-16)*q^15+(a^3-a^2-5*a+5)*q^16+(a^3+a^2-6*a)*q^17+(2*a^3+a^2-9*a+3)*q^18+(-2*a^3+10*a-2)*q^19+O(q^20), q-q^3-2*q^4-q^5-3*q^7+q^9-3*q^11+2*q^12-4*q^13+q^15+4*q^16+8*q^17-6*q^19+O(q^20), q-q^2-q^3-q^4+q^6+4*q^7+3*q^8+q^9+q^12+6*q^13-4*q^14-q^16-6*q^17-q^18+2*q^19+O(q^20), q+a*q^2-q^3+(-a+2)*q^4+(a+1)*q^5-a*q^6+(a+1)*q^7+(a-4)*q^8+q^9+4*q^10+(-a+3)*q^11+(a-2)*q^12+(-2*a-4)*q^13+4*q^14+(-a-1)*q^15-3*a*q^16-2*a*q^17+a*q^18+6*q^19+O(q^20)$ *] [* Number Field with defining polynomial $.1^4-.1^3-5*.1^2+5*.1$ - 1 over the Rational Field, Rational Field, Rational Field, Number Field with defining polynomial $.1^2+.1$ - 4 over the Rational Field *] [* 47, 141, 141, 141 *] 141d (rk=1),141b(rk=0). $n(141b,8) = 16-8$. Therefore is 141d.

53.2. $X_0(141)/w_{47}$**, genus 3.** [* $q-q^3-2*q^4-q^5-3*q^7+q^9-3*q^11+2*q^12-4*q^13+q^15+4*q^16+8*q^17-6*q^19+O(q^20), q-q^2+q^3-q^4+2*q^5-q^6+3*q^8+q^9-2*q^10+4*q^11-q^12-2*q^13+2*q^15-q^16+2*q^17-q^18+O(q^20), q+2*q^2+q^3+2*q^4-q^5+2*q^6-3*q^7+q^9-2*q^10+q^11+2*q^12-2*q^13-6*q^14-q^15-4*q^16+2*q^17+2*q^18+6*q^19+O(q^20)$ *] [* Rational Field, Rational Field, Rational Field *] [* 141, 141, 141 *] 141d,141c,141e $n(141e,4) = 12-10$. Petri equation.

54. $N = 142$

54.1. $X_0(142)/w_2$**, genus 9.** [* $q+a*q^2+(-a^2+3)*q^3+(a^2-2)*q^4+(-a-1)*q^5+(-2*a+3)*q^6+(2*a^2+2*a-6)*q^7+(a-3)*q^8+(-a^2-3*a+6)*q^9+(-a^2-a)*q^10+(-2*a^2-2*a+6)*q^11+(3*a-6)*q^12+4*q^13+(2*a^2+4*a-6)*q^14+(a^2+2*a-6)*q^15+(-a^2-3*a+4)*q^16+(2*a^2+2*a-6)*q^17+(-3*a^2+a+3)*q^18+(-a^2-a+7)*q^19+O(q^20), q+a*q^2-a*q^3+(a^2-2)*q^4+(-a^2+a+5)*q^5-a^2*q^6-2*a*q^7+(-a^2+3)*q^8+(a^2-3)*q^9+(2*a^2+a-3)*q^10+(2*a^2-6)*q^11+(a^2-2*a-3)*q^12+(-2*a^2+4)*q^13-2*a^2*q^14+(-2*a^2-a+3)*q^15+(-a^2-a+1)*q^16+(2*a^2+2*a-6)*q^17+(-a^2+a+3)*q^18+(a^2+2*a-2)*q^19+O(q^20), q-q^2-q^3+q^4-2*q^5+q^6-q^7-q^8-2*q^9+2*q^10-2*q^11-q^12-3*q^13+q^14+2*q^15+q^16-6*q^17+2*q^18+5*q^19+O(q^20), q-q^2+q^4+2*q^5-q^8-3*q^9-2*q^10+6*q^11+4*q^13+q^16+6*q^17+3*q^18-8*q^19+O(q^20), q-q^2+3*q^3+q^4+2*q^5-3*q^6-3*q^7-q^8+6*q^9-2*q^10-6*q^11+3*q^12-5*q^13+3*q^14+6*q^15+q^16+6*q^17-6*q^18+q^19+O(q^20)$ *] [* Number Field with defining polynomial $.1^3 - 5*.1 + 3$ over the Rational Field, Number Field with defining polynomial $.1^3 + .1^2 - 4*.1 - 3$ over the Rational Field, Rational Field, Rational Field, Rational Field *] [* 71, 71, 142, 142, 142 *]142b(rk=1),142c(rk0),142e(rk0), $n(142e, 9) = 28 - 14$, Usar equation Petri.

54.2. $X_0(142)/w_{71}$**, genus 2.** [* $q-q^2-q^3+q^4-2*q^5+q^6-q^7-q^8-2*q^9+2*q^10-2*q^11-q^12-3*q^13+q^14+2*q^15+q^16-6*q^17+2*q^18+5*q^19+O(q^20), q+q^2+q^3+q^4+q^6-q^7+q^8-2*q^9+q^12-q^13-q^14+q^16-2*q^18-q^19+O(q^20)$ *] [* Rational Field, Rational Field *] [* 142, 142 *]142b(rk=1),142d(rk=0). Si, dos bielliptiques.

55. $N = 143$

55.1. $X_0(143)/w_{11}$**,genus 7.** [* $q-q^3-2*q^4-q^5-2*q^7-2*q^9-q^11+2*q^12-q^13+q^15+4*q^16-4*q^17+2*q^19+O(q^20), q+a*q^2+(-a^5-a^4+8*a^3+6*a^2-11*a-5)*q^3+(a^2-2)*q^4+(a^5+2*a^4-8*a^3-14*a^2+12*a+15)*q^5+(-a^5-2*a^4+8*a^3+13*a^2-12*a-12)*q^6+(2*a^5+2*a^4-17*a^3-13*a^2+26*a+14)*q^7+(a^3-4*a)*q^8+(-3*a^5-4*a^4+25*a^3+27*a^2-38*a-26)*q^9+(2*a^5+2*a^4-16*a^3-12*a^2+22*a+12)*q^10-q^11+(-a^3+3*a-2)*q^12+q^13+(2*a^5+3*a^4-17*a^3-22*a^2+28*a+24)*q^14+(3*a^5+4*a^4-24*a^3-28*a^2+30*a+33)*q^15+(a^4-6*a^2+4)*q^16-2*a*q^17+(-4*a^5-5*a^4+33*a^3+34*a^2-47*a-36)*q^18+(-2*a^5-3*a^4+16*a^3+20*a^2-23*a-22)*q^19+O(q^20)$ *] [* Rational Field, Number Field with defining polynomial $.1^6 - 10 * .1^4 + 2 * .1^3 + 24 * .1^2 - 7*.1$ - 12 over the Rational Field *] [* 143, 143 *]143a,rk=1

55.2. $X_0(143)/w_{13}$**, genus 6.** [* $q-2*q^2-q^3+2*q^4+q^5+2*q^6-2*q^7-2*q^9-2*q^10+q^11-2*q^12+4*q^13+4*q^14-q^15-4*q^16-2*q^17+4*q^18+O(q^20), q-q^3-2*q^4-q^5-2*q^7-2*q^9-q^11+2*q^12-q^13+q^15+4*q^16-4*q^17+2*q^19+O(q^20), q+a*q^2+(-a^3+3*a^2-3)*q^3+(a^2-2)*q^4+(-2*a^2+2*a+4)*q^5+(-a^2+2*a+1)*q^6+(a^3-a^2-4*a+2)*q^7+(a^3-4*a)*q^8+(a^3-3*a^2-2*a+5)*q^9+(-2*a^3+2*a^2+4*a)*q^10+q^11+(a^3-4*a^2+a+6)*q^12-q^13+(2*a^3-3*a^2-3*a-1)*q^14+(-2*a^3+6*a^2+2*a-10)*q^15+(3*a^3-5*a^2-5*a+3)*q^16+(-4*a^2+6*a+8)*q^17+(-a^2-1)*q^18+(-3*a^3+7*a^2+2*a-3)*q^19+O(q^20)$ *] [* Rational Field, Rational Field, Number Field with defining polynomial $.1^4 - 3 * .1^3 - .1^2 + 5*.1 + 1$ over the Rational Field *] [* 11, 143, 143 *] 11a(rk=0), 143a,rk=1, $n(11a; 4) = 14 - 10$. Is 143a.

56. $N = 145$

56.1. $X_0(145)/w_5$**,genus 6.** [* $q+a*q^2-a*q^3+(-2*a-1)*q^4-q^5+(2*a-1)*q^6+(2*a+2)*q^7+(a-2)*q^8+(-2*a-2)*q^9-a*q^10+(a+2)*q^11+(-3*a+2)*q^12+(2*a+1)*q^13+(-2*a+2)*q^14+a*q^15+3*q^16+(-2*a-4)*q^17+(2*a-2)*q^18+6*q^19+O(q^20), q-q^2-q^4-q^5-2*q^7+3*q^8-3*q^9+q^10-6*q^11+2*q^13+2*q^14-q^16-2*q^17+3*q^18-2*q^19+O(q^20), q+a*q^2+(-a^2+2*a+1)*q^3+(a^2-2)*q^4-q^5+(-a^2+5)*q^6+(-a^2+3)*q^7+(3*a^2-3*a-5)*q^8+(-2*a+3)*q^9-a*q^10+(a^2-2*a+1)*q^11+(-a^2+3)*q^12+(2*a-4)*q^13+(-3*a^2+2*a+5)*q^14+(a^2-2*a-1)*q^15+(4*a^2-2*a-11)*q^16+(-3*a^2+2*a+9)*q^17+(-2*a^2+3*a)*q^18+(3*a^2-4*a-7)*q^19+O(q^20)$ *]

[* Number Field with defining polynomial $.1^2 + 2*.1$ - 1 over the Rational Field, Rational Field, Number Field with defining polynomial $.1^3 - 3 * .1^2 - .1 + 5$ over the Rational Field *] [* 29, 145, 145 *] 145a,rk=1

56.2. $X_0(145)/w_{29}$**, genus 4.** [* $q-q^2-q^4-q^5-2*q^7+3*q^8-3*q^9+q^10-6*q^11+2*q^13+2*q^14-q^16-2*q^17+3*q^18-2*q^19+O(q^20), q+a*q^2+(-a^2+3)*q^3+(a^2-2)*q^4+q^5+(-a^2+1)*q^6+(a^2-1)*q^7+(a^2-a-1)*q^8+(-2*a^2+2*a+5)*q^9+a*q^10+(a^2-2*a-1)*q^11+(a^2-2*a-5)*q^12-2*a*q^13+(a^2+2*a-1)*q^14+(-a^2+3)*q^15+(-2*a^2+2*a+3)*q^16+(3*a^2-4*a-7)*q^17+(-a+2)*q^18+(-a^2-1)*q^19+O(q^20)$ *] [* Rational Field, Number Field with defining polynomial $.1^3 - .1^2 - 3*.1 + 1$ over the Rational Field *] [* 145, 145 *]145a, rk=1.

57. $N = 155$

57.1. $X_0(155)/w_5$**, genus 8.** [* $q+a*q^2-2*a*q^3+(a-1)*q^4+q^5+(-2*a-2)*q^6+(2*a-3)*q^7+(-2*a+1)*q^8+(4*a+1)*q^9+a*q^10+2*q^11-2*q^12-2*a*q^13+(-a+2)*q^14-2*a*q^15-3*a*q^16+(-2*a+4)*q^17+(5*a+4)*q^18+(-2*a+1)*q^19+O(q^20), q-q^3-2*q^4-q^5-2*q^9-4*q^11+2*q^12-6*q^13+q^15+4*q^16+5*q^17-q^19+O(q^20), q-q^2+2*q^3-q^4-q^5-2*q^6+4*q^7+3*q^8+q^9+q^10+4*q^11-2*q^12-4*q^14-2*q^15-q^16-8*q^17-q^18+4*q^19+O(q^20), q+a*q^2+1/2*(-a^3-a^2+6*a+2)*q^3+(a^2-2)*q^4-q^5+(-a^2-a+6)*q^6+(a^2+a-4)*q^7+(a^3-4*a)*q^8+(-2*a^2-a+10)*q^9-a*q^10+(a^2-a-6)*q^11-2*q^12+(-a^2-a+8)*q^13+(a^3+a^2-4*a)*q^14+1/2*(a^3+a^2-6*a-2)*q^15+(-a^3+2*a^2+4*a-8)*q^16+1/2*(-a^3+a^2+4*a-6)*q^17+(-2*a^3-a^2+10*a)*q^18+(a^3+a^2-5*a-1)*q^19+O(q^20)$ *] [* Number Field with defining polynomial $.1^2 - .1 - 1$ over the Rational Field, Rational Field, Rational Field, Number Field with defining polynomial $.1^4 + .1^3 - 8*.1^2 - 4*.1 + 12$ over the Rational Field *] [* 31, 155, 155, 155 *] 155c(rk=1),155b(rk=0), $n(155b; 3) = 6 - 4$. Thus rk=1.

57.2. $X_0(155)/w_{31}$**, genus 5.** [* $q-q^3-2*q^4-q^5-2*q^9-4*q^11+2*q^12-6*q^13+q^15+4*q^16+5*q^17-q^19+O(q^20), q+a*q^2+1/2*(-a^3+a^2+4*a-2)*q^3+(a^2-2)*q^4+q^5+(-a^2+a+2)*q^6+(-a^2-a+4)*q^7+(a^3-4*a)*q^8-a*q^9+a*q^10+(-a^2+a+2)*q^11+(-2*a+2)*q^12+(a^3-5*a+2)*q^13+(-a^3-a^2+4*a)*q^14+1/2*(-a^3+a^2+4*a-2)*q^15+(a^3-4*a)*q^16+1/2*(a^3+a^2-6*a+2)*q^17-a^2*q^18+(-a^3+a^2+3*a-3)*q^19+O(q^20)$ *] [* Rational Field, Number Field with defining polynomial $.1^4 - .1^3 - 6*.1^2 + 4*.1 + 4$ over the Rational Field *] [* 155, 155 *] 155c (rk=1).

58. $N = 159$

58.1. $X_0(159)/w_3$**, genus 9.** [* $q-q^2-3*q^3-q^4+3*q^6-4*q^7+3*q^8+6*q^9+3*q^12-3*q^13+4*q^14-q^16-3*q^17-6*q^18-5*q^19+O(q^20), q+a*q^2+(-a^2-a+3)*q^3+(a^2-2)*q^4+(a^2-3)*q^5-q^6+(a^2-1)*q^7+(-a^2-a+1)*q^8+(-3*a^2-2*a+7)*q^9+(-a^2+1)*q^10+(a^2+2*a-3)*q^11+(2*a^2+a-6)*q^12+q^13+(-a^2+2*a+1)*q^14+(3*a^2+2*a-9)*q^15+(-2*a^2-2*a+3)*q^16+(2*a-1)*q^17+(a^2-2*a-3)*q^18+(a+4)*q^19+O(q^20), q+a*q^2-q^3+(a^2-2)*q^4+(-a^3-a^2+6*a+4)*q^5-a*q^6+1/3*(a^4+4*a^3-6*a^2-21*a+4)*q^7+(a^3-4*a)*q^8+q^9+(-a^4-a^3+6*a^2+4*a)*q^10+1/3*(-2*a^4-2*a^3+12*a^2+6*a-2)*q^11+(-a^2+2)*q^12+1/3*(2*a^4-a^3-15*a^2+6*a+20)*q^13+1/3*(4*a^4+4*a^3-21*a^2-18*a-5)*q^14+(a^3+a^2-6*a-4)*q^15+(a^4-6*a^2+4)*q^16-2*a*q^17+a*q^18+1/3*(-2*a^4-2*a^3+12*a^2+6*a-2)*q^19+O(q^20)$ *] [* Rational Field, Number Field with defining polynomial $.1^3 + .1^2 - 3*.1 - 1$ over the Rational Field, Number Field with defining polynomial $.1^5 - 10*.1^3 + 22*.1 + 5$ over the Rational Field *] [* 53, 53, 159 *] 53a, rk=1

58.2. $X_0(159)/w_{53}$**, genus 6.** [* $q-q^2-3*q^3-q^4+3*q^6-4*q^7+3*q^8+6*q^9+3*q^12-3*q^13+4*q^14-q^16-3*q^17-6*q^18-5*q^19+O(q^20), q+a*q^2+q^3+(a^2-2)*q^4+(-a^3+a^2+2*a)*q^5+a*q^6+(a^3-3*a^2-2*a+5)*q^7+(a^3-4*a)*q^8+q^9+(-2*a^3+a^2+7*a-3)*q^10+(4*a^3-6*a^2-12*a+12)*q^11+(a^2-2)*q^12+(-3*a^3+5*a^2+8*a-10)*q^13+(-a^2-2*a+3)*q^14+(-a^3+a^2+2*a)*q^15+(3*a^3-5*a^2-7*a+7)*q^16+(-4*a^3+8*a^2+10*a-12)*q^17+a*q^18+(2*a^2-4*a-4)*q^19+O(q^20), q-q^2-3*q^3-q^4+3*q^6-4*q^7+3*q^8+6*q^9+3*q^12-3*q^13+4*q^14-q^16-3*q^17-6*q^18-5*q^19+O(q^20)$ *] [* Rational Field, Number Field with defining polynomial $.1^4 - 3*.1^3 - .1^2 + 7*.1 - 3$ over the Rational Field, Rational Field *] [* 53, 159, 159 *] 53a (rep), rk=1.

CHAPTER 2

The modular curves $X_0(p_1p_2p_3)/W$ with $|W| = 4$

1. N=455

$X_0(N)/ < w_5, w_7, w_{35} >$, **genus 12**

```
[*
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 - q^5 + (a - 3)*q^6 + 2*q^7 - a*q^8 + (-2*a +
        1)*q^9 - a*q^10 + (a - 3)*q^11 + (-a + 1)*q^12 + q^13 + 2*a*q^14 + (a -
        1)*q^15 - 5*q^16 + 2*a*q^17 + (a - 6)*q^18 + (3*a - 1)*q^19 + O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q + a*q^2 + (-a^2 + a + 2)*q^3 + (a^2 - 2)*q^4 + (-a + 1)*q^5 + (-2*a +
        2)*q^6 - q^7 + (a^2 - 2)*q^8 + (-2*a + 3)*q^9 + (-a^2 + a)*q^10 + (a^2 -
        a - 2)*q^11 - 4*q^12 + q^13 - a*q^14 + (-a^2 + 3*a)*q^15 + (-a^2 + 2*a +
        2)*q^16 + (a^2 + a - 2)*q^17 + (-2*a^2 + 3*a)*q^18 + (-a - 1)*q^19 +
        O(q^20),
    q + q^2 - q^4 - q^5 - q^7 - 3*q^8 - 3*q^9 - q^10 - q^13 - q^14 - q^16 -
        2*q^17 - 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + (a^3 + a^2 - 4*a - 2)*q^3 + (a^2 - 2)*q^4 - q^5 + (a^2 + a -
        1)*q^6 - q^7 + (a^3 - 4*a)*q^8 + (-a^3 - a^2 + 6*a + 4)*q^9 - a*q^10 +
        (2*a^3 + 2*a^2 - 10*a - 4)*q^11 + (-a^3 - a^2 + 7*a + 4)*q^12 + q^13 -
        a*q^14 + (-a^3 - a^2 + 4*a + 2)*q^15 + (-a^3 - a^2 + 3*a + 3)*q^16 +
        (a^3 - a^2 - 6*a + 3)*q^17 + (a^2 + a + 1)*q^18 + (-a^3 - a^2 + 6*a +
        7)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 4*x + 2 over the Rational
    Field,
    Rational Field,
    Number Field with defining polynomial x^4 + x^3 - 5*x^2 - 3*x + 1 over the
    Rational Field
*]
[* 65, 65, 91, 91, 455, 455 *]
```

Not bielliptic $n(a_2 = 0; 4) = 21 - 16$ therefore with $|a_2| > 0$ more distance.

$X_0(455)/ < w_5, w_{13} >$, **genus 14**

```
[*
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
```

```
        4*q^16 + 3*q^17 + 2*q^19 + O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (a + 3)*q^5 - 2*q^6 + q^7 - 2*a*q^8 - q^9 + (3*a +
        2)*q^10 - 3*a*q^11 - q^13 + a*q^14 + (-3*a - 2)*q^15 - 4*q^16 - a*q^17 -
        a*q^18 + (3*a - 3)*q^19 + O(q^20),
    q + q^2 - q^4 - q^5 - q^7 - 3*q^8 - 3*q^9 - q^10 - q^13 - q^14 - q^16 -
        2*q^17 - 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + 1/14*(-a^6 - 5*a^5 + 18*a^4 + 46*a^3 - 88*a^2 - 73*a + 71)*q^3 +
        (a^2 - 2)*q^4 - q^5 + 1/14*(-5*a^6 + 3*a^5 + 48*a^4 - 22*a^3 - 90*a^2 -
        a + 19)*q^6 + q^7 + (a^3 - 4*a)*q^8 + 1/7*(-a^6 + 2*a^5 + 11*a^4 -
        24*a^3 - 32*a^2 + 60*a + 43)*q^9 - a*q^10 + 1/14*(3*a^6 + a^5 - 26*a^4 -
        12*a^3 + 26*a^2 + 51*a + 53)*q^11 + 1/14*(5*a^6 - 17*a^5 - 48*a^4 +
        148*a^3 + 90*a^2 - 195*a - 47)*q^12 - q^13 + a*q^14 + 1/14*(a^6 + 5*a^5
        - 18*a^4 - 46*a^3 + 88*a^2 + 73*a - 71)*q^15 + (a^4 - 6*a^2 + 4)*q^16 +
        1/7*(a^6 - 2*a^5 - 11*a^4 + 10*a^3 + 32*a^2 + 24*a - 36)*q^17 +
        1/7*(2*a^6 - 4*a^5 - 22*a^4 + 34*a^3 + 43*a^2 - 29*a + 19)*q^18 +
        1/14*(5*a^6 - 3*a^5 - 48*a^4 + 36*a^3 + 90*a^2 - 97*a + 9)*q^19 +
        O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^7 - 15*x^5 + 2*x^4 + 66*x^3 - 17*x^2
        - 72*x + 19 over the Rational Field,
    Rational Field
*]
[* 35, 65, 91, 91, 455, 455, 455 *]
```

Not bielliptic $n(a_2 = 0; 4) = 20 - 18$.
$X_0(455)/ < w_7, w_{13} >$**,genus 12**

```
[*
    q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
        (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
        a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
        - 2)*q^19 + O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a
        - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a -
```

```
        2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a +
        3)*q^19 + O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q + q^2 - q^4 - q^5 - q^7 - 3*q^8 - 3*q^9 - q^10 - q^13 - q^14 - q^16 -
        2*q^17 - 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + (-a^3 + 3*a^2 - 2)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^2 + 3*a +
        1)*q^6 - q^7 + (a^3 - 4*a)*q^8 + (-a^3 + 3*a^2 - 2*a)*q^9 + a*q^10 +
        (-2*a^3 + 2*a^2 + 6*a)*q^11 + (a^3 - 3*a^2 + a + 4)*q^12 - q^13 - a*q^14
        + (-a^3 + 3*a^2 - 2)*q^15 + (3*a^3 - 5*a^2 - 5*a + 3)*q^16 + (-a^3 + a^2
        + 2*a + 5)*q^17 + (-3*a^2 + 5*a + 1)*q^18 + (a^3 + a^2 - 6*a - 3)*q^19 +
        O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 - 3*x^3 - x^2 + 5*x + 1 over the
    Rational Field,
    Rational Field
*]
[* 35, 65, 65, 91, 455, 455, 455 *]
```

Not bielliptic $n(a_2 = 0; 2) = 7 - 6$, $n(|a_2| = 1; 4) = 17 - 16$.

$X_0(N)/ < w_{35}, w_{13}, w_{455} >$, **genus 7**

```
[ <5, 1>, <7, 1>, <13, 1> ] 455 [*
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a
        - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a -
        2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a +
        3)*q^19 + O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (a + 3)*q^5 - 2*q^6 + q^7 - 2*a*q^8 - q^9 + (3*a +
        2)*q^10 - 3*a*q^11 - q^13 + a*q^14 + (-3*a - 2)*q^15 - 4*q^16 - a*q^17 -
        a*q^18 + (3*a - 3)*q^19 + O(q^20),
    q + q^2 - q^4 - q^5 - q^7 - 3*q^8 - 3*q^9 - q^10 - q^13 - q^14 - q^16 -
        2*q^17 - 3*q^18 - 4*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
```

```
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Rational Field
*]
[* 65, 65, 91, 91, 455 *] x^14 +
4.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000000*x^13 + 14.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000*x^12 +
    34.0000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000000*x^11 + 77.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000*x^10 +
    142.0000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000000*x^9 + 244.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000*x^8 +
    355.9999999999999999999999999999999999999999999999999999999999999999999999\
    9999999999999999999999999*x^7 + 488.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000*x^6 +
    567.9999999999999999999999999999999999999999999999999999999999999999999999\
    9999999999999999999999999*x^5 + 616.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000*x^4 +
    544.0000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000000*x^3 + 448.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000*x^2 +
    256.0000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000000*x + 128.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000
1 [* 7, 17, 7, 29, 7, 41, 147, 317, 511, 937, 2163, 3965, 8015,
16649, 33187, 65405, 130431, 262121, 522963, 1052349 *] 455 [* 1, 1,
0, 1, 0, 0, 0, 0, 0, 0 *] 1 1 2 [* 7, 17, 7, 29, 7, 41, 147, 317,
511, 937, 2163, 3965, 8015, 16649, 33187, 65405, 130431, 262121,
522963, 1052349 *] [* 8, 16, 8, 32, 88, 112, 232, 576, 1016, 1936,
4232, 8288, 16024, 32944, 66088, 130176, 261944, 526288, 1046984,
2094752 *]
```

Not bielliptic, $n(a_2 = 1, -1; 4) = 17 - 16$, $n(a_2 = -2, 4) = 17 - 10$.

$X_0(455)/ < w_5, w_{7*13}, w_{455} >$, **genus 7**

```
[ <5, 1>, <7, 1>, <13, 1> ] 455 [*
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
        4*q^16 + 3*q^17 + 2*q^19 + O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 - q^5 + (a - 3)*q^6 + 2*q^7 - a*q^8 + (-2*a +
        1)*q^9 - a*q^10 + (a - 3)*q^11 + (-a + 1)*q^12 + q^13 + 2*a*q^14 + (a -
        1)*q^15 - 5*q^16 + 2*a*q^17 + (a - 6)*q^18 + (3*a - 1)*q^19 + O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
```

```
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 - 3*q^5 + q^7 + q^9 + 4*q^12 + q^13 + 6*q^15 + 4*q^16 -
        6*q^17 - 7*q^19 + O(q^20),
    q + q^2 - q^4 - q^5 - q^7 - 3*q^8 - 3*q^9 - q^10 - q^13 - q^14 - q^16 -
        2*q^17 - 3*q^18 - 4*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 35, 65, 65, 91, 91, 455 *] x^14 +
2.0000000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000000*x^13 + 10.0000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^12 +
    16.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^11 + 47.0000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^10 +
    62.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^9 + 138.0000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^8 +
    152.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^7 + 276.0000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^6 +
    248.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^5 + 376.0000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^4 +
    256.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^3 + 320.0000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^2 +
    128.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x + 128.0000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000
1 [* 5, 21, 5, 21, 25, 57, 145, 189, 545, 1161, 1985, 4077, 8065,
16569, 33025, 63549, 131585, 267753, 523265, 1040781 *] 455 [* 1, 0,
0, 0, 0, 0, 0, 1, 0, 0 *] 1 1 2 [* 5, 21, 5, 21, 25, 57, 145, 189,
545, 1161, 1985, 4077, 8065, 16569, 33025, 63549, 131585, 267753,
523265, 1040781 *] [* 6, 18, 18, 18, 66, 162, 258, 450, 1026, 2178,
4098, 7938, 16386, 33282, 65538, 130050, 262146, 526338, 1048578,
2093058 *]
```

Not bielliptic; $n(a_2; 4) \geq n(a_2 = 0; 4) = 21 - 18$.

$X_0(455)/ < w_{5*13}, W_7, W_{455} >$ **genus 9**

```
[ <5, 1>, <7, 1>, <13, 1> ] 455 [*
    q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
        (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
        a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
        - 2)*q^19 + O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q + a*q^2 + (-a^2 + a + 2)*q^3 + (a^2 - 2)*q^4 + (-a + 1)*q^5 + (-2*a +
        2)*q^6 - q^7 + (a^2 - 2)*q^8 + (-2*a + 3)*q^9 + (-a^2 + a)*q^10 + (a^2 -
        a - 2)*q^11 - 4*q^12 + q^13 - a*q^14 + (-a^2 + 3*a)*q^15 + (-a^2 + 2*a +
        2)*q^16 + (a^2 + a - 2)*q^17 + (-2*a^2 + 3*a)*q^18 + (-a - 1)*q^19 +
        O(q^20),
    q + q^2 - q^4 - q^5 - q^7 - 3*q^8 - 3*q^9 - q^10 - q^13 - q^14 - q^16 -
        2*q^17 - 3*q^18 - 4*q^19 + O(q^20),
    q - q^2 - q^4 + q^5 - q^7 + 3*q^8 - 3*q^9 - q^10 + q^13 + q^14 - q^16 -
        6*q^17 + 3*q^18 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 4*x + 2 over the Rational
    Field,
    Rational Field,
    Rational Field
*]
[* 35, 65, 91, 91, 455, 455 *] x^18 +
3.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000000*x^17 + 10.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000*x^16 +
    20.0000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000000*x^15 + 45.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000001*x^14 +
    75.0000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000000*x^13 + 136.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000001*x^12 +
    198.0000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000000*x^11 + 320.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000002*x^10 +
    431.9999999999999999999999999999999999999999999999999999999999999999999999\
    9999999999999999999999999999*x^9 + 640.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000004*x^8 +
    791.9999999999999999999999999999999999999999999999999999999999999999999999\
    9999999999999999999999999999*x^7 + 1088.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000*x^6 +
```

```
    1200.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000*x^5 + 1440.0000000000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000000*x^4 +
    1280.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000*x^3 + 1280.0000000000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000000*x^2 +
    768.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000001*x + 512.0000000000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000000
1 [* 6, 16, 6, 36, 26, 52, 146, 316, 474, 916, 1986, 4236, 7754,
16004, 34706, 64700, 131994, 265012, 519010, 1047276 *] 455 [* 0, 1,
0, 1, 0, 0, 0, 1, 0, 0 *] 0 0 2 [* 6, 16, 6, 36, 26, 52, 146, 316,
474, 916, 1986, 4236, 7754, 16004, 34706, 64700, 131994, 265012,
519010, 1047276 *] [* 4, 16, 28, 32, 44, 112, 284, 576, 1036, 1936,
3964, 8288, 16748, 32944, 64988, 130176, 262348, 526288, 1050172,
2094752 *]
```

Not bielliptic, $n(a_2 = 1; 2) = 6 - 4$, $n(a_2 = -1; 16) = 36 - 32$, $n(a_2 = -2, 4) = 16 - 10$.

$X_0(455)/ < w_{5*7}, W_{7*13}, W_{5*7} >$, **genus 10**

```
[ <5, 1>, <7, 1>, <13, 1> ] 455 [*
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 - 3*q^5 + q^7 + q^9 + 4*q^12 + q^13 + 6*q^15 + 4*q^16 -
        6*q^17 - 7*q^19 + O(q^20),
    q + q^2 - q^4 - q^5 - q^7 - 3*q^8 - 3*q^9 - q^10 - q^13 - q^14 - q^16 -
        2*q^17 - 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + (-a^3 + a^2 + 4*a - 2)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^4 + a^3 +
        4*a^2 - 2*a)*q^6 + q^7 + (a^3 - 4*a)*q^8 + (a^5 - a^4 - 8*a^3 + 6*a^2 +
        15*a - 8)*q^9 + a*q^10 + (-a^5 + 2*a^4 + 6*a^3 - 10*a^2 - 8*a + 9)*q^11
        + (-a^5 + a^4 + 6*a^3 - 4*a^2 - 8*a + 4)*q^12 + q^13 + a*q^14 + (-a^3 +
        a^2 + 4*a - 2)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + (a^5 - a^4 - 6*a^3 +
        2*a^2 + 7*a + 3)*q^17 + (2*a^5 - 2*a^4 - 14*a^3 + 9*a^2 + 23*a - 9)*q^18
        + (a^4 - 2*a^3 - 4*a^2 + 5*a + 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^6 - 3*$.1^5 - 6*$.1^4 + 20*$.1^3 +
        6*$.1^2 - 31*$.1 + 9 over the Rational Field
```

```
*]
[* 65, 91, 91, 455, 455 *] x^20 - x^19 +
7.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000000000002*x^18 -
    9.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000002*x^17 + 30.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000000000001*x^16 -
    45.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000004*x^15 + 101.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000000000002*x^14 -
    149.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000006*x^13 + 277.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000000000010*x^12 -
    372.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000008*x^11 + 620.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000000000000*x^10 -
    743.99999999999999999999999999999999999999999999999999999999999999999999\
    99999999999999999999999998*x^9 + 1108.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000000000002*x^8 -
    1192.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000004*x^7 + 1616.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000000000004*x^6 -
    1440.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000004*x^5 + 1920.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000000000002*x^4 -
    1152.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000000*x^3 + 1792.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000000000000*x^2 -
    512.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000000*x + 1024.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000000000000
1 [* 2, 18, 2, 30, 17, 66, 212, 238, 560, 1073, 1916, 4158, 8010,
17070, 34097, 63902, 129916, 262218, 522008, 1045545 *] 455 [* 0, 0,
0, 1, 1, 0, 0, 0, 0, 0 *] 5 5 2 [* 2, 18, 2, 30, 17, 66, 212, 238,
560, 1073, 1916, 4158, 8010, 17070, 34097, 63902, 129916, 262218,
522008, 1045545 *] [* 6, 18, 18, 18, 66, 162, 258, 450, 1026, 2178,
4098, 7938, 16386, 33282, 65538, 130050, 262146, 526338, 1048578,
2093058 *]
```

Not bielliptic, $n(a_2 = 0; 16) = 30 - 18$, $n(a_2 = -1 or 1; 4) = 18 - 16$, $n(a_2 = -2; 4) = 18 - 10$.

2. Level $N = 430$

$X_0(430)/ < w_2, w_5 >$, **genus 16**

```
[*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (-a + 2)*q^5 - 2*q^6 + (a - 2)*q^7 - 2*a*q^8 - q^9 +
        (2*a - 2)*q^10 + (2*a - 1)*q^11 + (2*a + 1)*q^13 + (-2*a + 2)*q^14 +
        (-2*a + 2)*q^15 - 4*q^16 + (2*a + 5)*q^17 - a*q^18 + (-2*a - 2)*q^19 +
        O(q^20),
```

```
    q - q^2 + a*q^3 + q^4 + (-a + 1)*q^5 - a*q^6 + 2*q^7 - q^8 + (-a + 2)*q^9 +
        (a - 1)*q^10 + a*q^12 + 2*q^13 - 2*q^14 + (2*a - 5)*q^15 + q^16 + (a -
        4)*q^17 + (a - 2)*q^18 + (-3*a - 1)*q^19 + O(q^20),
    q - 2*q^4 - q^5 - 2*q^7 - 3*q^9 - q^11 - q^13 + 4*q^16 - 3*q^17 - 2*q^19 +
        O(q^20),
    q + a*q^2 + (a^5 - 2*a^4 - 6*a^3 + 9*a^2 + 6*a - 2)*q^3 + (a^2 - 2)*q^4 -
        q^5 + (a^5 - a^4 - 8*a^3 + 3*a^2 + 15*a + 3)*q^6 + (-2*a^5 + 3*a^4 +
        13*a^3 - 12*a^2 - 16*a + 2)*q^7 + (a^3 - 4*a)*q^8 + (2*a^5 - 3*a^4 -
        13*a^3 + 10*a^2 + 16*a + 7)*q^9 - a*q^10 + (-3*a^5 + 3*a^4 + 23*a^3 -
        9*a^2 - 38*a - 9)*q^11 + (a^4 - 2*a^3 - 6*a^2 + 8*a + 7)*q^12 + (-2*a +
        2)*q^13 + (-3*a^5 + 3*a^4 + 22*a^3 - 10*a^2 - 32*a - 6)*q^14 + (-a^5 +
        2*a^4 + 6*a^3 - 9*a^2 - 6*a + 2)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + (4*a^5
        - 4*a^4 - 30*a^3 + 12*a^2 + 48*a + 12)*q^17 + (3*a^5 - 3*a^4 - 24*a^3 +
        10*a^2 + 41*a + 6)*q^18 + (2*a^5 - 2*a^4 - 16*a^3 + 6*a^2 + 28*a +
        8)*q^19 + O(q^20),
    q - q^2 + q^4 - q^5 + q^7 - q^8 - 3*q^9 + q^10 - 4*q^11 - q^13 - q^14 + q^16
        + 3*q^18 + q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 - q^5 - a*q^6 + 1/2*(a^2 + 2*a - 8)*q^7 - q^8 + (a^2 -
        3)*q^9 + q^10 + (-a^2 + a + 8)*q^11 + a*q^12 + 1/2*(-3*a^2 - 2*a +
        12)*q^13 + 1/2*(-a^2 - 2*a + 8)*q^14 - a*q^15 + q^16 + (a^2 + a -
        2)*q^17 + (-a^2 + 3)*q^18 + 1/2*(-a^2 + 2*a + 8)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 5 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^6 - 3*x^5 - 5*x^4 + 17*x^3 + 3*x^2 -
        17*x - 3 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 + 2*x^2 - 6*x - 8 over the
    Rational Field
*]
[* 43, 43, 86, 215, 215, 430, 430 *]
```

Not bielliptic $n(a_3 = 0; 9) = 45 - 32$.

$X_0(430)/ < w_2, w_{43} >$**,genus 14**

```
[*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q - 2*q^4 - q^5 - 2*q^7 - 3*q^9 - q^11 - q^13 + 4*q^16 - 3*q^17 - 2*q^19 +
        O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (a^2 - 2)*q^4 + q^5 + (a^2 + a)*q^6 + (-a^2 - 2*a
        + 1)*q^7 + (-2*a^2 - a + 3)*q^8 + (a^2 + 2*a - 2)*q^9 + a*q^10 + (-a^2 +
        a + 7)*q^11 + (-a^2 + a + 1)*q^12 + (-2*a - 2)*q^13 + (-2*a - 3)*q^14 +
        (a + 1)*q^15 + (a^2 - 3*a - 2)*q^16 + (-2*a + 2)*q^17 + (a + 3)*q^18 +
        (-2*a^2 - 4*a + 6)*q^19 + O(q^20),
    q + a*q^2 + (-a^3 + 5*a)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^4 + 5*a^2)*q^6 +
        (a^4 - a^3 - 6*a^2 + 6*a + 2)*q^7 + (a^3 - 4*a)*q^8 + (a^4 + a^3 - 6*a^2
        - 6*a + 5)*q^9 + a*q^10 + (a^3 - 6*a - 1)*q^11 + (-2*a^4 + 13*a^2 - 5*a
```

```
        - 4)*q^12 + (-a^4 + 5*a^2 + a + 3)*q^13 + (a^4 + a^3 - 7*a^2 - 3*a +
        4)*q^14 + (-a^3 + 5*a)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + (a^4 - 7*a^2 + a
        + 1)*q^17 + (3*a^4 + a^3 - 19*a^2 + 4)*q^18 + (-2*a^4 + 14*a^2 - 2*a -
        10)*q^19 + O(q^20),
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q - q^2 + q^4 - q^5 + q^7 - q^8 - 3*q^9 + q^10 - 4*q^11 - q^13 - q^14 + q^16
        + 3*q^18 + q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + q^5 - a*q^6 + (-2*a + 3)*q^7 - q^8 + (2*a - 1)*q^9 -
        q^10 + (-a + 2)*q^11 + a*q^12 + (2*a - 1)*q^13 + (2*a - 3)*q^14 + a*q^15
        + q^16 + (a + 4)*q^17 + (-2*a + 1)*q^18 + (2*a - 5)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 + 2*x^2 - 3*x - 3 over the
    Rational Field,
    Number Field with defining polynomial x^5 - 2*x^4 - 7*x^3 + 13*x^2 + 5*x - 4
    over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 2 over the Rational Field
*]
[* 43, 215, 215, 215, 215, 430, 430 *]
```

Not bielliptic, $n(a_3 = 0; 9) = 36 - 32$.

$X_0(430)/ < w_5, w_{43} >$**, genus 9**

```
[*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 + (-a - 1)*q^5 + a*q^6 + (-4*a + 2)*q^7 + q^8 + (a -
        2)*q^9 + (-a - 1)*q^10 + (4*a - 4)*q^11 + a*q^12 + (4*a - 2)*q^13 +
        (-4*a + 2)*q^14 + (-2*a - 1)*q^15 + q^16 - a*q^17 + (a - 2)*q^18 + (a +
        5)*q^19 + O(q^20),
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q - 2*q^4 - q^5 - 2*q^7 - 3*q^9 - q^11 - q^13 + 4*q^16 - 3*q^17 - 2*q^19 +
        O(q^20),
    q - q^2 + q^4 - q^5 + q^7 - q^8 - 3*q^9 + q^10 - 4*q^11 - q^13 - q^14 + q^16
        + 3*q^18 + q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 - q^5 + a*q^6 + q^7 + q^8 + 3*q^9 - q^10 + (-a +
        2)*q^11 + a*q^12 - q^13 + q^14 - a*q^15 + q^16 - a*q^17 + 3*q^18 + (-2*a
        + 1)*q^19 + O(q^20),
    q - 2*q^4 - q^5 - 2*q^7 - 3*q^9 - q^11 - q^13 + 4*q^16 - 3*q^17 - 2*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Rational Field,
```

```
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 6 over the Rational Field,
    Rational Field
*]
[* 43, 86, 86, 215, 430, 430, 430 *]
```

Not bielliptic, $n(a_3 = 0; 9) = 41 - 32$.

genus 7 $X_0(N)/ < W_{10}, W_{86}, W_{215} >$

```
[ <2, 1>, <5, 1>, <43, 1> ] 430 [*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (-a + 2)*q^5 - 2*q^6 + (a - 2)*q^7 - 2*a*q^8 - q^9 +
        (2*a - 2)*q^10 + (2*a - 1)*q^11 + (2*a + 1)*q^13 + (-2*a + 2)*q^14 +
        (-2*a + 2)*q^15 - 4*q^16 + (2*a + 5)*q^17 - a*q^18 + (-2*a - 2)*q^19 +
        O(q^20),
    q - 2*q^4 - q^5 - 2*q^7 - 3*q^9 - q^11 - q^13 + 4*q^16 - 3*q^17 - 2*q^19 +
        O(q^20),
    q - q^2 + q^4 - q^5 + q^7 - q^8 - 3*q^9 + q^10 - 4*q^11 - q^13 - q^14 + q^16
        + 3*q^18 + q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 + q^5 + a*q^6 + q^7 + q^8 - q^9 + q^10 + (-a + 2)*q^11
        + a*q^12 + (-2*a + 1)*q^13 + q^14 + a*q^15 + q^16 + a*q^17 - q^18 - q^19
        + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field
*]
[* 43, 43, 215, 430, 430 *] x^14 +
2.000000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000000*x^13 + 17.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^12 +
    28.000000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^11 + 133.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^10 +
    182.000000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^9 + 621.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000001*x^8 +
    696.000000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000001*x^7 + 1863.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^6 +
    1638.000000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^5 + 3591.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^4 +
    2268.000000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^3 + 4131.000000000000000000000000000000000000\
```

```
    000000000000000000000000000000000000000000000000000000000000*x^2 +
    1458.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x + 2187.000000000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000
1 [* 6, 40, 18, 68, 246, 616, 2274, 6836, 19494, 59560, 176754,
525956, 1596822, 4804840, 14347458, 42985076, 129120582, 387546664,
1162313682, 3486683588 *] 430 [* 0, 1, 0, 1, 0, 0, 0, 0, 0, 0 *] 0 0
3 [* 6, 40, 18, 68, 246, 616, 2274, 6836, 19494, 59560, 176754,
525956, 1596822, 4804840, 14347458, 42985076, 129120582, 387546664,
1162313682, 3486683588 *] [* 8, 32, 56, 128, 488, 1568, 4376, 12800,
39368, 119072, 354296, 1059968, 3188648, 9574688, 28697816,
86067200, 258280328, 774919712, 2324522936, 6973332608 *]
```

Not bielliptic, $n(a_3 = 0; 9) = 40 - 32$, $n(a_3 = -2; 9) = 40 - 24$.

genus 8, $X_0(N)/ < W_2, W_{215}, W_{430} >$

```
[*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (-a + 2)*q^5 - 2*q^6 + (a - 2)*q^7 - 2*a*q^8 - q^9 +
        (2*a - 2)*q^10 + (2*a - 1)*q^11 + (2*a + 1)*q^13 + (-2*a + 2)*q^14 +
        (-2*a + 2)*q^15 - 4*q^16 + (2*a + 5)*q^17 - a*q^18 + (-2*a - 2)*q^19 +
        O(q^20),
    q - q^2 + a*q^3 + q^4 + (-a + 1)*q^5 - a*q^6 + 2*q^7 - q^8 + (-a + 2)*q^9 +
        (a - 1)*q^10 + a*q^12 + 2*q^13 - 2*q^14 + (2*a - 5)*q^15 + q^16 + (a -
        4)*q^17 + (a - 2)*q^18 + (-3*a - 1)*q^19 + O(q^20),
    q - 2*q^4 - q^5 - 2*q^7 - 3*q^9 - q^11 - q^13 + 4*q^16 - 3*q^17 - 2*q^19 +
        O(q^20),
    q - q^2 + q^4 - q^5 + q^7 - q^8 - 3*q^9 + q^10 - 4*q^11 - q^13 - q^14 + q^16
        + 3*q^18 + q^19 + O(q^20),
    q - q^2 + q^4 + q^5 - 3*q^7 - q^8 - 3*q^9 - q^10 - 3*q^13 + 3*q^14 + q^16 -
        4*q^17 + 3*q^18 - q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 5 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 43, 43, 86, 215, 430, 430 *] x^16 +
3.000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^15 + 19.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000*x^14 +
    47.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^13 + 168.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000*x^12 +
    347.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000001*x^11 + 908.999999999999999999999999999999999999\
```

```
    99999999999999999999999999999999999999999999999999999999*x^10 +
    1575.000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^9 + 3294.000000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000*x^8 +
    4725.000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^7 + 8181.000000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000*x^6 +
    9368.999999999999999999999999999999999999999999999999999999999999999999999\
    999999999999999999999998*x^5 + 13608.000000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000*x^4 +
    11421.000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^3 + 13851.000000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000*x^2 +
    6560.999999999999999999999999999999999999999999999999999999999999999999999\
    999999999999999999999999*x + 6561.000000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000
1 [* 7, 39, 25, 71, 202, 747, 2191, 6359, 20005, 60354, 177151,
523679, 1594093, 4805931, 14347150, 43015319, 129111403, 387511263,
1162319269, 3486338186 *] 430 [* 1, 0, 0, 0, 1, 1, 0, 0, 0, 0 *] 5 6
3 [* 7, 39, 25, 71, 202, 747, 2191, 6359, 20005, 60354, 177151,
523679, 1594093, 4805931, 14347150, 43015319, 129111403, 387511263,
1162319269, 3486338186 *] [* 8, 32, 56, 128, 488, 1568, 4376, 12800,
39368, 119072, 354296, 1059968, 3188648, 9574688, 28697816,
86067200, 258280328, 774919712, 2324522936, 6973332608 *]
```

Not bielliptic, $n(a_3 = 0; 9) = 39 - 32$, therefore $n(a_3 = -2; 9) \geq 39 - 32$.
$X_0(430)/ < W_5, W_{86}, W_{430} >$**, genus 12**

```
[*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (-a + 2)*q^5 - 2*q^6 + (a - 2)*q^7 - 2*a*q^8 - q^9 +
        (2*a - 2)*q^10 + (2*a - 1)*q^11 + (2*a + 1)*q^13 + (-2*a + 2)*q^14 +
        (-2*a + 2)*q^15 - 4*q^16 + (2*a + 5)*q^17 - a*q^18 + (-2*a - 2)*q^19 +
        O(q^20),
    q - 2*q^4 - q^5 - 2*q^7 - 3*q^9 - q^11 - q^13 + 4*q^16 - 3*q^17 - 2*q^19 +
        O(q^20),
    q + a*q^2 + (a^5 - 2*a^4 - 6*a^3 + 9*a^2 + 6*a - 2)*q^3 + (a^2 - 2)*q^4 -
        q^5 + (a^5 - a^4 - 8*a^3 + 3*a^2 + 15*a + 3)*q^6 + (-2*a^5 + 3*a^4 +
        13*a^3 - 12*a^2 - 16*a + 2)*q^7 + (a^3 - 4*a)*q^8 + (2*a^5 - 3*a^4 -
        13*a^3 + 10*a^2 + 16*a + 7)*q^9 - a*q^10 + (-3*a^5 + 3*a^4 + 23*a^3 -
        9*a^2 - 38*a - 9)*q^11 + (a^4 - 2*a^3 - 6*a^2 + 8*a + 7)*q^12 + (-2*a +
        2)*q^13 + (-3*a^5 + 3*a^4 + 22*a^3 - 10*a^2 - 32*a - 6)*q^14 + (-a^5 +
        2*a^4 + 6*a^3 - 9*a^2 - 6*a + 2)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + (4*a^5
        - 4*a^4 - 30*a^3 + 12*a^2 + 48*a + 12)*q^17 + (3*a^5 - 3*a^4 - 24*a^3 +
        10*a^2 + 41*a + 6)*q^18 + (2*a^5 - 2*a^4 - 16*a^3 + 6*a^2 + 28*a +
        8)*q^19 + O(q^20),
    q - q^2 + q^4 - q^5 + q^7 - q^8 - 3*q^9 + q^10 - 4*q^11 - q^13 - q^14 + q^16
        + 3*q^18 + q^19 + O(q^20),
    q + q^2 - 2*q^3 + q^4 - q^5 - 2*q^6 - q^7 + q^8 + q^9 - q^10 - 6*q^11 -
        2*q^12 + 5*q^13 - q^14 + 2*q^15 + q^16 - 6*q^17 + q^18 - 7*q^19 +
```

```
        O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^6 - 3*x^5 - 5*x^4 + 17*x^3 + 3*x^2 -
        17*x - 3 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 43, 43, 215, 215, 430, 430 *] x^24 -
9.14389913025819985772626838644272767561527995858238928972100674582106 10\
    87253905565403584174881376237E-99*x^23 +
    17.00000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000*x^22 - 6.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000137*x^21 +
    138.00000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000*x^20 - 110.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000010*x^19 +
    742.00000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000004*x^18 - 928.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000047*x^17 +
    3123.00000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000004*x^16 - 4928.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000019*x^15 +
    11305.00000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000002*x^14 - 19008.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000007*x^13 +
    36132.00000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000006*x^12 - 57024.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000019*x^11 +
    101745.00000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000001*x^10 - 133056.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000005*x^9 +
    252963.00000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000003*x^8 - 225504.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000011*x^7 +
    540918.00000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000003*x^6 - 240570.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000020*x^5 +
    905417.99999999999999999999999999999999999999999999999999999999999999999999\
    9999999999999999999999999*x^4 - 118098.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000026*x^3 +
    1003833.00000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000*x^2 - 1.61798734818162374186506155742955862256 26\
    32065887269754126120924854148460418534278862581089709569857E-93*x +
    531441.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000000
```

```
1 [* 4, 44, 10, 56, 204, 824, 2440, 6088, 19180, 60704, 178666,
528272, 1601344, 4804886, 14344680, 43025976, 129095046, 387464648,
1162360724, 3486361336 *] 430 [* 0, 0, 0, 1, 0, 1, 0, 0, 0, 0 *] 0 0
3 [* 4, 44, 10, 56, 204, 824, 2440, 6088, 19180, 60704, 178666,
528272, 1601344, 4804886, 14344680, 43025976, 129095046, 387464648,
1162360724, 3486361336 *] [* 8, 32, 56, 128, 488, 1568, 4376, 12800,
39368, 119072, 354296, 1059968, 3188648, 9574688, 28697816,
86067200, 258280328, 774919712, 2324522936, 6973332608 *]
```

Not bielliptic $n(a_3; 9) \geq n(a_3 = 0; 9) = 44 - 32$.

genus 15, $X_0(N)/ < W_{43}, W_{10}, W_{430} >$

Cal retocar el programa Magma. En calcula una mes.

```
[*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 + (-a - 1)*q^5 + a*q^6 + (-4*a + 2)*q^7 + q^8 + (a -
        2)*q^9 + (-a - 1)*q^10 + (4*a - 4)*q^11 + a*q^12 + (4*a - 2)*q^13 +
        (-4*a + 2)*q^14 + (-2*a - 1)*q^15 + q^16 - a*q^17 + (a - 2)*q^18 + (a +
        5)*q^19 + O(q^20),
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q - 2*q^4 - q^5 - 2*q^7 - 3*q^9 - q^11 - q^13 + 4*q^16 - 3*q^17 - 2*q^19 +
        O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (a^2 - 2)*q^4 + q^5 + (a^2 + a)*q^6 + (-a^2 - 2*a
        + 1)*q^7 + (-2*a^2 - a + 3)*q^8 + (a^2 + 2*a - 2)*q^9 + a*q^10 + (-a^2 +
        a + 7)*q^11 + (-a^2 + a + 1)*q^12 + (-2*a - 2)*q^13 + (-2*a - 3)*q^14 +
        (a + 1)*q^15 + (a^2 - 3*a - 2)*q^16 + (-2*a + 2)*q^17 + (a + 3)*q^18 +
        (-2*a^2 - 4*a + 6)*q^19 + O(q^20),
    q + a*q^2 + (-a^3 + 5*a)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^4 + 5*a^2)*q^6 +
        (a^4 - a^3 - 6*a^2 + 6*a + 2)*q^7 + (a^3 - 4*a)*q^8 + (a^4 + a^3 - 6*a^2
        - 6*a + 5)*q^9 + a*q^10 + (a^3 - 6*a - 1)*q^11 + (-2*a^4 + 13*a^2 - 5*a
        - 4)*q^12 + (-a^4 + 5*a^2 + a + 3)*q^13 + (a^4 + a^3 - 7*a^2 - 3*a +
        4)*q^14 + (-a^3 + 5*a)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + (a^4 - 7*a^2 + a
        + 1)*q^17 + (3*a^4 + a^3 - 19*a^2 + 4)*q^18 + (-2*a^4 + 14*a^2 - 2*a -
        10)*q^19 + O(q^20),
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q - q^2 + q^4 - q^5 + q^7 - q^8 - 3*q^9 + q^10 - 4*q^11 - q^13 - q^14 + q^16
        + 3*q^18 + q^19 + O(q^20),
    q + q^2 - 2*q^3 + q^4 + q^5 - 2*q^6 - 5*q^7 + q^8 + q^9 + q^10 - 2*q^11 -
        2*q^12 - 5*q^13 - 5*q^14 - 2*q^15 + q^16 + 2*q^17 + q^18 + 3*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 + 2*x^2 - 3*x - 3 over the
    Rational Field,
```

```
    Number Field with defining polynomial x^5 - 2*x^4 - 7*x^3 + 13*x^2 + 5*x - 4
    over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 43, 86, 86, 215, 215, 215, 215, 430, 430 *]
```

3. Level $N = 430$

$X_0(N)/ < W_{10}, W_{86}, W_{215} >$, **genus 7**

```
[*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (-a + 2)*q^5 - 2*q^6 + (a - 2)*q^7 - 2*a*q^8 - q^9 +
        (2*a - 2)*q^10 + (2*a - 1)*q^11 + (2*a + 1)*q^13 + (-2*a + 2)*q^14 +
        (-2*a + 2)*q^15 - 4*q^16 + (2*a + 5)*q^17 - a*q^18 + (-2*a - 2)*q^19 +
        O(q^20),
    q - 2*q^4 - q^5 - 2*q^7 - 3*q^9 - q^11 - q^13 + 4*q^16 - 3*q^17 - 2*q^19 +
        O(q^20),
    q - q^2 + q^4 - q^5 + q^7 - q^8 - 3*q^9 + q^10 - 4*q^11 - q^13 - q^14 + q^16
        + 3*q^18 + q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 + q^5 + a*q^6 + q^7 + q^8 - q^9 + q^10 + (-a + 2)*q^11
        + a*q^12 + (-2*a + 1)*q^13 + q^14 + a*q^15 + q^16 + a*q^17 - q^18 - q^19
        + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field
*]
[* 43, 43, 215, 430, 430 *]
```

It is not bielliptic $n(a_3 = 0; 9) = 40-32$ and strongly $n(a_3 = -3, 9)$ is bigger. In particular $X_0(N)/ < W_{10} >$, $X_0(N)/ < W_{86} >$, $W_0(N)/ < W_{215} >$ are not bielliptic.

$X_0(430)/ < W_2, W_{215}, W_{430} >$, **genus 8**

```
430 [*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (-a + 2)*q^5 - 2*q^6 + (a - 2)*q^7 - 2*a*q^8 - q^9 +
        (2*a - 2)*q^10 + (2*a - 1)*q^11 + (2*a + 1)*q^13 + (-2*a + 2)*q^14 +
        (-2*a + 2)*q^15 - 4*q^16 + (2*a + 5)*q^17 - a*q^18 + (-2*a - 2)*q^19 +
        O(q^20),
    q - q^2 + a*q^3 + q^4 + (-a + 1)*q^5 - a*q^6 + 2*q^7 - q^8 + (-a + 2)*q^9 +
        (a - 1)*q^10 + a*q^12 + 2*q^13 - 2*q^14 + (2*a - 5)*q^15 + q^16 + (a -
        4)*q^17 + (a - 2)*q^18 + (-3*a - 1)*q^19 + O(q^20),
    q - 2*q^4 - q^5 - 2*q^7 - 3*q^9 - q^11 - q^13 + 4*q^16 - 3*q^17 - 2*q^19 +
        O(q^20),
    q - q^2 + q^4 - q^5 + q^7 - q^8 - 3*q^9 + q^10 - 4*q^11 - q^13 - q^14 + q^16
```

```
        + 3*q^18 + q^19 + O(q^20),
    q - q^2 + q^4 + q^5 - 3*q^7 - q^8 - 3*q^9 - q^10 - 3*q^13 + 3*q^14 + q^16 -
        4*q^17 + 3*q^18 - q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 5 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 43, 43, 86, 215, 430, 430 *] x^16 +
3.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000000*x^15 + 19.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000*x^14 +
    47.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000000*x^13 + 168.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000*x^12 +
    347.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000001*x^11 + 908.9999999999999999999999999999999999\
    9999999999999999999999999999999999999999999999999999999999999999999999*x^10 +
    1575.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000000*x^9 + 3294.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000*x^8 +
    4725.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000000*x^7 + 8181.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000*x^6 +
    9368.9999999999999999999999999999999999999999999999999999999999999999999999999999\
    9999999999999999999999998*x^5 + 13608.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000*x^4 +
    11421.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000000*x^3 + 13851.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000*x^2 +
    6560.9999999999999999999999999999999999999999999999999999999999999999999999999999\
    9999999999999999999999999*x + 6561.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000
1 [* 7, 39, 25, 71, 202, 747, 2191, 6359, 20005, 60354, 177151,
523679, 1594093, 4805931, 14347150, 43015319, 129111403, 387511263,
1162319269, 3486338186 *] 430 [* 1, 0, 0, 0, 1, 1, 0, 0, 0, 0 *] 5 6
3 [* 7, 39, 25, 71, 202, 747, 2191, 6359, 20005, 60354, 177151,
523679, 1594093, 4805931, 14347150, 43015319, 129111403, 387511263,
1162319269, 3486338186 *] [* 8, 32, 56, 128, 488, 1568, 4376, 12800,
39368, 119072, 354296, 1059968, 3188648, 9574688, 28697816,
86067200, 258280328, 774919712, 2324522936,
```

Observe that $n(a_3 = 0, 9) = 39 - 32$ in particular $n(a_3 = 0, 9) > 7$, therefore is not bielltiptic and by Harris-Silvermen we have $X_0(N)/ < W_{430} >, X_0(N)/W_2$, not bielliptic.

$X_0(430)/ < W_5, W_{86}, W_{430} >$**, genus 12**

```
430 [*
```

```
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (-a + 2)*q^5 - 2*q^6 + (a - 2)*q^7 - 2*a*q^8 - q^9 +
        (2*a - 2)*q^10 + (2*a - 1)*q^11 + (2*a + 1)*q^13 + (-2*a + 2)*q^14 +
        (-2*a + 2)*q^15 - 4*q^16 + (2*a + 5)*q^17 - a*q^18 + (-2*a - 2)*q^19 +
        O(q^20),
    q - 2*q^4 - q^5 - 2*q^7 - 3*q^9 - q^11 - q^13 + 4*q^16 - 3*q^17 - 2*q^19 +
        O(q^20),
    q + a*q^2 + (a^5 - 2*a^4 - 6*a^3 + 9*a^2 + 6*a - 2)*q^3 + (a^2 - 2)*q^4 -
        q^5 + (a^5 - a^4 - 8*a^3 + 3*a^2 + 15*a + 3)*q^6 + (-2*a^5 + 3*a^4 +
        13*a^3 - 12*a^2 - 16*a + 2)*q^7 + (a^3 - 4*a)*q^8 + (2*a^5 - 3*a^4 -
        13*a^3 + 10*a^2 + 16*a + 7)*q^9 - a*q^10 + (-3*a^5 + 3*a^4 + 23*a^3 -
        9*a^2 - 38*a - 9)*q^11 + (a^4 - 2*a^3 - 6*a^2 + 8*a + 7)*q^12 + (-2*a +
        2)*q^13 + (-3*a^5 + 3*a^4 + 22*a^3 - 10*a^2 - 32*a - 6)*q^14 + (-a^5 +
        2*a^4 + 6*a^3 - 9*a^2 - 6*a + 2)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + (4*a^5
        - 4*a^4 - 30*a^3 + 12*a^2 + 48*a + 12)*q^17 + (3*a^5 - 3*a^4 - 24*a^3 +
        10*a^2 + 41*a + 6)*q^18 + (2*a^5 - 2*a^4 - 16*a^3 + 6*a^2 + 28*a +
        8)*q^19 + O(q^20),
    q - q^2 + q^4 - q^5 + q^7 - q^8 - 3*q^9 + q^10 - 4*q^11 - q^13 - q^14 + q^16
        + 3*q^18 + q^19 + O(q^20),
    q + q^2 - 2*q^3 + q^4 - q^5 - 2*q^6 - q^7 + q^8 + q^9 - q^10 - 6*q^11 -
        2*q^12 + 5*q^13 - q^14 + 2*q^15 + q^16 - 6*q^17 + q^18 - 7*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^6 - 3*x^5 - 5*x^4 + 17*x^3 + 3*x^2 -
        17*x - 3 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 43, 43, 215, 215, 430, 430 *] x^24 -
9.14389913025819985772626838644272767561527995858238928972100674582106105\
    87253905565403584174881376237E-99*x^23 +
    17.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000000*x^22 - 6.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000000000137*x^21 +
    138.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000000*x^20 - 110.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000000000010*x^19 +
    742.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000004*x^18 - 928.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000000000047*x^17 +
    3123.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000004*x^16 - 4928.00000000000000000000000000000000000000\
    00000000000000000000000000000000000000000000000000000019*x^15 +
    11305.00000000000000000000000000000000000000000000000000000000000000000000\
```

```
    000000000000000000000000002*x^14 - 19008.000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000007*x^13 +
    36132.000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000006*x^12 - 57024.000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000019*x^11 +
    101745.000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000001*x^10 - 133056.000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000005*x^9 +
    252963.000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000003*x^8 - 225504.000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000011*x^7 +
    540918.000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000003*x^6 - 240570.000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000020*x^5 +
    905417.999999999999999999999999999999999999999999999999999999999999999999\
    999999999999999999999999*x^4 - 118098.000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000026*x^3 +
    1003833.000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^2 - 1.617987348181623741865061557429558622562 6\
    320658872697541261209248541484604185342788625810897095698 57E-93*x +
    531441.000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000
1 [* 4, 44, 10, 56, 204, 824, 2440, 6088, 19180, 60704, 178666,
528272, 1601344, 4804886, 14344680, 43025976, 129095046, 387464648,
1162360724, 3486361336 *] 430 [* 0, 0, 0, 1, 0, 1, 0, 0, 0, 0 *] 0 0
3 [* 4, 44, 10, 56, 204, 824, 2440, 6088, 19180, 60704, 178666,
528272, 1601344, 4804886, 14344680, 43025976, 129095046, 387464648,
1162360724, 3486361336 *] [* 8, 32, 56, 128, 488, 1568, 4376, 12800,
39368, 119072, 354296, 1059968, 3188648, 9574688, 28697816,
86067200, 258280328, 774919712, 2324522936, 6973332608 *]
```

Thus is not BIelliptic because $n(a_3 = 0; 9) = 44 - 32$, thus $n(a_3, 9) \geq 12$. By Harris-Silverman we obtain $X_0^*(430)/W_5$ is not bielliptic.

$X_0(N)/ < W_{43}, W_{10}, W_{430} >$, **genus 15**

```
 [ <2, 1>, <5, 1>, <43, 1> ]
430 [* 43, 86, 86, 215, 215, 215, 215, 430, 430 *] [*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 + (-a - 1)*q^5 + a*q^6 + (-4*a + 2)*q^7 + q^8 + (a -
        2)*q^9 + (-a - 1)*q^10 + (4*a - 4)*q^11 + a*q^12 + (4*a - 2)*q^13 +
        (-4*a + 2)*q^14 + (-2*a - 1)*q^15 + q^16 - a*q^17 + (a - 2)*q^18 + (a +
        5)*q^19 + O(q^20),
    q - 2*q^4 - q^5 - 2*q^7 - 3*q^9 - q^11 - q^13 + 4*q^16 - 3*q^17 - 2*q^19 +
        O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (a^2 - 2)*q^4 + q^5 + (a^2 + a)*q^6 + (-a^2 - 2*a
        + 1)*q^7 + (-2*a^2 - a + 3)*q^8 + (a^2 + 2*a - 2)*q^9 + a*q^10 + (-a^2 +
        a + 7)*q^11 + (-a^2 + a + 1)*q^12 + (-2*a - 2)*q^13 + (-2*a - 3)*q^14 +
        (a + 1)*q^15 + (a^2 - 3*a - 2)*q^16 + (-2*a + 2)*q^17 + (a + 3)*q^18 +
        (-2*a^2 - 4*a + 6)*q^19 + O(q^20),
```

```
    q + a*q^2 + (-a^3 + 5*a)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^4 + 5*a^2)*q^6 +
        (a^4 - a^3 - 6*a^2 + 6*a + 2)*q^7 + (a^3 - 4*a)*q^8 + (a^4 + a^3 - 6*a^2
        - 6*a + 5)*q^9 + a*q^10 + (a^3 - 6*a - 1)*q^11 + (-2*a^4 + 13*a^2 - 5*a
        - 4)*q^12 + (-a^4 + 5*a^2 + a + 3)*q^13 + (a^4 + a^3 - 7*a^2 - 3*a +
        4)*q^14 + (-a^3 + 5*a)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + (a^4 - 7*a^2 + a
        + 1)*q^17 + (3*a^4 + a^3 - 19*a^2 + 4)*q^18 + (-2*a^4 + 14*a^2 - 2*a -
        10)*q^19 + O(q^20),
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q - q^2 + q^4 - q^5 + q^7 - q^8 - 3*q^9 + q^10 - 4*q^11 - q^13 - q^14 + q^16
        + 3*q^18 + q^19 + O(q^20),
    q + q^2 - 2*q^3 + q^4 + q^5 - 2*q^6 - 5*q^7 + q^8 + q^9 + q^10 - 2*q^11 -
        2*q^12 - 5*q^13 - 5*q^14 - 2*q^15 + q^16 + 2*q^17 + q^18 + 3*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 + 2*x^2 - 3*x - 3 over the
    Rational Field,
    Number Field with defining polynomial x^5 - 2*x^4 - 7*x^3 + 13*x^2 + 5*x - 4
    over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
x^30 +
4.999999999999999999999999999999999999999999999999999999999999999999999999\
    999999999999999999999999998*x^29 + 29.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000*x^28 +
    101.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^27 + 362.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000001*x^26 +
    1015.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^25 + 2859.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000002*x^24 +
    6955.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000005*x^23 + 16840.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000002*x^22 +
    36851.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000004*x^21 + 79961.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000013*x^20 +
    160379.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000003*x^19 + 319168.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000007*x^18 +
    593873.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000014*x^17 + 1098414.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000003*x^16 +
```

```
    1907562.000000000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000005*x^15 + 3295242.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000008*x^14 +
    5344857.000000000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000012*x^13 + 8617536.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000019*x^12 +
    12990699.000000000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000002*x^11 + 19430523.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000003*x^10 +
    26864379.000000000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000003*x^9 + 36829080.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000004*x^8 +
    45631755.000000000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000003*x^7 + 56273697.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000004*x^6 +
    59934735.000000000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000002*x^5 + 64127214.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000003*x^4 +
    53675541.000000000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000000*x^3 + 46235367.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000*x^2 +
    23914844.999999999999999999999999999999999999999999999999999999999999999999999999\
    999999999999999999999999*x + 14348907.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000
1 [* 9, 43, 21, 103, 274, 811, 2221, 6119, 18543, 62778, 176515,
522103, 1600491, 4782289, 14355406, 43060439, 129069415, 387473383,
1162660663, 3486475338 *] 430 [* 1, 1, 0, 1, 1, 0, 0, 0, 0, 1 *] 5 6
3 [* 9, 43, 21, 103, 274, 811, 2221, 6119, 18543, 62778, 176515,
522103, 1600491, 4782289, 14355406, 43060439, 129069415, 387473383,
1162660663, 3486475338 *] [* 8, 32, 56, 128, 488, 1568, 4376, 12800,
39368, 119072, 354296, 1059968, 3188648, 9574688, 28697816,
86067200, 258280328, 774919712, 2324522936, 6973332608 *]
```

Not bielliptic $n(a_3; 9) \geq 43 - 32$, and by Harris-Silverman we have $X_0(N)/W_{43}$ is not bielliptic.

4. Level, N=370

4.1. $X_0(370)/ < w_2, w_5, w_{10} >$**. genus 13,** $X_0(370)/ < w_2, w_5, w_{10} >$**:**

```
[*
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
        2*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + (-a + 1)*q^5 - a*q^6 + (-2*a + 4)*q^7 - q^8 + (3*a -
        2)*q^9 + (a - 1)*q^10 + (-a + 1)*q^11 + a*q^12 + (a - 2)*q^13 + (2*a -
        4)*q^14 + (-2*a - 1)*q^15 + q^16 - 6*q^17 + (-3*a + 2)*q^18 + 2*q^19 +
        O(q^20),
    q + q^2 - 2*q^3 - q^4 - q^5 - 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^12 -
        2*q^13 - 2*q^14 + 2*q^15 - q^16 + 2*q^17 + q^18 + 2*q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 - q^5 - 2*q^6 - 5*q^7 - 2*q^9 + 2*q^10 + 3*q^11 +
```

```
        2*q^12 - 2*q^13 + 10*q^14 - q^15 - 4*q^16 - 4*q^17 + 4*q^18 - 4*q^19 +
        O(q^20),
    q + a*q^2 + 1/2*(-a^3 + 5*a + 2)*q^3 + (a^2 - 2)*q^4 - q^5 + 1/2*(-a^4 +
        5*a^2 + 2*a)*q^6 + 1/2*(a^4 - 7*a^2 - 2*a + 10)*q^7 + (a^3 - 4*a)*q^8 +
        1/2*(a^4 - a^3 - 7*a^2 + 5*a + 8)*q^9 - a*q^10 + (-a^2 + 3)*q^11 +
        1/2*(-2*a^4 - a^3 + 16*a^2 + a - 16)*q^12 + 1/2*(-a^4 + a^3 + 5*a^2 -
        5*a + 4)*q^13 + 1/2*(2*a^4 + a^3 - 16*a^2 - a + 12)*q^14 + 1/2*(a^3 -
        5*a - 2)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + 1/2*(-a^4 + a^3 + 9*a^2 - 9*a -
        12)*q^17 + 1/2*(a^4 + a^3 - 9*a^2 - 3*a + 12)*q^18 + 1/2*(-2*a^4 + a^3 +
        16*a^2 - 5*a - 20)*q^19 + O(q^20),
    q - q^2 + q^4 - q^5 - q^8 - 3*q^9 + q^10 - 4*q^11 + 2*q^13 + q^16 - 2*q^17 +
        3*q^18 - 4*q^19 + O(q^20),
    q - q^2 - 2*q^3 + q^4 - q^5 + 2*q^6 - q^7 - q^8 + q^9 + q^10 + 3*q^11 -
        2*q^12 - 4*q^13 + q^14 + 2*q^15 + q^16 + 3*q^17 - q^18 + 2*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 3*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^5 - 2*x^4 - 8*x^3 + 14*x^2 + 11*x -
        12 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 37, 37, 74, 185, 185, 185, 370, 370 *]
```

No bielliptic because $n(a_3 = -3; 9) = 37 - 14, n(a_3 = -2; 9) = 37 - 24, n(a_3 = 1; 9) = 37 - 30, n(a_3 = 0; 9) = 37 - 32$.

Therefore $X_0(370)/w_2$, $X_0(370)/w_5$ and $X_0(370)/w_{10}$ are not bielliptic.(Silverman-Harris prop.).

4.2. $X_0(370)/ < w_2, w_{37}, w_{74} >$. $X_0(370)/ < w_2, w_{37}, w_{74} >$**, genus 11:**

```
[*
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^2 - 2*q^3 - q^4 - q^5 - 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^12 -
        2*q^13 - 2*q^14 + 2*q^15 - q^16 + 2*q^17 + q^18 + 2*q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 - q^5 - 2*q^6 - 5*q^7 - 2*q^9 + 2*q^10 + 3*q^11 +
        2*q^12 - 2*q^13 + 10*q^14 - q^15 - 4*q^16 - 4*q^17 + 4*q^18 - 4*q^19 +
        O(q^20),
    q + a*q^2 + 1/2*(-a^4 + 7*a^2 - 2*a - 6)*q^3 + (a^2 - 2)*q^4 + q^5 +
        1/2*(-a^3 + 5*a - 2)*q^6 + 1/2*(-a^3 - 2*a^2 + 5*a + 8)*q^7 + (a^3 -
        4*a)*q^8 + 1/2*(a^4 + a^3 - 9*a^2 - 5*a + 14)*q^9 + a*q^10 + (a^4 + a^3
        - 6*a^2 - 3*a + 5)*q^11 + 1/2*(a^4 - 9*a^2 + 2*a + 12)*q^12 + 1/2*(-a^4
        - a^3 + 7*a^2 + a - 6)*q^13 + 1/2*(-a^4 - 2*a^3 + 5*a^2 + 8*a)*q^14 +
        1/2*(-a^4 + 7*a^2 - 2*a - 6)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + 1/2*(-a^4 -
        a^3 + 7*a^2 + 5*a - 10)*q^17 + 1/2*(a^4 - a^3 - 7*a^2 + 3*a + 2)*q^18 +
        1/2*(-a^4 - 2*a^3 + 5*a^2 + 4*a + 4)*q^19 + O(q^20),
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
```

```
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q - q^2 + q^4 - q^5 - q^8 - 3*q^9 + q^10 - 4*q^11 + 2*q^13 + q^16 - 2*q^17 +
        3*q^18 - 4*q^19 + O(q^20),
    q - q^2 + 2*q^3 + q^4 + q^5 - 2*q^6 + q^7 - q^8 + q^9 - q^10 + 3*q^11 +
        2*q^12 - q^14 + 2*q^15 + q^16 + 3*q^17 - q^18 - 6*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^5 - 8*x^3 + 2*x^2 + 11*x - 2 over
    the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 37, 185, 185, 185, 185, 370, 370 *]
```

Not bielliptic, $n(a_3 = -3; 9) = 32 - 14, n(a_3 = 2, -2; 9) = 32 - 24, n(a_3 = 0; 9) = 10 - 8, n(a_3 = 1; 3) = 10 - 6$. Therefore $X_0(370)/w_{37}$ and $X_0(370)/w_{74}$ are not bielliptic (Harris-Silverman).

4.3. $X_0(370)/ < w_5, w_{37}, w_{185} >$**.** Genus 12.

```
[*
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^2 + a*q^3 + q^4 + (-3*a - 1)*q^5 + a*q^6 + 2*a*q^7 + q^8 + (-a -
        2)*q^9 + (-3*a - 1)*q^10 + (-a - 3)*q^11 + a*q^12 + (3*a + 2)*q^13 +
        2*a*q^14 + (2*a - 3)*q^15 + q^16 + (4*a + 2)*q^17 + (-a - 2)*q^18 +
        (-4*a - 2)*q^19 + O(q^20),
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^2 - 2*q^3 - q^4 - q^5 - 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^12 -
        2*q^13 - 2*q^14 + 2*q^15 - q^16 + 2*q^17 + q^18 + 2*q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 - q^5 - 2*q^6 - 5*q^7 - 2*q^9 + 2*q^10 + 3*q^11 +
        2*q^12 - 2*q^13 + 10*q^14 - q^15 - 4*q^16 - 4*q^17 + 4*q^18 - 4*q^19 +
        O(q^20),
    q - q^2 + q^4 - q^5 - q^8 - 3*q^9 + q^10 - 4*q^11 + 2*q^13 + q^16 - 2*q^17 +
        3*q^18 - 4*q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 - q^5 + a*q^6 + 1/2*(-a^2 + 6)*q^7 + q^8 + (a^2 -
        3)*q^9 - q^10 + 1/2*(-a^2 - 2*a + 14)*q^11 + a*q^12 - 2*a*q^13 +
        1/2*(-a^2 + 6)*q^14 - a*q^15 + q^16 + 1/2*(a^2 + 2*a - 6)*q^17 + (a^2 -
        3)*q^18 + a*q^19 + O(q^20),
    q + q^2 - 2*q^3 - q^4 - q^5 - 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^12 -
        2*q^13 - 2*q^14 + 2*q^15 - q^16 + 2*q^17 + q^18 + 2*q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 - q^5 - 2*q^6 - 5*q^7 - 2*q^9 + 2*q^10 + 3*q^11 +
        2*q^12 - 2*q^13 + 10*q^14 - q^15 - 4*q^16 - 4*q^17 + 4*q^18 - 4*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
```

```
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - 10*x + 4 over the Rational
    Field,
    Rational Field,
    Rational Field
*]
[* 37, 74, 74, 185, 185, 370, 370, 370, 370 *]
```

Not bielliptic, $n(a_3 = -3; 9) = 31 - 14, n(a_3 = -2; 9) = 31 - 24, n(a_3 = 1; 3) = 13 - 6, n(a_3 = 0; 3) = 13 - 8$. Therefore $X_0(370)/w_{185}$ is not bielliptic.

4.4. $X_0(370)/ < w_{10}, w_{74}, w_{740} >$. genus 9;

```
[*
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
        2*q^19 + O(q^20),
    q + q^2 - 2*q^3 - q^4 - q^5 - 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^12 -
        2*q^13 - 2*q^14 + 2*q^15 - q^16 + 2*q^17 + q^18 + 2*q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 - q^5 - 2*q^6 - 5*q^7 - 2*q^9 + 2*q^10 + 3*q^11 +
        2*q^12 - 2*q^13 + 10*q^14 - q^15 - 4*q^16 - 4*q^17 + 4*q^18 - 4*q^19 +
        O(q^20),
    q - q^3 - 2*q^4 + q^5 - 3*q^7 - 2*q^9 - 5*q^11 + 2*q^12 + 4*q^13 - q^15 +
        4*q^16 - 4*q^17 - 8*q^19 + O(q^20),
    q - q^2 + q^4 - q^5 - q^8 - 3*q^9 + q^10 - 4*q^11 + 2*q^13 + q^16 - 2*q^17 +
        3*q^18 - 4*q^19 + O(q^20),
    q + q^2 - 2*q^3 + q^4 + q^5 - 2*q^6 + 2*q^7 + q^8 + q^9 + q^10 - 2*q^12 +
        2*q^13 + 2*q^14 - 2*q^15 + q^16 + 6*q^17 + q^18 + 2*q^19 + O(q^20),
    q + q^2 + 2*q^3 + q^4 + q^5 + 2*q^6 + (a - 2)*q^7 + q^8 + q^9 + q^10 -
        a*q^11 + 2*q^12 + (-2*a + 2)*q^13 + (a - 2)*q^14 + 2*q^15 + q^16 + (-a -
        2)*q^17 + q^18 - 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - x - 8 over the Rational Field
*]
[* 37, 37, 185, 185, 185, 370, 370, 370 *]
```

Not bielliptic, $n(a_3 = -3; 9) = 36-14, n(a_3 = 1, -1; 9) = 36-30, n(a_3 = -2; 9) = 36-24, n(a_3 = 0; 9) = 36-32$.

4.5. $X_0(370)/ < w_2, w_{185}, w_{370} >$. genus 10

```
[*
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
```

```
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
        2*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + (-a + 1)*q^5 - a*q^6 + (-2*a + 4)*q^7 - q^8 + (3*a -
        2)*q^9 + (a - 1)*q^10 + (-a + 1)*q^11 + a*q^12 + (a - 2)*q^13 + (2*a -
        4)*q^14 + (-2*a - 1)*q^15 + q^16 - 6*q^17 + (-3*a + 2)*q^18 + 2*q^19 +
        O(q^20),
    q + q^2 - 2*q^3 - q^4 - q^5 - 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^12 -
        2*q^13 - 2*q^14 + 2*q^15 - q^16 + 2*q^17 + q^18 + 2*q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 - q^5 - 2*q^6 - 5*q^7 - 2*q^9 + 2*q^10 + 3*q^11 +
        2*q^12 - 2*q^13 + 10*q^14 - q^15 - 4*q^16 - 4*q^17 + 4*q^18 - 4*q^19 +
        O(q^20),
    q - q^3 - 2*q^4 + q^5 - 3*q^7 - 2*q^9 - 5*q^11 + 2*q^12 + 4*q^13 - q^15 +
        4*q^16 - 4*q^17 - 8*q^19 + O(q^20),
    q - q^2 + q^4 - q^5 - q^8 - 3*q^9 + q^10 - 4*q^11 + 2*q^13 + q^16 - 2*q^17 +
        3*q^18 - 4*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + q^5 - a*q^6 + (-a - 4)*q^7 - q^8 + (-2*a - 1)*q^9 -
        q^10 + (-2*a - 4)*q^11 + a*q^12 + (-2*a - 4)*q^13 + (a + 4)*q^14 +
        a*q^15 + q^16 + (2*a + 4)*q^17 + (2*a + 1)*q^18 + (3*a + 4)*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 3*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 2 over the Rational Field
*]
[* 37, 37, 74, 185, 185, 185, 370, 370 *]
```

Not bielliptic, $n(a_3 = 0; 9) = 35 - 32, n(a_3 =; 9) \geq 35 - 32$. Therefore $X_0^+(370)$ is not bielliptic.

4.6. $X_0(370)/ < w_5, w_{74}, w_{370} >$. genus 10

```
[*
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
        2*q^19 + O(q^20),
    q + q^2 - 2*q^3 - q^4 - q^5 - 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^12 -
        2*q^13 - 2*q^14 + 2*q^15 - q^16 + 2*q^17 + q^18 + 2*q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 - q^5 - 2*q^6 - 5*q^7 - 2*q^9 + 2*q^10 + 3*q^11 +
        2*q^12 - 2*q^13 + 10*q^14 - q^15 - 4*q^16 - 4*q^17 + 4*q^18 - 4*q^19 +
        O(q^20),
    q + a*q^2 + 1/2*(-a^3 + 5*a + 2)*q^3 + (a^2 - 2)*q^4 - q^5 + 1/2*(-a^4 +
        5*a^2 + 2*a)*q^6 + 1/2*(a^4 - 7*a^2 - 2*a + 10)*q^7 + (a^3 - 4*a)*q^8 +
        1/2*(a^4 - a^3 - 7*a^2 + 5*a + 8)*q^9 - a*q^10 + (-a^2 + 3)*q^11 +
        1/2*(-2*a^4 - a^3 + 16*a^2 + a - 16)*q^12 + 1/2*(-a^4 + a^3 + 5*a^2 -
        5*a + 4)*q^13 + 1/2*(2*a^4 + a^3 - 16*a^2 - a + 12)*q^14 + 1/2*(a^3 -
```

```
        5*a - 2)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + 1/2*(-a^4 + a^3 + 9*a^2 - 9*a -
        12)*q^17 + 1/2*(a^4 + a^3 - 9*a^2 - 3*a + 12)*q^18 + 1/2*(-2*a^4 + a^3 +
        16*a^2 - 5*a - 20)*q^19 + O(q^20),
    q - q^2 + q^4 - q^5 - q^8 - 3*q^9 + q^10 - 4*q^11 + 2*q^13 + q^16 - 2*q^17 +
        3*q^18 - 4*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^5 - 2*x^4 - 8*x^3 + 14*x^2 + 11*x -
        12 over the Rational Field,
    Rational Field
*]
[* 37, 37, 185, 185, 185, 370 *]
```

Not bielliptic, $n(a_3 = 0; 9) = 34 - 32$, thus $n(a_3; 9) \geq 2$.

4.7. $X_0(370)/ < w_{37}, w_{10}, w_{370} >$. genus 12,
TINC ELIMINAR-NE UNA!!! Taules Cremona! Ja ho faig manual 37a en trec una.

```
[*
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^2 + a*q^3 + q^4 + (-3*a - 1)*q^5 + a*q^6 + 2*a*q^7 + q^8 + (-a -
        2)*q^9 + (-3*a - 1)*q^10 + (-a - 3)*q^11 + a*q^12 + (3*a + 2)*q^13 +
        2*a*q^14 + (2*a - 3)*q^15 + q^16 + (4*a + 2)*q^17 + (-a - 2)*q^18 +
        (-4*a - 2)*q^19 + O(q^20),

    q + q^2 - 2*q^3 - q^4 - q^5 - 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^12 -
        2*q^13 - 2*q^14 + 2*q^15 - q^16 + 2*q^17 + q^18 + 2*q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 - q^5 - 2*q^6 - 5*q^7 - 2*q^9 + 2*q^10 + 3*q^11 +
        2*q^12 - 2*q^13 + 10*q^14 - q^15 - 4*q^16 - 4*q^17 + 4*q^18 - 4*q^19 +
        O(q^20),
    q + a*q^2 + 1/2*(-a^4 + 7*a^2 - 2*a - 6)*q^3 + (a^2 - 2)*q^4 + q^5 +
        1/2*(-a^3 + 5*a - 2)*q^6 + 1/2*(-a^3 - 2*a^2 + 5*a + 8)*q^7 + (a^3 -
        4*a)*q^8 + 1/2*(a^4 + a^3 - 9*a^2 - 5*a + 14)*q^9 + a*q^10 + (a^4 + a^3
        - 6*a^2 - 3*a + 5)*q^11 + 1/2*(a^4 - 9*a^2 + 2*a + 12)*q^12 + 1/2*(-a^4
        - a^3 + 7*a^2 + a - 6)*q^13 + 1/2*(-a^4 - 2*a^3 + 5*a^2 + 8*a)*q^14 +
        1/2*(-a^4 + 7*a^2 - 2*a - 6)*q^15 + (a^4 - 6*a^2 + 4)*q^16 + 1/2*(-a^4 -
        a^3 + 7*a^2 + 5*a - 10)*q^17 + 1/2*(a^4 - a^3 - 7*a^2 + 3*a + 2)*q^18 +
        1/2*(-a^4 - 2*a^3 + 5*a^2 + 4*a + 4)*q^19 + O(q^20),
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q - q^2 + q^4 - q^5 - q^8 - 3*q^9 + q^10 - 4*q^11 + 2*q^13 + q^16 - 2*q^17 +
        3*q^18 - 4*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
```

```
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^5 - 8*x^3 + 2*x^2 + 11*x - 2 over
    the Rational Field,
    Rational Field,
    Rational Field
*]
[* 37, 74, 185, 185, 185, 185, 370 *]
```

Not bielliptic, $n(a_3 = -3; 3) = 16 - 14$, thus $n(a_3; 3) \geq 2$.

As a corolloray $X_0(370)/w_d$ is not bielliptic by Harris-Silverman, and this concludes the case of Level $N = 370$.

5. Level $N = 318$

$X_0(318)/ < w_2, w_3 >$, **genus 13**

```
[*
    q - q^2 - 3*q^3 - q^4 + 3*q^6 - 4*q^7 + 3*q^8 + 6*q^9 + 3*q^12 - 3*q^13 +
        4*q^14 - q^16 - 3*q^17 - 6*q^18 - 5*q^19 + O(q^20),
    q + a*q^2 + (-a^2 - a + 3)*q^3 + (a^2 - 2)*q^4 + (a^2 - 3)*q^5 - q^6 + (a^2
        - 1)*q^7 + (-a^2 - a + 1)*q^8 + (-3*a^2 - 2*a + 7)*q^9 + (-a^2 + 1)*q^10
        + (a^2 + 2*a - 3)*q^11 + (2*a^2 + a - 6)*q^12 + q^13 + (-a^2 + 2*a +
        1)*q^14 + (3*a^2 + 2*a - 9)*q^15 + (-2*a^2 - 2*a + 3)*q^16 + (2*a -
        1)*q^17 + (a^2 - 2*a - 3)*q^18 + (a + 4)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - q^8 - 2*q^9 + 4*q^10 - 4*q^11 - q^12 +
        q^13 + 4*q^15 + q^16 + 5*q^17 + 2*q^18 - 7*q^19 + O(q^20),
    q - q^2 + 2*q^3 + q^4 + q^5 - 2*q^6 - 2*q^7 - q^8 + q^9 - q^10 + 5*q^11 +
        2*q^12 - 4*q^13 + 2*q^14 + 2*q^15 + q^16 + 3*q^17 - q^18 - 4*q^19 +
        O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-a^3 - a^2 + 6*a + 4)*q^5 - a*q^6 +
        1/3*(a^4 + 4*a^3 - 6*a^2 - 21*a + 4)*q^7 + (a^3 - 4*a)*q^8 + q^9 + (-a^4
        - a^3 + 6*a^2 + 4*a)*q^10 + 1/3*(-2*a^4 - 2*a^3 + 12*a^2 + 6*a - 2)*q^11
        + (-a^2 + 2)*q^12 + 1/3*(2*a^4 - a^3 - 15*a^2 + 6*a + 20)*q^13 +
        1/3*(4*a^4 + 4*a^3 - 21*a^2 - 18*a - 5)*q^14 + (a^3 + a^2 - 6*a -
        4)*q^15 + (a^4 - 6*a^2 + 4)*q^16 - 2*a*q^17 + a*q^18 + 1/3*(-2*a^4 -
        2*a^3 + 12*a^2 + 6*a - 2)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^11 - q^12 - 2*q^13 +
        q^15 + q^16 - 7*q^17 - q^18 + 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 + 4*q^5 + q^6 + q^7 - q^8 + q^9 - 4*q^10 - q^11 - q^12 -
        4*q^13 - q^14 - 4*q^15 + q^16 + 6*q^17 - q^18 - q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^3 + x^2 - 3*x - 1 over the Rational
    Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^5 - 10*x^3 + 22*x + 5 over the
    Rational Field,
    Rational Field,
    Rational Field
```

```
*]
[* 53, 53, 106, 106, 159, 318, 318 *]
```

Not bielliptic: $n(a_5; 25) = 0$, remain first 53 e.c. and $n(a_7 = -4; 49) = 133 - 96$

$X_0(318)/ < w_2, w_{53} >$, **genus 12**

```
[*
    q - q^2 - 3*q^3 - q^4 + 3*q^6 - 4*q^7 + 3*q^8 + 6*q^9 + 3*q^12 - 3*q^13 +
        4*q^14 - q^16 - 3*q^17 - 6*q^18 - 5*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - q^8 - 2*q^9 + 4*q^10 - 4*q^11 - q^12 +
        q^13 + 4*q^15 + q^16 + 5*q^17 + 2*q^18 - 7*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a^3 + a^2 + 2*a)*q^5 + a*q^6 + (a^3 -
        3*a^2 - 2*a + 5)*q^7 + (a^3 - 4*a)*q^8 + q^9 + (-2*a^3 + a^2 + 7*a -
        3)*q^10 + (4*a^3 - 6*a^2 - 12*a + 12)*q^11 + (a^2 - 2)*q^12 + (-3*a^3 +
        5*a^2 + 8*a - 10)*q^13 + (-a^2 - 2*a + 3)*q^14 + (-a^3 + a^2 + 2*a)*q^15
        + (3*a^3 - 5*a^2 - 7*a + 7)*q^16 + (-4*a^3 + 8*a^2 + 10*a - 12)*q^17 +
        a*q^18 + (2*a^2 - 4*a - 4)*q^19 + O(q^20),
    q - q^2 - 3*q^3 - q^4 + 3*q^6 - 4*q^7 + 3*q^8 + 6*q^9 + 3*q^12 - 3*q^13 +
        4*q^14 - q^16 - 3*q^17 - 6*q^18 - 5*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^11 - q^12 - 2*q^13 +
        q^15 + q^16 - 7*q^17 - q^18 + 2*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 + 5*q^7 - q^8 + q^9 - 3*q^11 + q^12 - 4*q^13 -
        5*q^14 + q^16 + 6*q^17 - q^18 + 5*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 + a*q^5 - q^6 - q^8 + q^9 - a*q^10 + (-a + 2)*q^11 +
        q^12 + 6*q^13 + a*q^15 + q^16 + (-a - 4)*q^17 - q^18 - 2*a*q^19 +
        O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - q^8 - 2*q^9 + 4*q^10 - 4*q^11 - q^12 +
        q^13 + 4*q^15 + q^16 + 5*q^17 + 2*q^18 - 7*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 - 3*x^3 - x^2 + 7*x - 3 over the
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - x - 10 over the Rational Field,
    Rational Field
*]
[* 53, 106, 159, 159, 318, 318, 318, 318 *]
```

Not bielliptic $n(a_7 = 0 : 49) = 131 - 128$.

$X_0(318)/ < w_3, w_{53} >$, **genus 7**

```
[*
    q - q^2 - 3*q^3 - q^4 + 3*q^6 - 4*q^7 + 3*q^8 + 6*q^9 + 3*q^12 - 3*q^13 +
        4*q^14 - q^16 - 3*q^17 - 6*q^18 - 5*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - q^8 - 2*q^9 + 4*q^10 - 4*q^11 - q^12 +
        q^13 + 4*q^15 + q^16 + 5*q^17 + 2*q^18 - 7*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + q^6 - 4*q^7 + q^8 - 2*q^9 + q^12 + 5*q^13 - 4*q^14 +
        q^16 - 3*q^17 - 2*q^18 - q^19 + O(q^20),
```

```
    q + q^2 - 2*q^3 + q^4 + 3*q^5 - 2*q^6 + 2*q^7 + q^8 + q^9 + 3*q^10 - 3*q^11
        - 2*q^12 - 4*q^13 + 2*q^14 - 6*q^15 + q^16 + 3*q^17 + q^18 - 4*q^19 +
        O(q^20),
    q - q^2 - 3*q^3 - q^4 + 3*q^6 - 4*q^7 + 3*q^8 + 6*q^9 + 3*q^12 - 3*q^13 +
        4*q^14 - q^16 - 3*q^17 - 6*q^18 - 5*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^11 - q^12 - 2*q^13 +
        q^15 + q^16 - 7*q^17 - q^18 + 2*q^19 + O(q^20),
    q + q^2 - q^3 + q^4 - q^6 + q^7 + q^8 + q^9 + 5*q^11 - q^12 + q^14 + q^16 +
        2*q^17 + q^18 - q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 53, 106, 106, 106, 106, 318, 318 *]
```

?? I can not discard $a_5 = 0, a_7 = -4$, the others yes, $n(a_7 \geq 0; 7) \geq 17 - 16$, $n(|a_7| \geq 5; 49) > 0$.

$X_0(318)/ < W_6, W_{106}, W_{159} >$**, genus 8**

```
318 [* 53, 53, 106, 318, 318 *] [*
    q - q^2 - 3*q^3 - q^4 + 3*q^6 - 4*q^7 + 3*q^8 + 6*q^9 + 3*q^12 - 3*q^13 +
        4*q^14 - q^16 - 3*q^17 - 6*q^18 - 5*q^19 + O(q^20),
    q + a*q^2 + (-a^2 - a + 3)*q^3 + (a^2 - 2)*q^4 + (a^2 - 3)*q^5 - q^6 + (a^2
        - 1)*q^7 + (-a^2 - a + 1)*q^8 + (-3*a^2 - 2*a + 7)*q^9 + (-a^2 + 1)*q^10
        + (a^2 + 2*a - 3)*q^11 + (2*a^2 + a - 6)*q^12 + q^13 + (-a^2 + 2*a +
        1)*q^14 + (3*a^2 + 2*a - 9)*q^15 + (-2*a^2 - 2*a + 3)*q^16 + (2*a -
        1)*q^17 + (a^2 - 2*a - 3)*q^18 + (a + 4)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - q^8 - 2*q^9 + 4*q^10 - 4*q^11 - q^12 +
        q^13 + 4*q^15 + q^16 + 5*q^17 + 2*q^18 - 7*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^11 - q^12 - 2*q^13 +
        q^15 + q^16 - 7*q^17 - q^18 + 2*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + a*q^5 + q^6 + (-a + 1)*q^7 + q^8 + q^9 + a*q^10 - q^11
        + q^12 - 2*a*q^13 + (-a + 1)*q^14 + a*q^15 + q^16 + (-a - 2)*q^17 + q^18
        + (a + 1)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^3 + x^2 - 3*x - 1 over the Rational
    Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - x - 4 over the Rational Field
*]
x^16 +
3.000000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000000000*x^15 + 82.000000000000000000000000000000000000\
```

```
    0000000000000000000000000000000000000000000000000000000000000000*x^14 +
    243.0000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000*x^13 + 3165.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000*x^12 +
    8680.0000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000*x^11 + 74706.000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000*x^10 +
    179638.0000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000*x^9 + 1175676.000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000*x^8 +
    2335294.0000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000*x^7 + 12625314.000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000*x^6 +
    19069960.0000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000*x^5 + 90395565.000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000*x^4 +
    90224199.0000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000002*x^3 + 395798338.000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000*x^2 +
    188245551.0000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000*x + 815730721.000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000
```

????????? We have that for level 53, $n(a_7 = -4; 49) = 121 - 96$ discard. For level $N = 106$, $n(a_5 = -4; 25) = 68 - 40$ discard. Remains the level 318, corresponding to E318c. We cannot discard.

$X_0(318)/ < W_2, W_{159}, W_{318} >$**, genus 7**

```
[* 53, 53, 106, 106, 318 *] [*
    q - q^2 - 3*q^3 - q^4 + 3*q^6 - 4*q^7 + 3*q^8 + 6*q^9 + 3*q^12 - 3*q^13 +
        4*q^14 - q^16 - 3*q^17 - 6*q^18 - 5*q^19 + O(q^20),
    q + a*q^2 + (-a^2 - a + 3)*q^3 + (a^2 - 2)*q^4 + (a^2 - 3)*q^5 - q^6 + (a^2
        - 1)*q^7 + (-a^2 - a + 1)*q^8 + (-3*a^2 - 2*a + 7)*q^9 + (-a^2 + 1)*q^10
        + (a^2 + 2*a - 3)*q^11 + (2*a^2 + a - 6)*q^12 + q^13 + (-a^2 + 2*a +
        1)*q^14 + (3*a^2 + 2*a - 9)*q^15 + (-2*a^2 - 2*a + 3)*q^16 + (2*a -
        1)*q^17 + (a^2 - 2*a - 3)*q^18 + (a + 4)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - q^8 - 2*q^9 + 4*q^10 - 4*q^11 - q^12 +
        q^13 + 4*q^15 + q^16 + 5*q^17 + 2*q^18 - 7*q^19 + O(q^20),
    q - q^2 + 2*q^3 + q^4 + q^5 - 2*q^6 - 2*q^7 - q^8 + q^9 - q^10 + 5*q^11 +
        2*q^12 - 4*q^13 + 2*q^14 + 2*q^15 + q^16 + 3*q^17 - q^18 - 4*q^19 +
        O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^11 - q^12 - 2*q^13 +
        q^15 + q^16 - 7*q^17 - q^18 + 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^3 + x^2 - 3*x - 1 over the Rational
    Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
```

```
x^14 +
3.0000000000000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000000*x^13 + 67.0000000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000*x^12 +
    236.0000000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000*x^11 + 2686.0000000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000*x^10 +
    8419.9999999999999999999999999999999999999999999999999999999999999999999999999\
    999999999999999999999999*x^9 + 70781.9999999999999999999999999999999999999\
    999999999999999999999999999999999999999999999999999999999999999999999*x^8 +
    201082.0000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000*x^7 + 1344858.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000*x^6 +
    3039620.0000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000*x^5 + 18423274.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000*x^4 +
    30755756.0000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000*x^3 + 165898633.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000*x^2 +
    141137643.0000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000000*x + 893871739.0000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000000000
```

??? We can discard all except E318c. The others we discard 53, $n(a_7 = -4, 49) = 112 - 96$, discard 106 both because $n(a_5 = -4; 25) = 66 - 40$, $n(a_5 = 1; 5) = 12 - 10$.

$X_0(318)/ < W_3, W_{106}, W_{318} >$**, genus 12**

```
* 53, 53, 106, 159, 318, 318 *]
[*
    q - q^2 - 3*q^3 - q^4 + 3*q^6 - 4*q^7 + 3*q^8 + 6*q^9 + 3*q^12 - 3*q^13 +
        4*q^14 - q^16 - 3*q^17 - 6*q^18 - 5*q^19 + O(q^20),
    q + a*q^2 + (-a^2 - a + 3)*q^3 + (a^2 - 2)*q^4 + (a^2 - 3)*q^5 - q^6 + (a^2
        - 1)*q^7 + (-a^2 - a + 1)*q^8 + (-3*a^2 - 2*a + 7)*q^9 + (-a^2 + 1)*q^10
        + (a^2 + 2*a - 3)*q^11 + (2*a^2 + a - 6)*q^12 + q^13 + (-a^2 + 2*a +
        1)*q^14 + (3*a^2 + 2*a - 9)*q^15 + (-2*a^2 - 2*a + 3)*q^16 + (2*a -
        1)*q^17 + (a^2 - 2*a - 3)*q^18 + (a + 4)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - q^8 - 2*q^9 + 4*q^10 - 4*q^11 - q^12 +
        q^13 + 4*q^15 + q^16 + 5*q^17 + 2*q^18 - 7*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-a^3 - a^2 + 6*a + 4)*q^5 - a*q^6 +
        1/3*(a^4 + 4*a^3 - 6*a^2 - 21*a + 4)*q^7 + (a^3 - 4*a)*q^8 + q^9 + (-a^4
        - a^3 + 6*a^2 + 4*a)*q^10 + 1/3*(-2*a^4 - 2*a^3 + 12*a^2 + 6*a - 2)*q^11
        + (-a^2 + 2)*q^12 + 1/3*(2*a^4 - a^3 - 15*a^2 + 6*a + 20)*q^13 +
        1/3*(4*a^4 + 4*a^3 - 21*a^2 - 18*a - 5)*q^14 + (a^3 + a^2 - 6*a -
        4)*q^15 + (a^4 - 6*a^2 + 4)*q^16 - 2*a*q^17 + a*q^18 + 1/3*(-2*a^4 -
        2*a^3 + 12*a^2 + 6*a - 2)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^11 - q^12 - 2*q^13 +
        q^15 + q^16 - 7*q^17 - q^18 + 2*q^19 + O(q^20),
    q + q^2 - q^3 + q^4 - 3*q^5 - q^6 - 4*q^7 + q^8 + q^9 - 3*q^10 - 5*q^11 -
        q^12 - 2*q^13 - 4*q^14 + 3*q^15 + q^16 + 5*q^17 + q^18 + 6*q^19 +
        O(q^20)
*]
```

```
[*
    Rational Field,
    Number Field with defining polynomial x^3 + x^2 - 3*x - 1 over the Rational
    Field,
    Rational Field,
    Number Field with defining polynomial x^5 - 10*x^3 + 22*x + 5 over the
    Rational Field,
    Rational Field,
    Rational Field
*]
x^24 +
10.000000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^23 + 72.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000*x^22 +
    380.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000001*x^21 + 1704.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000*x^20 +
    6564.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000002*x^19 + 22849.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000001*x^18 +
    72218.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000003*x^17 + 212357.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000001*x^16 +
    580872.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000005*x^15 + 1498647.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000001*x^14 +
    3635676.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000003*x^13 + 8369620.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000009*x^12 +
    18178380.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000002*x^11 + 37466175.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000004*x^10 +
    72609000.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000006*x^9 + 132723125.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000001*x^8 +
    225681250.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000002*x^7 + 357015625.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000002*x^6 +
    512812500.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000002*x^5 + 665625000.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000001*x^4 +
    742187500.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000001*x^3 + 703125000.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000*x^2 +
    488281250.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x + 244140625.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000
```

Not bielliptic, because $n(a_5 = 0; 5) = 16 - 12$, $n(a_5 = -4; 25) = 70 - 40$, $n(a_5 = -1; 5) = 16 - 14$, $n(a_5 = -3; 25) = 70 - 54$. Thus for Harris-Silverman $X_0(318)/W_3$, $X_0(318)/W_{106}$, $X_0(318)/W_{318}$ are not bielliptic.

$X_0(318)/ < W_6, W_{53}, W_{318} >$**, genus 10**

```
318 [* 53, 106, 106, 106, 106, 159, 159, 318 *] [*
    q - q^2 - 3*q^3 - q^4 + 3*q^6 - 4*q^7 + 3*q^8 + 6*q^9 + 3*q^12 - 3*q^13 +
        4*q^14 - q^16 - 3*q^17 - 6*q^18 - 5*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - q^8 - 2*q^9 + 4*q^10 - 4*q^11 - q^12 +
        q^13 + 4*q^15 + q^16 + 5*q^17 + 2*q^18 - 7*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + q^6 - 4*q^7 + q^8 - 2*q^9 + q^12 + 5*q^13 - 4*q^14 +
        q^16 - 3*q^17 - 2*q^18 - q^19 + O(q^20),
    q + q^2 - 2*q^3 + q^4 + 3*q^5 - 2*q^6 + 2*q^7 + q^8 + q^9 + 3*q^10 - 3*q^11
        - 2*q^12 - 4*q^13 + 2*q^14 - 6*q^15 + q^16 + 3*q^17 + q^18 - 4*q^19 +
        O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a^3 + a^2 + 2*a)*q^5 + a*q^6 + (a^3 -
        3*a^2 - 2*a + 5)*q^7 + (a^3 - 4*a)*q^8 + q^9 + (-2*a^3 + a^2 + 7*a -
        3)*q^10 + (4*a^3 - 6*a^2 - 12*a + 12)*q^11 + (a^2 - 2)*q^12 + (-3*a^3 +
        5*a^2 + 8*a - 10)*q^13 + (-a^2 - 2*a + 3)*q^14 + (-a^3 + a^2 + 2*a)*q^15
        + (3*a^3 - 5*a^2 - 7*a + 7)*q^16 + (-4*a^3 + 8*a^2 + 10*a - 12)*q^17 +
        a*q^18 + (2*a^2 - 4*a - 4)*q^19 + O(q^20),
    q - q^2 - 3*q^3 - q^4 + 3*q^6 - 4*q^7 + 3*q^8 + 6*q^9 + 3*q^12 - 3*q^13 +
        4*q^14 - q^16 - 3*q^17 - 6*q^18 - 5*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^11 - q^12 - 2*q^13 +
        q^15 + q^16 - 7*q^17 - q^18 + 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 - 3*x^3 - x^2 + 7*x - 3 over the
    Rational Field,
    Rational Field,
    Rational Field
*]
x^20 -
9.14389913025819985772626838644272767561527995858238928972100674582106\10\
    87253905565403584174881376237E-100*x^19 +
    24.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000000*x^18 + 19.99999999999999999999999999999999999\
    99999999999999999999999999999999999999999999999997*x^17 +
    273.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000000*x^16 + 437.99999999999999999999999999999999999\
    99999999999999999999999999999999999999999999999995*x^15 +
    2287.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000000*x^14 + 4201.99999999999999999999999999999999999\
    99999999999999999999999999999999999999999999999996*x^13 +
    16690.00000000000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000000*x^12 + 25779.99999999999999999999999999999999\
```

```
    99999999999999999999999999999999999999999999999999999998*x^11 +
    97049.99999999999999999999999999999999999999999999999999999999999999999\
    99999999999999999999999*x^10 + 128899.99999999999999999999999999999999\
    99999999999999999999999999999999999999999999999999999999*x^9 +
    417249.99999999999999999999999999999999999999999999999999999999999999999\
    99999999999999999999999*x^8 + 525249.99999999999999999999999999999999\
    99999999999999999999999999999999999999999999999999999996*x^7 +
    1429375.00000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000*x^6 + 1368749.99999999999999999999999999999999\
    99999999999999999999999999999999999999999999999999999999*x^5 +
    4265625.00000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000*x^4 + 1562499.99999999999999999999999999999999\
    99999999999999999999999999999999999999999999999999999998*x^3 +
    9375000.00000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000002*x^2 - 2.27716737892228526632860515490086028360 66\
    67351989490765066392412757690425774233429510299311443098318E-93*x +
    9765625.00000000000000000000000000000000000000000000000000000000000000\
    00000000000000000000000
1 [* 6, 74, 186, 566, 2916, 16484, 76376, 388078, 1954680, 9774004,
48855186, 244093952, 1220844592, 6103473562, 30516699186,
152585871390, 762939351364, 3814718771732, 19073501005702,
95367374377716 *] 318 [* 0, 0, 0, 1, 0, 1, 0, 1, 0, 0 *] 0 0 5 [* 6,
74, 186, 566, 2916, 16484, 76376, 388078, 1954680, 9774004,
48855186, 244093952, 1220844592, 6103473562, 30516699186,
152585871390, 762939351364, 3814718771732, 19073501005702,
95367374377716 *] [* 12, 72, 252, 1152, 6252, 31752, 156252, 778752,
3906252, 19543752, 97656252, 488218752, 2441406252, 12207343752,
61035156252, 305174218752, 1525878906252, 7629402343752,
38146972656252, 190734824218752 *]
```

Not bielliptic because $n(a_5 = 0; 25) = 74 - 72$, therefore also $X_0(318)/W_{53}, X_0(318)/W_6, X_0^+(318)$ are not bielliptic.

$X_0(318)/ < w_{159} >$, **genus16**

```
[*
    q - q^2 - 3*q^3 - q^4 + 3*q^6 - 4*q^7 + 3*q^8 + 6*q^9 + 3*q^12 - 3*q^13 +
        4*q^14 - q^16 - 3*q^17 - 6*q^18 - 5*q^19 + O(q^20),
    q + a*q^2 + (-a^2 - a + 3)*q^3 + (a^2 - 2)*q^4 + (a^2 - 3)*q^5 - q^6 + (a^2
        - 1)*q^7 + (-a^2 - a + 1)*q^8 + (-3*a^2 - 2*a + 7)*q^9 + (-a^2 + 1)*q^10
        + (a^2 + 2*a - 3)*q^11 + (2*a^2 + a - 6)*q^12 + q^13 + (-a^2 + 2*a +
        1)*q^14 + (3*a^2 + 2*a - 9)*q^15 + (-2*a^2 - 2*a + 3)*q^16 + (2*a -
        1)*q^17 + (a^2 - 2*a - 3)*q^18 + (a + 4)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - q^8 - 2*q^9 + 4*q^10 - 4*q^11 - q^12 +
        q^13 + 4*q^15 + q^16 + 5*q^17 + 2*q^18 - 7*q^19 + O(q^20),
    q - q^2 + 2*q^3 + q^4 + q^5 - 2*q^6 - 2*q^7 - q^8 + q^9 - q^10 + 5*q^11 +
        2*q^12 - 4*q^13 + 2*q^14 + 2*q^15 + q^16 + 3*q^17 - q^18 - 4*q^19 +
        O(q^20),
    q + q^2 + q^3 + q^4 + q^6 - 4*q^7 + q^8 - 2*q^9 + q^12 + 5*q^13 - 4*q^14 +
        q^16 - 3*q^17 - 2*q^18 - q^19 + O(q^20),
    q + q^2 - 2*q^3 + q^4 + 3*q^5 - 2*q^6 + 2*q^7 + q^8 + q^9 + 3*q^10 - 3*q^11
        - 2*q^12 - 4*q^13 + 2*q^14 - 6*q^15 + q^16 + 3*q^17 + q^18 - 4*q^19 +
```

```
        O(q^20),
    q - q^2 - 3*q^3 - q^4 + 3*q^6 - 4*q^7 + 3*q^8 + 6*q^9 + 3*q^12 - 3*q^13 +
        4*q^14 - q^16 - 3*q^17 - 6*q^18 - 5*q^19 + O(q^20),
    q + a*q^2 + (-a^2 - a + 3)*q^3 + (a^2 - 2)*q^4 + (a^2 - 3)*q^5 - q^6 + (a^2
        - 1)*q^7 + (-a^2 - a + 1)*q^8 + (-3*a^2 - 2*a + 7)*q^9 + (-a^2 + 1)*q^10
        + (a^2 + 2*a - 3)*q^11 + (2*a^2 + a - 6)*q^12 + q^13 + (-a^2 + 2*a +
        1)*q^14 + (3*a^2 + 2*a - 9)*q^15 + (-2*a^2 - 2*a + 3)*q^16 + (2*a -
        1)*q^17 + (a^2 - 2*a - 3)*q^18 + (a + 4)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^11 - q^12 - 2*q^13 +
        q^15 + q^16 - 7*q^17 - q^18 + 2*q^19 + O(q^20),
    q + q^2 - q^3 + q^4 - q^6 + q^7 + q^8 + q^9 + 5*q^11 - q^12 + q^14 + q^16 +
        2*q^17 + q^18 - q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + a*q^5 + q^6 + (-a + 1)*q^7 + q^8 + q^9 + a*q^10 - q^11
        + q^12 - 2*a*q^13 + (-a + 1)*q^14 + a*q^15 + q^16 + (-a - 2)*q^17 + q^18
        + (a + 1)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^3 + x^2 - 3*x - 1 over the Rational
    Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 + x^2 - 3*x - 1 over the Rational
    Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - x - 4 over the Rational Field
*]
[* 53, 53, 106, 106, 106, 106, 106, 106, 318, 318, 318 *]
```

Not bielliptic, $n(a_5 = 0; 25) = 126 - 72$.

6. Level $N = 290$

$X_0(390)/ < w_2, w_3 >$, **genus 10**

```
[*
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 +
        (2*a - 2)*q^18 + 6*q^19 + O(q^20),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q - q^2 - q^4 - q^5 - 2*q^7 + 3*q^8 - 3*q^9 + q^10 - 6*q^11 + 2*q^13 +
        2*q^14 - q^16 - 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 + (-a^2 + 2*a + 1)*q^3 + (a^2 - 2)*q^4 - q^5 + (-a^2 + 5)*q^6 +
        (-a^2 + 3)*q^7 + (3*a^2 - 3*a - 5)*q^8 + (-2*a + 3)*q^9 - a*q^10 + (a^2
        - 2*a + 1)*q^11 + (-a^2 + 3)*q^12 + (2*a - 4)*q^13 + (-3*a^2 + 2*a +
```

```
        5)*q^14 + (a^2 - 2*a - 1)*q^15 + (4*a^2 - 2*a - 11)*q^16 + (-3*a^2 + 2*a
        + 9)*q^17 + (-2*a^2 + 3*a)*q^18 + (3*a^2 - 4*a - 7)*q^19 + O(q^20),
    q - q^2 + q^4 - q^5 - 2*q^7 - q^8 - 3*q^9 + q^10 + 2*q^11 - 6*q^13 + 2*q^14
        + q^16 + 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 - q^5 - a*q^6 + (-a + 3)*q^7 - q^8 + a*q^9 + q^10 +
        (2*a - 2)*q^11 + a*q^12 + (a + 4)*q^13 + (a - 3)*q^14 - a*q^15 + q^16 +
        (-3*a + 3)*q^17 - a*q^18 + (-2*a + 4)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - 3*x^2 - x + 5 over the Rational
    Field,
    Rational Field,
    Number Field with defining polynomial x^2 - x - 3 over the Rational Field
*]
[* 29, 58, 145, 145, 290, 290 *]
```

Not bielliptic, $n(a_3 = 0; 9) = 36 - 32$.

$X_0(290)/ < w_2, w_{29} >$**, genus 9**

```
[*
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q - q^2 - q^4 - q^5 - 2*q^7 + 3*q^8 - 3*q^9 + q^10 - 6*q^11 + 2*q^13 +
        2*q^14 - q^16 - 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 + (-a^2 + 3)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^2 + 1)*q^6 + (a^2 -
        1)*q^7 + (a^2 - a - 1)*q^8 + (-2*a^2 + 2*a + 5)*q^9 + a*q^10 + (a^2 -
        2*a - 1)*q^11 + (a^2 - 2*a - 5)*q^12 - 2*a*q^13 + (a^2 + 2*a - 1)*q^14 +
        (-a^2 + 3)*q^15 + (-2*a^2 + 2*a + 3)*q^16 + (3*a^2 - 4*a - 7)*q^17 + (-a
        + 2)*q^18 + (-a^2 - 1)*q^19 + O(q^20),
    q - q^2 + q^4 - q^5 - 2*q^7 - q^8 - 3*q^9 + q^10 + 2*q^11 - 6*q^13 + 2*q^14
        + q^16 + 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + q^5 - a*q^6 + (a + 1)*q^7 - q^8 + a*q^9 - q^10 +
        (-2*a + 2)*q^11 + a*q^12 - a*q^13 + (-a - 1)*q^14 + a*q^15 + q^16 + (a -
        1)*q^17 - a*q^18 + (-2*a + 4)*q^19 + O(q^20),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 3*x + 1 over the Rational
    Field,
    Rational Field,
    Number Field with defining polynomial x^2 - x - 3 over the Rational Field,
    Rational Field
*]
```

```
[* 58, 145, 145, 290, 290, 290 *]
```

Not bielliptic, $n(a_7 = -2; 49) = 133 - 120$.

$X_0(290)/ < w_5, w_{29} >$, **genus 8**

```
[*
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q + q^2 - q^3 + q^4 + q^5 - q^6 - 2*q^7 + q^8 - 2*q^9 + q^10 - 3*q^11 - q^12
        - q^13 - 2*q^14 - q^15 + q^16 + 8*q^17 - 2*q^18 + O(q^20),
    q - q^2 - q^4 - q^5 - 2*q^7 + 3*q^8 - 3*q^9 + q^10 - 6*q^11 + 2*q^13 +
        2*q^14 - q^16 - 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q - q^2 + q^4 - q^5 - 2*q^7 - q^8 - 3*q^9 + q^10 + 2*q^11 - 6*q^13 + 2*q^14
        + q^16 + 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 - q^5 + a*q^6 + (-a^2 + 6)*q^7 + q^8 + (a^2 - 3)*q^9 -
        q^10 + (-2*a + 2)*q^11 + a*q^12 + (2*a^2 - 3*a - 6)*q^13 + (-a^2 +
        6)*q^14 - a*q^15 + q^16 + (a^2 - 2*a - 2)*q^17 + (a^2 - 3)*q^18 +
        (-2*a^2 + 4*a + 6)*q^19 + O(q^20),
    q - q^2 - q^4 - q^5 - 2*q^7 + 3*q^8 - 3*q^9 + q^10 - 6*q^11 + 2*q^13 +
        2*q^14 - q^16 - 2*q^17 + 3*q^18 - 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - 3*x^2 - 3*x + 8 over the
    Rational Field,
    Rational Field
*]
[* 58, 58, 145, 290, 290, 290 *]
```

Not bielliptic. $n(a_3 = 0; 9) = 33 - 32$.

$X_0(290)/ < W_{10}, W_{58}, W_{145} >$, **genus 10**

```
[ <2, 1>, <5, 1>, <29, 1> ] 290 [*
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 +
        (2*a - 2)*q^18 + 6*q^19 + O(q^20),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q - q^2 - q^4 - q^5 - 2*q^7 + 3*q^8 - 3*q^9 + q^10 - 6*q^11 + 2*q^13 +
        2*q^14 - q^16 - 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - 2*q^3 + (-2*a - 1)*q^4 + q^5 - 2*a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 + a*q^10 + 2*a*q^11 + (4*a + 2)*q^12 - 2*q^13 - 2*q^14 -
        2*q^15 + 3*q^16 + (2*a + 2)*q^17 + a*q^18 + (-2*a - 4)*q^19 + O(q^20),
    q - q^2 + q^4 - q^5 - 2*q^7 - q^8 - 3*q^9 + q^10 + 2*q^11 - 6*q^13 + 2*q^14
        + q^16 + 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 + q^5 + a*q^6 + (-a^2 - 2*a + 4)*q^7 + q^8 + (a^2 -
        3)*q^9 + q^10 + (2*a^2 + 4*a - 8)*q^11 + a*q^12 + (-2*a^2 - 5*a +
```

```
        10)*q^13 + (-a^2 - 2*a + 4)*q^14 + a*q^15 + q^16 + (a^2 + 2*a - 6)*q^17
        + (a^2 - 3)*q^18 - 2*a*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 + x^2 - 7*x + 4 over the Rational
    Field
*]
[* 29, 58, 145, 145, 290, 290 *] x^20 +
6.000000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000000*x^19 + 29.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^18 +
    105.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^17 + 327.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^16 +
    888.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000001*x^15 + 2176.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^14 +
    4874.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000001*x^13 + 10080.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^12 +
    19377.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^11 + 34758.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^10 +
    58131.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000001*x^9 + 90720.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^8 +
    131598.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^7 + 176256.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^6 +
    215784.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000001*x^5 + 238383.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^4 +
    229635.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000001*x^3 + 190269.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000*x^2 +
    118098.000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x + 59049.000000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000000
1 [* 10, 32, 37, 68, 235, 779, 2208, 6076, 19351, 61167, 176780,
527531, 1601038, 4784308, 14333812, 43074524, 129059726, 387496535,
1162183308, 3486821663 *] 290 [* 0, 1, 1, 1, 1, 0, 0, 1, 0, 1 *] 5 8
3 [* 10, 32, 37, 68, 235, 779, 2208, 6076, 19351, 61167, 176780,
527531, 1601038, 4784308, 14333812, 43074524, 129059726, 387496535,
1162183308, 3486821663 *] [* 8, 32, 56, 128, 488, 1568, 4376, 12800,
```

```
39368, 119072, 354296, 1059968, 3188648, 9574688, 28697816,
86067200, 258280328, 774919712, 2324522936, 6973332608 *]
```

Not bielliptic because $n(a_3 \neq 0, 9) > 32-32$ and $n(a_3 = 0; 3) = 10-8$. In particular $X_0(290)/W_{10}, X_0(290)/W_{58}, X_0(290)/W_{1}$ are not bielliptic.

$X_0(290)/ < W_2, W_{145}, W_{290} >$**, genus 7**

```
[*
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 +
        (2*a - 2)*q^18 + 6*q^19 + O(q^20),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q - q^2 - q^4 - q^5 - 2*q^7 + 3*q^8 - 3*q^9 + q^10 - 6*q^11 + 2*q^13 +
        2*q^14 - q^16 - 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - 2*q^3 + (-2*a - 1)*q^4 + q^5 - 2*a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 + a*q^10 + 2*a*q^11 + (4*a + 2)*q^12 - 2*q^13 - 2*q^14 -
        2*q^15 + 3*q^16 + (2*a + 2)*q^17 + a*q^18 + (-2*a - 4)*q^19 + O(q^20),
    q - q^2 + q^4 - q^5 - 2*q^7 - q^8 - 3*q^9 + q^10 + 2*q^11 - 6*q^13 + 2*q^14
        + q^16 + 2*q^17 + 3*q^18 - 2*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field
*]
[* 29, 58, 145, 145, 290 *] x^14 +
5.0000000000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000000*x^13 + 22.0000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000*x^12 +
    63.0000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^11 + 164.0000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000*x^10 +
    339.0000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^9 + 675.0000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000*x^8 +
    1170.0000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^7 + 2025.0000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000*x^6 +
    3051.0000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^5 + 4428.0000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000*x^4 +
    5103.0000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000001*x^3 + 5346.0000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000*x^2 +
    3645.0000000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x + 2187.0000000000000000000000000000000000\
```

```
    0000000000000000000000000000000000000000000000000000000000
1
```

Not bielliptic, $n(a_3 = 0; 3) = 9-8, n(a_3 = -3; 9) = 29-14$. In particular $X_0^*(290)/W_2, X_0(290)/W_{145}, X_0(290)/W_{290}$ are not bielliptic.

$X_0(290)/ < W_5, W_{58}, W_{290} >$**, genus 8**

```
[*
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 +
        (2*a - 2)*q^18 + 6*q^19 + O(q^20),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q - q^2 - q^4 - q^5 - 2*q^7 + 3*q^8 - 3*q^9 + q^10 - 6*q^11 + 2*q^13 +
        2*q^14 - q^16 - 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 + (-a^2 + 2*a + 1)*q^3 + (a^2 - 2)*q^4 - q^5 + (-a^2 + 5)*q^6 +
        (-a^2 + 3)*q^7 + (3*a^2 - 3*a - 5)*q^8 + (-2*a + 3)*q^9 - a*q^10 + (a^2
        - 2*a + 1)*q^11 + (-a^2 + 3)*q^12 + (2*a - 4)*q^13 + (-3*a^2 + 2*a +
        5)*q^14 + (a^2 - 2*a - 1)*q^15 + (4*a^2 - 2*a - 11)*q^16 + (-3*a^2 + 2*a
        + 9)*q^17 + (-2*a^2 + 3*a)*q^18 + (3*a^2 - 4*a - 7)*q^19 + O(q^20),
    q - q^2 + q^4 - q^5 - 2*q^7 - q^8 - 3*q^9 + q^10 + 2*q^11 - 6*q^13 + 2*q^14
        + q^16 + 2*q^17 + 3*q^18 - 2*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - 3*x^2 - x + 5 over the Rational
    Field,
    Rational Field
*]
[* 29, 58, 145, 145, 290 *] x^16 +
7.999999999999999999999999999999999999999999999999999999999999999999999999\
    999999999999999999999999998*x^15 + 104.000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000*x^14 +
    659.999999999999999999999999999999999999999999999999999999999999999999999\
    999999999999999999999999999*x^13 + 5068.000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000*x^12 +
    26324.000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^11 + 151000.000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000*x^10 +
    657743.999999999999999999999999999999999999999999999999999999999999999999\
    999999999999999999999999999*x^9 + 3064422.000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000*x^8 +
    11181648.000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^7 + 43639000.000000000000000000000000000000000\
    000000000000000000000000000000000000000000000000000000000000000000*x^6 +
    129329812.000000000000000000000000000000000000000000000000000000000000000000\
    000000000000000000000000*x^5 + 423284428.000000000000000000000000000000000\
```

```
    0000000000000000000000000000000000000000000000000000000000000000*x^4 +
    937105619.9999999999999999999999999999999999999999999999999999999999999999\
    99999999999999999999999998*x^3 + 2510307176.0000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000*x^2 +
    3282709383.9999999999999999999999999999999999999999999999999999999999999999\
    99999999999999999999999999*x + 6975757441.0000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000000
```

???remains E290a, $n(a_3 = -3; 9) = 31 - 14$, $n(a_{11} = -6; 121) = 219 - 216$.

$X_0(290)/ < W_{29}, W_{10}, W_{290} >$**, genus 7**

```
290 [*
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q + q^2 - q^3 + q^4 + q^5 - q^6 - 2*q^7 + q^8 - 2*q^9 + q^10 - 3*q^11 - q^12
        - q^13 - 2*q^14 - q^15 + q^16 + 8*q^17 - 2*q^18 + O(q^20),
    q - q^2 - q^4 - q^5 - 2*q^7 + 3*q^8 - 3*q^9 + q^10 - 6*q^11 + 2*q^13 +
        2*q^14 - q^16 - 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 + (-a^2 + 3)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^2 + 1)*q^6 + (a^2 -
        1)*q^7 + (a^2 - a - 1)*q^8 + (-2*a^2 + 2*a + 5)*q^9 + a*q^10 + (a^2 -
        2*a - 1)*q^11 + (a^2 - 2*a - 5)*q^12 - 2*a*q^13 + (a^2 + 2*a - 1)*q^14 +
        (-a^2 + 3)*q^15 + (-2*a^2 + 2*a + 3)*q^16 + (3*a^2 - 4*a - 7)*q^17 + (-a
        + 2)*q^18 + (-a^2 - 1)*q^19 + O(q^20),
    q - q^2 + q^4 - q^5 - 2*q^7 - q^8 - 3*q^9 + q^10 + 2*q^11 - 6*q^13 + 2*q^14
        + q^16 + 2*q^17 + 3*q^18 - 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 3*x + 1 over the Rational
    Field,
    Rational Field
*]
[* 58, 58, 145, 145, 290 *] x^14 +
1.828779826051639971545253677288545535123055991716477857944201349164212\2
    1745078111308071683497627524 7E-99*x^13 +
    27.0000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000001*x^12 - 35.9999999999999999999999999999999999999\
    9999999999999999999999999999999999999999999999999999999999996*x^11 +
    153.0000000000000000000000000000000000000000000000000000000000000000000000\
    0000000000000000000000002*x^10 - 127.9999999999999999999999999999999999999\
    9999999999999999999999999999999999999999999999999999999999995*x^9 -
    3028.9999999999999999999999999999999999999999999999999999999999999999999999\
    9999999999999999999999997*x^8 + 7752.0000000000000000000000000000000000000\
    0000000000000000000000000000000000000000000000000000000000000*x^7 -
    51492.9999999999999999999999999999999999999999999999999999999999999999999999\
    9999999999999999999999993*x^6 - 36991.9999999999999999999999999999999999999\
    9999999999999999999999999999999999999999999999999999999999994*x^5 +
    751689.0000000000000000000000000000000000000000000000000000000000000000000000\
```

```
000000000000000000000018*x^4 - 3006755.999999999999999999999999999999\
999999999999999999999999999999999999999999999999999997*x^3 +
38336139.000000000000000000000000000000000000000000000000000000000000\
000000000000000000000002*x^2 + 4.218752196740233756566679023816330630681 8\
258310542144700177375225826685782764745641453966190945821 46E-92*x +
410338673.000000000000000000000000000000000000000000000000000000000000\
000000000000000000000000
```

???? 58b,145a, 290a, $n(58, a_3 = -3; 9) = 30 - 14$.

7. Level $N = 258$

$X_0(258)/ < w_2, w_3 >$, **genus 10**

```
[*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (-a + 2)*q^5 - 2*q^6 + (a - 2)*q^7 - 2*a*q^8 - q^9 +
        (2*a - 2)*q^10 + (2*a - 1)*q^11 + (2*a + 1)*q^13 + (-2*a + 2)*q^14 +
        (-2*a + 2)*q^15 - 4*q^16 + (2*a + 5)*q^17 - a*q^18 + (-2*a - 2)*q^19 +
        O(q^20),
    q - q^2 + a*q^3 + q^4 + (-a + 1)*q^5 - a*q^6 + 2*q^7 - q^8 + (-a + 2)*q^9 +
        (a - 1)*q^10 + a*q^12 + 2*q^13 - 2*q^14 + (2*a - 5)*q^15 + q^16 + (a -
        4)*q^17 + (a - 2)*q^18 + (-3*a - 1)*q^19 + O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 2*q^7 + q^9 - 5*q^11 + 2*q^12 + 3*q^13 + 2*q^15 +
        4*q^16 - 3*q^17 + 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + (2*a - 1)*q^4 + (-a + 2)*q^5 - a*q^6 + (-2*a + 3)*q^7 + (a
        + 2)*q^8 + q^9 - q^10 + (-a + 4)*q^11 + (-2*a + 1)*q^12 - 5*q^13 + (-a -
        2)*q^14 + (a - 2)*q^15 + 3*q^16 - 2*a*q^17 + a*q^18 + (4*a - 5)*q^19 +
        O(q^20),
    q - q^2 - q^3 + q^4 + q^5 + q^6 - 5*q^7 - q^8 + q^9 - q^10 + q^11 - q^12 -
        3*q^13 + 5*q^14 - q^15 + q^16 - q^18 - 7*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 + 2*q^7 - q^8 + q^9 + 2*q^10 - q^12 +
        2*q^13 - 2*q^14 + 2*q^15 + q^16 + 6*q^17 - q^18 + 4*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 5 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 43, 43, 86, 129, 129, 258, 258 *]
```

Not bielliptic, $n(|a_5| \geq 2; 25) \geq 68 - 64$, $n(a_7 = -5; 49) = 119 - 78$.

$X_0(258)/ < w_2, w_{43} >$, **genus 8**

```
[*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 2*q^7 + q^9 - 5*q^11 + 2*q^12 + 3*q^13 + 2*q^15 +
        4*q^16 - 3*q^17 + 2*q^19 + O(q^20),
```

```
    q + q^2 + q^3 - q^4 + 2*q^5 + q^6 - 3*q^8 + q^9 + 2*q^10 - q^12 - 2*q^13 +
        2*q^15 - q^16 - 6*q^17 + q^18 + 4*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a - 2)*q^5 + a*q^6 + (-a^2 + 6)*q^7 +
        (-2*a^2 + a + 8)*q^8 + q^9 + (-a^2 - 2*a)*q^10 + (a^2 - a - 5)*q^11 +
        (a^2 - 2)*q^12 + 3*q^13 + (2*a^2 + a - 8)*q^14 + (-a - 2)*q^15 + (3*a^2
        - 2*a - 12)*q^16 + (-a^2 + 5)*q^17 + a*q^18 + (-a^2 - 2*a + 2)*q^19 +
        O(q^20),
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 + q^5 + q^6 - 5*q^7 - q^8 + q^9 - q^10 + q^11 - q^12 -
        3*q^13 + 5*q^14 - q^15 + q^16 - q^18 - 7*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 + 2*x^2 - 5*x - 8 over the
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 43, 129, 129, 129, 129, 258 *]
```

Not bielliptic. $n(a_5 \geq 0; 5) \geq 17 - 12$, $n(a_5 = -2; 5) = 17 - 16$, $n(a_5 = -4; 25) = 47 - 40$.

$X_0(258)/ < w_3, w_{43} >$**, genus 9**

```
[*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 + (-a - 1)*q^5 + a*q^6 + (-4*a + 2)*q^7 + q^8 + (a -
        2)*q^9 + (-a - 1)*q^10 + (4*a - 4)*q^11 + a*q^12 + (4*a - 2)*q^13 +
        (-4*a + 2)*q^14 + (-2*a - 1)*q^15 + q^16 - a*q^17 + (a - 2)*q^18 + (a +
        5)*q^19 + O(q^20),
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 2*q^7 + q^9 - 5*q^11 + 2*q^12 + 3*q^13 + 2*q^15 +
        4*q^16 - 3*q^17 + 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 + q^5 + q^6 - 5*q^7 - q^8 + q^9 - q^10 + q^11 - q^12 -
        3*q^13 + 5*q^14 - q^15 + q^16 - q^18 - 7*q^19 + O(q^20),
    q + q^2 - q^3 + q^4 - 2*q^5 - q^6 + 4*q^7 + q^8 + q^9 - 2*q^10 + 4*q^11 -
        q^12 + 6*q^13 + 4*q^14 + 2*q^15 + q^16 - 6*q^17 + q^18 - 4*q^19 +
        O(q^20),
    q + q^2 - q^3 + q^4 + 3*q^5 - q^6 - q^7 + q^8 + q^9 + 3*q^10 - q^11 - q^12 +
        q^13 - q^14 - 3*q^15 + q^16 + 4*q^17 + q^18 + q^19 + O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 2*q^7 + q^9 - 5*q^11 + 2*q^12 + 3*q^13 + 2*q^15 +
        4*q^16 - 3*q^17 + 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Rational Field,
```

```
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 43, 86, 86, 129, 258, 258, 258, 258 *]
```

Not bielliptic $n(a_5 \geq 0; 5) \geq 19 - 12$,$n(a_5 = -2; 5) = 19 - 16$,$n(a_5 = -4; 25) = 55 - 40$.

$X_0(258)/ < W_6, W_{86}, W_{129} >$, **genus 7**

```
258 [*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (-a + 2)*q^5 - 2*q^6 + (a - 2)*q^7 - 2*a*q^8 - q^9 +
        (2*a - 2)*q^10 + (2*a - 1)*q^11 + (2*a + 1)*q^13 + (-2*a + 2)*q^14 +
        (-2*a + 2)*q^15 - 4*q^16 + (2*a + 5)*q^17 - a*q^18 + (-2*a - 2)*q^19 +
        O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 2*q^7 + q^9 - 5*q^11 + 2*q^12 + 3*q^13 + 2*q^15 +
        4*q^16 - 3*q^17 + 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 + q^5 + q^6 - 5*q^7 - q^8 + q^9 - q^10 + q^11 - q^12 -
        3*q^13 + 5*q^14 - q^15 + q^16 - q^18 - 7*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 - q^5 + q^6 + q^7 + q^8 + q^9 - q^10 + 5*q^11 + q^12 -
        7*q^13 + q^14 - q^15 + q^16 + 4*q^17 + q^18 - q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + 2*q^5 + q^6 - 2*q^7 + q^8 + q^9 + 2*q^10 - 4*q^11 +
        q^12 + 2*q^13 - 2*q^14 + 2*q^15 + q^16 - 2*q^17 + q^18 - 4*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 43, 43, 129, 258, 258, 258 *]
```

??? 129a, 258g, the others $n(43, a_5 = -4; 25) = 58 - 40$, $n(258, a_7 = -5; 49) = 102 - 78$, $n(258, a_7 = 1; 7) = 20 - 14$.

$X_0(258)/ < W_2, W_{129}, W_{258} >$, **genus 8**

```
[*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (-a + 2)*q^5 - 2*q^6 + (a - 2)*q^7 - 2*a*q^8 - q^9 +
        (2*a - 2)*q^10 + (2*a - 1)*q^11 + (2*a + 1)*q^13 + (-2*a + 2)*q^14 +
        (-2*a + 2)*q^15 - 4*q^16 + (2*a + 5)*q^17 - a*q^18 + (-2*a - 2)*q^19 +
        O(q^20),
    q - q^2 + a*q^3 + q^4 + (-a + 1)*q^5 - a*q^6 + 2*q^7 - q^8 + (-a + 2)*q^9 +
        (a - 1)*q^10 + a*q^12 + 2*q^13 - 2*q^14 + (2*a - 5)*q^15 + q^16 + (a -
        4)*q^17 + (a - 2)*q^18 + (-3*a - 1)*q^19 + O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 2*q^7 + q^9 - 5*q^11 + 2*q^12 + 3*q^13 + 2*q^15 +
```

```
        4*q^16 - 3*q^17 + 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 + q^5 + q^6 - 5*q^7 - q^8 + q^9 - q^10 + q^11 - q^12 -
        3*q^13 + 5*q^14 - q^15 + q^16 - q^18 - 7*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - 3*q^7 - q^8 + q^9 + 3*q^10 - 5*q^11 +
        q^12 - 3*q^13 + 3*q^14 - 3*q^15 + q^16 - q^18 + 7*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 5 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 43, 43, 86, 129, 258, 258 *]
```

????, 129a, 258c, the others $n(43, a_5 = -4; 25) = 49 - 40$, $n(258, a_7 = -5; 49) = 104 - 78$.

$X_0(258)/ < W_3, W_{86}, W_{258} >$**, genus 7**

```
[*
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (-a + 2)*q^5 - 2*q^6 + (a - 2)*q^7 - 2*a*q^8 - q^9 +
        (2*a - 2)*q^10 + (2*a - 1)*q^11 + (2*a + 1)*q^13 + (-2*a + 2)*q^14 +
        (-2*a + 2)*q^15 - 4*q^16 + (2*a + 5)*q^17 - a*q^18 + (-2*a - 2)*q^19 +
        O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 2*q^7 + q^9 - 5*q^11 + 2*q^12 + 3*q^13 + 2*q^15 +
        4*q^16 - 3*q^17 + 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + (2*a - 1)*q^4 + (-a + 2)*q^5 - a*q^6 + (-2*a + 3)*q^7 + (a
        + 2)*q^8 + q^9 - q^10 + (-a + 4)*q^11 + (-2*a + 1)*q^12 - 5*q^13 + (-a -
        2)*q^14 + (a - 2)*q^15 + 3*q^16 - 2*a*q^17 + a*q^18 + (4*a - 5)*q^19 +
        O(q^20),
    q - q^2 - q^3 + q^4 + q^5 + q^6 - 5*q^7 - q^8 + q^9 - q^10 + q^11 - q^12 -
        3*q^13 + 5*q^14 - q^15 + q^16 - q^18 - 7*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 1 over the Rational Field,
    Rational Field
*]
[* 43, 43, 129, 129, 258 *]
```

Not bielliptic, $n(a_5 = -4; 25) = 57 - 40$, $n(a_{13} = 3, 13) = 27 - 22$,$n(a_7 = -5, 49) = 89 - 78$. Therefore $X_0(258)/W_3, X_0(258)/W_{86}, X_0^+(258)$ are not bielliptic.

$X_0(258)/ < W_6, W_{43}, W_{258} >$**, genus 10**

```
[*
    q + q^2 + a*q^3 + q^4 + (-a - 1)*q^5 + a*q^6 + (-4*a + 2)*q^7 + q^8 + (a -
        2)*q^9 + (-a - 1)*q^10 + (4*a - 4)*q^11 + a*q^12 + (4*a - 2)*q^13 +
        (-4*a + 2)*q^14 + (-2*a - 1)*q^15 + q^16 - a*q^17 + (a - 2)*q^18 + (a +
        5)*q^19 + O(q^20),
```

```
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 2*q^7 + q^9 - 5*q^11 + 2*q^12 + 3*q^13 + 2*q^15 +
        4*q^16 - 3*q^17 + 2*q^19 + O(q^20),
    q + q^2 + q^3 - q^4 + 2*q^5 + q^6 - 3*q^8 + q^9 + 2*q^10 - q^12 - 2*q^13 +
        2*q^15 - q^16 - 6*q^17 + q^18 + 4*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a - 2)*q^5 + a*q^6 + (-a^2 + 6)*q^7 +
        (-2*a^2 + a + 8)*q^8 + q^9 + (-a^2 - 2*a)*q^10 + (a^2 - a - 5)*q^11 +
        (a^2 - 2)*q^12 + 3*q^13 + (2*a^2 + a - 8)*q^14 + (-a - 2)*q^15 + (3*a^2
        - 2*a - 12)*q^16 + (-a^2 + 5)*q^17 + a*q^18 + (-a^2 - 2*a + 2)*q^19 +
        O(q^20),
    q - 2*q^2 - 2*q^3 + 2*q^4 - 4*q^5 + 4*q^6 + q^9 + 8*q^10 + 3*q^11 - 4*q^12 -
        5*q^13 + 8*q^15 - 4*q^16 - 3*q^17 - 2*q^18 - 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 + q^5 + q^6 - 5*q^7 - q^8 + q^9 - q^10 + q^11 - q^12 -
        3*q^13 + 5*q^14 - q^15 + q^16 - q^18 - 7*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 + 2*x^2 - 5*x - 8 over the
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 43, 86, 86, 129, 129, 129, 129, 258 *]
```

Not bielliptic, $n(a_5 = -4; 25) = 60 - 20$, $n(a_5 = -2, 5) = 20 - 16$, $n(a_5 = 0, 5) = 20 - 12$, $n(a_5 = 1; 5) = 20 - 10$. In particular $X_0(258)/W_{43}, X_0(258)/W_6$ are not bielliptic.

8. Level $N = 246$

$X_0(246)/ < w_2, w_3, w_6 >$, **genus 10**

```
[*
    q + a*q^2 + 1/2*(-a^2 - 2*a + 3)*q^3 + (a^2 - 2)*q^4 + (-a - 1)*q^5 +
        1/2*(-a^2 - 2*a - 1)*q^6 + 1/2*(a^2 + 2*a + 1)*q^7 + (-a^2 + a + 1)*q^8
        + a*q^9 + (-a^2 - a)*q^10 + 1/2*(3*a^2 + 2*a - 9)*q^11 + 1/2*(a^2 - 2*a
        - 7)*q^12 + (-a^2 + 3)*q^13 + 1/2*(a^2 + 6*a + 1)*q^14 + (a^2 + 2*a -
        1)*q^15 + (-4*a + 3)*q^16 - 2*q^17 + a^2*q^18 + 1/2*(-3*a^2 - 2*a +
        13)*q^19 + O(q^20),
    q - q^2 - 2*q^3 + q^4 - 2*q^5 + 2*q^6 - 4*q^7 - q^8 + q^9 + 2*q^10 - 2*q^11
        - 2*q^12 + 4*q^13 + 4*q^14 + 4*q^15 + q^16 - 2*q^17 - q^18 + 6*q^19 +
        O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 4*q^7 + q^9 + 5*q^11 + 2*q^12 - 4*q^13 + 2*q^15 +
        4*q^16 - 5*q^17 - 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-a^2 + a + 4)*q^5 - a*q^6 + (-a^2 - a +
        4)*q^7 + (a^2 - 2)*q^8 + q^9 + 2*q^10 + (-a - 1)*q^11 + (-a^2 + 2)*q^12
        + (a^2 - a)*q^13 + (-2*a^2 + 2)*q^14 + (a^2 - a - 4)*q^15 + (-a^2 + 2*a
        + 2)*q^16 + (2*a^2 - a - 5)*q^17 + a*q^18 + (a^2 - a - 2)*q^19 +
        O(q^20),
```

```
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 + 2*q^7 - q^8 + q^9 + 2*q^10 - 4*q^11 -
        q^12 - 4*q^13 - 2*q^14 + 2*q^15 + q^16 - 2*q^17 - q^18 - 8*q^19 +
        O(q^20),
    q - q^2 - q^3 + q^4 + 3*q^5 + q^6 - 2*q^7 - q^8 + q^9 - 3*q^10 + 2*q^11 -
        q^12 + q^13 + 2*q^14 - 3*q^15 + q^16 + 5*q^17 - q^18 - q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^3 + x^2 - 5*x - 1 over the Rational
    Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 4*x + 2 over the Rational
    Field,
    Rational Field,
    Rational Field
*]
[* 41, 82, 123, 123, 246, 246 *]
```

Not bielliptic. $n(a_5 = 0; 25) = 73 - 72$.

$X_0(246)/ < w_2, w_{41} >$, **genus 8**

```
[*
    q - q^2 - 2*q^3 + q^4 - 2*q^5 + 2*q^6 - 4*q^7 - q^8 + q^9 + 2*q^10 - 2*q^11
        - 2*q^12 + 4*q^13 + 4*q^14 + 4*q^15 + q^16 - 2*q^17 - q^18 + 6*q^19 +
        O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 4*q^7 + q^9 + 5*q^11 + 2*q^12 - 4*q^13 + 2*q^15 +
        4*q^16 - 5*q^17 - 2*q^19 + O(q^20),
    q + a*q^2 + q^3 + (-a + 2)*q^5 + a*q^6 + (a - 2)*q^7 - 2*a*q^8 + q^9 + (2*a
        - 2)*q^10 + (-a + 1)*q^11 + (-3*a + 2)*q^13 + (-2*a + 2)*q^14 + (-a +
        2)*q^15 - 4*q^16 + (a + 1)*q^17 + a*q^18 + (a - 4)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 + 2*q^7 - q^8 + q^9 + 2*q^10 - 4*q^11 -
        q^12 - 4*q^13 - 2*q^14 + 2*q^15 + q^16 - 2*q^17 - q^18 - 8*q^19 +
        O(q^20),
    q - q^2 + q^3 + q^4 - 2*q^5 - q^6 + 2*q^7 - q^8 + q^9 + 2*q^10 + 4*q^11 +
        q^12 + 4*q^13 - 2*q^14 - 2*q^15 + q^16 - 2*q^17 - q^18 + O(q^20),
    q - q^2 + q^3 + q^4 + 3*q^5 - q^6 + 2*q^7 - q^8 + q^9 - 3*q^10 - 6*q^11 +
        q^12 - q^13 - 2*q^14 + 3*q^15 + q^16 + 3*q^17 - q^18 + 5*q^19 + O(q^20),
    q - q^2 - 2*q^3 + q^4 - 2*q^5 + 2*q^6 - 4*q^7 - q^8 + q^9 + 2*q^10 - 2*q^11
        - 2*q^12 + 4*q^13 + 4*q^14 + 4*q^15 + q^16 - 2*q^17 - q^18 + 6*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 82, 123, 123, 246, 246, 246, 246 *]
```

Not bielliptic $n(|a_5| \geq 2; 25) \geq 65 - 64$.

$X_0(246)/ < w_3, w_{41} >$, **genus 7**

```
[*
    q - q^2 - 2*q^3 + q^4 - 2*q^5 + 2*q^6 - 4*q^7 - q^8 + q^9 + 2*q^10 - 2*q^11
        - 2*q^12 + 4*q^13 + 4*q^14 + 4*q^15 + q^16 - 2*q^17 - q^18 + 6*q^19 +
        O(q^20),
    q + q^2 + a*q^3 + q^4 - 2*a*q^5 + a*q^6 + (-a - 2)*q^7 + q^8 - q^9 -
        2*a*q^10 + 3*a*q^11 + a*q^12 + (-a - 2)*q^14 - 4*q^15 + q^16 + (4*a +
        2)*q^17 - q^18 + (-a - 4)*q^19 + O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 4*q^7 + q^9 + 5*q^11 + 2*q^12 - 4*q^13 + 2*q^15 +
        4*q^16 - 5*q^17 - 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 + 2*q^7 - q^8 + q^9 + 2*q^10 - 4*q^11 -
        q^12 - 4*q^13 - 2*q^14 + 2*q^15 + q^16 - 2*q^17 - q^18 - 8*q^19 +
        O(q^20),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + 2*q^7 + q^8 + q^9 + q^10 + 2*q^11 - q^12 -
        7*q^13 + 2*q^14 - q^15 + q^16 + 7*q^17 + q^18 + 7*q^19 + O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 4*q^7 + q^9 + 5*q^11 + 2*q^12 - 4*q^13 + 2*q^15 +
        4*q^16 - 5*q^17 - 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 82, 82, 123, 246, 246, 246 *]
```

$n(a_7 \geq 0; 7) \geq 20 - 16$,?????

$X_0(246)/ < W_6, W_{82}, W_{123} >$, **genus 9**

```
[*
    q + a*q^2 + 1/2*(-a^2 - 2*a + 3)*q^3 + (a^2 - 2)*q^4 + (-a - 1)*q^5 +
        1/2*(-a^2 - 2*a - 1)*q^6 + 1/2*(a^2 + 2*a + 1)*q^7 + (-a^2 + a + 1)*q^8
        + a*q^9 + (-a^2 - a)*q^10 + 1/2*(3*a^2 + 2*a - 9)*q^11 + 1/2*(a^2 - 2*a
        - 7)*q^12 + (-a^2 + 3)*q^13 + 1/2*(a^2 + 6*a + 1)*q^14 + (a^2 + 2*a -
        1)*q^15 + (-4*a + 3)*q^16 - 2*q^17 + a^2*q^18 + 1/2*(-3*a^2 - 2*a +
        13)*q^19 + O(q^20),
    q - q^2 - 2*q^3 + q^4 - 2*q^5 + 2*q^6 - 4*q^7 - q^8 + q^9 + 2*q^10 - 2*q^11
        - 2*q^12 + 4*q^13 + 4*q^14 + 4*q^15 + q^16 - 2*q^17 - q^18 + 6*q^19 +
        O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 4*q^7 + q^9 + 5*q^11 + 2*q^12 - 4*q^13 + 2*q^15 +
        4*q^16 - 5*q^17 - 2*q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 - 4*q^5 - 2*q^6 - 2*q^7 + q^9 + 8*q^10 - 3*q^11 +
        2*q^12 - 6*q^13 + 4*q^14 - 4*q^15 - 4*q^16 + 3*q^17 - 2*q^18 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 + 2*q^7 - q^8 + q^9 + 2*q^10 - 4*q^11 -
        q^12 - 4*q^13 - 2*q^14 + 2*q^15 + q^16 - 2*q^17 - q^18 - 8*q^19 +
        O(q^20),
    q + q^2 + q^3 + q^4 + q^5 + q^6 - 2*q^7 + q^8 + q^9 + q^10 + 2*q^11 + q^12 -
        q^13 - 2*q^14 + q^15 + q^16 - 7*q^17 + q^18 + 5*q^19 + O(q^20),
```

```
    q + q^2 + q^3 + q^4 - 2*q^5 + q^6 + 4*q^7 + q^8 + q^9 - 2*q^10 - 4*q^11 +
        q^12 + 2*q^13 + 4*q^14 - 2*q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 +
        O(q^20)
*]
[*
    Number Field with defining polynomial x^3 + x^2 - 5*x - 1 over the Rational
    Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 41, 82, 123, 123, 246, 246, 246 *]
```

Not bielliptic, $n(a_5 = 0; 5) = 19-12$, $n(a_5 \neq 0; 25) \geq n(a_5 = \pm 1; 25) = 71-70$, therefore $X_0(246)/W_6, X_0(246)/W_{82}, X_0(246)$ are not bielliptic.

$X_0(246)/ < W_2, W_{123}, W_{246} >$**, genus 7**

```
[*
    q + a*q^2 + 1/2*(-a^2 - 2*a + 3)*q^3 + (a^2 - 2)*q^4 + (-a - 1)*q^5 +
        1/2*(-a^2 - 2*a - 1)*q^6 + 1/2*(a^2 + 2*a + 1)*q^7 + (-a^2 + a + 1)*q^8
        + a*q^9 + (-a^2 - a)*q^10 + 1/2*(3*a^2 + 2*a - 9)*q^11 + 1/2*(a^2 - 2*a
        - 7)*q^12 + (-a^2 + 3)*q^13 + 1/2*(a^2 + 6*a + 1)*q^14 + (a^2 + 2*a -
        1)*q^15 + (-4*a + 3)*q^16 - 2*q^17 + a^2*q^18 + 1/2*(-3*a^2 - 2*a +
        13)*q^19 + O(q^20),
    q - q^2 - 2*q^3 + q^4 - 2*q^5 + 2*q^6 - 4*q^7 - q^8 + q^9 + 2*q^10 - 2*q^11
        - 2*q^12 + 4*q^13 + 4*q^14 + 4*q^15 + q^16 - 2*q^17 - q^18 + 6*q^19 +
        O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 4*q^7 + q^9 + 5*q^11 + 2*q^12 - 4*q^13 + 2*q^15 +
        4*q^16 - 5*q^17 - 2*q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 - 4*q^5 - 2*q^6 - 2*q^7 + q^9 + 8*q^10 - 3*q^11 +
        2*q^12 - 6*q^13 + 4*q^14 - 4*q^15 - 4*q^16 + 3*q^17 - 2*q^18 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 + 2*q^7 - q^8 + q^9 + 2*q^10 - 4*q^11 -
        q^12 - 4*q^13 - 2*q^14 + 2*q^15 + q^16 - 2*q^17 - q^18 - 8*q^19 +
        O(q^20)
*]
[*
    Number Field with defining polynomial x^3 + x^2 - 5*x - 1 over the Rational
    Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 41, 82, 123, 123, 246 *]
```

Not bielliptic, because $n(a_5 \leq -2, 5) \geq n(a_5 = -2; 5) = 18 - 16$ and $n(|a_5| \geq 3, 25) \geq n(a_5 = \pm 3; 25) = 56 - 54$. In particular $X_0(246)/W_2, X_0(246)/W_{246}$ are not bielliptic.

$X_0(246)/ < W_3, W_{82}, W_{246} >$**, genus 9**

```
[*
```

```
    q + a*q^2 + 1/2*(-a^2 - 2*a + 3)*q^3 + (a^2 - 2)*q^4 + (-a - 1)*q^5 +
        1/2*(-a^2 - 2*a - 1)*q^6 + 1/2*(a^2 + 2*a + 1)*q^7 + (-a^2 + a + 1)*q^8
        + a*q^9 + (-a^2 - a)*q^10 + 1/2*(3*a^2 + 2*a - 9)*q^11 + 1/2*(a^2 - 2*a
        - 7)*q^12 + (-a^2 + 3)*q^13 + 1/2*(a^2 + 6*a + 1)*q^14 + (a^2 + 2*a -
        1)*q^15 + (-4*a + 3)*q^16 - 2*q^17 + a^2*q^18 + 1/2*(-3*a^2 - 2*a +
        13)*q^19 + O(q^20),
    q - q^2 - 2*q^3 + q^4 - 2*q^5 + 2*q^6 - 4*q^7 - q^8 + q^9 + 2*q^10 - 2*q^11
        - 2*q^12 + 4*q^13 + 4*q^14 + 4*q^15 + q^16 - 2*q^17 - q^18 + 6*q^19 +
        O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 4*q^7 + q^9 + 5*q^11 + 2*q^12 - 4*q^13 + 2*q^15 +
        4*q^16 - 5*q^17 - 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-a^2 + a + 4)*q^5 - a*q^6 + (-a^2 - a +
        4)*q^7 + (a^2 - 2)*q^8 + q^9 + 2*q^10 + (-a - 1)*q^11 + (-a^2 + 2)*q^12
        + (a^2 - a)*q^13 + (-2*a^2 + 2)*q^14 + (a^2 - a - 4)*q^15 + (-a^2 + 2*a
        + 2)*q^16 + (2*a^2 - a - 5)*q^17 + a*q^18 + (a^2 - a - 2)*q^19 +
        O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 + 2*q^7 - q^8 + q^9 + 2*q^10 - 4*q^11 -
        q^12 - 4*q^13 - 2*q^14 + 2*q^15 + q^16 - 2*q^17 - q^18 - 8*q^19 +
        O(q^20)
*]
[*
    Number Field with defining polynomial x^3 + x^2 - 5*x - 1 over the Rational
    Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 4*x + 2 over the Rational
    Field,
    Rational Field
*]
[* 41, 82, 123, 123, 246 *]
```

Not bielliptic and $n(a_5 \neq 0; 25) > n(a_5 = 0; 25) = 72 - 72 = 0$, the case $a_5 = 0$ does not appear above modular forms. Therefore $X_0(246)/W_3$ is not bielliptic.

$X_0(246)/ < W_6, W_{41}, W_{246} >$**, genus 7**

```
[*
    q - q^2 - 2*q^3 + q^4 - 2*q^5 + 2*q^6 - 4*q^7 - q^8 + q^9 + 2*q^10 - 2*q^11
        - 2*q^12 + 4*q^13 + 4*q^14 + 4*q^15 + q^16 - 2*q^17 - q^18 + 6*q^19 +
        O(q^20),
    q + q^2 + a*q^3 + q^4 - 2*a*q^5 + a*q^6 + (-a - 2)*q^7 + q^8 - q^9 -
        2*a*q^10 + 3*a*q^11 + a*q^12 + (-a - 2)*q^14 - 4*q^15 + q^16 + (4*a +
        2)*q^17 - q^18 + (-a - 4)*q^19 + O(q^20),
    q - q^3 - 2*q^4 - 2*q^5 - 4*q^7 + q^9 + 5*q^11 + 2*q^12 - 4*q^13 + 2*q^15 +
        4*q^16 - 5*q^17 - 2*q^19 + O(q^20),
    q + a*q^2 + q^3 + (-a + 2)*q^5 + a*q^6 + (a - 2)*q^7 - 2*a*q^8 + q^9 + (2*a
        - 2)*q^10 + (-a + 1)*q^11 + (-3*a + 2)*q^13 + (-2*a + 2)*q^14 + (-a +
        2)*q^15 - 4*q^16 + (a + 1)*q^17 + a*q^18 + (a - 4)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 + 2*q^7 - q^8 + q^9 + 2*q^10 - 4*q^11 -
        q^12 - 4*q^13 - 2*q^14 + 2*q^15 + q^16 - 2*q^17 - q^18 - 8*q^19 +
        O(q^20)
*]
```

```
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Rational Field
*]
[* 82, 82, 123, 123, 246 *]
```

???? 123b, $n(82a, a_{19} = 6; 19) = 40 - 28$, $n(246, a_7 = 2; 7) = 22 - 12$.

9. Level $N = 357$

$X_0(357)/ < w_3, w_7 >$, **genus 11**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a + 2)*q^4 + (-a + 1)*q^5 - a*q^6 + (a - 4)*q^8 + q^9 +
        (2*a - 4)*q^10 + (-a - 1)*q^11 + (a - 2)*q^12 + (a + 3)*q^13 + (a -
        1)*q^15 - 3*a*q^16 + q^17 + a*q^18 + (3*a + 3)*q^19 + O(q^20),
    q + a*q^2 + (-a^4 + 6*a^2 + a - 4)*q^3 + (a^2 - 2)*q^4 + (2*a^4 + a^3 -
        15*a^2 - 6*a + 18)*q^5 + (-2*a^4 - 2*a^3 + 15*a^2 + 10*a - 17)*q^6 - q^7
        + (a^3 - 4*a)*q^8 + (2*a^4 + a^3 - 13*a^2 - 8*a + 13)*q^9 + (5*a^4 + a^3
        - 34*a^2 - 10*a + 34)*q^10 + (-2*a^4 - 2*a^3 + 14*a^2 + 12*a - 14)*q^11
        + (-4*a^4 - a^3 + 26*a^2 + 9*a - 26)*q^12 + (-2*a^4 + 14*a^2 - 14)*q^13
        - a*q^14 + (-a^4 - a^3 + 7*a^2 + 3*a - 4)*q^15 + (a^4 - 6*a^2 + 4)*q^16
        + q^17 + (5*a^4 + 3*a^3 - 36*a^2 - 15*a + 34)*q^18 + (-2*a^4 + 14*a^2 +
        2*a - 14)*q^19 + O(q^20),
    q - q^3 - 2*q^4 + q^5 - q^7 + q^9 + 3*q^11 + 2*q^12 + 3*q^13 - q^15 + 4*q^16
        + q^17 + 3*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a - 1)*q^5 - a*q^6 - q^7 - 2*a*q^8 + q^9 + (-a - 2)*q^10
        + q^11 + (-a - 3)*q^13 - a*q^14 + (a + 1)*q^15 - 4*q^16 - q^17 + a*q^18
        + (-a - 5)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Number Field with defining polynomial x^5 - 2*x^4 - 8*x^3 + 14*x^2 + 14*x -
        17 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field
*]
[* 17, 51, 119, 357, 357 *]
```

Not bielliptic, $n(a_2 = 0; 16) = 39 - 18$, $n(a_2 = -1; 8) = 9 - 8$.

$X_0(357)/ < w_3, w_{17} >$, **genus 10**

```
[*
    q + a*q^2 + (-a^3 - a^2 + 4*a + 1)*q^3 + (a^2 - 2)*q^4 + (a^3 + a^2 -
        4*a)*q^5 + (-a^2 + 3)*q^6 + q^7 + (a^3 - 4*a)*q^8 + (-a^3 - 3*a^2 + 2*a
        + 7)*q^9 + (a^2 + a - 3)*q^10 - 2*a*q^11 + (a^3 + 2*a^2 - 5*a - 2)*q^12
        + (2*a^3 + 4*a^2 - 6*a - 4)*q^13 + a*q^14 + (2*a^2 + 2*a - 9)*q^15 +
        (-a^3 - a^2 + a + 1)*q^16 - q^17 + (-2*a^3 - 3*a^2 + 6*a + 3)*q^18 +
```

```
        (-2*a^3 - 4*a^2 + 4*a + 8)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a - 1)*q^5 - a*q^6 - q^7 - 2*a*q^8 + q^9 + (-a - 2)*q^10
        + q^11 + (-a - 3)*q^13 - a*q^14 + (a + 1)*q^15 - 4*q^16 - q^17 + a*q^18
        + (-a - 5)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-a^3 + a^2 + 5*a - 3)*q^5 - a*q^6 + q^7 +
        (a^3 - 4*a)*q^8 + q^9 + (-a^3 + 5*a + 2)*q^10 + (-a^2 + 2*a + 3)*q^11 +
        (-a^2 + 2)*q^12 + (a^3 - a^2 - 5*a + 3)*q^13 + a*q^14 + (a^3 - a^2 - 5*a
        + 3)*q^15 + (2*a^3 - a^2 - 8*a + 2)*q^16 - q^17 + a*q^18 + (-a^3 + a^2 +
        3*a + 1)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^4 + x^3 - 5*x^2 - x + 3 over the
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Number Field with defining polynomial x^4 - 2*x^3 - 5*x^2 + 8*x + 2 over the
    Rational Field
*]
[* 119, 357, 357 *]
```

Not bielliptic.

$X_0(357)/ < w_7, w_{17} >$, **genus 5**

```
[*
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + O(q^20),
    q + q^3 - 2*q^4 + 3*q^5 - 4*q^7 + q^9 - 3*q^11 - 2*q^12 - q^13 + 3*q^15 +
        4*q^16 - q^17 - q^19 + O(q^20),
    q + 2*q^2 + q^3 + 2*q^4 + q^5 + 2*q^6 - q^7 + q^9 + 2*q^10 + q^11 + 2*q^12 +
        q^13 - 2*q^14 + q^15 - 4*q^16 - q^17 + 2*q^18 + q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a - 1)*q^5 - a*q^6 - q^7 - 2*a*q^8 + q^9 + (-a - 2)*q^10
        + q^11 + (-a - 3)*q^13 - a*q^14 + (a + 1)*q^15 - 4*q^16 - q^17 + a*q^18
        + (-a - 5)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field
*]
[* 21, 51, 357, 357 *]
```

La primers?, $n(a_2 = 0; 16) = 24 - 18$, $n(a_2 = 2; 4) = 16 - 10$.

$X_0(357)/ < W_{21}, W_{51}, W_{119} >$, **genus 6**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a - 1)*q^5 - a*q^6 - q^7 - 2*a*q^8 + q^9 + (-a - 2)*q^10
        + q^11 + (-a - 3)*q^13 - a*q^14 + (a + 1)*q^15 - 4*q^16 - q^17 + a*q^18
        + (-a - 5)*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a + 1)*q^5 + a*q^6 + q^7 + (a^2 - 2)*q^8
        + q^9 + (-a^2 + a)*q^10 + (-a^2 + 5)*q^11 + (a^2 - 2)*q^12 + (-2*a^2 + a
        + 5)*q^13 + a*q^14 + (-a + 1)*q^15 + (-a^2 + 2*a + 2)*q^16 + q^17 +
```

```
        a*q^18 + (-a - 1)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 4*x + 2 over the Rational
    Field
*]
[* 17, 357, 357 *]
```

Not bielliptic, $n(a_5 = -2; 25) = 66-64$. Therefore $X_0(357)/W_{21}, X_0(357)/W_{51}, X_0(357)/W_{119}$ are not bielliptic.

$X_0(357)/ < W_3, W_{119}, W_{357} >$, **genus 6**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a + 2)*q^4 + (-a + 1)*q^5 - a*q^6 + (a - 4)*q^8 + q^9 +
        (2*a - 4)*q^10 + (-a - 1)*q^11 + (a - 2)*q^12 + (a + 3)*q^13 + (a -
        1)*q^15 - 3*a*q^16 + q^17 + a*q^18 + (3*a + 3)*q^19 + O(q^20),
    q - q^3 - 2*q^4 + q^5 + q^7 + q^9 - 5*q^11 + 2*q^12 - 5*q^13 - q^15 + 4*q^16
        + q^17 - 5*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a - 1)*q^5 - a*q^6 - q^7 - 2*a*q^8 + q^9 + (-a - 2)*q^10
        + q^11 + (-a - 3)*q^13 - a*q^14 + (a + 1)*q^15 - 4*q^16 - q^17 + a*q^18
        + (-a - 5)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field
*]
[* 17, 51, 357, 357 *]
```

???? 17a,357b.

$X_0(357)/ < W_7, W_{51}, W_{357} >$, **genus 11**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + O(q^20),
    q + a*q^2 + (-a^4 + 6*a^2 + a - 4)*q^3 + (a^2 - 2)*q^4 + (2*a^4 + a^3 -
        15*a^2 - 6*a + 18)*q^5 + (-2*a^4 - 2*a^3 + 15*a^2 + 10*a - 17)*q^6 - q^7
        + (a^3 - 4*a)*q^8 + (2*a^4 + a^3 - 13*a^2 - 8*a + 13)*q^9 + (5*a^4 + a^3
        - 34*a^2 - 10*a + 34)*q^10 + (-2*a^4 - 2*a^3 + 14*a^2 + 12*a - 14)*q^11
        + (-4*a^4 - a^3 + 26*a^2 + 9*a - 26)*q^12 + (-2*a^4 + 14*a^2 - 14)*q^13
        - a*q^14 + (-a^4 - a^3 + 7*a^2 + 3*a - 4)*q^15 + (a^4 - 6*a^2 + 4)*q^16
        + q^17 + (5*a^4 + 3*a^3 - 36*a^2 - 15*a + 34)*q^18 + (-2*a^4 + 14*a^2 +
        2*a - 14)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a - 1)*q^5 - a*q^6 - q^7 - 2*a*q^8 + q^9 + (-a - 2)*q^10
        + q^11 + (-a - 3)*q^13 - a*q^14 + (a + 1)*q^15 - 4*q^16 - q^17 + a*q^18
        + (-a - 5)*q^19 + O(q^20),
    q + a*q^2 + q^3 - 2*a*q^4 + (-a - 3)*q^5 + a*q^6 - q^7 + (2*a - 4)*q^8 + q^9
```

```
        + (-a - 2)*q^10 - 5*q^11 - 2*a*q^12 + (3*a + 1)*q^13 - a*q^14 + (-a -
        3)*q^15 + (-4*a + 4)*q^16 + q^17 + a*q^18 + (-3*a - 5)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^5 - 2*x^4 - 8*x^3 + 14*x^2 + 14*x -
        17 over the Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 2 over the Rational Field
*]
[* 17, 21, 119, 357, 357 *]
```

???17a $n(21 : a_1 1 = 4; 11) = 18 - 16$.

$X_0(357)/ < W_{17}, W_{21}, W_{357} >$, **genus 8**

```
[*
    q + q^3 - 2*q^4 + 3*q^5 - 4*q^7 + q^9 - 3*q^11 - 2*q^12 - q^13 + 3*q^15 +
        4*q^16 - q^17 - q^19 + O(q^20),
    q + a*q^2 + (-a^3 - a^2 + 4*a + 1)*q^3 + (a^2 - 2)*q^4 + (a^3 + a^2 -
        4*a)*q^5 + (-a^2 + 3)*q^6 + q^7 + (a^3 - 4*a)*q^8 + (-a^3 - 3*a^2 + 2*a
        + 7)*q^9 + (a^2 + a - 3)*q^10 - 2*a*q^11 + (a^3 + 2*a^2 - 5*a - 2)*q^12
        + (2*a^3 + 4*a^2 - 6*a - 4)*q^13 + a*q^14 + (2*a^2 + 2*a - 9)*q^15 +
        (-a^3 - a^2 + a + 1)*q^16 - q^17 + (-2*a^3 - 3*a^2 + 6*a + 3)*q^18 +
        (-2*a^3 - 4*a^2 + 4*a + 8)*q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 - 3*q^5 - 2*q^6 + q^7 + q^9 + 6*q^10 - 3*q^11 +
        2*q^12 + q^13 - 2*q^14 - 3*q^15 - 4*q^16 - q^17 - 2*q^18 - 7*q^19 +
        O(q^20),
    q + a*q^2 - q^3 + (-a - 1)*q^5 - a*q^6 - q^7 - 2*a*q^8 + q^9 + (-a - 2)*q^10
        + q^11 + (-a - 3)*q^13 - a*q^14 + (a + 1)*q^15 - 4*q^16 - q^17 + a*q^18
        + (-a - 5)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^4 + x^3 - 5*x^2 - x + 3 over the
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2 over the Rational Field
*]
[* 51, 119, 357, 357 *]
```

Not bielliptic, $n(a_2 = 0; 2^4) = 26-18$, $n(a_2 = -2, 4) = 18-10$. Therefore $X_0(357)/W_{17}, X_0(357)/W_{21}, X_0(357)/W_{357}$ not bielliptic.

10. Level $N = 286$

$X_0(286)/ < w_2, w_{11}$, **genus 10**

```
[*
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 + O(q^20),
    q - q^3 - 2*q^4 - q^5 - 2*q^7 - 2*q^9 - q^11 + 2*q^12 - q^13 + q^15 + 4*q^16
        - 4*q^17 + 2*q^19 + O(q^20),
    q + a*q^2 + (-a^5 - a^4 + 8*a^3 + 6*a^2 - 11*a - 5)*q^3 + (a^2 - 2)*q^4 +
```

```
        (a^5 + 2*a^4 - 8*a^3 - 14*a^2 + 12*a + 15)*q^5 + (-a^5 - 2*a^4 + 8*a^3 +
        13*a^2 - 12*a - 12)*q^6 + (2*a^5 + 2*a^4 - 17*a^3 - 13*a^2 + 26*a +
        14)*q^7 + (a^3 - 4*a)*q^8 + (-3*a^5 - 4*a^4 + 25*a^3 + 27*a^2 - 38*a -
        26)*q^9 + (2*a^5 + 2*a^4 - 16*a^3 - 12*a^2 + 22*a + 12)*q^10 - q^11 +
        (-a^3 + 3*a - 2)*q^12 + q^13 + (2*a^5 + 3*a^4 - 17*a^3 - 22*a^2 + 28*a +
        24)*q^14 + (3*a^5 + 4*a^4 - 24*a^3 - 28*a^2 + 30*a + 33)*q^15 + (a^4 -
        6*a^2 + 4)*q^16 - 2*a*q^17 + (-4*a^5 - 5*a^4 + 33*a^3 + 34*a^2 - 47*a -
        36)*q^18 + (-2*a^5 - 3*a^4 + 16*a^3 + 20*a^2 - 23*a - 22)*q^19 +
        O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 + q^7 - q^8 - 2*q^9 + q^10 - q^11 - q^12 -
        q^13 - q^14 + q^15 + q^16 - q^17 + 2*q^18 - 4*q^19 + O(q^20),
    q - q^2 - 2*q^3 + q^4 + 3*q^5 + 2*q^6 - q^7 - q^8 + q^9 - 3*q^10 - q^11 -
        2*q^12 + q^13 + q^14 - 6*q^15 + q^16 + 6*q^17 - q^18 + 8*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^6 - 10*x^4 + 2*x^3 + 24*x^2 - 7*x -
        12 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 26, 143, 143, 286, 286 *]
```

Not bielliptic $n(|a_3| \geq 1; 9) \geq 32 - 30$.

$X_0(286)/ < w_2, w_{13} >$, **tenus 10**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - q^3 - 2*q^4 - q^5 - 2*q^7 - 2*q^9 - q^11 + 2*q^12 - q^13 + q^15 + 4*q^16
        - 4*q^17 + 2*q^19 + O(q^20),
    q + a*q^2 + (-a^3 + 3*a^2 - 3)*q^3 + (a^2 - 2)*q^4 + (-2*a^2 + 2*a + 4)*q^5
        + (-a^2 + 2*a + 1)*q^6 + (a^3 - a^2 - 4*a + 2)*q^7 + (a^3 - 4*a)*q^8 +
        (a^3 - 3*a^2 - 2*a + 5)*q^9 + (-2*a^3 + 2*a^2 + 4*a)*q^10 + q^11 + (a^3
        - 4*a^2 + a + 6)*q^12 - q^13 + (2*a^3 - 3*a^2 - 3*a - 1)*q^14 + (-2*a^3
        + 6*a^2 + 2*a - 10)*q^15 + (3*a^3 - 5*a^2 - 5*a + 3)*q^16 + (-4*a^2 +
        6*a + 8)*q^17 + (-a^2 - 1)*q^18 + (-3*a^3 + 7*a^2 + 2*a - 3)*q^19 +
        O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 + q^7 - q^8 - 2*q^9 + q^10 - q^11 - q^12 -
        q^13 - q^14 + q^15 + q^16 - q^17 + 2*q^18 - 4*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + 1/2*(-a^2 + a + 8)*q^5 - a*q^6 + 1/2*(-a^2 + a +
        4)*q^7 - q^8 + (a^2 - 3)*q^9 + 1/2*(a^2 - a - 8)*q^10 + q^11 + a*q^12 -
        q^13 + 1/2*(a^2 - a - 4)*q^14 + (-a + 4)*q^15 + q^16 + (a^2 - 6)*q^17 +
        (-a^2 + 3)*q^18 - 4*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 - 3*x^3 - x^2 + 5*x + 1 over the
    Rational Field,
```

```
    Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 10*x + 8 over the Rational
    Field
*]
[* 11, 143, 143, 286, 286 *]
```

Not bielliptic $n(|a_3| \geq 1; 9) \geq 32 - 30$.

$X_0(286)/ < w_{11}, w_{13} >$, **genus 5**

```
[*
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 +
        O(q^20),
    q - q^3 - 2*q^4 - q^5 - 2*q^7 - 2*q^9 - q^11 + 2*q^12 - q^13 + q^15 + 4*q^16
        - 4*q^17 + 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 + q^7 - q^8 - 2*q^9 + q^10 - q^11 - q^12 -
        q^13 - q^14 + q^15 + q^16 - q^17 + 2*q^18 - 4*q^19 + O(q^20),
    q + q^2 + 2*q^3 + q^4 - q^5 + 2*q^6 + q^7 + q^8 + q^9 - q^10 - q^11 + 2*q^12
        - q^13 + q^14 - 2*q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 + O(q^20),
    q - q^3 - 2*q^4 - q^5 - 2*q^7 - 2*q^9 - q^11 + 2*q^12 - q^13 + q^15 + 4*q^16
        - 4*q^17 + 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 26, 143, 286, 286, 286 *]
```

Not bielliptic $n(a_5 = -1; 25) = 71 - 70$.

$X_0(286)/ < W_{22}, W_{26}, W_{143} >$, **genus 5**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - q^3 - 2*q^4 - q^5 - 2*q^7 - 2*q^9 - q^11 + 2*q^12 - q^13 + q^15 + 4*q^16
        - 4*q^17 + 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 + q^7 - q^8 - 2*q^9 + q^10 - q^11 - q^12 -
        q^13 - q^14 + q^15 + q^16 - q^17 + 2*q^18 - 4*q^19 + O(q^20),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 + q^10 + q^11 - q^12 +
        q^13 + 3*q^14 - q^15 + q^16 + 3*q^17 - 2*q^18 + O(q^20),
    q + q^2 + 2*q^3 + q^4 + q^5 + 2*q^6 - 3*q^7 + q^8 + q^9 + q^10 + q^11 +
        2*q^12 + q^13 - 3*q^14 + 2*q^15 + q^16 - 6*q^17 + q^18 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
```

```
[* 11, 143, 286, 286, 286 *]
```

Not bielliptic over $\mathbb{Q}$, $n(a_3 = -1; 9) = 32 - 30$, $n(a_3 = 2; 9) = 32 - 24$.

$X_0(286)/ < W_2, W_{143}, W_{286} >$, **genus 4**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 + O(q^20),
    q - q^3 - 2*q^4 - q^5 - 2*q^7 - 2*q^9 - q^11 + 2*q^12 - q^13 + q^15 + 4*q^16
        - 4*q^17 + 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 + q^7 - q^8 - 2*q^9 + q^10 - q^11 - q^12 -
        q^13 - q^14 + q^15 + q^16 - q^17 + 2*q^18 - 4*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 11, 26, 143, 286 *]
```

????,11a,26a,143a,286c

$X_0(286)/ < W_{11}, W_{26}, W_{286} >$, **genus 9**

```
[*
    q - q^3 - 2*q^4 - q^5 - 2*q^7 - 2*q^9 - q^11 + 2*q^12 - q^13 + q^15 + 4*q^16
        - 4*q^17 + 2*q^19 + O(q^20),
    q + a*q^2 + (-a^5 - a^4 + 8*a^3 + 6*a^2 - 11*a - 5)*q^3 + (a^2 - 2)*q^4 +
        (a^5 + 2*a^4 - 8*a^3 - 14*a^2 + 12*a + 15)*q^5 + (-a^5 - 2*a^4 + 8*a^3 +
        13*a^2 - 12*a - 12)*q^6 + (2*a^5 + 2*a^4 - 17*a^3 - 13*a^2 + 26*a +
        14)*q^7 + (a^3 - 4*a)*q^8 + (-3*a^5 - 4*a^4 + 25*a^3 + 27*a^2 - 38*a -
        26)*q^9 + (2*a^5 + 2*a^4 - 16*a^3 - 12*a^2 + 22*a + 12)*q^10 - q^11 +
        (-a^3 + 3*a - 2)*q^12 + q^13 + (2*a^5 + 3*a^4 - 17*a^3 - 22*a^2 + 28*a +
        24)*q^14 + (3*a^5 + 4*a^4 - 24*a^3 - 28*a^2 + 30*a + 33)*q^15 + (a^4 -
        6*a^2 + 4)*q^16 - 2*a*q^17 + (-4*a^5 - 5*a^4 + 33*a^3 + 34*a^2 - 47*a -
        36)*q^18 + (-2*a^5 - 3*a^4 + 16*a^3 + 20*a^2 - 23*a - 22)*q^19 +
        O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 + q^7 - q^8 - 2*q^9 + q^10 - q^11 - q^12 -
        q^13 - q^14 + q^15 + q^16 - q^17 + 2*q^18 - 4*q^19 + O(q^20),
    q + q^2 - q^3 + q^4 - 3*q^5 - q^6 - 5*q^7 + q^8 - 2*q^9 - 3*q^10 - q^11 -
        q^12 + q^13 - 5*q^14 + 3*q^15 + q^16 + 7*q^17 - 2*q^18 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^6 - 10*x^4 + 2*x^3 + 24*x^2 - 7*x -
        12 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 143, 143, 286, 286 *]
```

??? 143a,286c $n(a_7 = -5, 49) = 84 - 78$

$X_0(286)/ < W_{13}, W_{22}, W_{286} >$, **genus 8**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 +
        O(q^20),
    q - q^3 - 2*q^4 - q^5 - 2*q^7 - 2*q^9 - q^11 + 2*q^12 - q^13 + q^15 + 4*q^16
        - 4*q^17 + 2*q^19 + O(q^20),
    q + a*q^2 + (-a^3 + 3*a^2 - 3)*q^3 + (a^2 - 2)*q^4 + (-2*a^2 + 2*a + 4)*q^5
        + (-a^2 + 2*a + 1)*q^6 + (a^3 - a^2 - 4*a + 2)*q^7 + (a^3 - 4*a)*q^8 +
        (a^3 - 3*a^2 - 2*a + 5)*q^9 + (-2*a^3 + 2*a^2 + 4*a)*q^10 + q^11 + (a^3
        - 4*a^2 + a + 6)*q^12 - q^13 + (2*a^3 - 3*a^2 - 3*a - 1)*q^14 + (-2*a^3
        + 6*a^2 + 2*a - 10)*q^15 + (3*a^3 - 5*a^2 - 5*a + 3)*q^16 + (-4*a^2 +
        6*a + 8)*q^17 + (-a^2 - 1)*q^18 + (-3*a^3 + 7*a^2 + 2*a - 3)*q^19 +
        O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 + q^7 - q^8 - 2*q^9 + q^10 - q^11 - q^12 -
        q^13 - q^14 + q^15 + q^16 - q^17 + 2*q^18 - 4*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 - 3*x^3 - x^2 + 5*x + 1 over the
    Rational Field,
    Rational Field
*]
[* 11, 26, 143, 143, 286 *]
```

Not bielliptic, $n(a_3 = 0; 3) = 10-8$, $n(a_3 \neq 0; 9) > n(a_3 = 0; 9) = 32-32$. Therefore $X_0(286)/W_{13}, X_0(286)/W_{22}, X_0(286)/W$ not bielliptic.

11. Label $N = 285$

$X_0(285)/ < w_3, w_5 >$, **genus 9**

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 +
        O(q^20),
    q + a*q^2 + (-a^3 + 5*a - 2)*q^3 + (a^2 - 2)*q^4 - q^5 + (2*a^3 - a^2 - 10*a
        + 9)*q^6 + (-2*a^2 - 2*a + 8)*q^7 + (a^3 - 4*a)*q^8 + (-2*a + 1)*q^9 -
        a*q^10 + (2*a^2 + 2*a - 6)*q^11 + (-3*a^3 + 2*a^2 + 15*a - 14)*q^12 +
        (a^3 + 2*a^2 - 3*a - 4)*q^13 + (-2*a^3 - 2*a^2 + 8*a)*q^14 + (a^3 - 5*a
        + 2)*q^15 + (-2*a^3 + 8*a - 5)*q^16 + (2*a^3 - 10*a + 6)*q^17 + (-2*a^2
        + a)*q^18 + q^19 + O(q^20),
    q + q^2 - q^3 - q^4 - q^5 - q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 - 2*q^11 + q^12
        - 4*q^13 - 2*q^14 + q^15 - q^16 + 2*q^17 + q^18 - q^19 + O(q^20),
    q + a*q^2 - q^3 + (2*a - 1)*q^4 - q^5 - a*q^6 + (a + 1)*q^7 + (a + 2)*q^8 +
```

```
        q^9 - a*q^10 + (-a + 1)*q^11 + (-2*a + 1)*q^12 + (-a + 5)*q^13 + (3*a +
        1)*q^14 + q^15 + 3*q^16 + (-2*a - 2)*q^17 + a*q^18 + q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 + 2*x^3 - 6*x^2 - 8*x + 9 over the
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 1 over the Rational Field
*]
[* 19, 57, 95, 285, 285 *]
```

$n(a_2 = 0; 16) = 26 - 18$,$n(a_2 = -2; 4) = 14 - 10$,
$X_0(285)/ < w_3, w_{19} >$, **genus 10**

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 +
        O(q^20),
    q + a*q^2 + (-a^2 + 3)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^2 + 1)*q^6 + (2*a^2 -
        2*a - 4)*q^7 + (a^2 - a - 1)*q^8 + (-2*a^2 + 2*a + 5)*q^9 + a*q^10 +
        (-2*a - 2)*q^11 + (a^2 - 2*a - 5)*q^12 + (a^2 - 2*a + 1)*q^13 + (2*a -
        2)*q^14 + (-a^2 + 3)*q^15 + (-2*a^2 + 2*a + 3)*q^16 + (-2*a^2 + 4*a +
        4)*q^17 + (-a + 2)*q^18 - q^19 + O(q^20),
    q + q^2 - q^3 - q^4 - q^5 - q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 - 2*q^11 + q^12
        - 4*q^13 - 2*q^14 + q^15 - q^16 + 2*q^17 + q^18 - q^19 + O(q^20),
    q + q^2 - q^3 - q^4 + q^5 - q^6 + 4*q^7 - 3*q^8 + q^9 + q^10 + 4*q^11 + q^12
        + 2*q^13 + 4*q^14 - q^15 - q^16 + 2*q^17 + q^18 - q^19 + O(q^20),
    q + a*q^2 - q^3 + 5*q^4 + q^5 - a*q^6 + (-a - 1)*q^7 + 3*a*q^8 + q^9 +
        a*q^10 + (a + 3)*q^11 - 5*q^12 + (-a - 3)*q^13 + (-a - 7)*q^14 - q^15 +
        11*q^16 - 4*q^17 + a*q^18 - q^19 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 3*x + 1 over the Rational
    Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 7 over the Rational Field,
    Rational Field
*]
[* 15, 57, 95, 285, 285, 285, 285 *]
```

Not bielliptic. $n(a_2 = 1; 2) = 5 - 4$,$n(a_2 = -2; 4) = 13 - 10$, $n(a_2 = -1; 16) = 33 - 32$.
$X_0(285)/ < w_5, w_{19} >$, **genus 6**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 +
        O(q^20),
    q + q^2 + q^3 - q^4 - 2*q^5 + q^6 - 3*q^8 + q^9 - 2*q^10 - q^12 + 6*q^13 -
        2*q^15 - q^16 - 6*q^17 + q^18 - q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 + q^5 - 2*q^6 + 3*q^7 + q^9 - 2*q^10 - 3*q^11 +
        2*q^12 - 6*q^13 - 6*q^14 + q^15 - 4*q^16 + 3*q^17 - 2*q^18 - q^19 +
        O(q^20),
    q + q^2 - q^3 - q^4 - q^5 - q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 - 2*q^11 + q^12
        - 4*q^13 - 2*q^14 + q^15 - q^16 + 2*q^17 + q^18 - q^19 + O(q^20),
    q + a*q^2 + q^3 + (2*a - 1)*q^4 - q^5 + a*q^6 + (-a + 1)*q^7 + (a + 2)*q^8 +
        q^9 - a*q^10 + (-3*a + 5)*q^11 + (2*a - 1)*q^12 + (-a - 1)*q^13 + (-a -
        1)*q^14 - q^15 + 3*q^16 + (-2*a + 6)*q^17 + a*q^18 - q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 1 over the Rational Field
*]
[* 57, 57, 57, 285, 285 *]
```

$???n(a_2 = -2; 4) = 13 - 10$,
$X_0(285)/ < w_{95} >$, **genus 11**

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 +
        O(q^20),
    q + q^2 + q^3 - q^4 - 2*q^5 + q^6 - 3*q^8 + q^9 - 2*q^10 - q^12 + 6*q^13 -
        2*q^15 - q^16 - 6*q^17 + q^18 - q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 + q^5 - 2*q^6 + 3*q^7 + q^9 - 2*q^10 - 3*q^11 +
        2*q^12 - 6*q^13 - 6*q^14 + q^15 - 4*q^16 + 3*q^17 - 2*q^18 - q^19 +
        O(q^20),
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q + q^2 - q^3 - q^4 - q^5 - q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 - 2*q^11 + q^12
        - 4*q^13 - 2*q^14 + q^15 - q^16 + 2*q^17 + q^18 - q^19 + O(q^20),
    q + a*q^2 + q^3 + (2*a - 1)*q^4 - q^5 + a*q^6 + (-a + 1)*q^7 + (a + 2)*q^8 +
        q^9 - a*q^10 + (-3*a + 5)*q^11 + (2*a - 1)*q^12 + (-a - 1)*q^13 + (-a -
        1)*q^14 - q^15 + 3*q^16 + (-2*a + 6)*q^17 + a*q^18 - q^19 + O(q^20),
    q + a*q^2 + q^3 + q^4 + q^5 + a*q^6 + (a - 1)*q^7 - a*q^8 + q^9 + a*q^10 +
        (-a + 3)*q^11 + q^12 + (-a - 1)*q^13 + (-a + 3)*q^14 + q^15 - 5*q^16 +
        a*q^18 + q^19 + O(q^20)
*]
```

```
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field
*]
[* 15, 19, 57, 57, 57, 57, 285, 285, 285 *]
```

Not bielliptic $n(|a_2| \geq 0; 4) = 26 - 18$.

$X_0(285)/ < W_{15}, W_{57}, W_{95} >$, **genus 5**

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 +
        O(q^20),
    q + q^2 - q^3 - q^4 - q^5 - q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 - 2*q^11 + q^12
        - 4*q^13 - 2*q^14 + q^15 - q^16 + 2*q^17 + q^18 - q^19 + O(q^20),
    q + a*q^2 + q^3 + q^4 + q^5 + a*q^6 + (a - 1)*q^7 - a*q^8 + q^9 + a*q^10 +
        (-a + 3)*q^11 + q^12 + (-a - 1)*q^13 + (-a + 3)*q^14 + q^15 - 5*q^16 +
        a*q^18 + q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field
*]
[* 19, 57, 285, 285 *]
```

??? 285b, $n(a_2 = 0; 2^4) = 30 - 18$, $n(a_2 = -2; 4) = 14 - 10$.

$X_0(285)/ < W_3, W_{95}, W_{285} >$, **genus 4**

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 +
        O(q^20),
    q + q^2 - q^3 - q^4 - q^5 - q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 - 2*q^11 + q^12
        - 4*q^13 - 2*q^14 + q^15 - q^16 + 2*q^17 + q^18 - q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
```

```
    Rational Field,
    Rational Field
*]
[* 15, 19, 57, 285 *
```

???????15a,19a $n(a_2 = -2; 4) = 15 - 10$, $n(a_2 = 1; 2) = 5 - 4$.

$X_0(285)/ < W_5, W_{57}, W_{285} >$, **genus 8**

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 +
        O(q^20),
    q + a*q^2 + (-a^3 + 5*a - 2)*q^3 + (a^2 - 2)*q^4 - q^5 + (2*a^3 - a^2 - 10*a
        + 9)*q^6 + (-2*a^2 - 2*a + 8)*q^7 + (a^3 - 4*a)*q^8 + (-2*a + 1)*q^9 -
        a*q^10 + (2*a^2 + 2*a - 6)*q^11 + (-3*a^3 + 2*a^2 + 15*a - 14)*q^12 +
        (a^3 + 2*a^2 - 3*a - 4)*q^13 + (-2*a^3 - 2*a^2 + 8*a)*q^14 + (a^3 - 5*a
        + 2)*q^15 + (-2*a^3 + 8*a - 5)*q^16 + (2*a^3 - 10*a + 6)*q^17 + (-2*a^2
        + a)*q^18 + q^19 + O(q^20),
    q + q^2 - q^3 - q^4 - q^5 - q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 - 2*q^11 + q^12
        - 4*q^13 - 2*q^14 + q^15 - q^16 + 2*q^17 + q^18 - q^19 + O(q^20),
    q - q^2 + q^3 - q^4 - q^5 - q^6 - 2*q^7 + 3*q^8 + q^9 + q^10 - 6*q^11 - q^12
        + 2*q^14 - q^15 - q^16 - 6*q^17 - q^18 + q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 + 2*x^3 - 6*x^2 - 8*x + 9 over the
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 19, 57, 95, 285, 285 *]
```

Not bielliptic: $n(a_2 = 0; 2) = 7 - 6$, $n(a_2 = 1; 2) = 7 - 4$, $n(a_2 = -2; 4) = 15 - 10$, $n(a_2 = -1; 8) = 13 - 8$. Therefore $X_0(285)/W_5, X_0(285)/W_{57}$ $X_0(285)/W_{285}$ are not bielliptic.

$X_0(285)/ < W_{19}, W_{15}, W_{285} >$, **genus 7**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 +
        O(q^20),
    q + q^2 + q^3 - q^4 - 2*q^5 + q^6 - 3*q^8 + q^9 - 2*q^10 - q^12 + 6*q^13 -
        2*q^15 - q^16 - 6*q^17 + q^18 - q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 + q^5 - 2*q^6 + 3*q^7 + q^9 - 2*q^10 - 3*q^11 +
        2*q^12 - 6*q^13 - 6*q^14 + q^15 - 4*q^16 + 3*q^17 - 2*q^18 - q^19 +
        O(q^20),
    q + a*q^2 + (-a^2 + 3)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^2 + 1)*q^6 + (2*a^2 -
        2*a - 4)*q^7 + (a^2 - a - 1)*q^8 + (-2*a^2 + 2*a + 5)*q^9 + a*q^10 +
        (-2*a - 2)*q^11 + (a^2 - 2*a - 5)*q^12 + (a^2 - 2*a + 1)*q^13 + (2*a -
        2)*q^14 + (-a^2 + 3)*q^15 + (-2*a^2 + 2*a + 3)*q^16 + (-2*a^2 + 4*a +
        4)*q^17 + (-a + 2)*q^18 - q^19 + O(q^20),
```

```
    q + q^2 - q^3 - q^4 - q^5 - q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 - 2*q^11 + q^12
        - 4*q^13 - 2*q^14 + q^15 - q^16 + 2*q^17 + q^18 - q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 3*x + 1 over the Rational
    Field,
    Rational Field
*]
[* 57, 57, 57, 95, 285 *]
```

Not bielliptic $n(a_2 = 1; 16) = 36 - 32$, $n(a_2 = -2; 4) = 16 - 10$. Therefore $X_0(285)/W_{19}$ $X_0(285)/W_{15}$ are not bielliptic.

12. Level $N = 266$

$X_0(266)/ < w_2, w_7 >$, **genus 9**

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 - q^7 - q^8 - 2*q^9 - 6*q^11 + q^12 + 5*q^13 +
        q^14 + q^16 + 3*q^17 + 2*q^18 + q^19 + O(q^20),
    q + a*q^2 + a*q^3 + (-3*a - 3)*q^4 + (-2*a - 3)*q^5 + (-3*a - 1)*q^6 - q^7 +
        (4*a + 3)*q^8 + (-3*a - 4)*q^9 + (3*a + 2)*q^10 + (a - 3)*q^11 + (6*a +
        3)*q^12 + q^13 - a*q^14 + (3*a + 2)*q^15 + (-3*a + 2)*q^16 + (3*a +
        3)*q^17 + (5*a + 3)*q^18 - q^19 + O(q^20),
    q + a*q^2 + (-a^2 + 5)*q^3 + (a^2 - 2)*q^4 + (a^2 - a - 4)*q^5 + (-2*a^2 + a
        + 7)*q^6 - q^7 + (2*a^2 - 7)*q^8 + (-2*a^2 + a + 8)*q^9 + (a^2 - 7)*q^10
        + (-a + 3)*q^11 + (-a^2 - a + 4)*q^12 + (a^2 - a - 4)*q^13 - a*q^14 +
        (3*a^2 - 2*a - 13)*q^15 + (2*a^2 + a - 10)*q^16 + (-2*a^2 - a + 11)*q^17
        + (-3*a^2 + 14)*q^18 + q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + (a - 1)*q^5 - a*q^6 - q^7 - q^8 + (a + 4)*q^9 + (-a
        + 1)*q^10 + (-a + 2)*q^11 + a*q^12 - 2*a*q^13 + q^14 + 7*q^15 + q^16 -
        4*q^17 + (-a - 4)*q^18 + q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 2*x^2 - 4*x + 7 over the
    Rational Field,
    Number Field with defining polynomial x^2 - x - 7 over the Rational Field
*]
[* 19, 38, 133, 133, 266 *]
```

$n(a_3 = -2; 9) = 26 - 24$,??

$X_0(266)/ < w_2, w_{19} >$, **genus 7**

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20),
```

```
    q + a*q^2 + a*q^3 + (-3*a - 3)*q^4 + (-2*a - 3)*q^5 + (-3*a - 1)*q^6 - q^7 +
        (4*a + 3)*q^8 + (-3*a - 4)*q^9 + (3*a + 2)*q^10 + (a - 3)*q^11 + (6*a +
        3)*q^12 + q^13 - a*q^14 + (3*a + 2)*q^15 + (-3*a + 2)*q^16 + (3*a +
        3)*q^17 + (5*a + 3)*q^18 - q^19 + O(q^20),
    q + a*q^2 + (-a + 2)*q^3 + (a - 1)*q^4 + q^5 + (a - 1)*q^6 + q^7 + (-2*a +
        1)*q^8 + (-3*a + 2)*q^9 + a*q^10 + (a - 1)*q^11 + (2*a - 3)*q^12 - q^13
        + a*q^14 + (-a + 2)*q^15 - 3*a*q^16 + (3*a - 1)*q^17 + (-a - 3)*q^18 -
        q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + (-3*a + 5)*q^5 - a*q^6 + q^7 - q^8 + (3*a - 4)*q^9 +
        (3*a - 5)*q^10 + (a + 2)*q^11 + a*q^12 + 2*a*q^13 - q^14 + (-4*a +
        3)*q^15 + q^16 + (4*a - 8)*q^17 + (-3*a + 4)*q^18 - q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 3*x + 1 over the Rational Field
*]
[* 14, 133, 133, 266 *]
```

Not bielliptic $n(a_3 = -2; 9) = 27 - 24$.

$X_0(266)/ < w_7, w_{19} >$, **genus 8**

```
[*
    q + q^2 - q^3 + q^4 - 4*q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 - 4*q^10 + 2*q^11 -
        q^12 - q^13 + 3*q^14 + 4*q^15 + q^16 + 3*q^17 - 2*q^18 - q^19 + O(q^20),
    q + a*q^2 + a*q^3 + (-3*a - 3)*q^4 + (-2*a - 3)*q^5 + (-3*a - 1)*q^6 - q^7 +
        (4*a + 3)*q^8 + (-3*a - 4)*q^9 + (3*a + 2)*q^10 + (a - 3)*q^11 + (6*a +
        3)*q^12 + q^13 - a*q^14 + (3*a + 2)*q^15 + (-3*a + 2)*q^16 + (3*a +
        3)*q^17 + (5*a + 3)*q^18 - q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 + (-a^2 - 2*a + 6)*q^5 + a*q^6 - q^7 + q^8 + (a^2 -
        3)*q^9 + (-a^2 - 2*a + 6)*q^10 + (2*a^2 + 3*a - 8)*q^11 + a*q^12 +
        (2*a^2 + 4*a - 10)*q^13 - q^14 + (-a^2 - a + 4)*q^15 + q^16 + (-2*a^2 -
        6*a + 10)*q^17 + (a^2 - 3)*q^18 - q^19 + O(q^20),
    q + a*q^2 + a*q^3 + (-3*a - 3)*q^4 + (-2*a - 3)*q^5 + (-3*a - 1)*q^6 - q^7 +
        (4*a + 3)*q^8 + (-3*a - 4)*q^9 + (3*a + 2)*q^10 + (a - 3)*q^11 + (6*a +
        3)*q^12 + q^13 - a*q^14 + (3*a + 2)*q^15 + (-3*a + 2)*q^16 + (3*a +
        3)*q^17 + (5*a + 3)*q^18 - q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^3 + x^2 - 7*x + 4 over the Rational
    Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field
*]
[* 38, 133, 266, 266 *]
```

Not bielliptic $n(a_3 = -1; 3) = 12 - 10$.

$X_0(266)/ < W_{14}, W_{38}, W_{133} >$, **genus 7**

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
```

```
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q + a*q^2 + a*q^3 + (-3*a - 3)*q^4 + (-2*a - 3)*q^5 + (-3*a - 1)*q^6 - q^7 +
        (4*a + 3)*q^8 + (-3*a - 4)*q^9 + (3*a + 2)*q^10 + (a - 3)*q^11 + (6*a +
        3)*q^12 + q^13 - a*q^14 + (3*a + 2)*q^15 + (-3*a + 2)*q^16 + (3*a +
        3)*q^17 + (5*a + 3)*q^18 - q^19 + O(q^20),
    q + a*q^2 + (-a - 2)*q^3 + (-a + 1)*q^4 - 3*q^5 + (-a - 3)*q^6 + q^7 - 3*q^8
        + (3*a + 4)*q^9 - 3*a*q^10 + (-a - 3)*q^11 + q^12 + (2*a - 1)*q^13 +
        a*q^14 + (3*a + 6)*q^15 + (-a - 2)*q^16 + (a - 3)*q^17 + (a + 9)*q^18 +
        q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 + (-a + 1)*q^5 + a*q^6 + q^7 + q^8 + a*q^9 + (-a +
        1)*q^10 + (-a - 2)*q^11 + a*q^12 + (-2*a + 4)*q^13 + q^14 - 3*q^15 +
        q^16 + a*q^18 + q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 3 over the Rational Field,
    Number Field with defining polynomial x^2 - x - 3 over the Rational Field
*]
```

Not bielliptic, $n(a_5 = 3, 5) = 8 - 6$. Therefore $X_0(266)/W_{14}, X_0(266)/W_{38}, X_0(266)/W_{133}$ are not bielliptic.

$X_0(266)/ < W_2, W_{133}, W_{266} >$**, genus 7**

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 - q^7 - q^8 - 2*q^9 - 6*q^11 + q^12 + 5*q^13 +
        q^14 + q^16 + 3*q^17 + 2*q^18 + q^19 + O(q^20),
    q + a*q^2 + a*q^3 + (-3*a - 3)*q^4 + (-2*a - 3)*q^5 + (-3*a - 1)*q^6 - q^7 +
        (4*a + 3)*q^8 + (-3*a - 4)*q^9 + (3*a + 2)*q^10 + (a - 3)*q^11 + (6*a +
        3)*q^12 + q^13 - a*q^14 + (3*a + 2)*q^15 + (-3*a + 2)*q^16 + (3*a +
        3)*q^17 + (5*a + 3)*q^18 - q^19 + O(q^20),
    q + a*q^2 + (-a - 2)*q^3 + (-a + 1)*q^4 - 3*q^5 + (-a - 3)*q^6 + q^7 - 3*q^8
        + (3*a + 4)*q^9 - 3*a*q^10 + (-a - 3)*q^11 + q^12 + (2*a - 1)*q^13 +
        a*q^14 + (3*a + 6)*q^15 + (-a - 2)*q^16 + (a - 3)*q^17 + (a + 9)*q^18 +
        q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 3 over the Rational Field
*]
[* 14, 19, 38, 133, 133 *]
```

Not bielliptic, $n(a_3 = -2; 3) = 13 - 12$, $n(a_3 = 1; 3) = 13 - 6$. Therefore $X_0(266)/W_2, X_0(266)/W_{266}$ not bielliptic.

$X_0(266)/ < W_7, W_{38}, W_{266} >$**, genus 6**

```
[*
```

```
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q + a*q^2 + a*q^3 + (-3*a - 3)*q^4 + (-2*a - 3)*q^5 + (-3*a - 1)*q^6 - q^7 +
        (4*a + 3)*q^8 + (-3*a - 4)*q^9 + (3*a + 2)*q^10 + (a - 3)*q^11 + (6*a +
        3)*q^12 + q^13 - a*q^14 + (3*a + 2)*q^15 + (-3*a + 2)*q^16 + (3*a +
        3)*q^17 + (5*a + 3)*q^18 - q^19 + O(q^20),
    q + a*q^2 + (-a^2 + 5)*q^3 + (a^2 - 2)*q^4 + (a^2 - a - 4)*q^5 + (-2*a^2 + a
        + 7)*q^6 - q^7 + (2*a^2 - 7)*q^8 + (-2*a^2 + a + 8)*q^9 + (a^2 - 7)*q^10
        + (-a + 3)*q^11 + (-a^2 - a + 4)*q^12 + (a^2 - a - 4)*q^13 - a*q^14 +
        (3*a^2 - 2*a - 13)*q^15 + (2*a^2 + a - 10)*q^16 + (-2*a^2 - a + 11)*q^17
        + (-3*a^2 + 14)*q^18 + q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 2*x^2 - 4*x + 7 over the
    Rational Field
*]
[* 19, 133, 133 *]
```

????, 19a

$X_0(266)/ < W_{19}, W_{14}, W_{266} >$, **genus 5**

```
[*
    q + q^2 - q^3 + q^4 - 4*q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 - 4*q^10 + 2*q^11 -
        q^12 - q^13 + 3*q^14 + 4*q^15 + q^16 + 3*q^17 - 2*q^18 - q^19 + O(q^20),
    q + a*q^2 + a*q^3 + (-3*a - 3)*q^4 + (-2*a - 3)*q^5 + (-3*a - 1)*q^6 - q^7 +
        (4*a + 3)*q^8 + (-3*a - 4)*q^9 + (3*a + 2)*q^10 + (a - 3)*q^11 + (6*a +
        3)*q^12 + q^13 - a*q^14 + (3*a + 2)*q^15 + (-3*a + 2)*q^16 + (3*a +
        3)*q^17 + (5*a + 3)*q^18 - q^19 + O(q^20),
    q + a*q^2 + (-a + 2)*q^3 + (a - 1)*q^4 + q^5 + (a - 1)*q^6 + q^7 + (-2*a +
        1)*q^8 + (-3*a + 2)*q^9 + a*q^10 + (a - 1)*q^11 + (2*a - 3)*q^12 - q^13
        + a*q^14 + (-a + 2)*q^15 - 3*a*q^16 + (3*a - 1)*q^17 + (-a - 3)*q^18 -
        q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field
*]
[* 38, 133, 133 *]
```

Not bielliptic over $\mathbb{Q}$???????, $n(a_5 = -4; 25) = 48 - 40$.

13. Level $N = 255$

$X_0(255)/ < w_3, w_5 >$, **genus 8**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a + 2)*q^4 + (-a + 1)*q^5 - a*q^6 + (a - 4)*q^8 + q^9 +
        (2*a - 4)*q^10 + (-a - 1)*q^11 + (a - 2)*q^12 + (a + 3)*q^13 + (a -
        1)*q^15 - 3*a*q^16 + q^17 + a*q^18 + (3*a + 3)*q^19 + O(q^20),
```

```
    q + q^2 + 2*q^3 - q^4 - q^5 + 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^11 -
        2*q^12 + 2*q^13 - 2*q^14 - 2*q^15 - q^16 + q^17 + q^18 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a + 1)*q^4 - q^5 - a*q^6 + (2*a - 1)*q^7 + 3*q^8 + q^9 -
        a*q^10 + 5*q^11 + (-a - 1)*q^12 + (-2*a - 2)*q^13 + (a + 6)*q^14 + q^15
        + (a - 2)*q^16 + q^17 + a*q^18 + (-2*a - 1)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - x - 3 over the Rational Field
*]
[* 17, 51, 85, 85, 255 *]
```

Not bielliptic, $n(a_7 = 4; 7) = 10 - 8$, $n(a_2 = 1; 2) = 5 - 4$.

$X_0(255)/ < w_3, w_{17} >$, **genus 7**

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 + q^5 + (a - 3)*q^6 + (a - 1)*q^7 - a*q^8 +
        (-2*a + 1)*q^9 + a*q^10 + (-a + 3)*q^11 + (-a + 1)*q^12 - 4*q^13 + (-a +
        3)*q^14 + (-a + 1)*q^15 - 5*q^16 - q^17 + (a - 6)*q^18 + (2*a + 2)*q^19
        + O(q^20),
    q + a*q^2 - q^3 + (3*a - 3)*q^4 + q^5 - a*q^6 + (-2*a + 3)*q^7 + (4*a -
        3)*q^8 + q^9 + a*q^10 + (-4*a + 7)*q^11 + (-3*a + 3)*q^12 + (-2*a +
        6)*q^13 + (-3*a + 2)*q^14 - q^15 + (3*a + 2)*q^16 - q^17 + a*q^18 + (2*a
        - 9)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field,
    Number Field with defining polynomial x^2 - 3*x + 1 over the Rational Field
*]
[* 15, 85, 85, 255 *]
```

??

$X_0(255)/ < w_5, w_{17} >$, **genus 9**

```
[*
    q + q^3 - 2*q^4 + 3*q^5 - 4*q^7 + q^9 - 3*q^11 - 2*q^12 - q^13 + 3*q^15 +
        4*q^16 - q^17 - q^19 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
```

```
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 - q^5 + a*q^6 + (-a^3 - a^2 + 5*a + 5)*q^7 +
        (a^3 - 4*a)*q^8 + q^9 - a*q^10 + (a^3 + a^2 - 7*a - 3)*q^11 + (a^2 -
        2)*q^12 + (-2*a^2 + 8)*q^13 + (-2*a^3 - 3*a^2 + 12*a + 9)*q^14 - q^15 +
        (a^3 + 2*a^2 - 7*a - 5)*q^16 - q^17 + a*q^18 + (a^3 + a^2 - 5*a -
        1)*q^19 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^4 - x^3 - 8*x^2 + 7*x + 9 over the
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 51, 85, 255, 255 *]
```

???

$X_0(255)/ < W_{15}, W_{51}, W_{85} >$, **genus 6**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + q^5 + a*q^6 + (-a^2 - a + 4)*q^7 - q^8 +
        q^9 + a*q^10 + (-a^2 + a + 2)*q^11 + (a^2 - 2)*q^12 + (2*a^2 - 4)*q^13 +
        (-a^2 + 1)*q^14 + q^15 + (-2*a^2 - a + 4)*q^16 + q^17 + a*q^18 + (-3*a^2
        - 3*a + 8)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 4*x + 1 over the Rational Field
*]
[* 17, 85, 255 *]
```

Not bielliptic, $n(a_2 = -1; 8) = 9-8$. Therefore $X_0(255)/W_{15}, X_0(255)/W_{51}, X_0(255)/W_{85}$ are not bielliptic.

$X_0(255)/ < W_3, W_{85}, W_{255} >$, **genus 6**

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a + 2)*q^4 + (-a + 1)*q^5 - a*q^6 + (a - 4)*q^8 + q^9 +
```

```
        (2*a - 4)*q^10 + (-a - 1)*q^11 + (a - 2)*q^12 + (a + 3)*q^13 + (a -
        1)*q^15 - 3*a*q^16 + q^17 + a*q^18 + (3*a + 3)*q^19 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 15, 17, 51, 85 *]
```

????,15a

$n(17, a_7 = 4; 49) = 106 - 96$.

$X_0(255)/ < W_5, W_{51}, W_{255} >$, **genus 4**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + q^2 + 2*q^3 - q^4 - q^5 + 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^11 -
        2*q^12 + 2*q^13 - 2*q^14 - 2*q^15 - q^16 + q^17 + q^18 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 17, 85, 85 *]
```

Not over $\mathbb{Q}$????, $n(a_2 = 1; 2) = 5 - 4$, $n(a_2 = -1; 8) = 11 - 8$.

$X_0(255)/ < W_{17}, W_{15}, W_{255} >$, **genus 5**

```
[*
    q + q^3 - 2*q^4 + 3*q^5 - 4*q^7 + q^9 - 3*q^11 - 2*q^12 - q^13 + 3*q^15 +
        4*q^16 - q^17 - q^19 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 + q^5 + (a - 3)*q^6 + (a - 1)*q^7 - a*q^8 +
        (-2*a + 1)*q^9 + a*q^10 + (-a + 3)*q^11 + (-a + 1)*q^12 - 4*q^13 + (-a +
        3)*q^14 + (-a + 1)*q^15 - 5*q^16 - q^17 + (a - 6)*q^18 + (2*a + 2)*q^19
        + O(q^20)
*]
[*
    Rational Field,
```

```
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field
*]
[* 51, 85, 85 *]
```

Not bielliptic over $\mathbb{Q}$????, $n(a_2 = 0; 16) = 21 - 18$.

14. Level $N = 230$

$X_0(230)/ < w_2, w_5 >$, **genus 8**

```
[*
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q - q^2 + q^4 + 4*q^5 - 4*q^7 - q^8 - 3*q^9 - 4*q^10 + 2*q^11 - 2*q^13 +
        4*q^14 + q^16 - 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + 2*q^2 + 2*q^4 - q^5 + q^7 - 3*q^9 - 2*q^10 + 2*q^11 - 2*q^13 + 2*q^14 -
        4*q^16 + 3*q^17 - 6*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (4*a +
        3)*q^8 - 2*q^9 - a*q^10 + (2*a + 2)*q^11 + (3*a + 3)*q^12 + (2*a -
        1)*q^13 + (2*a + 2)*q^14 + q^15 + (-3*a + 2)*q^16 + (-4*a - 8)*q^17 -
        2*a*q^18 + (6*a + 10)*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 - q^5 - a*q^6 + (a + 1)*q^7 - q^8 + (-a + 2)*q^9 +
        q^10 + (a + 2)*q^11 + a*q^12 + (-a + 3)*q^13 + (-a - 1)*q^14 - a*q^15 +
        q^16 + (-a - 2)*q^17 + (a - 2)*q^18 + (-a + 3)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 5 over the Rational Field
*]
[* 23, 46, 115, 115, 230 *]
```

Not bielliptic, $n(a_3 = 0; 9) = 35 - 32$.

$X_0(230)/ < w_2, w_{23} >$, **genus 8**

```
[*
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (4*a +
        3)*q^8 - 2*q^9 - a*q^10 + (2*a + 2)*q^11 + (3*a + 3)*q^12 + (2*a -
        1)*q^13 + (2*a + 2)*q^14 + q^15 + (-3*a + 2)*q^16 + (-4*a - 8)*q^17 -
        2*a*q^18 + (6*a + 10)*q^19 + O(q^20),
    q + a*q^2 + (-a^2 + a + 2)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^3 + a^2 +
        2*a)*q^6 + (a^3 - 2*a^2 - 4*a + 3)*q^7 + (a^3 - 4*a)*q^8 + (a^2 - a -
        1)*q^9 + a*q^10 + (-2*a + 2)*q^11 + (-a^3 + 3*a - 2)*q^12 + (-2*a^3 +
        3*a^2 + 7*a - 4)*q^13 + (-2*a - 2)*q^14 + (-a^2 + a + 2)*q^15 + (2*a^3 -
        2*a^2 - 5*a + 2)*q^16 + (-a^3 + 2*a^2 + 2*a - 3)*q^17 + (a^3 - a^2 -
        a)*q^18 + (2*a - 2)*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + q^5 - a*q^6 + (-a + 3)*q^7 - q^8 + (3*a - 2)*q^9 -
        q^10 + (-a - 2)*q^11 + a*q^12 + (-a + 3)*q^13 + (a - 3)*q^14 + a*q^15 +
        q^16 + (-3*a + 6)*q^17 + (-3*a + 2)*q^18 + (-3*a + 5)*q^19 + O(q^20)
```

```
*]
[*
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^4 - 2*x^3 - 4*x^2 + 5*x + 2 over the
    Rational Field,
    Number Field with defining polynomial x^2 - 3*x - 1 over the Rational Field
*]
[* 115, 115, 230 *]
```

Not bielliptic,

$X_0(230)/ < w_5, w_{23} >$, **genus 7**

```
[*
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (4*a +
        3)*q^8 - 2*q^9 - a*q^10 + (2*a + 2)*q^11 + (3*a + 3)*q^12 + (2*a -
        1)*q^13 + (2*a + 2)*q^14 + q^15 + (-3*a + 2)*q^16 + (-4*a - 8)*q^17 -
        2*a*q^18 + (6*a + 10)*q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 - q^5 + a*q^6 + (-a^2 - 2*a + 8)*q^7 + q^8 + (a^2 -
        3)*q^9 - q^10 + (2*a^2 + a - 12)*q^11 + a*q^12 + (-a^2 + 6)*q^13 + (-a^2
        - 2*a + 8)*q^14 - a*q^15 + q^16 + (-a - 2)*q^17 + (a^2 - 3)*q^18 + (-a^2
        - 2*a + 8)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (4*a +
        3)*q^8 - 2*q^9 - a*q^10 + (2*a + 2)*q^11 + (3*a + 3)*q^12 + (2*a -
        1)*q^13 + (2*a + 2)*q^14 + q^15 + (-3*a + 2)*q^16 + (-4*a - 8)*q^17 -
        2*a*q^18 + (6*a + 10)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 9*x + 12 over the Rational
    Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field
*]
[* 115, 230, 230 *]
```

Not bielliptic.

$X_0(230)/ < W_{10}, W_{46}, W_{115} >$, **genus 6**

```
[*
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (4*a +
        3)*q^8 - 2*q^9 - a*q^10 + (2*a + 2)*q^11 + (3*a + 3)*q^12 + (2*a -
        1)*q^13 + (2*a + 2)*q^14 + q^15 + (-3*a + 2)*q^16 + (-4*a - 8)*q^17 -
        2*a*q^18 + (6*a + 10)*q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 + q^5 + a*q^6 + (-a + 1)*q^7 + q^8 + (a - 2)*q^9 +
        q^10 + (-3*a + 2)*q^11 + a*q^12 + (-5*a + 1)*q^13 + (-a + 1)*q^14 +
        a*q^15 + q^16 + (5*a - 2)*q^17 + (a - 2)*q^18 + (3*a - 3)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
```

```
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field
*]
[* 23, 115, 230 *]
```

Not bielliptic, therefore $X_0(230)/W_{10}, X_0(230)/W_{46}, X_0(246)/W_{115}$.

$X_0(230)/ < W_2, W_{115}, W_{230} >$, **genus 5**

```
[*
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q - q^2 + q^4 + 4*q^5 - 4*q^7 - q^8 - 3*q^9 - 4*q^10 + 2*q^11 - 2*q^13 +
        4*q^14 + q^16 - 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (4*a +
        3)*q^8 - 2*q^9 - a*q^10 + (2*a + 2)*q^11 + (3*a + 3)*q^12 + (2*a -
        1)*q^13 + (2*a + 2)*q^14 + q^15 + (-3*a + 2)*q^16 + (-4*a - 8)*q^17 -
        2*a*q^18 + (6*a + 10)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field
*]
[* 23, 46, 115 *]
```

bielliptic over $\mathbb{Q}$?, 46a.

$X_0(230)/ < W_5, W_{46}, W_{230} >$, **genus 5**

```
[*
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q + 2*q^2 + 2*q^4 - q^5 + q^7 - 3*q^9 - 2*q^10 + 2*q^11 - 2*q^13 + 2*q^14 -
        4*q^16 + 3*q^17 - 6*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (4*a +
        3)*q^8 - 2*q^9 - a*q^10 + (2*a + 2)*q^11 + (3*a + 3)*q^12 + (2*a -
        1)*q^13 + (2*a + 2)*q^14 + q^15 + (-3*a + 2)*q^16 + (-4*a - 8)*q^17 -
        2*a*q^18 + (6*a + 10)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field
*]
[* 23, 115, 115 *]
```

???115a

$X_0(230)/ < W_{23}, W_{10}, W_{230} >$, **genus 6**

```
[*
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (4*a +
        3)*q^8 - 2*q^9 - a*q^10 + (2*a + 2)*q^11 + (3*a + 3)*q^12 + (2*a -
```

```
        1)*q^13 + (2*a + 2)*q^14 + q^15 + (-3*a + 2)*q^16 + (-4*a - 8)*q^17 -
        2*a*q^18 + (6*a + 10)*q^19 + O(q^20),
    q + a*q^2 + (-a^2 + a + 2)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^3 + a^2 +
        2*a)*q^6 + (a^3 - 2*a^2 - 4*a + 3)*q^7 + (a^3 - 4*a)*q^8 + (a^2 - a -
        1)*q^9 + a*q^10 + (-2*a + 2)*q^11 + (-a^3 + 3*a - 2)*q^12 + (-2*a^3 +
        3*a^2 + 7*a - 4)*q^13 + (-2*a - 2)*q^14 + (-a^2 + a + 2)*q^15 + (2*a^3 -
        2*a^2 - 5*a + 2)*q^16 + (-a^3 + 2*a^2 + 2*a - 3)*q^17 + (a^3 - a^2 -
        a)*q^18 + (2*a - 2)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^4 - 2*x^3 - 4*x^2 + 5*x + 2 over the
    Rational Field
*]
[* 115, 115 *]
```

Not bielliptic, therefore $X_0(230)/W_{23}, X_0(230)/W_{10}$ and $X_0(230)/W_{230}$ are not bielliptic.

15. Level $N = 186$

$X_0(186)/ < w_2, w_3 >$, **genus 7**

```
[*
    q + a*q^2 - 2*a*q^3 + (a - 1)*q^4 + q^5 + (-2*a - 2)*q^6 + (2*a - 3)*q^7 +
        (-2*a + 1)*q^8 + (4*a + 1)*q^9 + a*q^10 + 2*q^11 - 2*q^12 - 2*a*q^13 +
        (-a + 2)*q^14 - 2*a*q^15 - 3*a*q^16 + (-2*a + 4)*q^17 + (5*a + 4)*q^18 +
        (-2*a + 1)*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + (-2*a + 2)*q^5 - a*q^6 + 2*q^7 - q^8 + (2*a - 1)*q^9
        + (2*a - 2)*q^10 + (a - 4)*q^11 + a*q^12 + (-3*a + 2)*q^13 - 2*q^14 +
        (-2*a - 4)*q^15 + q^16 + (2*a - 2)*q^17 + (-2*a + 1)*q^18 - 4*q^19 +
        O(q^20),
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 + (-2*a - 5)*q^5 - a*q^6 + (2*a + 1)*q^7 +
        (4*a + 3)*q^8 + q^9 + (a + 2)*q^10 + 2*a*q^11 + (3*a + 3)*q^12 + (2*a +
        2)*q^13 + (-5*a - 2)*q^14 + (2*a + 5)*q^15 + (-3*a + 2)*q^16 + (-4*a -
        8)*q^17 + a*q^18 + (-2*a - 7)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 + 2*q^7 - q^8 + q^9 + q^10 + 3*q^11 - q^12 +
        3*q^13 - 2*q^14 + q^15 + q^16 + q^17 - q^18 + 7*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 2 over the Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Rational Field
*]
[* 31, 62, 93, 186 *]
```

???

$X_0(186)/ < w_2, w_{31} >$, **genus 6**

```
[*
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 + (-2*a - 5)*q^5 - a*q^6 + (2*a + 1)*q^7 +
        (4*a + 3)*q^8 + q^9 + (a + 2)*q^10 + 2*a*q^11 + (3*a + 3)*q^12 + (2*a +
        2)*q^13 + (-5*a - 2)*q^14 + (2*a + 5)*q^15 + (-3*a + 2)*q^16 + (-4*a -
        8)*q^17 + a*q^18 + (-2*a - 7)*q^19 + O(q^20),
```

```
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a^2 - a + 2)*q^5 + a*q^6 + (-a^2 - a +
        4)*q^7 - q^8 + q^9 + (-a^2 - 2*a + 1)*q^10 + (2*a^2 - 6)*q^11 + (a^2 -
        2)*q^12 + (2*a^2 - 4)*q^13 + (-a^2 + 1)*q^14 + (-a^2 - a + 2)*q^15 +
        (-2*a^2 - a + 4)*q^16 + (2*a^2 + 2*a - 6)*q^17 + a*q^18 + (-a^2 + 3*a +
        4)*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 + 3*q^5 - q^6 - 2*q^7 - q^8 + q^9 - 3*q^10 + 5*q^11 +
        q^12 - 7*q^13 + 2*q^14 + 3*q^15 + q^16 - q^17 - q^18 + 7*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 4*x + 1 over the Rational Field,
    Rational Field
*]
[* 93, 93, 186 *]
```

Not bielliptic, $n(a_5 = 3; 5) = 9 - 6$

$X_0(186)/ < w_3, w_{31} >$, **genus 7**

```
[*
    q + q^2 + q^4 - 2*q^5 + q^8 - 3*q^9 - 2*q^10 + 2*q^13 + q^16 - 6*q^17 -
        3*q^18 + 4*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 + (-2*a - 5)*q^5 - a*q^6 + (2*a + 1)*q^7 +
        (4*a + 3)*q^8 + q^9 + (a + 2)*q^10 + 2*a*q^11 + (3*a + 3)*q^12 + (2*a +
        2)*q^13 + (-5*a - 2)*q^14 + (2*a + 5)*q^15 + (-3*a + 2)*q^16 + (-4*a -
        8)*q^17 + a*q^18 + (-2*a - 7)*q^19 + O(q^20),
    q + q^2 - q^3 + q^4 + a*q^5 - q^6 + (-2*a + 4)*q^7 + q^8 + q^9 + a*q^10 + (a
        - 2)*q^11 - q^12 + a*q^13 + (-2*a + 4)*q^14 - a*q^15 + q^16 + (-3*a +
        4)*q^17 + q^18 + (a - 2)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 + (-2*a - 5)*q^5 - a*q^6 + (2*a + 1)*q^7 +
        (4*a + 3)*q^8 + q^9 + (a + 2)*q^10 + 2*a*q^11 + (3*a + 3)*q^12 + (2*a +
        2)*q^13 + (-5*a - 2)*q^14 + (2*a + 5)*q^15 + (-3*a + 2)*q^16 + (-4*a -
        8)*q^17 + a*q^18 + (-2*a - 7)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 3*x - 2 over the Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field
*]
[* 62, 93, 186, 186 *]
```

Not bielliptic $n(a_{11} = 0; 11) = 25 - 24$.

$X_0(186)/ < w_6 >$, **genus 14**

```
[*
    q + a*q^2 - 2*a*q^3 + (a - 1)*q^4 + q^5 + (-2*a - 2)*q^6 + (2*a - 3)*q^7 +
        (-2*a + 1)*q^8 + (4*a + 1)*q^9 + a*q^10 + 2*q^11 - 2*q^12 - 2*a*q^13 +
        (-a + 2)*q^14 - 2*a*q^15 - 3*a*q^16 + (-2*a + 4)*q^17 + (5*a + 4)*q^18 +
        (-2*a + 1)*q^19 + O(q^20),
    q + q^2 + q^4 - 2*q^5 + q^8 - 3*q^9 - 2*q^10 + 2*q^13 + q^16 - 6*q^17 -
        3*q^18 + 4*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + (-2*a + 2)*q^5 - a*q^6 + 2*q^7 - q^8 + (2*a - 1)*q^9
        + (2*a - 2)*q^10 + (a - 4)*q^11 + a*q^12 + (-3*a + 2)*q^13 - 2*q^14 +
```

```
        (-2*a - 4)*q^15 + q^16 + (2*a - 2)*q^17 + (-2*a + 1)*q^18 - 4*q^19 +
        O(q^20),
    q + a*q^2 - 2*a*q^3 + (a - 1)*q^4 + q^5 + (-2*a - 2)*q^6 + (2*a - 3)*q^7 +
        (-2*a + 1)*q^8 + (4*a + 1)*q^9 + a*q^10 + 2*q^11 - 2*q^12 - 2*a*q^13 +
        (-a + 2)*q^14 - 2*a*q^15 - 3*a*q^16 + (-2*a + 4)*q^17 + (5*a + 4)*q^18 +
        (-2*a + 1)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 + (-2*a - 5)*q^5 - a*q^6 + (2*a + 1)*q^7 +
        (4*a + 3)*q^8 + q^9 + (a + 2)*q^10 + 2*a*q^11 + (3*a + 3)*q^12 + (2*a +
        2)*q^13 + (-5*a - 2)*q^14 + (2*a + 5)*q^15 + (-3*a + 2)*q^16 + (-4*a -
        8)*q^17 + a*q^18 + (-2*a - 7)*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a^2 - a + 2)*q^5 + a*q^6 + (-a^2 - a +
        4)*q^7 - q^8 + q^9 + (-a^2 - 2*a + 1)*q^10 + (2*a^2 - 6)*q^11 + (a^2 -
        2)*q^12 + (2*a^2 - 4)*q^13 + (-a^2 + 1)*q^14 + (-a^2 - a + 2)*q^15 +
        (-2*a^2 - a + 4)*q^16 + (2*a^2 + 2*a - 6)*q^17 + a*q^18 + (-a^2 + 3*a +
        4)*q^19 + O(q^20),
    q + a*q^2 - 2*a*q^3 + (a - 1)*q^4 + q^5 + (-2*a - 2)*q^6 + (2*a - 3)*q^7 +
        (-2*a + 1)*q^8 + (4*a + 1)*q^9 + a*q^10 + 2*q^11 - 2*q^12 - 2*a*q^13 +
        (-a + 2)*q^14 - 2*a*q^15 - 3*a*q^16 + (-2*a + 4)*q^17 + (5*a + 4)*q^18 +
        (-2*a + 1)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 + 2*q^7 - q^8 + q^9 + q^10 + 3*q^11 - q^12 +
        3*q^13 - 2*q^14 + q^15 + q^16 + q^17 - q^18 + 7*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + q^5 + q^6 - 2*q^7 + q^8 + q^9 + q^10 - 3*q^11 + q^12 -
        q^13 - 2*q^14 + q^15 + q^16 + 3*q^17 + q^18 - 5*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 2 over the Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 4*x + 1 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 31, 62, 62, 62, 93, 93, 93, 186, 186 *]
```

Not bielliptic, $n(|a_5| \geq 0; 25) \geq 100 - 72$.

$X_0(186)/ < W_6, W_{62}, W_{93} >$, **genus 5**

```
[*
    q + a*q^2 - 2*a*q^3 + (a - 1)*q^4 + q^5 + (-2*a - 2)*q^6 + (2*a - 3)*q^7 +
        (-2*a + 1)*q^8 + (4*a + 1)*q^9 + a*q^10 + 2*q^11 - 2*q^12 - 2*a*q^13 +
        (-a + 2)*q^14 - 2*a*q^15 - 3*a*q^16 + (-2*a + 4)*q^17 + (5*a + 4)*q^18 +
        (-2*a + 1)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 + (-2*a - 5)*q^5 - a*q^6 + (2*a + 1)*q^7 +
        (4*a + 3)*q^8 + q^9 + (a + 2)*q^10 + 2*a*q^11 + (3*a + 3)*q^12 + (2*a +
        2)*q^13 + (-5*a - 2)*q^14 + (2*a + 5)*q^15 + (-3*a + 2)*q^16 + (-4*a -
        8)*q^17 + a*q^18 + (-2*a - 7)*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + q^5 + q^6 - 2*q^7 + q^8 + q^9 + q^10 - 3*q^11 + q^12 -
        q^13 - 2*q^14 + q^15 + q^16 + 3*q^17 + q^18 - 5*q^19 + O(q^20)
*]
```

```
[*
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Rational Field
*]
[* 31, 93, 186 *]
```

??? also bielliptic over $\mathbb{Q}$:186b

$X_0(186)/ < W_2, W_{93}, W_{186} >$, **genus 6**

```
[*
    q + a*q^2 - 2*a*q^3 + (a - 1)*q^4 + q^5 + (-2*a - 2)*q^6 + (2*a - 3)*q^7 +
        (-2*a + 1)*q^8 + (4*a + 1)*q^9 + a*q^10 + 2*q^11 - 2*q^12 - 2*a*q^13 +
        (-a + 2)*q^14 - 2*a*q^15 - 3*a*q^16 + (-2*a + 4)*q^17 + (5*a + 4)*q^18 +
        (-2*a + 1)*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + (-2*a + 2)*q^5 - a*q^6 + 2*q^7 - q^8 + (2*a - 1)*q^9
        + (2*a - 2)*q^10 + (a - 4)*q^11 + a*q^12 + (-3*a + 2)*q^13 - 2*q^14 +
        (-2*a - 4)*q^15 + q^16 + (2*a - 2)*q^17 + (-2*a + 1)*q^18 - 4*q^19 +
        O(q^20),
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 + (-2*a - 5)*q^5 - a*q^6 + (2*a + 1)*q^7 +
        (4*a + 3)*q^8 + q^9 + (a + 2)*q^10 + 2*a*q^11 + (3*a + 3)*q^12 + (2*a +
        2)*q^13 + (-5*a - 2)*q^14 + (2*a + 5)*q^15 + (-3*a + 2)*q^16 + (-4*a -
        8)*q^17 + a*q^18 + (-2*a - 7)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 2 over the Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field
*]
[* 31, 62, 93 *]
```

Not bielliptic. Therefore $X_0(186)/W_2, X_0(186)/W_{93}, X_0(186)/W_{186}$ are not bielliptic.

$X_0(186)/ < W_3, W_{62}, W_{186} >$, **genus 4**

```
[*
    q + a*q^2 - 2*a*q^3 + (a - 1)*q^4 + q^5 + (-2*a - 2)*q^6 + (2*a - 3)*q^7 +
        (-2*a + 1)*q^8 + (4*a + 1)*q^9 + a*q^10 + 2*q^11 - 2*q^12 - 2*a*q^13 +
        (-a + 2)*q^14 - 2*a*q^15 - 3*a*q^16 + (-2*a + 4)*q^17 + (5*a + 4)*q^18 +
        (-2*a + 1)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 + (-2*a - 5)*q^5 - a*q^6 + (2*a + 1)*q^7 +
        (4*a + 3)*q^8 + q^9 + (a + 2)*q^10 + 2*a*q^11 + (3*a + 3)*q^12 + (2*a +
        2)*q^13 + (-5*a - 2)*q^14 + (2*a + 5)*q^15 + (-3*a + 2)*q^16 + (-4*a -
        8)*q^17 + a*q^18 + (-2*a - 7)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field
*]
[* 31, 93 *]
```

Not bielliptic over $\mathbb{Q}$

$X_0(186)/ < W_6, W_{31}, W_{186} >$, **genus 6**

```
[*
```

```
    q + q^2 + q^4 - 2*q^5 + q^8 - 3*q^9 - 2*q^10 + 2*q^13 + q^16 - 6*q^17 -
        3*q^18 + 4*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-3*a - 3)*q^4 + (-2*a - 5)*q^5 - a*q^6 + (2*a + 1)*q^7 +
        (4*a + 3)*q^8 + q^9 + (a + 2)*q^10 + 2*a*q^11 + (3*a + 3)*q^12 + (2*a +
        2)*q^13 + (-5*a - 2)*q^14 + (2*a + 5)*q^15 + (-3*a + 2)*q^16 + (-4*a -
        8)*q^17 + a*q^18 + (-2*a - 7)*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a^2 - a + 2)*q^5 + a*q^6 + (-a^2 - a +
        4)*q^7 - q^8 + q^9 + (-a^2 - 2*a + 1)*q^10 + (2*a^2 - 6)*q^11 + (a^2 -
        2)*q^12 + (2*a^2 - 4)*q^13 + (-a^2 + 1)*q^14 + (-a^2 - a + 2)*q^15 +
        (-2*a^2 - a + 4)*q^16 + (2*a^2 + 2*a - 6)*q^17 + a*q^18 + (-a^2 + 3*a +
        4)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 4*x + 1 over the Rational Field
*]
[* 62, 93, 93 *]
```

??? over $\mathbb{Q}$:62a

16. Level $N = 170$

$X_0(170)/ < w_2, w_5 >$, **genus 6**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + q^2 + 2*q^3 - q^4 - q^5 + 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^11 -
        2*q^12 + 2*q^13 - 2*q^14 - 2*q^15 - q^16 + q^17 + q^18 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q - q^2 - 2*q^3 + q^4 - q^5 + 2*q^6 + 2*q^7 - q^8 + q^9 + q^10 + 6*q^11 -
        2*q^12 + 2*q^13 - 2*q^14 + 2*q^15 + q^16 + q^17 - q^18 + 8*q^19 +
        O(q^20),
    q - q^2 + 3*q^3 + q^4 - q^5 - 3*q^6 + 2*q^7 - q^8 + 6*q^9 + q^10 - 4*q^11 +
        3*q^12 - 3*q^13 - 2*q^14 - 3*q^15 + q^16 + q^17 - 6*q^18 + 3*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 17, 85, 85, 170, 170 *]
```

?? $n(a_3 = 2; 2) = 5 - 4$,$n(a_3 = 3; 2) = 5 - 2$.

$X_0(170)/ < w_2, w_{17} >$, **genus 5**

```
[*
```

```
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 + q^5 + (a - 3)*q^6 + (a - 1)*q^7 - a*q^8 +
        (-2*a + 1)*q^9 + a*q^10 + (-a + 3)*q^11 + (-a + 1)*q^12 - 4*q^13 + (-a +
        3)*q^14 + (-a + 1)*q^15 - 5*q^16 - q^17 + (a - 6)*q^18 + (2*a + 2)*q^19
        + O(q^20),
    q - q^2 + q^3 + q^4 + q^5 - q^6 + 2*q^7 - q^8 - 2*q^9 - q^10 + q^12 + 5*q^13
        - 2*q^14 + q^15 + q^16 - q^17 + 2*q^18 - q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field,
    Rational Field
*]
[* 85, 85, 170 *]
```

???

$X_0(170)/ < w_5, w_{19} >$, **genus 6**

```
[*
    q + q^2 - 2*q^3 + q^4 - 2*q^6 - 4*q^7 + q^8 + q^9 + 6*q^11 - 2*q^12 + 2*q^13
        - 4*q^14 + q^16 - q^17 + q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 - q^5 + q^6 + 2*q^7 + q^8 - 2*q^9 - q^10 + q^12 - q^13 +
        2*q^14 - q^15 + q^16 - q^17 - 2*q^18 - q^19 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 34, 85, 170, 170 *]
```

Not bielliptic, $n(a_3 = 1; 3) = 13 - 6$, $n(a_3 = -2; 3) = 13 - 12$.

$X_0(170)/ < w_2 >$, **genus 11**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + q^2 + 2*q^3 - q^4 - q^5 + 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^11 -
        2*q^12 + 2*q^13 - 2*q^14 - 2*q^15 - q^16 + q^17 + q^18 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
```

```
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 + q^5 + (a - 3)*q^6 + (a - 1)*q^7 - a*q^8 +
        (-2*a + 1)*q^9 + a*q^10 + (-a + 3)*q^11 + (-a + 1)*q^12 - 4*q^13 + (-a +
        3)*q^14 + (-a + 1)*q^15 - 5*q^16 - q^17 + (a - 6)*q^18 + (2*a + 2)*q^19
        + O(q^20),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q - q^2 - 2*q^3 + q^4 - q^5 + 2*q^6 + 2*q^7 - q^8 + q^9 + q^10 + 6*q^11 -
        2*q^12 + 2*q^13 - 2*q^14 + 2*q^15 + q^16 + q^17 - q^18 + 8*q^19 +
        O(q^20),
    q - q^2 + 3*q^3 + q^4 - q^5 - 3*q^6 + 2*q^7 - q^8 + 6*q^9 + q^10 - 4*q^11 +
        3*q^12 - 3*q^13 - 2*q^14 - 3*q^15 + q^16 + q^17 - 6*q^18 + 3*q^19 +
        O(q^20),
    q - q^2 + q^3 + q^4 + q^5 - q^6 + 2*q^7 - q^8 - 2*q^9 - q^10 + q^12 + 5*q^13
        - 2*q^14 + q^15 + q^16 - q^17 + 2*q^18 - q^19 + O(q^20),
    q - q^2 - 2*q^3 + q^4 + q^5 + 2*q^6 - 2*q^7 - q^8 + q^9 - q^10 - 2*q^11 -
        2*q^12 - 6*q^13 + 2*q^14 - 2*q^15 + q^16 + q^17 - q^18 - 8*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 17, 85, 85, 85, 85, 170, 170, 170, 170 *]
```

Not bielliptic, $n(|a_3| \geq 0; 9) \geq 34 - 32$.

$X_0(170)/ < W_{10}, W_{34}, W_{85} >$**, genus 5**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 + q^5 + a*q^6 - 2*a*q^7 + q^8 + (-a + 1)*q^9 + q^10 -
        4*q^11 + a*q^12 + (-a + 2)*q^13 - 2*a*q^14 + a*q^15 + q^16 + q^17 + (-a
        + 1)*q^18 + a*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field
```

```
*]
[* 17, 85, 170 *]
```

Not bielliptic over $\mathbb{Q}$, $n(17, a_3 = 0, 3) = 9 - 8$

$X_0(170)/ < W_2, W_{85}.W_{170} >$, **genus 4**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q - q^2 - 2*q^3 + q^4 + q^5 + 2*q^6 - 2*q^7 - q^8 + q^9 - q^10 - 2*q^11 -
        2*q^12 - 6*q^13 + 2*q^14 - 2*q^15 + q^16 + q^17 - q^18 - 8*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field
*]
[* 17, 85, 170 *]
```

over $\mathbb{Q}$: 170a $n(17, a_3 = 0; 3) = 10 - 8$

$X_0(170)/ < W_5, W_{34}, W_{170} >$, **genus 4**

```
170 [*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + q^2 + 2*q^3 - q^4 - q^5 + 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^11 -
        2*q^12 + 2*q^13 - 2*q^14 - 2*q^15 - q^16 + q^17 + q^18 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 17, 85, 85 *]
```

Over $\overline{Q}$??? $n(85, a_3 = 3; 3) = 6 - 4$, $n(17, a_7 = 4; 7) = 10 - 8$.

$X_0(170)/ < W_{10}, W_{17}, W_{170}$, **genus 5**

```
[*
    q + q^2 - 2*q^3 + q^4 - 2*q^6 - 4*q^7 + q^8 + q^9 + 6*q^11 - 2*q^12 + 2*q^13
        - 4*q^14 + q^16 - q^17 + q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
```

```
    q + a*q^2 + (-a + 1)*q^3 + q^4 + q^5 + (a - 3)*q^6 + (a - 1)*q^7 - a*q^8 +
        (-2*a + 1)*q^9 + a*q^10 + (-a + 3)*q^11 + (-a + 1)*q^12 - 4*q^13 + (-a +
        3)*q^14 + (-a + 1)*q^15 - 5*q^16 - q^17 + (a - 6)*q^18 + (2*a + 2)*q^19
        + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field
*]
[* 34, 85, 85 *]
```

over $\mathbb{Q}$:34a

17. Level $N = 165$

$X_0(165)/ < w_3, w_5 >$,**genus 6**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q + q^2 - q^3 - q^4 - 2*q^5 - q^6 + 4*q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 +
        q^12 - 2*q^13 + 4*q^14 + 2*q^15 - q^16 - 2*q^17 + q^18 + O(q^20),
    q + a*q^2 + (-2*a + 2)*q^3 + (2*a - 1)*q^4 - q^5 + (-2*a - 2)*q^6 - 2*q^7 +
        (a + 2)*q^8 + 5*q^9 - a*q^10 + q^11 + (-2*a - 6)*q^12 + (2*a - 6)*q^13 -
        2*a*q^14 + (2*a - 2)*q^15 + 3*q^16 + (2*a + 2)*q^17 + 5*a*q^18 +
        O(q^20),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 11, 33, 55, 165 *]
```

Not bielliptic, $n(a_2 = -2; 4) = 12 - 10$,$n(a_7 = 4; 7) = 14 - 8$

$X_0(165)/ < w_3, w_{11} >$, **genus 4**

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q + q^2 - q^4 + q^5 - 3*q^8 - 3*q^9 + q^10 - q^11 + 2*q^13 - q^16 + 6*q^17 -
        3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
```

```
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 15, 55, 165 *]
```

Not bielliptic, $n(a_2 = -1; 8) = 11 - 8$, $n(a_2 = 1; 2) = 5 - 4$.

$X_0(165)/ < w_5, w_{11} >$, **genus 4**

```
[*
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + O(q^20),
    q + a*q^2 + q^3 + q^4 - q^5 + a*q^6 + 2*q^7 - a*q^8 + q^9 - a*q^10 - q^11 +
        q^12 + (-2*a + 2)*q^13 + 2*a*q^14 - q^15 - 5*q^16 + a*q^18 + (-2*a +
        2)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field
*]
[* 165, 165 *]
```

Not bielliptic.

$X_0(165)/ < W_{15}, W_{33}, W_{55} >$, **genus 6**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + q^5 + a*q^6 + (-a^2 - 2*a + 3)*q^7 + (-a^2
        + a + 1)*q^8 + q^9 + a*q^10 + q^11 + (a^2 - 2)*q^12 + (-a^2 + 3)*q^13 +
        (-a^2 - 2*a - 1)*q^14 + q^15 + (-4*a + 3)*q^16 + (a^2 - 2*a - 5)*q^17 +
        a*q^18 + (2*a^2 + 2*a - 4)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 + x^2 - 5*x - 1 over the Rational
    Field
*]
[* 11, 165, 165 *]
```

Not bielliptic, $n(11, a_2 = -2; 8) = 14 - 10$. Therefore $X_0(165)/W_{15}, X_0(165)/W_{33}, X_0(165)/W_{55}$ are not bielliptic.

$X_0(165)/ < W_3, W_{55}, W_{165} >$, **genus 5**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q + q^2 - q^3 - q^4 - 2*q^5 - q^6 + 4*q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 +
        q^12 - 2*q^13 + 4*q^14 + 2*q^15 - q^16 - 2*q^17 + q^18 + O(q^20),
```

```
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 11, 15, 33, 165 *]
```

????:15a $n(11, a_2 = -2; 4) = 13 - 10$, $n(33, a_2 = 1; 2) = 7 - 4$,
$X_0(165)/ < W_5, W_{33}, W_{165}$, **genus 5**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q + a*q^2 + (-2*a + 2)*q^3 + (2*a - 1)*q^4 - q^5 + (-2*a - 2)*q^6 - 2*q^7 +
        (a + 2)*q^8 + 5*q^9 - a*q^10 + q^11 + (-2*a - 6)*q^12 + (2*a - 6)*q^13 -
        2*a*q^14 + (2*a - 2)*q^15 + 3*q^16 + (2*a + 2)*q^17 + 5*a*q^18 +
        O(q^20),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 11, 55, 165 *]
```

???:11a
$X_0(165)/ < W_{15}, W_{11}, W_{165}$, **genus 3**

```
[*
    q + q^2 - q^4 + q^5 - 3*q^8 - 3*q^9 + q^10 - q^11 + 2*q^13 - q^16 + 6*q^17 -
        3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 55, 165 *]
```

???:55a
$X_0(165)/ < w_{165} >$, **genus 9**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
```

```
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q + q^2 - q^3 - q^4 - 2*q^5 - q^6 + 4*q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 +
        q^12 - 2*q^13 + 4*q^14 + 2*q^15 - q^16 - 2*q^17 + q^18 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q + q^2 - q^4 + q^5 - 3*q^8 - 3*q^9 + q^10 - q^11 + 2*q^13 - q^16 + 6*q^17 -
        3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + (-2*a + 2)*q^3 + (2*a - 1)*q^4 - q^5 + (-2*a - 2)*q^6 - 2*q^7 +
        (a + 2)*q^8 + 5*q^9 - a*q^10 + q^11 + (-2*a - 6)*q^12 + (2*a - 6)*q^13 -
        2*a*q^14 + (2*a - 2)*q^15 + 3*q^16 + (2*a + 2)*q^17 + 5*a*q^18 +
        O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 11, 15, 33, 33, 55, 55, 55, 165 *]
```

Not bielliptic, $n(|a_2| \geq 1; 4) \geq 18 - 16$,

18. Level $N = 154$

$X_0(154)/ < w_2, w_7 >$, **genus 5**

```
 [*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
    q + q^2 + 2*q^3 - q^4 - 2*q^5 + 2*q^6 - q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 - q^14 - 4*q^15 - q^16 + 4*q^17 + q^18 + O(q^20),
    q - q^2 + q^4 - 4*q^5 - q^7 - q^8 - 3*q^9 + 4*q^10 - q^11 + 2*q^13 + q^14 +
        q^16 - 4*q^17 + 3*q^18 - 6*q^19 + O(q^20),
    q - q^2 + 2*q^3 + q^4 + 2*q^5 - 2*q^6 - q^7 - q^8 + q^9 - 2*q^10 + q^11 +
        2*q^12 - 4*q^13 + q^14 + 4*q^15 + q^16 - q^18 + 4*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
```

```
    Rational Field,
    Rational Field,
    Rational Field
*]
```

???$n(a_3 = -3; 9) = 22 - 14$,$n(a_5 = -4; 25) = 50 - 40$,$n(a_5 = 2; 5) = 10 - 8$.
$X_0(154)/ < w_2, w_{11} >$**, genus 6**

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
    q + q^3 - 2*q^4 + 3*q^5 + q^7 - 2*q^9 - q^11 - 2*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 - 6*q^17 + 2*q^19 + O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + 3*q^4 - 2*q^5 + (a - 5)*q^6 + q^7 + a*q^8 + (-2*a
        + 3)*q^9 - 2*a*q^10 - q^11 + (-3*a + 3)*q^12 + (a + 1)*q^13 + a*q^14 +
        (2*a - 2)*q^15 - q^16 + (-a - 1)*q^17 + (3*a - 10)*q^18 + (2*a + 2)*q^19
        + O(q^20),
    q - q^2 + q^4 - 4*q^5 - q^7 - q^8 - 3*q^9 + 4*q^10 - q^11 + 2*q^13 + q^14 +
        q^16 - 4*q^17 + 3*q^18 - 6*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field
*]
[* 14, 77, 77, 77, 154 *]
```

?? $n(a_3 = -3; 9) = 20 - 14$,$n(a_5 = 3; 5) = 12 - 6$,$n(a_5 = -4; 25) = 52 - 40$.
$X_0(154)/ < w_7, w_{11} >$**, genus 4**

```
[*
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
    q - q^2 + q^4 - 4*q^5 - q^7 - q^8 - 3*q^9 + 4*q^10 - q^11 + 2*q^13 + q^14 +
        q^16 - 4*q^17 + 3*q^18 - 6*q^19 + O(q^20),
    q + q^2 + q^4 + 2*q^5 - q^7 + q^8 - 3*q^9 + 2*q^10 - q^11 + 2*q^13 - q^14 +
        q^16 + 2*q^17 - 3*q^18 + O(q^20),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 77, 154, 154, 154 *]
```

Not bielliptic, $n(a_3 = 0; 3) = 10 - 8$,$n(a_3 = -3; 9) = 16 - 14$.

$X_0(154)/ < W_{14}, W_{22}, W_{77} >$, **genus 5**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
    q - q^2 + q^4 - 4*q^5 - q^7 - q^8 - 3*q^9 + 4*q^10 - q^11 + 2*q^13 + q^14 +
        q^16 - 4*q^17 + 3*q^18 - 6*q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 - a*q^5 + a*q^6 + q^7 + q^8 + (-2*a + 1)*q^9 - a*q^10
        + q^11 + a*q^12 + (-a - 2)*q^13 + q^14 + (2*a - 4)*q^15 + q^16 +
        2*a*q^17 + (-2*a + 1)*q^18 + (-a - 6)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 4 over the Rational Field
*]
[* 11, 77, 154, 154 *]
```

???? 11a $n(154, a_3 = 0; 3) = 10 - 8$, $n(77, a_3 = -3; 9) = 18 - 14$.

$X_0(154)/ < W_2, W_{77}, W_{154} >$, **genus 4**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
    q - q^2 + q^4 - 4*q^5 - q^7 - q^8 - 3*q^9 + 4*q^10 - q^11 + 2*q^13 + q^14 +
        q^16 - 4*q^17 + 3*q^18 - 6*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 11, 14, 77, 154 *]
```

???11a,14a $n(77 : a_3 = -3; 9) = 20 - 14$, $n(154, a_3 = 0; 3) = 10 - 8$.

$X_0(154)/ < W_7, W_{22}, W_{154} >$, **genus 4**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
    q + q^2 + 2*q^3 - q^4 - 2*q^5 + 2*q^6 - q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 - q^14 - 4*q^15 - q^16 + 4*q^17 + q^18 + O(q^20),
    q - q^2 + q^4 - 4*q^5 - q^7 - q^8 - 3*q^9 + 4*q^10 - q^11 + 2*q^13 + q^14 +
```

```
        q^16 - 4*q^17 + 3*q^18 - 6*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 11, 77, 77, 154 *]
```

Bieliptic over $\overline{Q}$??? $n(77, a_3 = -3; 9) = 20 - 14$, $n(77, a_3 = 2; 3) = 6 - 4$, $n(11, a_5 = 1; 5) = 12 - 10$, $n(154, a_5 = -4; 25) = 44 - 40$.

$X_0(154)/ < W_{11}, W_{14}, W_{154} >$**, genus 5**

```
[*
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
    q + q^3 - 2*q^4 + 3*q^5 + q^7 - 2*q^9 - q^11 - 2*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 - 6*q^17 + 2*q^19 + O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + 3*q^4 - 2*q^5 + (a - 5)*q^6 + q^7 + a*q^8 + (-2*a
        + 3)*q^9 - 2*a*q^10 - q^11 + (-3*a + 3)*q^12 + (a + 1)*q^13 + a*q^14 +
        (2*a - 2)*q^15 - q^16 + (-a - 1)*q^17 + (3*a - 10)*q^18 + (2*a + 2)*q^19
        + O(q^20),
    q - q^2 + q^4 - 4*q^5 - q^7 - q^8 - 3*q^9 + 4*q^10 - q^11 + 2*q^13 + q^14 +
        q^16 - 4*q^17 + 3*q^18 - 6*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field
*]
[* 77, 77, 77, 154 *]
```

Bielliptic over $\overline{Q}$? $n(77, a_3 = -3; 9) = 18 - 14$, $n(77, a_5 = 3; 5) = 12 - 6$, $n(154, a_5 = -4; 25) = 42 - 40$.

$X_0(154)/ < w_2 >$**, genus 11**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
    q + q^2 + 2*q^3 - q^4 - 2*q^5 + 2*q^6 - q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 - q^14 - 4*q^15 - q^16 + 4*q^17 + q^18 + O(q^20),
    q + q^3 - 2*q^4 + 3*q^5 + q^7 - 2*q^9 - q^11 - 2*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 - 6*q^17 + 2*q^19 + O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + 3*q^4 - 2*q^5 + (a - 5)*q^6 + q^7 + a*q^8 + (-2*a
        + 3)*q^9 - 2*a*q^10 - q^11 + (-3*a + 3)*q^12 + (a + 1)*q^13 + a*q^14 +
        (2*a - 2)*q^15 - q^16 + (-a - 1)*q^17 + (3*a - 10)*q^18 + (2*a + 2)*q^19
        + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
```

```
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - q^2 + q^4 - 4*q^5 - q^7 - q^8 - 3*q^9 + 4*q^10 - q^11 + 2*q^13 + q^14 +
        q^16 - 4*q^17 + 3*q^18 - 6*q^19 + O(q^20),
    q - q^2 + 2*q^3 + q^4 + 2*q^5 - 2*q^6 - q^7 - q^8 + q^9 - 2*q^10 + q^11 +
        2*q^12 - 4*q^13 + q^14 + 4*q^15 + q^16 - q^18 + 4*q^19 + O(q^20),
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 11, 14, 77, 77, 77, 77, 77, 154, 154, 154 *]
```

Not bielliptic $n(|a_3| \geq 0; 9) \geq 36 - 32$.

**********++++

NEXT WITH $g_N^* = 1$ always BIELLITtic Cases with $|W| = 4$.

19. Level $N = 238$

$X_0(238)/ < w_2 >$, **genus 17**

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + (-a^3 - a^2 + 4*a + 1)*q^3 + (a^2 - 2)*q^4 + (a^3 + a^2 -
        4*a)*q^5 + (-a^2 + 3)*q^6 + q^7 + (a^3 - 4*a)*q^8 + (-a^3 - 3*a^2 + 2*a
        + 7)*q^9 + (a^2 + a - 3)*q^10 - 2*a*q^11 + (a^3 + 2*a^2 - 5*a - 2)*q^12
        + (2*a^3 + 4*a^2 - 6*a - 4)*q^13 + a*q^14 + (2*a^2 + 2*a - 9)*q^15 +
        (-a^3 - a^2 + a + 1)*q^16 - q^17 + (-2*a^3 - 3*a^2 + 6*a + 3)*q^18 +
        (-2*a^3 - 4*a^2 + 4*a + 8)*q^19 + O(q^20),
    q + a*q^2 + (-a^4 + 6*a^2 + a - 4)*q^3 + (a^2 - 2)*q^4 + (2*a^4 + a^3 -
        15*a^2 - 6*a + 18)*q^5 + (-2*a^4 - 2*a^3 + 15*a^2 + 10*a - 17)*q^6 - q^7
        + (a^3 - 4*a)*q^8 + (2*a^4 + a^3 - 13*a^2 - 8*a + 13)*q^9 + (5*a^4 + a^3
        - 34*a^2 - 10*a + 34)*q^10 + (-2*a^4 - 2*a^3 + 14*a^2 + 12*a - 14)*q^11
        + (-4*a^4 - a^3 + 26*a^2 + 9*a - 26)*q^12 + (-2*a^4 + 14*a^2 - 14)*q^13
        - a*q^14 + (-a^4 - a^3 + 7*a^2 + 3*a - 4)*q^15 + (a^4 - 6*a^2 + 4)*q^16
        + q^17 + (5*a^4 + 3*a^3 - 36*a^2 - 15*a + 34)*q^18 + (-2*a^4 + 14*a^2 +
        2*a - 14)*q^19 + O(q^20),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q - q^2 + q^4 - 2*q^5 - q^7 - q^8 - 3*q^9 + 2*q^10 - 2*q^11 + q^14 + q^16 -
```

```
        q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q - q^2 + 2*q^3 + q^4 + 4*q^5 - 2*q^6 + q^7 - q^8 + q^9 - 4*q^10 - 4*q^11 +
        2*q^12 - 4*q^13 - q^14 + 8*q^15 + q^16 - q^17 - q^18 - 6*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + (-a + 2)*q^5 - a*q^6 - q^7 - q^8 + (2*a + 1)*q^9 +
        (a - 2)*q^10 + (a + 2)*q^11 + a*q^12 + (-2*a + 4)*q^13 + q^14 - 4*q^15 +
        q^16 + q^17 + (-2*a - 1)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 + x^3 - 5*x^2 - x + 3 over the
    Rational Field,
    Number Field with defining polynomial x^5 - 2*x^4 - 8*x^3 + 14*x^2 + 14*x -
        17 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 4 over the Rational Field,
    Rational Field
*]
[* 14, 17, 119, 119, 119, 238, 238, 238, 238 *]
```

Not bielliptic, $n(|a_3| \geq 0; 9) \geq 44 - 32$.

$X_0(238)/ < w_7 >$,**genus 17**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + q^2 - 2*q^3 + q^4 - 2*q^6 - 4*q^7 + q^8 + q^9 + 6*q^11 - 2*q^12 + 2*q^13
        - 4*q^14 + q^16 - q^17 + q^18 - 4*q^19 + O(q^20),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + (-a^4 + 6*a^2 + a - 4)*q^3 + (a^2 - 2)*q^4 + (2*a^4 + a^3 -
        15*a^2 - 6*a + 18)*q^5 + (-2*a^4 - 2*a^3 + 15*a^2 + 10*a - 17)*q^6 - q^7
        + (a^3 - 4*a)*q^8 + (2*a^4 + a^3 - 13*a^2 - 8*a + 13)*q^9 + (5*a^4 + a^3
        - 34*a^2 - 10*a + 34)*q^10 + (-2*a^4 - 2*a^3 + 14*a^2 + 12*a - 14)*q^11
        + (-4*a^4 - a^3 + 26*a^2 + 9*a - 26)*q^12 + (-2*a^4 + 14*a^2 - 14)*q^13
        - a*q^14 + (-a^4 - a^3 + 7*a^2 + 3*a - 4)*q^15 + (a^4 - 6*a^2 + 4)*q^16
        + q^17 + (5*a^4 + 3*a^3 - 36*a^2 - 15*a + 34)*q^18 + (-2*a^4 + 14*a^2 +
        2*a - 14)*q^19 + O(q^20),
    q - q^2 + q^4 - 2*q^5 - q^7 - q^8 - 3*q^9 + 2*q^10 - 2*q^11 + q^14 + q^16 -
        q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + q^2 + 2*q^3 + q^4 + 2*q^6 - q^7 + q^8 + q^9 - 2*q^11 + 2*q^12 - 2*q^13 -
        q^14 + q^16 - q^17 + q^18 + O(q^20),
    q - q^2 + a*q^3 + q^4 + (-a + 2)*q^5 - a*q^6 - q^7 - q^8 + (2*a + 1)*q^9 +
        (a - 2)*q^10 + (a + 2)*q^11 + a*q^12 + (-2*a + 4)*q^13 + q^14 - 4*q^15 +
        q^16 + q^17 + (-2*a - 1)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q + a*q^2 + (-a^4 + 6*a^2 + a - 4)*q^3 + (a^2 - 2)*q^4 + (2*a^4 + a^3 -
        15*a^2 - 6*a + 18)*q^5 + (-2*a^4 - 2*a^3 + 15*a^2 + 10*a - 17)*q^6 - q^7
```

```
        + (a^3 - 4*a)*q^8 + (2*a^4 + a^3 - 13*a^2 - 8*a + 13)*q^9 + (5*a^4 + a^3
        - 34*a^2 - 10*a + 34)*q^10 + (-2*a^4 - 2*a^3 + 14*a^2 + 12*a - 14)*q^11
        + (-4*a^4 - a^3 + 26*a^2 + 9*a - 26)*q^12 + (-2*a^4 + 14*a^2 - 14)*q^13
        - a*q^14 + (-a^4 - a^3 + 7*a^2 + 3*a - 4)*q^15 + (a^4 - 6*a^2 + 4)*q^16
        + q^17 + (5*a^4 + 3*a^3 - 36*a^2 - 15*a + 34)*q^18 + (-2*a^4 + 14*a^2 +
        2*a - 14)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^5 - 2*x^4 - 8*x^3 + 14*x^2 + 14*x -
        17 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 4 over the Rational Field,
    Number Field with defining polynomial x^5 - 2*x^4 - 8*x^3 + 14*x^2 + 14*x -
        17 over the Rational Field
*]
[* 17, 34, 34, 119, 238, 238, 238, 238 *]
```

Not bielliptic $n(|a_3| \geq 0; 9) \geq 40 - 32$.

$X_0(238)/ < w_{17} >$, **genus 15**

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20),
    q + q^2 - 2*q^3 + q^4 - 2*q^6 - 4*q^7 + q^8 + q^9 + 6*q^11 - 2*q^12 + 2*q^13
        - 4*q^14 + q^16 - q^17 + q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + (-a^3 - a^2 + 4*a + 1)*q^3 + (a^2 - 2)*q^4 + (a^3 + a^2 -
        4*a)*q^5 + (-a^2 + 3)*q^6 + q^7 + (a^3 - 4*a)*q^8 + (-a^3 - 3*a^2 + 2*a
        + 7)*q^9 + (a^2 + a - 3)*q^10 - 2*a*q^11 + (a^3 + 2*a^2 - 5*a - 2)*q^12
        + (2*a^3 + 4*a^2 - 6*a - 4)*q^13 + a*q^14 + (2*a^2 + 2*a - 9)*q^15 +
        (-a^3 - a^2 + a + 1)*q^16 - q^17 + (-2*a^3 - 3*a^2 + 6*a + 3)*q^18 +
        (-2*a^3 - 4*a^2 + 4*a + 8)*q^19 + O(q^20),
    q - q^2 + q^4 - 2*q^5 - q^7 - q^8 - 3*q^9 + 2*q^10 - 2*q^11 + q^14 + q^16 -
        q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q - q^2 + 2*q^3 + q^4 + 4*q^5 - 2*q^6 + q^7 - q^8 + q^9 - 4*q^10 - 4*q^11 +
        2*q^12 - 4*q^13 - q^14 + 8*q^15 + q^16 - q^17 - q^18 - 6*q^19 + O(q^20),
    q + q^2 + 2*q^3 + q^4 + 2*q^6 - q^7 + q^8 + q^9 - 2*q^11 + 2*q^12 - 2*q^13 -
        q^14 + q^16 - q^17 + q^18 + O(q^20),
    q + q^2 - 2*q^3 + q^4 - 4*q^5 - 2*q^6 + q^7 + q^8 + q^9 - 4*q^10 - 6*q^11 -
        2*q^12 - 2*q^13 + q^14 + 8*q^15 + q^16 - q^17 + q^18 + O(q^20),
    q + a*q^2 + (-a^3 - a^2 + 4*a + 1)*q^3 + (a^2 - 2)*q^4 + (a^3 + a^2 -
        4*a)*q^5 + (-a^2 + 3)*q^6 + q^7 + (a^3 - 4*a)*q^8 + (-a^3 - 3*a^2 + 2*a
        + 7)*q^9 + (a^2 + a - 3)*q^10 - 2*a*q^11 + (a^3 + 2*a^2 - 5*a - 2)*q^12
        + (2*a^3 + 4*a^2 - 6*a - 4)*q^13 + a*q^14 + (2*a^2 + 2*a - 9)*q^15 +
        (-a^3 - a^2 + a + 1)*q^16 - q^17 + (-2*a^3 - 3*a^2 + 6*a + 3)*q^18 +
        (-2*a^3 - 4*a^2 + 4*a + 8)*q^19 + O(q^20),
    q + q^2 - 2*q^3 + q^4 - 2*q^6 - 4*q^7 + q^8 + q^9 + 6*q^11 - 2*q^12 + 2*q^13
        - 4*q^14 + q^16 - q^17 + q^18 - 4*q^19 + O(q^20)
```

```
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 + x^3 - 5*x^2 - x + 3 over the
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 + x^3 - 5*x^2 - x + 3 over the
    Rational Field,
    Rational Field
*]
[* 14, 34, 119, 238, 238, 238, 238, 238, 238 *]
```

Not bielliptic $n(|a_3| \geq 0; 9) \geq 40 - 32$.

$X_0(238)/ < w_{14} >$, **genus 17**

```
 (Need to small ad-hoc modification programme)
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + O(q^10),
    (REPEATED 3 TIMES INSTEAD OF 2, need to erase one in Jacobian
    decomposition and computation of points)
    q + q^2 - 2*q^3 + q^4 - 2*q^6 - 4*q^7 + q^8 + q^9 + O(q^10),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + O(q^10),
    q + a*q^2 + (-a^3 - a^2 + 4*a + 1)*q^3 + (a^2 - 2)*q^4 + (a^3 + a^2 -
        4*a)*q^5 + (-a^2 + 3)*q^6 + q^7 + (a^3 - 4*a)*q^8 + (-a^3 - 3*a^2 + 2*a
        + 7)*q^9 + O(q^10),
    q + a*q^2 + (-a^4 + 6*a^2 + a - 4)*q^3 + (a^2 - 2)*q^4 + (2*a^4 + a^3 -
        15*a^2 - 6*a + 18)*q^5 + (-2*a^4 - 2*a^3 + 15*a^2 + 10*a - 17)*q^6 - q^7
        + (a^3 - 4*a)*q^8 + (2*a^4 + a^3 - 13*a^2 - 8*a + 13)*q^9 + O(q^10),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + O(q^10),
    q - q^2 + q^4 - 2*q^5 - q^7 - q^8 - 3*q^9 + O(q^10),
    q + q^2 - 2*q^3 + q^4 - 4*q^5 - 2*q^6 + q^7 + q^8 + q^9 + O(q^10),
    q + q^2 + q^4 + 2*q^5 + q^7 + q^8 - 3*q^9 + O(q^10),
    q - q^2 + a*q^3 + q^4 + (-a + 2)*q^5 - a*q^6 - q^7 - q^8 + (2*a + 1)*q^9 +
        O(q^10)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 + x^3 - 5*x^2 - x + 3 over the
    Rational Field,
    Number Field with defining polynomial x^5 - 2*x^4 - 8*x^3 + 14*x^2 + 14*x -
        17 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
```

```
    Number Field with defining polynomial x^2 - 2*x - 4 over the Rational Field
*]
[* 17, 34, 34, 119, 119, 119, 238, 238, 238, 238 *]
```

Not bielliptic $n(|a_3| \geq 0; 9) \geq 48 - 32$

$X_0(238)/ < w_{34} >$, **genus 15**

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + (-a^3 - a^2 + 4*a + 1)*q^3 + (a^2 - 2)*q^4 + (a^3 + a^2 -
        4*a)*q^5 + (-a^2 + 3)*q^6 + q^7 + (a^3 - 4*a)*q^8 + (-a^3 - 3*a^2 + 2*a
        + 7)*q^9 + (a^2 + a - 3)*q^10 - 2*a*q^11 + (a^3 + 2*a^2 - 5*a - 2)*q^12
        + (2*a^3 + 4*a^2 - 6*a - 4)*q^13 + a*q^14 + (2*a^2 + 2*a - 9)*q^15 +
        (-a^3 - a^2 + a + 1)*q^16 - q^17 + (-2*a^3 - 3*a^2 + 6*a + 3)*q^18 +
        (-2*a^3 - 4*a^2 + 4*a + 8)*q^19 + O(q^20),
    q + a*q^2 + (-a^4 + 6*a^2 + a - 4)*q^3 + (a^2 - 2)*q^4 + (2*a^4 + a^3 -
        15*a^2 - 6*a + 18)*q^5 + (-2*a^4 - 2*a^3 + 15*a^2 + 10*a - 17)*q^6 - q^7
        + (a^3 - 4*a)*q^8 + (2*a^4 + a^3 - 13*a^2 - 8*a + 13)*q^9 + (5*a^4 + a^3
        - 34*a^2 - 10*a + 34)*q^10 + (-2*a^4 - 2*a^3 + 14*a^2 + 12*a - 14)*q^11
        + (-4*a^4 - a^3 + 26*a^2 + 9*a - 26)*q^12 + (-2*a^4 + 14*a^2 - 14)*q^13
        - a*q^14 + (-a^4 - a^3 + 7*a^2 + 3*a - 4)*q^15 + (a^4 - 6*a^2 + 4)*q^16
        + q^17 + (5*a^4 + 3*a^3 - 36*a^2 - 15*a + 34)*q^18 + (-2*a^4 + 14*a^2 +
        2*a - 14)*q^19 + O(q^20),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q - q^2 + q^4 - 2*q^5 - q^7 - q^8 - 3*q^9 + 2*q^10 - 2*q^11 + q^14 + q^16 -
        q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q - q^2 + 2*q^3 + q^4 + 4*q^5 - 2*q^6 + q^7 - q^8 + q^9 - 4*q^10 - 4*q^11 +
        2*q^12 - 4*q^13 - q^14 + 8*q^15 + q^16 - q^17 - q^18 - 6*q^19 + O(q^20),
    q + q^2 + q^4 + 2*q^5 + q^7 + q^8 - 3*q^9 + 2*q^10 - 2*q^13 + q^14 + q^16 +
        q^17 - 3*q^18 + 4*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 + x^3 - 5*x^2 - x + 3 over the
    Rational Field,
    Number Field with defining polynomial x^5 - 2*x^4 - 8*x^3 + 14*x^2 + 14*x -
        17 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 17, 119, 119, 119, 238, 238, 238 *]
```

Not bielliptic $n(|a_3| \geq 0; 9) \geq 48 - 32$.

$X_0(238)/ < w_{119} >$, **genus 7**

```
[*
```

```
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + q^2 - 2*q^3 + q^4 - 2*q^6 - 4*q^7 + q^8 + q^9 + 6*q^11 - 2*q^12 + 2*q^13
        - 4*q^14 + q^16 - q^17 + q^18 - 4*q^19 + O(q^20),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q - q^2 + q^4 - 2*q^5 - q^7 - q^8 - 3*q^9 + 2*q^10 - 2*q^11 + q^14 + q^16 -
        q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + q^2 + 2*q^3 + q^4 + 2*q^6 - q^7 + q^8 + q^9 - 2*q^11 + 2*q^12 - 2*q^13 -
        q^14 + q^16 - q^17 + q^18 + O(q^20),
    q + q^2 + q^4 + 2*q^5 + q^7 + q^8 - 3*q^9 + 2*q^10 - 2*q^13 + q^14 + q^16 +
        q^17 - 3*q^18 + 4*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 17, 34, 34, 238, 238, 238 *]
```

Not bielliptic $n(|a_3| \geq 0; 9) \geq 40 - 32$.

$X_0(238)/ < W_{14}, W_{34}, W_{119} >$, **genus 3**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q - q^2 + q^4 - 2*q^5 - q^7 - q^8 - 3*q^9 + 2*q^10 - 2*q^11 + q^14 + q^16 -
        q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + q^2 + q^4 + 2*q^5 + q^7 + q^8 - 3*q^9 + 2*q^10 - 2*q^13 + q^14 + q^16 +
        q^17 - 3*q^18 + 4*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 17, 238, 238 *]
```

??? 17a, 238b,238c

$X_0(238)/ < W_2, W_{119}, W_{238} >$, **genus 3**

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q - q^2 + q^4 - 2*q^5 - q^7 - q^8 - 3*q^9 + 2*q^10 - 2*q^11 + q^14 + q^16 -
```

```
        q^17 + 3*q^18 - 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 17, 238 *]
```

??? 14a, 17a, 238b

$X_0(238)/ < W_7, W_{34}, W_{238} >$, **genus 7**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + (-a^4 + 6*a^2 + a - 4)*q^3 + (a^2 - 2)*q^4 + (2*a^4 + a^3 -
        15*a^2 - 6*a + 18)*q^5 + (-2*a^4 - 2*a^3 + 15*a^2 + 10*a - 17)*q^6 - q^7
        + (a^3 - 4*a)*q^8 + (2*a^4 + a^3 - 13*a^2 - 8*a + 13)*q^9 + (5*a^4 + a^3
        - 34*a^2 - 10*a + 34)*q^10 + (-2*a^4 - 2*a^3 + 14*a^2 + 12*a - 14)*q^11
        + (-4*a^4 - a^3 + 26*a^2 + 9*a - 26)*q^12 + (-2*a^4 + 14*a^2 - 14)*q^13
        - a*q^14 + (-a^4 - a^3 + 7*a^2 + 3*a - 4)*q^15 + (a^4 - 6*a^2 + 4)*q^16
        + q^17 + (5*a^4 + 3*a^3 - 36*a^2 - 15*a + 34)*q^18 + (-2*a^4 + 14*a^2 +
        2*a - 14)*q^19 + O(q^20),
    q - q^2 + q^4 - 2*q^5 - q^7 - q^8 - 3*q^9 + 2*q^10 - 2*q^11 + q^14 + q^16 -
        q^17 + 3*q^18 - 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^5 - 2*x^4 - 8*x^3 + 14*x^2 + 14*x -
        17 over the Rational Field,
    Rational Field
*]
[* 17, 119, 238 *]
```

???17a, 238b

$X_0(238)/ < W_{17}, W_{14}, W_{238} >$, **genus 7**

```
[*
    q + q^2 - 2*q^3 + q^4 - 2*q^6 - 4*q^7 + q^8 + q^9 + 6*q^11 - 2*q^12 + 2*q^13
        - 4*q^14 + q^16 - q^17 + q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + (-a^3 - a^2 + 4*a + 1)*q^3 + (a^2 - 2)*q^4 + (a^3 + a^2 -
        4*a)*q^5 + (-a^2 + 3)*q^6 + q^7 + (a^3 - 4*a)*q^8 + (-a^3 - 3*a^2 + 2*a
        + 7)*q^9 + (a^2 + a - 3)*q^10 - 2*a*q^11 + (a^3 + 2*a^2 - 5*a - 2)*q^12
        + (2*a^3 + 4*a^2 - 6*a - 4)*q^13 + a*q^14 + (2*a^2 + 2*a - 9)*q^15 +
        (-a^3 - a^2 + a + 1)*q^16 - q^17 + (-2*a^3 - 3*a^2 + 6*a + 3)*q^18 +
        (-2*a^3 - 4*a^2 + 4*a + 8)*q^19 + O(q^20),
    q - q^2 + q^4 - 2*q^5 - q^7 - q^8 - 3*q^9 + 2*q^10 - 2*q^11 + q^14 + q^16 -
        q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + q^2 - 2*q^3 + q^4 - 4*q^5 - 2*q^6 + q^7 + q^8 + q^9 - 4*q^10 - 6*q^11 -
        2*q^12 - 2*q^13 + q^14 + 8*q^15 + q^16 - q^17 + q^18 + O(q^20)
*]
[*
    Rational Field,
```

```
    Number Field with defining polynomial x^4 + x^3 - 5*x^2 - x + 3 over the
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 34, 119, 238, 238 *]
```

??? 238b $n(a_3 = -2; 9) = 26 - 24$.

20. level $N = 231$

$X_0(231)/ < w_3 >$,**genus 15**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q + q^2 - q^3 - q^4 - 2*q^5 - q^6 + 4*q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 +
        q^12 - 2*q^13 + 4*q^14 + 2*q^15 - q^16 - 2*q^17 + q^18 + O(q^20),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
    q + q^2 + 2*q^3 - q^4 - 2*q^5 + 2*q^6 - q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 - q^14 - 4*q^15 - q^16 + 4*q^17 + q^18 + O(q^20),
    q + q^3 - 2*q^4 + 3*q^5 + q^7 - 2*q^9 - q^11 - 2*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 - 6*q^17 + 2*q^19 + O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + 3*q^4 - 2*q^5 + (a - 5)*q^6 + q^7 + a*q^8 + (-2*a
        + 3)*q^9 - 2*a*q^10 - q^11 + (-3*a + 3)*q^12 + (a + 1)*q^13 + a*q^14 +
        (2*a - 2)*q^15 - q^16 + (-a - 1)*q^17 + (3*a - 10)*q^18 + (2*a + 2)*q^19
        + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - q^2 - q^3 - q^4 - 2*q^5 + q^6 + q^7 + 3*q^8 + q^9 + 2*q^10 - q^11 + q^12
        + 6*q^13 - q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a + 3)*q^4 + 3*q^5 - a*q^6 + q^7 + (2*a - 5)*q^8 + q^9 +
        3*a*q^10 - q^11 + (a - 3)*q^12 + q^13 + a*q^14 - 3*q^15 + (-5*a +
        4)*q^16 + (2*a + 4)*q^17 + a*q^18 + (-2*a - 3)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-a^2 + a + 4)*q^5 - a*q^6 - q^7 + (2*a +
        1)*q^8 + q^9 + (a^2 - 2*a - 1)*q^10 + q^11 + (-a^2 + 2)*q^12 + (-a^2 + a
        + 4)*q^13 - a*q^14 + (a^2 - a - 4)*q^15 + (a + 4)*q^16 - 2*a*q^17 +
        a*q^18 + (-a^2 - a + 8)*q^19 + O(q^20),
    q + q^2 - q^3 - q^4 - 2*q^5 - q^6 + 4*q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 +
        q^12 - 2*q^13 + 4*q^14 + 2*q^15 - q^16 - 2*q^17 + q^18 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 5 over the Rational Field,
```

```
    Number Field with defining polynomial x^3 - 6*x - 1 over the Rational Field,
    Rational Field
*]
[* 11, 33, 77, 77, 77, 77, 77, 231, 231, 231, 231 *]
```

Not bielliptic $n(|a_2| \geq 0; 4) \geq 20 - 18$.

$X_0(231)/ < w_7 >$,**genus 15**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + O(q^20),
    q + q^2 - q^3 - q^4 - 2*q^5 - q^6 + 4*q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 +
        q^12 - 2*q^13 + 4*q^14 + 2*q^15 - q^16 - 2*q^17 + q^18 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
    q + q^2 + 2*q^3 - q^4 - 2*q^5 + 2*q^6 - q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 - q^14 - 4*q^15 - q^16 + 4*q^17 + q^18 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-a^2 + a + 4)*q^5 - a*q^6 - q^7 + (2*a +
        1)*q^8 + q^9 + (a^2 - 2*a - 1)*q^10 + q^11 + (-a^2 + 2)*q^12 + (-a^2 + a
        + 4)*q^13 - a*q^14 + (a^2 - a - 4)*q^15 + (a + 4)*q^16 - 2*a*q^17 +
        a*q^18 + (-a^2 - a + 8)*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a^2 - a + 6)*q^5 + a*q^6 - q^7 + (2*a^2
        - 7)*q^8 + q^9 + (-3*a^2 + 2*a + 7)*q^10 - q^11 + (a^2 - 2)*q^12 +
        (-3*a^2 + a + 10)*q^13 - a*q^14 + (-a^2 - a + 6)*q^15 + (2*a^2 + a -
        10)*q^16 + (4*a^2 - 2*a - 12)*q^17 + a*q^18 + (a^2 - a - 6)*q^19 +
        O(q^20),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
    q + q^2 + 2*q^3 - q^4 - 2*q^5 + 2*q^6 - q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 - q^14 - 4*q^15 - q^16 + 4*q^17 + q^18 + O(q^20),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - 6*x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 2*x^2 - 4*x + 7 over the
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
```

```
[* 11, 21, 33, 33, 77, 77, 231, 231, 231, 231, 231 *]
```

Not bielliptic $n(|a_2| \geq 0; 4) \geq 28 - 18$.

$X_0(231)/ < w_{11} >$**, genus 15**

```
[*
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + O(q^20),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
    q + q^3 - 2*q^4 + 3*q^5 + q^7 - 2*q^9 - q^11 - 2*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 - 6*q^17 + 2*q^19 + O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + 3*q^4 - 2*q^5 + (a - 5)*q^6 + q^7 + a*q^8 + (-2*a
        + 3)*q^9 - 2*a*q^10 - q^11 + (-3*a + 3)*q^12 + (a + 1)*q^13 + a*q^14 +
        (2*a - 2)*q^15 - q^16 + (-a - 1)*q^17 + (3*a - 10)*q^18 + (2*a + 2)*q^19
        + O(q^20),
    q - q^2 - q^3 - q^4 - 2*q^5 + q^6 + q^7 + 3*q^8 + q^9 + 2*q^10 - q^11 + q^12
        + 6*q^13 - q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a + 3)*q^4 + 3*q^5 - a*q^6 + q^7 + (2*a - 5)*q^8 + q^9 +
        3*a*q^10 - q^11 + (a - 3)*q^12 + q^13 + a*q^14 - 3*q^15 + (-5*a +
        4)*q^16 + (2*a + 4)*q^17 + a*q^18 + (-2*a - 3)*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a^2 - a + 6)*q^5 + a*q^6 - q^7 + (2*a^2
        - 7)*q^8 + q^9 + (-3*a^2 + 2*a + 7)*q^10 - q^11 + (a^2 - 2)*q^12 +
        (-3*a^2 + a + 10)*q^13 - a*q^14 + (-a^2 - a + 6)*q^15 + (2*a^2 + a -
        10)*q^16 + (4*a^2 - 2*a - 12)*q^17 + a*q^18 + (a^2 - a - 6)*q^19 +
        O(q^20),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
    q + q^3 - 2*q^4 + 3*q^5 + q^7 - 2*q^9 - q^11 - 2*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 - 6*q^17 + 2*q^19 + O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + 3*q^4 - 2*q^5 + (a - 5)*q^6 + q^7 + a*q^8 + (-2*a
        + 3)*q^9 - 2*a*q^10 - q^11 + (-3*a + 3)*q^12 + (a + 1)*q^13 + a*q^14 +
        (2*a - 2)*q^15 - q^16 + (-a - 1)*q^17 + (3*a - 10)*q^18 + (2*a + 2)*q^19
        + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 5 over the Rational Field,
    Number Field with defining polynomial x^3 - 2*x^2 - 4*x + 7 over the
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field
*]
[* 21, 77, 77, 77, 231, 231, 231, 231, 231, 231 *]
```

Not bielliptic $n(|a_2| \geq 0; 4) \geq 20 - 18$.

$X_0(231)/ < w_{21} >$**,genus 13**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 +
    O(q^10),(REPEATED 3 TIMES INSTEAD OF 2, NEED ERASE ONE FOR
    COMPUTAIONS OF POINTS AND JACOBIAN DECOMPOSITION)
    q + q^2 - q^3 - q^4 - 2*q^5 - q^6 + 4*q^7 - 3*q^8 + q^9 + O(q^10),
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 + O(q^10),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 + O(q^10),
    q + q^2 + 2*q^3 - q^4 - 2*q^5 + 2*q^6 - q^7 - 3*q^8 + q^9 + O(q^10),
    q + q^3 - 2*q^4 + 3*q^5 + q^7 - 2*q^9 + O(q^10),
    q + a*q^2 + (-a + 1)*q^3 + 3*q^4 - 2*q^5 + (a - 5)*q^6 + q^7 + a*q^8 + (-2*a
        + 3)*q^9 + O(q^10),
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 + O(q^10),
    q + a*q^2 + q^3 + (a - 1)*q^4 + q^5 + a*q^6 + q^7 + (-2*a + 1)*q^8 + q^9 +
        O(q^10),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-a^2 + a + 4)*q^5 - a*q^6 - q^7 + (2*a +
        1)*q^8 + q^9 + O(q^10)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 6*x - 1 over the Rational Field
*]
[* 11, 33, 33, 77, 77, 77, 77, 77, 231, 231 *]
```

Not bielliptic, $n(|a_2| \geq 0; 4) \geq 22 - 18$.

$X_0(231)/ < w_{33} >$, **genus 13**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + O(q^20),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
    q + q^2 + 2*q^3 - q^4 - 2*q^5 + 2*q^6 - q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 - q^14 - 4*q^15 - q^16 + 4*q^17 + q^18 + O(q^20),
    q + q^3 - 2*q^4 + 3*q^5 + q^7 - 2*q^9 - q^11 - 2*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 - 6*q^17 + 2*q^19 + O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + 3*q^4 - 2*q^5 + (a - 5)*q^6 + q^7 + a*q^8 + (-2*a
        + 3)*q^9 - 2*a*q^10 - q^11 + (-3*a + 3)*q^12 + (a + 1)*q^13 + a*q^14 +
        (2*a - 2)*q^15 - q^16 + (-a - 1)*q^17 + (3*a - 10)*q^18 + (2*a + 2)*q^19
        + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
```

```
    q - q^2 - q^3 - q^4 - 2*q^5 + q^6 + q^7 + 3*q^8 + q^9 + 2*q^10 - q^11 + q^12
        + 6*q^13 - q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a + 3)*q^4 + 3*q^5 - a*q^6 + q^7 + (2*a - 5)*q^8 + q^9 +
        3*a*q^10 - q^11 + (a - 3)*q^12 + q^13 + a*q^14 - 3*q^15 + (-5*a +
        4)*q^16 + (2*a + 4)*q^17 + a*q^18 + (-2*a - 3)*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a - 1)*q^4 + q^5 + a*q^6 + q^7 + (-2*a + 1)*q^8 + q^9 +
        a*q^10 + q^11 + (a - 1)*q^12 + (-4*a + 1)*q^13 + a*q^14 + q^15 -
        3*a*q^16 + (-2*a + 4)*q^17 + a*q^18 + (6*a - 3)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 5 over the Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field
*]
[* 11, 21, 77, 77, 77, 77, 77, 231, 231, 231 *]
```

Not bielliptic $n(|a_2| \geq 0; 4) \geq 22 - 18$.

$X_0(231)/ < w_{77} >$, **genus 11**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + O(q^20),
    q + q^2 - q^3 - q^4 - 2*q^5 - q^6 + 4*q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 +
        q^12 - 2*q^13 + 4*q^14 + 2*q^15 - q^16 - 2*q^17 + q^18 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a - 1)*q^4 + q^5 + a*q^6 + q^7 + (-2*a + 1)*q^8 + q^9 +
        a*q^10 + q^11 + (a - 1)*q^12 + (-4*a + 1)*q^13 + a*q^14 + q^15 -
        3*a*q^16 + (-2*a + 4)*q^17 + a*q^18 + (6*a - 3)*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a^2 - a + 6)*q^5 + a*q^6 - q^7 + (2*a^2
        - 7)*q^8 + q^9 + (-3*a^2 + 2*a + 7)*q^10 - q^11 + (a^2 - 2)*q^12 +
        (-3*a^2 + a + 10)*q^13 - a*q^14 + (-a^2 - a + 6)*q^15 + (2*a^2 + a -
        10)*q^16 + (4*a^2 - 2*a - 12)*q^17 + a*q^18 + (a^2 - a - 6)*q^19 +
        O(q^20),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
```

```
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 2*x^2 - 4*x + 7 over the
    Rational Field,
    Rational Field
*]
[* 11, 21, 33, 33, 77, 231, 231, 231 *]
```

Not bielliptic $n(|a_2| \geq 0; 4) \geq 24 - 18$.

$X_0(231)/ < W_{21}, W_{33}, W_{77} >$, **genus 4**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a - 1)*q^4 + q^5 + a*q^6 + q^7 + (-2*a + 1)*q^8 + q^9 +
        a*q^10 + q^11 + (a - 1)*q^12 + (-4*a + 1)*q^13 + a*q^14 + q^15 -
        3*a*q^16 + (-2*a + 4)*q^17 + a*q^18 + (6*a - 3)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field
*]
[* 11, 77, 231 *]
```

???? 77a $n(11, a_2 = -2; 4) = 14 - 10$

$X_0(231)/ < W_3, W_{77}, W_{231} >$, **genus 3**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q + q^2 - q^3 - q^4 - 2*q^5 - q^6 + 4*q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 +
        q^12 - 2*q^13 + 4*q^14 + 2*q^15 - q^16 - 2*q^17 + q^18 + O(q^20),
    q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
        4*q^16 + 2*q^17 - 6*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 11, 33, 77 *]
```

???33a,77a $n(11, a_2 = -2; 4) = 12 - 10$,

$X_0(231)/ < W_7, W_{33}, W_{231} >$, **genus 4**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + O(q^20),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
```

```
    q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + O(q^20),
  q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
    4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
  q + q^2 + 2*q^3 - q^4 - 2*q^5 + 2*q^6 - q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 -
    2*q^12 + 4*q^13 - q^14 - 4*q^15 - q^16 + 4*q^17 + q^18 + O(q^20)
*]
[*
  Rational Field,
  Rational Field,
  Rational Field,
  Rational Field
*]
[* 11, 21, 77, 77 *]
```

???21a,77a $n(77, a_2 = 1; 2) = 5 - 4$ $n(11, a_2 = -2; 4) = 15 - 10$.

$X_0(231)/ < W_{11}, W_{21}, W_{231} >$, **genus 4**

```
[*
  q - 3*q^3 - 2*q^4 - q^5 - q^7 + 6*q^9 - q^11 + 6*q^12 - 4*q^13 + 3*q^15 +
    4*q^16 + 2*q^17 - 6*q^19 + O(q^20),
  q + q^3 - 2*q^4 + 3*q^5 + q^7 - 2*q^9 - q^11 - 2*q^12 - 4*q^13 + 3*q^15 +
    4*q^16 - 6*q^17 + 2*q^19 + O(q^20),
  q + a*q^2 + (-a + 1)*q^3 + 3*q^4 - 2*q^5 + (a - 5)*q^6 + q^7 + a*q^8 + (-2*a
    + 3)*q^9 - 2*a*q^10 - q^11 + (-3*a + 3)*q^12 + (a + 1)*q^13 + a*q^14 +
    (2*a - 2)*q^15 - q^16 + (-a - 1)*q^17 + (3*a - 10)*q^18 + (2*a + 2)*q^19
    + O(q^20)
*]
[*
  Rational Field,
  Rational Field,
  Number Field with defining polynomial x^2 - 5 over the Rational Field
*]
[* 77, 77, 77 *]
```

???77a $n(77b, a_5 = 3, 5) = 8 - 6$,

21. Level $N = 222$

$X_0(222)/ < w_2 >$,**genus 18**

```
[*
  q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
    6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
  q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
    2*q^19 + O(q^20),
  q - q^2 + a*q^3 + q^4 + (-a + 1)*q^5 - a*q^6 + (-2*a + 4)*q^7 - q^8 + (3*a -
    2)*q^9 + (a - 1)*q^10 + (-a + 1)*q^11 + a*q^12 + (a - 2)*q^13 + (2*a -
    4)*q^14 + (-2*a - 1)*q^15 + q^16 - 6*q^17 + (-3*a + 2)*q^18 + 2*q^19 +
    O(q^20),
  q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-a^2 + 5)*q^5 - a*q^6 + (-2*a^2 + 2*a +
    4)*q^7 + (3*a^2 - 3*a - 5)*q^8 + q^9 + (-3*a^2 + 4*a + 5)*q^10 + (2*a^2
    - 4*a - 2)*q^11 + (-a^2 + 2)*q^12 + (2*a^2 - 4*a - 4)*q^13 + (-4*a^2 +
    2*a + 10)*q^14 + (a^2 - 5)*q^15 + (4*a^2 - 2*a - 11)*q^16 + (-a^2 + 4*a
    + 1)*q^17 + a*q^18 + (2*a^2 - 2*a - 8)*q^19 + O(q^20),
```

```
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a^3 - 2*a^2 + 3*a + 4)*q^5 + a*q^6 +
        (2*a^3 + 2*a^2 - 8*a - 2)*q^7 + (a^3 - 4*a)*q^8 + q^9 + (-2*a^3 - 3*a^2
        + 6*a + 5)*q^10 + (2*a^2 - 6)*q^11 + (a^2 - 2)*q^12 + (-2*a^3 - 4*a^2 +
        6*a + 10)*q^13 + (2*a^3 + 4*a^2 - 6*a - 10)*q^14 + (-a^3 - 2*a^2 + 3*a +
        4)*q^15 + (-2*a - 1)*q^16 + (-a^3 + 3*a - 2)*q^17 + a*q^18 + (2*a^2 +
        2*a - 4)*q^19 + O(q^20),
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
        2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 + 2*q^5 + q^6 - q^8 + q^9 - 2*q^10 - 4*q^11 - q^12 +
        6*q^13 - 2*q^15 + q^16 + 6*q^17 - q^18 + 8*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 + 3*q^7 - q^8 + q^9 + 4*q^10 + 5*q^11 -
        q^12 + 3*q^13 - 3*q^14 + 4*q^15 + q^16 + 3*q^17 - q^18 - 7*q^19 +
        O(q^20),
    q - q^2 + q^3 + q^4 + 4*q^5 - q^6 - q^7 - q^8 + q^9 - 4*q^10 - q^11 + q^12 -
        3*q^13 + q^14 + 4*q^15 + q^16 + 3*q^17 - q^18 - 5*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + (-a + 1)*q^5 - a*q^6 + (-2*a + 4)*q^7 - q^8 + (3*a -
        2)*q^9 + (a - 1)*q^10 + (-a + 1)*q^11 + a*q^12 + (a - 2)*q^13 + (2*a -
        4)*q^14 + (-2*a - 1)*q^15 + q^16 - 6*q^17 + (-3*a + 2)*q^18 + 2*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 3*x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 3*x^2 - x + 5 over the Rational
    Field,
    Number Field with defining polynomial x^4 - 6*x^2 + 2*x + 5 over the
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 3*x - 1 over the Rational Field
*]
[* 37, 37, 74, 111, 111, 111, 111, 222, 222, 222, 222 *]
```

Not bielliptic $n(|a_5| \geq 0; 25) \geq 104 - 72$.

$X_0(222)/ < w_3 >$**,genus 17**

```
[*
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
        2*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + (-a + 1)*q^5 - a*q^6 + (-2*a + 4)*q^7 - q^8 + (3*a -
        2)*q^9 + (a - 1)*q^10 + (-a + 1)*q^11 + a*q^12 + (a - 2)*q^13 + (2*a -
        4)*q^14 + (-2*a - 1)*q^15 + q^16 - 6*q^17 + (-3*a + 2)*q^18 + 2*q^19 +
        O(q^20),
```

```
    q + q^2 + a*q^3 + q^4 + (-3*a - 1)*q^5 + a*q^6 + 2*a*q^7 + q^8 + (-a -
        2)*q^9 + (-3*a - 1)*q^10 + (-a - 3)*q^11 + a*q^12 + (3*a + 2)*q^13 +
        2*a*q^14 + (2*a - 3)*q^15 + q^16 + (4*a + 2)*q^17 + (-a - 2)*q^18 +
        (-4*a - 2)*q^19 + O(q^20),
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
        2*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-a^2 + 5)*q^5 - a*q^6 + (-2*a^2 + 2*a +
        4)*q^7 + (3*a^2 - 3*a - 5)*q^8 + q^9 + (-3*a^2 + 4*a + 5)*q^10 + (2*a^2
        - 4*a - 2)*q^11 + (-a^2 + 2)*q^12 + (2*a^2 - 4*a - 4)*q^13 + (-4*a^2 +
        2*a + 10)*q^14 + (a^2 - 5)*q^15 + (4*a^2 - 2*a - 11)*q^16 + (-a^2 + 4*a
        + 1)*q^17 + a*q^18 + (2*a^2 - 2*a - 8)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 + 2*q^5 + q^6 - q^8 + q^9 - 2*q^10 - 4*q^11 - q^12 +
        6*q^13 - 2*q^15 + q^16 + 6*q^17 - q^18 + 8*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 + 3*q^7 - q^8 + q^9 + 4*q^10 + 5*q^11 -
        q^12 + 3*q^13 - 3*q^14 + 4*q^15 + q^16 + 3*q^17 - q^18 - 7*q^19 +
        O(q^20),
    q + q^2 - q^3 + q^4 - q^6 + 3*q^7 + q^8 + q^9 + q^11 - q^12 + q^13 + 3*q^14
        + q^16 - 3*q^17 + q^18 + 3*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-a^2 + 5)*q^5 - a*q^6 + (-2*a^2 + 2*a +
        4)*q^7 + (3*a^2 - 3*a - 5)*q^8 + q^9 + (-3*a^2 + 4*a + 5)*q^10 + (2*a^2
        - 4*a - 2)*q^11 + (-a^2 + 2)*q^12 + (2*a^2 - 4*a - 4)*q^13 + (-4*a^2 +
        2*a + 10)*q^14 + (a^2 - 5)*q^15 + (4*a^2 - 2*a - 11)*q^16 + (-a^2 + 4*a
        + 1)*q^17 + a*q^18 + (2*a^2 - 2*a - 8)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 3*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - 3*x^2 - x + 5 over the Rational
    Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - 3*x^2 - x + 5 over the Rational
    Field
*]
[* 37, 37, 74, 74, 74, 74, 111, 222, 222, 222, 222 *]
```

Not bielliptic $n(|a_5| \geq 0; 25) = 90 - 72$.

$X_0(222)/ < w_{37} >$, **genus 18**

```
[*
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^2 + a*q^3 + q^4 + (-3*a - 1)*q^5 + a*q^6 + 2*a*q^7 + q^8 + (-a -
        2)*q^9 + (-3*a - 1)*q^10 + (-a - 3)*q^11 + a*q^12 + (3*a + 2)*q^13 +
```

```
        2*a*q^14 + (2*a - 3)*q^15 + q^16 + (4*a + 2)*q^17 + (-a - 2)*q^18 +
        (-4*a - 2)*q^19 + O(q^20),
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a^3 - 2*a^2 + 3*a + 4)*q^5 + a*q^6 +
        (2*a^3 + 2*a^2 - 8*a - 2)*q^7 + (a^3 - 4*a)*q^8 + q^9 + (-2*a^3 - 3*a^2
        + 6*a + 5)*q^10 + (2*a^2 - 6)*q^11 + (a^2 - 2)*q^12 + (-2*a^3 - 4*a^2 +
        6*a + 10)*q^13 + (2*a^3 + 4*a^2 - 6*a - 10)*q^14 + (-a^3 - 2*a^2 + 3*a +
        4)*q^15 + (-2*a - 1)*q^16 + (-a^3 + 3*a - 2)*q^17 + a*q^18 + (2*a^2 +
        2*a - 4)*q^19 + O(q^20),
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q - q^2 + q^3 + q^4 + 4*q^5 - q^6 - q^7 - q^8 + q^9 - 4*q^10 - q^11 + q^12 -
        3*q^13 + q^14 + 4*q^15 + q^16 + 3*q^17 - q^18 - 5*q^19 + O(q^20),
    q + q^2 - q^3 + q^4 - q^6 + 3*q^7 + q^8 + q^9 + q^11 - q^12 + q^13 + 3*q^14
        + q^16 - 3*q^17 + q^18 + 3*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a^3 - 2*a^2 + 3*a + 4)*q^5 + a*q^6 +
        (2*a^3 + 2*a^2 - 8*a - 2)*q^7 + (a^3 - 4*a)*q^8 + q^9 + (-2*a^3 - 3*a^2
        + 6*a + 5)*q^10 + (2*a^2 - 6)*q^11 + (a^2 - 2)*q^12 + (-2*a^3 - 4*a^2 +
        6*a + 10)*q^13 + (2*a^3 + 4*a^2 - 6*a - 10)*q^14 + (-a^3 - 2*a^2 + 3*a +
        4)*q^15 + (-2*a - 1)*q^16 + (-a^3 + 3*a - 2)*q^17 + a*q^18 + (2*a^2 +
        2*a - 4)*q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 + (-3*a - 1)*q^5 + a*q^6 + 2*a*q^7 + q^8 + (-a -
        2)*q^9 + (-3*a - 1)*q^10 + (-a - 3)*q^11 + a*q^12 + (3*a + 2)*q^13 +
        2*a*q^14 + (2*a - 3)*q^15 + q^16 + (4*a + 2)*q^17 + (-a - 2)*q^18 +
        (-4*a - 2)*q^19 + O(q^20),
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 - 6*x^2 + 2*x + 5 over the
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 - 6*x^2 + 2*x + 5 over the
    Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field
*]
[* 37, 74, 74, 111, 111, 222, 222, 222, 222, 222 *]
```

Not bielliptic $n(|a_5| \geq 0; 25) \geq 88 - 72$.

$X_0(222)/ < w_6 >$, **genus 18**

```
[*
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 +
```

```
    O(q^20), (REPETED 3 TIMES INSTEAD OF 2, one needs to
    eliminate to compute points).
  q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
    2*q^19 + O(q^20),(REPEATED 3 TIMES INSTEAD OF 2)
  q - q^2 + a*q^3 + q^4 + (-a + 1)*q^5 - a*q^6 + (-2*a + 4)*q^7 - q^8 + (3*a -
    2)*q^9 + (a - 1)*q^10 + (-a + 1)*q^11 + a*q^12 + (a - 2)*q^13 + (2*a -
    4)*q^14 + (-2*a - 1)*q^15 + q^16 - 6*q^17 + (-3*a + 2)*q^18 + 2*q^19 +
    O(q^20),
  q + q^2 + a*q^3 + q^4 + (-3*a - 1)*q^5 + a*q^6 + 2*a*q^7 + q^8 + (-a -
    2)*q^9 + (-3*a - 1)*q^10 + (-a - 3)*q^11 + a*q^12 + (3*a + 2)*q^13 +
    2*a*q^14 + (2*a - 3)*q^15 + q^16 + (4*a + 2)*q^17 + (-a - 2)*q^18 +
    (-4*a - 2)*q^19 + O(q^20),
  q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
    6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
  q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
    2*q^19 + O(q^20),
  q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-a^2 + 5)*q^5 - a*q^6 + (-2*a^2 + 2*a +
    4)*q^7 + (3*a^2 - 3*a - 5)*q^8 + q^9 + (-3*a^2 + 4*a + 5)*q^10 + (2*a^2
    - 4*a - 2)*q^11 + (-a^2 + 2)*q^12 + (2*a^2 - 4*a - 4)*q^13 + (-4*a^2 +
    2*a + 10)*q^14 + (a^2 - 5)*q^15 + (4*a^2 - 2*a - 11)*q^16 + (-a^2 + 4*a
    + 1)*q^17 + a*q^18 + (2*a^2 - 2*a - 8)*q^19 + O(q^20),
  q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a^3 - 2*a^2 + 3*a + 4)*q^5 + a*q^6 +
    (2*a^3 + 2*a^2 - 8*a - 2)*q^7 + (a^3 - 4*a)*q^8 + q^9 + (-2*a^3 - 3*a^2
    + 6*a + 5)*q^10 + (2*a^2 - 6)*q^11 + (a^2 - 2)*q^12 + (-2*a^3 - 4*a^2 +
    6*a + 10)*q^13 + (2*a^3 + 4*a^2 - 6*a - 10)*q^14 + (-a^3 - 2*a^2 + 3*a +
    4)*q^15 + (-2*a - 1)*q^16 + (-a^3 + 3*a - 2)*q^17 + a*q^18 + (2*a^2 +
    2*a - 4)*q^19 + O(q^20),
  q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
    6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
  q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
    2*q^19 + O(q^20),
  q - q^2 - q^3 + q^4 + 2*q^5 + q^6 - q^8 + q^9 - 2*q^10 - 4*q^11 - q^12 +
    6*q^13 - 2*q^15 + q^16 + 6*q^17 - q^18 + 8*q^19 + O(q^20),
  q - q^2 - q^3 + q^4 - 4*q^5 + q^6 + 3*q^7 - q^8 + q^9 + 4*q^10 + 5*q^11 -
    q^12 + 3*q^13 - 3*q^14 + 4*q^15 + q^16 + 3*q^17 - q^18 - 7*q^19 +
    O(q^20),
  q + q^2 + q^3 + q^4 + q^6 - q^7 + q^8 + q^9 + 3*q^11 + q^12 - q^13 - q^14 +
    q^16 - 3*q^17 + q^18 - 7*q^19 + O(q^20)
*]
[*
  Rational Field,
  Rational Field,
  Number Field with defining polynomial x^2 - 3*x - 1 over the Rational Field,
  Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
  Rational Field,
  Rational Field,
  Number Field with defining polynomial x^3 - 3*x^2 - x + 5 over the Rational
  Field,
  Number Field with defining polynomial x^4 - 6*x^2 + 2*x + 5 over the
```

```
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 37, 37, 74, 74, 74, 74, 111, 111, 111, 111, 222, 222, 222 *]
```

Not bielliptic $n(|a_5| \geq 0; 25) \geq 104 - 72$.

$X_0(222)/ < w_{74} >$, **genus 13**

```
[*
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
        2*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-a^2 + 5)*q^5 - a*q^6 + (-2*a^2 + 2*a +
        4)*q^7 + (3*a^2 - 3*a - 5)*q^8 + q^9 + (-3*a^2 + 4*a + 5)*q^10 + (2*a^2
        - 4*a - 2)*q^11 + (-a^2 + 2)*q^12 + (2*a^2 - 4*a - 4)*q^13 + (-4*a^2 +
        2*a + 10)*q^14 + (a^2 - 5)*q^15 + (4*a^2 - 2*a - 11)*q^16 + (-a^2 + 4*a
        + 1)*q^17 + a*q^18 + (2*a^2 - 2*a - 8)*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a^3 - 2*a^2 + 3*a + 4)*q^5 + a*q^6 +
        (2*a^3 + 2*a^2 - 8*a - 2)*q^7 + (a^3 - 4*a)*q^8 + q^9 + (-2*a^3 - 3*a^2
        + 6*a + 5)*q^10 + (2*a^2 - 6)*q^11 + (a^2 - 2)*q^12 + (-2*a^3 - 4*a^2 +
        6*a + 10)*q^13 + (2*a^3 + 4*a^2 - 6*a - 10)*q^14 + (-a^3 - 2*a^2 + 3*a +
        4)*q^15 + (-2*a - 1)*q^16 + (-a^3 + 3*a - 2)*q^17 + a*q^18 + (2*a^2 +
        2*a - 4)*q^19 + O(q^20),
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
        2*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 + 4*q^5 - q^6 - q^7 - q^8 + q^9 - 4*q^10 - q^11 + q^12 -
        3*q^13 + q^14 + 4*q^15 + q^16 + 3*q^17 - q^18 - 5*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + q^6 - q^7 + q^8 + q^9 + 3*q^11 + q^12 - q^13 - q^14 +
        q^16 - 3*q^17 + q^18 - 7*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - 3*x^2 - x + 5 over the Rational
    Field,
    Number Field with defining polynomial x^4 - 6*x^2 + 2*x + 5 over the
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 37, 37, 111, 111, 111, 111, 222, 222 *]
```

Not bielliptic $n(|a_5| \geq 0; 25) \geq 88 - 72$.

$X_0(222)/ < w_{111} >$, **genuss 10**

```
[*
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
        2*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + (-a + 1)*q^5 - a*q^6 + (-2*a + 4)*q^7 - q^8 + (3*a -
        2)*q^9 + (a - 1)*q^10 + (-a + 1)*q^11 + a*q^12 + (a - 2)*q^13 + (2*a -
        4)*q^14 + (-2*a - 1)*q^15 + q^16 - 6*q^17 + (-3*a + 2)*q^18 + 2*q^19 +
        O(q^20),
    q + q^2 + a*q^3 + q^4 + (-3*a - 1)*q^5 + a*q^6 + 2*a*q^7 + q^8 + (-a -
        2)*q^9 + (-3*a - 1)*q^10 + (-a - 3)*q^11 + a*q^12 + (3*a + 2)*q^13 +
        2*a*q^14 + (2*a - 3)*q^15 + q^16 + (4*a + 2)*q^17 + (-a - 2)*q^18 +
        (-4*a - 2)*q^19 + O(q^20),
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
        2*q^19 + O(q^20),
    q + q^2 - q^3 + q^4 - q^6 + 3*q^7 + q^8 + q^9 + q^11 - q^12 + q^13 + 3*q^14
        + q^16 - 3*q^17 + q^18 + 3*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + q^6 - q^7 + q^8 + q^9 + 3*q^11 + q^12 - q^13 - q^14 +
        q^16 - 3*q^17 + q^18 - 7*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 3*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 37, 37, 74, 74, 74, 74, 222, 222 *]
```

Not bielliptic $n(|a_5| \geq 0; 25) \geq 88 - 72$.

$X_0(222)/ < W_6, W_{74}, W_{111} >$, **genus 3**

```
[*
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
        2*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + q^6 - q^7 + q^8 + q^9 + 3*q^11 + q^12 - q^13 - q^14 +
        q^16 - 3*q^17 + q^18 - 7*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field
*]
```

```
[* 37, 37, 222 *]
```

???37a,37b,222a

$X_0(222)/ < W_2, W_{111}, W_{222} >$, **genus 4**

```
[*
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
        2*q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + (-a + 1)*q^5 - a*q^6 + (-2*a + 4)*q^7 - q^8 + (3*a -
        2)*q^9 + (a - 1)*q^10 + (-a + 1)*q^11 + a*q^12 + (a - 2)*q^13 + (2*a -
        4)*q^14 + (-2*a - 1)*q^15 + q^16 - 6*q^17 + (-3*a + 2)*q^18 + 2*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 3*x - 1 over the Rational Field
*]
[* 37, 37, 74 *]
```

$X_0(222)/ < W_3, W_{74}, W_{222} >$, **genus 5**

```
[*
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^3 - 2*q^4 - q^7 - 2*q^9 + 3*q^11 - 2*q^12 - 4*q^13 + 4*q^16 + 6*q^17 +
        2*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-a^2 + 5)*q^5 - a*q^6 + (-2*a^2 + 2*a +
        4)*q^7 + (3*a^2 - 3*a - 5)*q^8 + q^9 + (-3*a^2 + 4*a + 5)*q^10 + (2*a^2
        - 4*a - 2)*q^11 + (-a^2 + 2)*q^12 + (2*a^2 - 4*a - 4)*q^13 + (-4*a^2 +
        2*a + 10)*q^14 + (a^2 - 5)*q^15 + (4*a^2 - 2*a - 11)*q^16 + (-a^2 + 4*a
        + 1)*q^17 + a*q^18 + (2*a^2 - 2*a - 8)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - 3*x^2 - x + 5 over the Rational
    Field
*]
[* 37, 37, 111 *]
```

$X_0(222)/ < W_{37}, W_6, W_{222} >$, **genus 8**

```
[*
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
    q + q^2 + a*q^3 + q^4 + (-3*a - 1)*q^5 + a*q^6 + 2*a*q^7 + q^8 + (-a -
        2)*q^9 + (-3*a - 1)*q^10 + (-a - 3)*q^11 + a*q^12 + (3*a + 2)*q^13 +
        2*a*q^14 + (2*a - 3)*q^15 + q^16 + (4*a + 2)*q^17 + (-a - 2)*q^18 +
        (-4*a - 2)*q^19 + O(q^20),
    q - 2*q^2 - 3*q^3 + 2*q^4 - 2*q^5 + 6*q^6 - q^7 + 6*q^9 + 4*q^10 - 5*q^11 -
        6*q^12 - 2*q^13 + 2*q^14 + 6*q^15 - 4*q^16 - 12*q^18 + O(q^20),
```

```
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + (-a^3 - 2*a^2 + 3*a + 4)*q^5 + a*q^6 +
        (2*a^3 + 2*a^2 - 8*a - 2)*q^7 + (a^3 - 4*a)*q^8 + q^9 + (-2*a^3 - 3*a^2
        + 6*a + 5)*q^10 + (2*a^2 - 6)*q^11 + (a^2 - 2)*q^12 + (-2*a^3 - 4*a^2 +
        6*a + 10)*q^13 + (2*a^3 + 4*a^2 - 6*a - 10)*q^14 + (-a^3 - 2*a^2 + 3*a +
        4)*q^15 + (-2*a - 1)*q^16 + (-a^3 + 3*a - 2)*q^17 + a*q^18 + (2*a^2 +
        2*a - 4)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 - 6*x^2 + 2*x + 5 over the
    Rational Field
*]
[* 37, 74, 74, 111 *]
```

???37a,repited

22. Level $N = 195$

$X_0(195)/ < w_3 >$, **genus 13**

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q + q^2 - q^3 - q^4 + 2*q^5 - q^6 - 4*q^7 - 3*q^8 + q^9 + 2*q^10 + 4*q^11 +
        q^12 + q^13 - 4*q^14 - 2*q^15 - q^16 + 2*q^17 + q^18 + O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 - q^5 + (a - 3)*q^6 + 2*q^7 - a*q^8 + (-2*a +
        1)*q^9 - a*q^10 + (a - 3)*q^11 + (-a + 1)*q^12 + q^13 + 2*a*q^14 + (a -
        1)*q^15 - 5*q^16 + 2*a*q^17 + (a - 6)*q^18 + (3*a - 1)*q^19 + O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a
        - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a -
        2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a +
        3)*q^19 + O(q^20),
    q + 2*q^2 - q^3 + 2*q^4 + q^5 - 2*q^6 + 3*q^7 + q^9 + 2*q^10 - q^11 - 2*q^12
        - q^13 + 6*q^14 - q^15 - 4*q^16 - q^17 + 2*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 - q^5 - a*q^6 + (-a^2 + 5)*q^7 + (3*a +
        2)*q^8 + q^9 - a*q^10 + (-a^2 + 5)*q^11 + (-a^2 + 2)*q^12 + q^13 + (-2*a
        - 2)*q^14 + q^15 + (a^2 + 2*a + 4)*q^16 + (a^2 - 2*a - 5)*q^17 + a*q^18
        + (-2*a + 2)*q^19 + O(q^20),
    q + q^2 - q^3 - q^4 + 2*q^5 - q^6 - 4*q^7 - 3*q^8 + q^9 + 2*q^10 + 4*q^11 +
        q^12 + q^13 - 4*q^14 - 2*q^15 - q^16 + 2*q^17 + q^18 + O(q^20),
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
```

```
    Number Field with defining polynomial $.1^2 - 3 over the Rational Field,
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field,
    Rational Field,
    Number Field with defining polynomial $.1^3 - 7*$.1 - 2 over the Rational
    Field,
    Rational Field,
    Rational Field
*]
[* 15, 39, 65, 65, 65, 195, 195, 195, 195 *]
```

Not bielliptic $n(|a_2| \geq 0; 4) \geq 22 - 18$.

$X_0(195)/ < w_5 >$,**genus 13**

```
[*
    q + q^2 - q^3 - q^4 + 2*q^5 - q^6 - 4*q^7 - 3*q^8 + q^9 + 2*q^10 + 4*q^11 +
        q^12 + q^13 - 4*q^14 - 2*q^15 - q^16 + 2*q^17 + q^18 + O(q^20),
    q + a*q^2 + q^3 + (-2*a - 1)*q^4 + (-2*a - 2)*q^5 + a*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + q^9 + (2*a - 2)*q^10 - 2*q^11 + (-2*a - 1)*q^12 - q^13 +
        (-2*a + 2)*q^14 + (-2*a - 2)*q^15 + 3*q^16 + (4*a + 6)*q^17 + a*q^18 +
        (-2*a - 2)*q^19 + O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 - q^5 + (a - 3)*q^6 + 2*q^7 - a*q^8 + (-2*a +
        1)*q^9 - a*q^10 + (a - 3)*q^11 + (-a + 1)*q^12 + q^13 + 2*a*q^14 + (a -
        1)*q^15 - 5*q^16 + 2*a*q^17 + (a - 6)*q^18 + (3*a - 1)*q^19 + O(q^20),
    q + 2*q^2 + q^3 + 2*q^4 - q^5 + 2*q^6 - q^7 + q^9 - 2*q^10 + 5*q^11 + 2*q^12
        - q^13 - 2*q^14 - q^15 - 4*q^16 - 7*q^17 + 2*q^18 - 6*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 - q^5 - a*q^6 + (-a^2 + 5)*q^7 + (3*a +
        2)*q^8 + q^9 - a*q^10 + (-a^2 + 5)*q^11 + (-a^2 + 2)*q^12 + q^13 + (-2*a
        - 2)*q^14 + q^15 + (a^2 + 2*a + 4)*q^16 + (a^2 - 2*a - 5)*q^17 + a*q^18
        + (-2*a + 2)*q^19 + O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 - q^5 + (a - 3)*q^6 + 2*q^7 - a*q^8 + (-2*a +
        1)*q^9 - a*q^10 + (a - 3)*q^11 + (-a + 1)*q^12 + q^13 + 2*a*q^14 + (a -
        1)*q^15 - 5*q^16 + 2*a*q^17 + (a - 6)*q^18 + (3*a - 1)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 - 3 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^3 - 7*$.1 - 2 over the Rational
    Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 - 3 over the Rational Field
```

```
*]
[* 39, 39, 65, 65, 195, 195, 195, 195 *]
```

Not bielliptic $n(|a_2| \geq 1; 4) > 0$ and $n(a_2 = 0; 16) = 38 - 18$.

$X_0(195)/ < w_{13} >$, **genus 13**

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q + a*q^2 + q^3 + (-2*a - 1)*q^4 + (-2*a - 2)*q^5 + a*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + q^9 + (2*a - 2)*q^10 - 2*q^11 + (-2*a - 1)*q^12 - q^13 +
        (-2*a + 2)*q^14 + (-2*a - 2)*q^15 + 3*q^16 + (4*a + 6)*q^17 + a*q^18 +
        (-2*a - 2)*q^19 + O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a
        - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a -
        2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a +
        3)*q^19 + O(q^20),
    q + 2*q^2 - q^3 + 2*q^4 + q^5 - 2*q^6 + 3*q^7 + q^9 + 2*q^10 - q^11 - 2*q^12
        - q^13 + 6*q^14 - q^15 - 4*q^16 - q^17 + 2*q^18 - 2*q^19 + O(q^20),
    q + 2*q^2 + q^3 + 2*q^4 - q^5 + 2*q^6 - q^7 + q^9 - 2*q^10 + 5*q^11 + 2*q^12
        - q^13 - 2*q^14 - q^15 - 4*q^16 - 7*q^17 + 2*q^18 - 6*q^19 + O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a
        - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a -
        2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a +
        3)*q^19 + O(q^20),
    q + a*q^2 + q^3 + (-2*a - 1)*q^4 + (-2*a - 2)*q^5 + a*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + q^9 + (2*a - 2)*q^10 - 2*q^11 + (-2*a - 1)*q^12 - q^13 +
        (-2*a + 2)*q^14 + (-2*a - 2)*q^15 + 3*q^16 + (4*a + 6)*q^17 + a*q^18 +
        (-2*a - 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field,
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field
*]
```

```
[* 15, 39, 65, 65, 195, 195, 195, 195, 195 *]
```

Not bielliptic $n(|a_2| \geq 0; 4) \geq 22 - 18$.

$X_0(195)/ < w_{15} >$, **genus 13**

```
[*
    q + q^2 - q^3 - q^4 + 2*q^5 - q^6 - 4*q^7 - 3*q^8 + q^9 + 2*q^10 + 4*q^11 +
        q^12 + q^13 - 4*q^14 - 2*q^15 - q^16 + 2*q^17 + q^18 + O(q^20),
    q + a*q^2 + q^3 + (-2*a - 1)*q^4 + (-2*a - 2)*q^5 + a*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + q^9 + (2*a - 2)*q^10 - 2*q^11 + (-2*a - 1)*q^12 - q^13 +
        (-2*a + 2)*q^14 + (-2*a - 2)*q^15 + 3*q^16 + (4*a + 6)*q^17 + a*q^18 +
        (-2*a - 2)*q^19 + O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 - q^5 + (a - 3)*q^6 + 2*q^7 - a*q^8 + (-2*a +
        1)*q^9 - a*q^10 + (a - 3)*q^11 + (-a + 1)*q^12 + q^13 + 2*a*q^14 + (a -
        1)*q^15 - 5*q^16 + 2*a*q^17 + (a - 6)*q^18 + (3*a - 1)*q^19 + O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a
        - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a -
        2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a +
        3)*q^19 + O(q^20),
    q - q^2 + q^3 - q^4 + q^5 - q^6 + 3*q^8 + q^9 - q^10 + 4*q^11 - q^12 + q^13
        + q^15 - q^16 + 2*q^17 - q^18 - 4*q^19 + O(q^20),
    q + 2*q^2 + q^3 + 2*q^4 + q^5 + 2*q^6 - 3*q^7 + q^9 + 2*q^10 - 5*q^11 +
        2*q^12 + q^13 - 6*q^14 + q^15 - 4*q^16 + 5*q^17 + 2*q^18 + 2*q^19 +
        O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 - q^5 - a*q^6 + (-a^2 + 5)*q^7 + (3*a +
        2)*q^8 + q^9 - a*q^10 + (-a^2 + 5)*q^11 + (-a^2 + 2)*q^12 + q^13 + (-2*a
        - 2)*q^14 + q^15 + (a^2 + 2*a + 4)*q^16 + (a^2 - 2*a - 5)*q^17 + a*q^18
        + (-2*a + 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 - 3 over the Rational Field,
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^3 - 7*$.1 - 2 over the Rational
    Field
*]
[* 39, 39, 65, 65, 65, 195, 195, 195 *]
```

Not bielliptic $n(|a_2| > 0; 4) > 0$ and $n(a_2 = 0; 16) = 22 - 18$.

$X_0(195)/ < w_{39} >$, **genus 9**

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
```

```
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 - q^5 + (a - 3)*q^6 + 2*q^7 - a*q^8 + (-2*a +
        1)*q^9 - a*q^10 + (a - 3)*q^11 + (-a + 1)*q^12 + q^13 + 2*a*q^14 + (a -
        1)*q^15 - 5*q^16 + 2*a*q^17 + (a - 6)*q^18 + (3*a - 1)*q^19 + O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a
        - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a -
        2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a +
        3)*q^19 + O(q^20),
    q + 2*q^2 - q^3 + 2*q^4 + q^5 - 2*q^6 + 3*q^7 + q^9 + 2*q^10 - q^11 - 2*q^12
        - q^13 + 6*q^14 - q^15 - 4*q^16 - q^17 + 2*q^18 - 2*q^19 + O(q^20),
    q - q^2 + q^3 - q^4 + q^5 - q^6 + 3*q^8 + q^9 - q^10 + 4*q^11 - q^12 + q^13
        + q^15 - q^16 + 2*q^17 - q^18 - 4*q^19 + O(q^20),
    q + 2*q^2 + q^3 + 2*q^4 + q^5 + 2*q^6 - 3*q^7 + q^9 + 2*q^10 - 5*q^11 +
        2*q^12 + q^13 - 6*q^14 + q^15 - 4*q^16 + 5*q^17 + 2*q^18 + 2*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 - 3 over the Rational Field,
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 15, 65, 65, 65, 195, 195, 195 *]
```

Not bielliptic $n(|a_2| > 0; 4) > 0$, $n(a_2 = 0; 16) = 42 - 18$.

$X_0(195)/ < w_{65} >$**, genus 9**

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q + q^2 - q^3 - q^4 + 2*q^5 - q^6 - 4*q^7 - 3*q^8 + q^9 + 2*q^10 + 4*q^11 +
        q^12 + q^13 - 4*q^14 - 2*q^15 - q^16 + 2*q^17 + q^18 + O(q^20),
    q + a*q^2 + q^3 + (-2*a - 1)*q^4 + (-2*a - 2)*q^5 + a*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + q^9 + (2*a - 2)*q^10 - 2*q^11 + (-2*a - 1)*q^12 - q^13 +
        (-2*a + 2)*q^14 + (-2*a - 2)*q^15 + 3*q^16 + (4*a + 6)*q^17 + a*q^18 +
        (-2*a - 2)*q^19 + O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + 2*q^2 + q^3 + 2*q^4 - q^5 + 2*q^6 - q^7 + q^9 - 2*q^10 + 5*q^11 + 2*q^12
        - q^13 - 2*q^14 - q^15 - 4*q^16 - 7*q^17 + 2*q^18 - 6*q^19 + O(q^20),
    q - q^2 + q^3 - q^4 + q^5 - q^6 + 3*q^8 + q^9 - q^10 + 4*q^11 - q^12 + q^13
        + q^15 - q^16 + 2*q^17 - q^18 - 4*q^19 + O(q^20),
    q + 2*q^2 + q^3 + 2*q^4 + q^5 + 2*q^6 - 3*q^7 + q^9 + 2*q^10 - 5*q^11 +
        2*q^12 + q^13 - 6*q^14 + q^15 - 4*q^16 + 5*q^17 + 2*q^18 + 2*q^19 +
```

```
        O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 15, 39, 39, 65, 195, 195, 195, 195 *]
```

Not bielliptic $n(|a_2| \geq 0; 4) \geq 22 - 18$.

$X_0(195)/ < W_{15}, W_{39}, W_{65} >$, **genus 3**

```
[*
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q - q^2 + q^3 - q^4 + q^5 - q^6 + 3*q^8 + q^9 - q^10 + 4*q^11 - q^12 + q^13
        + q^15 - q^16 + 2*q^17 - q^18 - 4*q^19 + O(q^20),
    q + 2*q^2 + q^3 + 2*q^4 + q^5 + 2*q^6 - 3*q^7 + q^9 + 2*q^10 - 5*q^11 +
        2*q^12 + q^13 - 6*q^14 + q^15 - 4*q^16 + 5*q^17 + 2*q^18 + 2*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 65, 195, 195 *]
```

??biellipitc,65a,195a $n(195, a_2 = 2; 2) = 3 - 2$.

$X_0(195)/ < W_3, W_{65}, W_{195} >$, **genus 3**

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q + q^2 - q^3 - q^4 + 2*q^5 - q^6 - 4*q^7 - 3*q^8 + q^9 + 2*q^10 + 4*q^11 +
        q^12 + q^13 - 4*q^14 - 2*q^15 - q^16 + 2*q^17 + q^18 + O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
```

```
    Rational Field
*]
[* 15, 39, 65 *]
```

$X_0(195)/ < W_5, W_{39}, W_{195} >$, **genus 3**

```
[*
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 - q^5 + (a - 3)*q^6 + 2*q^7 - a*q^8 + (-2*a +
        1)*q^9 - a*q^10 + (a - 3)*q^11 + (-a + 1)*q^12 + q^13 + 2*a*q^14 + (a -
        1)*q^15 - 5*q^16 + 2*a*q^17 + (a - 6)*q^18 + (3*a - 1)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field
*]
[* 65, 65 *]
```

$X_0(195)/ < W_{13}, W_{15}, W_{195} >$, **genus 5**

```
[*
    q + a*q^2 + q^3 + (-2*a - 1)*q^4 + (-2*a - 2)*q^5 + a*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + q^9 + (2*a - 2)*q^10 - 2*q^11 + (-2*a - 1)*q^12 - q^13 +
        (-2*a + 2)*q^14 + (-2*a - 2)*q^15 + 3*q^16 + (4*a + 6)*q^17 + a*q^18 +
        (-2*a - 2)*q^19 + O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a
        - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a -
        2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a +
        3)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 39, 65, 65 *]
```

23. Level $N = 190$

$X_0(190)/ < w_2 >$, **genus 14**

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 - q^7 - q^8 - 2*q^9 - 6*q^11 + q^12 + 5*q^13 +
        q^14 + q^16 + 3*q^17 + 2*q^18 + q^19 + O(q^20),
    q + a*q^2 + (-a^2 + 3)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^2 + 1)*q^6 + (2*a^2 -
        2*a - 4)*q^7 + (a^2 - a - 1)*q^8 + (-2*a^2 + 2*a + 5)*q^9 + a*q^10 +
        (-2*a - 2)*q^11 + (a^2 - 2*a - 5)*q^12 + (a^2 - 2*a + 1)*q^13 + (2*a -
```

```
        2)*q^14 + (-a^2 + 3)*q^15 + (-2*a^2 + 2*a + 3)*q^16 + (-2*a^2 + 4*a +
        4)*q^17 + (-a + 2)*q^18 - q^19 + O(q^20),
    q + a*q^2 + (-a^3 + 5*a - 2)*q^3 + (a^2 - 2)*q^4 - q^5 + (2*a^3 - a^2 - 10*a
        + 9)*q^6 + (-2*a^2 - 2*a + 8)*q^7 + (a^3 - 4*a)*q^8 + (-2*a + 1)*q^9 -
        a*q^10 + (2*a^2 + 2*a - 6)*q^11 + (-3*a^3 + 2*a^2 + 15*a - 14)*q^12 +
        (a^3 + 2*a^2 - 3*a - 4)*q^13 + (-2*a^3 - 2*a^2 + 8*a)*q^14 + (a^3 - 5*a
        + 2)*q^15 + (-2*a^3 + 8*a - 5)*q^16 + (2*a^3 - 10*a + 6)*q^17 + (-2*a^2
        + a)*q^18 + q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 - 2*q^9 + q^10 - q^12 - 3*q^13 +
        q^14 + q^15 + q^16 - 7*q^17 + 2*q^18 - q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + q^5 - a*q^6 + a*q^7 - q^8 + (-a + 1)*q^9 - q^10 +
        4*q^11 + a*q^12 + (-3*a - 2)*q^13 - a*q^14 + a*q^15 + q^16 + (a +
        6)*q^17 + (a - 1)*q^18 - q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 - q^7 - q^8 - 2*q^9 - 6*q^11 + q^12 + 5*q^13 +
        q^14 + q^16 + 3*q^17 + 2*q^18 + q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 3*x + 1 over the Rational
    Field,
    Number Field with defining polynomial x^4 + 2*x^3 - 6*x^2 - 8*x + 9 over the
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field
*]
[* 19, 38, 95, 95, 95, 190, 190, 190 *]
```

Not bielliptic $n(|a_3| \geq 0; 9) \geq 42 - 32$.

$X_0(190)/ < w_5 >$, **genus 14**

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 - q^7 - q^8 - 2*q^9 - 6*q^11 + q^12 + 5*q^13 +
        q^14 + q^16 + 3*q^17 + 2*q^18 + q^19 + O(q^20),
    q + q^2 - q^3 + q^4 - 4*q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 - 4*q^10 + 2*q^11 -
        q^12 - q^13 + 3*q^14 + 4*q^15 + q^16 + 3*q^17 - 2*q^18 - q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q + a*q^2 + (-a^3 + 5*a - 2)*q^3 + (a^2 - 2)*q^4 - q^5 + (2*a^3 - a^2 - 10*a
        + 9)*q^6 + (-2*a^2 - 2*a + 8)*q^7 + (a^3 - 4*a)*q^8 + (-2*a + 1)*q^9 -
        a*q^10 + (2*a^2 + 2*a - 6)*q^11 + (-3*a^3 + 2*a^2 + 15*a - 14)*q^12 +
        (a^3 + 2*a^2 - 3*a - 4)*q^13 + (-2*a^3 - 2*a^2 + 8*a)*q^14 + (a^3 - 5*a
        + 2)*q^15 + (-2*a^3 + 8*a - 5)*q^16 + (2*a^3 - 10*a + 6)*q^17 + (-2*a^2
        + a)*q^18 + q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 - 2*q^9 + q^10 - q^12 - 3*q^13 +
```

```
        q^14 + q^15 + q^16 - 7*q^17 + 2*q^18 - q^19 + O(q^20),
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 - 5*q^7 + q^8 + 6*q^9 - q^10 - 4*q^11 -
        3*q^12 - q^13 - 5*q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + q^19 +
        O(q^20),
    q + a*q^2 + (-a^3 + 5*a - 2)*q^3 + (a^2 - 2)*q^4 - q^5 + (2*a^3 - a^2 - 10*a
        + 9)*q^6 + (-2*a^2 - 2*a + 8)*q^7 + (a^3 - 4*a)*q^8 + (-2*a + 1)*q^9 -
        a*q^10 + (2*a^2 + 2*a - 6)*q^11 + (-3*a^3 + 2*a^2 + 15*a - 14)*q^12 +
        (a^3 + 2*a^2 - 3*a - 4)*q^13 + (-2*a^3 - 2*a^2 + 8*a)*q^14 + (a^3 - 5*a
        + 2)*q^15 + (-2*a^3 + 8*a - 5)*q^16 + (2*a^3 - 10*a + 6)*q^17 + (-2*a^2
        + a)*q^18 + q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 + 2*x^3 - 6*x^2 - 8*x + 9 over the
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 + 2*x^3 - 6*x^2 - 8*x + 9 over the
    Rational Field
*]
[* 19, 38, 38, 38, 95, 190, 190, 190 *]
```

Not bielliptic $n(|a_3| \geq 0; 9) \geq 34 - 32$

$X_0(190)/w_{19}$, **genus 11**

```
[*
    q + q^2 - q^3 + q^4 - 4*q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 - 4*q^10 + 2*q^11 -
        q^12 - q^13 + 3*q^14 + 4*q^15 + q^16 + 3*q^17 - 2*q^18 - q^19 + O(q^20),
    q + a*q^2 + (-a^2 + 3)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^2 + 1)*q^6 + (2*a^2 -
        2*a - 4)*q^7 + (a^2 - a - 1)*q^8 + (-2*a^2 + 2*a + 5)*q^9 + a*q^10 +
        (-2*a - 2)*q^11 + (a^2 - 2*a - 5)*q^12 + (a^2 - 2*a + 1)*q^13 + (2*a -
        2)*q^14 + (-a^2 + 3)*q^15 + (-2*a^2 + 2*a + 3)*q^16 + (-2*a^2 + 4*a +
        4)*q^17 + (-a + 2)*q^18 - q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 - 2*q^9 + q^10 - q^12 - 3*q^13 +
        q^14 + q^15 + q^16 - 7*q^17 + 2*q^18 - q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + q^5 - a*q^6 + a*q^7 - q^8 + (-a + 1)*q^9 - q^10 +
        4*q^11 + a*q^12 + (-3*a - 2)*q^13 - a*q^14 + a*q^15 + q^16 + (a +
        6)*q^17 + (a - 1)*q^18 - q^19 + O(q^20),
    q + a*q^2 + (-a^2 + 3)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^2 + 1)*q^6 + (2*a^2 -
        2*a - 4)*q^7 + (a^2 - a - 1)*q^8 + (-2*a^2 + 2*a + 5)*q^9 + a*q^10 +
        (-2*a - 2)*q^11 + (a^2 - 2*a - 5)*q^12 + (a^2 - 2*a + 1)*q^13 + (2*a -
        2)*q^14 + (-a^2 + 3)*q^15 + (-2*a^2 + 2*a + 3)*q^16 + (-2*a^2 + 4*a +
        4)*q^17 + (-a + 2)*q^18 - q^19 + O(q^20),
    q + q^2 - q^3 + q^4 - 4*q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 - 4*q^10 + 2*q^11 -
        q^12 - q^13 + 3*q^14 + 4*q^15 + q^16 + 3*q^17 - 2*q^18 - q^19 + O(q^20)
*]
[*
    Rational Field,
```

```
    Number Field with defining polynomial x^3 - x^2 - 3*x + 1 over the Rational
    Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 3*x + 1 over the Rational
    Field,
    Rational Field
*]
[* 38, 95, 190, 190, 190, 190 *]
```

Not bielliptic $n(|a_3| \geq 0; 9) \geq 40 - 32$.

$X_0(190)/w_{10}$, **genus 13**

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 + O(q^15),
    q - q^2 + q^3 + q^4 - q^6 - q^7 - q^8 - 2*q^9 - 6*q^11 + q^12 + 5*q^13 +
        q^14 + O(q^15),
    q + q^2 - q^3 + q^4 - 4*q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 - 4*q^10 + 2*q^11 -
        q^12 - q^13 + 3*q^14 + O(q^15),
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 + O(q^15),
    q + a*q^2 + (-a^2 + 3)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^2 + 1)*q^6 + (2*a^2 -
        2*a - 4)*q^7 + (a^2 - a - 1)*q^8 + (-2*a^2 + 2*a + 5)*q^9 + a*q^10 +
        (-2*a - 2)*q^11 + (a^2 - 2*a - 5)*q^12 + (a^2 - 2*a + 1)*q^13 + (2*a -
        2)*q^14 + O(q^15),
    q + a*q^2 + (-a^3 + 5*a - 2)*q^3 + (a^2 - 2)*q^4 - q^5 + (2*a^3 - a^2 - 10*a
        + 9)*q^6 + (-2*a^2 - 2*a + 8)*q^7 + (a^3 - 4*a)*q^8 + (-2*a + 1)*q^9 -
        a*q^10 + (2*a^2 + 2*a - 6)*q^11 + (-3*a^3 + 2*a^2 + 15*a - 14)*q^12 +
        (a^3 + 2*a^2 - 3*a - 4)*q^13 + (-2*a^3 - 2*a^2 + 8*a)*q^14 + O(q^15),
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 +
    O(q^15),(ERASED, APPEARED 3 TIMES INSTEAD OF 2)
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 - 2*q^9 + q^10 - q^12 - 3*q^13 +
        q^14 + O(q^15),
    q + q^2 + q^3 + q^4 + q^5 + q^6 - q^7 + q^8 - 2*q^9 + q^10 + q^12 - q^13 -
        q^14 + O(q^15)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 3*x + 1 over the Rational
    Field,
    Number Field with defining polynomial x^4 + 2*x^3 - 6*x^2 - 8*x + 9 over the
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 19, 38, 38, 38, 95, 95, 95, 190, 190 *]
```

Not bielliptic, $n(|a_5| \geq 0; 25) \geq 113 - 72$

$X_0(190)/w_{38}$, **genus 14**

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q + a*q^2 + (-a^2 + 3)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^2 + 1)*q^6 + (2*a^2 -
        2*a - 4)*q^7 + (a^2 - a - 1)*q^8 + (-2*a^2 + 2*a + 5)*q^9 + a*q^10 +
        (-2*a - 2)*q^11 + (a^2 - 2*a - 5)*q^12 + (a^2 - 2*a + 1)*q^13 + (2*a -
        2)*q^14 + (-a^2 + 3)*q^15 + (-2*a^2 + 2*a + 3)*q^16 + (-2*a^2 + 4*a +
        4)*q^17 + (-a + 2)*q^18 - q^19 + O(q^20),
    q + a*q^2 + (-a^3 + 5*a - 2)*q^3 + (a^2 - 2)*q^4 - q^5 + (2*a^3 - a^2 - 10*a
        + 9)*q^6 + (-2*a^2 - 2*a + 8)*q^7 + (a^3 - 4*a)*q^8 + (-2*a + 1)*q^9 -
        a*q^10 + (2*a^2 + 2*a - 6)*q^11 + (-3*a^3 + 2*a^2 + 15*a - 14)*q^12 +
        (a^3 + 2*a^2 - 3*a - 4)*q^13 + (-2*a^3 - 2*a^2 + 8*a)*q^14 + (a^3 - 5*a
        + 2)*q^15 + (-2*a^3 + 8*a - 5)*q^16 + (2*a^3 - 10*a + 6)*q^17 + (-2*a^2
        + a)*q^18 + q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 - 2*q^9 + q^10 - q^12 - 3*q^13 +
        q^14 + q^15 + q^16 - 7*q^17 + 2*q^18 - q^19 + O(q^20),
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 - 5*q^7 + q^8 + 6*q^9 - q^10 - 4*q^11 -
        3*q^12 - q^13 - 5*q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + q^19 +
        O(q^20),
    q + q^2 + q^3 + q^4 + q^5 + q^6 - q^7 + q^8 - 2*q^9 + q^10 + q^12 - q^13 -
        q^14 + q^15 + q^16 - 3*q^17 - 2*q^18 + q^19 + O(q^20),
    q - q^2 + a*q^3 + q^4 + q^5 - a*q^6 + a*q^7 - q^8 + (-a + 1)*q^9 - q^10 +
        4*q^11 + a*q^12 + (-3*a - 2)*q^13 - a*q^14 + a*q^15 + q^16 + (a +
        6)*q^17 + (a - 1)*q^18 - q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 3*x + 1 over the Rational
    Field,
    Number Field with defining polynomial x^4 + 2*x^3 - 6*x^2 - 8*x + 9 over the
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field
*]
[* 19, 95, 95, 95, 190, 190, 190, 190 *]
```

Not bielliptic $n(|a_3| \geq 0; 9) \geq 34 - 32$.

$X_0(190)/w_{95}$, **genus 6**

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 - q^7 - q^8 - 2*q^9 - 6*q^11 + q^12 + 5*q^13 +
        q^14 + q^16 + 3*q^17 + 2*q^18 + q^19 + O(q^20),
    q + q^2 - q^3 + q^4 - 4*q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 - 4*q^10 + 2*q^11 -
        q^12 - q^13 + 3*q^14 + 4*q^15 + q^16 + 3*q^17 - 2*q^18 - q^19 + O(q^20),
```

```
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 - 2*q^9 + q^10 - q^12 - 3*q^13 +
        q^14 + q^15 + q^16 - 7*q^17 + 2*q^18 - q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + q^5 + q^6 - q^7 + q^8 - 2*q^9 + q^10 + q^12 - q^13 -
        q^14 + q^15 + q^16 - 3*q^17 - 2*q^18 + q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 19, 38, 38, 38, 190, 190 *]
```

Not bielliptic $n(|a_3| \geq 0; 9) \geq 34 - 32$.

$X_0(190)/ < W_{10}, W_{38}, W_{95} >$, **genus 3**

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 - 2*q^9 + q^10 - q^12 - 3*q^13 +
        q^14 + q^15 + q^16 - 7*q^17 + 2*q^18 - q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + q^5 + q^6 - q^7 + q^8 - 2*q^9 + q^10 + q^12 - q^13 -
        q^14 + q^15 + q^16 - 3*q^17 - 2*q^18 + q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 19, 190, 190 *]
```

$X_0(190)/ < W_2, W_{95}, W_{190} >$, **genus 3**

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 - q^7 - q^8 - 2*q^9 - 6*q^11 + q^12 + 5*q^13 +
        q^14 + q^16 + 3*q^17 + 2*q^18 + q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 - 2*q^9 + q^10 - q^12 - 3*q^13 +
        q^14 + q^15 + q^16 - 7*q^17 + 2*q^18 - q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 19, 38, 190 *]
```

$X_0(190)/ < W_5, W_{38}, W_{190} >$, **genus 7**

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q + a*q^2 + (-a^3 + 5*a - 2)*q^3 + (a^2 - 2)*q^4 - q^5 + (2*a^3 - a^2 - 10*a
        + 9)*q^6 + (-2*a^2 - 2*a + 8)*q^7 + (a^3 - 4*a)*q^8 + (-2*a + 1)*q^9 -
        a*q^10 + (2*a^2 + 2*a - 6)*q^11 + (-3*a^3 + 2*a^2 + 15*a - 14)*q^12 +
        (a^3 + 2*a^2 - 3*a - 4)*q^13 + (-2*a^3 - 2*a^2 + 8*a)*q^14 + (a^3 - 5*a
        + 2)*q^15 + (-2*a^3 + 8*a - 5)*q^16 + (2*a^3 - 10*a + 6)*q^17 + (-2*a^2
        + a)*q^18 + q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 - 2*q^9 + q^10 - q^12 - 3*q^13 +
        q^14 + q^15 + q^16 - 7*q^17 + 2*q^18 - q^19 + O(q^20),
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 - 5*q^7 + q^8 + 6*q^9 - q^10 - 4*q^11 -
        3*q^12 - q^13 - 5*q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^4 + 2*x^3 - 6*x^2 - 8*x + 9 over the
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 19, 95, 190, 190 *]
```

$n(190, a_3 = -3; 9) = 18 - 14$

$X_0(190)/ < W_{19}, W_{10}, W_{190} >$, **genus 5**

```
[*
    q + q^2 - q^3 + q^4 - 4*q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 - 4*q^10 + 2*q^11 -
        q^12 - q^13 + 3*q^14 + 4*q^15 + q^16 + 3*q^17 - 2*q^18 - q^19 + O(q^20),
    q + a*q^2 + (-a^2 + 3)*q^3 + (a^2 - 2)*q^4 + q^5 + (-a^2 + 1)*q^6 + (2*a^2 -
        2*a - 4)*q^7 + (a^2 - a - 1)*q^8 + (-2*a^2 + 2*a + 5)*q^9 + a*q^10 +
        (-2*a - 2)*q^11 + (a^2 - 2*a - 5)*q^12 + (a^2 - 2*a + 1)*q^13 + (2*a -
        2)*q^14 + (-a^2 + 3)*q^15 + (-2*a^2 + 2*a + 3)*q^16 + (-2*a^2 + 4*a +
        4)*q^17 + (-a + 2)*q^18 - q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 - 2*q^9 + q^10 - q^12 - 3*q^13 +
        q^14 + q^15 + q^16 - 7*q^17 + 2*q^18 - q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^3 - x^2 - 3*x + 1 over the Rational
    Field,
    Rational Field
*]
[* 38, 95, 190 *]
```

24. Level $N = 182$

$X_0(182)/w_2$,**genus 13**

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20),
```

```
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 + O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 - 3*q^5 + q^7 + q^9 + 4*q^12 + q^13 + 6*q^15 + 4*q^16 -
        6*q^17 - 7*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (a + 3)*q^5 - 2*q^6 + q^7 - 2*a*q^8 - q^9 + (3*a +
        2)*q^10 - 3*a*q^11 - q^13 + a*q^14 + (-3*a - 2)*q^15 - 4*q^16 - a*q^17 -
        a*q^18 + (3*a - 3)*q^19 + O(q^20),
    q + a*q^2 + (-a^2 + a + 2)*q^3 + (a^2 - 2)*q^4 + (-a + 1)*q^5 + (-2*a +
        2)*q^6 - q^7 + (a^2 - 2)*q^8 + (-2*a + 3)*q^9 + (-a^2 + a)*q^10 + (a^2 -
        a - 2)*q^11 - 4*q^12 + q^13 - a*q^14 + (-a^2 + 3*a)*q^15 + (-a^2 + 2*a +
        2)*q^16 + (a^2 + a - 2)*q^17 + (-2*a^2 + 3*a)*q^18 + (-a - 1)*q^19 +
        O(q^20),
    q - q^2 + q^3 + q^4 + 4*q^5 - q^6 - q^7 - q^8 - 2*q^9 - 4*q^10 - q^11 + q^12
        + q^13 + q^14 + 4*q^15 + q^16 + 4*q^17 + 2*q^18 + 2*q^19 + O(q^20),
    q - q^2 + 3*q^3 + q^4 - 3*q^6 + q^7 - q^8 + 6*q^9 - 5*q^11 + 3*q^12 - q^13 -
        q^14 + q^16 - 4*q^17 - 6*q^18 + 2*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 + O(q^20),
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 - 2 over the Rational Field,
    Number Field with defining polynomial $.1^3 - $.1^2 - 4*$.1 + 2 over the
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 26, 91, 91, 91, 91, 182, 182, 182, 182 *]
```

Not bielliptic $n(|a_3| \geq 0; 9) \geq 44 - 32$.

$X_0(182)/w_7$, **genus 13**

```
[*
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 + O(q^20),
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 +
        O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q + a*q^2 + (-a^2 + a + 2)*q^3 + (a^2 - 2)*q^4 + (-a + 1)*q^5 + (-2*a +
        2)*q^6 - q^7 + (a^2 - 2)*q^8 + (-2*a + 3)*q^9 + (-a^2 + a)*q^10 + (a^2 -
```

```
        a - 2)*q^11 - 4*q^12 + q^13 - a*q^14 + (-a^2 + 3*a)*q^15 + (-a^2 + 2*a +
        2)*q^16 + (a^2 + a - 2)*q^17 + (-2*a^2 + 3*a)*q^18 + (-a - 1)*q^19 +
        O(q^20),
    q - q^2 + q^3 + q^4 + 4*q^5 - q^6 - q^7 - q^8 - 2*q^9 - 4*q^10 - q^11 + q^12
        + q^13 + q^14 + 4*q^15 + q^16 + 4*q^17 + 2*q^18 + 2*q^19 + O(q^20),
    q + q^2 + q^4 + 2*q^5 - q^7 + q^8 - 3*q^9 + 2*q^10 + 4*q^11 - q^13 - q^14 +
        q^16 - 6*q^17 - 3*q^18 + O(q^20),
    q + q^2 + 3*q^3 + q^4 - 4*q^5 + 3*q^6 - q^7 + q^8 + 6*q^9 - 4*q^10 + q^11 +
        3*q^12 - q^13 - q^14 - 12*q^15 + q^16 + 6*q^18 - 6*q^19 + O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q + a*q^2 + (-a^2 + a + 2)*q^3 + (a^2 - 2)*q^4 + (-a + 1)*q^5 + (-2*a +
        2)*q^6 - q^7 + (a^2 - 2)*q^8 + (-2*a + 3)*q^9 + (-a^2 + a)*q^10 + (a^2 -
        a - 2)*q^11 - 4*q^12 + q^13 - a*q^14 + (-a^2 + 3*a)*q^15 + (-a^2 + 2*a +
        2)*q^16 + (a^2 + a - 2)*q^17 + (-2*a^2 + 3*a)*q^18 + (-a - 1)*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^3 - $.1^2 - 4*$.1 + 2 over the
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^3 - $.1^2 - 4*$.1 + 2 over the
    Rational Field
*]
[* 26, 26, 91, 91, 182, 182, 182, 182, 182 *]
```

Not bielliptic $n(|a_3| \geq 0; 9) \geq 36 - 32$.

$X_0(182)/w_{13}$, **genus 12**

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20),
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 +
        O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (a + 3)*q^5 - 2*q^6 + q^7 - 2*a*q^8 - q^9 + (3*a +
        2)*q^10 - 3*a*q^11 - q^13 + a*q^14 + (-3*a - 2)*q^15 - 4*q^16 - a*q^17 -
        a*q^18 + (3*a - 3)*q^19 + O(q^20),
    q - q^2 + 3*q^3 + q^4 - 3*q^6 + q^7 - q^8 + 6*q^9 - 5*q^11 + 3*q^12 - q^13 -
        q^14 + q^16 - 4*q^17 - 6*q^18 + 2*q^19 + O(q^20),
    q + q^2 + q^4 + 2*q^5 - q^7 + q^8 - 3*q^9 + 2*q^10 + 4*q^11 - q^13 - q^14 +
        q^16 - 6*q^17 - 3*q^18 + O(q^20),
    q + q^2 + 3*q^3 + q^4 - 4*q^5 + 3*q^6 - q^7 + q^8 + 6*q^9 - 4*q^10 + q^11 +
```

```
        3*q^12 - q^13 - q^14 - 12*q^15 + q^16 + 6*q^18 - 6*q^19 + O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (a + 3)*q^5 - 2*q^6 + q^7 - 2*a*q^8 - q^9 + (3*a +
        2)*q^10 - 3*a*q^11 - q^13 + a*q^14 + (-3*a - 2)*q^15 - 4*q^16 - a*q^17 -
        a*q^18 + (3*a - 3)*q^19 + O(q^20),
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 - 2 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 - 2 over the Rational Field,
    Rational Field
*]
[* 14, 26, 91, 91, 182, 182, 182, 182, 182, 182 *]
```

Not bielliptic $n(|a_3| \geq 0; 9) \geq 34 - 32$.

$X_0(182)/w_{14}$, **genus 11**

```
[*
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 + O(q^20),
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 +
        O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 - 3*q^5 + q^7 + q^9 + 4*q^12 + q^13 + 6*q^15 + 4*q^16 -
        6*q^17 - 7*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (a + 3)*q^5 - 2*q^6 + q^7 - 2*a*q^8 - q^9 + (3*a +
        2)*q^10 - 3*a*q^11 - q^13 + a*q^14 + (-3*a - 2)*q^15 - 4*q^16 - a*q^17 -
        a*q^18 + (3*a - 3)*q^19 + O(q^20),
    q + a*q^2 + (-a^2 + a + 2)*q^3 + (a^2 - 2)*q^4 + (-a + 1)*q^5 + (-2*a +
        2)*q^6 - q^7 + (a^2 - 2)*q^8 + (-2*a + 3)*q^9 + (-a^2 + a)*q^10 + (a^2 -
        a - 2)*q^11 - 4*q^12 + q^13 - a*q^14 + (-a^2 + 3*a)*q^15 + (-a^2 + 2*a +
        2)*q^16 + (a^2 + a - 2)*q^17 + (-2*a^2 + 3*a)*q^18 + (-a - 1)*q^19 +
        O(q^20),
    q - q^2 + q^3 + q^4 + 4*q^5 - q^6 - q^7 - q^8 - 2*q^9 - 4*q^10 - q^11 + q^12
        + q^13 + q^14 + 4*q^15 + q^16 + 4*q^17 + 2*q^18 + 2*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + q^6 + q^7 + q^8 - 2*q^9 - 3*q^11 + q^12 + q^13 + q^14
        + q^16 - 2*q^18 + 2*q^19 + O(q^20)
*]
[*
```

```
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 - 2 over the Rational Field,
    Number Field with defining polynomial $.1^3 - $.1^2 - 4*$.1 + 2 over the
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 26, 26, 91, 91, 91, 91, 182, 182 *]
```

Not bielliptic, $n(|a_3| \geq 0; 9) \geq 40 - 32$.

$X_0(182)/w_{26}$, **genus 10**

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 - 3*q^5 + q^7 + q^9 + 4*q^12 + q^13 + 6*q^15 + 4*q^16 -
        6*q^17 - 7*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (a + 3)*q^5 - 2*q^6 + q^7 - 2*a*q^8 - q^9 + (3*a +
        2)*q^10 - 3*a*q^11 - q^13 + a*q^14 + (-3*a - 2)*q^15 - 4*q^16 - a*q^17 -
        a*q^18 + (3*a - 3)*q^19 + O(q^20),
    q + a*q^2 + (-a^2 + a + 2)*q^3 + (a^2 - 2)*q^4 + (-a + 1)*q^5 + (-2*a +
        2)*q^6 - q^7 + (a^2 - 2)*q^8 + (-2*a + 3)*q^9 + (-a^2 + a)*q^10 + (a^2 -
        a - 2)*q^11 - 4*q^12 + q^13 - a*q^14 + (-a^2 + 3*a)*q^15 + (-a^2 + 2*a +
        2)*q^16 + (a^2 + a - 2)*q^17 + (-2*a^2 + 3*a)*q^18 + (-a - 1)*q^19 +
        O(q^20),
    q - q^2 + 3*q^3 + q^4 - 3*q^6 + q^7 - q^8 + 6*q^9 - 5*q^11 + 3*q^12 - q^13 -
        q^14 + q^16 - 4*q^17 - 6*q^18 + 2*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + q^6 + q^7 + q^8 - 2*q^9 - 3*q^11 + q^12 + q^13 + q^14
        + q^16 - 2*q^18 + 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 - 2 over the Rational Field,
    Number Field with defining polynomial $.1^3 - $.1^2 - 4*$.1 + 2 over the
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 91, 91, 91, 91, 182, 182 *]
```

Not bielliptic. $n(|a_3| \geq 1; 9) > 0$, $a_3 = 0$ say nothing on $\mathbb{F}_{3^n}$-points. But $n(|a_5| \geq 0; 25) \geq 76 - 72$.

$X_0(182)/w_{91}$, **genus 10**

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20),
```

```
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 + O(q^20),
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 +
        O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 - 3*q^5 + q^7 + q^9 + 4*q^12 + q^13 + 6*q^15 + 4*q^16 -
        6*q^17 - 7*q^19 + O(q^20),
    q + q^2 + q^4 + 2*q^5 - q^7 + q^8 - 3*q^9 + 2*q^10 + 4*q^11 - q^13 - q^14 +
        q^16 - 6*q^17 - 3*q^18 + O(q^20),
    q + q^2 + 3*q^3 + q^4 - 4*q^5 + 3*q^6 - q^7 + q^8 + 6*q^9 - 4*q^10 + q^11 +
        3*q^12 - q^13 - q^14 - 12*q^15 + q^16 + 6*q^18 - 6*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + q^6 + q^7 + q^8 - 2*q^9 - 3*q^11 + q^12 + q^13 + q^14
        + q^16 - 2*q^18 + 2*q^19 + O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 - 3*q^5 + q^7 + q^9 + 4*q^12 + q^13 + 6*q^15 + 4*q^16 -
        6*q^17 - 7*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 26, 26, 91, 91, 182, 182, 182, 182, 182 *]
```

Not bielliptic $n(|a_3| \geq 0; 9) \geq 38 - 32$.

$X_0(182)/ < w_2, w_7, w_{14} >$, **genus 6**

```
[*
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 + O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q + a*q^2 + (-a^2 + a + 2)*q^3 + (a^2 - 2)*q^4 + (-a + 1)*q^5 + (-2*a +
        2)*q^6 - q^7 + (a^2 - 2)*q^8 + (-2*a + 3)*q^9 + (-a^2 + a)*q^10 + (a^2 -
        a - 2)*q^11 - 4*q^12 + q^13 - a*q^14 + (-a^2 + 3*a)*q^15 + (-a^2 + 2*a +
        2)*q^16 + (a^2 + a - 2)*q^17 + (-2*a^2 + 3*a)*q^18 + (-a - 1)*q^19 +
        O(q^20),
    q - q^2 + q^3 + q^4 + 4*q^5 - q^6 - q^7 - q^8 - 2*q^9 - 4*q^10 - q^11 + q^12
        + q^13 + q^14 + 4*q^15 + q^16 + 4*q^17 + 2*q^18 + 2*q^19 + O(q^20)
*]
[*
```

```
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^3 - $.1^2 - 4*$.1 + 2 over the
    Rational Field,
    Rational Field
*]
[* 26, 91, 91, 182 *]
```

$X_0(182)/ < w_2, w_{13}, w_{26} >$, **genus 5**

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (a + 3)*q^5 - 2*q^6 + q^7 - 2*a*q^8 - q^9 + (3*a +
        2)*q^10 - 3*a*q^11 - q^13 + a*q^14 + (-3*a - 2)*q^15 - 4*q^16 - a*q^17 -
        a*q^18 + (3*a - 3)*q^19 + O(q^20),
    q - q^2 + 3*q^3 + q^4 - 3*q^6 + q^7 - q^8 + 6*q^9 - 5*q^11 + 3*q^12 - q^13 -
        q^14 + q^16 - 4*q^17 - 6*q^18 + 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 - 2 over the Rational Field,
    Rational Field
*]
[* 14, 91, 91, 182 *]
```

$n(a_3 = 3 : 3) = 3 - 2,$

$X_0(182)/ < w_7, w_{13}, w_{91} >$, **genus 5**

```
[*
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 +
        O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q + q^2 + q^4 + 2*q^5 - q^7 + q^8 - 3*q^9 + 2*q^10 + 4*q^11 - q^13 - q^14 +
        q^16 - 6*q^17 - 3*q^18 + O(q^20),
    q + q^2 + 3*q^3 + q^4 - 4*q^5 + 3*q^6 - q^7 + q^8 + 6*q^9 - 4*q^10 + q^11 +
        3*q^12 - q^13 - q^14 - 12*q^15 + q^16 + 6*q^18 - 6*q^19 + O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 26, 91, 182, 182, 182 *]
```

$n(a_3 = \pm 3; 9) = 22 - 14$,
$X_0(182)/ < w_{14}, w_{26}, w_{91} >$**,genus 3**

```
[*
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 - 3*q^5 + q^7 + q^9 + 4*q^12 + q^13 + 6*q^15 + 4*q^16 -
        6*q^17 - 7*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + q^6 + q^7 + q^8 - 2*q^9 - 3*q^11 + q^12 + q^13 + q^14
        + q^16 - 2*q^18 + 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 91, 91, 182 *]
```

$X_0(182)/ < w_2, w_{91}, w_{182} >$**,genus 4**

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 + O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 - 3*q^5 + q^7 + q^9 + 4*q^12 + q^13 + 6*q^15 + 4*q^16 -
        6*q^17 - 7*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 26, 91, 91 *]
```

$n(a_3 = -2; 9) = 25 - 24.n(a_3 = 1; 3) = 7 - 6.$
$X_0(182)/ < w_7, w_{26}, w_{182} >$**, genus 4**

```
[*
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q + a*q^2 + (-a^2 + a + 2)*q^3 + (a^2 - 2)*q^4 + (-a + 1)*q^5 + (-2*a +
        2)*q^6 - q^7 + (a^2 - 2)*q^8 + (-2*a + 3)*q^9 + (-a^2 + a)*q^10 + (a^2 -
        a - 2)*q^11 - 4*q^12 + q^13 - a*q^14 + (-a^2 + 3*a)*q^15 + (-a^2 + 2*a +
        2)*q^16 + (a^2 + a - 2)*q^17 + (-2*a^2 + 3*a)*q^18 + (-a - 1)*q^19 +
        O(q^20)
*]
[*
    Rational Field,
```

```
    Number Field with defining polynomial $.1^3 - $.1^2 - 4*$.1 + 2 over the
    Rational Field
*]
[* 91, 91 *]
```

$X_0(182)/ < w_{13}, w_{14}, w_{182} >$,**genus 4**

```
[*
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 +
        O(q^20),
    q - 2*q^2 + 2*q^4 - 3*q^5 - q^7 - 3*q^9 + 6*q^10 - 6*q^11 - q^13 + 2*q^14 -
        4*q^16 + 4*q^17 + 6*q^18 + 5*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (a + 3)*q^5 - 2*q^6 + q^7 - 2*a*q^8 - q^9 + (3*a +
        2)*q^10 - 3*a*q^11 - q^13 + a*q^14 + (-3*a - 2)*q^15 - 4*q^16 - a*q^17 -
        a*q^18 + (3*a - 3)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 - 2 over the Rational Field
*]
[* 26, 91, 91 *]
```

$n(a_3 = -3; 9) = 21 - 14$,

25. $N = 174$

25.1. $X_0(174)/w_2$**, genus 14.**

```
[*
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 +
        (2*a - 2)*q^18 + 6*q^19 + O(q^20),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q + a*q^2 + q^3 + (a - 1)*q^4 + (-2*a + 2)*q^5 + a*q^6 + (-2*a - 1)*q^7 +
        (-2*a + 1)*q^8 + q^9 - 2*q^10 + (2*a + 1)*q^11 + (a - 1)*q^12 + (4*a -
        3)*q^13 + (-3*a - 2)*q^14 + (-2*a + 2)*q^15 - 3*a*q^16 + 3*q^17 + a*q^18
        + (2*a - 6)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-2*a^2 + 8)*q^5 - a*q^6 + (a^2 - a -
        2)*q^7 + (2*a^2 - 7)*q^8 + q^9 + (-4*a^2 + 14)*q^10 + (a^2 - a - 6)*q^11
        + (-a^2 + 2)*q^12 + (-a^2 - a + 6)*q^13 + (a^2 + 2*a - 7)*q^14 + (2*a^2
        - 8)*q^15 + (2*a^2 + a - 10)*q^16 + (3*a^2 - a - 10)*q^17 + a*q^18 +
        (2*a - 2)*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 +
        (2*a - 2)*q^18 + 6*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 + 3*q^5 + q^6 - 3*q^7 - q^8 + q^9 - 3*q^10 + 6*q^11 -
        q^12 + 3*q^14 - 3*q^15 + q^16 + 7*q^17 - q^18 + 5*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 + 2*q^5 - q^6 - q^8 + q^9 - 2*q^10 - 4*q^11 + q^12 +
```

```
        6*q^13 + 2*q^15 + q^16 - 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 + 5*q^7 - q^8 + q^9 + 3*q^10 + 6*q^11 +
        q^12 - 4*q^13 - 5*q^14 - 3*q^15 + q^16 + 3*q^17 - q^18 - q^19 + O(q^20),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 2*x^2 - 4*x + 7 over the
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 29, 58, 87, 87, 87, 174, 174, 174, 174 *]
```

Not bielliptic $n(|a_5| \geq 0; 25) \geq 78 - 72$.

25.2. $X_0(174)/w_3$**, genus 14.**

```
[*
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 +
        (2*a - 2)*q^18 + 6*q^19 + O(q^20),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q + q^2 - q^3 + q^4 + q^5 - q^6 - 2*q^7 + q^8 - 2*q^9 + q^10 - 3*q^11 - q^12
        - q^13 - 2*q^14 - q^15 + q^16 + 8*q^17 - 2*q^18 + O(q^20),
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 +
        (2*a - 2)*q^18 + 6*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-2*a^2 + 8)*q^5 - a*q^6 + (a^2 - a -
        2)*q^7 + (2*a^2 - 7)*q^8 + q^9 + (-4*a^2 + 14)*q^10 + (a^2 - a - 6)*q^11
        + (-a^2 + 2)*q^12 + (-a^2 - a + 6)*q^13 + (a^2 + 2*a - 7)*q^14 + (2*a^2
        - 8)*q^15 + (2*a^2 + a - 10)*q^16 + (3*a^2 - a - 10)*q^17 + a*q^18 +
        (2*a - 2)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 + 3*q^5 + q^6 - 3*q^7 - q^8 + q^9 - 3*q^10 + 6*q^11 -
        q^12 + 3*q^14 - 3*q^15 + q^16 + 7*q^17 - q^18 + 5*q^19 + O(q^20),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + q^7 + q^8 + q^9 + q^10 + 6*q^11 - q^12 -
        4*q^13 + q^14 - q^15 + q^16 - 7*q^17 + q^18 - 3*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-2*a^2 + 8)*q^5 - a*q^6 + (a^2 - a -
        2)*q^7 + (2*a^2 - 7)*q^8 + q^9 + (-4*a^2 + 14)*q^10 + (a^2 - a - 6)*q^11
        + (-a^2 + 2)*q^12 + (-a^2 - a + 6)*q^13 + (a^2 + 2*a - 7)*q^14 + (2*a^2
        - 8)*q^15 + (2*a^2 + a - 10)*q^16 + (3*a^2 - a - 10)*q^17 + a*q^18 +
```

```
        (2*a - 2)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 2*x^2 - 4*x + 7 over the
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - 2*x^2 - 4*x + 7 over the
    Rational Field
*]
[* 29, 58, 58, 58, 87, 174, 174, 174 *]
```

Not bielliptic $n(|a_5| \geq 0; 25) \geq 78 - 72$

25.3. $X_0(174)/w_{29}$**, genus 11.**

```
[*
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q + q^2 - q^3 + q^4 + q^5 - q^6 - 2*q^7 + q^8 - 2*q^9 + q^10 - 3*q^11 - q^12
        - q^13 - 2*q^14 - q^15 + q^16 + 8*q^17 - 2*q^18 + O(q^20),
    q + a*q^2 + q^3 + (a - 1)*q^4 + (-2*a + 2)*q^5 + a*q^6 + (-2*a - 1)*q^7 +
        (-2*a + 1)*q^8 + q^9 - 2*q^10 + (2*a + 1)*q^11 + (a - 1)*q^12 + (4*a -
        3)*q^13 + (-3*a - 2)*q^14 + (-2*a + 2)*q^15 - 3*a*q^16 + 3*q^17 + a*q^18
        + (2*a - 6)*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 + 2*q^5 - q^6 - q^8 + q^9 - 2*q^10 - 4*q^11 + q^12 +
        6*q^13 + 2*q^15 + q^16 - 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 + 5*q^7 - q^8 + q^9 + 3*q^10 + 6*q^11 +
        q^12 - 4*q^13 - 5*q^14 - 3*q^15 + q^16 + 3*q^17 - q^18 - q^19 + O(q^20),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + q^7 + q^8 + q^9 + q^10 + 6*q^11 - q^12 -
        4*q^13 + q^14 - q^15 + q^16 - 7*q^17 + q^18 - 3*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a - 1)*q^4 + (-2*a + 2)*q^5 + a*q^6 + (-2*a - 1)*q^7 +
        (-2*a + 1)*q^8 + q^9 - 2*q^10 + (2*a + 1)*q^11 + (a - 1)*q^12 + (4*a -
        3)*q^13 + (-3*a - 2)*q^14 + (-2*a + 2)*q^15 - 3*a*q^16 + 3*q^17 + a*q^18
        + (2*a - 6)*q^19 + O(q^20),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q + q^2 - q^3 + q^4 + q^5 - q^6 - 2*q^7 + q^8 - 2*q^9 + q^10 - 3*q^11 - q^12
        - q^13 - 2*q^14 - q^15 + q^16 + 8*q^17 - 2*q^18 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
```

```
    Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 58, 58, 87, 174, 174, 174, 174, 174, 174 *]
```

Not bielliptic, $n(|a_5| \geq 0; 25) \geq 78 - 72$.

25.4. $X_0(174)/w_6$**,genus 13.**

```
[*
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + O(q^15),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + O(q^15),
    q + q^2 - q^3 + q^4 + q^5 - q^6 - 2*q^7 + q^8 - 2*q^9 + q^10 - 3*q^11 - q^12
        - q^13 - 2*q^14 + O(q^15),
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + O(q^15),
    q + a*q^2 + q^3 + (a - 1)*q^4 + (-2*a + 2)*q^5 + a*q^6 + (-2*a - 1)*q^7 +
        (-2*a + 1)*q^8 + q^9 - 2*q^10 + (2*a + 1)*q^11 + (a - 1)*q^12 + (4*a -
        3)*q^13 + (-3*a - 2)*q^14 + O(q^15),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-2*a^2 + 8)*q^5 - a*q^6 + (a^2 - a -
        2)*q^7 + (2*a^2 - 7)*q^8 + q^9 + (-4*a^2 + 14)*q^10 + (a^2 - a - 6)*q^11
        + (-a^2 + 2)*q^12 + (-a^2 - a + 6)*q^13 + (a^2 + 2*a - 7)*q^14 +
        O(q^15),
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + O(q^15),(ERASED,REPEATED
        3TIMES INSTEAD OF 2).
    q - q^2 - q^3 + q^4 + 3*q^5 + q^6 - 3*q^7 - q^8 + q^9 - 3*q^10 + 6*q^11 -
        q^12 + 3*q^14 + O(q^15),
    q + q^2 + q^3 + q^4 - q^5 + q^6 + q^7 + q^8 + q^9 - q^10 - 2*q^11 + q^12 +
        q^14 + O(q^15)
*]
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 2*x^2 - 4*x + 7 over the
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 29, 58, 58, 58, 87, 87, 87, 174, 174 *]
```

Not bielliptic, $n(|a_5| \geq 0; 25) \geq 88 - 72$.

25.5. $X_0(174)/w_{58}$**, genus 14.**

```
[*
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 +
        (2*a - 2)*q^18 + 6*q^19 + O(q^20),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q + a*q^2 + q^3 + (a - 1)*q^4 + (-2*a + 2)*q^5 + a*q^6 + (-2*a - 1)*q^7 +
        (-2*a + 1)*q^8 + q^9 - 2*q^10 + (2*a + 1)*q^11 + (a - 1)*q^12 + (4*a -
        3)*q^13 + (-3*a - 2)*q^14 + (-2*a + 2)*q^15 - 3*a*q^16 + 3*q^17 + a*q^18
        + (2*a - 6)*q^19 + O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-2*a^2 + 8)*q^5 - a*q^6 + (a^2 - a -
        2)*q^7 + (2*a^2 - 7)*q^8 + q^9 + (-4*a^2 + 14)*q^10 + (a^2 - a - 6)*q^11
        + (-a^2 + 2)*q^12 + (-a^2 - a + 6)*q^13 + (a^2 + 2*a - 7)*q^14 + (2*a^2
        - 8)*q^15 + (2*a^2 + a - 10)*q^16 + (3*a^2 - a - 10)*q^17 + a*q^18 +
        (2*a - 2)*q^19 + O(q^20),
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 +
        (2*a - 2)*q^18 + 6*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 + 2*q^5 - q^6 - q^8 + q^9 - 2*q^10 - 4*q^11 + q^12 +
        6*q^13 + 2*q^15 + q^16 - 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 + 5*q^7 - q^8 + q^9 + 3*q^10 + 6*q^11 +
        q^12 - 4*q^13 - 5*q^14 - 3*q^15 + q^16 + 3*q^17 - q^18 - q^19 + O(q^20),
    q + q^2 + q^3 + q^4 - q^5 + q^6 + q^7 + q^8 + q^9 - q^10 - 2*q^11 + q^12 +
        q^14 - q^15 + q^16 - 3*q^17 + q^18 - q^19 + O(q^20),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 - 2*x^2 - 4*x + 7 over the
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 29, 58, 87, 87, 87, 174, 174, 174, 174 *]
```

Not bielliptic, $n(|a_5| \geq 0; 25) \geq 86 - 72$.

25.6. $X_0(174)/w_{87}$**, genus 8.**

```
[*
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 +
        (2*a - 2)*q^18 + 6*q^19 + O(q^20),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q + q^2 - q^3 + q^4 + q^5 - q^6 - 2*q^7 + q^8 - 2*q^9 + q^10 - 3*q^11 - q^12
        - q^13 - 2*q^14 - q^15 + q^16 + 8*q^17 - 2*q^18 + O(q^20),
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 +
        (2*a - 2)*q^18 + 6*q^19 + O(q^20),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + q^7 + q^8 + q^9 + q^10 + 6*q^11 - q^12 -
        4*q^13 + q^14 - q^15 + q^16 - 7*q^17 + q^18 - 3*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 - q^5 + q^6 + q^7 + q^8 + q^9 - q^10 - 2*q^11 + q^12 +
        q^14 - q^15 + q^16 - 3*q^17 + q^18 - q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 29, 58, 58, 58, 174, 174 *]
```

Not bielliptic $n(|a_5| \geq 0; 25) \geq 90 - 72$.

25.7. $X_0(174)/ < w_2, w_3 >$**,genus 7.**

```
[*
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 +
        (2*a - 2)*q^18 + 6*q^19 + O(q^20),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-2*a^2 + 8)*q^5 - a*q^6 + (a^2 - a -
        2)*q^7 + (2*a^2 - 7)*q^8 + q^9 + (-4*a^2 + 14)*q^10 + (a^2 - a - 6)*q^11
        + (-a^2 + 2)*q^12 + (-a^2 - a + 6)*q^13 + (a^2 + 2*a - 7)*q^14 + (2*a^2
        - 8)*q^15 + (2*a^2 + a - 10)*q^16 + (3*a^2 - a - 10)*q^17 + a*q^18 +
        (2*a - 2)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 + 3*q^5 + q^6 - 3*q^7 - q^8 + q^9 - 3*q^10 + 6*q^11 -
        q^12 + 3*q^14 - 3*q^15 + q^16 + 7*q^17 - q^18 + 5*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
```

```
    Number Field with defining polynomial x^3 - 2*x^2 - 4*x + 7 over the
    Rational Field,
    Rational Field
*]
[* 29, 58, 87, 174 *]
```

$n(a_5 = 3; 5) = 8 - 6,$

25.8. $X_0(174)/ < w_2, w_{29} >$**,genus 6.**

```
[*
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q + a*q^2 + q^3 + (a - 1)*q^4 + (-2*a + 2)*q^5 + a*q^6 + (-2*a - 1)*q^7 +
        (-2*a + 1)*q^8 + q^9 - 2*q^10 + (2*a + 1)*q^11 + (a - 1)*q^12 + (4*a -
        3)*q^13 + (-3*a - 2)*q^14 + (-2*a + 2)*q^15 - 3*a*q^16 + 3*q^17 + a*q^18
        + (2*a - 6)*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 + 2*q^5 - q^6 - q^8 + q^9 - 2*q^10 - 4*q^11 + q^12 +
        6*q^13 + 2*q^15 + q^16 - 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 + 5*q^7 - q^8 + q^9 + 3*q^10 + 6*q^11 +
        q^12 - 4*q^13 - 5*q^14 - 3*q^15 + q^16 + 3*q^17 - q^18 - q^19 + O(q^20),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 58, 87, 174, 174, 174 *]
```

$n(a_5 = 2; 5) = 11 - 8,\ n(a_7 = 5; 7) = 11 - 6,$

25.9. $X_0(174)/ < w_3, w_{19} >$**,genus 3.**

```
[*
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q + q^2 - q^3 + q^4 + q^5 - q^6 - 2*q^7 + q^8 - 2*q^9 + q^10 - 3*q^11 - q^12
        - q^13 - 2*q^14 - q^15 + q^16 + 8*q^17 - 2*q^18 + O(q^20),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + q^7 + q^8 + q^9 + q^10 + 6*q^11 - q^12 -
        4*q^13 + q^14 - q^15 + q^16 - 7*q^17 + q^18 - 3*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 58, 58, 174 *]
```

25.10. $X_0(174)/ < w_6, w_58 >$**,genus 4.**

```
[*
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 +
        (2*a - 2)*q^18 + 6*q^19 + O(q^20),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q + q^2 + q^3 + q^4 - q^5 + q^6 + q^7 + q^8 + q^9 - q^10 - 2*q^11 + q^12 +
        q^14 - q^15 + q^16 - 3*q^17 + q^18 - q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 29, 58, 174 *]
```

25.11. $X_0(174)/ < w_2, w_87 >$**,genus 3.**

```
[*
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 +
        (2*a - 2)*q^18 + 6*q^19 + O(q^20),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field
*]
[* 29, 58 *]
```

25.12. $X_0(174)/ < w_3, w_58 >$**, genus 6.**

```
[*
    q + a*q^2 - a*q^3 + (-2*a - 1)*q^4 - q^5 + (2*a - 1)*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + (-2*a - 2)*q^9 - a*q^10 + (a + 2)*q^11 + (-3*a + 2)*q^12 +
        (2*a + 1)*q^13 + (-2*a + 2)*q^14 + a*q^15 + 3*q^16 + (-2*a - 4)*q^17 +
        (2*a - 2)*q^18 + 6*q^19 + O(q^20),
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 + (-2*a^2 + 8)*q^5 - a*q^6 + (a^2 - a -
        2)*q^7 + (2*a^2 - 7)*q^8 + q^9 + (-4*a^2 + 14)*q^10 + (a^2 - a - 6)*q^11
        + (-a^2 + 2)*q^12 + (-a^2 - a + 6)*q^13 + (a^2 + 2*a - 7)*q^14 + (2*a^2
        - 8)*q^15 + (2*a^2 + a - 10)*q^16 + (3*a^2 - a - 10)*q^17 + a*q^18 +
        (2*a - 2)*q^19 + O(q^20)
*]
```

```
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - 2*x^2 - 4*x + 7 over the
    Rational Field
*]
[* 29, 58, 87 *]
```

25.13. $X_0(174)/ < w_{29}, w_6 >$, **genus 4.**

```
[*
    q - q^2 - 3*q^3 + q^4 - 3*q^5 + 3*q^6 - 2*q^7 - q^8 + 6*q^9 + 3*q^10 - q^11
        - 3*q^12 + 3*q^13 + 2*q^14 + 9*q^15 + q^16 - 4*q^17 - 6*q^18 - 8*q^19 +
        O(q^20),
    q + q^2 - q^3 + q^4 + q^5 - q^6 - 2*q^7 + q^8 - 2*q^9 + q^10 - 3*q^11 - q^12
        - q^13 - 2*q^14 - q^15 + q^16 + 8*q^17 - 2*q^18 + O(q^20),
    q + a*q^2 + q^3 + (a - 1)*q^4 + (-2*a + 2)*q^5 + a*q^6 + (-2*a - 1)*q^7 +
        (-2*a + 1)*q^8 + q^9 - 2*q^10 + (2*a + 1)*q^11 + (a - 1)*q^12 + (4*a -
        3)*q^13 + (-3*a - 2)*q^14 + (-2*a + 2)*q^15 - 3*a*q^16 + 3*q^17 + a*q^18
        + (2*a - 6)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - x - 1 over the Rational Field
*]
[* 58, 58, 87 *]
```

26. $N = 138$

26.1. $X_0(138)/w_2$, **genus 11.**

```
[*
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q - q^2 + q^4 + 4*q^5 - 4*q^7 - q^8 - 3*q^9 - 4*q^10 + 2*q^11 - 2*q^13 +
        4*q^14 + q^16 - 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + q^2 + q^3 - q^4 + q^6 - 2*q^7 - 3*q^8 + q^9 + 4*q^11 - q^12 - 6*q^13 -
        2*q^14 - q^16 + 4*q^17 + q^18 + 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + 3*q^4 + (-a - 1)*q^5 - a*q^6 + (-a + 1)*q^7 + a*q^8 + q^9
        + (-a - 5)*q^10 + 4*q^11 - 3*q^12 + 2*a*q^13 + (a - 5)*q^14 + (a +
        1)*q^15 - q^16 + (-a - 5)*q^17 + a*q^18 + (-a + 5)*q^19 + O(q^20),
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 2*q^10 - 6*q^11 -
        q^12 - 2*q^13 + 2*q^14 + 2*q^15 + q^16 - q^18 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 + 2*q^7 - q^8 + q^9 + q^12 + 2*q^13 - 2*q^14 +
        q^16 - q^18 + 2*q^19 + O(q^20),
```

```
    q - q^2 + q^4 + 4*q^5 - 4*q^7 - q^8 - 3*q^9 - 4*q^10 + 2*q^11 - 2*q^13 +
        4*q^14 + q^16 - 2*q^17 + 3*q^18 - 2*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 23, 46, 69, 69, 69, 138, 138, 138 *]
```

Not bielliptic, $n(a_5 = \pm 4; 25) = 64 - 49$, and $n(|a_7| \geq 2; 49) \geq 124 - 120$

26.2. $X_0(138)/w_3$**, genus 11.**

```
[*
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q - q^2 + q^4 + 4*q^5 - 4*q^7 - q^8 - 3*q^9 - 4*q^10 + 2*q^11 - 2*q^13 +
        4*q^14 + q^16 - 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + 3*q^4 + (-a - 1)*q^5 - a*q^6 + (-a + 1)*q^7 + a*q^8 + q^9
        + (-a - 5)*q^10 + 4*q^11 - 3*q^12 + 2*a*q^13 + (a - 5)*q^14 + (a +
        1)*q^15 - q^16 + (-a - 5)*q^17 + a*q^18 + (-a + 5)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 2*q^10 - 6*q^11 -
        q^12 - 2*q^13 + 2*q^14 + 2*q^15 + q^16 - q^18 + O(q^20),
    q + q^2 - q^3 + q^4 + 2*q^5 - q^6 + q^8 + q^9 + 2*q^10 - q^12 - 2*q^13 -
        2*q^15 + q^16 + 2*q^17 + q^18 - 8*q^19 + O(q^20),
    q + a*q^2 - q^3 + 3*q^4 + (-a - 1)*q^5 - a*q^6 + (-a + 1)*q^7 + a*q^8 + q^9
        + (-a - 5)*q^10 + 4*q^11 - 3*q^12 + 2*a*q^13 + (a - 5)*q^14 + (a +
        1)*q^15 - q^16 + (-a - 5)*q^17 + a*q^18 + (-a + 5)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field
*]
[* 23, 46, 46, 69, 138, 138, 138 *]
```

Not bielliptic $n(a_5 \geq 2; 5) \geq 10 - 8$ and $n(a_7 = -2; 49) = 136 - 120$.

26.3. $X_0(138)/w_{23}$**, genus 5.**

```
[*
    q + q^2 + q^3 - q^4 + q^6 - 2*q^7 - 3*q^8 + q^9 + 4*q^11 - q^12 - 6*q^13 -
        2*q^14 - q^16 + 4*q^17 + q^18 + 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 2*q^10 - 6*q^11 -
        q^12 - 2*q^13 + 2*q^14 + 2*q^15 + q^16 - q^18 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 + 2*q^7 - q^8 + q^9 + q^12 + 2*q^13 - 2*q^14 +
        q^16 - q^18 + 2*q^19 + O(q^20),
    q + q^2 - q^3 + q^4 + 2*q^5 - q^6 + q^8 + q^9 + 2*q^10 - q^12 - 2*q^13 -
        2*q^15 + q^16 + 2*q^17 + q^18 - 8*q^19 + O(q^20),
    q + q^2 + q^3 - q^4 + q^6 - 2*q^7 - 3*q^8 + q^9 + 4*q^11 - q^12 - 6*q^13 -
        2*q^14 - q^16 + 4*q^17 + q^18 + 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 69, 138, 138, 138, 138 *]
```

$???n(|a_5| \geq 2; 25) \geq 68 - 64$

26.4. $X_0(138)/w_6$**, genus 11.**

```
[*
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + O(q^15),
    q - q^2 + q^4 + 4*q^5 - 4*q^7 - q^8 - 3*q^9 - 4*q^10 + 2*q^11 - 2*q^13 +
        4*q^14 + O(q^15),
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + O(q^15),
    q + q^2 + q^3 - q^4 + q^6 - 2*q^7 - 3*q^8 + q^9 + 4*q^11 - q^12 - 6*q^13 -
        2*q^14 + O(q^15),
    q + a*q^2 - q^3 + 3*q^4 + (-a - 1)*q^5 - a*q^6 + (-a + 1)*q^7 + a*q^8 + q^9
        + (-a - 5)*q^10 + 4*q^11 - 3*q^12 + 2*a*q^13 + (a - 5)*q^14 + O(q^15),
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + O(q^15),(ERASED,APPEARED 3
        TIMES INSTEAD OF 2).
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 2*q^10 - 6*q^11 -
        q^12 - 2*q^13 + 2*q^14 + O(q^15),
    q + q^2 + q^3 + q^4 + a*q^5 + q^6 + (-2*a - 2)*q^7 + q^8 + q^9 + a*q^10 +
        (-a - 4)*q^11 + q^12 + (2*a + 2)*q^13 + (-2*a - 2)*q^14 + O(q^15)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
```

```
    Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 4 over the Rational Field
*]
[* 23, 46, 46, 69, 69, 69, 138, 138 *]
```

Not bielliptic, $n(|a_5| \geq 2; 25) \geq 68 - 64$,$n(a_{11} = 4; 11) = 22 - 16$.

26.5. $X_0(138)/w_{46}$**, genus 11.**

```
[*
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q + q^2 + q^3 - q^4 + q^6 - 2*q^7 - 3*q^8 + q^9 + 4*q^11 - q^12 - 6*q^13 -
        2*q^14 - q^16 + 4*q^17 + q^18 + 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + 3*q^4 + (-a - 1)*q^5 - a*q^6 + (-a + 1)*q^7 + a*q^8 + q^9
        + (-a - 5)*q^10 + 4*q^11 - 3*q^12 + 2*a*q^13 + (a - 5)*q^14 + (a +
        1)*q^15 - q^16 + (-a - 5)*q^17 + a*q^18 + (-a + 5)*q^19 + O(q^20),
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 2*q^10 - 6*q^11 -
        q^12 - 2*q^13 + 2*q^14 + 2*q^15 + q^16 - q^18 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 + 2*q^7 - q^8 + q^9 + q^12 + 2*q^13 - 2*q^14 +
        q^16 - q^18 + 2*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + a*q^5 + q^6 + (-2*a - 2)*q^7 + q^8 + q^9 + a*q^10 +
        (-a - 4)*q^11 + q^12 + (2*a + 2)*q^13 + (-2*a - 2)*q^14 + a*q^15 + q^16
        - 4*q^17 + q^18 + (3*a + 2)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 4 over the Rational Field
*]
[* 23, 69, 69, 69, 138, 138, 138 *]
```

Not bielliptic $n(|a_5| \geq 0; 25) \geq 84 - 72$

26.6. $X_0(138)/w_{69}$**, genus 9.**

```
[*
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
```

```
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q - q^2 + q^4 + 4*q^5 - 4*q^7 - q^8 - 3*q^9 - 4*q^10 + 2*q^11 - 2*q^13 +
        4*q^14 + q^16 - 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 2*q^10 - 6*q^11 -
        q^12 - 2*q^13 + 2*q^14 + 2*q^15 + q^16 - q^18 + O(q^20),
    q + q^2 - q^3 + q^4 + 2*q^5 - q^6 + q^8 + q^9 + 2*q^10 - q^12 - 2*q^13 -
        2*q^15 + q^16 + 2*q^17 + q^18 - 8*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + a*q^5 + q^6 + (-2*a - 2)*q^7 + q^8 + q^9 + a*q^10 +
        (-a - 4)*q^11 + q^12 + (2*a + 2)*q^13 + (-2*a - 2)*q^14 + a*q^15 + q^16
        - 4*q^17 + q^18 + (3*a + 2)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 4 over the Rational Field
*]
[* 23, 46, 46, 138, 138, 138 *]
```

??? $n(a_5 = 4; 5) = 8 - 4$.

26.7. $X_0(138)/ < w_2, w_3 >$**, genus 6.**

```
[*
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q - q^2 + q^4 + 4*q^5 - 4*q^7 - q^8 - 3*q^9 - 4*q^10 + 2*q^11 - 2*q^13 +
        4*q^14 + q^16 - 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + 3*q^4 + (-a - 1)*q^5 - a*q^6 + (-a + 1)*q^7 + a*q^8 + q^9
        + (-a - 5)*q^10 + 4*q^11 - 3*q^12 + 2*a*q^13 + (a - 5)*q^14 + (a +
        1)*q^15 - q^16 + (-a - 5)*q^17 + a*q^18 + (-a + 5)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 2*q^10 - 6*q^11 -
        q^12 - 2*q^13 + 2*q^14 + 2*q^15 + q^16 - q^18 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field
*]
[* 23, 46, 69, 138 *]
```

$n(a_5 = 4; 5) = 8 - 4$,

26.8. $X_0(138)/ < w_2, w_{23} >$**, genus 3.**

```
[*
    q + q^2 + q^3 - q^4 + q^6 - 2*q^7 - 3*q^8 + q^9 + 4*q^11 - q^12 - 6*q^13 -
        2*q^14 - q^16 + 4*q^17 + q^18 + 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 2*q^10 - 6*q^11 -
        q^12 - 2*q^13 + 2*q^14 + 2*q^15 + q^16 - q^18 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 + 2*q^7 - q^8 + q^9 + q^12 + 2*q^13 - 2*q^14 +
        q^16 - q^18 + 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 69, 138, 138 *]
```

26.9. $X_0(138)/ < w_3, w_{23} >$**, genus 2.**

```
[*
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 2*q^10 - 6*q^11 -
        q^12 - 2*q^13 + 2*q^14 + 2*q^15 + q^16 - q^18 + O(q^20),
    q + q^2 - q^3 + q^4 + 2*q^5 - q^6 + q^8 + q^9 + 2*q^10 - q^12 - 2*q^13 -
        2*q^15 + q^16 + 2*q^17 + q^18 - 8*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field
*]
[* 138, 138 *]
```

26.10. $X_0(138)/ < w_6, w_{46} >$**, genus 5.**

```
[*
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 2*q^10 - 6*q^11 -
        q^12 - 2*q^13 + 2*q^14 + 2*q^15 + q^16 - q^18 + O(q^20),
    q + q^2 + q^3 + q^4 + a*q^5 + q^6 + (-2*a - 2)*q^7 + q^8 + q^9 + a*q^10 +
        (-a - 4)*q^11 + q^12 + (2*a + 2)*q^13 + (-2*a - 2)*q^14 + a*q^15 + q^16
        - 4*q^17 + q^18 + (3*a + 2)*q^19 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 4 over the Rational Field
*]
[* 23, 138, 138 *]
```

26.11. $X_0(138)/ < w_2, w_{69} >$**, genus 4.**

```
[*
```

```
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q - q^2 + q^4 + 4*q^5 - 4*q^7 - q^8 - 3*q^9 - 4*q^10 + 2*q^11 - 2*q^13 +
        4*q^14 + q^16 - 2*q^17 + 3*q^18 - 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 2*q^10 - 6*q^11 -
        q^12 - 2*q^13 + 2*q^14 + 2*q^15 + q^16 - q^18 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 23, 46, 138 *]
```

$n(a_5 = 4; 5) = 6 - 4,$

26.12. $X_0(138)/ < w_3, w_{46} >$**, genus 5.**

```
[*
    q + a*q^2 + (-2*a - 1)*q^3 + (-a - 1)*q^4 + 2*a*q^5 + (a - 2)*q^6 + (2*a +
        2)*q^7 + (-2*a - 1)*q^8 + 2*q^9 + (-2*a + 2)*q^10 + (-2*a - 4)*q^11 + (a
        + 3)*q^12 + 3*q^13 + 2*q^14 + (2*a - 4)*q^15 + 3*a*q^16 + (-2*a +
        2)*q^17 + 2*a*q^18 - 2*q^19 + O(q^20),
    q + a*q^2 - q^3 + 3*q^4 + (-a - 1)*q^5 - a*q^6 + (-a + 1)*q^7 + a*q^8 + q^9
        + (-a - 5)*q^10 + 4*q^11 - 3*q^12 + 2*a*q^13 + (a - 5)*q^14 + (a +
        1)*q^15 - q^16 + (-a - 5)*q^17 + a*q^18 + (-a + 5)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 2*q^10 - 6*q^11 -
        q^12 - 2*q^13 + 2*q^14 + 2*q^15 + q^16 - q^18 + O(q^20)
*]
[*
    Number Field with defining polynomial x^2 + x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field
*]
[* 23, 69, 138 *]
```

26.13. $X_0(138)/ < w_{23}, w_6 >$**, genus 2.**

```
[*
    q + q^2 + q^3 - q^4 + q^6 - 2*q^7 - 3*q^8 + q^9 + 4*q^11 - q^12 - 6*q^13 -
        2*q^14 - q^16 + 4*q^17 + q^18 + 2*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 2*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 2*q^10 - 6*q^11 -
        q^12 - 2*q^13 + 2*q^14 + 2*q^15 + q^16 - q^18 + O(q^20)
*]
[*
    Rational Field,
    Rational Field
*]
[* 69, 138 *]
```

27. $N = 130$

27.1. $X_0(130)/w_2$,genus 8.

```
[*
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 + O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 - q^5 + (a - 3)*q^6 + 2*q^7 - a*q^8 + (-2*a +
        1)*q^9 - a*q^10 + (a - 3)*q^11 + (-a + 1)*q^12 + q^13 + 2*a*q^14 + (a -
        1)*q^15 - 5*q^16 + 2*a*q^17 + (a - 6)*q^18 + (3*a - 1)*q^19 + O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a
        - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a -
        2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a +
        3)*q^19 + O(q^20),
    q - q^2 - 2*q^3 + q^4 + q^5 + 2*q^6 - 4*q^7 - q^8 + q^9 - q^10 - 6*q^11 -
        2*q^12 + q^13 + 4*q^14 - 2*q^15 + q^16 - 6*q^17 - q^18 + 2*q^19 +
        O(q^20),
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 - 3 over the Rational Field,
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field,
    Rational Field,
    Rational Field
*]
[* 26, 65, 65, 65, 130, 130 *]
```

Not bielliptic $n(|a_3| \geq 0; 9) \geq 36 - 32$.

27.2. $X_0(130)/w_5$, genus 9.

```
[*
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 + O(q^20),
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 +
        O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 - q^5 + (a - 3)*q^6 + 2*q^7 - a*q^8 + (-2*a +
        1)*q^9 - a*q^10 + (a - 3)*q^11 + (-a + 1)*q^12 + q^13 + 2*a*q^14 + (a -
        1)*q^15 - 5*q^16 + 2*a*q^17 + (a - 6)*q^18 + (3*a - 1)*q^19 + O(q^20),
    q + q^2 + 2*q^3 + q^4 - q^5 + 2*q^6 - 4*q^7 + q^8 + q^9 - q^10 - 2*q^11 +
        2*q^12 - q^13 - 4*q^14 - 2*q^15 + q^16 + 2*q^17 + q^18 + 6*q^19 +
        O(q^20),
```

```
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 - q^5 + (a - 3)*q^6 + 2*q^7 - a*q^8 + (-2*a +
        1)*q^9 - a*q^10 + (a - 3)*q^11 + (-a + 1)*q^12 + q^13 + 2*a*q^14 + (a -
        1)*q^15 - 5*q^16 + 2*a*q^17 + (a - 6)*q^18 + (3*a - 1)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 - 3 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 - 3 over the Rational Field
*]
[* 26, 26, 65, 65, 130, 130, 130 *]
```

Not bielliptic, $n(a_{11}; 11) = 18 - 12$,(first), $n(|a_3| \geq 2; 9) \geq 26 - 24$.

27.3. $X_0(130)/w_{13}$**, genus 9.**

```
[*
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 +
        O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a
        - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a -
        2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a +
        3)*q^19 + O(q^20),
    q + q^2 + 2*q^3 + q^4 - q^5 + 2*q^6 - 4*q^7 + q^8 + q^9 - q^10 - 2*q^11 +
        2*q^12 - q^13 - 4*q^14 - 2*q^15 + q^16 + 2*q^17 + q^18 + 6*q^19 +
        O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a
        - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a -
        2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a +
        3)*q^19 + O(q^20),
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field,
```

```
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field,
    Rational Field
*]
[* 26, 65, 65, 130, 130, 130, 130 *]
```

Not bielliptic $n(|a_3| \geq 2; 9) \geq 26 - 24$.

27.4. $X_0(130)/w_{10}$**, genus 8.**

```
[*
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 + O(q^20),
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 +
        O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 - q^5 + (a - 3)*q^6 + 2*q^7 - a*q^8 + (-2*a +
        1)*q^9 - a*q^10 + (a - 3)*q^11 + (-a + 1)*q^12 + q^13 + 2*a*q^14 + (a -
        1)*q^15 - 5*q^16 + 2*a*q^17 + (a - 6)*q^18 + (3*a - 1)*q^19 + O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a
        - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a -
        2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a +
        3)*q^19 + O(q^20),
    q + q^2 + q^4 + q^5 + q^8 - 3*q^9 + q^10 + q^13 + q^16 + 2*q^17 - 3*q^18 -
        8*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial $.1^2 - 3 over the Rational Field,
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field,
    Rational Field
*]
[* 26, 26, 65, 65, 65, 130 *]
```

??? the last one? $n(|a_3| \geq 1; 9) > 0$,

27.5. $X_0(130)/w_{26}$**,genus 6.**

```
[*
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 - q^5 + (a - 3)*q^6 + 2*q^7 - a*q^8 + (-2*a +
        1)*q^9 - a*q^10 + (a - 3)*q^11 + (-a + 1)*q^12 + q^13 + 2*a*q^14 + (a -
        1)*q^15 - 5*q^16 + 2*a*q^17 + (a - 6)*q^18 + (3*a - 1)*q^19 + O(q^20),
    q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a
```

```
        - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a -
        2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a +
        3)*q^19 + O(q^20),
    q + q^2 + q^4 + q^5 + q^8 - 3*q^9 + q^10 + q^13 + q^16 + 2*q^17 - 3*q^18 -
        8*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial $.1^2 - 3 over the Rational Field,
    Number Field with defining polynomial $.1^2 + 2*$.1 - 1 over the Rational
    Field,
    Rational Field
*]
[* 65, 65, 65, 130 *]
```

The last one, $n(a_3 = -2; 9) > 0$.

27.6. $X_0(130)/w_{65}$**, genus 7.**

```
[*
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 + O(q^20),
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 +
        O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20),
    q - q^2 - 2*q^3 + q^4 + q^5 + 2*q^6 - 4*q^7 - q^8 + q^9 - q^10 - 6*q^11 -
        2*q^12 + q^13 + 4*q^14 - 2*q^15 + q^16 - 6*q^17 - q^18 + 2*q^19 +
        O(q^20),
    q + q^2 + 2*q^3 + q^4 - q^5 + 2*q^6 - 4*q^7 + q^8 + q^9 - q^10 - 2*q^11 +
        2*q^12 - q^13 - 4*q^14 - 2*q^15 + q^16 + 2*q^17 + q^18 + 6*q^19 +
        O(q^20),
    q + q^2 + q^4 + q^5 + q^8 - 3*q^9 + q^10 + q^13 + q^16 + 2*q^17 - 3*q^18 -
        8*q^19 + O(q^20),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 26, 26, 65, 130, 130, 130, 130 *]
```

Not bielliptic, $n(a_3 \geq 0; 3) \geq 10 - 8$, $n(a_3 \leq -2; 9) \geq 26 - 24$

28. $N = 114$

28.1. $X_0(114)/w_2$, genus 8.

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 - q^7 - q^8 - 2*q^9 - 6*q^11 + q^12 + 5*q^13 +
        q^14 + q^16 + 3*q^17 + 2*q^18 + q^19 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 +
        O(q^20),
    q + q^2 + q^3 - q^4 - 2*q^5 + q^6 - 3*q^8 + q^9 - 2*q^10 - q^12 + 6*q^13 -
        2*q^15 - q^16 - 6*q^17 + q^18 - q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 + q^5 - 2*q^6 + 3*q^7 + q^9 - 2*q^10 - 3*q^11 +
        2*q^12 - 6*q^13 - 6*q^14 + q^15 - 4*q^16 + 3*q^17 - 2*q^18 - q^19 +
        O(q^20),
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - q^2 - q^3 + q^4 + q^6 + 4*q^7 - q^8 + q^9 + 4*q^11 - q^12 - 4*q^14 +
        q^16 - 2*q^17 - q^18 + q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 - q^7 - q^8 - 2*q^9 - 6*q^11 + q^12 + 5*q^13 +
        q^14 + q^16 + 3*q^17 + 2*q^18 + q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 19, 38, 57, 57, 57, 57, 114, 114 *]
```

Not bielliptic $n(|a_5| \geq 0; 25) \geq 74 - 72$.

28.2. $X_0(114)/w_3$, genus 8.

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 - q^7 - q^8 - 2*q^9 - 6*q^11 + q^12 + 5*q^13 +
        q^14 + q^16 + 3*q^17 + 2*q^18 + q^19 + O(q^20),
    q + q^2 - q^3 + q^4 - 4*q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 - 4*q^10 + 2*q^11 -
        q^12 - q^13 + 3*q^14 + 4*q^15 + q^16 + 3*q^17 - 2*q^18 - q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 +
        O(q^20),
    q - q^2 - q^3 + q^4 + q^6 + 4*q^7 - q^8 + q^9 + 4*q^11 - q^12 - 4*q^14 +
```

```
        q^16 - 2*q^17 - q^18 + q^19 + O(q^20),
    q + q^2 - q^3 + q^4 + 2*q^5 - q^6 + q^8 + q^9 + 2*q^10 - 4*q^11 - q^12 +
        2*q^13 - 2*q^15 + q^16 - 6*q^17 + q^18 - q^19 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 19, 38, 38, 38, 57, 114, 114, 114 *]
```

??? $n(a_5 = 3; 5) = 8 - 6$,$n(a_5 = -4; 25) = 50 - 40$,$n(a_7 = 4; 7) = 14 - 8$.

28.3. $X_0(114)/w_{19}$**, genus 9.**

```
[*
    q + q^2 - q^3 + q^4 - 4*q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 - 4*q^10 + 2*q^11 -
        q^12 - q^13 + 3*q^14 + 4*q^15 + q^16 + 3*q^17 - 2*q^18 - q^19 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 +
        O(q^20),
    q + q^2 + q^3 - q^4 - 2*q^5 + q^6 - 3*q^8 + q^9 - 2*q^10 - q^12 + 6*q^13 -
        2*q^15 - q^16 - 6*q^17 + q^18 - q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 + q^5 - 2*q^6 + 3*q^7 + q^9 - 2*q^10 - 3*q^11 +
        2*q^12 - 6*q^13 - 6*q^14 + q^15 - 4*q^16 + 3*q^17 - 2*q^18 - q^19 +
        O(q^20),
    q + q^2 - q^3 + q^4 + 2*q^5 - q^6 + q^8 + q^9 + 2*q^10 - 4*q^11 - q^12 +
        2*q^13 - 2*q^15 + q^16 - 6*q^17 + q^18 - q^19 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 +
        O(q^20),
    q + q^2 + q^3 - q^4 - 2*q^5 + q^6 - 3*q^8 + q^9 - 2*q^10 - q^12 + 6*q^13 -
        2*q^15 - q^16 - 6*q^17 + q^18 - q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 + q^5 - 2*q^6 + 3*q^7 + q^9 - 2*q^10 - 3*q^11 +
        2*q^12 - 6*q^13 - 6*q^14 + q^15 - 4*q^16 + 3*q^17 - 2*q^18 - q^19 +
        O(q^20),
    q + q^2 - q^3 + q^4 - 4*q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 - 4*q^10 + 2*q^11 -
        q^12 - q^13 + 3*q^14 + 4*q^15 + q^16 + 3*q^17 - 2*q^18 - q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
```

```
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 38, 57, 57, 57, 114, 114, 114, 114, 114 *]
```

Not bielliptic $n(a_5 \geq -3; 5) \geq 20 - 18$, $n(|a_5| \geq 4; 25) \geq 52 - 40$.

28.4. $X_0(114)/w_6$**, genus 9.**

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 - 6*q^20 + 2*q^21 + 4*q^25 + 4*q^27 + 2*q^28 +
        6*q^29 - 4*q^31 - 6*q^33 - 3*q^35 - 2*q^36 + 2*q^37 + 8*q^39 - 6*q^41 -
        q^43 - 6*q^44 + 3*q^45 - 3*q^47 - 8*q^48 - 6*q^49 + O(q^50),
    q - q^2 + q^3 + q^4 - q^6 - q^7 - q^8 - 2*q^9 - 6*q^11 + q^12 + 5*q^13 +
        q^14 + q^16 + 3*q^17 + 2*q^18 + q^19 - q^21 + 6*q^22 + 3*q^23 - q^24 -
        5*q^25 - 5*q^26 - 5*q^27 - q^28 + 9*q^29 - 4*q^31 - q^32 - 6*q^33 -
        3*q^34 - 2*q^36 + 2*q^37 - q^38 + 5*q^39 + q^42 + 8*q^43 - 6*q^44 -
        3*q^46 + q^48 - 6*q^49 + O(q^50),
    q + q^2 - q^3 + q^4 - 4*q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 - 4*q^10 + 2*q^11 -
        q^12 - q^13 + 3*q^14 + 4*q^15 + q^16 + 3*q^17 - 2*q^18 - q^19 - 4*q^20 -
        3*q^21 + 2*q^22 - q^23 - q^24 + 11*q^25 - q^26 + 5*q^27 + 3*q^28 -
        5*q^29 + 4*q^30 - 8*q^31 + q^32 - 2*q^33 + 3*q^34 - 12*q^35 - 2*q^36 -
        2*q^37 - q^38 + q^39 - 4*q^40 - 8*q^41 - 3*q^42 + 4*q^43 + 2*q^44 +
        8*q^45 - q^46 + 8*q^47 - q^48 + 2*q^49 + O(q^50),
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 - 6*q^20 + 2*q^21 + 4*q^25 + 4*q^27 + 2*q^28 +
        6*q^29 - 4*q^31 - 6*q^33 - 3*q^35 - 2*q^36 + 2*q^37 + 8*q^39 - 6*q^41 -
        q^43 - 6*q^44 + 3*q^45 - 3*q^47 - 8*q^48 - 6*q^49 + O(q^50),
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 -
        6*q^20 + 5*q^21 - 2*q^22 - 4*q^23 + 4*q^25 - 4*q^26 - q^27 - 10*q^28 -
        2*q^29 - 6*q^30 - 6*q^31 + 8*q^32 - q^33 + 2*q^34 + 15*q^35 + 2*q^36 +
        2*q^38 - 2*q^39 - 10*q^42 - q^43 + 2*q^44 - 3*q^45 + 8*q^46 - 9*q^47 +
        4*q^48 + 18*q^49 + O(q^50),
    q + q^2 + q^3 - q^4 - 2*q^5 + q^6 - 3*q^8 + q^9 - 2*q^10 - q^12 + 6*q^13 -
        2*q^15 - q^16 - 6*q^17 + q^18 - q^19 + 2*q^20 + 4*q^23 - 3*q^24 - q^25 +
        6*q^26 + q^27 + 2*q^29 - 2*q^30 + 8*q^31 + 5*q^32 - 6*q^34 - q^36 -
        10*q^37 - q^38 + 6*q^39 + 6*q^40 - 2*q^41 - 4*q^43 - 2*q^45 + 4*q^46 +
        12*q^47 - q^48 - 7*q^49 + O(q^50),
    q - 2*q^2 + q^3 + 2*q^4 + q^5 - 2*q^6 + 3*q^7 + q^9 - 2*q^10 - 3*q^11 +
        2*q^12 - 6*q^13 - 6*q^14 + q^15 - 4*q^16 + 3*q^17 - 2*q^18 - q^19 +
        2*q^20 + 3*q^21 + 6*q^22 + 4*q^23 - 4*q^25 + 12*q^26 + q^27 + 6*q^28 -
        10*q^29 - 2*q^30 + 2*q^31 + 8*q^32 - 3*q^33 - 6*q^34 + 3*q^35 + 2*q^36 +
        8*q^37 + 2*q^38 - 6*q^39 - 8*q^41 - 6*q^42 - q^43 - 6*q^44 + q^45 -
        8*q^46 + 3*q^47 - 4*q^48 + 2*q^49 + O(q^50),
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 - 6*q^20 + 2*q^21 + 4*q^25 + 4*q^27 + 2*q^28 +
        6*q^29 - 4*q^31 - 6*q^33 - 3*q^35 - 2*q^36 + 2*q^37 + 8*q^39 - 6*q^41 -
```

```
        q^43 - 6*q^44 + 3*q^45 - 3*q^47 - 8*q^48 - 6*q^49 +
        O(q^50),(ERASED,REPEATED 3 TIMES INSTEAD OF 2).
    q - q^2 - q^3 + q^4 + q^6 + 4*q^7 - q^8 + q^9 + 4*q^11 - q^12 - 4*q^14 +
        q^16 - 2*q^17 - q^18 + q^19 - 4*q^21 - 4*q^22 - 2*q^23 + q^24 - 5*q^25 -
        q^27 + 4*q^28 - 6*q^29 + 6*q^31 - q^32 - 4*q^33 + 2*q^34 + q^36 - 8*q^37
        - q^38 + 10*q^41 + 4*q^42 - 12*q^43 + 4*q^44 + 2*q^46 + 10*q^47 - q^48 +
        9*q^49 + O(q^50),
    q + q^2 + q^3 + q^4 + q^6 - 4*q^7 + q^8 + q^9 + q^12 - 4*q^13 - 4*q^14 +
        q^16 + 6*q^17 + q^18 + q^19 - 4*q^21 - 6*q^23 + q^24 - 5*q^25 - 4*q^26 +
        q^27 - 4*q^28 + 6*q^29 + 2*q^31 + q^32 + 6*q^34 + q^36 - 4*q^37 + q^38 -
        4*q^39 + 6*q^41 - 4*q^42 - 4*q^43 - 6*q^46 + 6*q^47 + q^48 + 9*q^49 +
        O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 19, 38, 38, 38, 57, 57, 57, 57, 114, 114 *]
```

remains: $57c$,$n(|a_5| \geq 2; 25) \geq 68 - 64$, $n(|a_7| \geq 4; 49) \geq 98 - 96$.$n(a_1 1 = -6; 121) = 236 - 216$

28.5. $X_0(114)/w_{38}$**, genus 6.**

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 +
        O(q^20),
    q + q^2 + q^3 - q^4 - 2*q^5 + q^6 - 3*q^8 + q^9 - 2*q^10 - q^12 + 6*q^13 -
        2*q^15 - q^16 - 6*q^17 + q^18 - q^19 + O(q^20),
    q - 2*q^2 + q^3 + 2*q^4 + q^5 - 2*q^6 + 3*q^7 + q^9 - 2*q^10 - 3*q^11 +
        2*q^12 - 6*q^13 - 6*q^14 + q^15 - 4*q^16 + 3*q^17 - 2*q^18 - q^19 +
        O(q^20),
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + q^6 - 4*q^7 + q^8 + q^9 + q^12 - 4*q^13 - 4*q^14 +
        q^16 + 6*q^17 + q^18 + q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
```

```
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 19, 57, 57, 57, 57, 114 *]
```

??? $n(a_7 = 0; 7) = 0$,$n(a_7 = -5; 49) = 82 - 78$.

28.6. $X_0(114)/w_{57}$**, genus 8.**

```
[*
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 - q^7 - q^8 - 2*q^9 - 6*q^11 + q^12 + 5*q^13 +
        q^14 + q^16 + 3*q^17 + 2*q^18 + q^19 + O(q^20),
    q + q^2 - q^3 + q^4 - 4*q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 - 4*q^10 + 2*q^11 -
        q^12 - q^13 + 3*q^14 + 4*q^15 + q^16 + 3*q^17 - 2*q^18 - q^19 + O(q^20),
    q - 2*q^3 - 2*q^4 + 3*q^5 - q^7 + q^9 + 3*q^11 + 4*q^12 - 4*q^13 - 6*q^15 +
        4*q^16 - 3*q^17 + q^19 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 +
        O(q^20),
    q + q^2 - q^3 + q^4 + 2*q^5 - q^6 + q^8 + q^9 + 2*q^10 - 4*q^11 - q^12 +
        2*q^13 - 2*q^15 + q^16 - 6*q^17 + q^18 - q^19 + O(q^20),
    q + q^2 + q^3 + q^4 + q^6 - 4*q^7 + q^8 + q^9 + q^12 - 4*q^13 - 4*q^14 +
        q^16 + 6*q^17 + q^18 + q^19 + O(q^20),
    q - 2*q^2 - q^3 + 2*q^4 - 3*q^5 + 2*q^6 - 5*q^7 + q^9 + 6*q^10 + q^11 -
        2*q^12 + 2*q^13 + 10*q^14 + 3*q^15 - 4*q^16 - q^17 - 2*q^18 - q^19 +
        O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 19, 38, 38, 38, 57, 114, 114, 114 *]
```

??$a_5 = 0, a_7 = -4$,$n(a_5 = 3; 5) = 8 - 6$, $n(a_5 = -4; 25) = 50 - 40$,$n(a_7 \geq -1; 7) \geq 22 - 18$, $n(|a_7| \geq 5; 49) \geq 84 - 78$.

29. $N = 102$

29.1. $X_0(102)/w_2$**, genus 7.**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + 2*q^20 + 4*q^23 - q^25 + 2*q^26 - 4*q^28
        + 6*q^29 + 4*q^31 - 5*q^32 - q^34 - 8*q^35 + 3*q^36 - 2*q^37 + 4*q^38 -
        6*q^40 - 6*q^41 + 4*q^43 + 6*q^45 - 4*q^46 + 9*q^49 + O(q^50),
```

```
    q + q^3 - 2*q^4 + 3*q^5 - 4*q^7 + q^9 - 3*q^11 - 2*q^12 - q^13 + 3*q^15 +
        4*q^16 - q^17 - q^19 - 6*q^20 - 4*q^21 + 9*q^23 + 4*q^25 + q^27 + 8*q^28
        + 6*q^29 + 2*q^31 - 3*q^33 - 12*q^35 - 2*q^36 - 4*q^37 - q^39 - 3*q^41 -
        7*q^43 + 6*q^44 + 3*q^45 - 6*q^47 + 4*q^48 + 9*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-a + 2)*q^4 + (-a + 1)*q^5 - a*q^6 + (a - 4)*q^8 + q^9 +
        (2*a - 4)*q^10 + (-a - 1)*q^11 + (a - 2)*q^12 + (a + 3)*q^13 + (a -
        1)*q^15 - 3*a*q^16 + q^17 + a*q^18 + (3*a + 3)*q^19 + (-4*a + 6)*q^20 -
        4*q^22 + (-a - 5)*q^23 + (-a + 4)*q^24 - 3*a*q^25 + (2*a + 4)*q^26 -
        q^27 + (4*a + 2)*q^29 + (-2*a + 4)*q^30 + (-2*a - 2)*q^31 + (a - 4)*q^32
        + (a + 1)*q^33 + a*q^34 + (-a + 2)*q^36 + 2*a*q^37 + 12*q^38 + (-a -
        3)*q^39 + (6*a - 8)*q^40 + (a - 1)*q^41 + (-3*a - 3)*q^43 + (-2*a +
        2)*q^44 + (-a + 1)*q^45 + (-4*a - 4)*q^46 + (2*a - 6)*q^47 + 3*a*q^48 -
        7*q^49 + O(q^50),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + 2*q^20 + 4*q^23 - q^25 + 2*q^26 - 4*q^28
        + 6*q^29 + 4*q^31 - 5*q^32 - q^34 - 8*q^35 + 3*q^36 - 2*q^37 + 4*q^38 -
        6*q^40 - 6*q^41 + 4*q^43 + 6*q^45 - 4*q^46 + 9*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 - 4*q^20 + 2*q^21
        + 6*q^23 + q^24 + 11*q^25 + 6*q^26 - q^27 - 2*q^28 - 4*q^29 - 4*q^30 -
        6*q^31 - q^32 + q^34 + 8*q^35 + q^36 - 4*q^37 - 4*q^38 + 6*q^39 + 4*q^40
        - 10*q^41 - 2*q^42 - 4*q^43 - 4*q^45 - 6*q^46 + 4*q^47 - q^48 - 3*q^49 +
        O(q^50),
    q - q^2 + q^3 + q^4 - q^6 + 2*q^7 - q^8 + q^9 + q^12 + 2*q^13 - 2*q^14 +
        q^16 - q^17 - q^18 - 4*q^19 + 2*q^21 - 6*q^23 - q^24 - 5*q^25 - 2*q^26 +
        q^27 + 2*q^28 - 10*q^31 - q^32 + q^34 + q^36 + 8*q^37 + 4*q^38 + 2*q^39
        + 6*q^41 - 2*q^42 - 4*q^43 + 6*q^46 + 12*q^47 + q^48 - 3*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 17, 51, 51, 51, 102, 102 *]
```

??? $n(a_5 = 3; 5) = 8 - 6$, $n(a_5 = -4; 25) = 50 - 40$.

29.2. $X_0(102)/w_3$**, genus 8.**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + 2*q^20 + 4*q^23 - q^25 + 2*q^26 - 4*q^28
        + 6*q^29 + 4*q^31 - 5*q^32 - q^34 - 8*q^35 + 3*q^36 - 2*q^37 + 4*q^38 -
        6*q^40 - 6*q^41 + 4*q^43 + 6*q^45 - 4*q^46 + 9*q^49 + O(q^50),
    q + q^2 - 2*q^3 + q^4 - 2*q^6 - 4*q^7 + q^8 + q^9 + 6*q^11 - 2*q^12 + 2*q^13
        - 4*q^14 + q^16 - q^17 + q^18 - 4*q^19 + 8*q^21 + 6*q^22 - 2*q^24 -
        5*q^25 + 2*q^26 + 4*q^27 - 4*q^28 - 4*q^31 + q^32 - 12*q^33 - q^34 +
        q^36 - 4*q^37 - 4*q^38 - 4*q^39 + 6*q^41 + 8*q^42 + 8*q^43 + 6*q^44 -
        2*q^48 + 9*q^49 + O(q^50),
```

```
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + 2*q^20 + 4*q^23 - q^25 + 2*q^26 - 4*q^28
        + 6*q^29 + 4*q^31 - 5*q^32 - q^34 - 8*q^35 + 3*q^36 - 2*q^37 + 4*q^38 -
        6*q^40 - 6*q^41 + 4*q^43 + 6*q^45 - 4*q^46 + 9*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-a + 2)*q^4 + (-a + 1)*q^5 - a*q^6 + (a - 4)*q^8 + q^9 +
        (2*a - 4)*q^10 + (-a - 1)*q^11 + (a - 2)*q^12 + (a + 3)*q^13 + (a -
        1)*q^15 - 3*a*q^16 + q^17 + a*q^18 + (3*a + 3)*q^19 + (-4*a + 6)*q^20 -
        4*q^22 + (-a - 5)*q^23 + (-a + 4)*q^24 - 3*a*q^25 + (2*a + 4)*q^26 -
        q^27 + (4*a + 2)*q^29 + (-2*a + 4)*q^30 + (-2*a - 2)*q^31 + (a - 4)*q^32
        + (a + 1)*q^33 + a*q^34 + (-a + 2)*q^36 + 2*a*q^37 + 12*q^38 + (-a -
        3)*q^39 + (6*a - 8)*q^40 + (a - 1)*q^41 + (-3*a - 3)*q^43 + (-2*a +
        2)*q^44 + (-a + 1)*q^45 + (-4*a - 4)*q^46 + (2*a - 6)*q^47 + 3*a*q^48 -
        7*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 - 4*q^20 + 2*q^21
        + 6*q^23 + q^24 + 11*q^25 + 6*q^26 - q^27 - 2*q^28 - 4*q^29 - 4*q^30 -
        6*q^31 - q^32 + q^34 + 8*q^35 + q^36 - 4*q^37 - 4*q^38 + 6*q^39 + 4*q^40
        - 10*q^41 - 2*q^42 - 4*q^43 - 4*q^45 - 6*q^46 + 4*q^47 - q^48 - 3*q^49 +
        O(q^50),
    q + a*q^2 - q^3 + (-a + 2)*q^4 + (-a + 1)*q^5 - a*q^6 + (a - 4)*q^8 + q^9 +
        (2*a - 4)*q^10 + (-a - 1)*q^11 + (a - 2)*q^12 + (a + 3)*q^13 + (a -
        1)*q^15 - 3*a*q^16 + q^17 + a*q^18 + (3*a + 3)*q^19 + (-4*a + 6)*q^20 -
        4*q^22 + (-a - 5)*q^23 + (-a + 4)*q^24 - 3*a*q^25 + (2*a + 4)*q^26 -
        q^27 + (4*a + 2)*q^29 + (-2*a + 4)*q^30 + (-2*a - 2)*q^31 + (a - 4)*q^32
        + (a + 1)*q^33 + a*q^34 + (-a + 2)*q^36 + 2*a*q^37 + 12*q^38 + (-a -
        3)*q^39 + (6*a - 8)*q^40 + (a - 1)*q^41 + (-3*a - 3)*q^43 + (-2*a +
        2)*q^44 + (-a + 1)*q^45 + (-4*a - 4)*q^46 + (2*a - 6)*q^47 + 3*a*q^48 -
        7*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field
*]
[* 17, 34, 34, 51, 102, 102 *]
```

Not bielliptic, $n(a_5 = -4; 25) = 56 - 40$, $n(a_7 = \pm 4, 49) = 110 - 96$

29.3. $X_0(102)/w_{17}$**, genus 6.**

```
[*
    q + q^2 - 2*q^3 + q^4 - 2*q^6 - 4*q^7 + q^8 + q^9 + 6*q^11 - 2*q^12 + 2*q^13
        - 4*q^14 + q^16 - q^17 + q^18 - 4*q^19 + 8*q^21 + 6*q^22 - 2*q^24 -
        5*q^25 + 2*q^26 + 4*q^27 - 4*q^28 - 4*q^31 + q^32 - 12*q^33 - q^34 +
        q^36 - 4*q^37 - 4*q^38 - 4*q^39 + 6*q^41 + 8*q^42 + 8*q^43 + 6*q^44 -
        2*q^48 + 9*q^49 + O(q^50),
    q + q^3 - 2*q^4 + 3*q^5 - 4*q^7 + q^9 - 3*q^11 - 2*q^12 - q^13 + 3*q^15 +
        4*q^16 - q^17 - q^19 - 6*q^20 - 4*q^21 + 9*q^23 + 4*q^25 + q^27 + 8*q^28
```

```
        + 6*q^29 + 2*q^31 - 3*q^33 - 12*q^35 - 2*q^36 - 4*q^37 - q^39 - 3*q^41 -
        7*q^43 + 6*q^44 + 3*q^45 - 6*q^47 + 4*q^48 + 9*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 - 4*q^20 + 2*q^21
        + 6*q^23 + q^24 + 11*q^25 + 6*q^26 - q^27 - 2*q^28 - 4*q^29 - 4*q^30 -
        6*q^31 - q^32 + q^34 + 8*q^35 + q^36 - 4*q^37 - 4*q^38 + 6*q^39 + 4*q^40
        - 10*q^41 - 2*q^42 - 4*q^43 - 4*q^45 - 6*q^46 + 4*q^47 - q^48 - 3*q^49 +
        O(q^50),
    q - q^2 + q^3 + q^4 - q^6 + 2*q^7 - q^8 + q^9 + q^12 + 2*q^13 - 2*q^14 +
        q^16 - q^17 - q^18 - 4*q^19 + 2*q^21 - 6*q^23 - q^24 - 5*q^25 - 2*q^26 +
        q^27 + 2*q^28 - 10*q^31 - q^32 + q^34 + q^36 + 8*q^37 + 4*q^38 + 2*q^39
        + 6*q^41 - 2*q^42 - 4*q^43 + 6*q^46 + 12*q^47 + q^48 - 3*q^49 + O(q^50),
    q + q^3 - 2*q^4 + 3*q^5 - 4*q^7 + q^9 - 3*q^11 - 2*q^12 - q^13 + 3*q^15 +
        4*q^16 - q^17 - q^19 - 6*q^20 - 4*q^21 + 9*q^23 + 4*q^25 + q^27 + 8*q^28
        + 6*q^29 + 2*q^31 - 3*q^33 - 12*q^35 - 2*q^36 - 4*q^37 - q^39 - 3*q^41 -
        7*q^43 + 6*q^44 + 3*q^45 - 6*q^47 + 4*q^48 + 9*q^49 + O(q^50),
    q + q^2 - 2*q^3 + q^4 - 2*q^6 - 4*q^7 + q^8 + q^9 + 6*q^11 - 2*q^12 + 2*q^13
        - 4*q^14 + q^16 - q^17 + q^18 - 4*q^19 + 8*q^21 + 6*q^22 - 2*q^24 -
        5*q^25 + 2*q^26 + 4*q^27 - 4*q^28 - 4*q^31 + q^32 - 12*q^33 - q^34 +
        q^36 - 4*q^37 - 4*q^38 - 4*q^39 + 6*q^41 + 8*q^42 + 8*q^43 + 6*q^44 -
        2*q^48 + 9*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 34, 51, 102, 102, 102, 102 *]
```

?? $n(a_7 \geq -2; 7) \geq 24 - 20$.

29.4. $X_0(102)/w_6$**, genus 8.**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + 2*q^20 + 4*q^23 - q^25 + 2*q^26 - 4*q^28
        + 6*q^29 + 4*q^31 - 5*q^32 - q^34 - 8*q^35 + 3*q^36 - 2*q^37 + 4*q^38 -
        6*q^40 - 6*q^41 + 4*q^43 + 6*q^45 - 4*q^46 + 9*q^49 + O(q^50),
    q + q^2 - 2*q^3 + q^4 - 2*q^6 - 4*q^7 + q^8 + q^9 + 6*q^11 - 2*q^12 + 2*q^13
        - 4*q^14 + q^16 - q^17 + q^18 - 4*q^19 + 8*q^21 + 6*q^22 - 2*q^24 -
        5*q^25 + 2*q^26 + 4*q^27 - 4*q^28 - 4*q^31 + q^32 - 12*q^33 - q^34 +
        q^36 - 4*q^37 - 4*q^38 - 4*q^39 + 6*q^41 + 8*q^42 + 8*q^43 + 6*q^44 -
        2*q^48 + 9*q^49 + O(q^50),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + 2*q^20 + 4*q^23 - q^25 + 2*q^26 - 4*q^28
        + 6*q^29 + 4*q^31 - 5*q^32 - q^34 - 8*q^35 + 3*q^36 - 2*q^37 + 4*q^38 -
        6*q^40 - 6*q^41 + 4*q^43 + 6*q^45 - 4*q^46 + 9*q^49 + O(q^50),
    q + q^3 - 2*q^4 + 3*q^5 - 4*q^7 + q^9 - 3*q^11 - 2*q^12 - q^13 + 3*q^15 +
        4*q^16 - q^17 - q^19 - 6*q^20 - 4*q^21 + 9*q^23 + 4*q^25 + q^27 + 8*q^28
```

```
        + 6*q^29 + 2*q^31 - 3*q^33 - 12*q^35 - 2*q^36 - 4*q^37 - q^39 - 3*q^41 -
        7*q^43 + 6*q^44 + 3*q^45 - 6*q^47 + 4*q^48 + 9*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-a + 2)*q^4 + (-a + 1)*q^5 - a*q^6 + (a - 4)*q^8 + q^9 +
        (2*a - 4)*q^10 + (-a - 1)*q^11 + (a - 2)*q^12 + (a + 3)*q^13 + (a -
        1)*q^15 - 3*a*q^16 + q^17 + a*q^18 + (3*a + 3)*q^19 + (-4*a + 6)*q^20 -
        4*q^22 + (-a - 5)*q^23 + (-a + 4)*q^24 - 3*a*q^25 + (2*a + 4)*q^26 -
        q^27 + (4*a + 2)*q^29 + (-2*a + 4)*q^30 + (-2*a - 2)*q^31 + (a - 4)*q^32
        + (a + 1)*q^33 + a*q^34 + (-a + 2)*q^36 + 2*a*q^37 + 12*q^38 + (-a -
        3)*q^39 + (6*a - 8)*q^40 + (a - 1)*q^41 + (-3*a - 3)*q^43 + (-2*a +
        2)*q^44 + (-a + 1)*q^45 + (-4*a - 4)*q^46 + (2*a - 6)*q^47 + 3*a*q^48 -
        7*q^49 + O(q^50),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + 2*q^20 + 4*q^23 - q^25 + 2*q^26 - 4*q^28
        + 6*q^29 + 4*q^31 - 5*q^32 - q^34 - 8*q^35 + 3*q^36 - 2*q^37 + 4*q^38 -
        6*q^40 - 6*q^41 + 4*q^43 + 6*q^45 - 4*q^46 + 9*q^49 +
        O(q^50),(ERASED)
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 - 4*q^20 + 2*q^21
        + 6*q^23 + q^24 + 11*q^25 + 6*q^26 - q^27 - 2*q^28 - 4*q^29 - 4*q^30 -
        6*q^31 - q^32 + q^34 + 8*q^35 + q^36 - 4*q^37 - 4*q^38 + 6*q^39 + 4*q^40
        - 10*q^41 - 2*q^42 - 4*q^43 - 4*q^45 - 6*q^46 + 4*q^47 - q^48 - 3*q^49 +
        O(q^50),
    q + q^2 + q^3 + q^4 - 2*q^5 + q^6 + q^8 + q^9 - 2*q^10 - 4*q^11 + q^12 -
        2*q^13 - 2*q^15 + q^16 + q^17 + q^18 + 4*q^19 - 2*q^20 - 4*q^22 + q^24 -
        q^25 - 2*q^26 + q^27 - 10*q^29 - 2*q^30 + 8*q^31 + q^32 - 4*q^33 + q^34
        + q^36 - 2*q^37 + 4*q^38 - 2*q^39 - 2*q^40 + 10*q^41 + 12*q^43 - 4*q^44
        - 2*q^45 + q^48 - 7*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 17, 34, 34, 51, 51, 51, 102, 102 *]
```

Remains 102b ,$n(|a_5| \geq 3; 25) \geq 56 - 54$, $n(17, repeated, a_7 = 4; 7) = 10 - 8$, $n(34, a_{11}, 11) = 14 - 12$

29.5. $X_0(102)/w_{34}$**, genus 8.**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + 2*q^20 + 4*q^23 - q^25 + 2*q^26 - 4*q^28
        + 6*q^29 + 4*q^31 - 5*q^32 - q^34 - 8*q^35 + 3*q^36 - 2*q^37 + 4*q^38 -
        6*q^40 - 6*q^41 + 4*q^43 + 6*q^45 - 4*q^46 + 9*q^49 + O(q^50),
    q + q^3 - 2*q^4 + 3*q^5 - 4*q^7 + q^9 - 3*q^11 - 2*q^12 - q^13 + 3*q^15 +
        4*q^16 - q^17 - q^19 - 6*q^20 - 4*q^21 + 9*q^23 + 4*q^25 + q^27 + 8*q^28
        + 6*q^29 + 2*q^31 - 3*q^33 - 12*q^35 - 2*q^36 - 4*q^37 - q^39 - 3*q^41 -
```

```
        7*q^43 + 6*q^44 + 3*q^45 - 6*q^47 + 4*q^48 + 9*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-a + 2)*q^4 + (-a + 1)*q^5 - a*q^6 + (a - 4)*q^8 + q^9 +
        (2*a - 4)*q^10 + (-a - 1)*q^11 + (a - 2)*q^12 + (a + 3)*q^13 + (a -
        1)*q^15 - 3*a*q^16 + q^17 + a*q^18 + (3*a + 3)*q^19 + (-4*a + 6)*q^20 -
        4*q^22 + (-a - 5)*q^23 + (-a + 4)*q^24 - 3*a*q^25 + (2*a + 4)*q^26 -
        q^27 + (4*a + 2)*q^29 + (-2*a + 4)*q^30 + (-2*a - 2)*q^31 + (a - 4)*q^32
        + (a + 1)*q^33 + a*q^34 + (-a + 2)*q^36 + 2*a*q^37 + 12*q^38 + (-a -
        3)*q^39 + (6*a - 8)*q^40 + (a - 1)*q^41 + (-3*a - 3)*q^43 + (-2*a +
        2)*q^44 + (-a + 1)*q^45 + (-4*a - 4)*q^46 + (2*a - 6)*q^47 + 3*a*q^48 -
        7*q^49 + O(q^50),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + 2*q^20 + 4*q^23 - q^25 + 2*q^26 - 4*q^28
        + 6*q^29 + 4*q^31 - 5*q^32 - q^34 - 8*q^35 + 3*q^36 - 2*q^37 + 4*q^38 -
        6*q^40 - 6*q^41 + 4*q^43 + 6*q^45 - 4*q^46 + 9*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 - 4*q^20 + 2*q^21
        + 6*q^23 + q^24 + 11*q^25 + 6*q^26 - q^27 - 2*q^28 - 4*q^29 - 4*q^30 -
        6*q^31 - q^32 + q^34 + 8*q^35 + q^36 - 4*q^37 - 4*q^38 + 6*q^39 + 4*q^40
        - 10*q^41 - 2*q^42 - 4*q^43 - 4*q^45 - 6*q^46 + 4*q^47 - q^48 - 3*q^49 +
        O(q^50),
    q - q^2 + q^3 + q^4 - q^6 + 2*q^7 - q^8 + q^9 + q^12 + 2*q^13 - 2*q^14 +
        q^16 - q^17 - q^18 - 4*q^19 + 2*q^21 - 6*q^23 - q^24 - 5*q^25 - 2*q^26 +
        q^27 + 2*q^28 - 10*q^31 - q^32 + q^34 + q^36 + 8*q^37 + 4*q^38 + 2*q^39
        + 6*q^41 - 2*q^42 - 4*q^43 + 6*q^46 + 12*q^47 + q^48 - 3*q^49 + O(q^50),
    q + q^2 + q^3 + q^4 - 2*q^5 + q^6 + q^8 + q^9 - 2*q^10 - 4*q^11 + q^12 -
        2*q^13 - 2*q^15 + q^16 + q^17 + q^18 + 4*q^19 - 2*q^20 - 4*q^22 + q^24 -
        q^25 - 2*q^26 + q^27 - 10*q^29 - 2*q^30 + 8*q^31 + q^32 - 4*q^33 + q^34
        + q^36 - 2*q^37 + 4*q^38 - 2*q^39 - 2*q^40 + 10*q^41 + 12*q^43 - 4*q^44
        - 2*q^45 + q^48 - 7*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 17, 51, 51, 51, 102, 102, 102 *]
```

??? $n(|a_5| \geq 3; 25) \geq 56 - 54$, $n(|a_7| \geq 4; 49) \geq 106 - 96$.

29.6. $X_0(102)/w_{51}$**, genus 5.**

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + 2*q^20 + 4*q^23 - q^25 + 2*q^26 - 4*q^28
        + 6*q^29 + 4*q^31 - 5*q^32 - q^34 - 8*q^35 + 3*q^36 - 2*q^37 + 4*q^38 -
        6*q^40 - 6*q^41 + 4*q^43 + 6*q^45 - 4*q^46 + 9*q^49 + O(q^50),
    q + q^2 - 2*q^3 + q^4 - 2*q^6 - 4*q^7 + q^8 + q^9 + 6*q^11 - 2*q^12 + 2*q^13
        - 4*q^14 + q^16 - q^17 + q^18 - 4*q^19 + 8*q^21 + 6*q^22 - 2*q^24 -
```

```
        5*q^25 + 2*q^26 + 4*q^27 - 4*q^28 - 4*q^31 + q^32 - 12*q^33 - q^34 +
        q^36 - 4*q^37 - 4*q^38 - 4*q^39 + 6*q^41 + 8*q^42 + 8*q^43 + 6*q^44 -
        2*q^48 + 9*q^49 + O(q^50),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + 2*q^20 + 4*q^23 - q^25 + 2*q^26 - 4*q^28
        + 6*q^29 + 4*q^31 - 5*q^32 - q^34 - 8*q^35 + 3*q^36 - 2*q^37 + 4*q^38 -
        6*q^40 - 6*q^41 + 4*q^43 + 6*q^45 - 4*q^46 + 9*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 - 4*q^20 + 2*q^21
        + 6*q^23 + q^24 + 11*q^25 + 6*q^26 - q^27 - 2*q^28 - 4*q^29 - 4*q^30 -
        6*q^31 - q^32 + q^34 + 8*q^35 + q^36 - 4*q^37 - 4*q^38 + 6*q^39 + 4*q^40
        - 10*q^41 - 2*q^42 - 4*q^43 - 4*q^45 - 6*q^46 + 4*q^47 - q^48 - 3*q^49 +
        O(q^50),
    q + q^2 + q^3 + q^4 - 2*q^5 + q^6 + q^8 + q^9 - 2*q^10 - 4*q^11 + q^12 -
        2*q^13 - 2*q^15 + q^16 + q^17 + q^18 + 4*q^19 - 2*q^20 - 4*q^22 + q^24 -
        q^25 - 2*q^26 + q^27 - 10*q^29 - 2*q^30 + 8*q^31 + q^32 - 4*q^33 + q^34
        + q^36 - 2*q^37 + 4*q^38 - 2*q^39 - 2*q^40 + 10*q^41 + 12*q^43 - 4*q^44
        - 2*q^45 + q^48 - 7*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 17, 34, 34, 102, 102 *]
```

???? $n(a_5 \geq -1; 5) \geq 16 - 14$, $n(|a_5| \geq 4; 25) \geq 48 - 40$.

30. $N = 110$

30.1. $X_0(110)/w_{22}$, genus 8.

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + 2*q^20 +
        2*q^21 - 2*q^22 - q^23 - 4*q^25 - 8*q^26 + 5*q^27 - 4*q^28 + 2*q^30 +
        7*q^31 + 8*q^32 - q^33 + 4*q^34 - 2*q^35 - 4*q^36 + 3*q^37 - 4*q^39 -
        8*q^41 - 4*q^42 - 6*q^43 + 2*q^44 - 2*q^45 + 2*q^46 + 8*q^47 + 4*q^48 -
        3*q^49 + O(q^50),
    q + q^2 - q^4 + q^5 - 3*q^8 - 3*q^9 + q^10 - q^11 + 2*q^13 - q^16 + 6*q^17 -
        3*q^18 - 4*q^19 - q^20 - q^22 + 4*q^23 + q^25 + 2*q^26 + 6*q^29 - 8*q^31
        + 5*q^32 + 6*q^34 + 3*q^36 - 2*q^37 - 4*q^38 - 3*q^40 + 2*q^41 + 4*q^43
        + q^44 - 3*q^45 + 4*q^46 - 12*q^47 - 7*q^49 + O(q^50),
    q + a*q^2 + (-2*a + 2)*q^3 + (2*a - 1)*q^4 - q^5 + (-2*a - 2)*q^6 - 2*q^7 +
        (a + 2)*q^8 + 5*q^9 - a*q^10 + q^11 + (-2*a - 6)*q^12 + (2*a - 6)*q^13 -
        2*a*q^14 + (2*a - 2)*q^15 + 3*q^16 + (2*a + 2)*q^17 + 5*a*q^18 + (-2*a +
        1)*q^20 + (4*a - 4)*q^21 + a*q^22 + (-2*a + 2)*q^23 + (-6*a + 2)*q^24 +
        q^25 + (-2*a + 2)*q^26 + (-4*a + 4)*q^27 + (-4*a + 2)*q^28 + (-4*a +
        6)*q^29 + (2*a + 2)*q^30 + (a - 4)*q^32 + (-2*a + 2)*q^33 + (6*a +
        2)*q^34 + 2*q^35 + (10*a - 5)*q^36 + (-4*a + 2)*q^37 + (8*a - 16)*q^39 +
```

```
        (-a - 2)*q^40 + 6*q^41 + (4*a + 4)*q^42 - 6*q^43 + (2*a - 1)*q^44 -
        5*q^45 + (-2*a - 2)*q^46 + (2*a - 2)*q^47 + (-6*a + 6)*q^48 - 3*q^49 +
        O(q^50),
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + 2*q^20 +
        2*q^21 - 2*q^22 - q^23 - 4*q^25 - 8*q^26 + 5*q^27 - 4*q^28 + 2*q^30 +
        7*q^31 + 8*q^32 - q^33 + 4*q^34 - 2*q^35 - 4*q^36 + 3*q^37 - 4*q^39 -
        8*q^41 - 4*q^42 - 6*q^43 + 2*q^44 - 2*q^45 + 2*q^46 + 8*q^47 + 4*q^48 -
        3*q^49 + O(q^50),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 + q^10 + q^11 - q^12 -
        6*q^13 + 3*q^14 - q^15 + q^16 - 7*q^17 - 2*q^18 + 5*q^19 + q^20 - 3*q^21
        + q^22 - 6*q^23 - q^24 + q^25 - 6*q^26 + 5*q^27 + 3*q^28 + 5*q^29 - q^30
        - 3*q^31 + q^32 - q^33 - 7*q^34 + 3*q^35 - 2*q^36 + 3*q^37 + 5*q^38 +
        6*q^39 + q^40 + 2*q^41 - 3*q^42 + 4*q^43 + q^44 - 2*q^45 - 6*q^46 -
        2*q^47 - q^48 + 2*q^49 + O(q^50),
    q - q^2 + a*q^3 + q^4 + q^5 - a*q^6 - a*q^7 - q^8 + (-a + 5)*q^9 - q^10 -
        q^11 + a*q^12 + 2*q^13 + a*q^14 + a*q^15 + q^16 + (-a - 2)*q^17 + (a -
        5)*q^18 + (a + 4)*q^19 + q^20 + (a - 8)*q^21 + q^22 + (-2*a - 4)*q^23 -
        a*q^24 + q^25 - 2*q^26 + (3*a - 8)*q^27 - a*q^28 + (-a - 2)*q^29 -
        a*q^30 - a*q^31 - q^32 - a*q^33 + (a + 2)*q^34 - a*q^35 + (-a + 5)*q^36
        + (-a + 6)*q^37 + (-a - 4)*q^38 + 2*a*q^39 - q^40 + (4*a + 2)*q^41 + (-a
        + 8)*q^42 - 4*q^43 - q^44 + (-a + 5)*q^45 + (2*a + 4)*q^46 + (-2*a -
        4)*q^47 + a*q^48 + (-a + 1)*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 8 over the Rational Field
*]
[* 11, 55, 55, 55, 110, 110 *]
```

???

CHAPTER 3

The modular curves $X_0(p_1p_2p_3p_4)/W$

1. Jacobian decomposition of $X_0(p_1p_2p_3p_4)/W$

We are interested with $|W| = 8$ first. Fix a level, there are 15 subgroups of order 8 which are denoted by

```
H1:=[*p1,p2,p3,p1*p2,p1*p3,p2*p3,p1*p2*p3*];
H2:=[*p1,p2,p4,p1*p2,p1*p4,p2*p4,p1*p2*p4*];
H3:=[*p4,p2,p3,p4*p2,p4*p3,p2*p3,p4*p2*p3*];
H4:=[*p1,p4,p3,p1*p4,p1*p3,p4*p3,p1*p4*p3*];
H5:=[*p1*p2,p3*p4,p2*p3,p1*p4,p2*p4,p1*p3,p1*p2*p3*p4*];
H6:=[*p1*p2*p3,p1,p2*p3,p4,p1*p2*p3*p4,p1*p4,p2*p3*p4*];
H7:=[*p1*p2*p3,p2,p1*p3,p4,p1*p2*p3*p4,p2*p4,p1*p3*p4*];
H8:=[*p1*p2*p3,p3,p1*p2,p4,p1*p2*p3*p4,p3*p4,p1*p2*p4*];
H9:=[*p1*p3*p4,p1,p3*p4,p2,p1*p2*p3*p4,p1*p2,p2*p3*p4*];
H10:=[*p1*p3*p4,p3,p1*p4,p2,p1*p2*p3*p4,p2*p3,p1*p2*p4*];
H11:=[*p2*p3*p4,p3,p2*p4,p1,p1*p2*p3*p4,p1*p3,p1*p2*p4*];
H12:=[*p1*p2*p3,p2*p3*p4,p1*p3*p4,p2*p4,p1*p4,p1*p2,p3*];
H13:=[*p1*p2*p3,p2*p3*p4,p2*p1*p4,p1*p4,p3*p4,p1*p3,p2*];
H14:=[*p1*p2*p3,p1*p3*p4,p1*p2*p4,p2*p3,p2*p4,p3*p4,p1*];
H15:=[*p1*p2*p4,p2*p3*p4,p1*p3*p4,p1*p2,p1*p3,p2*p3,p4*];
```

1.1. $N = 210$**.** All $H1, \ldots, H15$ satisfy that $X_0(N)/Hi$ is bielliptic curve because $X_0^*(210)$ is an elliptic curve.

Need to study the 35 subgroups of order 4 in $B(N)$ and its Jacobian decomposition first, for such level. Such notation for order 4 groups is as follows with $p_1 < p_2 < p_3 < p_4$:

```
N1:=[*p1,p2,p1*p2*]; N2:=[*p1,p3,p1*p3*]; N3:=[*p1,p4,p4*p1*];
N4:=[*p2,p3,p2*p3*]; N5:=[*p2,p4,p2*p4*]; N6:=[*p3,p4,p3*p4*];
N7:=[*p1,p2*p3,p1*p2*p3*]; N8:=[*p1,p2*p4,p1*p2*p4*];
N9:=[*p1,p3*p4,p1*p3*p4*]; N10:=[*p1,p2*p3*p4,p1*p2*p3*p4*];
N11:=[*p2,p1*p3,p1*p2*p3*]; N12:=[*p2,p1*p4,p1*p2*p4*];
N13:=[*p2,p3*p4,p2*p3*p4*]; N14:=[*p2,p1*p3*p4,p1*p2*p3*p4*];
N15:=[*p3,p2*p1,p1*p2*p3*]; N16:=[*p3,p2*p4,p3*p2*p4*];
N17:=[*p3,p1*p4,p1*p3*p4*]; N18:=[*p3,p2*p1*p4,p1*p2*p3*p4*];
N19:=[*p4,p2*p3,p4*p2*p3*]; N20:=[*p4,p2*p1,p1*p2*p4*];
N21:=[*p4,p3*p1,p1*p3*p4*]; N22:=[*p4,p2*p3*p1,p1*p2*p3*p4*];
N23:=[*p1*p2,p1*p3,p2*p3*]; N24:=[*p1*p4,p1*p2,p2*p4*];
N25:=[*p1*p4,p1*p3,p3*p4*]; N26:=[*p2*p3,p3*p4,p2*p4*];
N27:=[*p1*p2,p3*p4,p1*p2*p3*p4*]; N28:=[*p1*p3,p2*p4,p1*p2*p3*p4*];
N29:=[*p1*p4,p2*p3,p1*p2*p3*p4*]; N30:=[*p1*p2,p1*p3*p4,p2*p3*p4*];
N31:=[*p1*p3,p1*p2*p4,p3*p2*p4*]; N32:=[*p1*p4,p1*p2*p3,p4*p2*p3*];
N33:=[*p2*p3,p2*p1*p4,p3*p1*p4*]; N34:=[*p2*p4,p2*p1*p3,p4*p1*p3*];
N35:=[*p3*p4,p3*p2*p1,p4*p2*p1*];
```

1.2. $N = 330$**.**

1.2.1. $H1$, *genus 8.*

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + 2*q^20 +
        2*q^21 - 2*q^22 - q^23 - 4*q^25 - 8*q^26 + 5*q^27 - 4*q^28 + 2*q^30 +
        7*q^31 + 8*q^32 - q^33 + 4*q^34 - 2*q^35 - 4*q^36 + 3*q^37 - 4*q^39 -
        8*q^41 - 4*q^42 - 6*q^43 + 2*q^44 - 2*q^45 + 2*q^46 + 8*q^47 + 4*q^48 -
        3*q^49 + O(q^50),
    q + q^2 - q^3 - q^4 - 2*q^5 - q^6 + 4*q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 +
        q^12 - 2*q^13 + 4*q^14 + 2*q^15 - q^16 - 2*q^17 + q^18 + 2*q^20 - 4*q^21
        + q^22 + 8*q^23 + 3*q^24 - q^25 - 2*q^26 - q^27 - 4*q^28 - 6*q^29 +
        2*q^30 - 8*q^31 + 5*q^32 - q^33 - 2*q^34 - 8*q^35 - q^36 + 6*q^37 +
        2*q^39 + 6*q^40 - 2*q^41 - 4*q^42 - q^44 - 2*q^45 + 8*q^46 + 8*q^47 +
        q^48 + 9*q^49 + O(q^50),
    q + a*q^2 + (-2*a + 2)*q^3 + (2*a - 1)*q^4 - q^5 + (-2*a - 2)*q^6 - 2*q^7 +
        (a + 2)*q^8 + 5*q^9 - a*q^10 + q^11 + (-2*a - 6)*q^12 + (2*a - 6)*q^13 -
        2*a*q^14 + (2*a - 2)*q^15 + 3*q^16 + (2*a + 2)*q^17 + 5*a*q^18 + (-2*a +
        1)*q^20 + (4*a - 4)*q^21 + a*q^22 + (-2*a + 2)*q^23 + (-6*a + 2)*q^24 +
        q^25 + (-2*a + 2)*q^26 + (-4*a + 4)*q^27 + (-4*a + 2)*q^28 + (-4*a +
        6)*q^29 + (2*a + 2)*q^30 + (a - 4)*q^32 + (-2*a + 2)*q^33 + (6*a +
        2)*q^34 + 2*q^35 + (10*a - 5)*q^36 + (-4*a + 2)*q^37 + (8*a - 16)*q^39 +
        (-a - 2)*q^40 + 6*q^41 + (4*a + 4)*q^42 - 6*q^43 + (2*a - 1)*q^44 -
        5*q^45 + (-2*a - 2)*q^46 + (2*a - 2)*q^47 + (-6*a + 6)*q^48 - 3*q^49 +
        O(q^50),
    q - q^2 + q^3 + q^4 - q^5 - q^6 + 5*q^7 - q^8 - 2*q^9 + q^10 + q^11 + q^12 +
        2*q^13 - 5*q^14 - q^15 + q^16 + 3*q^17 + 2*q^18 - 7*q^19 - q^20 + 5*q^21
        - q^22 - 6*q^23 - q^24 + q^25 - 2*q^26 - 5*q^27 + 5*q^28 - 3*q^29 + q^30
        - 7*q^31 - q^32 + q^33 - 3*q^34 - 5*q^35 - 2*q^36 - 7*q^37 + 7*q^38 +
        2*q^39 + q^40 + 6*q^41 - 5*q^42 + 8*q^43 + q^44 + 2*q^45 + 6*q^46 +
        6*q^47 + q^48 + 18*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + (2*a +
        1)*q^20 + (2*a + 4)*q^21 - a*q^22 - 4*q^23 + (-a + 2)*q^24 + q^25 +
        (-4*a + 4)*q^26 - q^27 + (2*a + 8)*q^28 + 2*a*q^29 + a*q^30 + (a +
        4)*q^32 + q^33 + (-2*a - 2)*q^34 + (2*a + 4)*q^35 + (-2*a - 1)*q^36 +
        (-4*a + 2)*q^37 + (-6*a + 2)*q^38 + (-4*a - 4)*q^39 + (-a + 2)*q^40 -
        2*a*q^41 + 2*q^42 + (2*a - 4)*q^43 + (2*a + 1)*q^44 - q^45 - 4*a*q^46 -
        4*q^47 - 3*q^48 + (8*a + 13)*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 + q^11 - q^12 + 2*q^13 +
        q^15 + q^16 - 2*q^17 - q^18 + 8*q^19 - q^20 - q^22 + 4*q^23 + q^24 +
        q^25 - 2*q^26 - q^27 + 2*q^29 - q^30 + 8*q^31 - q^32 - q^33 + 2*q^34 +
        q^36 - 2*q^37 - 8*q^38 - 2*q^39 + q^40 + 6*q^41 + 8*q^43 + q^44 - q^45 -
        4*q^46 - 4*q^47 - q^48 - 7*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 1 over the Rational Field,
```

```
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field
*]
[* 11, 33, 55, 110, 165, 330 *]
```

11,330??, $n(a_7 \geq 4; 7) \geq 9 - 8$.

1.2.2. *H*2*, genus 6.*

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 + O(q^50),
    q + q^2 - q^4 + q^5 - 3*q^8 - 3*q^9 + q^10 - q^11 + 2*q^13 - q^16 + 6*q^17 -
        3*q^18 - 4*q^19 - q^20 - q^22 + 4*q^23 + q^25 + 2*q^26 + 6*q^29 - 8*q^31
        + 5*q^32 + 6*q^34 + 3*q^36 - 2*q^37 - 4*q^38 - 3*q^40 + 2*q^41 + 4*q^43
        + q^44 - 3*q^45 + 4*q^46 - 12*q^47 - 7*q^49 + O(q^50),
    q - q^2 + a*q^3 + q^4 + q^5 - a*q^6 - a*q^7 - q^8 + (-a + 5)*q^9 - q^10 -
        q^11 + a*q^12 + 2*q^13 + a*q^14 + a*q^15 + q^16 + (-a - 2)*q^17 + (a -
        5)*q^18 + (a + 4)*q^19 + q^20 + (a - 8)*q^21 + q^22 + (-2*a - 4)*q^23 -
        a*q^24 + q^25 - 2*q^26 + (3*a - 8)*q^27 - a*q^28 + (-a - 2)*q^29 -
        a*q^30 - a*q^31 - q^32 - a*q^33 + (a + 2)*q^34 - a*q^35 + (-a + 5)*q^36
        + (-a + 6)*q^37 + (-a - 4)*q^38 + 2*a*q^39 - q^40 + (4*a + 2)*q^41 + (-a
        + 8)*q^42 - 4*q^43 - q^44 + (-a + 5)*q^45 + (2*a + 4)*q^46 + (-2*a -
        4)*q^47 + a*q^48 + (-a + 1)*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + (2*a +
        1)*q^20 + (2*a + 4)*q^21 - a*q^22 - 4*q^23 + (-a + 2)*q^24 + q^25 +
        (-4*a + 4)*q^26 - q^27 + (2*a + 8)*q^28 + 2*a*q^29 + a*q^30 + (a +
        4)*q^32 + q^33 + (-2*a - 2)*q^34 + (2*a + 4)*q^35 + (-2*a - 1)*q^36 +
        (-4*a + 2)*q^37 + (-6*a + 2)*q^38 + (-4*a - 4)*q^39 + (-a + 2)*q^40 -
        2*a*q^41 + 2*q^42 + (2*a - 4)*q^43 + (2*a + 1)*q^44 - q^45 - 4*a*q^46 -
        4*q^47 - 3*q^48 + (8*a + 13)*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 8 over the Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 15, 55, 110, 165 *]
```

15,55?

1.2.3. *H*3*, genus 7.*

```
[*
    q + q^2 - q^3 + q^4 + 2*q^5 - q^6 - 4*q^7 + q^8 + q^9 + 2*q^10 - q^11 - q^12
        - 6*q^13 - 4*q^14 - 2*q^15 + q^16 + 2*q^17 + q^18 + 4*q^19 + 2*q^20 +
        4*q^21 - q^22 + 4*q^23 - q^24 - q^25 - 6*q^26 - q^27 - 4*q^28 + 6*q^29 -
        2*q^30 + q^32 + q^33 + 2*q^34 - 8*q^35 + q^36 + 6*q^37 + 4*q^38 + 6*q^39
```

```
        + 2*q^40 - 6*q^41 + 4*q^42 + 4*q^43 - q^44 + 2*q^45 + 4*q^46 - 12*q^47 -
        q^48 + 9*q^49 + O(q^50),
    q + q^2 + q^3 + q^4 - q^5 + q^6 - q^7 + q^8 - 2*q^9 - q^10 - q^11 + q^12 +
        2*q^13 - q^14 - q^15 + q^16 - 3*q^17 - 2*q^18 - q^19 - q^20 - q^21 -
        q^22 + 6*q^23 + q^24 + q^25 + 2*q^26 - 5*q^27 - q^28 - 9*q^29 - q^30 +
        5*q^31 + q^32 - q^33 - 3*q^34 + q^35 - 2*q^36 + 5*q^37 - q^38 + 2*q^39 -
        q^40 - 6*q^41 - q^42 + 8*q^43 - q^44 + 2*q^45 + 6*q^46 + 6*q^47 + q^48 -
        6*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + (2*a +
        1)*q^20 + (2*a + 4)*q^21 - a*q^22 - 4*q^23 + (-a + 2)*q^24 + q^25 +
        (-4*a + 4)*q^26 - q^27 + (2*a + 8)*q^28 + 2*a*q^29 + a*q^30 + (a +
        4)*q^32 + q^33 + (-2*a - 2)*q^34 + (2*a + 4)*q^35 + (-2*a - 1)*q^36 +
        (-4*a + 2)*q^37 + (-6*a + 2)*q^38 + (-4*a - 4)*q^39 + (-a + 2)*q^40 -
        2*a*q^41 + 2*q^42 + (2*a - 4)*q^43 + (2*a + 1)*q^44 - q^45 - 4*a*q^46 -
        4*q^47 - 3*q^48 + (8*a + 13)*q^49 + O(q^50),
    q + q^2 - q^3 + q^4 - q^5 - q^6 + 4*q^7 + q^8 + q^9 - q^10 - q^11 - q^12 +
        2*q^13 + 4*q^14 + q^15 + q^16 + 2*q^17 + q^18 + 4*q^19 - q^20 - 4*q^21 -
        q^22 - 4*q^23 - q^24 + q^25 + 2*q^26 - q^27 + 4*q^28 + 6*q^29 + q^30 +
        q^32 + q^33 + 2*q^34 - 4*q^35 + q^36 - 10*q^37 + 4*q^38 - 2*q^39 - q^40
        - 6*q^41 - 4*q^42 - 12*q^43 - q^44 - q^45 - 4*q^46 - 4*q^47 - q^48 +
        9*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + (2*a +
        1)*q^20 + (2*a + 4)*q^21 - a*q^22 - 4*q^23 + (-a + 2)*q^24 + q^25 +
        (-4*a + 4)*q^26 - q^27 + (2*a + 8)*q^28 + 2*a*q^29 + a*q^30 + (a +
        4)*q^32 + q^33 + (-2*a - 2)*q^34 + (2*a + 4)*q^35 + (-2*a - 1)*q^36 +
        (-4*a + 2)*q^37 + (-6*a + 2)*q^38 + (-4*a - 4)*q^39 + (-a + 2)*q^40 -
        2*a*q^41 + 2*q^42 + (2*a - 4)*q^43 + (2*a + 1)*q^44 - q^45 - 4*a*q^46 -
        4*q^47 - 3*q^48 + (8*a + 13)*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 66, 110, 165, 330, 330 *]
```

66,110? $n(a_7 \geq 0; 7) \geq 9 - 8$.

1.2.4. *H4, genus 6.*

```
[*
    q - q^2 + q^3 + q^4 - q^5 - q^6 - 4*q^7 - q^8 + q^9 + q^10 + q^12 + 2*q^13 +
        4*q^14 - q^15 + q^16 + 6*q^17 - q^18 - 4*q^19 - q^20 - 4*q^21 - q^24 +
        q^25 - 2*q^26 + q^27 - 4*q^28 - 6*q^29 + q^30 + 8*q^31 - q^32 - 6*q^34 +
        4*q^35 + q^36 + 2*q^37 + 4*q^38 + 2*q^39 + q^40 - 6*q^41 + 4*q^42 -
        4*q^43 - q^45 + q^48 + 9*q^49 + O(q^50),
```

```
    q - q^2 + q^3 + q^4 - q^6 + 2*q^7 - q^8 + q^9 - q^11 + q^12 - 4*q^13 -
        2*q^14 + q^16 - 6*q^17 - q^18 - 4*q^19 + 2*q^21 + q^22 + 6*q^23 - q^24 -
        5*q^25 + 4*q^26 + q^27 + 2*q^28 + 6*q^29 + 8*q^31 - q^32 - q^33 + 6*q^34
        + q^36 - 10*q^37 + 4*q^38 - 4*q^39 + 6*q^41 - 2*q^42 + 8*q^43 - q^44 -
        6*q^46 - 6*q^47 + q^48 - 3*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + (2*a +
        1)*q^20 + (2*a + 4)*q^21 - a*q^22 - 4*q^23 + (-a + 2)*q^24 + q^25 +
        (-4*a + 4)*q^26 - q^27 + (2*a + 8)*q^28 + 2*a*q^29 + a*q^30 + (a +
        4)*q^32 + q^33 + (-2*a - 2)*q^34 + (2*a + 4)*q^35 + (-2*a - 1)*q^36 +
        (-4*a + 2)*q^37 + (-6*a + 2)*q^38 + (-4*a - 4)*q^39 + (-a + 2)*q^40 -
        2*a*q^41 + 2*q^42 + (2*a - 4)*q^43 + (2*a + 1)*q^44 - q^45 - 4*a*q^46 -
        4*q^47 - 3*q^48 + (8*a + 13)*q^49 + O(q^50),
    q + a*q^2 + q^3 + q^4 - q^5 + a*q^6 + 2*q^7 - a*q^8 + q^9 - a*q^10 - q^11 +
        q^12 + (-2*a + 2)*q^13 + 2*a*q^14 - q^15 - 5*q^16 + a*q^18 + (-2*a +
        2)*q^19 - q^20 + 2*q^21 - a*q^22 - 4*a*q^23 - a*q^24 + q^25 + (2*a -
        6)*q^26 + q^27 + 2*q^28 + 2*a*q^29 - a*q^30 + (4*a - 4)*q^31 - 3*a*q^32
        - q^33 - 2*q^35 + q^36 + (4*a + 2)*q^37 + (2*a - 6)*q^38 + (-2*a +
        2)*q^39 + a*q^40 - 2*a*q^41 + 2*a*q^42 + (4*a + 2)*q^43 - q^44 - q^45 -
        12*q^46 + 4*a*q^47 - 5*q^48 - 3*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field
*]
[* 30, 66, 165, 165 *]
```

30,66?

1.2.5. *$H5$, genus 8.*

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + 2*q^20 +
        2*q^21 - 2*q^22 - q^23 - 4*q^25 - 8*q^26 + 5*q^27 - 4*q^28 + 2*q^30 +
        7*q^31 + 8*q^32 - q^33 + 4*q^34 - 2*q^35 - 4*q^36 + 3*q^37 - 4*q^39 -
        8*q^41 - 4*q^42 - 6*q^43 + 2*q^44 - 2*q^45 + 2*q^46 + 8*q^47 + 4*q^48 -
        3*q^49 + O(q^50),
    q + q^2 + q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 + q^8 + q^9 - 4*q^10 + q^11 + q^12
        + 4*q^13 - 2*q^14 - 4*q^15 + q^16 - 2*q^17 + q^18 - 4*q^20 - 2*q^21 +
        q^22 - 6*q^23 + q^24 + 11*q^25 + 4*q^26 + q^27 - 2*q^28 + 10*q^29 -
        4*q^30 - 8*q^31 + q^32 + q^33 - 2*q^34 + 8*q^35 + q^36 - 2*q^37 + 4*q^39
        - 4*q^40 + 2*q^41 - 2*q^42 + 4*q^43 + q^44 - 4*q^45 - 6*q^46 - 2*q^47 +
        q^48 - 3*q^49 + O(q^50),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 + q^10 + q^11 - q^12 -
        6*q^13 + 3*q^14 - q^15 + q^16 - 7*q^17 - 2*q^18 + 5*q^19 + q^20 - 3*q^21
        + q^22 - 6*q^23 - q^24 + q^25 - 6*q^26 + 5*q^27 + 3*q^28 + 5*q^29 - q^30
        - 3*q^31 + q^32 - q^33 - 7*q^34 + 3*q^35 - 2*q^36 + 3*q^37 + 5*q^38 +
        6*q^39 + q^40 + 2*q^41 - 3*q^42 + 4*q^43 + q^44 - 2*q^45 - 6*q^46 -
```

```
        2*q^47 - q^48 + 2*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + (2*a +
        1)*q^20 + (2*a + 4)*q^21 - a*q^22 - 4*q^23 + (-a + 2)*q^24 + q^25 +
        (-4*a + 4)*q^26 - q^27 + (2*a + 8)*q^28 + 2*a*q^29 + a*q^30 + (a +
        4)*q^32 + q^33 + (-2*a - 2)*q^34 + (2*a + 4)*q^35 + (-2*a - 1)*q^36 +
        (-4*a + 2)*q^37 + (-6*a + 2)*q^38 + (-4*a - 4)*q^39 + (-a + 2)*q^40 -
        2*a*q^41 + 2*q^42 + (2*a - 4)*q^43 + (2*a + 1)*q^44 - q^45 - 4*a*q^46 -
        4*q^47 - 3*q^48 + (8*a + 13)*q^49 + O(q^50),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + q^5 + a*q^6 + (-a^2 - 2*a + 3)*q^7 + (-a^2
        + a + 1)*q^8 + q^9 + a*q^10 + q^11 + (a^2 - 2)*q^12 + (-a^2 + 3)*q^13 +
        (-a^2 - 2*a - 1)*q^14 + q^15 + (-4*a + 3)*q^16 + (a^2 - 2*a - 5)*q^17 +
        a*q^18 + (2*a^2 + 2*a - 4)*q^19 + (a^2 - 2)*q^20 + (-a^2 - 2*a + 3)*q^21
        + a*q^22 + (2*a^2 + 4*a - 6)*q^23 + (-a^2 + a + 1)*q^24 + q^25 + (a^2 -
        2*a - 1)*q^26 + q^27 + (a^2 - 2*a - 7)*q^28 + (-2*a - 4)*q^29 + a*q^30 +
        (-2*a^2 + 10)*q^31 + (-2*a^2 + a - 2)*q^32 + q^33 + (-3*a^2 + 1)*q^34 +
        (-a^2 - 2*a + 3)*q^35 + (a^2 - 2)*q^36 - 2*q^37 + (6*a + 2)*q^38 + (-a^2
        + 3)*q^39 + (-a^2 + a + 1)*q^40 + (2*a - 4)*q^41 + (-a^2 - 2*a - 1)*q^42
        + (3*a^2 + 2*a - 9)*q^43 + (a^2 - 2)*q^44 + q^45 + (2*a^2 + 4*a +
        2)*q^46 + (2*a^2 - 10)*q^47 + (-4*a + 3)*q^48 + (4*a + 5)*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 + x^2 - 5*x - 1 over the Rational
    Field
*]
[* 11, 66, 110, 165, 165 *]
```

11,66?? $n(a_7 = 3; 7) = 13 - 10$.

1.2.6. *$H6$, genus 6.*

```
[*
    q + q^2 - q^4 + q^5 - 3*q^8 - 3*q^9 + q^10 - q^11 + 2*q^13 - q^16 + 6*q^17 -
        3*q^18 - 4*q^19 - q^20 - q^22 + 4*q^23 + q^25 + 2*q^26 + 6*q^29 - 8*q^31
        + 5*q^32 + 6*q^34 + 3*q^36 - 2*q^37 - 4*q^38 - 3*q^40 + 2*q^41 + 4*q^43
        + q^44 - 3*q^45 + 4*q^46 - 12*q^47 - 7*q^49 + O(q^50),
    q - q^2 + q^3 + q^4 - q^6 + 2*q^7 - q^8 + q^9 - q^11 + q^12 - 4*q^13 -
        2*q^14 + q^16 - 6*q^17 - q^18 - 4*q^19 + 2*q^21 + q^22 + 6*q^23 - q^24 -
        5*q^25 + 4*q^26 + q^27 + 2*q^28 + 6*q^29 + 8*q^31 - q^32 - q^33 + 6*q^34
        + q^36 - 10*q^37 + 4*q^38 - 4*q^39 + 6*q^41 - 2*q^42 + 8*q^43 - q^44 -
        6*q^46 - 6*q^47 + q^48 - 3*q^49 + O(q^50),
    q - q^2 + a*q^3 + q^4 + q^5 - a*q^6 - a*q^7 - q^8 + (-a + 5)*q^9 - q^10 -
        q^11 + a*q^12 + 2*q^13 + a*q^14 + a*q^15 + q^16 + (-a - 2)*q^17 + (a -
        5)*q^18 + (a + 4)*q^19 + q^20 + (a - 8)*q^21 + q^22 + (-2*a - 4)*q^23 -
        a*q^24 + q^25 - 2*q^26 + (3*a - 8)*q^27 - a*q^28 + (-a - 2)*q^29 -
        a*q^30 - a*q^31 - q^32 - a*q^33 + (a + 2)*q^34 - a*q^35 + (-a + 5)*q^36
        + (-a + 6)*q^37 + (-a - 4)*q^38 + 2*a*q^39 - q^40 + (4*a + 2)*q^41 + (-a
```

```
        + 8)*q^42 - 4*q^43 - q^44 + (-a + 5)*q^45 + (2*a + 4)*q^46 + (-2*a -
        4)*q^47 + a*q^48 + (-a + 1)*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + (2*a +
        1)*q^20 + (2*a + 4)*q^21 - a*q^22 - 4*q^23 + (-a + 2)*q^24 + q^25 +
        (-4*a + 4)*q^26 - q^27 + (2*a + 8)*q^28 + 2*a*q^29 + a*q^30 + (a +
        4)*q^32 + q^33 + (-2*a - 2)*q^34 + (2*a + 4)*q^35 + (-2*a - 1)*q^36 +
        (-4*a + 2)*q^37 + (-6*a + 2)*q^38 + (-4*a - 4)*q^39 + (-a + 2)*q^40 -
        2*a*q^41 + 2*q^42 + (2*a - 4)*q^43 + (2*a + 1)*q^44 - q^45 - 4*a*q^46 -
        4*q^47 - 3*q^48 + (8*a + 13)*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 8 over the Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 55, 66, 110, 165 *]
```

55,66??

1.2.7. *H7, genus 5.*

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 + O(q^50),
    q + q^2 - q^4 + q^5 - 3*q^8 - 3*q^9 + q^10 - q^11 + 2*q^13 - q^16 + 6*q^17 -
        3*q^18 - 4*q^19 - q^20 - q^22 + 4*q^23 + q^25 + 2*q^26 + 6*q^29 - 8*q^31
        + 5*q^32 + 6*q^34 + 3*q^36 - 2*q^37 - 4*q^38 - 3*q^40 + 2*q^41 + 4*q^43
        + q^44 - 3*q^45 + 4*q^46 - 12*q^47 - 7*q^49 + O(q^50),
    q + q^2 - q^3 + q^4 + 2*q^5 - q^6 - 4*q^7 + q^8 + q^9 + 2*q^10 - q^11 - q^12
        - 6*q^13 - 4*q^14 - 2*q^15 + q^16 + 2*q^17 + q^18 + 4*q^19 + 2*q^20 +
        4*q^21 - q^22 + 4*q^23 - q^24 - q^25 - 6*q^26 - q^27 - 4*q^28 + 6*q^29 -
        2*q^30 + q^32 + q^33 + 2*q^34 - 8*q^35 + q^36 + 6*q^37 + 4*q^38 + 6*q^39
        + 2*q^40 - 6*q^41 + 4*q^42 + 4*q^43 - q^44 + 2*q^45 + 4*q^46 - 12*q^47 -
        q^48 + 9*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + (2*a +
        1)*q^20 + (2*a + 4)*q^21 - a*q^22 - 4*q^23 + (-a + 2)*q^24 + q^25 +
        (-4*a + 4)*q^26 - q^27 + (2*a + 8)*q^28 + 2*a*q^29 + a*q^30 + (a +
        4)*q^32 + q^33 + (-2*a - 2)*q^34 + (2*a + 4)*q^35 + (-2*a - 1)*q^36 +
        (-4*a + 2)*q^37 + (-6*a + 2)*q^38 + (-4*a - 4)*q^39 + (-a + 2)*q^40 -
        2*a*q^41 + 2*q^42 + (2*a - 4)*q^43 + (2*a + 1)*q^44 - q^45 - 4*a*q^46 -
        4*q^47 - 3*q^48 + (8*a + 13)*q^49 + O(q^50)
*]
[*
    Rational Field,
```

```
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 15, 55, 66, 165 *]
```

15,55,66??

1.2.8. *H*8, *genus 5.*

```
[*
    q + q^2 + q^3 + q^4 - q^5 + q^6 - q^7 + q^8 - 2*q^9 - q^10 - q^11 + q^12 +
        2*q^13 - q^14 - q^15 + q^16 - 3*q^17 - 2*q^18 - q^19 - q^20 - q^21 -
        q^22 + 6*q^23 + q^24 + q^25 + 2*q^26 - 5*q^27 - q^28 - 9*q^29 - q^30 +
        5*q^31 + q^32 - q^33 - 3*q^34 + q^35 - 2*q^36 + 5*q^37 - q^38 + 2*q^39 -
        q^40 - 6*q^41 - q^42 + 8*q^43 - q^44 + 2*q^45 + 6*q^46 + 6*q^47 + q^48 -
        6*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + (2*a +
        1)*q^20 + (2*a + 4)*q^21 - a*q^22 - 4*q^23 + (-a + 2)*q^24 + q^25 +
        (-4*a + 4)*q^26 - q^27 + (2*a + 8)*q^28 + 2*a*q^29 + a*q^30 + (a +
        4)*q^32 + q^33 + (-2*a - 2)*q^34 + (2*a + 4)*q^35 + (-2*a - 1)*q^36 +
        (-4*a + 2)*q^37 + (-6*a + 2)*q^38 + (-4*a - 4)*q^39 + (-a + 2)*q^40 -
        2*a*q^41 + 2*q^42 + (2*a - 4)*q^43 + (2*a + 1)*q^44 - q^45 - 4*a*q^46 -
        4*q^47 - 3*q^48 + (8*a + 13)*q^49 + O(q^50),
    q + a*q^2 + q^3 + q^4 - q^5 + a*q^6 + 2*q^7 - a*q^8 + q^9 - a*q^10 - q^11 +
        q^12 + (-2*a + 2)*q^13 + 2*a*q^14 - q^15 - 5*q^16 + a*q^18 + (-2*a +
        2)*q^19 - q^20 + 2*q^21 - a*q^22 - 4*a*q^23 - a*q^24 + q^25 + (2*a -
        6)*q^26 + q^27 + 2*q^28 + 2*a*q^29 - a*q^30 + (4*a - 4)*q^31 - 3*a*q^32
        - q^33 - 2*q^35 + q^36 + (4*a + 2)*q^37 + (2*a - 6)*q^38 + (-2*a +
        2)*q^39 + a*q^40 - 2*a*q^41 + 2*a*q^42 + (4*a + 2)*q^43 - q^44 - q^45 -
        12*q^46 + 4*a*q^47 - 5*q^48 - 3*q^49 + O(q^50)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field
*]
[* 110, 165, 165 *]
```

110?

1.2.9. *H*9, *genus 6.*

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + 2*q^20 +
        2*q^21 - 2*q^22 - q^23 - 4*q^25 - 8*q^26 + 5*q^27 - 4*q^28 + 2*q^30 +
        7*q^31 + 8*q^32 - q^33 + 4*q^34 - 2*q^35 - 4*q^36 + 3*q^37 - 4*q^39 -
        8*q^41 - 4*q^42 - 6*q^43 + 2*q^44 - 2*q^45 + 2*q^46 + 8*q^47 + 4*q^48 -
        3*q^49 + O(q^50),
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
```

```
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 + O(q^50),
    q + q^2 - q^3 - q^4 - 2*q^5 - q^6 + 4*q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 +
        q^12 - 2*q^13 + 4*q^14 + 2*q^15 - q^16 - 2*q^17 + q^18 + 2*q^20 - 4*q^21
        + q^22 + 8*q^23 + 3*q^24 - q^25 - 2*q^26 - q^27 - 4*q^28 - 6*q^29 +
        2*q^30 - 8*q^31 + 5*q^32 - q^33 - 2*q^34 - 8*q^35 - q^36 + 6*q^37 +
        2*q^39 + 6*q^40 - 2*q^41 - 4*q^42 - q^44 - 2*q^45 + 8*q^46 + 8*q^47 +
        q^48 + 9*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + (2*a +
        1)*q^20 + (2*a + 4)*q^21 - a*q^22 - 4*q^23 + (-a + 2)*q^24 + q^25 +
        (-4*a + 4)*q^26 - q^27 + (2*a + 8)*q^28 + 2*a*q^29 + a*q^30 + (a +
        4)*q^32 + q^33 + (-2*a - 2)*q^34 + (2*a + 4)*q^35 + (-2*a - 1)*q^36 +
        (-4*a + 2)*q^37 + (-6*a + 2)*q^38 + (-4*a - 4)*q^39 + (-a + 2)*q^40 -
        2*a*q^41 + 2*q^42 + (2*a - 4)*q^43 + (2*a + 1)*q^44 - q^45 - 4*a*q^46 -
        4*q^47 - 3*q^48 + (8*a + 13)*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 + q^5 + q^6 - 4*q^7 - q^8 + q^9 - q^10 + q^11 - q^12 -
        2*q^13 + 4*q^14 - q^15 + q^16 - 2*q^17 - q^18 - 8*q^19 + q^20 + 4*q^21 -
        q^22 + q^24 + q^25 + 2*q^26 - q^27 - 4*q^28 + 2*q^29 + q^30 - 8*q^31 -
        q^32 - q^33 + 2*q^34 - 4*q^35 + q^36 - 10*q^37 + 8*q^38 + 2*q^39 - q^40
        - 10*q^41 - 4*q^42 + q^44 + q^45 - q^48 + 9*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field
*]
[* 11, 15, 33, 165, 330 *]
```

11,15,330? $n(a_7 = 4; 7) = 14 - 8$.

1.3. $H10$**, genus 6.**

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + 2*q^20 +
        2*q^21 - 2*q^22 - q^23 - 4*q^25 - 8*q^26 + 5*q^27 - 4*q^28 + 2*q^30 +
        7*q^31 + 8*q^32 - q^33 + 4*q^34 - 2*q^35 - 4*q^36 + 3*q^37 - 4*q^39 -
        8*q^41 - 4*q^42 - 6*q^43 + 2*q^44 - 2*q^45 + 2*q^46 + 8*q^47 + 4*q^48 -
        3*q^49 + O(q^50),
    q + q^2 - q^3 - q^4 - 2*q^5 - q^6 + 4*q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 +
        q^12 - 2*q^13 + 4*q^14 + 2*q^15 - q^16 - 2*q^17 + q^18 + 2*q^20 - 4*q^21
        + q^22 + 8*q^23 + 3*q^24 - q^25 - 2*q^26 - q^27 - 4*q^28 - 6*q^29 +
        2*q^30 - 8*q^31 + 5*q^32 - q^33 - 2*q^34 - 8*q^35 - q^36 + 6*q^37 +
        2*q^39 + 6*q^40 - 2*q^41 - 4*q^42 - q^44 - 2*q^45 + 8*q^46 + 8*q^47 +
        q^48 + 9*q^49 + O(q^50),
    q + a*q^2 + (-2*a + 2)*q^3 + (2*a - 1)*q^4 - q^5 + (-2*a - 2)*q^6 - 2*q^7 +
        (a + 2)*q^8 + 5*q^9 - a*q^10 + q^11 + (-2*a - 6)*q^12 + (2*a - 6)*q^13 -
        2*a*q^14 + (2*a - 2)*q^15 + 3*q^16 + (2*a + 2)*q^17 + 5*a*q^18 + (-2*a +
```

```
        1)*q^20 + (4*a - 4)*q^21 + a*q^22 + (-2*a + 2)*q^23 + (-6*a + 2)*q^24 +
        q^25 + (-2*a + 2)*q^26 + (-4*a + 4)*q^27 + (-4*a + 2)*q^28 + (-4*a +
        6)*q^29 + (2*a + 2)*q^30 + (a - 4)*q^32 + (-2*a + 2)*q^33 + (6*a +
        2)*q^34 + 2*q^35 + (10*a - 5)*q^36 + (-4*a + 2)*q^37 + (8*a - 16)*q^39 +
        (-a - 2)*q^40 + 6*q^41 + (4*a + 4)*q^42 - 6*q^43 + (2*a - 1)*q^44 -
        5*q^45 + (-2*a - 2)*q^46 + (2*a - 2)*q^47 + (-6*a + 6)*q^48 - 3*q^49 +
        O(q^50),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + (2*a +
        1)*q^20 + (2*a + 4)*q^21 - a*q^22 - 4*q^23 + (-a + 2)*q^24 + q^25 +
        (-4*a + 4)*q^26 - q^27 + (2*a + 8)*q^28 + 2*a*q^29 + a*q^30 + (a +
        4)*q^32 + q^33 + (-2*a - 2)*q^34 + (2*a + 4)*q^35 + (-2*a - 1)*q^36 +
        (-4*a + 2)*q^37 + (-6*a + 2)*q^38 + (-4*a - 4)*q^39 + (-a + 2)*q^40 -
        2*a*q^41 + 2*q^42 + (2*a - 4)*q^43 + (2*a + 1)*q^44 - q^45 - 4*a*q^46 -
        4*q^47 - 3*q^48 + (8*a + 13)*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 11, 33, 55, 165 *]
```

11?, $n(a_7 = 4; 7) = 14 - 8$.

1.3.1. *$H11$, genus 7.*

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + 2*q^20 +
        2*q^21 - 2*q^22 - q^23 - 4*q^25 - 8*q^26 + 5*q^27 - 4*q^28 + 2*q^30 +
        7*q^31 + 8*q^32 - q^33 + 4*q^34 - 2*q^35 - 4*q^36 + 3*q^37 - 4*q^39 -
        8*q^41 - 4*q^42 - 6*q^43 + 2*q^44 - 2*q^45 + 2*q^46 + 8*q^47 + 4*q^48 -
        3*q^49 + O(q^50),
    q - q^2 + q^3 + q^4 - q^5 - q^6 - 4*q^7 - q^8 + q^9 + q^10 + q^12 + 2*q^13 +
        4*q^14 - q^15 + q^16 + 6*q^17 - q^18 - 4*q^19 - q^20 - 4*q^21 - q^24 +
        q^25 - 2*q^26 + q^27 - 4*q^28 - 6*q^29 + q^30 + 8*q^31 - q^32 - 6*q^34 +
        4*q^35 + q^36 + 2*q^37 + 4*q^38 + 2*q^39 + q^40 - 6*q^41 + 4*q^42 -
        4*q^43 - q^45 + q^48 + 9*q^49 + O(q^50),
    q + a*q^2 + (-2*a + 2)*q^3 + (2*a - 1)*q^4 - q^5 + (-2*a - 2)*q^6 - 2*q^7 +
        (a + 2)*q^8 + 5*q^9 - a*q^10 + q^11 + (-2*a - 6)*q^12 + (2*a - 6)*q^13 -
        2*a*q^14 + (2*a - 2)*q^15 + 3*q^16 + (2*a + 2)*q^17 + 5*a*q^18 + (-2*a +
        1)*q^20 + (4*a - 4)*q^21 + a*q^22 + (-2*a + 2)*q^23 + (-6*a + 2)*q^24 +
        q^25 + (-2*a + 2)*q^26 + (-4*a + 4)*q^27 + (-4*a + 2)*q^28 + (-4*a +
        6)*q^29 + (2*a + 2)*q^30 + (a - 4)*q^32 + (-2*a + 2)*q^33 + (6*a +
        2)*q^34 + 2*q^35 + (10*a - 5)*q^36 + (-4*a + 2)*q^37 + (8*a - 16)*q^39 +
        (-a - 2)*q^40 + 6*q^41 + (4*a + 4)*q^42 - 6*q^43 + (2*a - 1)*q^44 -
        5*q^45 + (-2*a - 2)*q^46 + (2*a - 2)*q^47 + (-6*a + 6)*q^48 - 3*q^49 +
        O(q^50),
    q - q^2 + q^3 + q^4 - q^5 - q^6 + 5*q^7 - q^8 - 2*q^9 + q^10 + q^11 + q^12 +
```

```
        2*q^13 - 5*q^14 - q^15 + q^16 + 3*q^17 + 2*q^18 - 7*q^19 - q^20 + 5*q^21
        - q^22 - 6*q^23 - q^24 + q^25 - 2*q^26 - 5*q^27 + 5*q^28 - 3*q^29 + q^30
        - 7*q^31 - q^32 + q^33 - 3*q^34 - 5*q^35 - 2*q^36 - 7*q^37 + 7*q^38 +
        2*q^39 + q^40 + 6*q^41 - 5*q^42 + 8*q^43 + q^44 + 2*q^45 + 6*q^46 +
        6*q^47 + q^48 + 18*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + (2*a +
        1)*q^20 + (2*a + 4)*q^21 - a*q^22 - 4*q^23 + (-a + 2)*q^24 + q^25 +
        (-4*a + 4)*q^26 - q^27 + (2*a + 8)*q^28 + 2*a*q^29 + a*q^30 + (a +
        4)*q^32 + q^33 + (-2*a - 2)*q^34 + (2*a + 4)*q^35 + (-2*a - 1)*q^36 +
        (-4*a + 2)*q^37 + (-6*a + 2)*q^38 + (-4*a - 4)*q^39 + (-a + 2)*q^40 -
        2*a*q^41 + 2*q^42 + (2*a - 4)*q^43 + (2*a + 1)*q^44 - q^45 - 4*a*q^46 -
        4*q^47 - 3*q^48 + (8*a + 13)*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 11, 30, 55, 110, 165 *]
```

11,30? $n(a_7 \geq 0; 7) \geq 17 - 16$.

1.3.2. *$H12$, genus 6.*

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + 2*q^20 +
        2*q^21 - 2*q^22 - q^23 - 4*q^25 - 8*q^26 + 5*q^27 - 4*q^28 + 2*q^30 +
        7*q^31 + 8*q^32 - q^33 + 4*q^34 - 2*q^35 - 4*q^36 + 3*q^37 - 4*q^39 -
        8*q^41 - 4*q^42 - 6*q^43 + 2*q^44 - 2*q^45 + 2*q^46 + 8*q^47 + 4*q^48 -
        3*q^49 + O(q^50),
    q + a*q^2 + (-2*a + 2)*q^3 + (2*a - 1)*q^4 - q^5 + (-2*a - 2)*q^6 - 2*q^7 +
        (a + 2)*q^8 + 5*q^9 - a*q^10 + q^11 + (-2*a - 6)*q^12 + (2*a - 6)*q^13 -
        2*a*q^14 + (2*a - 2)*q^15 + 3*q^16 + (2*a + 2)*q^17 + 5*a*q^18 + (-2*a +
        1)*q^20 + (4*a - 4)*q^21 + a*q^22 + (-2*a + 2)*q^23 + (-6*a + 2)*q^24 +
        q^25 + (-2*a + 2)*q^26 + (-4*a + 4)*q^27 + (-4*a + 2)*q^28 + (-4*a +
        6)*q^29 + (2*a + 2)*q^30 + (a - 4)*q^32 + (-2*a + 2)*q^33 + (6*a +
        2)*q^34 + 2*q^35 + (10*a - 5)*q^36 + (-4*a + 2)*q^37 + (8*a - 16)*q^39 +
        (-a - 2)*q^40 + 6*q^41 + (4*a + 4)*q^42 - 6*q^43 + (2*a - 1)*q^44 -
        5*q^45 + (-2*a - 2)*q^46 + (2*a - 2)*q^47 + (-6*a + 6)*q^48 - 3*q^49 +
        O(q^50),
    q + q^2 + q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 + q^8 + q^9 - 4*q^10 + q^11 + q^12
        + 4*q^13 - 2*q^14 - 4*q^15 + q^16 - 2*q^17 + q^18 - 4*q^20 - 2*q^21 +
        q^22 - 6*q^23 + q^24 + 11*q^25 + 4*q^26 + q^27 - 2*q^28 + 10*q^29 -
        4*q^30 - 8*q^31 + q^32 + q^33 - 2*q^34 + 8*q^35 + q^36 - 2*q^37 + 4*q^39
        - 4*q^40 + 2*q^41 - 2*q^42 + 4*q^43 + q^44 - 4*q^45 - 6*q^46 - 2*q^47 +
        q^48 - 3*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
```

```
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + (2*a +
        1)*q^20 + (2*a + 4)*q^21 - a*q^22 - 4*q^23 + (-a + 2)*q^24 + q^25 +
        (-4*a + 4)*q^26 - q^27 + (2*a + 8)*q^28 + 2*a*q^29 + a*q^30 + (a +
        4)*q^32 + q^33 + (-2*a - 2)*q^34 + (2*a + 4)*q^35 + (-2*a - 1)*q^36 +
        (-4*a + 2)*q^37 + (-6*a + 2)*q^38 + (-4*a - 4)*q^39 + (-a + 2)*q^40 -
        2*a*q^41 + 2*q^42 + (2*a - 4)*q^43 + (2*a + 1)*q^44 - q^45 - 4*a*q^46 -
        4*q^47 - 3*q^48 + (8*a + 13)*q^49 + O(q^50)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 2*x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 11, 55, 66, 165 *]
```

11,66?

1.3.3. $H13$, *genus 7.*

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + 2*q^20 +
        2*q^21 - 2*q^22 - q^23 - 4*q^25 - 8*q^26 + 5*q^27 - 4*q^28 + 2*q^30 +
        7*q^31 + 8*q^32 - q^33 + 4*q^34 - 2*q^35 - 4*q^36 + 3*q^37 - 4*q^39 -
        8*q^41 - 4*q^42 - 6*q^43 + 2*q^44 - 2*q^45 + 2*q^46 + 8*q^47 + 4*q^48 -
        3*q^49 + O(q^50),
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 + O(q^50),
    q + q^2 - q^3 - q^4 - 2*q^5 - q^6 + 4*q^7 - 3*q^8 + q^9 - 2*q^10 + q^11 +
        q^12 - 2*q^13 + 4*q^14 + 2*q^15 - q^16 - 2*q^17 + q^18 + 2*q^20 - 4*q^21
        + q^22 + 8*q^23 + 3*q^24 - q^25 - 2*q^26 - q^27 - 4*q^28 - 6*q^29 +
        2*q^30 - 8*q^31 + 5*q^32 - q^33 - 2*q^34 - 8*q^35 - q^36 + 6*q^37 +
        2*q^39 + 6*q^40 - 2*q^41 - 4*q^42 - q^44 - 2*q^45 + 8*q^46 + 8*q^47 +
        q^48 + 9*q^49 + O(q^50),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + 3*q^7 + q^8 - 2*q^9 + q^10 + q^11 - q^12 -
        6*q^13 + 3*q^14 - q^15 + q^16 - 7*q^17 - 2*q^18 + 5*q^19 + q^20 - 3*q^21
        + q^22 - 6*q^23 - q^24 + q^25 - 6*q^26 + 5*q^27 + 3*q^28 + 5*q^29 - q^30
        - 3*q^31 + q^32 - q^33 - 7*q^34 + 3*q^35 - 2*q^36 + 3*q^37 + 5*q^38 +
        6*q^39 + q^40 + 2*q^41 - 3*q^42 + 4*q^43 + q^44 - 2*q^45 - 6*q^46 -
        2*q^47 - q^48 + 2*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + (2*a +
        1)*q^20 + (2*a + 4)*q^21 - a*q^22 - 4*q^23 + (-a + 2)*q^24 + q^25 +
        (-4*a + 4)*q^26 - q^27 + (2*a + 8)*q^28 + 2*a*q^29 + a*q^30 + (a +
        4)*q^32 + q^33 + (-2*a - 2)*q^34 + (2*a + 4)*q^35 + (-2*a - 1)*q^36 +
        (-4*a + 2)*q^37 + (-6*a + 2)*q^38 + (-4*a - 4)*q^39 + (-a + 2)*q^40 -
```

```
        2*a*q^41 + 2*q^42 + (2*a - 4)*q^43 + (2*a + 1)*q^44 - q^45 - 4*a*q^46 -
        4*q^47 - 3*q^48 + (8*a + 13)*q^49 + O(q^50),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + q^8 + q^9 + q^10 + q^11 - q^12 + 6*q^13 -
        q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 + q^20 + q^22 - q^24 + q^25 +
        6*q^26 - q^27 - 10*q^29 - q^30 + q^32 - q^33 + 2*q^34 + q^36 + 6*q^37 -
        4*q^38 - 6*q^39 + q^40 + 2*q^41 + 4*q^43 + q^44 + q^45 - 8*q^47 - q^48 -
        7*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field
*]
[* 11, 15, 33, 110, 165, 330 *]
```

all possible.

1.3.4. *$H14$, genus 6.*

```
[*
    q - 2*q^2 - q^3 + 2*q^4 + q^5 + 2*q^6 - 2*q^7 - 2*q^9 - 2*q^10 + q^11 -
        2*q^12 + 4*q^13 + 4*q^14 - q^15 - 4*q^16 - 2*q^17 + 4*q^18 + 2*q^20 +
        2*q^21 - 2*q^22 - q^23 - 4*q^25 - 8*q^26 + 5*q^27 - 4*q^28 + 2*q^30 +
        7*q^31 + 8*q^32 - q^33 + 4*q^34 - 2*q^35 - 4*q^36 + 3*q^37 - 4*q^39 -
        8*q^41 - 4*q^42 - 6*q^43 + 2*q^44 - 2*q^45 + 2*q^46 + 8*q^47 + 4*q^48 -
        3*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + (2*a +
        1)*q^20 + (2*a + 4)*q^21 - a*q^22 - 4*q^23 + (-a + 2)*q^24 + q^25 +
        (-4*a + 4)*q^26 - q^27 + (2*a + 8)*q^28 + 2*a*q^29 + a*q^30 + (a +
        4)*q^32 + q^33 + (-2*a - 2)*q^34 + (2*a + 4)*q^35 + (-2*a - 1)*q^36 +
        (-4*a + 2)*q^37 + (-6*a + 2)*q^38 + (-4*a - 4)*q^39 + (-a + 2)*q^40 -
        2*a*q^41 + 2*q^42 + (2*a - 4)*q^43 + (2*a + 1)*q^44 - q^45 - 4*a*q^46 -
        4*q^47 - 3*q^48 + (8*a + 13)*q^49 + O(q^50),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + q^5 + a*q^6 + (-a^2 - 2*a + 3)*q^7 + (-a^2
        + a + 1)*q^8 + q^9 + a*q^10 + q^11 + (a^2 - 2)*q^12 + (-a^2 + 3)*q^13 +
        (-a^2 - 2*a - 1)*q^14 + q^15 + (-4*a + 3)*q^16 + (a^2 - 2*a - 5)*q^17 +
        a*q^18 + (2*a^2 + 2*a - 4)*q^19 + (a^2 - 2)*q^20 + (-a^2 - 2*a + 3)*q^21
        + a*q^22 + (2*a^2 + 4*a - 6)*q^23 + (-a^2 + a + 1)*q^24 + q^25 + (a^2 -
        2*a - 1)*q^26 + q^27 + (a^2 - 2*a - 7)*q^28 + (-2*a - 4)*q^29 + a*q^30 +
        (-2*a^2 + 10)*q^31 + (-2*a^2 + a - 2)*q^32 + q^33 + (-3*a^2 + 1)*q^34 +
        (-a^2 - 2*a + 3)*q^35 + (a^2 - 2)*q^36 - 2*q^37 + (6*a + 2)*q^38 + (-a^2
        + 3)*q^39 + (-a^2 + a + 1)*q^40 + (2*a - 4)*q^41 + (-a^2 - 2*a - 1)*q^42
        + (3*a^2 + 2*a - 9)*q^43 + (a^2 - 2)*q^44 + q^45 + (2*a^2 + 4*a +
        2)*q^46 + (2*a^2 - 10)*q^47 + (-4*a + 3)*q^48 + (4*a + 5)*q^49 + O(q^50)
*]
[*
    Rational Field,
```

```
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^3 + x^2 - 5*x - 1 over the Rational
    Field
*]
[* 11, 165, 165 *]
```

11?

1.3.5. *H15, genus 4.*

```
[*
    q + q^2 - q^4 + q^5 - 3*q^8 - 3*q^9 + q^10 - q^11 + 2*q^13 - q^16 + 6*q^17 -
        3*q^18 - 4*q^19 - q^20 - q^22 + 4*q^23 + q^25 + 2*q^26 + 6*q^29 - 8*q^31
        + 5*q^32 + 6*q^34 + 3*q^36 - 2*q^37 - 4*q^38 - 3*q^40 + 2*q^41 + 4*q^43
        + q^44 - 3*q^45 + 4*q^46 - 12*q^47 - 7*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-2*a - 1)*q^4 - q^5 - a*q^6 + (-2*a - 4)*q^7 + (a -
        2)*q^8 + q^9 - a*q^10 - q^11 + (2*a + 1)*q^12 + (4*a + 4)*q^13 - 2*q^14
        + q^15 + 3*q^16 + (-2*a - 6)*q^17 + a*q^18 + (2*a - 2)*q^19 + (2*a +
        1)*q^20 + (2*a + 4)*q^21 - a*q^22 - 4*q^23 + (-a + 2)*q^24 + q^25 +
        (-4*a + 4)*q^26 - q^27 + (2*a + 8)*q^28 + 2*a*q^29 + a*q^30 + (a +
        4)*q^32 + q^33 + (-2*a - 2)*q^34 + (2*a + 4)*q^35 + (-2*a - 1)*q^36 +
        (-4*a + 2)*q^37 + (-6*a + 2)*q^38 + (-4*a - 4)*q^39 + (-a + 2)*q^40 -
        2*a*q^41 + 2*q^42 + (2*a - 4)*q^43 + (2*a + 1)*q^44 - q^45 - 4*a*q^46 -
        4*q^47 - 3*q^48 + (8*a + 13)*q^49 + O(q^50),
    q + q^2 + q^3 + q^4 + q^5 + q^6 + q^8 + q^9 + q^10 - q^11 + q^12 - 2*q^13 +
        q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 + q^20 - q^22 + q^24 + q^25 -
        2*q^26 + q^27 - 2*q^29 + q^30 + q^32 - q^33 + 2*q^34 + q^36 - 2*q^37 -
        4*q^38 - 2*q^39 + q^40 + 2*q^41 - 12*q^43 - q^44 + q^45 + 8*q^47 + q^48
        - 7*q^49 + O(q^50)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field
*]
[* 55, 165, 330 *]
```

55,330?

1.4. $N = 390 = 2 \cdot 3 \cdot 5 \cdot 13$**.**

1.4.1. *H1, genus 10.*

```
[*
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 - 3*q^20 -
        q^21 - 6*q^22 - q^24 + 4*q^25 - q^26 - 5*q^27 - q^28 + 6*q^29 + 3*q^30 -
        4*q^31 - q^32 + 6*q^33 + 3*q^34 + 3*q^35 - 2*q^36 - 7*q^37 - 2*q^38 +
        q^39 + 3*q^40 + q^42 - q^43 + 6*q^44 + 6*q^45 + 3*q^47 + q^48 - 6*q^49 +
        O(q^50),
    q + q^2 - q^3 - q^4 + 2*q^5 - q^6 - 4*q^7 - 3*q^8 + q^9 + 2*q^10 + 4*q^11 +
        q^12 + q^13 - 4*q^14 - 2*q^15 - q^16 + 2*q^17 + q^18 - 2*q^20 + 4*q^21 +
        4*q^22 + 3*q^24 - q^25 + q^26 - q^27 + 4*q^28 - 10*q^29 - 2*q^30 +
        4*q^31 + 5*q^32 - 4*q^33 + 2*q^34 - 8*q^35 - q^36 - 2*q^37 - q^39 -
        6*q^40 + 6*q^41 + 4*q^42 - 12*q^43 - 4*q^44 + 2*q^45 + q^48 + 9*q^49 +
```

```
        O(q^50),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q + a*q^2 + (-a + 1)*q^3 + q^4 - q^5 + (a - 3)*q^6 + 2*q^7 - a*q^8 + (-2*a +
        1)*q^9 - a*q^10 + (a - 3)*q^11 + (-a + 1)*q^12 + q^13 + 2*a*q^14 + (a -
        1)*q^15 - 5*q^16 + 2*a*q^17 + (a - 6)*q^18 + (3*a - 1)*q^19 - q^20 +
        (-2*a + 2)*q^21 + (-3*a + 3)*q^22 + (a + 3)*q^23 + (-a + 3)*q^24 + q^25
        + a*q^26 + 4*q^27 + 2*q^28 + (-2*a - 6)*q^29 + (-a + 3)*q^30 + (-3*a +
        5)*q^31 - 3*a*q^32 + (4*a - 6)*q^33 + 6*q^34 - 2*q^35 + (-2*a + 1)*q^36
        - 4*q^37 + (-a + 9)*q^38 + (-a + 1)*q^39 + a*q^40 - 2*a*q^41 + (2*a -
        6)*q^42 + (3*a + 5)*q^43 + (a - 3)*q^44 + (2*a - 1)*q^45 + (3*a +
        3)*q^46 + 6*q^47 + (5*a - 5)*q^48 - 3*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 + 2*q^5 + q^6 + 4*q^7 - q^8 + q^9 - 2*q^10 - 4*q^11 -
        q^12 + q^13 - 4*q^14 - 2*q^15 + q^16 + 2*q^17 - q^18 - 8*q^19 + 2*q^20 -
        4*q^21 + 4*q^22 + q^24 - q^25 - q^26 - q^27 + 4*q^28 + 6*q^29 + 2*q^30 -
        4*q^31 - q^32 + 4*q^33 - 2*q^34 + 8*q^35 + q^36 - 2*q^37 + 8*q^38 - q^39
        - 2*q^40 - 10*q^41 + 4*q^42 + 4*q^43 - 4*q^44 + 2*q^45 + 8*q^47 - q^48 +
        9*q^49 + O(q^50),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 - q^5 - a*q^6 + (-a^2 + 5)*q^7 + (3*a +
        2)*q^8 + q^9 - a*q^10 + (-a^2 + 5)*q^11 + (-a^2 + 2)*q^12 + q^13 + (-2*a
        - 2)*q^14 + q^15 + (a^2 + 2*a + 4)*q^16 + (a^2 - 2*a - 5)*q^17 + a*q^18
        + (-2*a + 2)*q^19 + (-a^2 + 2)*q^20 + (a^2 - 5)*q^21 + (-2*a - 2)*q^22 +
        (a^2 - 2*a - 7)*q^23 + (-3*a - 2)*q^24 + q^25 + a*q^26 - q^27 + (-2*a -
        10)*q^28 + 6*q^29 + a*q^30 + (2*a + 2)*q^31 + (2*a^2 + 5*a - 2)*q^32 +
        (a^2 - 5)*q^33 + (-2*a^2 + 2*a + 2)*q^34 + (a^2 - 5)*q^35 + (a^2 -
        2)*q^36 + (-a^2 - 2*a + 9)*q^37 + (-2*a^2 + 2*a)*q^38 - q^39 + (-3*a -
        2)*q^40 + (-a^2 + 2*a + 5)*q^41 + (2*a + 2)*q^42 + 4*a*q^43 + (-2*a -
        10)*q^44 - q^45 + (-2*a^2 + 2)*q^46 + (-2*a - 6)*q^47 + (-a^2 - 2*a -
        4)*q^48 + (-3*a^2 + 2*a + 18)*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^12 - q^13 + q^15 +
        q^16 - 6*q^17 - q^18 - q^20 - 4*q^23 + q^24 + q^25 + q^26 - q^27 -
        10*q^29 - q^30 - q^32 + 6*q^34 + q^36 - 6*q^37 + q^39 + q^40 + 2*q^41 -
        4*q^43 - q^45 + 4*q^46 - q^48 - 7*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - 7*x - 2 over the Rational Field,
    Rational Field
*]
[* 26, 39, 65, 65, 78, 195, 390 *]
```

???26,390, $n(a_7 = \pm 4; 49) = 100 - 96$.

1.4.2. *H2, genus 7.*

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 + O(q^50),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a
        - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a -
        2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a +
        3)*q^19 + (-2*a - 1)*q^20 + (2*a - 2)*q^21 + (3*a - 1)*q^22 + (-a -
        1)*q^23 + (-3*a - 1)*q^24 + q^25 - a*q^26 + (-4*a - 4)*q^27 + (-6*a +
        4)*q^28 + (4*a + 4)*q^29 + (-a + 1)*q^30 + (3*a + 9)*q^31 + (a + 4)*q^32
        + 2*a*q^33 - 2*q^34 - 2*a*q^35 + (2*a + 1)*q^36 + (6*a + 6)*q^37 + (a +
        1)*q^38 + (-a - 1)*q^39 + (a - 2)*q^40 + (-2*a - 8)*q^41 + (-6*a +
        2)*q^42 + (5*a + 1)*q^43 + (-5*a + 1)*q^44 - q^45 + (a - 1)*q^46 +
        2*a*q^47 + (3*a + 3)*q^48 + (-8*a - 3)*q^49 + O(q^50),
    q + 2*q^2 - q^3 + 2*q^4 + q^5 - 2*q^6 + 3*q^7 + q^9 + 2*q^10 - q^11 - 2*q^12
        - q^13 + 6*q^14 - q^15 - 4*q^16 - q^17 + 2*q^18 - 2*q^19 + 2*q^20 -
        3*q^21 - 2*q^22 - 3*q^23 + q^25 - 2*q^26 - q^27 + 6*q^28 - 2*q^29 -
        2*q^30 - 6*q^31 - 8*q^32 + q^33 - 2*q^34 + 3*q^35 + 2*q^36 + 11*q^37 -
        4*q^38 + q^39 - 5*q^41 - 6*q^42 + 4*q^43 - 2*q^44 + q^45 - 6*q^46 -
        10*q^47 + 4*q^48 + 2*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^12 - q^13 + q^15 +
        q^16 - 6*q^17 - q^18 - q^20 - 4*q^23 + q^24 + q^25 + q^26 - q^27 -
        10*q^29 - q^30 - q^32 + 6*q^34 + q^36 - 6*q^37 + q^39 + q^40 + 2*q^41 -
        4*q^43 - q^45 + 4*q^46 - q^48 - 7*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 + q^5 + q^6 - 2*q^7 - q^8 + q^9 - q^10 + 4*q^11 - q^12 -
        q^13 + 2*q^14 - q^15 + q^16 + 4*q^17 - q^18 - 2*q^19 + q^20 + 2*q^21 -
        4*q^22 + 2*q^23 + q^24 + q^25 + q^26 - q^27 - 2*q^28 + 8*q^29 + q^30 +
        4*q^31 - q^32 - 4*q^33 - 4*q^34 - 2*q^35 + q^36 + 6*q^37 + 2*q^38 + q^39
        - q^40 + 10*q^41 - 2*q^42 + 4*q^43 + 4*q^44 + q^45 - 2*q^46 - q^48 -
        3*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 15, 65, 65, 195, 390, 390 *]
```

All possible!!

1.4.3. *H3, genus 6.*

```
[*
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 - q^20 -
        3*q^21 - 2*q^22 - 4*q^23 - 3*q^24 - 4*q^25 - q^26 - 9*q^27 + q^28 +
        2*q^29 + 3*q^30 + 4*q^31 + q^32 + 6*q^33 - 3*q^34 - q^35 + 6*q^36 +
        3*q^37 + 6*q^38 + 3*q^39 - q^40 - 3*q^42 - 5*q^43 - 2*q^44 - 6*q^45 -
        4*q^46 + 13*q^47 - 3*q^48 - 6*q^49 + O(q^50),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q + q^2 + 2*q^3 + q^4 - q^5 + 2*q^6 - 4*q^7 + q^8 + q^9 - q^10 - 2*q^11 +
        2*q^12 - q^13 - 4*q^14 - 2*q^15 + q^16 + 2*q^17 + q^18 + 6*q^19 - q^20 -
        8*q^21 - 2*q^22 + 6*q^23 + 2*q^24 + q^25 - q^26 - 4*q^27 - 4*q^28 +
        2*q^29 - 2*q^30 - 6*q^31 + q^32 - 4*q^33 + 2*q^34 + 4*q^35 + q^36 -
        2*q^37 + 6*q^38 - 2*q^39 - q^40 + 10*q^41 - 8*q^42 - 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 - 12*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^12 - q^13 + q^15 +
        q^16 - 6*q^17 - q^18 - q^20 - 4*q^23 + q^24 + q^25 + q^26 - q^27 -
        10*q^29 - q^30 - q^32 + 6*q^34 + q^36 - 6*q^37 + q^39 + q^40 + 2*q^41 -
        4*q^43 - q^45 + 4*q^46 - q^48 - 7*q^49 + O(q^50),
    q + q^2 - q^3 + q^4 - q^5 - q^6 + 2*q^7 + q^8 + q^9 - q^10 + 4*q^11 - q^12 -
        q^13 + 2*q^14 + q^15 + q^16 + 8*q^17 + q^18 - 6*q^19 - q^20 - 2*q^21 +
        4*q^22 + 6*q^23 - q^24 + q^25 - q^26 - q^27 + 2*q^28 - 4*q^29 + q^30 +
        q^32 - 4*q^33 + 8*q^34 - 2*q^35 + q^36 - 2*q^37 - 6*q^38 + q^39 - q^40 -
        2*q^41 - 2*q^42 - 4*q^43 + 4*q^44 - q^45 + 6*q^46 - q^48 - 3*q^49 +
        O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 26, 65, 130, 130, 390, 390 *]
```

??130,130??? $n(a_7 \geq 0; 7) \geq 17 - 16$.

1.4.4. *H4, genus 8.*

```
[*
    q - q^2 + q^3 + q^4 - q^5 - q^6 - 4*q^7 - q^8 + q^9 + q^10 + q^12 + 2*q^13 +
        4*q^14 - q^15 + q^16 + 6*q^17 - q^18 - 4*q^19 - q^20 - 4*q^21 - q^24 +
        q^25 - 2*q^26 + q^27 - 4*q^28 - 6*q^29 + q^30 + 8*q^31 - q^32 - 6*q^34 +
        4*q^35 + q^36 + 2*q^37 + 4*q^38 + 2*q^39 + q^40 - 6*q^41 + 4*q^42 -
        4*q^43 - q^45 + q^48 + 9*q^49 + O(q^50),
    q + a*q^2 + q^3 + (-2*a - 1)*q^4 + (-2*a - 2)*q^5 + a*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + q^9 + (2*a - 2)*q^10 - 2*q^11 + (-2*a - 1)*q^12 - q^13 +
        (-2*a + 2)*q^14 + (-2*a - 2)*q^15 + 3*q^16 + (4*a + 6)*q^17 + a*q^18 +
        (-2*a - 2)*q^19 + (-2*a + 6)*q^20 + (2*a + 2)*q^21 - 2*a*q^22 - 4*q^23 +
        (a - 2)*q^24 + 3*q^25 - a*q^26 + q^27 + (2*a - 6)*q^28 + 2*q^29 + (2*a -
        2)*q^30 + (2*a - 2)*q^31 + (a + 4)*q^32 - 2*q^33 + (-2*a + 4)*q^34 -
        8*q^35 + (-2*a - 1)*q^36 + (-4*a - 6)*q^37 + (2*a - 2)*q^38 - q^39 +
        (6*a + 2)*q^40 + (-2*a + 6)*q^41 + (-2*a + 2)*q^42 - 4*a*q^43 + (4*a +
        2)*q^44 + (-2*a - 2)*q^45 - 4*a*q^46 + (-4*a - 10)*q^47 + 3*q^48 + q^49
        + O(q^50),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q + 2*q^2 + q^3 + 2*q^4 - q^5 + 2*q^6 - q^7 + q^9 - 2*q^10 + 5*q^11 + 2*q^12
        - q^13 - 2*q^14 - q^15 - 4*q^16 - 7*q^17 + 2*q^18 - 6*q^19 - 2*q^20 -
        q^21 + 10*q^22 + 3*q^23 + q^25 - 2*q^26 + q^27 - 2*q^28 + 2*q^29 -
        2*q^30 + 2*q^31 - 8*q^32 + 5*q^33 - 14*q^34 + q^35 + 2*q^36 + 7*q^37 -
        12*q^38 - q^39 + 9*q^41 - 2*q^42 - 8*q^43 + 10*q^44 - q^45 + 6*q^46 +
        10*q^47 - 4*q^48 - 6*q^49 + O(q^50),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^12 - q^13 + q^15 +
        q^16 - 6*q^17 - q^18 - q^20 - 4*q^23 + q^24 + q^25 + q^26 - q^27 -
        10*q^29 - q^30 - q^32 + 6*q^34 + q^36 - 6*q^37 + q^39 + q^40 + 2*q^41 -
        4*q^43 - q^45 + 4*q^46 - q^48 - 7*q^49 + O(q^50),
    q - q^2 + q^3 + q^4 - q^5 - q^6 + 4*q^7 - q^8 + q^9 + q^10 + q^12 - q^13 -
        4*q^14 - q^15 + q^16 - 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^21 + 8*q^23 -
        q^24 + q^25 + q^26 + q^27 + 4*q^28 + 2*q^29 + q^30 - 8*q^31 - q^32 +
        2*q^34 - 4*q^35 + q^36 + 2*q^37 - 4*q^38 - q^39 + q^40 - 6*q^41 - 4*q^42
        + 12*q^43 - q^45 - 8*q^46 + q^48 + 9*q^49 + O(q^50)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
```

```
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 30, 39, 65, 195, 195, 390, 390 *]
```

30,65,??? $n(a_7 \geq 0; 7) \geq 17 - 16$,$n(a_{11} = 5; 121) = 257 - 238$.

1.4.5. *$H5$, genus 5.*

```
[*
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q + q^2 + q^4 + q^5 + q^8 - 3*q^9 + q^10 + q^13 + q^16 + 2*q^17 - 3*q^18 -
        8*q^19 + q^20 - 4*q^23 + q^25 + q^26 - 2*q^29 - 4*q^31 + q^32 + 2*q^34 -
        3*q^36 + 6*q^37 - 8*q^38 + q^40 + 10*q^41 - 3*q^45 - 4*q^46 + 8*q^47 -
        7*q^49 + O(q^50),
    q - q^2 + q^3 - q^4 + q^5 - q^6 + 3*q^8 + q^9 - q^10 + 4*q^11 - q^12 + q^13
        + q^15 - q^16 + 2*q^17 - q^18 - 4*q^19 - q^20 - 4*q^22 + 8*q^23 + 3*q^24
        + q^25 - q^26 + q^27 - 2*q^29 - q^30 - 8*q^31 - 5*q^32 + 4*q^33 - 2*q^34
        - q^36 + 6*q^37 + 4*q^38 + q^39 + 3*q^40 - 6*q^41 - 4*q^43 - 4*q^44 +
        q^45 - 8*q^46 - 8*q^47 - q^48 - 7*q^49 + O(q^50),
    q + 2*q^2 + q^3 + 2*q^4 + q^5 + 2*q^6 - 3*q^7 + q^9 + 2*q^10 - 5*q^11 +
        2*q^12 + q^13 - 6*q^14 + q^15 - 4*q^16 + 5*q^17 + 2*q^18 + 2*q^19 +
        2*q^20 - 3*q^21 - 10*q^22 - q^23 + q^25 + 2*q^26 + q^27 - 6*q^28 +
        10*q^29 + 2*q^30 - 2*q^31 - 8*q^32 - 5*q^33 + 10*q^34 - 3*q^35 + 2*q^36
        - 3*q^37 + 4*q^38 + q^39 - 9*q^41 - 6*q^42 - 4*q^43 - 10*q^44 + q^45 -
        2*q^46 + 10*q^47 - 4*q^48 + 2*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^12 - q^13 + q^15 +
        q^16 - 6*q^17 - q^18 - q^20 - 4*q^23 + q^24 + q^25 + q^26 - q^27 -
        10*q^29 - q^30 - q^32 + 6*q^34 + q^36 - 6*q^37 + q^39 + q^40 + 2*q^41 -
        4*q^43 - q^45 + 4*q^46 - q^48 - 7*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 65, 130, 195, 195, 390 *]
```

All possible.

1.4.6. *$H6$, genus 6.*

```
[*
    q + a*q^2 + q^3 + (-2*a - 1)*q^4 + (-2*a - 2)*q^5 + a*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + q^9 + (2*a - 2)*q^10 - 2*q^11 + (-2*a - 1)*q^12 - q^13 +
        (-2*a + 2)*q^14 + (-2*a - 2)*q^15 + 3*q^16 + (4*a + 6)*q^17 + a*q^18 +
        (-2*a - 2)*q^19 + (-2*a + 6)*q^20 + (2*a + 2)*q^21 - 2*a*q^22 - 4*q^23 +
```

```
        (a - 2)*q^24 + 3*q^25 - a*q^26 + q^27 + (2*a - 6)*q^28 + 2*q^29 + (2*a -
        2)*q^30 + (2*a - 2)*q^31 + (a + 4)*q^32 - 2*q^33 + (-2*a + 4)*q^34 -
        8*q^35 + (-2*a - 1)*q^36 + (-4*a - 6)*q^37 + (2*a - 2)*q^38 - q^39 +
        (6*a + 2)*q^40 + (-2*a + 6)*q^41 + (-2*a + 2)*q^42 - 4*a*q^43 + (4*a +
        2)*q^44 + (-2*a - 2)*q^45 - 4*a*q^46 + (-4*a - 10)*q^47 + 3*q^48 + q^49
        + O(q^50),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a
        - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a -
        2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a +
        3)*q^19 + (-2*a - 1)*q^20 + (2*a - 2)*q^21 + (3*a - 1)*q^22 + (-a -
        1)*q^23 + (-3*a - 1)*q^24 + q^25 - a*q^26 + (-4*a - 4)*q^27 + (-6*a +
        4)*q^28 + (4*a + 4)*q^29 + (-a + 1)*q^30 + (3*a + 9)*q^31 + (a + 4)*q^32
        + 2*a*q^33 - 2*q^34 - 2*a*q^35 + (2*a + 1)*q^36 + (6*a + 6)*q^37 + (a +
        1)*q^38 + (-a - 1)*q^39 + (a - 2)*q^40 + (-2*a - 8)*q^41 + (-6*a +
        2)*q^42 + (5*a + 1)*q^43 + (-5*a + 1)*q^44 - q^45 + (a - 1)*q^46 +
        2*a*q^47 + (3*a + 3)*q^48 + (-8*a - 3)*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^12 - q^13 + q^15 +
        q^16 - 6*q^17 - q^18 - q^20 - 4*q^23 + q^24 + q^25 + q^26 - q^27 -
        10*q^29 - q^30 - q^32 + 6*q^34 + q^36 - 6*q^37 + q^39 + q^40 + 2*q^41 -
        4*q^43 - q^45 + 4*q^46 - q^48 - 7*q^49 + O(q^50)
*]
[*
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field
*]
[* 39, 65, 65, 390 *]
```

65?,390?

1.4.7. *H*7*, genus 7.*

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 + O(q^50),
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 - q^20 -
        3*q^21 - 2*q^22 - 4*q^23 - 3*q^24 - 4*q^25 - q^26 - 9*q^27 + q^28 +
        2*q^29 + 3*q^30 + 4*q^31 + q^32 + 6*q^33 - 3*q^34 - q^35 + 6*q^36 +
        3*q^37 + 6*q^38 + 3*q^39 - q^40 - 3*q^42 - 5*q^43 - 2*q^44 - 6*q^45 -
        4*q^46 + 13*q^47 - 3*q^48 - 6*q^49 + O(q^50),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
```

```
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a
        - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a -
        2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a +
        3)*q^19 + (-2*a - 1)*q^20 + (2*a - 2)*q^21 + (3*a - 1)*q^22 + (-a -
        1)*q^23 + (-3*a - 1)*q^24 + q^25 - a*q^26 + (-4*a - 4)*q^27 + (-6*a +
        4)*q^28 + (4*a + 4)*q^29 + (-a + 1)*q^30 + (3*a + 9)*q^31 + (a + 4)*q^32
        + 2*a*q^33 - 2*q^34 - 2*a*q^35 + (2*a + 1)*q^36 + (6*a + 6)*q^37 + (a +
        1)*q^38 + (-a - 1)*q^39 + (a - 2)*q^40 + (-2*a - 8)*q^41 + (-6*a +
        2)*q^42 + (5*a + 1)*q^43 + (-5*a + 1)*q^44 - q^45 + (a - 1)*q^46 +
        2*a*q^47 + (3*a + 3)*q^48 + (-8*a - 3)*q^49 + O(q^50),
    q + 2*q^2 - q^3 + 2*q^4 + q^5 - 2*q^6 + 3*q^7 + q^9 + 2*q^10 - q^11 - 2*q^12
        - q^13 + 6*q^14 - q^15 - 4*q^16 - q^17 + 2*q^18 - 2*q^19 + 2*q^20 -
        3*q^21 - 2*q^22 - 3*q^23 + q^25 - 2*q^26 - q^27 + 6*q^28 - 2*q^29 -
        2*q^30 - 6*q^31 - 8*q^32 + q^33 - 2*q^34 + 3*q^35 + 2*q^36 + 11*q^37 -
        4*q^38 + q^39 - 5*q^41 - 6*q^42 + 4*q^43 - 2*q^44 + q^45 - 6*q^46 -
        10*q^47 + 4*q^48 + 2*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^12 - q^13 + q^15 +
        q^16 - 6*q^17 - q^18 - q^20 - 4*q^23 + q^24 + q^25 + q^26 - q^27 -
        10*q^29 - q^30 - q^32 + 6*q^34 + q^36 - 6*q^37 + q^39 + q^40 + 2*q^41 -
        4*q^43 - q^45 + 4*q^46 - q^48 - 7*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 15, 26, 65, 65, 195, 390 *]
```

All possible

1.4.8. *H*8*, genus 8.* (need to eliminate, eliminate L[3] a form that appears too much repeated in the programme)

```
[*
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 - q^20 -
        3*q^21 - 2*q^22 - 4*q^23 - 3*q^24 - 4*q^25 - q^26 - 9*q^27 + q^28 +
        2*q^29 + 3*q^30 + 4*q^31 + q^32 + 6*q^33 - 3*q^34 - q^35 + 6*q^36 +
        3*q^37 + 6*q^38 + 3*q^39 - q^40 - 3*q^42 - 5*q^43 - 2*q^44 - 6*q^45 -
        4*q^46 + 13*q^47 - 3*q^48 - 6*q^49 + O(q^50),
    q + a*q^2 + q^3 + (-2*a - 1)*q^4 + (-2*a - 2)*q^5 + a*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + q^9 + (2*a - 2)*q^10 - 2*q^11 + (-2*a - 1)*q^12 - q^13 +
        (-2*a + 2)*q^14 + (-2*a - 2)*q^15 + 3*q^16 + (4*a + 6)*q^17 + a*q^18 +
        (-2*a - 2)*q^19 + (-2*a + 6)*q^20 + (2*a + 2)*q^21 - 2*a*q^22 - 4*q^23 +
```

```
        (a - 2)*q^24 + 3*q^25 - a*q^26 + q^27 + (2*a - 6)*q^28 + 2*q^29 + (2*a -
        2)*q^30 + (2*a - 2)*q^31 + (a + 4)*q^32 - 2*q^33 + (-2*a + 4)*q^34 -
        8*q^35 + (-2*a - 1)*q^36 + (-4*a - 6)*q^37 + (2*a - 2)*q^38 - q^39 +
        (6*a + 2)*q^40 + (-2*a + 6)*q^41 + (-2*a + 2)*q^42 - 4*a*q^43 + (4*a +
        2)*q^44 + (-2*a - 2)*q^45 - 4*a*q^46 + (-4*a - 10)*q^47 + 3*q^48 + q^49
        + O(q^50),
    q + q^2 + 2*q^3 + q^4 - q^5 + 2*q^6 - 4*q^7 + q^8 + q^9 - q^10 - 2*q^11 +
        2*q^12 - q^13 - 4*q^14 - 2*q^15 + q^16 + 2*q^17 + q^18 + 6*q^19 - q^20 -
        8*q^21 - 2*q^22 + 6*q^23 + 2*q^24 + q^25 - q^26 - 4*q^27 - 4*q^28 +
        2*q^29 - 2*q^30 - 6*q^31 + q^32 - 4*q^33 + 2*q^34 + 4*q^35 + q^36 -
        2*q^37 + 6*q^38 - 2*q^39 - q^40 + 10*q^41 - 8*q^42 - 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 - 12*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q + 2*q^2 + q^3 + 2*q^4 - q^5 + 2*q^6 - q^7 + q^9 - 2*q^10 + 5*q^11 + 2*q^12
        - q^13 - 2*q^14 - q^15 - 4*q^16 - 7*q^17 + 2*q^18 - 6*q^19 - 2*q^20 -
        q^21 + 10*q^22 + 3*q^23 + q^25 - 2*q^26 + q^27 - 2*q^28 + 2*q^29 -
        2*q^30 + 2*q^31 - 8*q^32 + 5*q^33 - 14*q^34 + q^35 + 2*q^36 + 7*q^37 -
        12*q^38 - q^39 + 9*q^41 - 2*q^42 - 8*q^43 + 10*q^44 - q^45 + 6*q^46 +
        10*q^47 - 4*q^48 - 6*q^49 + O(q^50),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^12 - q^13 + q^15 +
        q^16 - 6*q^17 - q^18 - q^20 - 4*q^23 + q^24 + q^25 + q^26 - q^27 -
        10*q^29 - q^30 - q^32 + 6*q^34 + q^36 - 6*q^37 + q^39 + q^40 + 2*q^41 -
        4*q^43 - q^45 + 4*q^46 - q^48 - 7*q^49 + O(q^50)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 26, 39, 65, 130, 130, 195, 195, 390 *]
```

65,130,195, $n(a_7 \geq -1; 7) \geq 20 - 18$.

1.4.9. *H*9*, genus 7.*

```
*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
```

```
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 + O(q^50),
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 - 3*q^20 -
        q^21 - 6*q^22 - q^24 + 4*q^25 - q^26 - 5*q^27 - q^28 + 6*q^29 + 3*q^30 -
        4*q^31 - q^32 + 6*q^33 + 3*q^34 + 3*q^35 - 2*q^36 - 7*q^37 - 2*q^38 +
        q^39 + 3*q^40 + q^42 - q^43 + 6*q^44 + 6*q^45 + 3*q^47 + q^48 - 6*q^49 +
        O(q^50),
    q + q^2 - q^3 - q^4 + 2*q^5 - q^6 - 4*q^7 - 3*q^8 + q^9 + 2*q^10 + 4*q^11 +
        q^12 + q^13 - 4*q^14 - 2*q^15 - q^16 + 2*q^17 + q^18 - 2*q^20 + 4*q^21 +
        4*q^22 + 3*q^24 - q^25 + q^26 - q^27 + 4*q^28 - 10*q^29 - 2*q^30 +
        4*q^31 + 5*q^32 - 4*q^33 + 2*q^34 - 8*q^35 - q^36 - 2*q^37 - q^39 -
        6*q^40 + 6*q^41 + 4*q^42 - 12*q^43 - 4*q^44 + 2*q^45 + q^48 + 9*q^49 +
        O(q^50),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 + 2*q^5 + q^6 + 4*q^7 - q^8 + q^9 - 2*q^10 - 4*q^11 -
        q^12 + q^13 - 4*q^14 - 2*q^15 + q^16 + 2*q^17 - q^18 - 8*q^19 + 2*q^20 -
        4*q^21 + 4*q^22 + q^24 - q^25 - q^26 - q^27 + 4*q^28 + 6*q^29 + 2*q^30 -
        4*q^31 - q^32 + 4*q^33 - 2*q^34 + 8*q^35 + q^36 - 2*q^37 + 8*q^38 - q^39
        - 2*q^40 - 10*q^41 + 4*q^42 + 4*q^43 - 4*q^44 + 2*q^45 + 8*q^47 - q^48 +
        9*q^49 + O(q^50),
    q - q^2 - 2*q^3 + q^4 + q^5 + 2*q^6 - 4*q^7 - q^8 + q^9 - q^10 - 6*q^11 -
        2*q^12 + q^13 + 4*q^14 - 2*q^15 + q^16 - 6*q^17 - q^18 + 2*q^19 + q^20 +
        8*q^21 + 6*q^22 + 6*q^23 + 2*q^24 + q^25 - q^26 + 4*q^27 - 4*q^28 -
        6*q^29 + 2*q^30 + 2*q^31 - q^32 + 12*q^33 + 6*q^34 - 4*q^35 + q^36 +
        2*q^37 - 2*q^38 - 2*q^39 - q^40 - 6*q^41 - 8*q^42 + 2*q^43 - 6*q^44 +
        q^45 - 6*q^46 - 12*q^47 - 2*q^48 + 9*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^12 - q^13 + q^15 +
        q^16 - 6*q^17 - q^18 - q^20 - 4*q^23 + q^24 + q^25 + q^26 - q^27 -
        10*q^29 - q^30 - q^32 + 6*q^34 + q^36 - 6*q^37 + q^39 + q^40 + 2*q^41 -
        4*q^43 - q^45 + 4*q^46 - q^48 - 7*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 15, 26, 39, 65, 78, 130, 390 *]
```

39,65,130? $n(a_7 \geq 0; 7) \geq 17 - 16$, $n(a_{11} = 6; 11) = 14 - 12$.

1.4.10. *H*10*, genus 8.*

```
[*
    q + q^2 - q^3 - q^4 + 2*q^5 - q^6 - 4*q^7 - 3*q^8 + q^9 + 2*q^10 + 4*q^11 +
        q^12 + q^13 - 4*q^14 - 2*q^15 - q^16 + 2*q^17 + q^18 - 2*q^20 + 4*q^21 +
        4*q^22 + 3*q^24 - q^25 + q^26 - q^27 + 4*q^28 - 10*q^29 - 2*q^30 +
        4*q^31 + 5*q^32 - 4*q^33 + 2*q^34 - 8*q^35 - q^36 - 2*q^37 - q^39 -
        6*q^40 + 6*q^41 + 4*q^42 - 12*q^43 - 4*q^44 + 2*q^45 + q^48 + 9*q^49 +
        O(q^50),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q + a*q^2 + (-a + 1)*q^3 + q^4 - q^5 + (a - 3)*q^6 + 2*q^7 - a*q^8 + (-2*a +
        1)*q^9 - a*q^10 + (a - 3)*q^11 + (-a + 1)*q^12 + q^13 + 2*a*q^14 + (a -
        1)*q^15 - 5*q^16 + 2*a*q^17 + (a - 6)*q^18 + (3*a - 1)*q^19 - q^20 +
        (-2*a + 2)*q^21 + (-3*a + 3)*q^22 + (a + 3)*q^23 + (-a + 3)*q^24 + q^25
        + a*q^26 + 4*q^27 + 2*q^28 + (-2*a - 6)*q^29 + (-a + 3)*q^30 + (-3*a +
        5)*q^31 - 3*a*q^32 + (4*a - 6)*q^33 + 6*q^34 - 2*q^35 + (-2*a + 1)*q^36
        - 4*q^37 + (-a + 9)*q^38 + (-a + 1)*q^39 + a*q^40 - 2*a*q^41 + (2*a -
        6)*q^42 + (3*a + 5)*q^43 + (a - 3)*q^44 + (2*a - 1)*q^45 + (3*a +
        3)*q^46 + 6*q^47 + (5*a - 5)*q^48 - 3*q^49 + O(q^50),
    q + a*q^2 - q^3 + (a^2 - 2)*q^4 - q^5 - a*q^6 + (-a^2 + 5)*q^7 + (3*a +
        2)*q^8 + q^9 - a*q^10 + (-a^2 + 5)*q^11 + (-a^2 + 2)*q^12 + q^13 + (-2*a
        - 2)*q^14 + q^15 + (a^2 + 2*a + 4)*q^16 + (a^2 - 2*a - 5)*q^17 + a*q^18
        + (-2*a + 2)*q^19 + (-a^2 + 2)*q^20 + (a^2 - 5)*q^21 + (-2*a - 2)*q^22 +
        (a^2 - 2*a - 7)*q^23 + (-3*a - 2)*q^24 + q^25 + a*q^26 - q^27 + (-2*a -
        10)*q^28 + 6*q^29 + a*q^30 + (2*a + 2)*q^31 + (2*a^2 + 5*a - 2)*q^32 +
        (a^2 - 5)*q^33 + (-2*a^2 + 2*a + 2)*q^34 + (a^2 - 5)*q^35 + (a^2 -
        2)*q^36 + (-a^2 - 2*a + 9)*q^37 + (-2*a^2 + 2*a)*q^38 - q^39 + (-3*a -
        2)*q^40 + (-a^2 + 2*a + 5)*q^41 + (2*a + 2)*q^42 + 4*a*q^43 + (-2*a -
        10)*q^44 - q^45 + (-2*a^2 + 2)*q^46 + (-2*a - 6)*q^47 + (-a^2 - 2*a -
        4)*q^48 + (-3*a^2 + 2*a + 18)*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^12 - q^13 + q^15 +
        q^16 - 6*q^17 - q^18 - q^20 - 4*q^23 + q^24 + q^25 + q^26 - q^27 -
        10*q^29 - q^30 - q^32 + 6*q^34 + q^36 - 6*q^37 + q^39 + q^40 + 2*q^41 -
        4*q^43 - q^45 + 4*q^46 - q^48 - 7*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field,
    Number Field with defining polynomial x^3 - 7*x - 2 over the Rational Field,
    Rational Field
*]
[* 39, 65, 65, 195, 390 *]
```

Could be 39,65 or 390.

1.4.11. *H*11*, genus 6.*

```
[*
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 - 3*q^20 -
        q^21 - 6*q^22 - q^24 + 4*q^25 - q^26 - 5*q^27 - q^28 + 6*q^29 + 3*q^30 -
        4*q^31 - q^32 + 6*q^33 + 3*q^34 + 3*q^35 - 2*q^36 - 7*q^37 - 2*q^38 +
        q^39 + 3*q^40 + q^42 - q^43 + 6*q^44 + 6*q^45 + 3*q^47 + q^48 - 6*q^49 +
        O(q^50),
    q - q^2 + q^3 + q^4 - q^5 - q^6 - 4*q^7 - q^8 + q^9 + q^10 + q^12 + 2*q^13 +
        4*q^14 - q^15 + q^16 + 6*q^17 - q^18 - 4*q^19 - q^20 - 4*q^21 - q^24 +
        q^25 - 2*q^26 + q^27 - 4*q^28 - 6*q^29 + q^30 + 8*q^31 - q^32 - 6*q^34 +
        4*q^35 + q^36 + 2*q^37 + 4*q^38 + 2*q^39 + q^40 - 6*q^41 + 4*q^42 -
        4*q^43 - q^45 + q^48 + 9*q^49 + O(q^50),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q + a*q^2 + (-a + 1)*q^3 + q^4 - q^5 + (a - 3)*q^6 + 2*q^7 - a*q^8 + (-2*a +
        1)*q^9 - a*q^10 + (a - 3)*q^11 + (-a + 1)*q^12 + q^13 + 2*a*q^14 + (a -
        1)*q^15 - 5*q^16 + 2*a*q^17 + (a - 6)*q^18 + (3*a - 1)*q^19 - q^20 +
        (-2*a + 2)*q^21 + (-3*a + 3)*q^22 + (a + 3)*q^23 + (-a + 3)*q^24 + q^25
        + a*q^26 + 4*q^27 + 2*q^28 + (-2*a - 6)*q^29 + (-a + 3)*q^30 + (-3*a +
        5)*q^31 - 3*a*q^32 + (4*a - 6)*q^33 + 6*q^34 - 2*q^35 + (-2*a + 1)*q^36
        - 4*q^37 + (-a + 9)*q^38 + (-a + 1)*q^39 + a*q^40 - 2*a*q^41 + (2*a -
        6)*q^42 + (3*a + 5)*q^43 + (a - 3)*q^44 + (2*a - 1)*q^45 + (3*a +
        3)*q^46 + 6*q^47 + (5*a - 5)*q^48 - 3*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^12 - q^13 + q^15 +
        q^16 - 6*q^17 - q^18 - q^20 - 4*q^23 + q^24 + q^25 + q^26 - q^27 -
        10*q^29 - q^30 - q^32 + 6*q^34 + q^36 - 6*q^37 + q^39 + q^40 + 2*q^41 -
        4*q^43 - q^45 + 4*q^46 - q^48 - 7*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field,
    Rational Field
*]
[* 26, 30, 65, 65, 390 *]
```

26,30,65, or 390...

1.4.12. *H*12*, genus 5.*

```
[*
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
```

```
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q + a*q^2 + (-a + 1)*q^3 + q^4 - q^5 + (a - 3)*q^6 + 2*q^7 - a*q^8 + (-2*a +
        1)*q^9 - a*q^10 + (a - 3)*q^11 + (-a + 1)*q^12 + q^13 + 2*a*q^14 + (a -
        1)*q^15 - 5*q^16 + 2*a*q^17 + (a - 6)*q^18 + (3*a - 1)*q^19 - q^20 +
        (-2*a + 2)*q^21 + (-3*a + 3)*q^22 + (a + 3)*q^23 + (-a + 3)*q^24 + q^25
        + a*q^26 + 4*q^27 + 2*q^28 + (-2*a - 6)*q^29 + (-a + 3)*q^30 + (-3*a +
        5)*q^31 - 3*a*q^32 + (4*a - 6)*q^33 + 6*q^34 - 2*q^35 + (-2*a + 1)*q^36
        - 4*q^37 + (-a + 9)*q^38 + (-a + 1)*q^39 + a*q^40 - 2*a*q^41 + (2*a -
        6)*q^42 + (3*a + 5)*q^43 + (a - 3)*q^44 + (2*a - 1)*q^45 + (3*a +
        3)*q^46 + 6*q^47 + (5*a - 5)*q^48 - 3*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^12 - q^13 + q^15 +
        q^16 - 6*q^17 - q^18 - q^20 - 4*q^23 + q^24 + q^25 + q^26 - q^27 -
        10*q^29 - q^30 - q^32 + 6*q^34 + q^36 - 6*q^37 + q^39 + q^40 + 2*q^41 -
        4*q^43 - q^45 + 4*q^46 - q^48 - 7*q^49 + O(q^50),
    q + q^2 + q^3 + q^4 - q^5 + q^6 + 2*q^7 + q^8 + q^9 - q^10 + q^12 + q^13 +
        2*q^14 - q^15 + q^16 + q^18 + 2*q^19 - q^20 + 2*q^21 - 6*q^23 + q^24 +
        q^25 + q^26 + q^27 + 2*q^28 - q^30 - 4*q^31 + q^32 - 2*q^35 + q^36 +
        2*q^37 + 2*q^38 + q^39 - q^40 - 6*q^41 + 2*q^42 - 4*q^43 - q^45 - 6*q^46
        + q^48 - 3*q^49 + O(q^50)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 65, 65, 390, 390 *]
```

65,390,390?

1.4.13. *H*13*, genus 6.*

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 + O(q^50),
    q + q^2 - q^3 - q^4 + 2*q^5 - q^6 - 4*q^7 - 3*q^8 + q^9 + 2*q^10 + 4*q^11 +
        q^12 + q^13 - 4*q^14 - 2*q^15 - q^16 + 2*q^17 + q^18 - 2*q^20 + 4*q^21 +
        4*q^22 + 3*q^24 - q^25 + q^26 - q^27 + 4*q^28 - 10*q^29 - 2*q^30 +
        4*q^31 + 5*q^32 - 4*q^33 + 2*q^34 - 8*q^35 - q^36 - 2*q^37 - q^39 -
        6*q^40 + 6*q^41 + 4*q^42 - 12*q^43 - 4*q^44 + 2*q^45 + q^48 + 9*q^49 +
        O(q^50),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q + q^2 + q^4 + q^5 + q^8 - 3*q^9 + q^10 + q^13 + q^16 + 2*q^17 - 3*q^18 -
        8*q^19 + q^20 - 4*q^23 + q^25 + q^26 - 2*q^29 - 4*q^31 + q^32 + 2*q^34 -
```

```
        3*q^36 + 6*q^37 - 8*q^38 + q^40 + 10*q^41 - 3*q^45 - 4*q^46 + 8*q^47 -
        7*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^12 - q^13 + q^15 +
        q^16 - 6*q^17 - q^18 - q^20 - 4*q^23 + q^24 + q^25 + q^26 - q^27 -
        10*q^29 - q^30 - q^32 + 6*q^34 + q^36 - 6*q^37 + q^39 + q^40 + 2*q^41 -
        4*q^43 - q^45 + 4*q^46 - q^48 - 7*q^49 + O(q^50),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + q^8 + q^9 + q^10 + 4*q^11 - q^12 + q^13 -
        q^15 + q^16 - 6*q^17 + q^18 + 4*q^19 + q^20 + 4*q^22 + 8*q^23 - q^24 +
        q^25 + q^26 - q^27 + 6*q^29 - q^30 - 8*q^31 + q^32 - 4*q^33 - 6*q^34 +
        q^36 - 10*q^37 + 4*q^38 - q^39 + q^40 - 6*q^41 + 4*q^43 + 4*q^44 + q^45
        + 8*q^46 - q^48 - 7*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 15, 39, 65, 130, 390, 390 *]
```

15,130,130,390, ? $n(a_7 = -4; 49) = 102 - 96$.

1.4.14. *$H14$, genus 7.*

```
[*
    q - q^2 + q^3 + q^4 - 3*q^5 - q^6 - q^7 - q^8 - 2*q^9 + 3*q^10 + 6*q^11 +
        q^12 + q^13 + q^14 - 3*q^15 + q^16 - 3*q^17 + 2*q^18 + 2*q^19 - 3*q^20 -
        q^21 - 6*q^22 - q^24 + 4*q^25 - q^26 - 5*q^27 - q^28 + 6*q^29 + 3*q^30 -
        4*q^31 - q^32 + 6*q^33 + 3*q^34 + 3*q^35 - 2*q^36 - 7*q^37 - 2*q^38 +
        q^39 + 3*q^40 + q^42 - q^43 + 6*q^44 + 6*q^45 + 3*q^47 + q^48 - 6*q^49 +
        O(q^50),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
    q - q^2 - 2*q^3 + q^4 + q^5 + 2*q^6 - 4*q^7 - q^8 + q^9 - q^10 - 6*q^11 -
        2*q^12 + q^13 + 4*q^14 - 2*q^15 + q^16 - 6*q^17 - q^18 + 2*q^19 + q^20 +
        8*q^21 + 6*q^22 + 6*q^23 + 2*q^24 + q^25 - q^26 + 4*q^27 - 4*q^28 -
        6*q^29 + 2*q^30 + 2*q^31 - q^32 + 12*q^33 + 6*q^34 - 4*q^35 + q^36 +
        2*q^37 - 2*q^38 - 2*q^39 - q^40 - 6*q^41 - 8*q^42 + 2*q^43 - 6*q^44 +
        q^45 - 6*q^46 - 12*q^47 - 2*q^48 + 9*q^49 + O(q^50),
    q - q^2 + q^3 - q^4 + q^5 - q^6 + 3*q^8 + q^9 - q^10 + 4*q^11 - q^12 + q^13
        + q^15 - q^16 + 2*q^17 - q^18 - 4*q^19 - q^20 - 4*q^22 + 8*q^23 + 3*q^24
        + q^25 - q^26 + q^27 - 2*q^29 - q^30 - 8*q^31 - 5*q^32 + 4*q^33 - 2*q^34
        - q^36 + 6*q^37 + 4*q^38 + q^39 + 3*q^40 - 6*q^41 - 4*q^43 - 4*q^44 +
        q^45 - 8*q^46 - 8*q^47 - q^48 - 7*q^49 + O(q^50),
    q + 2*q^2 + q^3 + 2*q^4 + q^5 + 2*q^6 - 3*q^7 + q^9 + 2*q^10 - 5*q^11 +
        2*q^12 + q^13 - 6*q^14 + q^15 - 4*q^16 + 5*q^17 + 2*q^18 + 2*q^19 +
```

```
        2*q^20 - 3*q^21 - 10*q^22 - q^23 + q^25 + 2*q^26 + q^27 - 6*q^28 +
        10*q^29 + 2*q^30 - 2*q^31 - 8*q^32 - 5*q^33 + 10*q^34 - 3*q^35 + 2*q^36
        - 3*q^37 + 4*q^38 + q^39 - 9*q^41 - 6*q^42 - 4*q^43 - 10*q^44 + q^45 -
        2*q^46 + 10*q^47 - 4*q^48 + 2*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^12 - q^13 + q^15 +
        q^16 - 6*q^17 - q^18 - q^20 - 4*q^23 + q^24 + q^25 + q^26 - q^27 -
        10*q^29 - q^30 - q^32 + 6*q^34 + q^36 - 6*q^37 + q^39 + q^40 + 2*q^41 -
        4*q^43 - q^45 + 4*q^46 - q^48 - 7*q^49 + O(q^50),
    q - q^2 + q^3 + q^4 + q^5 - q^6 + 2*q^7 - q^8 + q^9 - q^10 + q^12 + q^13 -
        2*q^14 + q^15 + q^16 - q^18 + 2*q^19 + q^20 + 2*q^21 - 6*q^23 - q^24 +
        q^25 - q^26 + q^27 + 2*q^28 - q^30 + 8*q^31 - q^32 + 2*q^35 + q^36 +
        2*q^37 - 2*q^38 + q^39 - q^40 + 6*q^41 - 2*q^42 - 4*q^43 + q^45 + 6*q^46
        + q^48 - 3*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 26, 65, 130, 195, 195, 390, 390 *]
```

26,195,2on? $n(a_7 = -4; 49) = 102 - 96$, $n(a_7 \geq 0; 7) > 0$.

1.4.15. *$H15$, genus 9.*

```
[*
    q + q^2 - 3*q^3 + q^4 - q^5 - 3*q^6 + q^7 + q^8 + 6*q^9 - q^10 - 2*q^11 -
        3*q^12 - q^13 + q^14 + 3*q^15 + q^16 - 3*q^17 + 6*q^18 + 6*q^19 - q^20 -
        3*q^21 - 2*q^22 - 4*q^23 - 3*q^24 - 4*q^25 - q^26 - 9*q^27 + q^28 +
        2*q^29 + 3*q^30 + 4*q^31 + q^32 + 6*q^33 - 3*q^34 - q^35 + 6*q^36 +
        3*q^37 + 6*q^38 + 3*q^39 - q^40 - 3*q^42 - 5*q^43 - 2*q^44 - 6*q^45 -
        4*q^46 + 13*q^47 - 3*q^48 - 6*q^49 + O(q^50),
    q + a*q^2 + q^3 + (-2*a - 1)*q^4 + (-2*a - 2)*q^5 + a*q^6 + (2*a + 2)*q^7 +
        (a - 2)*q^8 + q^9 + (2*a - 2)*q^10 - 2*q^11 + (-2*a - 1)*q^12 - q^13 +
        (-2*a + 2)*q^14 + (-2*a - 2)*q^15 + 3*q^16 + (4*a + 6)*q^17 + a*q^18 +
        (-2*a - 2)*q^19 + (-2*a + 6)*q^20 + (2*a + 2)*q^21 - 2*a*q^22 - 4*q^23 +
        (a - 2)*q^24 + 3*q^25 - a*q^26 + q^27 + (2*a - 6)*q^28 + 2*q^29 + (2*a -
        2)*q^30 + (2*a - 2)*q^31 + (a + 4)*q^32 - 2*q^33 + (-2*a + 4)*q^34 -
        8*q^35 + (-2*a - 1)*q^36 + (-4*a - 6)*q^37 + (2*a - 2)*q^38 - q^39 +
        (6*a + 2)*q^40 + (-2*a + 6)*q^41 + (-2*a + 2)*q^42 - 4*a*q^43 + (4*a +
        2)*q^44 + (-2*a - 2)*q^45 - 4*a*q^46 + (-4*a - 10)*q^47 + 3*q^48 + q^49
        + O(q^50),
    q - q^2 - 2*q^3 - q^4 - q^5 + 2*q^6 - 4*q^7 + 3*q^8 + q^9 + q^10 + 2*q^11 +
        2*q^12 - q^13 + 4*q^14 + 2*q^15 - q^16 + 2*q^17 - q^18 - 6*q^19 + q^20 +
        8*q^21 - 2*q^22 - 6*q^23 - 6*q^24 + q^25 + q^26 + 4*q^27 + 4*q^28 +
        2*q^29 - 2*q^30 - 10*q^31 - 5*q^32 - 4*q^33 - 2*q^34 + 4*q^35 - q^36 -
        2*q^37 + 6*q^38 + 2*q^39 - 3*q^40 - 6*q^41 - 8*q^42 + 10*q^43 - 2*q^44 -
        q^45 + 6*q^46 + 4*q^47 + 2*q^48 + 9*q^49 + O(q^50),
```

```
    q + a*q^2 + (a + 1)*q^3 + (-2*a - 1)*q^4 + q^5 + (-a + 1)*q^6 - 2*a*q^7 + (a
        - 2)*q^8 - q^9 + a*q^10 + (-a + 1)*q^11 + (a - 3)*q^12 - q^13 + (4*a -
        2)*q^14 + (a + 1)*q^15 + 3*q^16 + (-2*a - 4)*q^17 - a*q^18 + (a +
        3)*q^19 + (-2*a - 1)*q^20 + (2*a - 2)*q^21 + (3*a - 1)*q^22 + (-a -
        1)*q^23 + (-3*a - 1)*q^24 + q^25 - a*q^26 + (-4*a - 4)*q^27 + (-6*a +
        4)*q^28 + (4*a + 4)*q^29 + (-a + 1)*q^30 + (3*a + 9)*q^31 + (a + 4)*q^32
        + 2*a*q^33 - 2*q^34 - 2*a*q^35 + (2*a + 1)*q^36 + (6*a + 6)*q^37 + (a +
        1)*q^38 + (-a - 1)*q^39 + (a - 2)*q^40 + (-2*a - 8)*q^41 + (-6*a +
        2)*q^42 + (5*a + 1)*q^43 + (-5*a + 1)*q^44 - q^45 + (a - 1)*q^46 +
        2*a*q^47 + (3*a + 3)*q^48 + (-8*a - 3)*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^8 + q^9 + q^10 - q^12 - q^13 + q^15 +
        q^16 - 6*q^17 - q^18 - q^20 - 4*q^23 + q^24 + q^25 + q^26 - q^27 -
        10*q^29 - q^30 - q^32 + 6*q^34 + q^36 - 6*q^37 + q^39 + q^40 + 2*q^41 -
        4*q^43 - q^45 + 4*q^46 - q^48 - 7*q^49 + O(q^50),
    q + q^2 + q^3 + q^4 + q^5 + q^6 + a*q^7 + q^8 + q^9 + q^10 - 2*a*q^11 + q^12
        - q^13 + a*q^14 + q^15 + q^16 + (-a - 2)*q^17 + q^18 - a*q^19 + q^20 +
        a*q^21 - 2*a*q^22 + 3*a*q^23 + q^24 + q^25 - q^26 + q^27 + a*q^28 + (a -
        6)*q^29 + q^30 + 4*q^31 + q^32 - 2*a*q^33 + (-a - 2)*q^34 + a*q^35 +
        q^36 + (2*a - 6)*q^37 - a*q^38 - q^39 + q^40 + (2*a - 2)*q^41 + a*q^42 +
        (-2*a + 4)*q^43 - 2*a*q^44 + q^45 + 3*a*q^46 - 8*q^47 + q^48 + q^49 +
        O(q^50)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 8 over the Rational Field
*]
[* 26, 39, 65, 65, 390, 390 *]
```

26,390? $n(a_7 = -4; 49) = 103 - 96$.

1.5. $N = 510$.

1.5.1. *$H1$, genus 12.*

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 - q^3 + (-a + 2)*q^4 + (-a + 1)*q^5 - a*q^6 + (a - 4)*q^8 + q^9 +
        (2*a - 4)*q^10 + (-a - 1)*q^11 + (a - 2)*q^12 + (a + 3)*q^13 + (a -
        1)*q^15 - 3*a*q^16 + q^17 + a*q^18 + (3*a + 3)*q^19 + O(q^20),
    q + q^2 + 2*q^3 - q^4 - q^5 + 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^11 -
        2*q^12 + 2*q^13 - 2*q^14 - 2*q^15 - q^16 + q^17 + q^18 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 + O(q^20),
    q - q^2 - 2*q^3 + q^4 - q^5 + 2*q^6 + 2*q^7 - q^8 + q^9 + q^10 + 6*q^11 -
```

```
        2*q^12 + 2*q^13 - 2*q^14 + 2*q^15 + q^16 + q^17 - q^18 + 8*q^19 +
        O(q^20),
    q - q^2 + 3*q^3 + q^4 - q^5 - 3*q^6 + 2*q^7 - q^8 + 6*q^9 + q^10 - 4*q^11 +
        3*q^12 - 3*q^13 - 2*q^14 - 3*q^15 + q^16 + q^17 - 6*q^18 + 3*q^19 +
        O(q^20),
    q + a*q^2 - q^3 + (a + 1)*q^4 - q^5 - a*q^6 + (2*a - 1)*q^7 + 3*q^8 + q^9 -
        a*q^10 + 5*q^11 + (-a - 1)*q^12 + (-2*a - 2)*q^13 + (a + 6)*q^14 + q^15
        + (a - 2)*q^16 + q^17 + a*q^18 + (-2*a - 1)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - q^5 + q^6 + 2*q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 +
        4*q^13 - 2*q^14 + q^15 + q^16 + q^17 - q^18 - 4*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - x - 3 over the Rational Field,
    Rational Field
*]
[* 17, 51, 85, 85, 102, 170, 170, 255, 510 *] [* 2, 3, 5, 6, 10, 15,
30 *]
```

Not bielliptic, $n(|a_7| \geq 0,49) \geq 144 - 128$

1.5.2. *H2, genus 12.*

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 + q^5 + (a - 3)*q^6 + (a - 1)*q^7 - a*q^8 +
        (-2*a + 1)*q^9 + a*q^10 + (-a + 3)*q^11 + (-a + 1)*q^12 - 4*q^13 + (-a +
        3)*q^14 + (-a + 1)*q^15 - 5*q^16 - q^17 + (a - 6)*q^18 + (2*a + 2)*q^19
        + O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 + q^5 - q^6 + 2*q^7 - q^8 - 2*q^9 - q^10 + q^12 + 5*q^13
        - 2*q^14 + q^15 + q^16 - q^17 + 2*q^18 - q^19 + O(q^20),
    q + a*q^2 - q^3 + (3*a - 3)*q^4 + q^5 - a*q^6 + (-2*a + 3)*q^7 + (4*a -
        3)*q^8 + q^9 + a*q^10 + (-4*a + 7)*q^11 + (-3*a + 3)*q^12 + (-2*a +
        6)*q^13 + (-3*a + 2)*q^14 - q^15 + (3*a + 2)*q^16 - q^17 + a*q^18 + (2*a
        - 9)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 + q^5 + q^6 + a*q^7 - q^8 + q^9 - q^10 - q^12 + (-a +
        2)*q^13 - a*q^14 - q^15 + q^16 - q^17 - q^18 + 4*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 + O(q^20)
```

```
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 3*x + 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 24 over the Rational Field,
    Rational Field
*]
[* 15, 85, 85, 102, 170, 255, 510, 510 *]
```

Not bielliptic, $n(|a_7| > 0; 49) > 0$ $n(a_{11} = -4; 121) = 268 - 256$.

1.5.3. *H3, genus 8.*

```
[*
    q + q^2 - 2*q^3 + q^4 - 2*q^6 - 4*q^7 + q^8 + q^9 + 6*q^11 - 2*q^12 + 2*q^13
        - 4*q^14 + q^16 - q^17 + q^18 - 4*q^19 + 8*q^21 + 6*q^22 - 2*q^24 -
        5*q^25 + 2*q^26 + 4*q^27 - 4*q^28 - 4*q^31 + q^32 - 12*q^33 - q^34 +
        q^36 - 4*q^37 - 4*q^38 - 4*q^39 + 6*q^41 + 8*q^42 + 8*q^43 + 6*q^44 -
        2*q^48 + 9*q^49 + O(q^50),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + (2*a + 1)*q^20 + 2*q^21 + (-5*a + 1)*q^22 +
        (-a - 3)*q^23 + (a + 5)*q^24 + q^25 + (2*a - 2)*q^26 + (-8*a - 16)*q^27
        + (5*a - 1)*q^28 + (-2*a - 4)*q^29 + (a + 1)*q^30 + (3*a + 3)*q^31 + (a
        + 4)*q^32 + (2*a + 8)*q^33 - a*q^34 + (-a + 1)*q^35 + (-2*a - 15)*q^36 +
        (6*a + 4)*q^37 + (2*a - 2)*q^38 + (4*a + 8)*q^39 + (-a + 2)*q^40 + (-6*a
        - 4)*q^41 + 2*a*q^42 + (4*a + 6)*q^43 + (9*a + 1)*q^44 + (-4*a - 7)*q^45
        + (-a - 1)*q^46 + (-2*a - 4)*q^47 + (-3*a - 9)*q^48 + (-4*a - 5)*q^49 +
        O(q^50),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 - 4*q^20 + 2*q^21
        + 6*q^23 + q^24 + 11*q^25 + 6*q^26 - q^27 - 2*q^28 - 4*q^29 - 4*q^30 -
        6*q^31 - q^32 + q^34 + 8*q^35 + q^36 - 4*q^37 - 4*q^38 + 6*q^39 + 4*q^40
        - 10*q^41 - 2*q^42 - 4*q^43 - 4*q^45 - 6*q^46 + 4*q^47 - q^48 - 3*q^49 +
        O(q^50),
    q + q^2 + q^3 + q^4 - q^5 + q^6 + 2*q^7 + q^8 - 2*q^9 - q^10 + q^12 - q^13 +
        2*q^14 - q^15 + q^16 - q^17 - 2*q^18 - q^19 - q^20 + 2*q^21 - 6*q^23 +
        q^24 + q^25 - q^26 - 5*q^27 + 2*q^28 - 3*q^29 - q^30 + 5*q^31 + q^32 -
        q^34 - 2*q^35 - 2*q^36 + 8*q^37 - q^38 - q^39 - q^40 + 6*q^41 + 2*q^42 -
        10*q^43 + 2*q^45 - 6*q^46 - 3*q^47 + q^48 - 3*q^49 + O(q^50),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + (2*a + 1)*q^20 + 2*q^21 + (-5*a + 1)*q^22 +
        (-a - 3)*q^23 + (a + 5)*q^24 + q^25 + (2*a - 2)*q^26 + (-8*a - 16)*q^27
        + (5*a - 1)*q^28 + (-2*a - 4)*q^29 + (a + 1)*q^30 + (3*a + 3)*q^31 + (a
```

```
        + 4)*q^32 + (2*a + 8)*q^33 - a*q^34 + (-a + 1)*q^35 + (-2*a - 15)*q^36 +
        (6*a + 4)*q^37 + (2*a - 2)*q^38 + (4*a + 8)*q^39 + (-a + 2)*q^40 + (-6*a
        - 4)*q^41 + 2*a*q^42 + (4*a + 6)*q^43 + (9*a + 1)*q^44 + (-4*a - 7)*q^45
        + (-a - 1)*q^46 + (-2*a - 4)*q^47 + (-3*a - 9)*q^48 + (-4*a - 5)*q^49 +
        O(q^50),
    q + q^2 - q^3 + q^4 - q^5 - q^6 + 2*q^7 + q^8 + q^9 - q^10 - q^12 + 4*q^13 +
        2*q^14 + q^15 + q^16 - q^17 + q^18 + 4*q^19 - q^20 - 2*q^21 + 4*q^23 -
        q^24 + q^25 + 4*q^26 - q^27 + 2*q^28 + 2*q^29 + q^30 + q^32 - q^34 -
        2*q^35 + q^36 - 2*q^37 + 4*q^38 - 4*q^39 - q^40 - 4*q^41 - 2*q^42 +
        10*q^43 - q^45 + 4*q^46 - 8*q^47 - q^48 - 3*q^49 + O(q^50)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field
*]
[* 34, 85, 102, 170, 170, 510 *]
```

?? 102; $n(a_7 \geq 0; 7) \geq 2$, $n(a_7 = -4; 49) = 110 - 96$.

1.5.4. *H4, genus 12.*

```
[*
    q - q^2 + q^3 + q^4 - q^5 - q^6 - 4*q^7 - q^8 + q^9 + q^10 + q^12 + 2*q^13 +
        4*q^14 - q^15 + q^16 + 6*q^17 - q^18 - 4*q^19 + O(q^20),
    q + q^3 - 2*q^4 + 3*q^5 - 4*q^7 + q^9 - 3*q^11 - 2*q^12 - q^13 + 3*q^15 +
        4*q^16 - q^17 - q^19 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 + O(q^20),
    q - q^2 + q^3 + q^4 - q^6 + 2*q^7 - q^8 + q^9 + q^12 + 2*q^13 - 2*q^14 +
        q^16 - q^17 - q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 - q^5 + a*q^6 + (-a^3 - a^2 + 5*a + 5)*q^7 +
        (a^3 - 4*a)*q^8 + q^9 - a*q^10 + (a^3 + a^2 - 7*a - 3)*q^11 + (a^2 -
        2)*q^12 + (-2*a^2 + 8)*q^13 + (-2*a^3 - 3*a^2 + 12*a + 9)*q^14 - q^15 +
        (a^3 + 2*a^2 - 7*a - 5)*q^16 - q^17 + a*q^18 + (a^3 + a^2 - 5*a -
        1)*q^19 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
```

```
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^4 - x^3 - 8*x^2 + 7*x + 9 over the
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 30, 51, 85, 102, 102, 255, 255 *]
```

Not bielliptic, $n(a_{11} = 0; 11) = 29 - 24$,$n(a_7 \geq -1; 7) \geq 20 - 18$, $n(|a_7| \geq 4; 49) \geq 104 - 96$.

1.5.5. *H*5, *genus 10.*

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + O(q^20),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 + O(q^20),
    q + q^2 + q^3 + q^4 - 2*q^5 + q^6 + q^8 + q^9 - 2*q^10 - 4*q^11 + q^12 -
        2*q^13 - 2*q^15 + q^16 + q^17 + q^18 + 4*q^19 + O(q^20),
    q + q^2 + a*q^3 + q^4 + q^5 + a*q^6 - 2*a*q^7 + q^8 + (-a + 1)*q^9 + q^10 -
        4*q^11 + a*q^12 + (-a + 2)*q^13 - 2*a*q^14 + a*q^15 + q^16 + q^17 + (-a
        + 1)*q^18 + a*q^19 + O(q^20),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + q^5 + a*q^6 + (-a^2 - a + 4)*q^7 - q^8 +
        q^9 + a*q^10 + (-a^2 + a + 2)*q^11 + (a^2 - 2)*q^12 + (2*a^2 - 4)*q^13 +
        (-a^2 + 1)*q^14 + q^15 + (-2*a^2 - a + 4)*q^16 + q^17 + a*q^18 + (-3*a^2
        - 3*a + 8)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Number Field with defining polynomial x^3 - 4*x + 1 over the Rational Field
*]
[* 17, 85, 102, 102, 170, 255 *]
```

Not bielliptic $n(a_{11} \geq -4; 11) \geq 34 - 32$

1.5.6. *H*6, *genus 8.*

```
[*
    q + q^3 - 2*q^4 + 3*q^5 - 4*q^7 + q^9 - 3*q^11 - 2*q^12 - q^13 + 3*q^15 +
        4*q^16 - q^17 - q^19 - 6*q^20 - 4*q^21 + 9*q^23 + 4*q^25 + q^27 + 8*q^28
        + 6*q^29 + 2*q^31 - 3*q^33 - 12*q^35 - 2*q^36 - 4*q^37 - q^39 - 3*q^41 -
        7*q^43 + 6*q^44 + 3*q^45 - 6*q^47 + 4*q^48 + 9*q^49 + O(q^50),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + (2*a + 1)*q^20 + 2*q^21 + (-5*a + 1)*q^22 +
        (-a - 3)*q^23 + (a + 5)*q^24 + q^25 + (2*a - 2)*q^26 + (-8*a - 16)*q^27
```

```
        + (5*a - 1)*q^28 + (-2*a - 4)*q^29 + (a + 1)*q^30 + (3*a + 3)*q^31 + (a
        + 4)*q^32 + (2*a + 8)*q^33 - a*q^34 + (-a + 1)*q^35 + (-2*a - 15)*q^36 +
        (6*a + 4)*q^37 + (2*a - 2)*q^38 + (4*a + 8)*q^39 + (-a + 2)*q^40 + (-6*a
        - 4)*q^41 + 2*a*q^42 + (4*a + 6)*q^43 + (9*a + 1)*q^44 + (-4*a - 7)*q^45
        + (-a - 1)*q^46 + (-2*a - 4)*q^47 + (-3*a - 9)*q^48 + (-4*a - 5)*q^49 +
        O(q^50),
    q + a*q^2 + (-a + 1)*q^3 + q^4 + q^5 + (a - 3)*q^6 + (a - 1)*q^7 - a*q^8 +
        (-2*a + 1)*q^9 + a*q^10 + (-a + 3)*q^11 + (-a + 1)*q^12 - 4*q^13 + (-a +
        3)*q^14 + (-a + 1)*q^15 - 5*q^16 - q^17 + (a - 6)*q^18 + (2*a + 2)*q^19
        + q^20 + (2*a - 4)*q^21 + (3*a - 3)*q^22 + (3*a - 3)*q^23 + (-a +
        3)*q^24 + q^25 - 4*a*q^26 + 4*q^27 + (a - 1)*q^28 + 2*a*q^29 + (a -
        3)*q^30 + (a + 5)*q^31 - 3*a*q^32 + (-4*a + 6)*q^33 - a*q^34 + (a -
        1)*q^35 + (-2*a + 1)*q^36 + (-2*a - 4)*q^37 + (2*a + 6)*q^38 + (4*a -
        4)*q^39 - a*q^40 + 2*a*q^41 + (-4*a + 6)*q^42 + (-2*a - 4)*q^43 + (-a +
        3)*q^44 + (-2*a + 1)*q^45 + (-3*a + 9)*q^46 + (-4*a + 6)*q^47 + (5*a -
        5)*q^48 + (-2*a - 3)*q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 - 4*q^20 + 2*q^21
        + 6*q^23 + q^24 + 11*q^25 + 6*q^26 - q^27 - 2*q^28 - 4*q^29 - 4*q^30 -
        6*q^31 - q^32 + q^34 + 8*q^35 + q^36 - 4*q^37 - 4*q^38 + 6*q^39 + 4*q^40
        - 10*q^41 - 2*q^42 - 4*q^43 - 4*q^45 - 6*q^46 + 4*q^47 - q^48 - 3*q^49 +
        O(q^50),
    q - q^2 + q^3 + q^4 - q^6 + 2*q^7 - q^8 + q^9 + q^12 + 2*q^13 - 2*q^14 +
        q^16 - q^17 - q^18 - 4*q^19 + 2*q^21 - 6*q^23 - q^24 - 5*q^25 - 2*q^26 +
        q^27 + 2*q^28 - 10*q^31 - q^32 + q^34 + q^36 + 8*q^37 + 4*q^38 + 2*q^39
        + 6*q^41 - 2*q^42 - 4*q^43 + 6*q^46 + 12*q^47 + q^48 - 3*q^49 + O(q^50),
    q - q^2 + q^3 + q^4 + q^5 - q^6 + 2*q^7 - q^8 - 2*q^9 - q^10 + q^12 + 5*q^13
        - 2*q^14 + q^15 + q^16 - q^17 + 2*q^18 - q^19 + q^20 + 2*q^21 + 6*q^23 -
        q^24 + q^25 - 5*q^26 - 5*q^27 + 2*q^28 - 9*q^29 - q^30 - q^31 - q^32 +
        q^34 + 2*q^35 - 2*q^36 - 4*q^37 + q^38 + 5*q^39 - q^40 - 6*q^41 - 2*q^42
        + 2*q^43 - 2*q^45 - 6*q^46 - 9*q^47 + q^48 - 3*q^49 + O(q^50)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 51, 85, 85, 102, 102, 170 *]
```

???102 $(a_{13} = -6!)!$, $n(a_7 \geq 1; 7) > 0$, $n(a_7 = -4; 49) = 114 - 96$.

1.5.7. *H7, genus 9.*

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 + O(q^20),
    q + q^2 - 2*q^3 + q^4 - 2*q^6 - 4*q^7 + q^8 + q^9 + 6*q^11 - 2*q^12 + 2*q^13
        - 4*q^14 + q^16 - q^17 + q^18 - 4*q^19 + O(q^20),
```

```
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + O(q^20),
    q + a*q^2 + (-a + 1)*q^3 + q^4 + q^5 + (a - 3)*q^6 + (a - 1)*q^7 - a*q^8 +
        (-2*a + 1)*q^9 + a*q^10 + (-a + 3)*q^11 + (-a + 1)*q^12 - 4*q^13 + (-a +
        3)*q^14 + (-a + 1)*q^15 - 5*q^16 - q^17 + (a - 6)*q^18 + (2*a + 2)*q^19
        + O(q^20),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 + O(q^20),
    q + a*q^2 - q^3 + (3*a - 3)*q^4 + q^5 - a*q^6 + (-2*a + 3)*q^7 + (4*a -
        3)*q^8 + q^9 + a*q^10 + (-4*a + 7)*q^11 + (-3*a + 3)*q^12 + (-2*a +
        6)*q^13 + (-3*a + 2)*q^14 - q^15 + (3*a + 2)*q^16 - q^17 + a*q^18 + (2*a
        - 9)*q^19 + O(q^20)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 3*x + 1 over the Rational Field
*]
[* 15, 34, 85, 85, 102, 255 *]
```

Not bielliptic, $n(a_7 \geq -1; 7) \geq 20 - 18$, $n(|a_7| \geq 2; 49) \geq 126 - 120$.

1.5.8. *H*8*, genus 12.*

```
(Modificat programa, tret el tercer que es repetia 3
cops, per els nivells no estan modificats!!!)

[*
    q + q^2 - 2*q^3 + q^4 - 2*q^6 - 4*q^7 + q^8 + q^9 + 6*q^11 - 2*q^12 + 2*q^13
        - 4*q^14 + q^16 - q^17 + q^18 - 4*q^19 + 8*q^21 + 6*q^22 - 2*q^24 -
        5*q^25 + 2*q^26 + 4*q^27 - 4*q^28 - 4*q^31 + q^32 - 12*q^33 - q^34 +
        q^36 - 4*q^37 - 4*q^38 - 4*q^39 + 6*q^41 + 8*q^42 + 8*q^43 + 6*q^44 -
        2*q^48 + 9*q^49 - 5*q^50 + 2*q^51 + 2*q^52 - 6*q^53 + 4*q^54 - 4*q^56 +
        8*q^57 - 4*q^61 - 4*q^62 - 4*q^63 + q^64 - 12*q^66 + 8*q^67 - q^68 +
        O(q^70),
    q + q^3 - 2*q^4 + 3*q^5 - 4*q^7 + q^9 - 3*q^11 - 2*q^12 - q^13 + 3*q^15 +
        4*q^16 - q^17 - q^19 - 6*q^20 - 4*q^21 + 9*q^23 + 4*q^25 + q^27 + 8*q^28
        + 6*q^29 + 2*q^31 - 3*q^33 - 12*q^35 - 2*q^36 - 4*q^37 - q^39 - 3*q^41 -
        7*q^43 + 6*q^44 + 3*q^45 - 6*q^47 + 4*q^48 + 9*q^49 - q^51 + 2*q^52 -
        6*q^53 - 9*q^55 - q^57 + 6*q^59 - 6*q^60 + 8*q^61 - 4*q^63 - 8*q^64 -
        3*q^65 - 4*q^67 + 2*q^68 + 9*q^69 + O(q^70),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 - 4*q^20 + 2*q^21
        + 6*q^23 + q^24 + 11*q^25 + 6*q^26 - q^27 - 2*q^28 - 4*q^29 - 4*q^30 -
        6*q^31 - q^32 + q^34 + 8*q^35 + q^36 - 4*q^37 - 4*q^38 + 6*q^39 + 4*q^40
        - 10*q^41 - 2*q^42 - 4*q^43 - 4*q^45 - 6*q^46 + 4*q^47 - q^48 - 3*q^49 -
        11*q^50 + q^51 - 6*q^52 - 2*q^53 + q^54 + 2*q^56 - 4*q^57 + 4*q^58 +
```

```
    12*q^59 + 4*q^60 - 4*q^61 + 6*q^62 - 2*q^63 + q^64 + 24*q^65 - 12*q^67 -
    q^68 - 6*q^69 + O(q^70),
q + q^2 + q^3 + q^4 - q^5 + q^6 + 2*q^7 + q^8 - 2*q^9 - q^10 + q^12 - q^13 +
    2*q^14 - q^15 + q^16 - q^17 - 2*q^18 - q^19 - q^20 + 2*q^21 - 6*q^23 +
    q^24 + q^25 - q^26 - 5*q^27 + 2*q^28 - 3*q^29 - q^30 + 5*q^31 + q^32 -
    q^34 - 2*q^35 - 2*q^36 + 8*q^37 - q^38 - q^39 - q^40 + 6*q^41 + 2*q^42 -
    10*q^43 + 2*q^45 - 6*q^46 - 3*q^47 + q^48 - 3*q^49 + q^50 - q^51 - q^52
    - 3*q^53 - 5*q^54 + 2*q^56 - q^57 - 3*q^58 + 3*q^59 - q^60 + 11*q^61 +
    5*q^62 - 4*q^63 + q^64 + q^65 + 2*q^67 - q^68 - 6*q^69 + O(q^70),
q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
    + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
    (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
    4)*q^18 + (-2*a - 2)*q^19 + (2*a + 1)*q^20 + 2*q^21 + (-5*a + 1)*q^22 +
    (-a - 3)*q^23 + (a + 5)*q^24 + q^25 + (2*a - 2)*q^26 + (-8*a - 16)*q^27
    + (5*a - 1)*q^28 + (-2*a - 4)*q^29 + (a + 1)*q^30 + (3*a + 3)*q^31 + (a
    + 4)*q^32 + (2*a + 8)*q^33 - a*q^34 + (-a + 1)*q^35 + (-2*a - 15)*q^36 +
    (6*a + 4)*q^37 + (2*a - 2)*q^38 + (4*a + 8)*q^39 + (-a + 2)*q^40 + (-6*a
    - 4)*q^41 + 2*a*q^42 + (4*a + 6)*q^43 + (9*a + 1)*q^44 + (-4*a - 7)*q^45
    + (-a - 1)*q^46 + (-2*a - 4)*q^47 + (-3*a - 9)*q^48 + (-4*a - 5)*q^49 +
    a*q^50 + (a + 3)*q^51 + (-2*a + 6)*q^52 + (-4*a + 2)*q^53 - 8*q^54 + (-a
    + 3)*q^55 + (-5*a + 3)*q^56 + (4*a + 8)*q^57 - 2*q^58 + (2*a - 10)*q^59
    + (-3*a - 5)*q^60 + (4*a + 6)*q^61 + (-3*a + 3)*q^62 + (-5*a - 3)*q^63 +
    (2*a - 5)*q^64 + (2*a + 2)*q^65 + (4*a + 2)*q^66 + (2*a - 4)*q^67 + (2*a
    + 1)*q^68 + (4*a + 10)*q^69 + O(q^70),
q + a*q^2 + q^3 + (a^2 - 2)*q^4 - q^5 + a*q^6 + (-a^3 - a^2 + 5*a + 5)*q^7 +
    (a^3 - 4*a)*q^8 + q^9 - a*q^10 + (a^3 + a^2 - 7*a - 3)*q^11 + (a^2 -
    2)*q^12 + (-2*a^2 + 8)*q^13 + (-2*a^3 - 3*a^2 + 12*a + 9)*q^14 - q^15 +
    (a^3 + 2*a^2 - 7*a - 5)*q^16 - q^17 + a*q^18 + (a^3 + a^2 - 5*a -
    1)*q^19 + (-a^2 + 2)*q^20 + (-a^3 - a^2 + 5*a + 5)*q^21 + (2*a^3 + a^2 -
    10*a - 9)*q^22 + (-2*a^3 + 10*a)*q^23 + (a^3 - 4*a)*q^24 + q^25 +
    (-2*a^3 + 8*a)*q^26 + q^27 + (-3*a^3 - 2*a^2 + 13*a + 8)*q^28 + (a^3 +
    a^2 - 5*a - 3)*q^29 - a*q^30 + (-2*a + 2)*q^31 + (a^3 + a^2 - 4*a -
    9)*q^32 + (a^3 + a^2 - 7*a - 3)*q^33 - a*q^34 + (a^3 + a^2 - 5*a -
    5)*q^35 + (a^2 - 2)*q^36 + (a^3 + 3*a^2 - 5*a - 13)*q^37 + (2*a^3 +
    3*a^2 - 8*a - 9)*q^38 + (-2*a^2 + 8)*q^39 + (-a^3 + 4*a)*q^40 + (a^3 +
    a^2 - 9*a - 3)*q^41 + (-2*a^3 - 3*a^2 + 12*a + 9)*q^42 + (2*a^3 - 12*a +
    2)*q^43 + (a^3 + 4*a^2 - 9*a - 12)*q^44 - q^45 + (-2*a^3 - 6*a^2 + 14*a
    + 18)*q^46 + (-a^3 - a^2 + 7*a + 3)*q^47 + (a^3 + 2*a^2 - 7*a - 5)*q^48
    + (-a^3 + a^2 + 9*a)*q^49 + a*q^50 - q^51 + (-2*a^3 - 4*a^2 + 14*a +
    2)*q^52 + (-a^3 - a^2 + 9*a + 3)*q^53 + a*q^54 + (-a^3 - a^2 + 7*a +
    3)*q^55 + (-a^3 - 5*a^2 + 5*a + 9)*q^56 + (a^3 + a^2 - 5*a - 1)*q^57 +
    (2*a^3 + 3*a^2 - 10*a - 9)*q^58 + (-2*a^2 + 6)*q^59 + (-a^2 + 2)*q^60 +
    (-2*a^2 + 8)*q^61 + (-2*a^2 + 2*a)*q^62 + (-a^3 - a^2 + 5*a + 5)*q^63 +
    (-2*a + 1)*q^64 + (2*a^2 - 8)*q^65 + (2*a^3 + a^2 - 10*a - 9)*q^66 +
    (-2*a^3 + 2*a^2 + 12*a - 4)*q^67 + (-a^2 + 2)*q^68 + (-2*a^3 +
    10*a)*q^69 + O(q^70),
q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
    + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
    (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
```

```
        4)*q^18 + (-2*a - 2)*q^19 + (2*a + 1)*q^20 + 2*q^21 + (-5*a + 1)*q^22 +
        (-a - 3)*q^23 + (a + 5)*q^24 + q^25 + (2*a - 2)*q^26 + (-8*a - 16)*q^27
        + (5*a - 1)*q^28 + (-2*a - 4)*q^29 + (a + 1)*q^30 + (3*a + 3)*q^31 + (a
        + 4)*q^32 + (2*a + 8)*q^33 - a*q^34 + (-a + 1)*q^35 + (-2*a - 15)*q^36 +
        (6*a + 4)*q^37 + (2*a - 2)*q^38 + (4*a + 8)*q^39 + (-a + 2)*q^40 + (-6*a
        - 4)*q^41 + 2*a*q^42 + (4*a + 6)*q^43 + (9*a + 1)*q^44 + (-4*a - 7)*q^45
        + (-a - 1)*q^46 + (-2*a - 4)*q^47 + (-3*a - 9)*q^48 + (-4*a - 5)*q^49 +
        a*q^50 + (a + 3)*q^51 + (-2*a + 6)*q^52 + (-4*a + 2)*q^53 - 8*q^54 + (-a
        + 3)*q^55 + (-5*a + 3)*q^56 + (4*a + 8)*q^57 - 2*q^58 + (2*a - 10)*q^59
        + (-3*a - 5)*q^60 + (4*a + 6)*q^61 + (-3*a + 3)*q^62 + (-5*a - 3)*q^63 +
        (2*a - 5)*q^64 + (2*a + 2)*q^65 + (4*a + 2)*q^66 + (2*a - 4)*q^67 + (2*a
        + 1)*q^68 + (4*a + 10)*q^69 + O(q^70)
*]


[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^4 - x^3 - 8*x^2 + 7*x + 9 over the
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field
*]
[* 34, 51, 85, 102, 170, 170, 255, 255 *]
```

102,(??) $n(a_7 \geq -1; 7) \geq 20 - 18$, $n(a_7 = -4; 49) = 104 - 96$

1.5.9. *H*9*, genus 8.*

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 + O(q^50),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + 2*q^20 + 4*q^23 - q^25 + 2*q^26 - 4*q^28
        + 6*q^29 + 4*q^31 - 5*q^32 - q^34 - 8*q^35 + 3*q^36 - 2*q^37 + 4*q^38 -
        6*q^40 - 6*q^41 + 4*q^43 + 6*q^45 - 4*q^46 + 9*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-a + 2)*q^4 + (-a + 1)*q^5 - a*q^6 + (a - 4)*q^8 + q^9 +
        (2*a - 4)*q^10 + (-a - 1)*q^11 + (a - 2)*q^12 + (a + 3)*q^13 + (a -
        1)*q^15 - 3*a*q^16 + q^17 + a*q^18 + (3*a + 3)*q^19 + (-4*a + 6)*q^20 -
        4*q^22 + (-a - 5)*q^23 + (-a + 4)*q^24 - 3*a*q^25 + (2*a + 4)*q^26 -
        q^27 + (4*a + 2)*q^29 + (-2*a + 4)*q^30 + (-2*a - 2)*q^31 + (a - 4)*q^32
        + (a + 1)*q^33 + a*q^34 + (-a + 2)*q^36 + 2*a*q^37 + 12*q^38 + (-a -
        3)*q^39 + (6*a - 8)*q^40 + (a - 1)*q^41 + (-3*a - 3)*q^43 + (-2*a +
        2)*q^44 + (-a + 1)*q^45 + (-4*a - 4)*q^46 + (2*a - 6)*q^47 + 3*a*q^48 -
        7*q^49 + O(q^50),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
```

```
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + (2*a + 1)*q^20 + 2*q^21 + (-5*a + 1)*q^22 +
        (-a - 3)*q^23 + (a + 5)*q^24 + q^25 + (2*a - 2)*q^26 + (-8*a - 16)*q^27
        + (5*a - 1)*q^28 + (-2*a - 4)*q^29 + (a + 1)*q^30 + (3*a + 3)*q^31 + (a
        + 4)*q^32 + (2*a + 8)*q^33 - a*q^34 + (-a + 1)*q^35 + (-2*a - 15)*q^36 +
        (6*a + 4)*q^37 + (2*a - 2)*q^38 + (4*a + 8)*q^39 + (-a + 2)*q^40 + (-6*a
        - 4)*q^41 + 2*a*q^42 + (4*a + 6)*q^43 + (9*a + 1)*q^44 + (-4*a - 7)*q^45
        + (-a - 1)*q^46 + (-2*a - 4)*q^47 + (-3*a - 9)*q^48 + (-4*a - 5)*q^49 +
        O(q^50),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 - 4*q^20 + 2*q^21
        + 6*q^23 + q^24 + 11*q^25 + 6*q^26 - q^27 - 2*q^28 - 4*q^29 - 4*q^30 -
        6*q^31 - q^32 + q^34 + 8*q^35 + q^36 - 4*q^37 - 4*q^38 + 6*q^39 + 4*q^40
        - 10*q^41 - 2*q^42 - 4*q^43 - 4*q^45 - 6*q^46 + 4*q^47 - q^48 - 3*q^49 +
        O(q^50),
    q - q^2 - 2*q^3 + q^4 + q^5 + 2*q^6 - 2*q^7 - q^8 + q^9 - q^10 - 2*q^11 -
        2*q^12 - 6*q^13 + 2*q^14 - 2*q^15 + q^16 + q^17 - q^18 - 8*q^19 + q^20 +
        4*q^21 + 2*q^22 - 2*q^23 + 2*q^24 + q^25 + 6*q^26 + 4*q^27 - 2*q^28 +
        6*q^29 + 2*q^30 - 2*q^31 - q^32 + 4*q^33 - q^34 - 2*q^35 + q^36 + 6*q^37
        + 8*q^38 + 12*q^39 - q^40 + 2*q^41 - 4*q^42 - 4*q^43 - 2*q^44 + q^45 +
        2*q^46 + 4*q^47 - 2*q^48 - 3*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 15, 17, 51, 85, 102, 170 *]
```

15???, $n(|a_7| \geq 2; 49) > 0$,

1.5.10. *H_{10}, genus 10.*

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + 2*q^20 + 4*q^23 - q^25 + 2*q^26 - 4*q^28
        + 6*q^29 + 4*q^31 - 5*q^32 - q^34 - 8*q^35 + 3*q^36 - 2*q^37 + 4*q^38 -
        6*q^40 - 6*q^41 + 4*q^43 + 6*q^45 - 4*q^46 + 9*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-a + 2)*q^4 + (-a + 1)*q^5 - a*q^6 + (a - 4)*q^8 + q^9 +
        (2*a - 4)*q^10 + (-a - 1)*q^11 + (a - 2)*q^12 + (a + 3)*q^13 + (a -
        1)*q^15 - 3*a*q^16 + q^17 + a*q^18 + (3*a + 3)*q^19 + (-4*a + 6)*q^20 -
        4*q^22 + (-a - 5)*q^23 + (-a + 4)*q^24 - 3*a*q^25 + (2*a + 4)*q^26 -
        q^27 + (4*a + 2)*q^29 + (-2*a + 4)*q^30 + (-2*a - 2)*q^31 + (a - 4)*q^32
        + (a + 1)*q^33 + a*q^34 + (-a + 2)*q^36 + 2*a*q^37 + 12*q^38 + (-a -
        3)*q^39 + (6*a - 8)*q^40 + (a - 1)*q^41 + (-3*a - 3)*q^43 + (-2*a +
        2)*q^44 + (-a + 1)*q^45 + (-4*a - 4)*q^46 + (2*a - 6)*q^47 + 3*a*q^48 -
        7*q^49 + O(q^50),
    q + q^2 + 2*q^3 - q^4 - q^5 + 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^11 -
        2*q^12 + 2*q^13 - 2*q^14 - 2*q^15 - q^16 + q^17 + q^18 + q^20 - 4*q^21 +
```

```
        2*q^22 + 6*q^23 - 6*q^24 + q^25 + 2*q^26 - 4*q^27 + 2*q^28 - 6*q^29 -
        2*q^30 - 10*q^31 + 5*q^32 + 4*q^33 + q^34 + 2*q^35 - q^36 + 2*q^37 +
        4*q^39 + 3*q^40 + 10*q^41 - 4*q^42 + 4*q^43 - 2*q^44 - q^45 + 6*q^46 +
        12*q^47 - 2*q^48 - 3*q^49 + O(q^50),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + (2*a + 1)*q^20 + 2*q^21 + (-5*a + 1)*q^22 +
        (-a - 3)*q^23 + (a + 5)*q^24 + q^25 + (2*a - 2)*q^26 + (-8*a - 16)*q^27
        + (5*a - 1)*q^28 + (-2*a - 4)*q^29 + (a + 1)*q^30 + (3*a + 3)*q^31 + (a
        + 4)*q^32 + (2*a + 8)*q^33 - a*q^34 + (-a + 1)*q^35 + (-2*a - 15)*q^36 +
        (6*a + 4)*q^37 + (2*a - 2)*q^38 + (4*a + 8)*q^39 + (-a + 2)*q^40 + (-6*a
        - 4)*q^41 + 2*a*q^42 + (4*a + 6)*q^43 + (9*a + 1)*q^44 + (-4*a - 7)*q^45
        + (-a - 1)*q^46 + (-2*a - 4)*q^47 + (-3*a - 9)*q^48 + (-4*a - 5)*q^49 +
        O(q^50),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 - 4*q^20 + 2*q^21
        + 6*q^23 + q^24 + 11*q^25 + 6*q^26 - q^27 - 2*q^28 - 4*q^29 - 4*q^30 -
        6*q^31 - q^32 + q^34 + 8*q^35 + q^36 - 4*q^37 - 4*q^38 + 6*q^39 + 4*q^40
        - 10*q^41 - 2*q^42 - 4*q^43 - 4*q^45 - 6*q^46 + 4*q^47 - q^48 - 3*q^49 +
        O(q^50),
    q + a*q^2 - q^3 + (a + 1)*q^4 - q^5 - a*q^6 + (2*a - 1)*q^7 + 3*q^8 + q^9 -
        a*q^10 + 5*q^11 + (-a - 1)*q^12 + (-2*a - 2)*q^13 + (a + 6)*q^14 + q^15
        + (a - 2)*q^16 + q^17 + a*q^18 + (-2*a - 1)*q^19 + (-a - 1)*q^20 + (-2*a
        + 1)*q^21 + 5*a*q^22 + (-2*a + 2)*q^23 - 3*q^24 + q^25 + (-4*a - 6)*q^26
        - q^27 + (3*a + 5)*q^28 + (-2*a + 5)*q^29 + a*q^30 + (-2*a - 2)*q^31 +
        (-a - 3)*q^32 - 5*q^33 + a*q^34 + (-2*a + 1)*q^35 + (a + 1)*q^36 + (-4*a
        + 3)*q^37 + (-3*a - 6)*q^38 + (2*a + 2)*q^39 - 3*q^40 + (2*a + 5)*q^41 +
        (-a - 6)*q^42 + 4*a*q^43 + (5*a + 5)*q^44 - q^45 - 6*q^46 + (4*a -
        7)*q^47 + (-a + 2)*q^48 + 6*q^49 + O(q^50),
    q + q^2 - q^3 + q^4 - q^5 - q^6 - 4*q^7 + q^8 + q^9 - q^10 - 4*q^11 - q^12 -
        2*q^13 - 4*q^14 + q^15 + q^16 + q^17 + q^18 - 4*q^19 - q^20 + 4*q^21 -
        4*q^22 - 4*q^23 - q^24 + q^25 - 2*q^26 - q^27 - 4*q^28 + 2*q^29 + q^30 +
        4*q^31 + q^32 + 4*q^33 + q^34 + 4*q^35 + q^36 - 6*q^37 - 4*q^38 + 2*q^39
        - q^40 + 2*q^41 + 4*q^42 - 12*q^43 - 4*q^44 - q^45 - 4*q^46 + 8*q^47 -
        q^48 + 9*q^49 + O(q^50)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - x - 3 over the Rational Field,
    Rational Field
*]
[* 17, 51, 85, 85, 102, 255, 510 *]
```

??85,102, $n(a_7 = \pm 4; 49) = 112 - 96$.

1.5.11. *H_{11}, genus 8.*

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + 2*q^20 + 4*q^23 - q^25 + 2*q^26 - 4*q^28
        + 6*q^29 + 4*q^31 - 5*q^32 - q^34 - 8*q^35 + 3*q^36 - 2*q^37 + 4*q^38 -
        6*q^40 - 6*q^41 + 4*q^43 + 6*q^45 - 4*q^46 + 9*q^49 + O(q^50),
    q - q^2 + q^3 + q^4 - q^5 - q^6 - 4*q^7 - q^8 + q^9 + q^10 + q^12 + 2*q^13 +
        4*q^14 - q^15 + q^16 + 6*q^17 - q^18 - 4*q^19 - q^20 - 4*q^21 - q^24 +
        q^25 - 2*q^26 + q^27 - 4*q^28 - 6*q^29 + q^30 + 8*q^31 - q^32 - 6*q^34 +
        4*q^35 + q^36 + 2*q^37 + 4*q^38 + 2*q^39 + q^40 - 6*q^41 + 4*q^42 -
        4*q^43 - q^45 + q^48 + 9*q^49 + O(q^50),
    q + q^2 + 2*q^3 - q^4 - q^5 + 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^11 -
        2*q^12 + 2*q^13 - 2*q^14 - 2*q^15 - q^16 + q^17 + q^18 + q^20 - 4*q^21 +
        2*q^22 + 6*q^23 - 6*q^24 + q^25 + 2*q^26 - 4*q^27 + 2*q^28 - 6*q^29 -
        2*q^30 - 10*q^31 + 5*q^32 + 4*q^33 + q^34 + 2*q^35 - q^36 + 2*q^37 +
        4*q^39 + 3*q^40 + 10*q^41 - 4*q^42 + 4*q^43 - 2*q^44 - q^45 + 6*q^46 +
        12*q^47 - 2*q^48 - 3*q^49 + O(q^50),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + (2*a + 1)*q^20 + 2*q^21 + (-5*a + 1)*q^22 +
        (-a - 3)*q^23 + (a + 5)*q^24 + q^25 + (2*a - 2)*q^26 + (-8*a - 16)*q^27
        + (5*a - 1)*q^28 + (-2*a - 4)*q^29 + (a + 1)*q^30 + (3*a + 3)*q^31 + (a
        + 4)*q^32 + (2*a + 8)*q^33 - a*q^34 + (-a + 1)*q^35 + (-2*a - 15)*q^36 +
        (6*a + 4)*q^37 + (2*a - 2)*q^38 + (4*a + 8)*q^39 + (-a + 2)*q^40 + (-6*a
        - 4)*q^41 + 2*a*q^42 + (4*a + 6)*q^43 + (9*a + 1)*q^44 + (-4*a - 7)*q^45
        + (-a - 1)*q^46 + (-2*a - 4)*q^47 + (-3*a - 9)*q^48 + (-4*a - 5)*q^49 +
        O(q^50),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 - 4*q^20 + 2*q^21
        + 6*q^23 + q^24 + 11*q^25 + 6*q^26 - q^27 - 2*q^28 - 4*q^29 - 4*q^30 -
        6*q^31 - q^32 + q^34 + 8*q^35 + q^36 - 4*q^37 - 4*q^38 + 6*q^39 + 4*q^40
        - 10*q^41 - 2*q^42 - 4*q^43 - 4*q^45 - 6*q^46 + 4*q^47 - q^48 - 3*q^49 +
        O(q^50),
    q - q^2 - 2*q^3 + q^4 - q^5 + 2*q^6 + 2*q^7 - q^8 + q^9 + q^10 + 6*q^11 -
        2*q^12 + 2*q^13 - 2*q^14 + 2*q^15 + q^16 + q^17 - q^18 + 8*q^19 - q^20 -
        4*q^21 - 6*q^22 - 6*q^23 + 2*q^24 + q^25 - 2*q^26 + 4*q^27 + 2*q^28 -
        6*q^29 - 2*q^30 + 2*q^31 - q^32 - 12*q^33 - q^34 - 2*q^35 + q^36 +
        2*q^37 - 8*q^38 - 4*q^39 + q^40 - 6*q^41 + 4*q^42 - 4*q^43 + 6*q^44 -
        q^45 + 6*q^46 + 12*q^47 - 2*q^48 - 3*q^49 + O(q^50),
    q - q^2 + 3*q^3 + q^4 - q^5 - 3*q^6 + 2*q^7 - q^8 + 6*q^9 + q^10 - 4*q^11 +
        3*q^12 - 3*q^13 - 2*q^14 - 3*q^15 + q^16 + q^17 - 6*q^18 + 3*q^19 - q^20
        + 6*q^21 + 4*q^22 - 6*q^23 - 3*q^24 + q^25 + 3*q^26 + 9*q^27 + 2*q^28 +
        9*q^29 + 3*q^30 - 3*q^31 - q^32 - 12*q^33 - q^34 - 2*q^35 + 6*q^36 -
        8*q^37 - 3*q^38 - 9*q^39 + q^40 - 6*q^41 - 6*q^42 + 6*q^43 - 4*q^44 -
        6*q^45 + 6*q^46 - 13*q^47 + 3*q^48 - 3*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
```

```
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 17, 30, 85, 85, 102, 170, 170 *]
```

???85, 102, 170, 170, $n(a_{11} = \pm 4; 11) = 16 - 12$

1.5.12. *H_{12}, genus 7.*

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + 2*q^20 + 4*q^23 - q^25 + 2*q^26 - 4*q^28
        + 6*q^29 + 4*q^31 - 5*q^32 - q^34 - 8*q^35 + 3*q^36 - 2*q^37 + 4*q^38 -
        6*q^40 - 6*q^41 + 4*q^43 + 6*q^45 - 4*q^46 + 9*q^49 + O(q^50),
    q + q^2 + 2*q^3 - q^4 - q^5 + 2*q^6 - 2*q^7 - 3*q^8 + q^9 - q^10 + 2*q^11 -
        2*q^12 + 2*q^13 - 2*q^14 - 2*q^15 - q^16 + q^17 + q^18 + q^20 - 4*q^21 +
        2*q^22 + 6*q^23 - 6*q^24 + q^25 + 2*q^26 - 4*q^27 + 2*q^28 - 6*q^29 -
        2*q^30 - 10*q^31 + 5*q^32 + 4*q^33 + q^34 + 2*q^35 - q^36 + 2*q^37 +
        4*q^39 + 3*q^40 + 10*q^41 - 4*q^42 + 4*q^43 - 2*q^44 - q^45 + 6*q^46 +
        12*q^47 - 2*q^48 - 3*q^49 + O(q^50),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + (2*a + 1)*q^20 + 2*q^21 + (-5*a + 1)*q^22 +
        (-a - 3)*q^23 + (a + 5)*q^24 + q^25 + (2*a - 2)*q^26 + (-8*a - 16)*q^27
        + (5*a - 1)*q^28 + (-2*a - 4)*q^29 + (a + 1)*q^30 + (3*a + 3)*q^31 + (a
        + 4)*q^32 + (2*a + 8)*q^33 - a*q^34 + (-a + 1)*q^35 + (-2*a - 15)*q^36 +
        (6*a + 4)*q^37 + (2*a - 2)*q^38 + (4*a + 8)*q^39 + (-a + 2)*q^40 + (-6*a
        - 4)*q^41 + 2*a*q^42 + (4*a + 6)*q^43 + (9*a + 1)*q^44 + (-4*a - 7)*q^45
        + (-a - 1)*q^46 + (-2*a - 4)*q^47 + (-3*a - 9)*q^48 + (-4*a - 5)*q^49 +
        O(q^50),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 - 4*q^20 + 2*q^21
        + 6*q^23 + q^24 + 11*q^25 + 6*q^26 - q^27 - 2*q^28 - 4*q^29 - 4*q^30 -
        6*q^31 - q^32 + q^34 + 8*q^35 + q^36 - 4*q^37 - 4*q^38 + 6*q^39 + 4*q^40
        - 10*q^41 - 2*q^42 - 4*q^43 - 4*q^45 - 6*q^46 + 4*q^47 - q^48 - 3*q^49 +
        O(q^50),
    q + q^2 + q^3 + q^4 - 2*q^5 + q^6 + q^8 + q^9 - 2*q^10 - 4*q^11 + q^12 -
        2*q^13 - 2*q^15 + q^16 + q^17 + q^18 + 4*q^19 - 2*q^20 - 4*q^22 + q^24 -
        q^25 - 2*q^26 + q^27 - 10*q^29 - 2*q^30 + 8*q^31 + q^32 - 4*q^33 + q^34
        + q^36 - 2*q^37 + 4*q^38 - 2*q^39 - 2*q^40 + 10*q^41 + 12*q^43 - 4*q^44
        - 2*q^45 + q^48 - 7*q^49 + O(q^50),
    q + q^2 + q^3 + q^4 - q^5 + q^6 + q^8 + q^9 - q^10 + 4*q^11 + q^12 + 2*q^13
        - q^15 + q^16 + q^17 + q^18 - 4*q^19 - q^20 + 4*q^22 + 4*q^23 + q^24 +
        q^25 + 2*q^26 + q^27 + 2*q^29 - q^30 - 4*q^31 + q^32 + 4*q^33 + q^34 +
        q^36 - 6*q^37 - 4*q^38 + 2*q^39 - q^40 - 10*q^41 - 8*q^43 + 4*q^44 -
        q^45 + 4*q^46 + q^48 - 7*q^49 + O(q^50)
*]
[*
```

```
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 17, 85, 85, 102, 102, 510 *]
```

85,102,102?,$n(a_7 = 4; 7) = 12 - 8$, $n(a_{11} = 4; 11) = 18 - 16$.

1.5.13. *H_{13}, genus 10.*

```
[*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 + O(q^50),
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + 2*q^20 + 4*q^23 - q^25 + 2*q^26 - 4*q^28
        + 6*q^29 + 4*q^31 - 5*q^32 - q^34 - 8*q^35 + 3*q^36 - 2*q^37 + 4*q^38 -
        6*q^40 - 6*q^41 + 4*q^43 + 6*q^45 - 4*q^46 + 9*q^49 + O(q^50),
    q + a*q^2 - q^3 + (-a + 2)*q^4 + (-a + 1)*q^5 - a*q^6 + (a - 4)*q^8 + q^9 +
        (2*a - 4)*q^10 + (-a - 1)*q^11 + (a - 2)*q^12 + (a + 3)*q^13 + (a -
        1)*q^15 - 3*a*q^16 + q^17 + a*q^18 + (3*a + 3)*q^19 + (-4*a + 6)*q^20 -
        4*q^22 + (-a - 5)*q^23 + (-a + 4)*q^24 - 3*a*q^25 + (2*a + 4)*q^26 -
        q^27 + (4*a + 2)*q^29 + (-2*a + 4)*q^30 + (-2*a - 2)*q^31 + (a - 4)*q^32
        + (a + 1)*q^33 + a*q^34 + (-a + 2)*q^36 + 2*a*q^37 + 12*q^38 + (-a -
        3)*q^39 + (6*a - 8)*q^40 + (a - 1)*q^41 + (-3*a - 3)*q^43 + (-2*a +
        2)*q^44 + (-a + 1)*q^45 + (-4*a - 4)*q^46 + (2*a - 6)*q^47 + 3*a*q^48 -
        7*q^49 + O(q^50),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + (2*a + 1)*q^20 + 2*q^21 + (-5*a + 1)*q^22 +
        (-a - 3)*q^23 + (a + 5)*q^24 + q^25 + (2*a - 2)*q^26 + (-8*a - 16)*q^27
        + (5*a - 1)*q^28 + (-2*a - 4)*q^29 + (a + 1)*q^30 + (3*a + 3)*q^31 + (a
        + 4)*q^32 + (2*a + 8)*q^33 - a*q^34 + (-a + 1)*q^35 + (-2*a - 15)*q^36 +
        (6*a + 4)*q^37 + (2*a - 2)*q^38 + (4*a + 8)*q^39 + (-a + 2)*q^40 + (-6*a
        - 4)*q^41 + 2*a*q^42 + (4*a + 6)*q^43 + (9*a + 1)*q^44 + (-4*a - 7)*q^45
        + (-a - 1)*q^46 + (-2*a - 4)*q^47 + (-3*a - 9)*q^48 + (-4*a - 5)*q^49 +
        O(q^50),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 - 4*q^20 + 2*q^21
        + 6*q^23 + q^24 + 11*q^25 + 6*q^26 - q^27 - 2*q^28 - 4*q^29 - 4*q^30 -
        6*q^31 - q^32 + q^34 + 8*q^35 + q^36 - 4*q^37 - 4*q^38 + 6*q^39 + 4*q^40
        - 10*q^41 - 2*q^42 - 4*q^43 - 4*q^45 - 6*q^46 + 4*q^47 - q^48 - 3*q^49 +
        O(q^50),
    q + q^2 + a*q^3 + q^4 + q^5 + a*q^6 - 2*a*q^7 + q^8 + (-a + 1)*q^9 + q^10 -
        4*q^11 + a*q^12 + (-a + 2)*q^13 - 2*a*q^14 + a*q^15 + q^16 + q^17 + (-a
        + 1)*q^18 + a*q^19 + q^20 + (2*a - 8)*q^21 - 4*q^22 + 2*a*q^23 + a*q^24
```

```
        + q^25 + (-a + 2)*q^26 + (-a - 4)*q^27 - 2*a*q^28 + (3*a + 2)*q^29 +
        a*q^30 + (a - 4)*q^31 + q^32 - 4*a*q^33 + q^34 - 2*a*q^35 + (-a +
        1)*q^36 + (2*a - 2)*q^37 + a*q^38 + (3*a - 4)*q^39 + q^40 + (-4*a -
        6)*q^41 + (2*a - 8)*q^42 + (2*a + 4)*q^43 - 4*q^44 + (-a + 1)*q^45 +
        2*a*q^46 + (-a + 4)*q^47 + a*q^48 + (-4*a + 9)*q^49 + O(q^50),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + q^8 + q^9 + q^10 + 4*q^11 - q^12 - 2*q^13
        - q^15 + q^16 + q^17 + q^18 + 4*q^19 + q^20 + 4*q^22 - q^24 + q^25 -
        2*q^26 - q^27 - 2*q^29 - q^30 + 8*q^31 + q^32 - 4*q^33 + q^34 + q^36 +
        6*q^37 + 4*q^38 + 2*q^39 + q^40 - 6*q^41 - 4*q^43 + 4*q^44 + q^45 - q^48
        - 7*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field
*]
[* 15, 17, 51, 85, 102, 170, 510 *]
```

15???$n(|a_7| \geq 2; 49) \geq 122 - 120$, $n(a_{11} = 4; 11) = 29 - 16$.

1.5.14. *H_{14}, genus 9.*

```
[*
    q - q^2 - q^4 - 2*q^5 + 4*q^7 + 3*q^8 - 3*q^9 + 2*q^10 - 2*q^13 - 4*q^14 -
        q^16 + q^17 + 3*q^18 - 4*q^19 + 2*q^20 + 4*q^23 - q^25 + 2*q^26 - 4*q^28
        + 6*q^29 + 4*q^31 - 5*q^32 - q^34 - 8*q^35 + 3*q^36 - 2*q^37 + 4*q^38 -
        6*q^40 - 6*q^41 + 4*q^43 + 6*q^45 - 4*q^46 + 9*q^49 + O(q^50),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 - a*q^10 + (a - 3)*q^11 + (3*a + 5)*q^12 +
        (-2*a - 2)*q^13 + (-3*a + 1)*q^14 + (a + 3)*q^15 + 3*q^16 - q^17 + (-a +
        4)*q^18 + (-2*a - 2)*q^19 + (2*a + 1)*q^20 + 2*q^21 + (-5*a + 1)*q^22 +
        (-a - 3)*q^23 + (a + 5)*q^24 + q^25 + (2*a - 2)*q^26 + (-8*a - 16)*q^27
        + (5*a - 1)*q^28 + (-2*a - 4)*q^29 + (a + 1)*q^30 + (3*a + 3)*q^31 + (a
        + 4)*q^32 + (2*a + 8)*q^33 - a*q^34 + (-a + 1)*q^35 + (-2*a - 15)*q^36 +
        (6*a + 4)*q^37 + (2*a - 2)*q^38 + (4*a + 8)*q^39 + (-a + 2)*q^40 + (-6*a
        - 4)*q^41 + 2*a*q^42 + (4*a + 6)*q^43 + (9*a + 1)*q^44 + (-4*a - 7)*q^45
        + (-a - 1)*q^46 + (-2*a - 4)*q^47 + (-3*a - 9)*q^48 + (-4*a - 5)*q^49 +
        O(q^50),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + 4*q^10 - q^12 -
        6*q^13 + 2*q^14 + 4*q^15 + q^16 - q^17 - q^18 + 4*q^19 - 4*q^20 + 2*q^21
        + 6*q^23 + q^24 + 11*q^25 + 6*q^26 - q^27 - 2*q^28 - 4*q^29 - 4*q^30 -
        6*q^31 - q^32 + q^34 + 8*q^35 + q^36 - 4*q^37 - 4*q^38 + 6*q^39 + 4*q^40
        - 10*q^41 - 2*q^42 - 4*q^43 - 4*q^45 - 6*q^46 + 4*q^47 - q^48 - 3*q^49 +
        O(q^50),
    q - q^2 - 2*q^3 + q^4 + q^5 + 2*q^6 - 2*q^7 - q^8 + q^9 - q^10 - 2*q^11 -
        2*q^12 - 6*q^13 + 2*q^14 - 2*q^15 + q^16 + q^17 - q^18 - 8*q^19 + q^20 +
        4*q^21 + 2*q^22 - 2*q^23 + 2*q^24 + q^25 + 6*q^26 + 4*q^27 - 2*q^28 +
        6*q^29 + 2*q^30 - 2*q^31 - q^32 + 4*q^33 - q^34 - 2*q^35 + q^36 + 6*q^37
```

```
        + 8*q^38 + 12*q^39 - q^40 + 2*q^41 - 4*q^42 - 4*q^43 - 2*q^44 + q^45 +
        2*q^46 + 4*q^47 - 2*q^48 - 3*q^49 + O(q^50),
    q + a*q^2 + q^3 + (a^2 - 2)*q^4 + q^5 + a*q^6 + (-a^2 - a + 4)*q^7 - q^8 +
        q^9 + a*q^10 + (-a^2 + a + 2)*q^11 + (a^2 - 2)*q^12 + (2*a^2 - 4)*q^13 +
        (-a^2 + 1)*q^14 + q^15 + (-2*a^2 - a + 4)*q^16 + q^17 + a*q^18 + (-3*a^2
        - 3*a + 8)*q^19 + (a^2 - 2)*q^20 + (-a^2 - a + 4)*q^21 + (a^2 - 2*a +
        1)*q^22 + (-2*a - 2)*q^23 - q^24 + q^25 + (4*a - 2)*q^26 + q^27 + (2*a^2
        - a - 7)*q^28 + (3*a^2 - a - 10)*q^29 + a*q^30 + (4*a^2 + 2*a - 10)*q^31
        + (-a^2 - 4*a + 4)*q^32 + (-a^2 + a + 2)*q^33 + a*q^34 + (-a^2 - a +
        4)*q^35 + (a^2 - 2)*q^36 + (-a^2 - 3*a + 8)*q^37 + (-3*a^2 - 4*a +
        3)*q^38 + (2*a^2 - 4)*q^39 - q^40 + (3*a^2 + 3*a - 10)*q^41 + (-a^2 +
        1)*q^42 + 4*a*q^43 + (3*a - 5)*q^44 + q^45 + (-2*a^2 - 2*a)*q^46 + (a^2
        - a - 6)*q^47 + (-2*a^2 - a + 4)*q^48 + (-3*a^2 - a + 7)*q^49 + O(q^50),
    q - q^2 + q^3 + q^4 + q^5 - q^6 - 2*q^7 - q^8 + q^9 - q^10 + 4*q^11 + q^12 +
        2*q^14 + q^15 + q^16 + q^17 - q^18 + 4*q^19 + q^20 - 2*q^21 - 4*q^22 +
        4*q^23 - q^24 + q^25 + q^27 - 2*q^28 + 6*q^29 - q^30 - 8*q^31 - q^32 +
        4*q^33 - q^34 - 2*q^35 + q^36 - 6*q^37 - 4*q^38 - q^40 + 8*q^41 + 2*q^42
        + 2*q^43 + 4*q^44 + q^45 - 4*q^46 - 8*q^47 + q^48 - 3*q^49 + O(q^50)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^3 - 4*x + 1 over the Rational Field,
    Rational Field
*]
[* 17, 85, 102, 170, 255, 510 *]
```

???102,170,$n(a_7 = 4; 7) = 10 - 8$,$n(a_{11} = 4; 11) = 20 - 16$.

1.5.15. *H_{15}, genus 8.*

```
[*
    q + q^2 - 2*q^3 + q^4 - 2*q^6 - 4*q^7 + q^8 + q^9 + O(q^10),
    q + q^3 - 2*q^4 + 3*q^5 - 4*q^7 + q^9 + O(q^10),
    q + a*q^2 + (-a - 3)*q^3 + (-2*a - 1)*q^4 - q^5 + (-a - 1)*q^6 + (a - 1)*q^7
        + (a - 2)*q^8 + (4*a + 7)*q^9 + O(q^10),
    q + a*q^2 + (-a + 1)*q^3 + q^4 + q^5 + (a - 3)*q^6 + (a - 1)*q^7 - a*q^8 +
        (-2*a + 1)*q^9 + O(q^10),
    q - q^2 - q^3 + q^4 - 4*q^5 + q^6 - 2*q^7 - q^8 + q^9 + O(q^10),
    q + q^2 + q^3 + q^4 + q^5 + q^6 + 2*q^7 + q^8 + q^9 + O(q^10)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + 2*x - 1 over the Rational Field,
    Number Field with defining polynomial x^2 - 3 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 34, 51, 85, 85, 102, 510 *]
```

Not bielliptic, $n(a_7 \geq -2; 7) \geq 22 - 20$, $n(a_7 = -4; 49) = 102 - 96$.

1.6. $N = 210$**.**

1.7. $N1$**.**

```
[*
   q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
       q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
       4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
       + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + O(q^50),
   q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
       2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
       q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
       10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
       8*q^47 + q^48 - 7*q^49 + O(q^50),
   q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
       4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
       2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
       12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + O(q^50),
   q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
       (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
       a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
       - 2)*q^19 + (-a + 2)*q^20 + (a + 1)*q^21 + 4*q^22 + (-2*a - 2)*q^23 +
       4*a*q^24 + q^25 + (2*a + 4)*q^26 + (a - 3)*q^27 + (a - 2)*q^28 + (-3*a -
       1)*q^29 - 4*q^30 + (a - 4)*q^32 + (-a - 5)*q^33 + (-2*a - 4)*q^34 - q^35
       + a*q^36 + 6*q^37 + (-4*a + 8)*q^38 + (-3*a - 7)*q^39 + (a - 4)*q^40 -
       2*a*q^41 + 4*q^42 + (2*a + 6)*q^43 + (2*a - 2)*q^44 + (a + 2)*q^45 -
       8*q^46 + (3*a - 1)*q^47 + 12*q^48 + q^49 + O(q^50),
   q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
       q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
       4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
       + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + O(q^50),
   q + a*q^2 - q^3 + 3*q^4 - q^5 - a*q^6 + q^7 + a*q^8 + q^9 - a*q^10 + (-2*a +
       2)*q^11 - 3*q^12 - 2*a*q^13 + a*q^14 + q^15 - q^16 - 2*q^17 + a*q^18 +
       (2*a + 2)*q^19 - 3*q^20 - q^21 + (2*a - 10)*q^22 + 4*q^23 - a*q^24 +
       q^25 - 10*q^26 - q^27 + 3*q^28 - 2*q^29 + a*q^30 + (2*a + 6)*q^31 -
       3*a*q^32 + (2*a - 2)*q^33 - 2*a*q^34 - q^35 + 3*q^36 + (4*a + 2)*q^37 +
       (2*a + 10)*q^38 + 2*a*q^39 - a*q^40 - 2*q^41 - a*q^42 - 4*a*q^43 + (-6*a
       + 6)*q^44 - q^45 + 4*a*q^46 + (-4*a + 4)*q^47 + q^48 + q^49 + O(q^50),
   q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
       2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
       q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
       10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
       8*q^47 + q^48 - 7*q^49 + O(q^50),
   q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
       2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
       8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
       q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
       q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 + O(q^50)
*]
```

```
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 15, 35, 35, 70, 105, 105, 210 *]
```

1.7.1. $N2$.

```
 [*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
        4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
        + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + O(q^50),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
        q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
        5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
        6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 +
        O(q^50),
    q - q^2 + q^3 + q^4 - q^5 - q^6 - 4*q^7 - q^8 + q^9 + q^10 + q^12 + 2*q^13 +
        4*q^14 - q^15 + q^16 + 6*q^17 - q^18 - 4*q^19 - q^20 - 4*q^21 - q^24 +
        q^25 - 2*q^26 + q^27 - 4*q^28 - 6*q^29 + q^30 + 8*q^31 - q^32 - 6*q^34 +
        4*q^35 + q^36 + 2*q^37 + 4*q^38 + 2*q^39 + q^40 - 6*q^41 + 4*q^42 -
        4*q^43 - q^45 + q^48 + 9*q^49 + O(q^50),
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
        4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
        2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
        12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + O(q^50),
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
        4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
        + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + O(q^50),
    q + a*q^2 - q^3 + 3*q^4 - q^5 - a*q^6 + q^7 + a*q^8 + q^9 - a*q^10 + (-2*a +
        2)*q^11 - 3*q^12 - 2*a*q^13 + a*q^14 + q^15 - q^16 - 2*q^17 + a*q^18 +
        (2*a + 2)*q^19 - 3*q^20 - q^21 + (2*a - 10)*q^22 + 4*q^23 - a*q^24 +
        q^25 - 10*q^26 - q^27 + 3*q^28 - 2*q^29 + a*q^30 + (2*a + 6)*q^31 -
        3*a*q^32 + (2*a - 2)*q^33 - 2*a*q^34 - q^35 + 3*q^36 + (4*a + 2)*q^37 +
        (2*a + 10)*q^38 + 2*a*q^39 - a*q^40 - 2*q^41 - a*q^42 - 4*a*q^43 + (-6*a
        + 6)*q^44 - q^45 + 4*a*q^46 + (-4*a + 4)*q^47 + q^48 + q^49 + O(q^50),
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
        4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
        2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
        12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
```

```
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 + O(q^50),
    q - q^2 + q^3 + q^4 - q^5 - q^6 - 4*q^7 - q^8 + q^9 + q^10 + q^12 + 2*q^13 +
        4*q^14 - q^15 + q^16 + 6*q^17 - q^18 - 4*q^19 - q^20 - 4*q^21 - q^24 +
        q^25 - 2*q^26 + q^27 - 4*q^28 - 6*q^29 + q^30 + 8*q^31 - q^32 - 6*q^34 +
        4*q^35 + q^36 + 2*q^37 + 4*q^38 + 2*q^39 + q^40 - 6*q^41 + 4*q^42 -
        4*q^43 - q^45 + q^48 + 9*q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 21, 30, 35, 42, 105, 105, 210, 210 *]
```

1.7.2. $N3$.

```
 [*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 + O(q^50),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
        q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
        5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
        6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 +
        O(q^50),
    q - q^2 + q^3 + q^4 - q^5 - q^6 - 4*q^7 - q^8 + q^9 + q^10 + q^12 + 2*q^13 +
        4*q^14 - q^15 + q^16 + 6*q^17 - q^18 - 4*q^19 - q^20 - 4*q^21 - q^24 +
        q^25 - 2*q^26 + q^27 - 4*q^28 - 6*q^29 + q^30 + 8*q^31 - q^32 - 6*q^34 +
        4*q^35 + q^36 + 2*q^37 + 4*q^38 + 2*q^39 + q^40 - 6*q^41 + 4*q^42 -
        4*q^43 - q^45 + q^48 + 9*q^49 + O(q^50),
    q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
        (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
        a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
        - 2)*q^19 + (-a + 2)*q^20 + (a + 1)*q^21 + 4*q^22 + (-2*a - 2)*q^23 +
        4*a*q^24 + q^25 + (2*a + 4)*q^26 + (a - 3)*q^27 + (a - 2)*q^28 + (-3*a -
        1)*q^29 - 4*q^30 + (a - 4)*q^32 + (-a - 5)*q^33 + (-2*a - 4)*q^34 - q^35
        + a*q^36 + 6*q^37 + (-4*a + 8)*q^38 + (-3*a - 7)*q^39 + (a - 4)*q^40 -
        2*a*q^41 + 4*q^42 + (2*a + 6)*q^43 + (2*a - 2)*q^44 + (a + 2)*q^45 -
        8*q^46 + (3*a - 1)*q^47 + 12*q^48 + q^49 + O(q^50),
    q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
```

```
        (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
        a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
        - 2)*q^19 + (-a + 2)*q^20 + (a + 1)*q^21 + 4*q^22 + (-2*a - 2)*q^23 +
        4*a*q^24 + q^25 + (2*a + 4)*q^26 + (a - 3)*q^27 + (a - 2)*q^28 + (-3*a -
        1)*q^29 - 4*q^30 + (a - 4)*q^32 + (-a - 5)*q^33 + (-2*a - 4)*q^34 - q^35
        + a*q^36 + 6*q^37 + (-4*a + 8)*q^38 + (-3*a - 7)*q^39 + (a - 4)*q^40 -
        2*a*q^41 + 4*q^42 + (2*a + 6)*q^43 + (2*a - 2)*q^44 + (a + 2)*q^45 -
        8*q^46 + (3*a - 1)*q^47 + 12*q^48 + q^49 + O(q^50),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
        q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
        5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
        6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 +
        O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 15, 21, 30, 35, 105, 105, 210 *]
```

1.7.3. $N4$.

```
 [*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
        4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
        + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + O(q^50),
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
        4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
        2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
        12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + O(q^50),
    q + q^2 - q^3 + q^4 - 2*q^5 - q^6 - q^7 + q^8 + q^9 - 2*q^10 - 4*q^11 - q^12
        + 6*q^13 - q^14 + 2*q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 - 2*q^20 + q^21
        - 4*q^22 + 8*q^23 - q^24 - q^25 + 6*q^26 - q^27 - q^28 - 2*q^29 + 2*q^30
        + q^32 + 4*q^33 + 2*q^34 + 2*q^35 + q^36 - 10*q^37 - 4*q^38 - 6*q^39 -
        2*q^40 - 6*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 + 8*q^46 - q^48 + q^49
        + O(q^50),
    q + q^2 + q^4 - q^5 - q^7 + q^8 - 3*q^9 - q^10 + 4*q^11 - 6*q^13 - q^14 +
        q^16 + 2*q^17 - 3*q^18 - q^20 + 4*q^22 + q^25 - 6*q^26 - q^28 + 6*q^29 +
        8*q^31 + q^32 + 2*q^34 + q^35 - 3*q^36 - 10*q^37 - q^40 + 2*q^41 +
```

```
        4*q^43 + 4*q^44 + 3*q^45 + 8*q^47 + q^49 + O(q^50),
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
        4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
        2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
        12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + O(q^50),
    q + a*q^2 - q^3 + 3*q^4 - q^5 - a*q^6 + q^7 + a*q^8 + q^9 - a*q^10 + (-2*a +
        2)*q^11 - 3*q^12 - 2*a*q^13 + a*q^14 + q^15 - q^16 - 2*q^17 + a*q^18 +
        (2*a + 2)*q^19 - 3*q^20 - q^21 + (2*a - 10)*q^22 + 4*q^23 - a*q^24 +
        q^25 - 10*q^26 - q^27 + 3*q^28 - 2*q^29 + a*q^30 + (2*a + 6)*q^31 -
        3*a*q^32 + (2*a - 2)*q^33 - 2*a*q^34 - q^35 + 3*q^36 + (4*a + 2)*q^37 +
        (2*a + 10)*q^38 + 2*a*q^39 - a*q^40 - 2*q^41 - a*q^42 - 4*a*q^43 + (-6*a
        + 6)*q^44 - q^45 + 4*a*q^46 + (-4*a + 4)*q^47 + q^48 + q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 + O(q^50),
    q + a*q^2 - q^3 + 3*q^4 - q^5 - a*q^6 + q^7 + a*q^8 + q^9 - a*q^10 + (-2*a +
        2)*q^11 - 3*q^12 - 2*a*q^13 + a*q^14 + q^15 - q^16 - 2*q^17 + a*q^18 +
        (2*a + 2)*q^19 - 3*q^20 - q^21 + (2*a - 10)*q^22 + 4*q^23 - a*q^24 +
        q^25 - 10*q^26 - q^27 + 3*q^28 - 2*q^29 + a*q^30 + (2*a + 6)*q^31 -
        3*a*q^32 + (2*a - 2)*q^33 - 2*a*q^34 - q^35 + 3*q^36 + (4*a + 2)*q^37 +
        (2*a + 10)*q^38 + 2*a*q^39 - a*q^40 - 2*q^41 - a*q^42 - 4*a*q^43 + (-6*a
        + 6)*q^44 - q^45 + 4*a*q^46 + (-4*a + 4)*q^47 + q^48 + q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field
*]
[* 14, 35, 42, 70, 70, 105, 210, 210 *]
```

1.7.4. $N5$.

```
 [*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 + O(q^50),
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 + O(q^50),
    q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
```

```
        (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
        a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
        - 2)*q^19 + (-a + 2)*q^20 + (a + 1)*q^21 + 4*q^22 + (-2*a - 2)*q^23 +
        4*a*q^24 + q^25 + (2*a + 4)*q^26 + (a - 3)*q^27 + (a - 2)*q^28 + (-3*a -
        1)*q^29 - 4*q^30 + (a - 4)*q^32 + (-a - 5)*q^33 + (-2*a - 4)*q^34 - q^35
        + a*q^36 + 6*q^37 + (-4*a + 8)*q^38 + (-3*a - 7)*q^39 + (a - 4)*q^40 -
        2*a*q^41 + 4*q^42 + (2*a + 6)*q^43 + (2*a - 2)*q^44 + (a + 2)*q^45 -
        8*q^46 + (3*a - 1)*q^47 + 12*q^48 + q^49 + O(q^50),
    q + q^2 - q^3 + q^4 - 2*q^5 - q^6 - q^7 + q^8 + q^9 - 2*q^10 - 4*q^11 - q^12
        + 6*q^13 - q^14 + 2*q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 - 2*q^20 + q^21
        - 4*q^22 + 8*q^23 - q^24 - q^25 + 6*q^26 - q^27 - q^28 - 2*q^29 + 2*q^30
        + q^32 + 4*q^33 + 2*q^34 + 2*q^35 + q^36 - 10*q^37 - 4*q^38 - 6*q^39 -
        2*q^40 - 6*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 + 8*q^46 - q^48 + q^49
        + O(q^50),
    q + q^2 + q^4 - q^5 - q^7 + q^8 - 3*q^9 - q^10 + 4*q^11 - 6*q^13 - q^14 +
        q^16 + 2*q^17 - 3*q^18 - q^20 + 4*q^22 + q^25 - 6*q^26 - q^28 + 6*q^29 +
        8*q^31 + q^32 + 2*q^34 + q^35 - 3*q^36 - 10*q^37 - q^40 + 2*q^41 +
        4*q^43 + 4*q^44 + 3*q^45 + 8*q^47 + q^49 + O(q^50),
    q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
        (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
        a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
        - 2)*q^19 + (-a + 2)*q^20 + (a + 1)*q^21 + 4*q^22 + (-2*a - 2)*q^23 +
        4*a*q^24 + q^25 + (2*a + 4)*q^26 + (a - 3)*q^27 + (a - 2)*q^28 + (-3*a -
        1)*q^29 - 4*q^30 + (a - 4)*q^32 + (-a - 5)*q^33 + (-2*a - 4)*q^34 - q^35
        + a*q^36 + 6*q^37 + (-4*a + 8)*q^38 + (-3*a - 7)*q^39 + (a - 4)*q^40 -
        2*a*q^41 + 4*q^42 + (2*a + 6)*q^43 + (2*a - 2)*q^44 + (a + 2)*q^45 -
        8*q^46 + (3*a - 1)*q^47 + 12*q^48 + q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 + O(q^50),
    q + q^2 - q^3 + q^4 - 2*q^5 - q^6 - q^7 + q^8 + q^9 - 2*q^10 - 4*q^11 - q^12
        + 6*q^13 - q^14 + 2*q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 - 2*q^20 + q^21
        - 4*q^22 + 8*q^23 - q^24 - q^25 + 6*q^26 - q^27 - q^28 - 2*q^29 + 2*q^30
        + q^32 + 4*q^33 + 2*q^34 + 2*q^35 + q^36 - 10*q^37 - 4*q^38 - 6*q^39 -
        2*q^40 - 6*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 + 8*q^46 - q^48 + q^49
        + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Rational Field
*]
```

```
[* 15, 30, 35, 42, 70, 70, 210, 210 *]
```

1.7.5. $N6$.

```
[*
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
        q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
        5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
        6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 +
        O(q^50),
    q - q^2 + q^3 + q^4 - q^5 - q^6 - 4*q^7 - q^8 + q^9 + q^10 + q^12 + 2*q^13 +
        4*q^14 - q^15 + q^16 + 6*q^17 - q^18 - 4*q^19 - q^20 - 4*q^21 - q^24 +
        q^25 - 2*q^26 + q^27 - 4*q^28 - 6*q^29 + q^30 + 8*q^31 - q^32 - 6*q^34 +
        4*q^35 + q^36 + 2*q^37 + 4*q^38 + 2*q^39 + q^40 - 6*q^41 + 4*q^42 -
        4*q^43 - q^45 + q^48 + 9*q^49 + O(q^50),
    q + q^2 - q^3 + q^4 - 2*q^5 - q^6 - q^7 + q^8 + q^9 - 2*q^10 - 4*q^11 - q^12
        + 6*q^13 - q^14 + 2*q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 - 2*q^20 + q^21
        - 4*q^22 + 8*q^23 - q^24 - q^25 + 6*q^26 - q^27 - q^28 - 2*q^29 + 2*q^30
        + q^32 + 4*q^33 + 2*q^34 + 2*q^35 + q^36 - 10*q^37 - 4*q^38 - 6*q^39 -
        2*q^40 - 6*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 + 8*q^46 - q^48 + q^49
        + O(q^50),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
        q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
        5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
        6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 +
        O(q^50),
    q + q^2 + q^4 - q^5 - q^7 + q^8 - 3*q^9 - q^10 + 4*q^11 - 6*q^13 - q^14 +
        q^16 + 2*q^17 - 3*q^18 - q^20 + 4*q^22 + q^25 - 6*q^26 - q^28 + 6*q^29 +
        8*q^31 + q^32 + 2*q^34 + q^35 - 3*q^36 - 10*q^37 - q^40 + 2*q^41 +
        4*q^43 + 4*q^44 + 3*q^45 + 8*q^47 + q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 + O(q^50),
    q + q^2 + q^4 - q^5 - q^7 + q^8 - 3*q^9 - q^10 + 4*q^11 - 6*q^13 - q^14 +
        q^16 + 2*q^17 - 3*q^18 - q^20 + 4*q^22 + q^25 - 6*q^26 - q^28 + 6*q^29 +
        8*q^31 + q^32 + 2*q^34 + q^35 - 3*q^36 - 10*q^37 - q^40 + 2*q^41 +
        4*q^43 + 4*q^44 + 3*q^45 + 8*q^47 + q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
```

```
*]
[* 21, 30, 42, 42, 70, 210, 210 *]
```

1.7.6. *N7, genus 11.* The programme compute one more differential, need an ad-hoc modification for the Jacobian decomposition.

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
        4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
        + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + 5*q^50 -
        12*q^51 - 4*q^52 + 6*q^53 - 4*q^54 + O(q^55),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
        q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
        5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
        6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 + q^50 -
        6*q^51 + 2*q^52 + 6*q^53 - q^54 + O(q^55),
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
        4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
        2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
        12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + 3*q^51 -
        10*q^52 + 12*q^53 + O(q^55),
    q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
        (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
        a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
        - 2)*q^19 + (-a + 2)*q^20 + (a + 1)*q^21 + 4*q^22 + (-2*a - 2)*q^23 +
        4*a*q^24 + q^25 + (2*a + 4)*q^26 + (a - 3)*q^27 + (a - 2)*q^28 + (-3*a -
        1)*q^29 - 4*q^30 + (a - 4)*q^32 + (-a - 5)*q^33 + (-2*a - 4)*q^34 - q^35
        + a*q^36 + 6*q^37 + (-4*a + 8)*q^38 + (-3*a - 7)*q^39 + (a - 4)*q^40 -
        2*a*q^41 + 4*q^42 + (2*a + 6)*q^43 + (2*a - 2)*q^44 + (a + 2)*q^45 -
        8*q^46 + (3*a - 1)*q^47 + 12*q^48 + q^49 + a*q^50 + (3*a + 7)*q^51 +
        2*q^52 + 2*a*q^53 + (-4*a + 4)*q^54 + O(q^55),
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
        4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
        + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + 5*q^50 -
        12*q^51 - 4*q^52 + 6*q^53 - 4*q^54 + O(q^55),
    q + q^2 + q^3 - q^4 + q^5 + q^6 + q^7 - 3*q^8 + q^9 + q^10 - q^12 - 6*q^13 +
        q^14 + q^15 - q^16 + 2*q^17 + q^18 - 8*q^19 - q^20 + q^21 + 8*q^23 -
        3*q^24 + q^25 - 6*q^26 + q^27 - q^28 - 2*q^29 + q^30 + 4*q^31 + 5*q^32 +
        2*q^34 + q^35 - q^36 - 2*q^37 - 8*q^38 - 6*q^39 - 3*q^40 - 6*q^41 + q^42
        + 4*q^43 + q^45 + 8*q^46 + 8*q^47 - q^48 + q^49 + q^50 + 2*q^51 + 6*q^52
        + 10*q^53 + q^54 + O(q^55),
    q + a*q^2 - q^3 + 3*q^4 - q^5 - a*q^6 + q^7 + a*q^8 + q^9 - a*q^10 + (-2*a +
        2)*q^11 - 3*q^12 - 2*a*q^13 + a*q^14 + q^15 - q^16 - 2*q^17 + a*q^18 +
        (2*a + 2)*q^19 - 3*q^20 - q^21 + (2*a - 10)*q^22 + 4*q^23 - a*q^24 +
        q^25 - 10*q^26 - q^27 + 3*q^28 - 2*q^29 + a*q^30 + (2*a + 6)*q^31 -
        3*a*q^32 + (2*a - 2)*q^33 - 2*a*q^34 - q^35 + 3*q^36 + (4*a + 2)*q^37 +
        (2*a + 10)*q^38 + 2*a*q^39 - a*q^40 - 2*q^41 - a*q^42 - 4*a*q^43 + (-6*a
        + 6)*q^44 - q^45 + 4*a*q^46 + (-4*a + 4)*q^47 + q^48 + q^49 + a*q^50 +
```

```
        2*q^51 - 6*a*q^52 + (-2*a - 8)*q^53 - a*q^54 + O(q^55),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
        6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55),
    q - q^2 + q^3 + q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^10 + q^12 + 2*q^13 -
        q^14 + q^15 + q^16 - 6*q^17 - q^18 + 8*q^19 + q^20 + q^21 - q^24 + q^25
        - 2*q^26 + q^27 + q^28 + 6*q^29 - q^30 - 4*q^31 - q^32 + 6*q^34 + q^35 +
        q^36 - 10*q^37 - 8*q^38 + 2*q^39 - q^40 - 6*q^41 - q^42 - 4*q^43 + q^45
        + q^48 + q^49 - q^50 - 6*q^51 + 2*q^52 - 6*q^53 - q^54 + O(q^55)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 21, 35, 35, 42, 70, 105, 105, 210, 210 *]
```

Eliminated 6 which coincides with 5 but we keept all the levels before arising the extra term in the Jacobian decomposition that provides the Magma code programme.

1.7.7. *$N8$, genus 10.* Not bielliptic. $n(|a_{11}| \geq 0; 121) \geq 4$.

1.7.8. *$N9$.*

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
        4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
        + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + 5*q^50 -
        12*q^51 - 4*q^52 + 6*q^53 - 4*q^54 + O(q^55),
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 - q^50 - 2*q^51 + 2*q^52 - 10*q^53 + q^54 +
        O(q^55),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
        q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
        5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
        6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 + q^50 -
        6*q^51 + 2*q^52 + 6*q^53 - q^54 + O(q^55),
    q - q^2 + q^3 + q^4 - q^5 - q^6 - 4*q^7 - q^8 + q^9 + q^10 + q^12 + 2*q^13 +
        4*q^14 - q^15 + q^16 + 6*q^17 - q^18 - 4*q^19 - q^20 - 4*q^21 - q^24 +
```

```
        q^25 - 2*q^26 + q^27 - 4*q^28 - 6*q^29 + q^30 + 8*q^31 - q^32 - 6*q^34 +
        4*q^35 + q^36 + 2*q^37 + 4*q^38 + 2*q^39 + q^40 - 6*q^41 + 4*q^42 -
        4*q^43 - q^45 + q^48 + 9*q^49 - q^50 + 6*q^51 + 2*q^52 - 6*q^53 - q^54 +
        O(q^55),
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
        4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
        + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + 5*q^50 -
        12*q^51 - 4*q^52 + 6*q^53 - 4*q^54 + O(q^55),
    q + q^2 + q^3 - q^4 + q^5 + q^6 + q^7 - 3*q^8 + q^9 + q^10 - q^12 - 6*q^13 +
        q^14 + q^15 - q^16 + 2*q^17 + q^18 - 8*q^19 - q^20 + q^21 + 8*q^23 -
        3*q^24 + q^25 - 6*q^26 + q^27 - q^28 - 2*q^29 + q^30 + 4*q^31 + 5*q^32 +
        2*q^34 + q^35 - q^36 - 2*q^37 - 8*q^38 - 6*q^39 - 3*q^40 - 6*q^41 + q^42
        + 4*q^43 + q^45 + 8*q^46 + 8*q^47 - q^48 + q^49 + q^50 + 2*q^51 + 6*q^52
        + 10*q^53 + q^54 + O(q^55),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
        6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55),
    q - q^2 + q^3 + q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^10 + q^12 + 2*q^13 -
        q^14 + q^15 + q^16 - 6*q^17 - q^18 + 8*q^19 + q^20 + q^21 - q^24 + q^25
        - 2*q^26 + q^27 + q^28 + 6*q^29 - q^30 - 4*q^31 - q^32 + 6*q^34 + q^35 +
        q^36 - 10*q^37 - 8*q^38 + 2*q^39 - q^40 - 6*q^41 - q^42 - 4*q^43 + q^45
        + q^48 + q^49 - q^50 - 6*q^51 + 2*q^52 - 6*q^53 - q^54 + O(q^55)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 15, 21, 30, 42, 105, 210, 210 *]
```

1.7.9. $N10$. Need modify Jacobian programme.

1.7.10. $N11$.

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
        4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
        + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + 5*q^50 -
        12*q^51 - 4*q^52 + 6*q^53 - 4*q^54 + O(q^55),
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
```

```
    10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
    8*q^47 + q^48 - 7*q^49 - q^50 - 2*q^51 + 2*q^52 - 10*q^53 + q^54 +
    O(q^55),
q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
    4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
    2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
    12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + 3*q^51 -
    10*q^52 + 12*q^53 + O(q^55),
q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
    (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
    a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
    - 2)*q^19 + (-a + 2)*q^20 + (a + 1)*q^21 + 4*q^22 + (-2*a - 2)*q^23 +
    4*a*q^24 + q^25 + (2*a + 4)*q^26 + (a - 3)*q^27 + (a - 2)*q^28 + (-3*a -
    1)*q^29 - 4*q^30 + (a - 4)*q^32 + (-a - 5)*q^33 + (-2*a - 4)*q^34 - q^35
    + a*q^36 + 6*q^37 + (-4*a + 8)*q^38 + (-3*a - 7)*q^39 + (a - 4)*q^40 -
    2*a*q^41 + 4*q^42 + (2*a + 6)*q^43 + (2*a - 2)*q^44 + (a + 2)*q^45 -
    8*q^46 + (3*a - 1)*q^47 + 12*q^48 + q^49 + a*q^50 + (3*a + 7)*q^51 +
    2*q^52 + 2*a*q^53 + (-4*a + 4)*q^54 + O(q^55),
q + q^2 - q^3 + q^4 - 2*q^5 - q^6 - q^7 + q^8 + q^9 - 2*q^10 - 4*q^11 - q^12
    + 6*q^13 - q^14 + 2*q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 - 2*q^20 + q^21
    - 4*q^22 + 8*q^23 - q^24 - q^25 + 6*q^26 - q^27 - q^28 - 2*q^29 + 2*q^30
    + q^32 + 4*q^33 + 2*q^34 + 2*q^35 + q^36 - 10*q^37 - 4*q^38 - 6*q^39 -
    2*q^40 - 6*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 + 8*q^46 - q^48 + q^49
    - q^50 - 2*q^51 + 6*q^52 + 6*q^53 - q^54 + O(q^55),
q + a*q^2 - q^3 + 3*q^4 - q^5 - a*q^6 + q^7 + a*q^8 + q^9 - a*q^10 + (-2*a +
    2)*q^11 - 3*q^12 - 2*a*q^13 + a*q^14 + q^15 - q^16 - 2*q^17 + a*q^18 +
    (2*a + 2)*q^19 - 3*q^20 - q^21 + (2*a - 10)*q^22 + 4*q^23 - a*q^24 +
    q^25 - 10*q^26 - q^27 + 3*q^28 - 2*q^29 + a*q^30 + (2*a + 6)*q^31 -
    3*a*q^32 + (2*a - 2)*q^33 - 2*a*q^34 - q^35 + 3*q^36 + (4*a + 2)*q^37 +
    (2*a + 10)*q^38 + 2*a*q^39 - a*q^40 - 2*q^41 - a*q^42 - 4*a*q^43 + (-6*a
    + 6)*q^44 - q^45 + 4*a*q^46 + (-4*a + 4)*q^47 + q^48 + q^49 + a*q^50 +
    2*q^51 - 6*a*q^52 + (-2*a - 8)*q^53 - a*q^54 + O(q^55),
q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
    2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
    q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
    10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
    8*q^47 + q^48 - 7*q^49 - q^50 - 2*q^51 + 2*q^52 - 10*q^53 + q^54 +
    O(q^55),
q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
    2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
    8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
    q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
    q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
    6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55),
q + q^2 - q^3 + q^4 + q^5 - q^6 + q^7 + q^8 + q^9 + q^10 + 4*q^11 - q^12 -
    2*q^13 + q^14 - q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 + q^20 - q^21 +
    4*q^22 - 8*q^23 - q^24 + q^25 - 2*q^26 - q^27 + q^28 + 6*q^29 - q^30 -
    8*q^31 + q^32 - 4*q^33 + 2*q^34 + q^35 + q^36 - 2*q^37 - 4*q^38 + 2*q^39
    + q^40 + 2*q^41 - q^42 - 12*q^43 + 4*q^44 + q^45 - 8*q^46 - 8*q^47 -
```

```
        q^48 + q^49 + q^50 - 2*q^51 - 2*q^52 + 6*q^53 - q^54 + O(q^55)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 15, 35, 35, 42, 105, 105, 210, 210 *]
```

1.7.11. $N12$. Need modify Jacobian

1.7.12. $N13$.

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
        4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
        + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + 5*q^50 -
        12*q^51 - 4*q^52 + 6*q^53 - 4*q^54 + O(q^55),
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 - q^50 - 2*q^51 + 2*q^52 - 10*q^53 + q^54 +
        O(q^55),
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 - q^50 - 2*q^51 + 2*q^52 - 10*q^53 + q^54 +
        O(q^55),
    q + q^2 - q^3 + q^4 - 2*q^5 - q^6 - q^7 + q^8 + q^9 - 2*q^10 - 4*q^11 - q^12
        + 6*q^13 - q^14 + 2*q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 - 2*q^20 + q^21
        - 4*q^22 + 8*q^23 - q^24 - q^25 + 6*q^26 - q^27 - q^28 - 2*q^29 + 2*q^30
        + q^32 + 4*q^33 + 2*q^34 + 2*q^35 + q^36 - 10*q^37 - 4*q^38 - 6*q^39 -
        2*q^40 - 6*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 + 8*q^46 - q^48 + q^49
        - q^50 - 2*q^51 + 6*q^52 + 6*q^53 - q^54 + O(q^55),
    q + q^2 + q^4 - q^5 - q^7 + q^8 - 3*q^9 - q^10 + 4*q^11 - 6*q^13 - q^14 +
        q^16 + 2*q^17 - 3*q^18 - q^20 + 4*q^22 + q^25 - 6*q^26 - q^28 + 6*q^29 +
        8*q^31 + q^32 + 2*q^34 + q^35 - 3*q^36 - 10*q^37 - q^40 + 2*q^41 +
        4*q^43 + 4*q^44 + 3*q^45 + 8*q^47 + q^49 + q^50 - 6*q^52 - 2*q^53 +
        O(q^55),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
```

```
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
        6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + q^7 + q^8 + q^9 + q^10 + 4*q^11 - q^12 -
        2*q^13 + q^14 - q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 + q^20 - q^21 +
        4*q^22 - 8*q^23 - q^24 + q^25 - 2*q^26 - q^27 + q^28 + 6*q^29 - q^30 -
        8*q^31 + q^32 - 4*q^33 + 2*q^34 + q^35 + q^36 - 2*q^37 - 4*q^38 + 2*q^39
        + q^40 + 2*q^41 - q^42 - 12*q^43 + 4*q^44 + q^45 - 8*q^46 - 8*q^47 -
        q^48 + q^49 + q^50 - 2*q^51 - 2*q^52 + 6*q^53 - q^54 + O(q^55)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 15, 30, 42, 70, 210, 210 *]
```

1.7.13. $N14$. Modify programme.

1.7.14. $N15$. Modify programme.

1.7.15. $N16$.

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
        4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
        + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + 5*q^50 -
        12*q^51 - 4*q^52 + 6*q^53 - 4*q^54 + O(q^55),
    q - q^2 + q^3 + q^4 - q^5 - q^6 - 4*q^7 - q^8 + q^9 + q^10 + q^12 + 2*q^13 +
        4*q^14 - q^15 + q^16 + 6*q^17 - q^18 - 4*q^19 - q^20 - 4*q^21 - q^24 +
        q^25 - 2*q^26 + q^27 - 4*q^28 - 6*q^29 + q^30 + 8*q^31 - q^32 - 6*q^34 +
        4*q^35 + q^36 + 2*q^37 + 4*q^38 + 2*q^39 + q^40 - 6*q^41 + 4*q^42 -
        4*q^43 - q^45 + q^48 + 9*q^49 - q^50 + 6*q^51 + 2*q^52 - 6*q^53 - q^54 +
        O(q^55),
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
        4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
        2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
        12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + 3*q^51 -
        10*q^52 + 12*q^53 + O(q^55),
    q + q^2 - q^3 + q^4 - 2*q^5 - q^6 - q^7 + q^8 + q^9 - 2*q^10 - 4*q^11 - q^12
        + 6*q^13 - q^14 + 2*q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 - 2*q^20 + q^21
        - 4*q^22 + 8*q^23 - q^24 - q^25 + 6*q^26 - q^27 - q^28 - 2*q^29 + 2*q^30
        + q^32 + 4*q^33 + 2*q^34 + 2*q^35 + q^36 - 10*q^37 - 4*q^38 - 6*q^39 -
        2*q^40 - 6*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 + 8*q^46 - q^48 + q^49
        - q^50 - 2*q^51 + 6*q^52 + 6*q^53 - q^54 + O(q^55),
    q + q^2 + q^4 - q^5 - q^7 + q^8 - 3*q^9 - q^10 + 4*q^11 - 6*q^13 - q^14 +
        q^16 + 2*q^17 - 3*q^18 - q^20 + 4*q^22 + q^25 - 6*q^26 - q^28 + 6*q^29 +
        8*q^31 + q^32 + 2*q^34 + q^35 - 3*q^36 - 10*q^37 - q^40 + 2*q^41 +
        4*q^43 + 4*q^44 + 3*q^45 + 8*q^47 + q^49 + q^50 - 6*q^52 - 2*q^53 +
```

```
        O(q^55),
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
        4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
        2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
        12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + 3*q^51 -
        10*q^52 + 12*q^53 + O(q^55),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
        6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55),
    q + q^2 + q^3 + q^4 - q^5 + q^6 + q^7 + q^8 + q^9 - q^10 + q^12 + 2*q^13 +
        q^14 - q^15 + q^16 - 6*q^17 + q^18 - 4*q^19 - q^20 + q^21 + q^24 + q^25
        + 2*q^26 + q^27 + q^28 - 6*q^29 - q^30 - 4*q^31 + q^32 - 6*q^34 - q^35 +
        q^36 + 2*q^37 - 4*q^38 + 2*q^39 - q^40 + 6*q^41 + q^42 + 8*q^43 - q^45 -
        12*q^47 + q^48 + q^49 + q^50 - 6*q^51 + 2*q^52 + 6*q^53 + q^54 + O(q^55)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 30, 35, 42, 70, 70, 210, 210 *]
```

1.7.16. $N17$.

```
[*
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
        q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
        5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
        6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 + q^50 -
        6*q^51 + 2*q^52 + 6*q^53 - q^54 + O(q^55),
    q - q^2 + q^3 + q^4 - q^5 - q^6 - 4*q^7 - q^8 + q^9 + q^10 + q^12 + 2*q^13 +
        4*q^14 - q^15 + q^16 + 6*q^17 - q^18 - 4*q^19 - q^20 - 4*q^21 - q^24 +
        q^25 - 2*q^26 + q^27 - 4*q^28 - 6*q^29 + q^30 + 8*q^31 - q^32 - 6*q^34 +
        4*q^35 + q^36 + 2*q^37 + 4*q^38 + 2*q^39 + q^40 - 6*q^41 + 4*q^42 -
        4*q^43 - q^45 + q^48 + 9*q^49 - q^50 + 6*q^51 + 2*q^52 - 6*q^53 - q^54 +
        O(q^55),
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
        4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
        2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
        12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + 3*q^51 -
        10*q^52 + 12*q^53 + O(q^55),
    q + a*q^2 - q^3 + 3*q^4 - q^5 - a*q^6 + q^7 + a*q^8 + q^9 - a*q^10 + (-2*a +
```

```
        2)*q^11 - 3*q^12 - 2*a*q^13 + a*q^14 + q^15 - q^16 - 2*q^17 + a*q^18 +
        (2*a + 2)*q^19 - 3*q^20 - q^21 + (2*a - 10)*q^22 + 4*q^23 - a*q^24 +
        q^25 - 10*q^26 - q^27 + 3*q^28 - 2*q^29 + a*q^30 + (2*a + 6)*q^31 -
        3*a*q^32 + (2*a - 2)*q^33 - 2*a*q^34 - q^35 + 3*q^36 + (4*a + 2)*q^37 +
        (2*a + 10)*q^38 + 2*a*q^39 - a*q^40 - 2*q^41 - a*q^42 - 4*a*q^43 + (-6*a
        + 6)*q^44 - q^45 + 4*a*q^46 + (-4*a + 4)*q^47 + q^48 + q^49 + a*q^50 +
        2*q^51 - 6*a*q^52 + (-2*a - 8)*q^53 - a*q^54 + O(q^55),
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
        4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
        2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
        12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + 3*q^51 -
        10*q^52 + 12*q^53 + O(q^55),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
        6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55),
    q + q^2 + q^3 + q^4 - q^5 + q^6 + q^7 + q^8 + q^9 - q^10 + q^12 + 2*q^13 +
        q^14 - q^15 + q^16 - 6*q^17 + q^18 - 4*q^19 - q^20 + q^21 + q^24 + q^25
        + 2*q^26 + q^27 + q^28 - 6*q^29 - q^30 - 4*q^31 + q^32 - 6*q^34 - q^35 +
        q^36 + 2*q^37 - 4*q^38 + 2*q^39 - q^40 + 6*q^41 + q^42 + 8*q^43 - q^45 -
        12*q^47 + q^48 + q^49 + q^50 - 6*q^51 + 2*q^52 + 6*q^53 + q^54 + O(q^55)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 21, 30, 35, 105, 105, 210, 210 *]
```

1.7.17. $N18$. Need modification programme.

1.7.18. $N19$.

```
[*
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
        q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
        5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
        6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 + q^50 -
        6*q^51 + 2*q^52 + 6*q^53 - q^54 + O(q^55),
    q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
        (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
        a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
        - 2)*q^19 + (-a + 2)*q^20 + (a + 1)*q^21 + 4*q^22 + (-2*a - 2)*q^23 +
        4*a*q^24 + q^25 + (2*a + 4)*q^26 + (a - 3)*q^27 + (a - 2)*q^28 + (-3*a -
        1)*q^29 - 4*q^30 + (a - 4)*q^32 + (-a - 5)*q^33 + (-2*a - 4)*q^34 - q^35
```

```
        + a*q^36 + 6*q^37 + (-4*a + 8)*q^38 + (-3*a - 7)*q^39 + (a - 4)*q^40 -
        2*a*q^41 + 4*q^42 + (2*a + 6)*q^43 + (2*a - 2)*q^44 + (a + 2)*q^45 -
        8*q^46 + (3*a - 1)*q^47 + 12*q^48 + q^49 + a*q^50 + (3*a + 7)*q^51 +
        2*q^52 + 2*a*q^53 + (-4*a + 4)*q^54 + O(q^55),
    q + q^2 - q^3 + q^4 - 2*q^5 - q^6 - q^7 + q^8 + q^9 - 2*q^10 - 4*q^11 - q^12
        + 6*q^13 - q^14 + 2*q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 - 2*q^20 + q^21
        - 4*q^22 + 8*q^23 - q^24 - q^25 + 6*q^26 - q^27 - q^28 - 2*q^29 + 2*q^30
        + q^32 + 4*q^33 + 2*q^34 + 2*q^35 + q^36 - 10*q^37 - 4*q^38 - 6*q^39 -
        2*q^40 - 6*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 + 8*q^46 - q^48 + q^49
        - q^50 - 2*q^51 + 6*q^52 + 6*q^53 - q^54 + O(q^55),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
        q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
        5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
        6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 + q^50 -
        6*q^51 + 2*q^52 + 6*q^53 - q^54 + O(q^55),
    q + q^2 + q^4 - q^5 - q^7 + q^8 - 3*q^9 - q^10 + 4*q^11 - 6*q^13 - q^14 +
        q^16 + 2*q^17 - 3*q^18 - q^20 + 4*q^22 + q^25 - 6*q^26 - q^28 + 6*q^29 +
        8*q^31 + q^32 + 2*q^34 + q^35 - 3*q^36 - 10*q^37 - q^40 + 2*q^41 +
        4*q^43 + 4*q^44 + 3*q^45 + 8*q^47 + q^49 + q^50 - 6*q^52 - 2*q^53 +
        O(q^55),
    q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
        (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
        a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
        - 2)*q^19 + (-a + 2)*q^20 + (a + 1)*q^21 + 4*q^22 + (-2*a - 2)*q^23 +
        4*a*q^24 + q^25 + (2*a + 4)*q^26 + (a - 3)*q^27 + (a - 2)*q^28 + (-3*a -
        1)*q^29 - 4*q^30 + (a - 4)*q^32 + (-a - 5)*q^33 + (-2*a - 4)*q^34 - q^35
        + a*q^36 + 6*q^37 + (-4*a + 8)*q^38 + (-3*a - 7)*q^39 + (a - 4)*q^40 -
        2*a*q^41 + 4*q^42 + (2*a + 6)*q^43 + (2*a - 2)*q^44 + (a + 2)*q^45 -
        8*q^46 + (3*a - 1)*q^47 + 12*q^48 + q^49 + a*q^50 + (3*a + 7)*q^51 +
        2*q^52 + 2*a*q^53 + (-4*a + 4)*q^54 + O(q^55),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
        6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55),
    q + q^2 + q^3 + q^4 + q^5 + q^6 - q^7 + q^8 + q^9 + q^10 - 4*q^11 + q^12 -
        2*q^13 - q^14 + q^15 + q^16 + 2*q^17 + q^18 + 4*q^19 + q^20 - q^21 -
        4*q^22 - 8*q^23 + q^24 + q^25 - 2*q^26 + q^27 - q^28 - 2*q^29 + q^30 +
        q^32 - 4*q^33 + 2*q^34 - q^35 + q^36 + 6*q^37 + 4*q^38 - 2*q^39 + q^40 -
        6*q^41 - q^42 - 4*q^43 - 4*q^44 + q^45 - 8*q^46 + q^48 + q^49 + q^50 +
        2*q^51 - 2*q^52 - 10*q^53 + q^54 + O(q^55)
*]
[*
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Rational Field,
```

```
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 21, 35, 42, 42, 70, 70, 210, 210 *]
```

1.7.19. $N20$. Need modify programme.

1.7.20. $N21$. Need to modify programme.

1.7.21. $N22$. Need to modify ad-hoc programme.

1.7.22. $N23$.

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
        4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
        + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + 5*q^50 -
        12*q^51 - 4*q^52 + 6*q^53 - 4*q^54 + O(q^55),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
        q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
        5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
        6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 + q^50 -
        6*q^51 + 2*q^52 + 6*q^53 - q^54 + O(q^55),
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
        4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
        2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
        12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + 3*q^51 -
        10*q^52 + 12*q^53 + O(q^55),
    q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
        (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
        a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
        - 2)*q^19 + (-a + 2)*q^20 + (a + 1)*q^21 + 4*q^22 + (-2*a - 2)*q^23 +
        4*a*q^24 + q^25 + (2*a + 4)*q^26 + (a - 3)*q^27 + (a - 2)*q^28 + (-3*a -
        1)*q^29 - 4*q^30 + (a - 4)*q^32 + (-a - 5)*q^33 + (-2*a - 4)*q^34 - q^35
        + a*q^36 + 6*q^37 + (-4*a + 8)*q^38 + (-3*a - 7)*q^39 + (a - 4)*q^40 -
        2*a*q^41 + 4*q^42 + (2*a + 6)*q^43 + (2*a - 2)*q^44 + (a + 2)*q^45 -
        8*q^46 + (3*a - 1)*q^47 + 12*q^48 + q^49 + a*q^50 + (3*a + 7)*q^51 +
        2*q^52 + 2*a*q^53 + (-4*a + 4)*q^54 + O(q^55),
    q + q^2 + q^3 - q^4 + q^5 + q^6 + q^7 - 3*q^8 + q^9 + q^10 - q^12 - 6*q^13 +
        q^14 + q^15 - q^16 + 2*q^17 + q^18 - 8*q^19 - q^20 + q^21 + 8*q^23 -
        3*q^24 + q^25 - 6*q^26 + q^27 - q^28 - 2*q^29 + q^30 + 4*q^31 + 5*q^32 +
        2*q^34 + q^35 - q^36 - 2*q^37 - 8*q^38 - 6*q^39 - 3*q^40 - 6*q^41 + q^42
        + 4*q^43 + q^45 + 8*q^46 + 8*q^47 - q^48 + q^49 + q^50 + 2*q^51 + 6*q^52
        + 10*q^53 + q^54 + O(q^55),
    q + a*q^2 - q^3 + 3*q^4 - q^5 - a*q^6 + q^7 + a*q^8 + q^9 - a*q^10 + (-2*a +
        2)*q^11 - 3*q^12 - 2*a*q^13 + a*q^14 + q^15 - q^16 - 2*q^17 + a*q^18 +
        (2*a + 2)*q^19 - 3*q^20 - q^21 + (2*a - 10)*q^22 + 4*q^23 - a*q^24 +
        q^25 - 10*q^26 - q^27 + 3*q^28 - 2*q^29 + a*q^30 + (2*a + 6)*q^31 -
        3*a*q^32 + (2*a - 2)*q^33 - 2*a*q^34 - q^35 + 3*q^36 + (4*a + 2)*q^37 +
        (2*a + 10)*q^38 + 2*a*q^39 - a*q^40 - 2*q^41 - a*q^42 - 4*a*q^43 + (-6*a
```

```
        + 6)*q^44 - q^45 + 4*a*q^46 + (-4*a + 4)*q^47 + q^48 + q^49 + a*q^50 +
        2*q^51 - 6*a*q^52 + (-2*a - 8)*q^53 - a*q^54 + O(q^55),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
        6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55),
    q + q^2 + q^3 + q^4 + q^5 + q^6 - q^7 + q^8 + q^9 + q^10 - 4*q^11 + q^12 -
        2*q^13 - q^14 + q^15 + q^16 + 2*q^17 + q^18 + 4*q^19 + q^20 - q^21 -
        4*q^22 - 8*q^23 + q^24 + q^25 - 2*q^26 + q^27 - q^28 - 2*q^29 + q^30 +
        q^32 - 4*q^33 + 2*q^34 - q^35 + q^36 + 6*q^37 + 4*q^38 - 2*q^39 + q^40 -
        6*q^41 - q^42 - 4*q^43 - 4*q^44 + q^45 - 8*q^46 + q^48 + q^49 + q^50 +
        2*q^51 - 2*q^52 - 10*q^53 + q^54 + O(q^55)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 21, 35, 35, 105, 105, 210, 210 *]
```

1.7.23. $N24$.

```
 [*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 - q^50 - 2*q^51 + 2*q^52 - 10*q^53 + q^54 +
        O(q^55),
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
        4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
        2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
        12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + 3*q^51 -
        10*q^52 + 12*q^53 + O(q^55),
    q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
        (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
        a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
        - 2)*q^19 + (-a + 2)*q^20 + (a + 1)*q^21 + 4*q^22 + (-2*a - 2)*q^23 +
        4*a*q^24 + q^25 + (2*a + 4)*q^26 + (a - 3)*q^27 + (a - 2)*q^28 + (-3*a -
        1)*q^29 - 4*q^30 + (a - 4)*q^32 + (-a - 5)*q^33 + (-2*a - 4)*q^34 - q^35
        + a*q^36 + 6*q^37 + (-4*a + 8)*q^38 + (-3*a - 7)*q^39 + (a - 4)*q^40 -
        2*a*q^41 + 4*q^42 + (2*a + 6)*q^43 + (2*a - 2)*q^44 + (a + 2)*q^45 -
        8*q^46 + (3*a - 1)*q^47 + 12*q^48 + q^49 + a*q^50 + (3*a + 7)*q^51 +
        2*q^52 + 2*a*q^53 + (-4*a + 4)*q^54 + O(q^55),
```

```
    q + q^2 + q^3 - q^4 + q^5 + q^6 + q^7 - 3*q^8 + q^9 + q^10 - q^12 - 6*q^13 +
        q^14 + q^15 - q^16 + 2*q^17 + q^18 - 8*q^19 - q^20 + q^21 + 8*q^23 -
        3*q^24 + q^25 - 6*q^26 + q^27 - q^28 - 2*q^29 + q^30 + 4*q^31 + 5*q^32 +
        2*q^34 + q^35 - q^36 - 2*q^37 - 8*q^38 - 6*q^39 - 3*q^40 - 6*q^41 + q^42
        + 4*q^43 + q^45 + 8*q^46 + 8*q^47 - q^48 + q^49 + q^50 + 2*q^51 + 6*q^52
        + 10*q^53 + q^54 + O(q^55),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
        6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55),
    q + q^2 + q^3 + q^4 - q^5 + q^6 + q^7 + q^8 + q^9 - q^10 + q^12 + 2*q^13 +
        q^14 - q^15 + q^16 - 6*q^17 + q^18 - 4*q^19 - q^20 + q^21 + q^24 + q^25
        + 2*q^26 + q^27 + q^28 - 6*q^29 - q^30 - 4*q^31 + q^32 - 6*q^34 - q^35 +
        q^36 + 2*q^37 - 4*q^38 + 2*q^39 - q^40 + 6*q^41 + q^42 + 8*q^43 - q^45 -
        12*q^47 + q^48 + q^49 + q^50 - 6*q^51 + 2*q^52 + 6*q^53 + q^54 + O(q^55)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 15, 35, 35, 105, 210, 210 *]
```

1.7.24. $N = 25$.

```
 [*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 - q^50 - 2*q^51 + 2*q^52 - 10*q^53 + q^54 +
        O(q^55),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
        q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
        5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
        6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 + q^50 -
        6*q^51 + 2*q^52 + 6*q^53 - q^54 + O(q^55),
    q - q^2 + q^3 + q^4 - q^5 - q^6 - 4*q^7 - q^8 + q^9 + q^10 + q^12 + 2*q^13 +
        4*q^14 - q^15 + q^16 + 6*q^17 - q^18 - 4*q^19 - q^20 - 4*q^21 - q^24 +
        q^25 - 2*q^26 + q^27 - 4*q^28 - 6*q^29 + q^30 + 8*q^31 - q^32 - 6*q^34 +
        4*q^35 + q^36 + 2*q^37 + 4*q^38 + 2*q^39 + q^40 - 6*q^41 + 4*q^42 -
        4*q^43 - q^45 + q^48 + 9*q^49 - q^50 + 6*q^51 + 2*q^52 - 6*q^53 - q^54 +
        O(q^55),
    q + q^2 + q^3 - q^4 + q^5 + q^6 + q^7 - 3*q^8 + q^9 + q^10 - q^12 - 6*q^13 +
        q^14 + q^15 - q^16 + 2*q^17 + q^18 - 8*q^19 - q^20 + q^21 + 8*q^23 -
```

```
        3*q^24 + q^25 - 6*q^26 + q^27 - q^28 - 2*q^29 + q^30 + 4*q^31 + 5*q^32 +
        2*q^34 + q^35 - q^36 - 2*q^37 - 8*q^38 - 6*q^39 - 3*q^40 - 6*q^41 + q^42
        + 4*q^43 + q^45 + 8*q^46 + 8*q^47 - q^48 + q^49 + q^50 + 2*q^51 + 6*q^52
        + 10*q^53 + q^54 + O(q^55),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
        6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + q^7 + q^8 + q^9 + q^10 + 4*q^11 - q^12 -
        2*q^13 + q^14 - q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 + q^20 - q^21 +
        4*q^22 - 8*q^23 - q^24 + q^25 - 2*q^26 - q^27 + q^28 + 6*q^29 - q^30 -
        8*q^31 + q^32 - 4*q^33 + 2*q^34 + q^35 + q^36 - 2*q^37 - 4*q^38 + 2*q^39
        + q^40 + 2*q^41 - q^42 - 12*q^43 + 4*q^44 + q^45 - 8*q^46 - 8*q^47 -
        q^48 + q^49 + q^50 - 2*q^51 - 2*q^52 + 6*q^53 - q^54 + O(q^55)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 15, 21, 30, 105, 210, 210 *]
```

1.7.25. $N26$.

```
 [*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
        4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
        + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + 5*q^50 -
        12*q^51 - 4*q^52 + 6*q^53 - 4*q^54 + O(q^55),
    q + q^2 - q^3 + q^4 - 2*q^5 - q^6 - q^7 + q^8 + q^9 - 2*q^10 - 4*q^11 - q^12
        + 6*q^13 - q^14 + 2*q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 - 2*q^20 + q^21
        - 4*q^22 + 8*q^23 - q^24 - q^25 + 6*q^26 - q^27 - q^28 - 2*q^29 + 2*q^30
        + q^32 + 4*q^33 + 2*q^34 + 2*q^35 + q^36 - 10*q^37 - 4*q^38 - 6*q^39 -
        2*q^40 - 6*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 + 8*q^46 - q^48 + q^49
        - q^50 - 2*q^51 + 6*q^52 + 6*q^53 - q^54 + O(q^55),
    q + q^2 + q^4 - q^5 - q^7 + q^8 - 3*q^9 - q^10 + 4*q^11 - 6*q^13 - q^14 +
        q^16 + 2*q^17 - 3*q^18 - q^20 + 4*q^22 + q^25 - 6*q^26 - q^28 + 6*q^29 +
        8*q^31 + q^32 + 2*q^34 + q^35 - 3*q^36 - 10*q^37 - q^40 + 2*q^41 +
        4*q^43 + 4*q^44 + 3*q^45 + 8*q^47 + q^49 + q^50 - 6*q^52 - 2*q^53 +
        O(q^55),
    q + q^2 + q^3 - q^4 + q^5 + q^6 + q^7 - 3*q^8 + q^9 + q^10 - q^12 - 6*q^13 +
        q^14 + q^15 - q^16 + 2*q^17 + q^18 - 8*q^19 - q^20 + q^21 + 8*q^23 -
        3*q^24 + q^25 - 6*q^26 + q^27 - q^28 - 2*q^29 + q^30 + 4*q^31 + 5*q^32 +
        2*q^34 + q^35 - q^36 - 2*q^37 - 8*q^38 - 6*q^39 - 3*q^40 - 6*q^41 + q^42
        + 4*q^43 + q^45 + 8*q^46 + 8*q^47 - q^48 + q^49 + q^50 + 2*q^51 + 6*q^52
```

```
        + 10*q^53 + q^54 + O(q^55),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
        6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55),
    q - q^2 + q^3 + q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^10 + q^12 + 2*q^13 -
        q^14 + q^15 + q^16 - 6*q^17 - q^18 + 8*q^19 + q^20 + q^21 - q^24 + q^25
        - 2*q^26 + q^27 + q^28 + 6*q^29 - q^30 - 4*q^31 - q^32 + 6*q^34 + q^35 +
        q^36 - 10*q^37 - 8*q^38 + 2*q^39 - q^40 - 6*q^41 - q^42 - 4*q^43 + q^45
        + q^48 + q^49 - q^50 - 6*q^51 + 2*q^52 - 6*q^53 - q^54 + O(q^55),
    q + q^2 + q^3 - q^4 + q^5 + q^6 + q^7 - 3*q^8 + q^9 + q^10 - q^12 - 6*q^13 +
        q^14 + q^15 - q^16 + 2*q^17 + q^18 - 8*q^19 - q^20 + q^21 + 8*q^23 -
        3*q^24 + q^25 - 6*q^26 + q^27 - q^28 - 2*q^29 + q^30 + 4*q^31 + 5*q^32 +
        2*q^34 + q^35 - q^36 - 2*q^37 - 8*q^38 - 6*q^39 - 3*q^40 - 6*q^41 + q^42
        + 4*q^43 + q^45 + 8*q^46 + 8*q^47 - q^48 + q^49 + q^50 + 2*q^51 + 6*q^52
        + 10*q^53 + q^54 + O(q^55)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 42, 70, 105, 210, 210, 210 *]
```

1.7.26. $N27$.

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
        4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
        + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + 5*q^50 -
        12*q^51 - 4*q^52 + 6*q^53 - 4*q^54 + O(q^55),
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 - q^50 - 2*q^51 + 2*q^52 - 10*q^53 + q^54 +
        O(q^55),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
        q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
        5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
        6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 + q^50 -
        6*q^51 + 2*q^52 + 6*q^53 - q^54 + O(q^55),
    q + q^2 + q^4 - q^5 - q^7 + q^8 - 3*q^9 - q^10 + 4*q^11 - 6*q^13 - q^14 +
```

```
        q^16 + 2*q^17 - 3*q^18 - q^20 + 4*q^22 + q^25 - 6*q^26 - q^28 + 6*q^29 +
        8*q^31 + q^32 + 2*q^34 + q^35 - 3*q^36 - 10*q^37 - q^40 + 2*q^41 +
        4*q^43 + 4*q^44 + 3*q^45 + 8*q^47 + q^49 + q^50 - 6*q^52 - 2*q^53 +
        O(q^55),
    q + q^2 + q^3 - q^4 + q^5 + q^6 + q^7 - 3*q^8 + q^9 + q^10 - q^12 - 6*q^13 +
        q^14 + q^15 - q^16 + 2*q^17 + q^18 - 8*q^19 - q^20 + q^21 + 8*q^23 -
        3*q^24 + q^25 - 6*q^26 + q^27 - q^28 - 2*q^29 + q^30 + 4*q^31 + 5*q^32 +
        2*q^34 + q^35 - q^36 - 2*q^37 - 8*q^38 - 6*q^39 - 3*q^40 - 6*q^41 + q^42
        + 4*q^43 + q^45 + 8*q^46 + 8*q^47 - q^48 + q^49 + q^50 + 2*q^51 + 6*q^52
        + 10*q^53 + q^54 + O(q^55),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
        6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 15, 21, 70, 105, 210 *]
```

1.7.27. $N28$.

```
 [*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
        4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
        + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + 5*q^50 -
        12*q^51 - 4*q^52 + 6*q^53 - 4*q^54 + O(q^55),
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 - q^50 - 2*q^51 + 2*q^52 - 10*q^53 + q^54 +
        O(q^55),
    q - q^2 + q^3 + q^4 - q^5 - q^6 - 4*q^7 - q^8 + q^9 + q^10 + q^12 + 2*q^13 +
        4*q^14 - q^15 + q^16 + 6*q^17 - q^18 - 4*q^19 - q^20 - 4*q^21 - q^24 +
        q^25 - 2*q^26 + q^27 - 4*q^28 - 6*q^29 + q^30 + 8*q^31 - q^32 - 6*q^34 +
        4*q^35 + q^36 + 2*q^37 + 4*q^38 + 2*q^39 + q^40 - 6*q^41 + 4*q^42 -
        4*q^43 - q^45 + q^48 + 9*q^49 - q^50 + 6*q^51 + 2*q^52 - 6*q^53 - q^54 +
        O(q^55),
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
        4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
        2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
        12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + 3*q^51 -
```

```
        10*q^52 + 12*q^53 + O(q^55),
    q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
        (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
        a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
        - 2)*q^19 + (-a + 2)*q^20 + (a + 1)*q^21 + 4*q^22 + (-2*a - 2)*q^23 +
        4*a*q^24 + q^25 + (2*a + 4)*q^26 + (a - 3)*q^27 + (a - 2)*q^28 + (-3*a -
        1)*q^29 - 4*q^30 + (a - 4)*q^32 + (-a - 5)*q^33 + (-2*a - 4)*q^34 - q^35
        + a*q^36 + 6*q^37 + (-4*a + 8)*q^38 + (-3*a - 7)*q^39 + (a - 4)*q^40 -
        2*a*q^41 + 4*q^42 + (2*a + 6)*q^43 + (2*a - 2)*q^44 + (a + 2)*q^45 -
        8*q^46 + (3*a - 1)*q^47 + 12*q^48 + q^49 + a*q^50 + (3*a + 7)*q^51 +
        2*q^52 + 2*a*q^53 + (-4*a + 4)*q^54 + O(q^55),
    q + q^2 - q^3 + q^4 - 2*q^5 - q^6 - q^7 + q^8 + q^9 - 2*q^10 - 4*q^11 - q^12
        + 6*q^13 - q^14 + 2*q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 - 2*q^20 + q^21
        - 4*q^22 + 8*q^23 - q^24 - q^25 + 6*q^26 - q^27 - q^28 - 2*q^29 + 2*q^30
        + q^32 + 4*q^33 + 2*q^34 + 2*q^35 + q^36 - 10*q^37 - 4*q^38 - 6*q^39 -
        2*q^40 - 6*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 + 8*q^46 - q^48 + q^49
        - q^50 - 2*q^51 + 6*q^52 + 6*q^53 - q^54 + O(q^55),
    q + q^2 + q^3 - q^4 + q^5 + q^6 + q^7 - 3*q^8 + q^9 + q^10 - q^12 - 6*q^13 +
        q^14 + q^15 - q^16 + 2*q^17 + q^18 - 8*q^19 - q^20 + q^21 + 8*q^23 -
        3*q^24 + q^25 - 6*q^26 + q^27 - q^28 - 2*q^29 + q^30 + 4*q^31 + 5*q^32 +
        2*q^34 + q^35 - q^36 - 2*q^37 - 8*q^38 - 6*q^39 - 3*q^40 - 6*q^41 + q^42
        + 4*q^43 + q^45 + 8*q^46 + 8*q^47 - q^48 + q^49 + q^50 + 2*q^51 + 6*q^52
        + 10*q^53 + q^54 + O(q^55),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
        6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 15, 30, 35, 35, 42, 105, 210 *]
```

1.7.28. $N29$.

```
 [*
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
        q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
        5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
        6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 + q^50 -
        6*q^51 + 2*q^52 + 6*q^53 - q^54 + O(q^55),
```

```
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
        4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
        2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
        12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + 3*q^51 -
        10*q^52 + 12*q^53 + O(q^55),
    q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
        (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
        a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
        - 2)*q^19 + (-a + 2)*q^20 + (a + 1)*q^21 + 4*q^22 + (-2*a - 2)*q^23 +
        4*a*q^24 + q^25 + (2*a + 4)*q^26 + (a - 3)*q^27 + (a - 2)*q^28 + (-3*a -
        1)*q^29 - 4*q^30 + (a - 4)*q^32 + (-a - 5)*q^33 + (-2*a - 4)*q^34 - q^35
        + a*q^36 + 6*q^37 + (-4*a + 8)*q^38 + (-3*a - 7)*q^39 + (a - 4)*q^40 -
        2*a*q^41 + 4*q^42 + (2*a + 6)*q^43 + (2*a - 2)*q^44 + (a + 2)*q^45 -
        8*q^46 + (3*a - 1)*q^47 + 12*q^48 + q^49 + a*q^50 + (3*a + 7)*q^51 +
        2*q^52 + 2*a*q^53 + (-4*a + 4)*q^54 + O(q^55),
    q + q^2 + q^3 - q^4 + q^5 + q^6 + q^7 - 3*q^8 + q^9 + q^10 - q^12 - 6*q^13 +
        q^14 + q^15 - q^16 + 2*q^17 + q^18 - 8*q^19 - q^20 + q^21 + 8*q^23 -
        3*q^24 + q^25 - 6*q^26 + q^27 - q^28 - 2*q^29 + q^30 + 4*q^31 + 5*q^32 +
        2*q^34 + q^35 - q^36 - 2*q^37 - 8*q^38 - 6*q^39 - 3*q^40 - 6*q^41 + q^42
        + 4*q^43 + q^45 + 8*q^46 + 8*q^47 - q^48 + q^49 + q^50 + 2*q^51 + 6*q^52
        + 10*q^53 + q^54 + O(q^55),
    q + a*q^2 - q^3 + 3*q^4 - q^5 - a*q^6 + q^7 + a*q^8 + q^9 - a*q^10 + (-2*a +
        2)*q^11 - 3*q^12 - 2*a*q^13 + a*q^14 + q^15 - q^16 - 2*q^17 + a*q^18 +
        (2*a + 2)*q^19 - 3*q^20 - q^21 + (2*a - 10)*q^22 + 4*q^23 - a*q^24 +
        q^25 - 10*q^26 - q^27 + 3*q^28 - 2*q^29 + a*q^30 + (2*a + 6)*q^31 -
        3*a*q^32 + (2*a - 2)*q^33 - 2*a*q^34 - q^35 + 3*q^36 + (4*a + 2)*q^37 +
        (2*a + 10)*q^38 + 2*a*q^39 - a*q^40 - 2*q^41 - a*q^42 - 4*a*q^43 + (-6*a
        + 6)*q^44 - q^45 + 4*a*q^46 + (-4*a + 4)*q^47 + q^48 + q^49 + a*q^50 +
        2*q^51 - 6*a*q^52 + (-2*a - 8)*q^53 - a*q^54 + O(q^55),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
        6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55)
*]
[*
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field
*]
[* 21, 35, 35, 105, 105, 210 *]
```

1.7.29. $N30$.

```
 [*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
```

```
    4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
    + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + 5*q^50 -
    12*q^51 - 4*q^52 + 6*q^53 - 4*q^54 + O(q^55),
q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
    2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
    q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
    10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
    8*q^47 + q^48 - 7*q^49 - q^50 - 2*q^51 + 2*q^52 - 10*q^53 + q^54 +
    O(q^55),
q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
    q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
    q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
    5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
    6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 + q^50 -
    6*q^51 + 2*q^52 + 6*q^53 - q^54 + O(q^55),
q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
    4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
    2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
    12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + 3*q^51 -
    10*q^52 + 12*q^53 + O(q^55),
q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
    (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
    a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
    - 2)*q^19 + (-a + 2)*q^20 + (a + 1)*q^21 + 4*q^22 + (-2*a - 2)*q^23 +
    4*a*q^24 + q^25 + (2*a + 4)*q^26 + (a - 3)*q^27 + (a - 2)*q^28 + (-3*a -
    1)*q^29 - 4*q^30 + (a - 4)*q^32 + (-a - 5)*q^33 + (-2*a - 4)*q^34 - q^35
    + a*q^36 + 6*q^37 + (-4*a + 8)*q^38 + (-3*a - 7)*q^39 + (a - 4)*q^40 -
    2*a*q^41 + 4*q^42 + (2*a + 6)*q^43 + (2*a - 2)*q^44 + (a + 2)*q^45 -
    8*q^46 + (3*a - 1)*q^47 + 12*q^48 + q^49 + a*q^50 + (3*a + 7)*q^51 +
    2*q^52 + 2*a*q^53 + (-4*a + 4)*q^54 + O(q^55),
q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
    2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
    8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
    q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
    q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
    6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55),
q + q^2 + q^3 + q^4 - q^5 + q^6 + q^7 + q^8 + q^9 - q^10 + q^12 + 2*q^13 +
    q^14 - q^15 + q^16 - 6*q^17 + q^18 - 4*q^19 - q^20 + q^21 + q^24 + q^25
    + 2*q^26 + q^27 + q^28 - 6*q^29 - q^30 - 4*q^31 + q^32 - 6*q^34 - q^35 +
    q^36 + 2*q^37 - 4*q^38 + 2*q^39 - q^40 + 6*q^41 + q^42 + 8*q^43 - q^45 -
    12*q^47 + q^48 + q^49 + q^50 - 6*q^51 + 2*q^52 + 6*q^53 + q^54 +
    O(q^55),
q + q^2 + q^3 + q^4 + q^5 + q^6 - q^7 + q^8 + q^9 + q^10 - 4*q^11 + q^12 -
    2*q^13 - q^14 + q^15 + q^16 + 2*q^17 + q^18 + 4*q^19 + q^20 - q^21 -
    4*q^22 - 8*q^23 + q^24 + q^25 - 2*q^26 + q^27 - q^28 - 2*q^29 + q^30 +
    q^32 - 4*q^33 + 2*q^34 - q^35 + q^36 + 6*q^37 + 4*q^38 - 2*q^39 + q^40 -
    6*q^41 - q^42 - 4*q^43 - 4*q^44 + q^45 - 8*q^46 + q^48 + q^49 + q^50 +
    2*q^51 - 2*q^52 - 10*q^53 + q^54 + O(q^55)
*]
```

```
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 15, 21, 35, 35, 210, 210, 210 *]
```

1.7.30. $N = 31$.

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
        4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
        + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + 5*q^50 -
        12*q^51 - 4*q^52 + 6*q^53 - 4*q^54 + O(q^55),
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 - q^50 - 2*q^51 + 2*q^52 - 10*q^53 + q^54 +
        O(q^55),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
        q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
        5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
        6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 + q^50 -
        6*q^51 + 2*q^52 + 6*q^53 - q^54 + O(q^55),
    q - q^2 + q^3 + q^4 - q^5 - q^6 - 4*q^7 - q^8 + q^9 + q^10 + q^12 + 2*q^13 +
        4*q^14 - q^15 + q^16 + 6*q^17 - q^18 - 4*q^19 - q^20 - 4*q^21 - q^24 +
        q^25 - 2*q^26 + q^27 - 4*q^28 - 6*q^29 + q^30 + 8*q^31 - q^32 - 6*q^34 +
        4*q^35 + q^36 + 2*q^37 + 4*q^38 + 2*q^39 + q^40 - 6*q^41 + 4*q^42 -
        4*q^43 - q^45 + q^48 + 9*q^49 - q^50 + 6*q^51 + 2*q^52 - 6*q^53 - q^54 +
        O(q^55),
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
        4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
        2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
        12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + 3*q^51 -
        10*q^52 + 12*q^53 + O(q^55),
    q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
        (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
        a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
        - 2)*q^19 + (-a + 2)*q^20 + (a + 1)*q^21 + 4*q^22 + (-2*a - 2)*q^23 +
        4*a*q^24 + q^25 + (2*a + 4)*q^26 + (a - 3)*q^27 + (a - 2)*q^28 + (-3*a -
        1)*q^29 - 4*q^30 + (a - 4)*q^32 + (-a - 5)*q^33 + (-2*a - 4)*q^34 - q^35
        + a*q^36 + 6*q^37 + (-4*a + 8)*q^38 + (-3*a - 7)*q^39 + (a - 4)*q^40 -
        2*a*q^41 + 4*q^42 + (2*a + 6)*q^43 + (2*a - 2)*q^44 + (a + 2)*q^45 -
```

```
        8*q^46 + (3*a - 1)*q^47 + 12*q^48 + q^49 + a*q^50 + (3*a + 7)*q^51 +
        2*q^52 + 2*a*q^53 + (-4*a + 4)*q^54 + O(q^55),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
        6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + q^7 + q^8 + q^9 + q^10 + 4*q^11 - q^12 -
        2*q^13 + q^14 - q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 + q^20 - q^21 +
        4*q^22 - 8*q^23 - q^24 + q^25 - 2*q^26 - q^27 + q^28 + 6*q^29 - q^30 -
        8*q^31 + q^32 - 4*q^33 + 2*q^34 + q^35 + q^36 - 2*q^37 - 4*q^38 + 2*q^39
        + q^40 + 2*q^41 - q^42 - 12*q^43 + 4*q^44 + q^45 - 8*q^46 - 8*q^47 -
        q^48 + q^49 + q^50 - 2*q^51 - 2*q^52 + 6*q^53 - q^54 + O(q^55),
    q + q^2 + q^3 + q^4 + q^5 + q^6 - q^7 + q^8 + q^9 + q^10 - 4*q^11 + q^12 -
        2*q^13 - q^14 + q^15 + q^16 + 2*q^17 + q^18 + 4*q^19 + q^20 - q^21 -
        4*q^22 - 8*q^23 + q^24 + q^25 - 2*q^26 + q^27 - q^28 - 2*q^29 + q^30 +
        q^32 - 4*q^33 + 2*q^34 - q^35 + q^36 + 6*q^37 + 4*q^38 - 2*q^39 + q^40 -
        6*q^41 - q^42 - 4*q^43 - 4*q^44 + q^45 - 8*q^46 + q^48 + q^49 + q^50 +
        2*q^51 - 2*q^52 - 10*q^53 + q^54 + O(q^55)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 15, 21, 30, 35, 35, 210, 210, 210 *]
```

1.7.31. $N32$.

```
 [*
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 - q^50 - 2*q^51 + 2*q^52 - 10*q^53 + q^54 +
        O(q^55),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
        q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
        5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
        6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 + q^50 -
        6*q^51 + 2*q^52 + 6*q^53 - q^54 + O(q^55),
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
        4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
```

```
        2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
        12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + 3*q^51 -
        10*q^52 + 12*q^53 + O(q^55),
    q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
        (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
        a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
        - 2)*q^19 + (-a + 2)*q^20 + (a + 1)*q^21 + 4*q^22 + (-2*a - 2)*q^23 +
        4*a*q^24 + q^25 + (2*a + 4)*q^26 + (a - 3)*q^27 + (a - 2)*q^28 + (-3*a -
        1)*q^29 - 4*q^30 + (a - 4)*q^32 + (-a - 5)*q^33 + (-2*a - 4)*q^34 - q^35
        + a*q^36 + 6*q^37 + (-4*a + 8)*q^38 + (-3*a - 7)*q^39 + (a - 4)*q^40 -
        2*a*q^41 + 4*q^42 + (2*a + 6)*q^43 + (2*a - 2)*q^44 + (a + 2)*q^45 -
        8*q^46 + (3*a - 1)*q^47 + 12*q^48 + q^49 + a*q^50 + (3*a + 7)*q^51 +
        2*q^52 + 2*a*q^53 + (-4*a + 4)*q^54 + O(q^55),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
        6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + q^7 + q^8 + q^9 + q^10 + 4*q^11 - q^12 -
        2*q^13 + q^14 - q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 + q^20 - q^21 +
        4*q^22 - 8*q^23 - q^24 + q^25 - 2*q^26 - q^27 + q^28 + 6*q^29 - q^30 -
        8*q^31 + q^32 - 4*q^33 + 2*q^34 + q^35 + q^36 - 2*q^37 - 4*q^38 + 2*q^39
        + q^40 + 2*q^41 - q^42 - 12*q^43 + 4*q^44 + q^45 - 8*q^46 - 8*q^47 -
        q^48 + q^49 + q^50 - 2*q^51 - 2*q^52 + 6*q^53 - q^54 + O(q^55),
    q + q^2 + q^3 + q^4 - q^5 + q^6 + q^7 + q^8 + q^9 - q^10 + q^12 + 2*q^13 +
        q^14 - q^15 + q^16 - 6*q^17 + q^18 - 4*q^19 - q^20 + q^21 + q^24 + q^25
        + 2*q^26 + q^27 + q^28 - 6*q^29 - q^30 - 4*q^31 + q^32 - 6*q^34 - q^35 +
        q^36 + 2*q^37 - 4*q^38 + 2*q^39 - q^40 + 6*q^41 + q^42 + 8*q^43 - q^45 -
        12*q^47 + q^48 + q^49 + q^50 - 6*q^51 + 2*q^52 + 6*q^53 + q^54 + O(q^55)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 15, 21, 35, 35, 210, 210, 210 *]
```

1.7.32. $N33$.

```
 [*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
        4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
        + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + 5*q^50 -
        12*q^51 - 4*q^52 + 6*q^53 - 4*q^54 + O(q^55),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
```

```
    q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
    q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
    5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
    6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 + q^50 -
    6*q^51 + 2*q^52 + 6*q^53 - q^54 + O(q^55),
q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 - 2*q^12 + 5*q^13 - q^15 +
    4*q^16 + 3*q^17 + 2*q^19 + 2*q^20 + q^21 - 6*q^23 + q^25 - 5*q^27 -
    2*q^28 + 3*q^29 - 4*q^31 - 3*q^33 - q^35 + 4*q^36 + 2*q^37 + 5*q^39 -
    12*q^41 - 10*q^43 + 6*q^44 + 2*q^45 + 9*q^47 + 4*q^48 + q^49 + 3*q^51 -
    10*q^52 + 12*q^53 + O(q^55),
q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
    (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + (-2*a + 2)*q^12 + (a + 3)*q^13 -
    a*q^14 + (-a - 1)*q^15 - 3*a*q^16 + (-a - 3)*q^17 + (a + 4)*q^18 + (2*a
    - 2)*q^19 + (-a + 2)*q^20 + (a + 1)*q^21 + 4*q^22 + (-2*a - 2)*q^23 +
    4*a*q^24 + q^25 + (2*a + 4)*q^26 + (a - 3)*q^27 + (a - 2)*q^28 + (-3*a -
    1)*q^29 - 4*q^30 + (a - 4)*q^32 + (-a - 5)*q^33 + (-2*a - 4)*q^34 - q^35
    + a*q^36 + 6*q^37 + (-4*a + 8)*q^38 + (-3*a - 7)*q^39 + (a - 4)*q^40 -
    2*a*q^41 + 4*q^42 + (2*a + 6)*q^43 + (2*a - 2)*q^44 + (a + 2)*q^45 -
    8*q^46 + (3*a - 1)*q^47 + 12*q^48 + q^49 + a*q^50 + (3*a + 7)*q^51 +
    2*q^52 + 2*a*q^53 + (-4*a + 4)*q^54 + O(q^55),
q + q^2 + q^3 - q^4 + q^5 + q^6 + q^7 - 3*q^8 + q^9 + q^10 - q^12 - 6*q^13 +
    q^14 + q^15 - q^16 + 2*q^17 + q^18 - 8*q^19 - q^20 + q^21 + 8*q^23 -
    3*q^24 + q^25 - 6*q^26 + q^27 - q^28 - 2*q^29 + q^30 + 4*q^31 + 5*q^32 +
    2*q^34 + q^35 - q^36 - 2*q^37 - 8*q^38 - 6*q^39 - 3*q^40 - 6*q^41 + q^42
    + 4*q^43 + q^45 + 8*q^46 + 8*q^47 - q^48 + q^49 + q^50 + 2*q^51 + 6*q^52
    + 10*q^53 + q^54 + O(q^55),
q + a*q^2 - q^3 + 3*q^4 - q^5 - a*q^6 + q^7 + a*q^8 + q^9 - a*q^10 + (-2*a +
    2)*q^11 - 3*q^12 - 2*a*q^13 + a*q^14 + q^15 - q^16 - 2*q^17 + a*q^18 +
    (2*a + 2)*q^19 - 3*q^20 - q^21 + (2*a - 10)*q^22 + 4*q^23 - a*q^24 +
    q^25 - 10*q^26 - q^27 + 3*q^28 - 2*q^29 + a*q^30 + (2*a + 6)*q^31 -
    3*a*q^32 + (2*a - 2)*q^33 - 2*a*q^34 - q^35 + 3*q^36 + (4*a + 2)*q^37 +
    (2*a + 10)*q^38 + 2*a*q^39 - a*q^40 - 2*q^41 - a*q^42 - 4*a*q^43 + (-6*a
    + 6)*q^44 - q^45 + 4*a*q^46 + (-4*a + 4)*q^47 + q^48 + q^49 + a*q^50 +
    2*q^51 - 6*a*q^52 + (-2*a - 8)*q^53 - a*q^54 + O(q^55),
q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
    2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
    8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
    q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
    q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 - q^50 +
    6*q^51 - 2*q^52 + 10*q^53 + q^54 + O(q^55),
q - q^2 + q^3 + q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^10 + q^12 + 2*q^13 -
    q^14 + q^15 + q^16 - 6*q^17 - q^18 + 8*q^19 + q^20 + q^21 - q^24 + q^25
    - 2*q^26 + q^27 + q^28 + 6*q^29 - q^30 - 4*q^31 - q^32 + 6*q^34 + q^35 +
    q^36 - 10*q^37 - 8*q^38 + 2*q^39 - q^40 - 6*q^41 - q^42 - 4*q^43 + q^45
    + q^48 + q^49 - q^50 - 6*q^51 + 2*q^52 - 6*q^53 - q^54 + O(q^55),
q + q^2 + q^3 + q^4 + q^5 + q^6 - q^7 + q^8 + q^9 + q^10 - 4*q^11 + q^12 -
    2*q^13 - q^14 + q^15 + q^16 + 2*q^17 + q^18 + 4*q^19 + q^20 - q^21 -
    4*q^22 - 8*q^23 + q^24 + q^25 - 2*q^26 + q^27 - q^28 - 2*q^29 + q^30 +
    q^32 - 4*q^33 + 2*q^34 - q^35 + q^36 + 6*q^37 + 4*q^38 - 2*q^39 + q^40 -
```

```
        6*q^41 - q^42 - 4*q^43 - 4*q^44 + q^45 - 8*q^46 + q^48 + q^49 + q^50 +
        2*q^51 - 2*q^52 - 10*q^53 + q^54 + O(q^55)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 - 5 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 21, 35, 35, 105, 105, 210, 210, 210 *]
```

1.7.33. $N34$.

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 + O(q^12),
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + O(q^12),
    q + q^3 - 2*q^4 - q^5 + q^7 - 2*q^9 - 3*q^11 + O(q^12),
    q + a*q^2 + (-a - 1)*q^3 + (-a + 2)*q^4 + q^5 - 4*q^6 - q^7 + (a - 4)*q^8 +
        (a + 2)*q^9 + a*q^10 + (a + 1)*q^11 + O(q^12),
    q + q^2 - q^3 + q^4 - 2*q^5 - q^6 - q^7 + q^8 + q^9 - 2*q^10 - 4*q^11 +
        O(q^12),
    q + q^2 + q^3 - q^4 + q^5 + q^6 + q^7 - 3*q^8 + q^9 + q^10 + O(q^12),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 + O(q^12),
    q - q^2 + q^3 + q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^10 + O(q^12),
    q + q^2 + q^3 + q^4 - q^5 + q^6 + q^7 + q^8 + q^9 - q^10 + O(q^12)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Number Field with defining polynomial x^2 + x - 4 over the Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 15, 35, 35, 42, 105, 210, 210, 210 *]
```

Not bielliptic, $n(a_{11} = 0; 11) = 26 - 24$, $n(|a_{11}| \geq 3, 121) \geq 276 - 270$.

1.7.34. $N35$.

```
[*
    q - q^2 - 2*q^3 + q^4 + 2*q^6 + q^7 - q^8 + q^9 - 2*q^12 - 4*q^13 - q^14 +
        q^16 + 6*q^17 - q^18 + 2*q^19 - 2*q^21 + 2*q^24 - 5*q^25 + 4*q^26 +
        4*q^27 + q^28 - 6*q^29 - 4*q^31 - q^32 - 6*q^34 + q^36 + 2*q^37 - 2*q^38
        + 8*q^39 + 6*q^41 + 2*q^42 + 8*q^43 - 12*q^47 - 2*q^48 + q^49 + O(q^50),
    q - q^2 - q^3 - q^4 + q^5 + q^6 + 3*q^8 + q^9 - q^10 - 4*q^11 + q^12 -
```

```
        2*q^13 - q^15 - q^16 + 2*q^17 - q^18 + 4*q^19 - q^20 + 4*q^22 - 3*q^24 +
        q^25 + 2*q^26 - q^27 - 2*q^29 + q^30 - 5*q^32 + 4*q^33 - 2*q^34 - q^36 -
        10*q^37 - 4*q^38 + 2*q^39 + 3*q^40 + 10*q^41 + 4*q^43 + 4*q^44 + q^45 +
        8*q^47 + q^48 - 7*q^49 + O(q^50),
    q - q^2 + q^3 - q^4 - 2*q^5 - q^6 - q^7 + 3*q^8 + q^9 + 2*q^10 + 4*q^11 -
        q^12 - 2*q^13 + q^14 - 2*q^15 - q^16 - 6*q^17 - q^18 + 4*q^19 + 2*q^20 -
        q^21 - 4*q^22 + 3*q^24 - q^25 + 2*q^26 + q^27 + q^28 - 2*q^29 + 2*q^30 -
        5*q^32 + 4*q^33 + 6*q^34 + 2*q^35 - q^36 + 6*q^37 - 4*q^38 - 2*q^39 -
        6*q^40 + 2*q^41 + q^42 - 4*q^43 - 4*q^44 - 2*q^45 - q^48 + q^49 +
        O(q^50),
    q + q^2 + q^4 - q^5 - q^7 + q^8 - 3*q^9 - q^10 + 4*q^11 - 6*q^13 - q^14 +
        q^16 + 2*q^17 - 3*q^18 - q^20 + 4*q^22 + q^25 - 6*q^26 - q^28 + 6*q^29 +
        8*q^31 + q^32 + 2*q^34 + q^35 - 3*q^36 - 10*q^37 - q^40 + 2*q^41 +
        4*q^43 + 4*q^44 + 3*q^45 + 8*q^47 + q^49 + O(q^50),
    q + q^2 + q^3 - q^4 + q^5 + q^6 + q^7 - 3*q^8 + q^9 + q^10 - q^12 - 6*q^13 +
        q^14 + q^15 - q^16 + 2*q^17 + q^18 - 8*q^19 - q^20 + q^21 + 8*q^23 -
        3*q^24 + q^25 - 6*q^26 + q^27 - q^28 - 2*q^29 + q^30 + 4*q^31 + 5*q^32 +
        2*q^34 + q^35 - q^36 - 2*q^37 - 8*q^38 - 6*q^39 - 3*q^40 - 6*q^41 + q^42
        + 4*q^43 + q^45 + 8*q^46 + 8*q^47 - q^48 + q^49 + O(q^50),
    q - q^2 - q^3 + q^4 - q^5 + q^6 - q^7 - q^8 + q^9 + q^10 - 4*q^11 - q^12 -
        2*q^13 + q^14 + q^15 + q^16 - 6*q^17 - q^18 - q^20 + q^21 + 4*q^22 -
        8*q^23 + q^24 + q^25 + 2*q^26 - q^27 - q^28 + 10*q^29 - q^30 - 8*q^31 -
        q^32 + 4*q^33 + 6*q^34 + q^35 + q^36 + 2*q^37 + 2*q^39 + q^40 - 2*q^41 -
        q^42 + 8*q^43 - 4*q^44 - q^45 + 8*q^46 + 4*q^47 - q^48 + q^49 + O(q^50),
    q - q^2 + q^3 + q^4 + q^5 - q^6 + q^7 - q^8 + q^9 - q^10 + q^12 + 2*q^13 -
        q^14 + q^15 + q^16 - 6*q^17 - q^18 + 8*q^19 + q^20 + q^21 - q^24 + q^25
        - 2*q^26 + q^27 + q^28 + 6*q^29 - q^30 - 4*q^31 - q^32 + 6*q^34 + q^35 +
        q^36 - 10*q^37 - 8*q^38 + 2*q^39 - q^40 - 6*q^41 - q^42 - 4*q^43 + q^45
        + q^48 + q^49 + O(q^50),
    q + q^2 - q^3 + q^4 + q^5 - q^6 + q^7 + q^8 + q^9 + q^10 + 4*q^11 - q^12 -
        2*q^13 + q^14 - q^15 + q^16 + 2*q^17 + q^18 - 4*q^19 + q^20 - q^21 +
        4*q^22 - 8*q^23 - q^24 + q^25 - 2*q^26 - q^27 + q^28 + 6*q^29 - q^30 -
        8*q^31 + q^32 - 4*q^33 + 2*q^34 + q^35 + q^36 - 2*q^37 - 4*q^38 + 2*q^39
        + q^40 + 2*q^41 - q^42 - 12*q^43 + 4*q^44 + q^45 - 8*q^46 - 8*q^47 -
        q^48 + q^49 + O(q^50)
*]
[*
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field,
    Rational Field
*]
[* 14, 15, 21, 70, 105, 210, 210, 210 *]
```

1.8. Magma programme by use in general, with Atkin-Lehner action.

```
L:=[* *];F:=[* *];Level:=[* *];  N:=186; Factorization(N);N;
```

```
Nd:=Divisors(N);

p1:=2; p2:=3; p3:=31; p4:=7;

N1:=[*p1,p2,p1*p2*]; N2:=[*p1,p3,p1*p3*]; N3:=[*p1,p4,p4*p1*];
N4:=[*p2,p3,p2*p3*]; N5:=[*p2,p4,p2*p4*]; N6:=[*p3,p4,p3*p4*];
N7:=[*p1,p2*p3,p1*p2*p3*]; N8:=[*p1,p2*p4,p1*p2*p4*];
N9:=[*p1,p3*p4,p1*p3*p4*]; N10:=[*p1,p2*p3*p4,p1*p2*p3*p4*];
N11:=[*p2,p1*p3,p1*p2*p3*]; N12:=[*p2,p1*p4,p1*p2*p4*];
N13:=[*p2,p3*p4,p2*p3*p4*]; N14:=[*p2,p1*p3*p4,p1*p2*p3*p4*];
N15:=[*p3,p2*p1,p1*p2*p3*]; N16:=[*p3,p2*p4,p3*p2*p4*];
N17:=[*p3,p1*p4,p1*p3*p4*]; N18:=[*p3,p2*p1*p4,p1*p2*p3*p4*];
N19:=[*p4,p2*p3,p4*p2*p3*]; N20:=[*p4,p2*p1,p1*p2*p4*];
N21:=[*p4,p3*p1,p1*p3*p4*]; N22:=[*p4,p2*p3*p1,p1*p2*p3*p4*];
N23:=[*p1*p2,p1*p3,p2*p3*]; N24:=[*p1*p4,p1*p2,p2*p4*];
N25:=[*p1*p4,p1*p3,p3*p4*]; N26:=[*p2*p3,p3*p4,p2*p4*];
N27:=[*p1*p2,p3*p4,p1*p2*p3*p4*]; N28:=[*p1*p3,p2*p4,p1*p2*p3*p4*];
N29:=[*p1*p4,p2*p3,p1*p2*p3*p4*]; N30:=[*p1*p2,p1*p3*p4,p2*p3*p4*];
N31:=[*p1*p3,p1*p2*p4,p3*p2*p4*]; N32:=[*p1*p4,p1*p2*p3,p4*p2*p3*];
N33:=[*p2*p3,p2*p1*p4,p3*p1*p4*]; N34:=[*p2*p4,p2*p1*p3,p4*p1*p3*];
N35:=[*p3*p4,p3*p2*p1,p4*p2*p1*];




H:=[*N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,N14,N15,N16,N17,N18,N19,N20,
N21,N22,N23,N24,N25,N26,N27,N28,N29,N30,N31,N32,N33,N34,N35*];


for subgroup in [7..7] do
 subgroup;
 Hh:=H[subgroup];
AtkinLehnerfix:=Hh; Involutions:=#AtkinLehnerfix;

for j in Nd do
MS:=NewformDecomposition(CuspidalSubspace(ModularSymbols(j,2,1)));
     m:=#MS;
     M:=PrimeDivisors(j);
     Nr:=Numerator(N/j);
     divi:=GCD(j,Nr);
     jj:=Numerator(j/divi);
     Mm:=PrimeDivisors(jj);
     Nn:=Divisors(jj);
     mm:=#Mm;
     mn:=#Nn;
     D:=Factorization(jj);
       for i in [1..m] do
          f:=Eigenform(MS[i],30);
          f2:=MS[i];
```

```
          K:=Parent(Coefficient(f,3)); d:=Dimension(MS[i]);
          X:=IdentityMatrix(Rationals(), d);
          u:=0;

            if GCD(divi,j) eq 1 then
              for jo in [1..Involutions] do
                dd:=GCD(j,AtkinLehnerfix[jo]);
               if dd eq AtkinLehnerfix[jo] then
                 Y:=AtkinLehner(MS[i],dd);
                  if Y eq X then
                  else
                  u:=1;
                  end if;
               else
                  if dd eq 1 then
                  else
                  end if;
               end if;
             end for;
            end if;

        if u eq 0 then
           if GCD(p1,j) eq p1 then
       AtkinLehner(f2,p1);
           end if;
           if GCD(p2,j) eq p2 then
        AtkinLehner(f2,p2);
           end if;
          if GCD(p3,j) eq p3 then
        AtkinLehner(f2,p3);
          end if;
        L:=Append(L,f);
        f;
        F:=Append(F,K);
        Level:=Append(Level,j);
              else
               end if;




    end for;
 end for;

L;F;Level;
 felm:=# F;

p:=11;
```

```
C:=ComplexField(100); R<x>:=PolynomialRing(C); pj:=0*x+1; Roo:=[*
*]; for j in [1 .. felm] do
  if Degree(F[j]) eq 1 then
    cc:=Roots(x^2-Coefficient(L[j],p)*x+p,C);
    Roo:=Append(Roo,cc);
    pj:=pj*(x^2-Coefficient(L[j],p)*x+p);
  else
   dd:=Degree(F[j]);
   u:=Roots(DefiningPolynomial(F[j]),C); uu:= # u;
      for m in [1 .. uu] do
      f := hom< F[j] -> C | u[m][1]>;
      cc2:=Roots(x^2-f(Coefficient(L[j],p))*x+p,C);
      Roo:=Append(Roo,cc2);
      pj:=pj*(x^2-f(Coefficient(L[j],p))*x+p);
      end for;
  end if;
end for; pjdegree:=Degree(pj); pjdegree; PR:=[* *];
 d2:=Degree(pj);
long:= # Roo;


for nn in [1 .. 20] do s:=0;
  for i in [1 .. long] do
    for j in [1..2] do
     if Roo[i][j][2] gt 0 then
s:=s+(Roo[i][j][2])*(Roo[i][j][1])^(nn) ;
  else
  s:=s;
  end if;
end for; end for;
 a:=Round(1+p^(nn)-s); PR:=Append(PR,a); end for;
Jj:=[* *]; for aaa in [1..20] do
 ss:=0;
  adiv:=Divisors(aaa);
    for kk in adiv do
       vv:=aaa/kk;vv:=Numerator(vv);
      ss:=ss+(MoebiusMu(vv))*(PR[kk]);
     end for;

  vvv:=ss/aaa;
  Rr:=Integers(2); bb:=Rr!vvv;
  Jj:=Append(Jj,bb);
end for; jjel:=# Jj; ssum:=0; var:=0; for t in [1..jjel] do
  if Jj[t] eq 1 then
     tred:=Rr!t;
     if tred eq 1 then
     ssum:=ssum+t;
     var:=t;
     else
```

```
     ssum:=ssum;
     end if;
  else
     ssum:=ssum;
  end if;
end for;

PR2:=[* *]; a3:=0; cearrels:=Roots(x^2-a3*x+p,C);

for i in [1..20] do

b:=2*(p^i+1-Round(cearrels[1][1]^i+ p^i/cearrels[1][1]^i));
PR2:=Append(PR2,b); end for;

PR;PR2;

L:=[**]; F:=[**]; Level:=[**]; end for;
```

1.9. Magma programme by 4 primes, 15 subgroups order 8.

```
L:=[* *];F:=[* *];Level:=[* *];  N:=390; Factorization(N);N;
Nd:=Divisors(N);

p1:=2; p2:=3; p3:=5; p4:=13;

H1:=[*p1,p2,p3,p1*p2,p1*p3,p2*p3,p1*p2*p3*];
H2:=[*p1,p2,p4,p1*p2,p1*p4,p2*p4,p1*p2*p4*];
H3:=[*p4,p2,p3,p4*p2,p4*p3,p2*p3,p4*p2*p3*];
H4:=[*p1,p4,p3,p1*p4,p1*p3,p4*p3,p1*p4*p3*];
H5:=[*p1*p2,p3*p4,p2*p3,p1*p4,p2*p4,p1*p3,p1*p2*p3*p4*];
H6:=[*p1*p2*p3,p1,p2*p3,p4,p1*p2*p3*p4,p1*p4,p2*p3*p4*];
H7:=[*p1*p2*p3,p2,p1*p3,p4,p1*p2*p3*p4,p2*p4,p1*p3*p4*];
H8:=[*p1*p2*p3,p3,p1*p2,p4,p1*p2*p3*p4,p3*p4,p1*p2*p4*];
H9:=[*p1*p3*p4,p1,p3*p4,p2,p1*p2*p3*p4,p1*p2,p2*p3*p4*];
H10:=[*p1*p3*p4,p3,p1*p4,p2,p1*p2*p3*p4,p2*p3,p1*p2*p4*];
H11:=[*p2*p3*p4,p3,p2*p4,p1,p1*p2*p3*p4,p1*p3,p1*p2*p4*];
H12:=[*p1*p2*p3,p2*p3*p4,p1*p3*p4,p2*p4,p1*p4,p1*p2,p3*];
H13:=[*p1*p2*p3,p2*p3*p4,p2*p1*p4,p1*p4,p3*p4,p1*p3,p2*];
H14:=[*p1*p2*p3,p1*p3*p4,p1*p2*p4,p2*p3,p2*p4,p3*p4,p1*];
H15:=[*p1*p2*p4,p2*p3*p4,p1*p3*p4,p1*p2,p1*p3,p2*p3,p4*];

H:=[*H1,H2,H3,H4,H5,H6,H7,H8,H9,H10,H11,H12,H13,H14,H15*];

for subgroup in [8..8] do
 subgroup;
 Hh:=H[subgroup];
AtkinLehnerfix:=Hh; Involutions:=#AtkinLehnerfix;

for j in Nd do
MS:=NewformDecomposition(CuspidalSubspace(ModularSymbols(j,2,1)));
```

```
     m:=#MS;
     M:=PrimeDivisors(j);
     Nr:=Numerator(N/j);
     divi:=GCD(j,Nr);
     jj:=Numerator(j/divi);
     Mm:=PrimeDivisors(jj);
     Nn:=Divisors(jj);
     mm:=#Mm;
     mn:=#Nn;
     D:=Factorization(jj);
       for i in [1..m] do
          f:=Eigenform(MS[i],50);
          K:=Parent(Coefficient(f,3)); d:=Dimension(MS[i]);
          X:=IdentityMatrix(Rationals(), d);
          u:=0;

            if GCD(divi,j) eq 1 then
              for jo in [1..Involutions] do
                dd:=GCD(j,AtkinLehnerfix[jo]);
               if dd eq AtkinLehnerfix[jo] then
                 Y:=AtkinLehner(MS[i],dd);
                  if Y eq X then
                  else
                  u:=1;
                  end if;
              else
                 if dd eq 1 then
                 else
                 end if;
              end if;
             end for;
            end if;

         if u eq 0 then
         L:=Append(L,f);
         F:=Append(F,K);
         Level:=Append(Level,j);
               else
                end if;




     end for;
 end for;
Ll:=[*L[1],L[2],L[4],L[5],L[6],L[7],L[8]*]; L:=Ll;L;
Ff:=[*F[1],F[2],F[4],F[5],F[6],F[7],F[8]*]; F:=Ff; F; Level;Hh;
 felm:=# F;
```

```
p:=11;

C:=ComplexField(100); R<x>:=PolynomialRing(C); pj:=0*x+1; Roo:=[*
*]; for j in [1 .. felm] do
  if Degree(F[j]) eq 1 then
    cc:=Roots(x^2-Coefficient(L[j],p)*x+p,C);
    Roo:=Append(Roo,cc);
    pj:=pj*(x^2-Coefficient(L[j],p)*x+p);
  else
   dd:=Degree(F[j]);
   u:=Roots(DefiningPolynomial(F[j]),C); uu:= # u;
      for m in [1 .. uu] do
      f := hom< F[j] -> C | u[m][1]>;
      cc2:=Roots(x^2-f(Coefficient(L[j],p))*x+p,C);
      Roo:=Append(Roo,cc2);
      pj:=pj*(x^2-f(Coefficient(L[j],p))*x+p);
      end for;
  end if;
end for; pjdegree:=Degree(pj); pjdegree; PR:=[* *];
 d2:=Degree(pj);
long:= # Roo;


for nn in [1 .. 20] do s:=0;
  for i in [1 .. long] do
    for j in [1..2] do
     if Roo[i][j][2] gt 0 then
s:=s+(Roo[i][j][2])*(Roo[i][j][1])^(nn) ;
  else
  s:=s;
  end if;
end for; end for;
 a:=Round(1+p^(nn)-s); PR:=Append(PR,a); end for;
Jj:=[* *]; for aaa in [1..20] do
 ss:=0;
  adiv:=Divisors(aaa);
    for kk in adiv do
       vv:=aaa/kk;vv:=Numerator(vv);
      ss:=ss+(MoebiusMu(vv))*(PR[kk]);
     end for;

  vvv:=ss/aaa;
  Rr:=Integers(2); bb:=Rr!vvv;
  Jj:=Append(Jj,bb);
end for; jjel:=# Jj; ssum:=0; var:=0; for t in [1..jjel] do
  if Jj[t] eq 1 then
     tred:=Rr!t;
     if tred eq 1 then
     ssum:=ssum+t;
```

```
     var:=t;
     else
     ssum:=ssum;
     end if;
  else
     ssum:=ssum;
  end if;
end for;

PR2:=[* *]; a3:=2; cearrels:=Roots(x^2-a3*x+p,C);

for i in [1..20] do

b:=2*(p^i+1-Round(cearrels[1][1]^i+ p^i/cearrels[1][1]^i));
PR2:=Append(PR2,b); end for;

pr2:=PR2[2]; pr:=PR[2]; resta:=pr-pr2;

resta;resta;

PR; PR2;

end for;
```

```
L:=[* *];F:=[* *];Level:=[* *];  N:=210; Factorization(N);N;
Nd:=Divisors(N);

p1:=2; p2:=3; p3:=5; p4:=7; N1:=[*p1,p2,p1*p2*];
N2:=[*p1,p3,p1*p3*]; N3:=[*p1,p4,p4*p1*]; N4:=[*p2,p3,p2*p3*];
N5:=[*p2,p4,p2*p4*]; N6:=[*p3,p4,p3*p4*]; N7:=[*p1,p2*p3,p1*p2*p3*];
N8:=[*p1,p2*p4,p1*p2*p4*]; N9:=[*p1,p3*p4,p1*p3*p4*];
N10:=[*p1,p2*p3*p4,p1*p2*p3*p4*]; N11:=[*p2,p1*p3,p1*p2*p3*];
N12:=[*p2,p1*p4,p1*p2*p4*]; N13:=[*p2,p3*p4,p2*p3*p4*];
N14:=[*p2,p1*p3*p4,p1*p2*p3*p4*]; N15:=[*p3,p2*p1,p1*p2*p3*];
N16:=[*p3,p2*p4,p3*p2*p4*]; N17:=[*p3,p1*p4,p1*p3*p4*];
N18:=[*p3,p2*p1*p4,p1*p2*p3*p4*]; N19:=[*p4,p2*p3,p4*p2*p3*];
N20:=[*p4,p2*p1,p1*p2*p4*]; N21:=[*p4,p3*p1,p1*p3*p4*];
N22:=[*p4,p2*p3*p1,p1*p2*p3*p4*]; N23:=[*p1*p2,p1*p3,p2*p3*];
N24:=[*p1*p4,p1*p2,p2*p4*]; N25:=[*p1*p4,p1*p3,p3*p4*];
N26:=[*p2*p3,p3*p4,p2*p4*]; N27:=[*p1*p2,p3*p4,p1*p2*p3*p4*];
N28:=[*p1*p3,p2*p4,p1*p2*p3*p4*]; N29:=[*p1*p4,p2*p3,p1*p2*p3*p4*];
N30:=[*p1*p2,p1*p3*p4,p2*p3*p4*]; N31:=[*p1*p3,p1*p2*p4,p3*p2*p4*];
N32:=[*p1*p4,p1*p2*p3,p4*p2*p3*]; N33:=[*p2*p3,p2*p1*p4,p3*p1*p4*];
N34:=[*p2*p4,p2*p1*p3,p4*p1*p3*]; N35:=[*p3*p4,p3*p2*p1,p4*p2*p1*];




H:=[*N1,N2,N3,N4,N5,N6,N7,N8,N9,N10,N11,N12,N13,N14,N15,N16,N17,N18,N19,N20,
N21,N22,N23,N24,N25,N26,N27,N28,N29,N30,N31,N32,N33,N34,N35*];


for subgroup in [35..35] do
 subgroup;
 Hh:=H[subgroup];
AtkinLehnerfix:=Hh; Involutions:=#AtkinLehnerfix;

for j in Nd do
MS:=NewformDecomposition(CuspidalSubspace(ModularSymbols(j,2,1)));
     m:=#MS;
     M:=PrimeDivisors(j);
     Nr:=Numerator(N/j);
     divi:=GCD(j,Nr);
     jj:=Numerator(j/divi);
     Mm:=PrimeDivisors(jj);
     Nn:=Divisors(jj);
     mm:=#Mm;
     mn:=#Nn;
     D:=Factorization(jj);
       for i in [1..m] do
          f:=Eigenform(MS[i],50);
```

```
          K:=Parent(Coefficient(f,3)); d:=Dimension(MS[i]);
          X:=IdentityMatrix(Rationals(), d);
          u:=0;

            if GCD(divi,j) eq 1 then
              for jo in [1..Involutions] do
                dd:=GCD(j,AtkinLehnerfix[jo]);
               if dd eq AtkinLehnerfix[jo] then
                 Y:=AtkinLehner(MS[i],dd);
                  if Y eq X then
                  else
                  u:=1;
                  end if;
               else
                  if dd eq 1 then
                  else
                  end if;
               end if;
             end for;
            end if;

        if u eq 0 then
        L:=Append(L,f);
        F:=Append(F,K);
        Level:=Append(Level,j);
              else
               end if;




     end for;
 end for;

L;F;Level;
 felm:=# F;

p:=11;

C:=ComplexField(100); R<x>:=PolynomialRing(C); pj:=0*x+1; Roo:=[*
*]; for j in [1 .. felm] do
  if Degree(F[j]) eq 1 then
    cc:=Roots(x^2-Coefficient(L[j],p)*x+p,C);
    Roo:=Append(Roo,cc);
    pj:=pj*(x^2-Coefficient(L[j],p)*x+p);
  else
   dd:=Degree(F[j]);
   u:=Roots(DefiningPolynomial(F[j]),C); uu:= # u;
      for m in [1 .. uu] do
```

```
      f := hom< F[j] -> C | u[m][1]>;
      cc2:=Roots(x^2-f(Coefficient(L[j],p))*x+p,C);
      Roo:=Append(Roo,cc2);
      pj:=pj*(x^2-f(Coefficient(L[j],p))*x+p);
      end for;
  end if;
end for; pjdegree:=Degree(pj); pjdegree; PR:=[* *];
 d2:=Degree(pj);
long:= # Roo;


for nn in [1 .. 20] do s:=0;
  for i in [1 .. long] do
    for j in [1..2] do
     if Roo[i][j][2] gt 0 then
s:=s+(Roo[i][j][2])*(Roo[i][j][1])^(nn) ;
  else
  s:=s;
  end if;
end for; end for;
 a:=Round(1+p^(nn)-s); PR:=Append(PR,a); end for;
Jj:=[* *]; for aaa in [1..20] do
 ss:=0;
  adiv:=Divisors(aaa);
    for kk in adiv do
       vv:=aaa/kk;vv:=Numerator(vv);
      ss:=ss+(MoebiusMu(vv))*(PR[kk]);
     end for;

  vvv:=ss/aaa;
  Rr:=Integers(2); bb:=Rr!vvv;
  Jj:=Append(Jj,bb);
end for; jjel:=# Jj; ssum:=0; var:=0; for t in [1..jjel] do
  if Jj[t] eq 1 then
     tred:=Rr!t;
     if tred eq 1 then
     ssum:=ssum+t;
     var:=t;
     else
     ssum:=ssum;
     end if;
  else
     ssum:=ssum;
  end if;
end for;

PR2:=[* *]; a3:=0; cearrels:=Roots(x^2-a3*x+p,C);

for i in [1..20] do
```

```
b:=2*(p^i+1-Round(cearrels[1][1]^i+ p^i/cearrels[1][1]^i));
PR2:=Append(PR2,b); end for;

PR;PR2;

L:=[**]; F:=[**]; Level:=[**]; end for;
```