GPU Computing

Laurea **Magistrale in Informatica** - AA 2018/19

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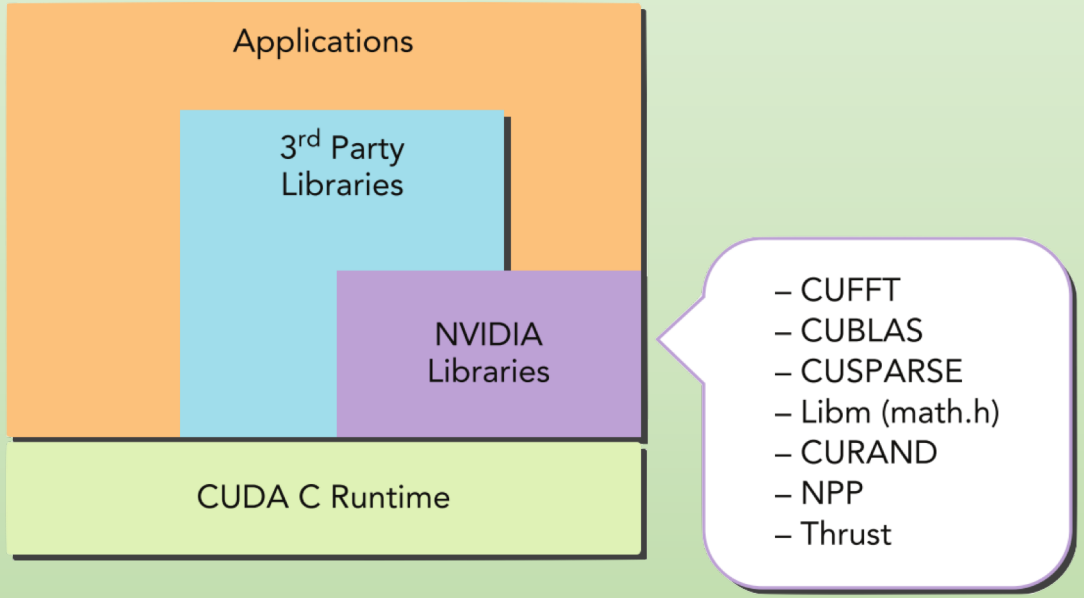
Lezione 9 – Librerie CUDA

Sommario

- ✓ Librerie CUDA
- ✓ cuBLAS
- ✓ cuRAND
- ✓ cuFFT
- ✓ demo

GPU-accelerated CUDA library

- ✓ Tutte le computazioni implementate nelle **librerie** CUDA sono **accelerate** dalla GPU
- ✓ Per molte applicazioni, le librerie CUDA offrono un buon **bilanciamento** tra **usabilità** e **prestazioni**
- ✓ Le API di molte librerie CUDA sono volutamente **simili** a quelle della **libreria standard** nello stesso dominio
- ✓ Il vantaggio è il minimo sforzo per il **porting** del codice sequenziale a parallelo (meno tempo e complessità)
- ✓ Nessun costo di **mantenimento** che è a **carico** degli **sviluppatori** della libreria (compito svolto in genere da esperti in CUDA programming)
- ✓ Alcune sono sviluppate da NVIDIA direttamente altre da terze parti
- ✓ Tutte si appoggiano sopra il livello Runtime al pari delle applicazioni sviluppate da utenti
- ✓ Esistono reference guide ottimamente redatte per illustrarne uso e potenzialità



Esempi di librerie CUDA

Esempi di librerie sviluppate in CUDA a supporto di determinati domini, scaricabili su **`developer.nvidia.com`**

Libreria	Dominio
cuFFT (NVIDIA)	Fast Fourier Transforms Linear
cuBLAS (NVIDIA)	Linear Algebra (BLAS Library)
cuSPARSE (NVIDIA)	Sparse Linear Algebra
cuRAND (NVIDIA)	Random Number Generation
NPP (NVIDIA)	Image and Signal Processing
CUSP (NVIDIA)	Sparse Linear Algebra and Graph Computations
CUDA Math Library (NVIDIA)	Mathematics
Trust (terze parti)	Parallel Algorithms and Data Structures
MAGMA (terze parti)	Next generation Linear Algebra

Workflow tipico

1. Creare un **handle** specifico della libreria

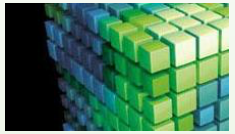
   (per la gestione delle informazioni e relativo contesto in cui essa opera, es. uso degli stream)

2. Allocare la **device memory** per gli **input** e **output** alle funzioni della libreria

   (Se gli input non sono già in un formato supportato dalla libreria, convertirli al **formato specifico** di uso della liberia, es. converti array 2D in column-major order)

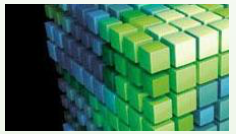
3. **Popolare** con i **dati** nel formato specifico
4. **Configurare** le computazioni per l'esecuzione (es. dimensione dei dati)
5. **Eseguire** la **chiamata** della funzione di libreria che avvia la computazione sulla GPU
6. **Recuperare** i **risultati** dalla device memory
7. Se necessario, **(ri)convertire** i dati nel formato specifico o nativo dell'applicazione
8. **Rilasciare** le risorse CUDA allocate per la data libreria

Libreria cuBLAS



- ✓ **Basic Linear Algebra Subprograms** (**BLAS**)
- ✓ All operations are done on **dense** cuBLAS **vectors** or **matrices**
- ✓ Like BLAS, cuBLAS subroutines are split into **multiple classes** based on the data types on which they operate.
  - **cuBLAS Level 1** contains **vector-only** operations like vector addition.
  - **cuBLAS Level 2** contains **matrix-vector** operations like matrix-vector multiplication.
  - **cuBLAS Level 3** contains **matrix-matrix** operations like matrix-multiplication.
- ✓ Building blocks for higher level numerical linear algebra such as in **LAPACK**
- ✓ Both **BLAS** and **LAPACK** were developed in **Fortran**
- ✓ Because the original **BLAS** library was written in **fortran**, it historically uses **column-major** array storage and one-based indexing
- ✓ The new **CuBLAS interface** is entirely **asynchronous** in nature, meaning even functions that return values do it in such a way as the value may not be available unless the programmer specifically waits for the asynchronous GPU operation to complete.

Da… docs.nvidia.com



Error status

All cuBLAS library function calls return the error status `cublasStatus_t`

cuBLAS context

The application must initialize the **handle** to the cuBLAS library context by calling the `cublasCreate()` function. Then, the is explicitly passed to every subsequent library function call. Once the application finishes using the library, it must call the function `cublasDestory()` to release the resources associated with the cuBLAS library context

Thread Safety

The library is thread safe and its functions can be called from **multiple host threads**, even with the same handle… it is not recommended that multiple thread share the same CUBLAS handle

Library support

For example, on Linux, to compile a small application using cuBLAS, against the **dynamic library**, the following command can be used:

```
nvcc myCublasApp.c -lcublas -o myCublasApp
```

Read more at: http://docs.nvidia.com/cuda/cublas/index.html#ixzz4hR2vT2yG

cuBLAS lib

Implementazione delle librerie BLAS (Basic Linear Algebra Subprograms)

- ✓ **Livello 1**: operazioni basate su **vettori** e **scalari** (BLAS1)
  - ES: $y[i] = \alpha * x[i] + y[i], \qquad i = 1, \ldots, n$
- ✓ **Livello 2**: operazioni basate su **vettori** e **matrici** (BLAS2)
  - ES: $y = \alpha A x + \beta y, \qquad \text{dove: } \alpha, \beta \text{ scalari, } A \text{ matrice, } x, y \text{ vettori}$
- ✓ **Livello 3**: operazioni basate su **matrici** e **matrici** (BLAS3)
  - ES: $C = \alpha A B + \beta C, \qquad \text{dove: } \alpha, \beta \text{ scalari, } A, B, C \text{ matrici}$
- ✓ Precisione
  - single: real & complex
  - double: real & complex (not all functions)
- ✓ Non è richiesta nessuna invocazione di kernel, no uso della shared memory, etc

Operare con cuBLAS

Passi:

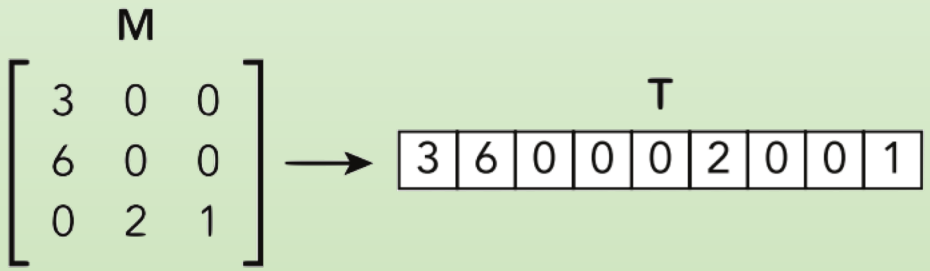
1. **Create** a cuBLAS handle using **`cublasCreateHandle`**
2. **Allocate** device memory for inputs and outputs using **`cudaMalloc`**
3. **Populate** the allocated device memory with inputs using **`cublasSetVector`** and **`cublasSetMatrix`**
4. **Execute** (e.g.) **`cublasSgemv`** library call to offload a matrix-vector multiplication operation to the GPU
5. **Retrieve** results from device memory using **`cublasGetVector`**
6. **Release** CUDA and cuBLAS resources using **`cudaFree`** and **`cublasDestroy`**.

Column-major order in cuBLAS

- ✓ Per ragioni di compatibilità le cuBLAS library scelgono di usare la memorizzazione **column-major**
- ✓ Se ho una matrice di **M** righe e **N** colonne, una entry di posto **(m,n)** nei due casi si ottiene:

$$\text{Row-Major: } f(m, n) = m \times N + n$$

$$\text{Column-Major: } f(m, n) = n \times M + m$$



- ✓ Compliazione e linking di lib dinamica:

```
#include "cublas_v2.h"
```

```
$ nvcc -arch=sm_20 -lcublas kernel.cu -o kernel
```

cuBLAS datatypes

Every cuBLAS API function comes in four different data types

- single precision floating point (S)
- double precision floating point (D)
- complex single precision floating point (C)
- complex double precision floating point (Z)

<type>	<t>	Meaning
`float`	's' or 'S'	real single-precision
`double`	'd' or 'D'	real double-precision
`cuComplex`	'c' or 'C'	complex single-precision
`cuDoubleComplex`	'z' or 'Z'	complex double-precision

- **SGEMM**: Single Precision General Matrix-Matrix Multiply
- **DGEMM**: Double Precision General Matrix-Matrix Multiply
- **CGEMM**: Complex Single Precision General Matrix-Matrix Multiply
- **ZGEMM**: Complex Double Precision General Matrix-Matrix Multiply

Operazioni

Value	Meaning
`CUBLAS_OP_N`	the non-transpose operation is selected
`CUBLAS_OP_T`	the transpose operation is selected
`CUBLAS_OP_C`	the conjugate transpose operation is selected

CUBLAS APIs: Set & Get

Copies ***n* elements** from a vector ***cpumem*** on the CPU memory to a vector ***gpumem*** on the GPU memory

Prototipo->

```
cublasSetVector(int n, int elemSize, const void *cpumem, int incx, void *gpumem, int incy)
```

- ***cpumem***: pointer of an array to send data ***gpumem***: pointer of an array to receive data
- ***elemSize***: Each element size in byte ***incx***, ***incy***: Storage spacing size (= 1 for a vector)

Copies *n* elements from a vector ***gpumem*** on the GPU memory to a vector ***cpumem*** on the CPU memory

Prototipo->

```
cublasGetVector(int n, int elemSize, const void *gpumem, int incx, void *cpumem, int incy)
```

Prototipo->

```
cublasSetMatrix(int rows, int cols, int elemSize, const void *A, int lda, void *B, int ldb)
```

- This function copies a tile of `rows x cols` elements from a matrix `A` in host memory space to a matrix `B` in GPU memory space. It is assumed that each element requires storage of `elemSize` bytes and that both matrices are stored in column-major format, with the leading dimension of the source matrix `A` and destination matrix `B` given in `lda` and `ldb`, respectively

Prototipo->

```
cublasGetMatrixAsync(int rows, int cols, int elemSize, const void *A, int lda, void *B,
                                                                    int ldb, cudaStream_t stream)
```

- This function has the same functionality as cublasGetMatrix(), with the exception that the data transfer is done asynchronously (with respect to the host) using the given CUDA™ stream parameter

Read more at: http://http://docs.nvidia.com/cuda/cublas/index.html#cublas-helper-function-reference

Esempi: copia di matrici e vettori

- ✓ Transferring a **single column** with length M of a column-major matrix A to a vector dV could be done using:

```
cublasSetVector(M, sizeof(float), A, 1, dV, 1);
```

- ✓ To transfer a **single row** of that matrix A to a vector dV on the device:

```
cublasSetVector(M, sizeof(float), A, M, dV, 1);
```

- ✓ This function copies N elements from A to dV, skipping M elements in A at a time. Because A is column-major, this command would copy the first row of A to the device. Copying **row i** would be implemented as:

```
cublasSetVector(N, sizeof(float), A + i, M, dV, 1);
```

- ✓ If you were given a dense **two-dimensional column-major matrix A** of single-precision floating-point values on the host with M rows and N columns, you could use:

```
cublasSetMatrix(M, N, sizeof(float), A, M, dA, M);
```

CUBLAS(2) APIs: mat-vect multiplication

Prototipo->

```
cublasStatus_t cublasSgemv(cublasHandle_t handle,
                           cublasOperation_t trans,
                                   int m, int n,
                                   const float *alpha,
                                   const float *A,
                                   int lda,
                                   const float *x,
                                   int incx,
                                   const float *beta,
                                   float *y,
                                   int incy)
```

Read more at: http://docs.nvidia.com/cuda/cublas/index.html#ixzz496uNjCKh

Param.	Memory	In/out	Meaning
handle		input	handle to the cuBLAS library context
trans		input	operation op(A) that is non- or (conj.) transpose
m		input	number of rows of matrix A
n		input	number of columns of matrix A
alpha	host/dev	input	<type> scalar used for multiplication
A	device	input	<type> array of dimension lda x n with lda >= max(1,m) if transa==CUBLAS_OP_N and lda x m with lda >= max(1,n) otherwise
lda		input	leading dimension of two-dimensional array used to store matrix A
x	device	input	<type> vector with n elements if transa==CUBLAS_OP_N and m elements otherwise
incx		input	stride between consecutive elements of x
beta	host/dev	input	<type> scalar used for multiplication, if beta==0 then y does not have to be a valid input
y	device	in/out	<type> vector with m elements if transa==CUBLAS_OP_N and n elements otherwise
incy		input	stride between consecutive elements of y

CUBLAS(3) APIs: mat-mat multiplication

Prototipo->

```
cublasStatus_t cublasSgemm(cublasHandle_t handle, cublasOperation_t transa,
                           cublasOperation_t transb,
                           int m, int n, int k,
                           const float *alpha,
                           const float *A, int lda,
                           const float *B, int ldb,
                           const float *beta,
                           float *C, int ldc)
```

Param.	Memory	In/out	Meaning
handle		input	handle to the cuBLAS library context.
transa		input	operation op(A) that is non- or (conj.) transpose.
transb		input	operation op(B) that is non- or (conj.) transpose.
m		input	number of rows of matrix op(A) and C.
n		input	number of columns of matrix op(B) and C.
k		input	number of columns of op(A) and rows of op(B).
alpha	host/dev	input	<type> scalar used for multiplication.
A	device	input	<type> array of dimensions lda x k with lda>=max(1,m) if transa == CUBLAS_OP_N and lda x m with lda>=max(1,k) otherwise.
lda		input	leading dimension of two-dimensional array used to store the matrix A.
B	device	input	<type> array of dimension ldb x n with ldb>=max(1,k) if transa == CUBLAS_OP_N and ldb x k with ldb>=max(1,n) otherwise.
ldb		input	leading dimension of two-dimensional array used to store matrix B.
beta	host/devi	input	<type> scalar used for multiplication. If beta==0, C does not have to be a valid input.
C	device	in/out	<type> array of dimensions ldc x n with ldc>=max(1,m).
ldc		input	leading dimension of a two-dimensional array used to store the matrix C

Read more at: http://docs.nvidia.com/cuda/cublas/index.html#ixzz496tehfHp

Copiare una sottomatrice

Estrazione di sottomatrice da una matrice [**100 x 100**] di dimensioni: **rows [10:99], cols [90:99]**

allocazione sul device di **rowsB * colsB (90 x 10)** float

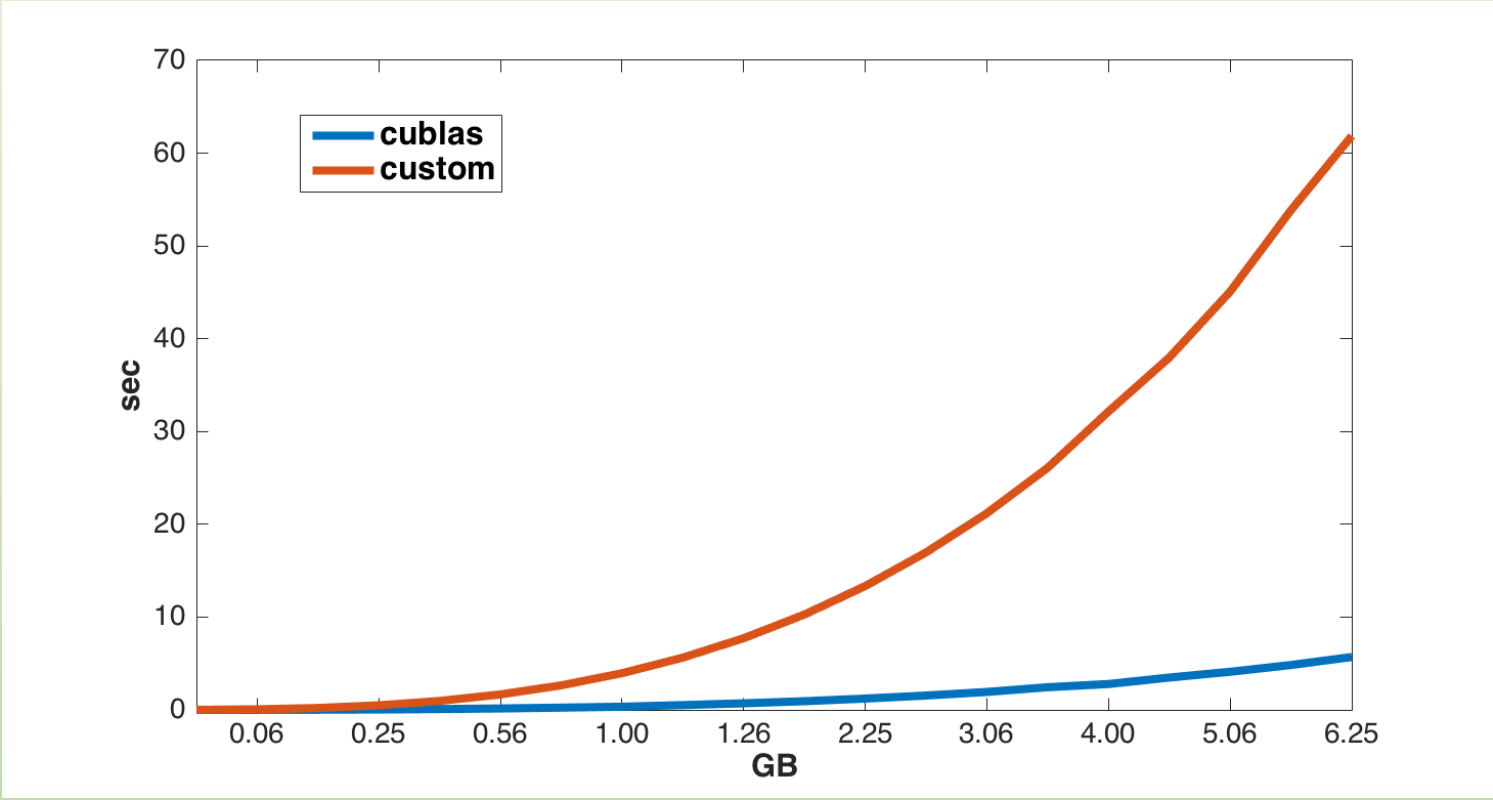
salta 10 righe e 90 colonne

```
// The matrix in host memory
int rowsA = 100;
int colsA = 100;
float *A = (float*)malloc(rowsA*colsA*sizeof(float ));
...
// The sub-matrix that should be copied to the device.
// The minimum index is INCLUSIVE
// The maximum index is EXCLUSIVE
int minRowA = 10;
int maxRowA = 100;
int minColA = 90;
int maxColA = 100;
int rowsB = maxRowA-minRowA; // = 90
int colsB = maxColA-minColA; // = 10

// Allocate the device matrix
float *dB = nullptr;
cudaMalloc(&dB, rowsB * colsB * sizeof(float));

// pointers and sizes
float *subA = A + (minRowA + minColA * rowsA);
cublasSetMatrix(rowsB, colsB, sizeof(float), subA, rowsA, dB, rowsB);
//                90    10                      100        90
```

Risultati

Risultati su prodotti tra matrici quadrate su Tesla K40c:

- Dimensione delle matrici: da **0.06 GByte** a **6.25 Gbyte** (size N x N = 20480 x 20480)
- **cublas**: funzione di libreria cuBLAS
- **custom**: kernel sviluppato a lezione



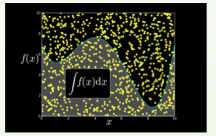
Esercitazione

Effettuare moltiplicazioni tra matrici MQDB:

Utilizzare cuBLAS e verificare efficienza:

- Indicizzare sottomatrici
- Effettuare un loop su host senza kernel
- Provare approccio complessivo o blocco-a-blocco

cuRAND



La libreria cuRAND fornisce semplici ed efficienti generatori di numeri

- ✓ **Pseudorandom***:* **sequenza** di numeri che **soddisfa** molte delle **proprietà statistiche** di una vera sequenza casuale… es. la seq. generata da un algoritmo deterministico:

$$A_{n+1} = (Z * A_n + I) MOD(M)$$

- ✓ **Quasirandom***:* sequenza di punti **n-dimensionali uniformemente generati** da un algoritmo deterministico (non si formano cluster nello spazio di generazione)

- ✓ **cuRAND** si compone di due parti: una libreria per l'host (**curand.h**) e una per il device (**curand_kernel.h**)

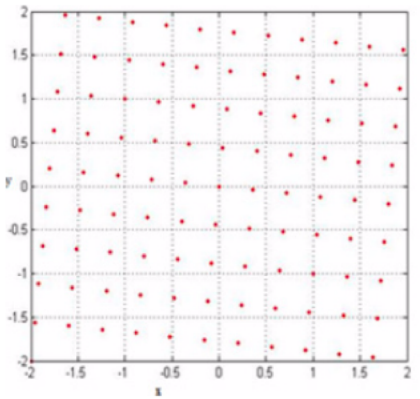


Fig-1a:Quasi-random distribution

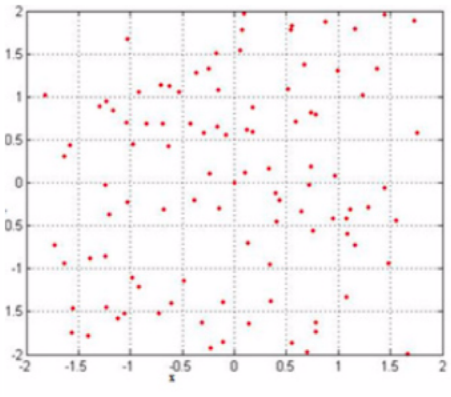


Fig-1b:Pseudo-random distribution

cuRAND host

✓Using on the host:

- Call from host
- Allocates memory on GPU
- Generates random numbers on GPU

✓Several pseudorandom generators available

✓Several random distributions available

Host API

- Functions to know:
  - `curandCreateGenerator(&g, GEN_TYPE)`
    - `GEN_TYPE = CURAND_RNG_PSEUDO_DEFAULT, CURAND_RNG_PSEUDO_XORWOW`
    - Doesn't particularly matter, differences are small
  - `curandSetRandomGeneratorSeed(g, SEED)`
    - Again, `SEED` doesn't matter too much, just pick one (ex.: time(NULL))
  - `curandGenerate_____(…)`
    - Depends on distribution
    - Ex.: `curandGenerate(g, src, n), curandGenerateNormal(g, src, n, mean, stddev)`
  - `curandDestroyGenerator(g)`

Svantaggi host API

✓`curandGenerate()` launches asynchronously

  ✓Much faster than serial CPU generation

✓ However, we still need to copy data to GPU

- `src` in curandGenerate() is host pointer, not device pointer!
- Introduces some undesired overhead
- Might need more memory than we can pass in one go

✓Solution: **cuRAND device API**

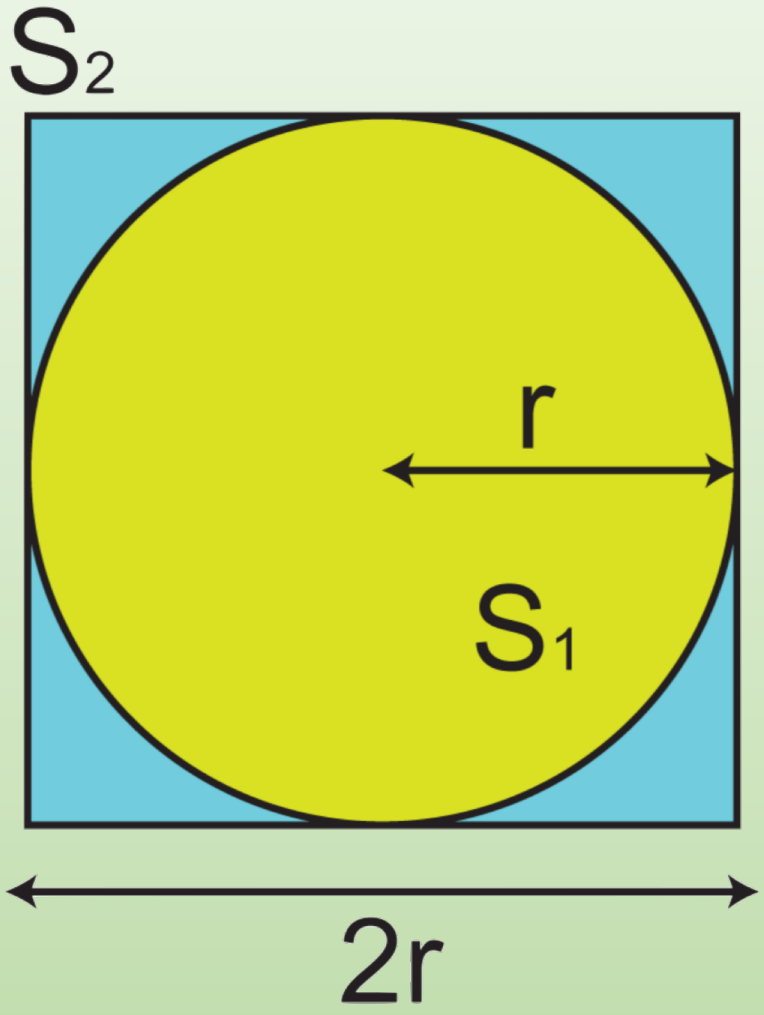
cuRAND device

- ✓Supports RNG on kernels
- ✓Do not need to generate random data before kernel
  - We don’t have to copy and store all data at once
- ✓Stores RNG states completely on GPU
  - Still need to allocate memory for it on host

Step

1. **Pre-allocating** a set of **cuRAND state objects** in device memory for each thread to manage its RNG's state
2. Optionally, **pre-allocating device memory** to store the **random values** generated by cuRAND (if they are intended to be copied back to the host or must be persisted for later kernels)
3. **Initializing** the **state** of all cuRAND state objects in **device memory** with a CUDA kernel call
4. 4. Executing a CUDA kernel that calls a cuRAND device function (for example., **`curand_uniform`**) and generates random values using the pre-allocated cuRAND state objects
5. Optionally, **transferring random values back** to the host if device memory was pre-allocated in step 2 for retrieving random values

Calcolo di π con il metodo Monte Carlo



Circle: $S_1 = \pi r^2$

Square: $S_2 = (2r)^2 = 4r^2$

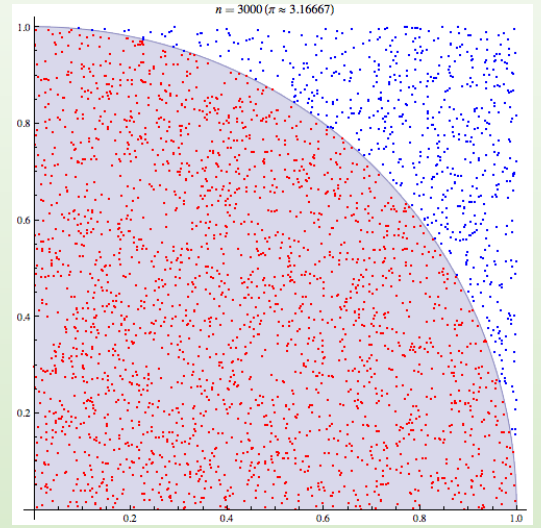
$$\frac{S_1}{S_2} = \frac{\pi r^2}{4r^2} \rightarrow \pi = \frac{4S_1}{S_2}$$

Stima su CPU

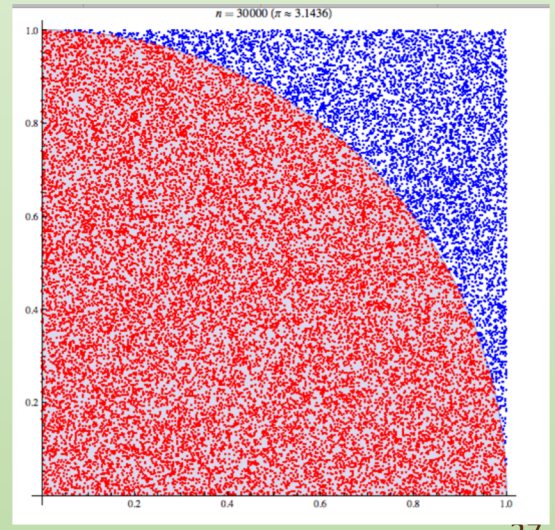
Tecnica numerica usando i numeri casuali:

- ✓ Generare un **elevato** numero di **punti random** all'interno del **quadrato** ([0,1] x [0,1])
- ✓ **L'area** del **cerchio** può essere ottenuta come il **rapporto** tra i numero di **punti all'interno** del cerchio e il numero **totale** di punti
- ✓ Esempio di codice in CPU:

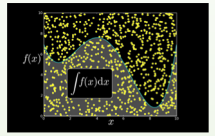
```
float pi_mc_CPU(long trials) {
        float x, y;
        long points_in_circle=0;
        for(long i = 0; i < trials; i++) {
                x = rand() / (float) RAND_MAX;
                y = rand() / (float) RAND_MAX;
                points_in_circle += (x*x + y*y <= 1.0f);
        }
        return 4.0f * points_in_circle/trials;
}
```



$\pi \approx 3.1415926535897932$



cuRAND device API



Genera numeri casuali nella memoria GPU

Passi per la generazione:

1. Includere **curand_kernel.h**
2. Allocare spazio di memoria sul device
3. Inizializzare lo stato con un “seed”
4. Generare la sequenza di numeri random

```
__device__ void curand_init ( unsigned long long seed,
                              unsigned long long sequence,
                              unsigned long long offset,
                              curandState_t *state)
```

```
// host
curandState *devStates;
cudaMalloc( (void **)&devStates, BLOCKS*THREADS*sizeof(curandState));

// kernel
int tid = threadIdx.x + blockDim.x*blockIdx.x;
curand_init(tid, 0, 0, &states[tid]);

x = curand_uniform(&states[tid]);
y = curand_uniform(&states[tid]);
```

code ex.

```
#define TRIALS_PER_THREAD 10000
#define BLOCKS  264
#define THREADS 264
#define PI 3.1415926535 // known value of pi

float host[BLOCKS * THREADS];
float *dev;

// GPU procedure
curandState *devStates;
cudaMalloc((void **) &dev, BLOCKS * THREADS * sizeof(float));
cudaMalloc((void **) &devStates, BLOCKS * THREADS * sizeof(curandState));
cudaEventRecord(start);
pi_mc_GPU<<<BLOCKS, THREADS>>>(dev, devStates);
```

kernel code

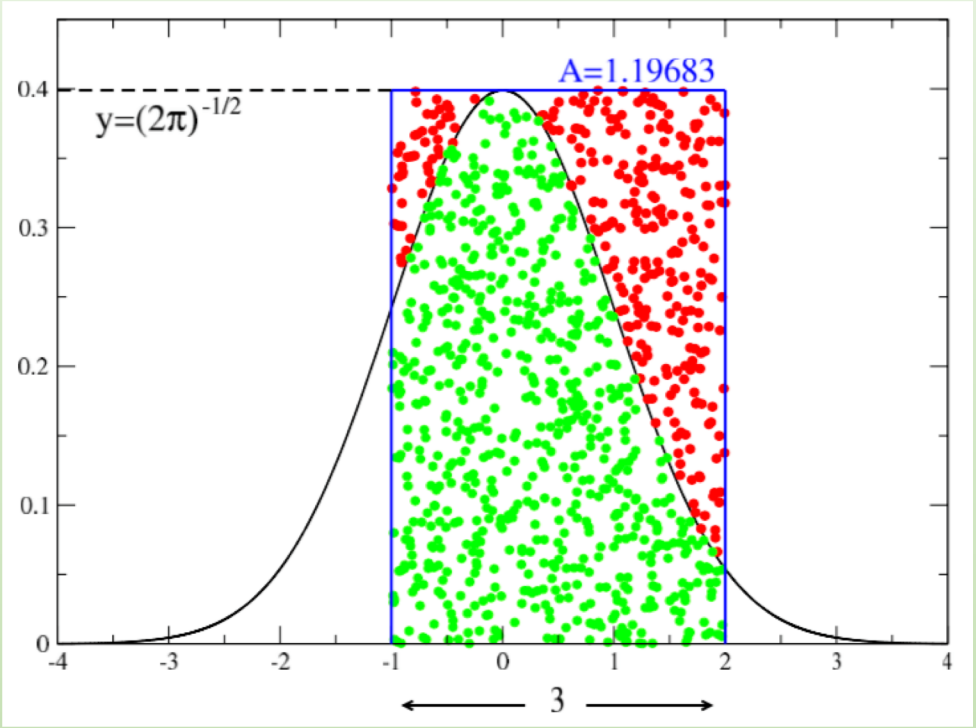
```
__global__ void pi_mc_GPU(float *estimate, curandState *states) {
        unsigned int tid = threadIdx.x + blockDim.x * blockIdx.x;
        int points_in_circle = 0;
        curand_init(tid, 0, 0, &states[tid]);
        for (int i = 0; i < TRIALS_PER_THREAD; i++) {
                float x = curand_uniform(&states[tid]);
                float y = curand_uniform(&states[tid]);
                points_in_circle += (x * x + y * y <= 1.0f);
        }
        estimate[tid] = 4.0f * points_in_circle / (float) TRIAL_x_TH;
}
```

Calcolo di un'area con Monte Carlo

Supponiamo di avere una funzione *g* definita su ***[a, b]*** a valori in ***[c, d], c, d ≥ 0***, e di volerne calcolare l'integrale $\int_a^g f(x)dx$

Metodo Monte Carlo:

1. poniamo **s = 0, A = (b – a) ∗ (d – c)**
2. si estrare un numero casuale uniforme **u ∈ [a, b]**
3. si estrare un numero casuale uniforme **v ∈ [c,d]**
4. Se **v < f(u)** allora si pone **s = s+1**
5. si itera la procedura ***n*** volte
6. Il valore dell'integrale è circa: $\int_a^g f(x)dx \approx \frac{A \cdot s}{n}$



Esercitazione

MONTE CARLO SU GPU:

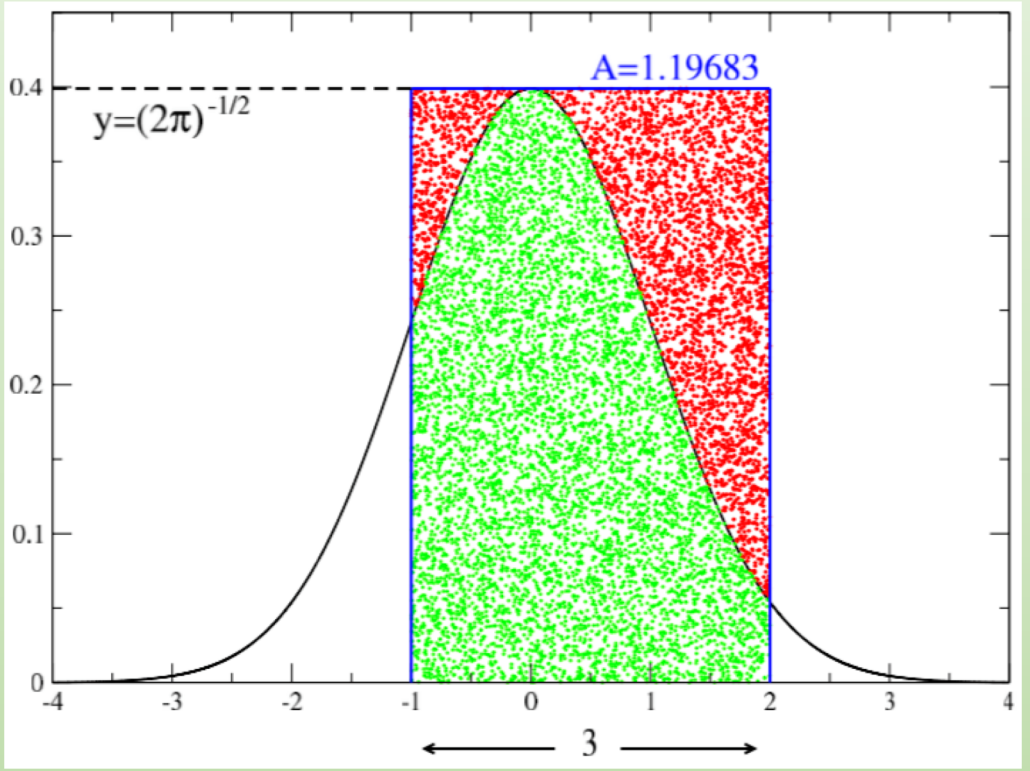
Disegnare un kernel per la stima dell'area della campana di gauss con il metodo Monte Carlo usando la libreria curand

$$P = \int_{-1}^{2} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$

$$A = 1.196826841$$

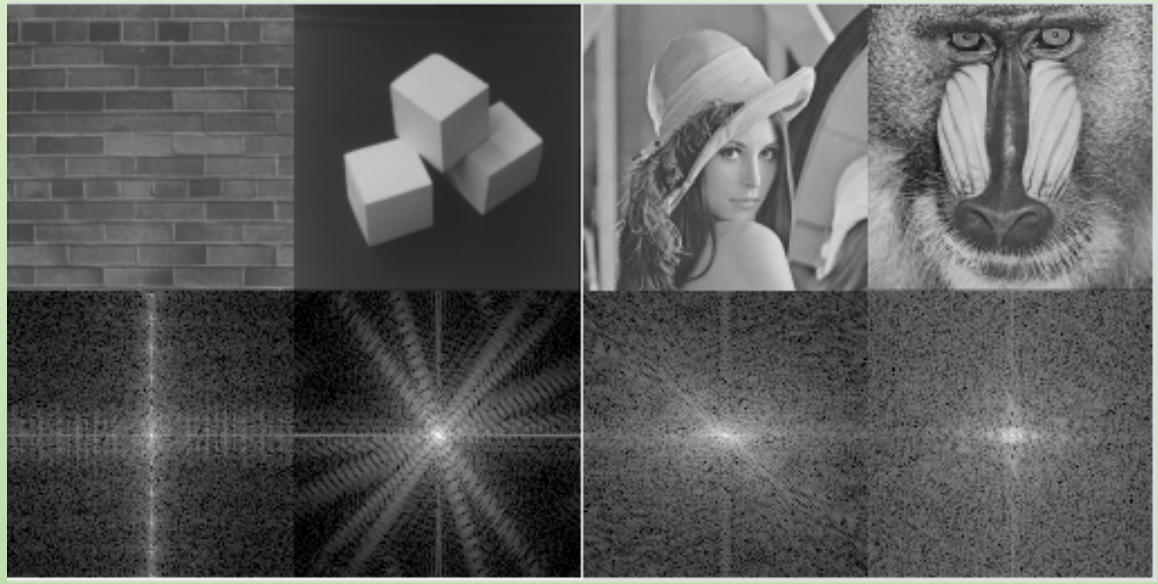
$$P = 0.8185946141$$

$$P/A = 0.683970801738252$$



Fourier Transform (cuFFT)

- ✓Another concept with lots of application, scalability, and parallelizability: Fourier Transformation
- ✓Commonly used in physics, signal processing, etc.
- ✓Oftentimes needs to be real-time (makes great use of GPU)



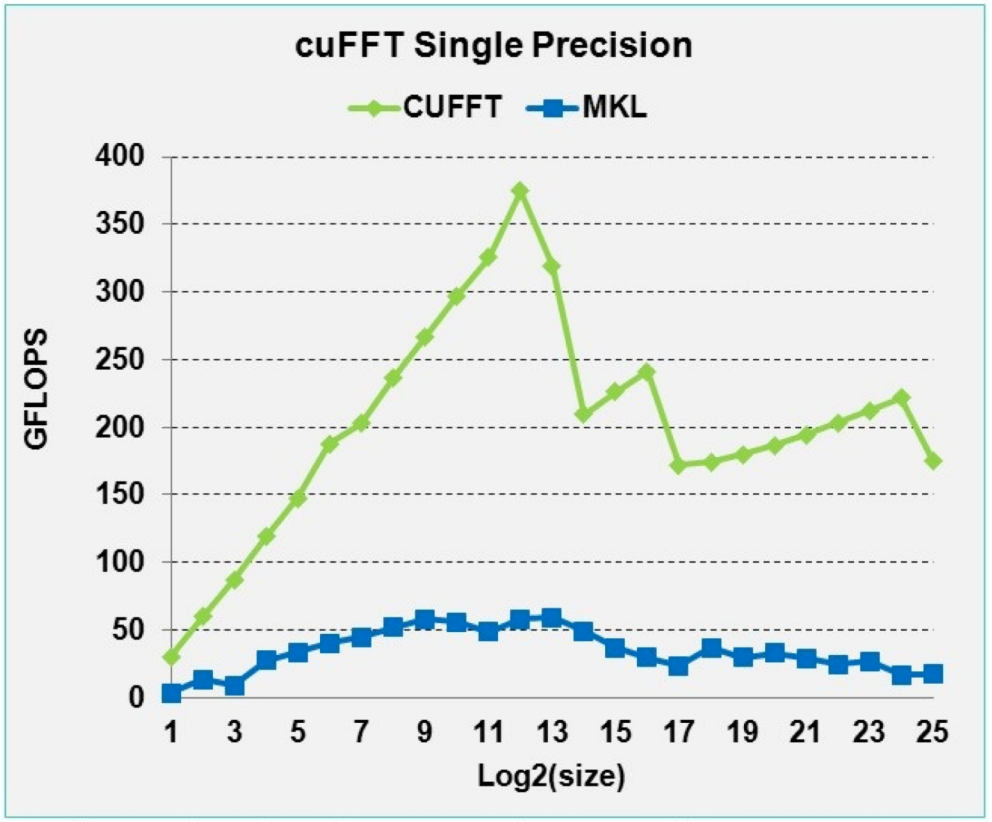
cuFFT lib

- ✓ The **cuFFT** library provides an optimized, CUDA-based implementation of the fast Fourier transform (**FFT**)
- ✓ An **FFT** is a **transformation** in signal processing that converts a signal from the **time domain** to the **frequency domain**
- ✓ An **inverse FFT** does the opposite
- ✓ FFT receives as input a **sequence** of **samples** taken **from** a **signal** at regular time intervals
- ✓ It uses those samples to **generate** a **set** of **component frequencies** that are superimposed to create the signal that **generated** the **input samples**

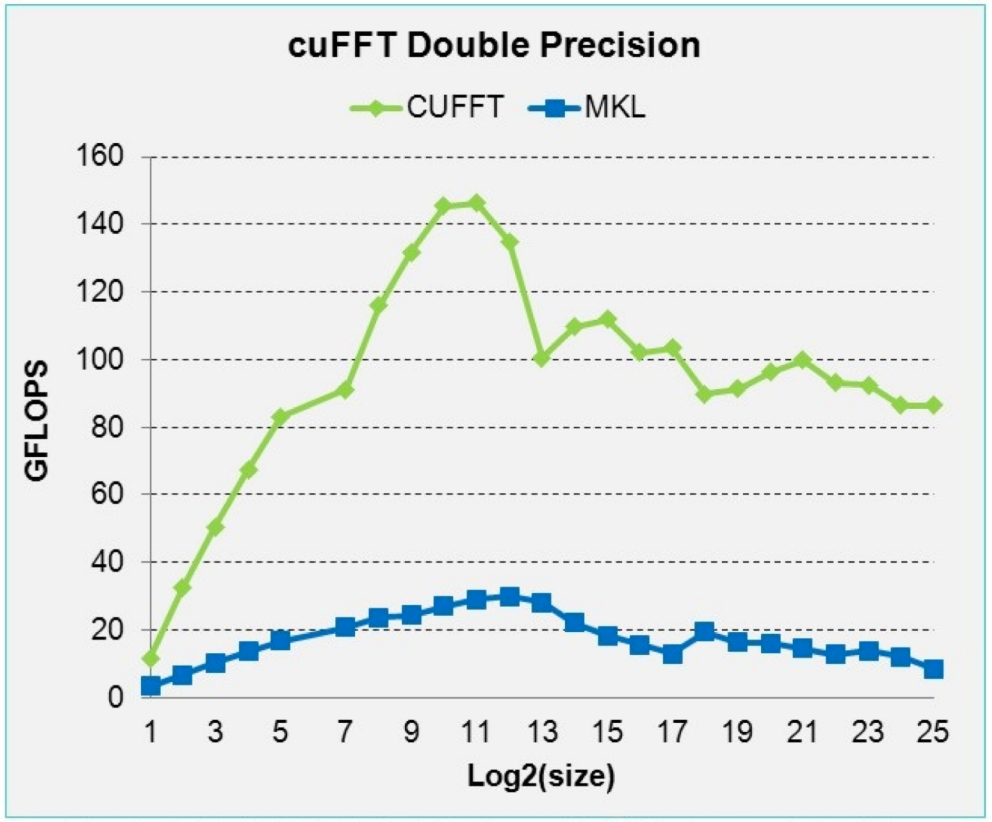
Lib. features

- ✓Supports 1D, 2D, or 3D Fourier Transforms
- ✓1D transforms can have up to 128 million elements
- ✓Based on Cooley-Tukey and Bluestein FFT algorithms
- ✓Similar API to FFTW, if familiar
- ✓Thread-safe, streamed, asynchronous execution
- ✓Supports both in-place and out-of-place transforms
- ✓Supports real, complex, float, double data

cuFFT vs MKL



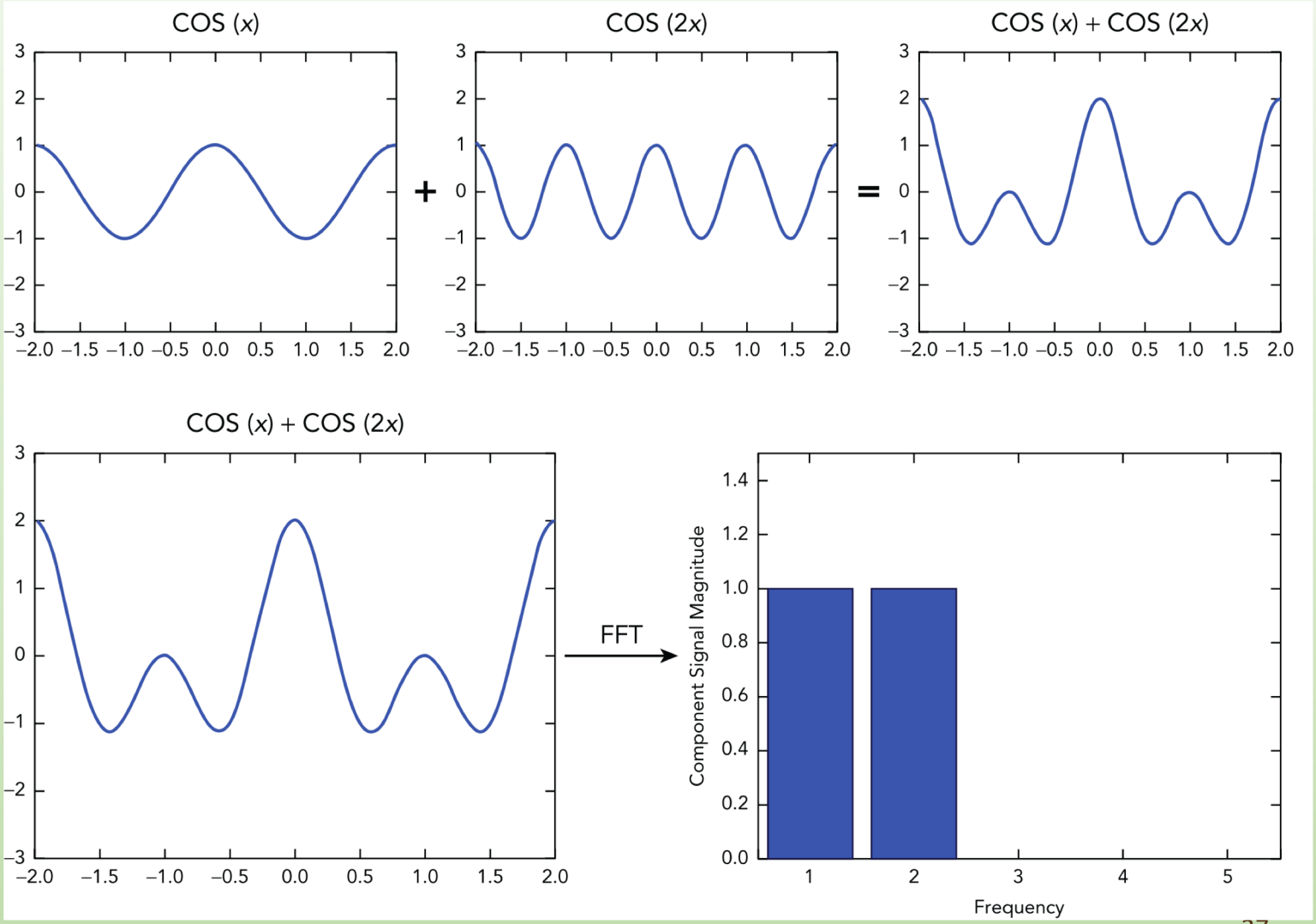
- Measured on sizes that are exactly powers-of-2
- cuFFT 4.1 on Tesla M2090, ECC on
- MKL 10.2.3, TYAN FT72-B7015 Xeon x5680 Six-Core @ 3.33 GHz



- MKL 10.2.3, TYAN FT72-B7015 Xeon x5680 Six-Core @ 3.33 GHz
- Performance may vary based on OS version and motherboard configuration

Example

Sum of two signals to form the signal cos(x)+cos(2x) and its decomposition by FFT into frequencies of 1.0 and 2.0



Trasformata di Fourier (DTFT)

Dal processo di campionamento, con freq espressa in [cicli/campione]:

$$X(\nu) = \sum_{n=-\infty}^{+\infty} x(n) e^{-i2\pi\nu n}, \qquad 0 \leq \nu < 1$$

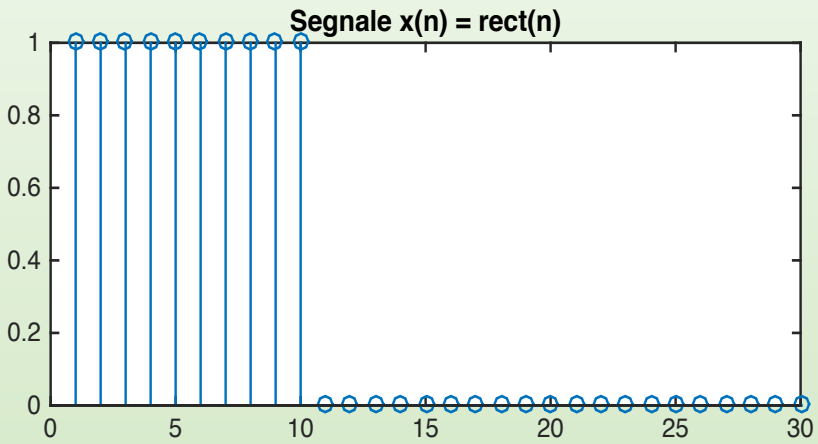
$$x(n) = \int_0^1 X(\nu) e^{i2\pi\nu n} d\nu$$

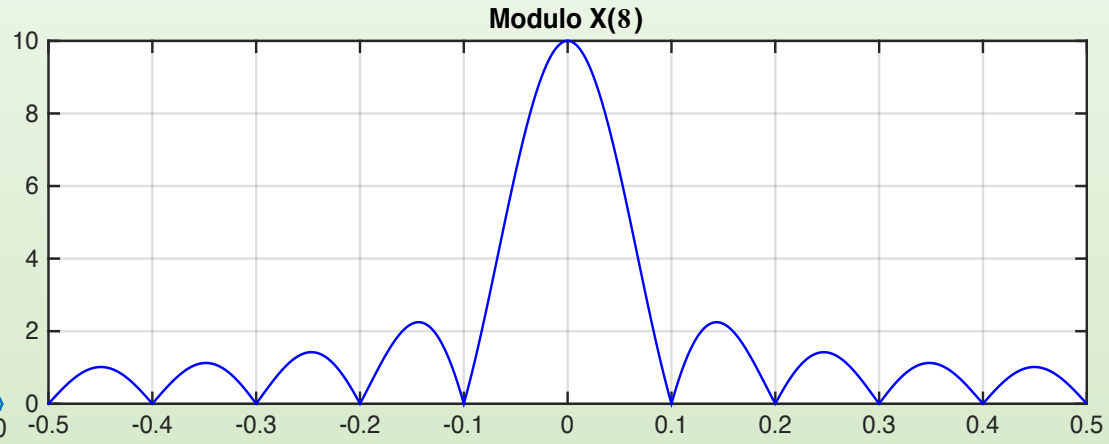
Dalla risposta in frequenza, con freq espressa in [rad/campione]:

$$X(e^{i\omega}) = \sum_{n=-\infty}^{+\infty} x(n) e^{-i\omega n}, \qquad 0 \leq \omega < 2\pi$$

$$x(n) = \frac{1}{2\pi} \int_0^{2\pi} X(e^{i\omega}) e^{i\omega n} d\omega$$

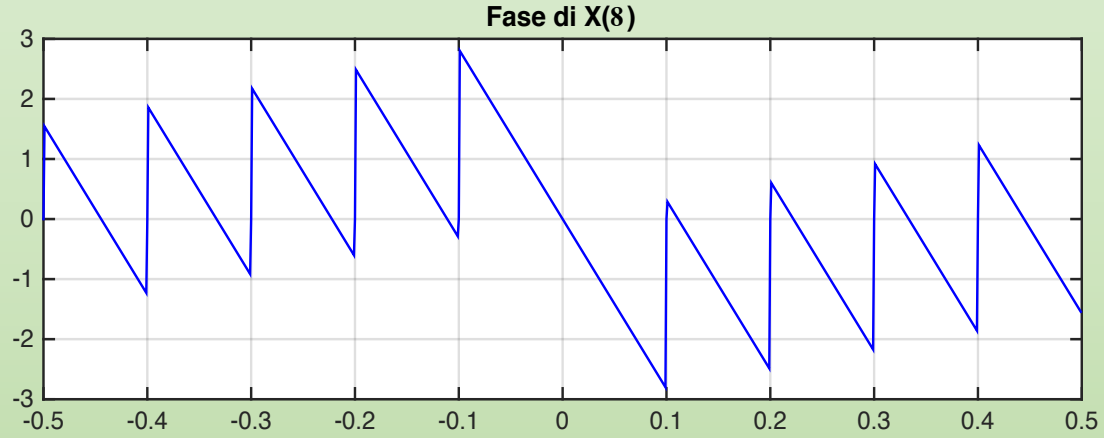
Esempio: DTFT di *rect(n)*





$$x(n) = \mathrm{rect}(n)$$

$$X(\nu) = \frac{\sin(\pi N \nu)}{\sin(\pi \nu)}$$



Trasformata di Fourier discreta - DFT

Se *x(n)* è un segnale periodico di periodo *per = NT*, trasformata e antitrasformata di Fourier sono date da:

$$X(kF) = T \sum_{n=0}^{N-1} x(nT) e^{-j2\pi \frac{nk}{N}}, \quad f \in \{0, F, \ldots, F(N-1)\}, \quad F = \frac{1}{NT}$$

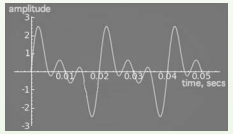
$$x(nT) = F \sum_{k=0}^{N-1} X(kF) e^{j2\pi \frac{nk}{N}}, \quad x = [x(0), x(T), x(2T) \ldots, x((N-1)T)\}$$

Ponendo *T = 1*, si ha rispettivamente:

$$X(k+1) = \sum_{n=0}^{N-1} x(n+1) e^{-j2\pi \frac{nk}{N}}, \qquad x(n+1) = \frac{1}{N} \sum_{k=0}^{N-1} X(k+1) e^{j2\pi \frac{nk}{N}}$$

Se *T ≠ 1*, basta porre : $X(k+1) = X(kF)/T$

Libreria cuFFT



- ✓ **Half-precision** (16-bit floating point), **single-precision** (32-bit) and **double-precision** (64-bit) input and output
- ✓ Types supported are:
  - ✓ **C2C** - Complex input to complex output
  - ✓ **R2C** - Real input to complex output
  - ✓ **C2R** - Symmetric complex input to real output
- ✓ Execution of multiple **1D**, **2D** and **3D** transforms simultaneously. These **batched** transforms have **higher performance** than single transforms. In-place and out-of-place transforms
- ✓ Arbitrary intra- and inter-dimension element **strides** (strided layout)
- ✓ **FFTW** compatible data layout
- ✓ Execution of transforms across multiple GPUs
- ✓ **Streamed** execution, enabling **asynchronous** computation and data movement

Read more at: http://docs.nvidia.com/cuda/cufft/index.html#ixzz4hV8LC4Sc

cuFFt types

- ✓ `cufftHandle`

  Handle type to store CUFFT plans

- ✓ `cufftResult`

  Return values, like **CUFFT_SUCCESS**, **CUFFT_INVALID_PLAN**, **CUFFT_ALLOC_FAILED**, **CUFFT_INVALID_TYPE**, etc

- ✓ Types:

  `cufftReal, cufftDoubleReal, cufftComplex, cufftDoubleComplex`

- ✓ Transform types:

- ✓ **R2C**: real to complex

  C2R: Complex to real

  C2C: complex to complex

  D2Z: double to double complex

  Z2D: double complex to double

  Z2Z: double complex to double complex

- ✓ Plans:

  cufftPlan1d(), cufftPlan2d(), cufftPlan3d(), cufftPlanMany()

cuFFT plans

- ✓ **Configuration** of the cuFFT library is done with FFT **plans** (the terminology that cuFFT uses to refer to its handles)
- ✓ A **plan** defines a **single transform operation** to be performed
- ✓ cuFFT uses plans to **derive** the **internal memory allocations**, **transfers**, and kernel **launches** that must occur to perform the requested transformation

```
cufftResult cufftPlan1d(cufftHandle *plan, int nx, cufftType type, int batch);

cufftResult cufftPlan2d(cufftHandle *plan, int nx, int ny, cufftType type);

cufftResult cufftPlan3d(cufftHandle *plan, int nx, int ny, int nz, cufftType type);
```

Schema

1. Create and configure a **cuFFT plan**
2. Allocate device memory to store the input samples and output frequencies from cuFFT using **`cudaMalloc`**
3. Populate that device memory with the input signal samples using **`cudaMemcpy`**
4. Execute the plan using a **`cufftExec`*** function
5. Retrieve the result from device memory using **`cudaMemcpy`**
6. Release CUDA and cuFFT resources using cudaFree and **`cufftDestroy`**

✓ Compliazione e linking di lib dinamica:

```
#include "cufft.h"
```

```
$ nvcc -arch=sm_20 -lcufft kernel.cu -o kernel
```

Esempio d'uso

```
// Setup the cuFFT plan
cufftPlan1d(&plan, N, CUFFT_C2C, 1);
// Allocate device memory
cudaMalloc((void **)&dComplexSamples, sizeof(cufftComplex) * N);
// Transfer inputs into device memory
cudaMemcpy(dComplexSamples, complexSamples, sizeof(cufftComplex) * N,
                                                cudaMemcpyHostToDevice);
// Execute a complex-to-complex 1D FFT
cufftExecC2C(plan, dComplexSamples, dComplexSamples, CUFFT_FORWARD);
// Retrieve the results into host memory
cudaMemcpy(complexFreq, dComplexSamples, sizeof(cufftComplex) * N,
                                                cudaMemcpyDeviceToHost);
```

Conversione a valori complessi

```
/*
 * Convert a real-valued vector r of length Nto a complex-valued vector.
 */
void real_to_complex(float *r, cufftComplex **complx, int N) {

   (*complx) = (cufftComplex *) malloc(sizeof(cufftComplex) * N);

   for (int  i = 0; i < N; i++) {
      (*complx)[i].x = r[i];
      (*complx)[i].y = 0;
   }
}
```

Esempio

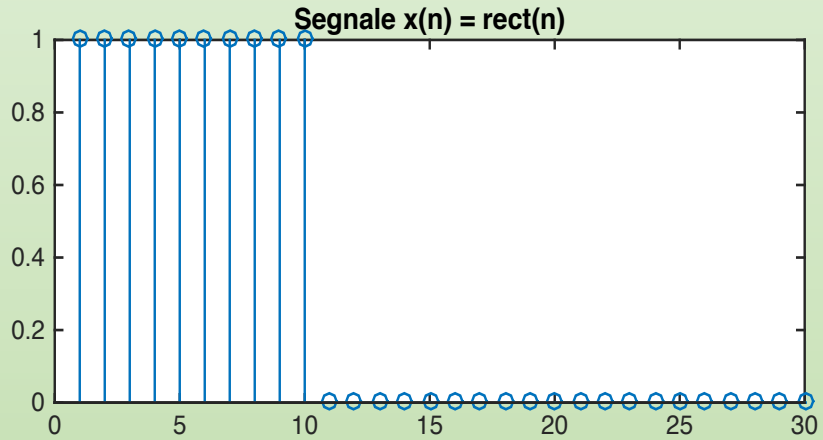
```
int main(int argc, char **argv) {


int i;
int N = 2048;
float *samples;
cufftHandle plan = 0;
cufftComplex *dComplexSamples, *complexSamples, *complexFreq;

// Input Generation
generate_fake_samples(N, &samples);
real_to_complex(samples, &complexSamples, N);
complexFreq = (cufftComplex *) malloc(sizeof(cufftComplex) * N);
printf("Initial Samples:\n");

// Setup the cuFFT plan
cufftPlan1d(&plan, N, CUFFT_C2C, 1);

// Allocate device memory
cudaMalloc((void ** )&dComplexSamples, sizeof(cufftComplex) * N);

// Transfer inputs into device memory
cudaMemcpy(dComplexSamples, complexSamples, sizeof(cufftComplex) * N, cudaMemcpyHostToDevice);

// Execute a complex-to-complex 1D FFT
cufftExecC2C(plan, dComplexSamples, dComplexSamples, CUFFT_FORWARD);

// Retrieve the results into host memory
cudaMemcpy(complexFreq, dComplexSamples, sizeof(cufftComplex) * N, cudaMemcpyDeviceToHost);

cudaFree(dComplexSamples);
cufftDestroy(plan);
```

Esercitazione

FFT di un impulso rettangolare:

Effettuare il calcolo della trasformata e della trasformata inversa verificando che **IFFT(FFT(x)) = x**:

- Allocare il vettore con impulso di durata par a ¼, ½, ¾ del segnale (di lunghezza **N**)
- Verificare l'efficienza usando tipi reali e complessi



← x = IFFT(FFT(x)) →