Research Track 2 - Assignment 3
Statistical Evaluation

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1 Introduction

The goal of this assignment is to make a statistical evaluation. In particular the goal is to compare two different algorithms developed for the first Assignment of Research Track 1. The two algorithms taken into account are:

1. Robot 1: `https://github.com/Francesca-Corrao/Assignment1`
2. Robot 2: `https://github.com/Tabi43/First-Assignment-RT1`

In the following paragraphs the steps made in order to make a statistical evaluation are shown. In particular an Hypothesis is made and to prove it, whit a certain significance level, a test is designed. In order to found the data to perform the test some experiment are done.

2 Hypothesis

The Hypothesis made is that:

> "the algorithm developed by me (1) has a higher rate of success than the one developed by my colleague (2) when the position and number of tokens in the map change"

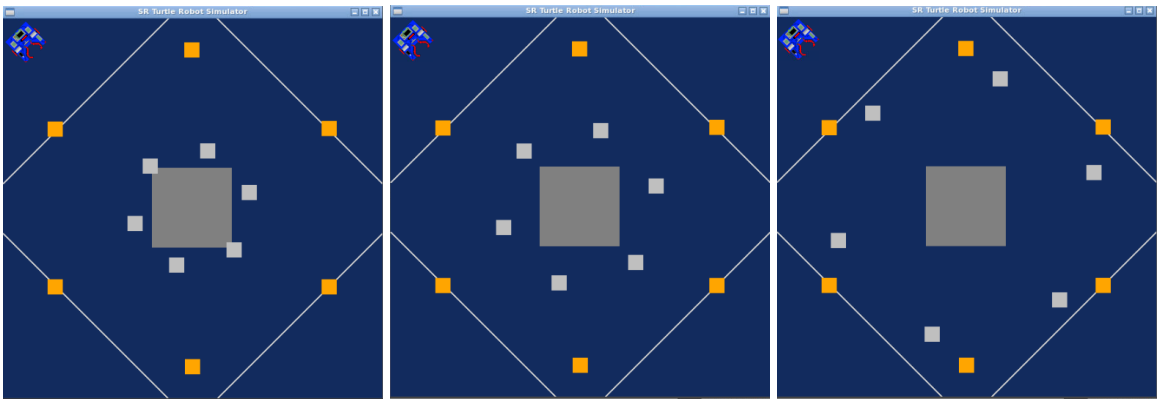
The *rate of success* is the number of silver-golden tokens paired over the number of tokens in the environment. A couple of silver-gold tokens is considered *paired* if during its execution the robot takes the silver token and releases it to the gold, once a couple is paired what happens to it doesn't matter,and for the sake of experiments will be considered paired in the end. This is lead to the following definition of null and alternative Hypothesis:

- $H_0 : \mu_1 = \mu_2$ is the **null Hypothesis**;
- $H_1 : \mu_1 > \mu_2$ is the **alternative Hypothesis**.

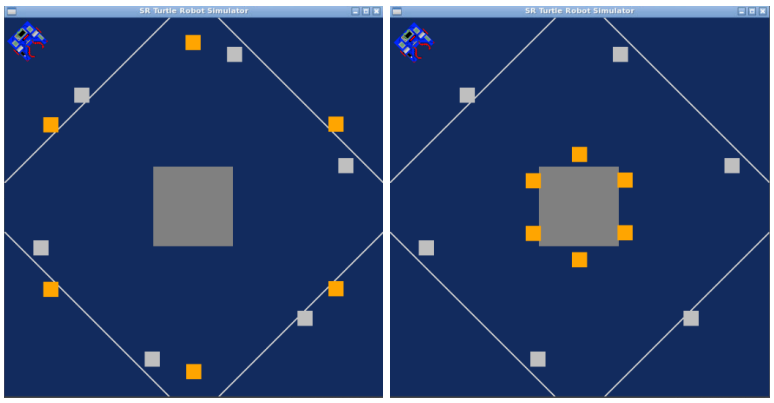
This means that the test that should be used is a **one-tailed test**, in particular a right-tailed test because the goal is to prove that the rate of success of algorithm 1 is greater than the rate of success of algorithm 2.
To prove H_1 the null Hypothesis H_0 must be rejected with a **significant level** of 5%. An alternative could be to use a 1% level of significance but, since the values of rate of success go between [0,1] and it's not expect to vary too much, it's better to choose 5%.
The distribution we aim to use is a t-distribution and in order to apply the central limit theorem, so that the shape of the sampling distribution is more similar that the one of a t-distribution, we need a number of sampling greater than 30. This must be kept in mind designing the number of experiments to make.
Lastly a paired T-test is chosen to test the hypothesis since we are comparing two different approaches applied to the same scenario.

3 Experiments

In the experiments both algorithms are executed whit different disposition of tokens in the environment and different numbers of tokens, and for each execution the rate of success is computed as defined above: number of tokens paired over total number of tokens. Let's see what are the different configurations taken into account. To take into account how the position of tokens effects the performance of the algorithms, 5 different maps have been chosen. In the first four the radius of the inner circle, in which the silver tokens are placed, is increased until both tokens are almost on the same circle. In the last one the position of silver tokens and gold tokens is swapped, so the gold token are on the inner circle and the silver token are on the outer circle. The different position of token in the environment taken into account for the experiment can be seen in Figure 1.



(a) original configuration (b) inner circle +0.3 (c) inner circle +1



(d) inner and outer circle the same (e) gold and silver swapped

Figure 1: Position of tokens in the environment

To see how the rate of success changes with respect to the number of token on the map, the 5 different environments designed above are run whit 3 different number of token in the map: the original number and the case in which 2 tokens are added and 2 tokens are removed. In the end the experiment is performed with 6 tokens, 4 tokens and 8 tokens. To have a more robust measurement of the rate of success for each configuration (position-number of tokens), the algorithms are run 3 times. In the end the over all states evaluated are 45 (5x3x3) which is greater that 30 and therefore the central limit theorem allow us to say that the sample distribution has the shape of a t-distribution. The results obtained are shown in the table 1.

Position	n_token	Robot 1			Robot 2			Difference		
		rate_success			rate_success			$x_1 - x_2$		
original map		1	1	1	1	1	1	0	0	0
inner 1.2		1	1	1	1	1	1	0	0	0
inner 2	6	1	1	1	0	0	0	1	1	1
inner = outer		0,5	1	0.5	0	0	0	0.5	1	0.5
swap silver-gold		0,167	0,667	0.5	0,333	1	0.5	-0.167	-0.333	0
original map		1	1	1	1	1	1	0	0	0
inner 1.2		1	1	1	1	1	1	0	0	0
inner 2	4	1	1	1	1	1	1	0	0	0
inner = outer		1	1	1	1	1	1	0	0	0
swap silver-gold		0,5	0,5	0,75	1	0,75	1	-0.5	-0.25	-0.5
original map		0,75	0,75	0,75	0	0,625	0	0.75	0.125	0.75
inner 1.2		0,75	0,75	0,75	0,875	1	1	-0.125	-0.25	-0.25
inner 2	8	0,25	0,25	0,25	1	1	0,875	-0.75	-0.75	-0.625
inner = outer		0,625	0,75	0,75	0,125	0,125	0,125	0.5	0.625	0.625
swap silver-gold		0,125	0,125	0,125	0,125	0,625	0,625	0	-0.5	-0.5

Table 1: Experiment Result

4 Paired T-test

To test the Hypothesis a Paired T-test is performed with the data obtained from the experiments (Table 1). Since the goal is to prove $H_1 : \mu_1 > \mu_2$, so to reject the null Hypothesis $H_0 : \mu_1 = \mu_2$ with a significant level of 5%.
To prove this the following values are computed:

- the difference $d = x_1 - x_2$ between the two observation of each pair, last column of Table 1;
- the **mean** of the difference it $\bar{d} = 0,070$
- the **standard deviation** of the difference $s_d = 0,450$
- the **standard error** of the difference $SE(d) = \frac{s_d}{\sqrt{N}} = 0,067$
- the **t-value** $t = \frac{\bar{d}}{SE(d)} = 1,045$

Once the t-value is found it is compered with the value on the t-table in this case (44 DOF and 5% level of significance) 1,680. Since the t-value computed is less then the one on the t-table, the null Hypothesis H_0 can't be rejected with a confidence of 95%. In particular the p-value associated to the t-value it is 0.15 which means that the null hypothesis can be rejected whit a confidence of 85% which means a 15% level of significant which is 3 times the one requested.

5 Conclusion

In the end whit the experiments done so far the Hypothesis isn't proven. To make sure that the null Hypothesis can't be rejected, the power of the test in detecting false null hypothesis could be increased by increasing the sample size and therefore make more experiments. This is because, as the sample size increases, the mean and the standard error will stabilize to the true parameter and therefore the value of the t-test will be more correct. The experiment could be extended:

- by designing another map in which the position of the tokens is changed, by taking into account another configuration the sample size is 54 (6x3x3);
- by taking more measurement for each map, in fact by adding another measurement for each environment configuration the sample size became 60 (5x4x3);
- by considering others numbers of tokens in the map, for example take into account also when one token is added or removed from the map (5 token and 7 token) this will bring the sample size to 75 (5x3x5).

To see if it's worth to take other samples or however the result obtained with the paired t-test is the same, another measurement for each map is assumed with an optimistic approach. In the optimistic approach all the new measurement will advantage Robot 1, this means that for each set of data taken (3 for each map) the highest rate of success is taken for Robot 1 and the lowest for Robot 2. With this optimistic assumption and a new ideal data set of 60 samples, the result found is that the t-value will be $t = 1.73$ that is greater than the one in the t-table and the corresponding p-value is $p = 0.044$ so the null Hypothesis could be discarded with a level of significance of 5%. This scenario is very difficult to happen, because, even if only a few of the experiments benefit Robot 2, then the result found is similar to the one obtained before and so again the null Hypothesis can't be discarded with a confidence of 95%.