1. Models and entailment in propositional logic

1.1 Validity and soundness

a) VOCABULARY

V: "Peter's argument is valid"
S: "Peter's argument is sound"
E: "The premises of Peter's argument entail the conclusion of Peter's argument"
PT: "All the premises of Peter's argument are true"

b) TRANSLATION

(P1) $(V \land PT) \Rightarrow S$
(P2) $E \Rightarrow V$
(P3) E
(C) S

c) Using Modus Ponens,

$$\frac{\begin{array}{l} E \Rightarrow V \\ E \end{array}}{V}$$

Therefore to make the conclusion valid, we add
(P4) PT

1.2 Modelling

a) $(p \Rightarrow q) \Rightarrow ((p \Rightarrow r) \Rightarrow (q \Rightarrow r))$

p	q	r	$(p \Rightarrow q)$	$\Rightarrow$	$(p \Rightarrow r)$	$\Rightarrow$	$(q \Rightarrow r)$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	0	1	1
1	0	1	0	1	1	1	1
1	1	0	1	1	0	1	0
1	1	1	1	1	1	1	1

The statement is satisfiable

b) $(p \lor (\neg q \Rightarrow r)) \Rightarrow (q \lor (\neg p \Rightarrow r))$

$(p \lor (\neg q \Rightarrow r)) \Rightarrow (q \lor (\neg p \Rightarrow r))$

p	q	r							
0	0	0	0	1	0	1	0	1	0
0	0	1	1	1	1	1	1	1	1
0	1	0	1	0	1	1	1	1	0
0	1	1	1	0	1	1	1	1	1
1	0	0	1	1	0	1	1	0	1
1	0	1	1	1	1	1	1	0	1
1	1	0	1	0	1	1	1	0	1
1	1	1	1	0	1	1	1	0	1

The statement is a tautology

c) $(\neg(p \land (q \Rightarrow \neg r))) \Rightarrow ((p \Rightarrow q) \land (p \Rightarrow r))$

$(\neg(p \land (q \Rightarrow \neg r))) \Rightarrow ((p \Rightarrow q) \land (p \Rightarrow r))$

p	q	r								
0	0	0	1	0	1	1	1	1	1	1
0	0	1	1	0	1	0	1	1	1	1
0	1	0	1	0	1	1	1	1	1	1
0	1	1	1	0	0	0	1	1	1	1
1	0	0	0	1	1	1	1	0	0	0
1	0	1	0	1	1	0	1	0	0	1
1	1	0	0	1	1	1	1	1	0	0
1	1	1	1	0	0	0	1	1	1	1

The statement is a tautology

d) $(\neg(\neg p \Rightarrow (q \land r))) \Rightarrow (\neg(p \lor q) \land r)$

$(\neg(\neg p \Rightarrow (q \land r))) \Rightarrow (\neg(p \lor q) \land r)$

p	q	r								
0	0	0	1	1	0	0	0	1	0	0
0	0	1	1	1	0	0	1	1	0	1
0	1	0	1	1	0	0	0	0	1	0
0	1	1	0	1	1	1	1	0	1	0
1	0	0	0	0	1	0	1	0	1	0
1	0	1	0	0	1	0	1	0	1	0
1	1	0	0	0	1	0	1	0	1	0
1	1	1	0	0	1	1	1	0	1	0

The statement is satisfiable

1.3 Modelling 2

$(\neg p \vee q \vee \neg r) \wedge (p \vee q \vee \neg r)$

2. Resolution in propositional logic

2.1 Conjunctive Normal Form

a) $p \Leftrightarrow q$

$(p \Rightarrow q) \wedge (q \Rightarrow p)$ biconditional elimination

$(\neg p \vee q) \wedge (\neg q \vee p)$ implication elimination

b) $\neg((p \Rightarrow q) \wedge r)$

$\neg((\neg p \vee q) \wedge r)$ implication elimination

$\neg(\neg p \vee q) \vee \neg r$ De Morgan

$(p \wedge \neg q) \vee \neg r$ De Morgan

$(\neg r \vee p) \wedge (\neg r \vee \neg q)$ distributivity of $\vee$ over $\wedge$

c) $((p \vee q) \vee (r \wedge (\neg(q \Rightarrow r))))$

$((p \vee q) \vee (r \wedge (\neg(\neg q \vee r))))$ implication elimination

$((p \vee q) \vee (r \wedge q \wedge \neg r))$ De Morgan

$(p \vee q)$ $r \wedge \neg r \equiv F$

d) Yes. By definition, a formula is in CNF if it is a conjunction of one or more disjunctive clauses. The solution to c) has one clause, which is a disjunction of literals, therefore is a CNF.

2.2 Inference in propositional logic

CONVERSION TO CNF

a) $(p \Rightarrow q) \Rightarrow q$

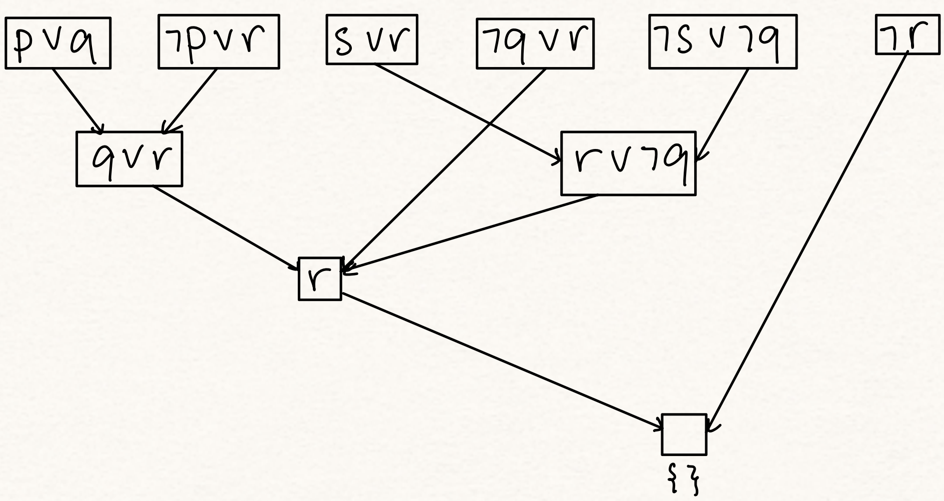
$(\neg p \vee q) \Rightarrow q$ implication elimination

$\neg(\neg p \lor q) \lor q$ implication elimination
$(p \land \neg q) \lor q$ De Morgan
$(p \lor q) \land (\neg q \lor q)$ distributivity of $\lor$ over $\land$
$(p \lor q)$ $\neg q \lor q \equiv T$

b) $p \Rightarrow r$
$(\neg p \lor r)$ implication elimination

c) $(r \Rightarrow s) \Rightarrow (\neg(s \Rightarrow q))$
$\neg(r \Rightarrow s) \lor (\neg(s \Rightarrow q))$ implication elimination
$\neg(\neg r \lor s) \lor (\neg(\neg s \lor q))$ implication elimination
$(r \land \neg s) \lor (s \land \neg q)$ De Morgan
$((s \land \neg q) \lor r) \land ((s \land \neg q) \lor \neg s)$ distributivity of $\lor$ over $\land$
$((s \lor r) \land (\neg q \lor r)) \land ((s \lor \neg s) \land (\neg s \lor \neg q))$ distributivity of $\lor$ over $\land$
$(s \lor r) \land (\neg q \lor r) \land (\neg s \lor \neg q)$ $s \lor \neg s \equiv T$

KB: $p \lor q, \neg p \lor r, s \lor r, \neg q \lor r, \neg s \lor \neg q$
does $KB \models r$?
disproving $KB \land \neg r$ proves $KB \models r$



Therefore $KB \models r$

3. Representation in First-Order Logic (FOL)

a) Samson is a pitcher
Pitcher(Samson)

b) Jones is not a friend of Robinson
$\neg$friend(Jones, Robinson)

4. Resolution in FOL

a) Argument A

CONVERSION TO CNF

- $\forall x : [\text{flies}(x, \text{Robinson}) \Rightarrow \text{Pitcher}(x)]$

  $\forall x : [\neg \text{flies}(x, \text{Robinson}) \vee \text{Pitcher}(x)]$ eliminate implication

  $\neg \text{flies}(x, \text{Robinson}) \vee \text{Pitcher}(x)$ drop universal quantif.

- $\text{scores}(\text{Crabb}) \Rightarrow (\text{flies}(\text{Samson}, \text{Robinson}) \wedge \text{Centerfielder}(\text{Robinson}))$

  $\neg\text{scores}(\text{Crabb}) \vee (\text{flies}(\text{Samson}, \text{Robinson}) \wedge \text{Centerfielder}(\text{Robinson}))$ eliminate implication

  $(\neg\text{scores}(\text{Crabb}) \vee \text{flies}(\text{Samson}, \text{Robinson})) \wedge (\neg\text{scores}(\text{Crabb}) \vee \text{Centerfielder}(\text{Rob.}))$ distribute ∨ over ∧

- $\text{scores}(\text{Crabb})$

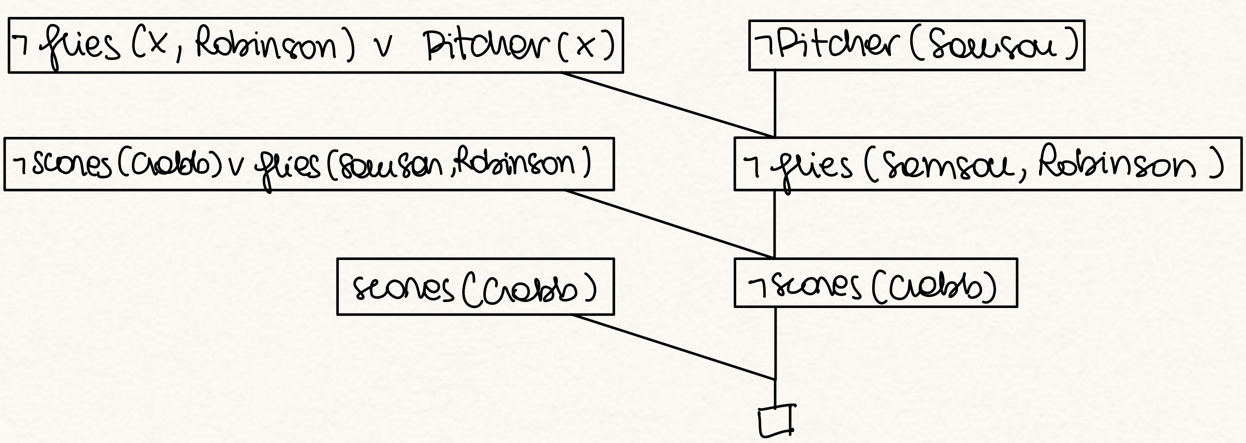
KNOWLEDGE BASE

1. $\neg\text{flies}(x, \text{Robinson}) \vee \text{Pitcher}(x)$
2. $\neg\text{scores}(\text{Crabb}) \vee \text{flies}(\text{Samson}, \text{Robinson})$
3. $\neg\text{Scores}(\text{Crabb}) \vee \text{Centerfielder}(\text{Robinson})$

Facts: $\text{scores}(\text{Crabb})$

Add: $\neg\text{Pitcher}(\text{Samson})$

RESOLUTION



b) Argument B

CONVERSION TO CNF

- $\forall x : [(Centerfielder(x) \land \neg scores(x)) \Rightarrow \neg \exists y : [friend(y,x)]]$

  $\forall x : [\neg(Centerfielder(x) \land \neg scores(x)) \lor \neg \exists y : [friend(y,x)]]$ eliminate implication

  $\forall x : [\neg Centerfielder(x) \lor scores(x) \lor \forall y : \neg friend(y,x)]$ De Morgan

  $\neg Centerfielder(x) \lor scores(x) \lor \neg friend(y,x)$ drop universal quantifiers

- $Centerfielder(Robinson) \land Centerfielder(Jones)$

- $\forall x : [(Centerfielder(x) \land flies(x, Jones)) \Rightarrow \neg scores(x)]$

  $\forall x : [\neg(Centerfielder(x) \land flies(x, Jones)) \lor \neg scores(x)]$ eliminate implication

  $\forall x : [\neg Centerfielder(x) \lor \neg flies(x, Jones) \lor \neg scores(x)]$ De Morgan

  $\neg Centerfielder(x) \lor \neg flies(x, Jones) \lor \neg scores(x)$ drop universal quantifiers

- $flies(Robinson, Jones)$

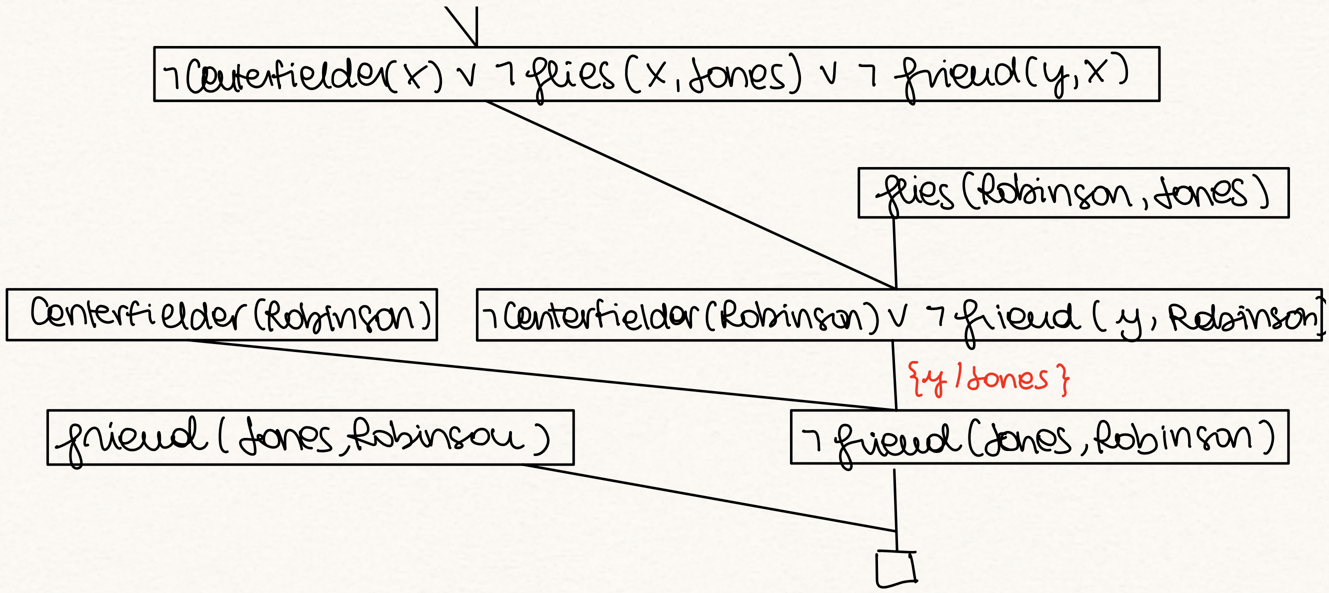
KNOWLEDGE BASE

① $\neg Centerfielder(x) \lor scores(x) \lor \neg friend(y,x)$
② $\neg Centerfielder(x) \lor \neg flies(x, Jones) \lor \neg scores(x)$
Facts: $Centerfielder(Robinson)$
$Centerfielder(Jones)$
$flies(Robinson, Jones)$
Add: $friend(Jones, Robinson)$

RESOLUTION

$\boxed{\neg Centerfielder(x) \lor \neg flies(x, Jones) \lor \neg scores(x)}$

$\boxed{\neg Centerfielder(x) \lor scores(x) \lor \neg friend(y,x)}$



ASSIGNMENT 5
TDT4136 - Introduction to Artificial Intelligence

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ELENA FILIPPINI
FRANCESCA GRIMALDI