Progetto bayesian

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Struttura dell'elaborato

Initial problem

Problem: computationally prohibitive (or unavailable) evaluation of the likelihood function

Given the posterior distribution $\pi(\theta|y_{obs}) = \frac{p(y_{obs}|\theta)\pi(\theta)}{\int_{\Theta} p(y_{obs}|\theta)\pi(\theta)d\theta}$

Approach

To solve this issue we can use methods based on the approximation of the likelihood function, called *Likelihood-free methods*.

algo generale

Is this an efficient method for complex analysis?

Likelihood-free

punto 3 modificato $<$ h

This adjust the potentially very low (or zero) probability requirement that $y = y_{obs}$ exactly.

ABC

We focused on a particular case of the Likelihood-free methods: the *Approximate Bayesian Computation (ABC)*.

The aim: find a practical way of performing Bayesian analysis, while keeping the approximation and the computation to a minimum.

The likelihood-free rejection algorithm is sampling from the joint distribution $\propto \mathbb{I}(\| y - y_{obs} \| \leq h) p(y|\theta)\pi(\theta)$
$\Longrightarrow$ replace the indicator function with a standard smoothing kernel function $K_h(u)$, with $u = \| y - y_{obs} \|$:

$$K_h(u) = \frac{1}{h} K(\frac{u}{h})$$

Hence:

$$\pi_{ABC}(\theta, y | y_{obs}) \propto K_h(u) p(y|\theta)\pi(\theta)$$

ABC - summary statistics

Is this feasible in practice?

In practice: difficult to have $y \approx y_{obs}$ from $p(y|\theta)$, unless y_{obs} very low dimensional or $p(y|\theta)$ factorises into low-dimensional components.
Thus we should use a large h, obtaining a poor posterior approximation!
$\Longrightarrow$ use summary statistics $s = S(y)$

$$\pi_{ABC}(\theta|s_{obs})$$

Summary statistics

Critical decision: choice of summary statistics

Dimension of summary statistics:

- large enough to contain as much as informations about observed data as possible
- low enough to avoid curse of dimensionality of matching s and s_{obs}

$\Longrightarrow$ choose sufficient statistics, such that:

$$\pi(\theta|s_{obs}) \equiv \pi(\theta|y_{obs})$$

Distance measure

Distance measure: substantial impact on ABC algorithm efficiency

$$\| s - s_{obs} \| = (s - s_{obs})^\top \Sigma^{-1} (s - s_{obs})$$

- $\Sigma =$ identity matrix $\rightarrow$ Euclidean distance
- $\Sigma =$ diagonal matrix of non-zero weights $\rightarrow$ Weighted Euclidean distance
- $\Sigma =$ full covariance matrix of s $\rightarrow$ Mahalanobis distance

ABC algorithm

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we can add a stopping rule to the ABC Rejection Sampling Algorithm

Grazie per l'attenzione!