1. Models and entailement in propositional logic

1.1 Validity and soundness

a) VOCABULARY

V: "Peter's argument is valid"
S: "Peter's argument is sound"
E: "The premises of Peter's argument entail the conclusion of Peter's argument"
PT: "All the premises of Peter's argument are true"

b) TRANSLATION

(P1) $V \wedge PT \Rightarrow S$
(P2) $E \Rightarrow V$
(P3) E
(C) S
(P1 ∧ P2 ∧ P3 ⇒ C) $((V \wedge PT \Rightarrow S) \wedge (E \Rightarrow V) \wedge E) \Rightarrow S$

c) Using Modus Ponens,

$$\frac{\begin{array}{c} E \Rightarrow V \\ E \end{array}}{V}$$

Therefore to make the conclusion valid, we add
(P4) PT

1.2 Modelling

a) $(p \Rightarrow q) \Rightarrow ((p \Rightarrow r) \Rightarrow (q \Rightarrow r))$

p	q	r	$(p \Rightarrow q)$	$\Rightarrow$	$(p \Rightarrow r)$	$\Rightarrow$	$(q \Rightarrow r)$
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	1	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	0	1	1
1	0	1	0	1	1	1	1
1	1	0	1	1	0	1	0
1	1	1	1	1	1	1	1

The statement is satisfiable

b) $(p \vee (\neg q \Rightarrow r)) \Rightarrow (q \vee (\neg p \Rightarrow r))$

p	q	r	$(p \vee (\neg q \Rightarrow r))$			$\Rightarrow$	$(q \vee (\neg p \Rightarrow r))$		
0	0	0	0	1	0	1	0	1	0
0	0	1	1	1	1	1	1	1	1
0	1	0	1	0	1	1	1	1	0
0	1	1	1	0	1	1	1	1	1
1	0	0	1	1	0	1	1	0	1
1	0	1	1	1	1	1	1	0	1
1	1	0	1	0	1	1	1	0	1
1	1	1	1	0	1	1	1	0	1

The statement is a tautology

c) $(\neg(p \wedge (q \Rightarrow \neg r))) \Rightarrow ((p \Rightarrow q) \wedge (p \Rightarrow r))$

p	q	r	$(\neg(p \wedge (q \Rightarrow \neg r)))$				$\Rightarrow$	$((p \Rightarrow q) \wedge (p \Rightarrow r))$		
0	0	0	1	0	1	1	1	1	1	1
0	0	1	1	0	1	0	1	1	1	1
0	1	0	1	0	1	1	1	1	1	1
0	1	1	1	0	0	0	1	1	1	1
1	0	0	0	1	1	1	1	0	0	0
1	0	1	0	1	1	0	1	0	0	1
1	1	0	0	1	1	1	1	1	0	0
1	1	1	1	0	0	0	1	1	1	1

The statement is a tautology

d) $(\neg(\neg p \Rightarrow (q \wedge r))) \Rightarrow (\neg(p \vee q) \wedge r)$

p	q	r	$(\neg(\neg p \Rightarrow (q \wedge r)))$				$\Rightarrow$	$(\neg(p \vee q) \wedge r)$		
0	0	0	1	1	0	0	0	1	0	0
0	0	1	1	1	0	0	1	1	0	1
0	1	0	1	1	0	0	0	0	1	0
0	1	1	0	1	1	1	1	0	1	0
1	0	0	0	0	1	0	1	0	1	0
1	0	1	0	0	1	0	1	0	1	0
1	1	0	0	0	1	0	1	0	1	0
1	1	1	0	0	1	1	1	0	1	0

The statement is satisfiable

1.3 Modelling 2

$(\neg p \vee q \vee \neg r) \wedge (p \vee q \vee \neg r) \rightarrow (r \Rightarrow q) \wedge (p \vee \neg p)$

2. Resolution in propositional logic

2.1 Conjunctive Normal Form

a) $p \Leftrightarrow q$

$(p \Rightarrow q) \wedge (q \Rightarrow p)$ biconditional elimination

$(\neg p \vee q) \wedge (\neg q \vee p)$ implication elimination

b) $\neg((p \Rightarrow q) \wedge r)$

$\neg((\neg p \vee q) \wedge r)$ implication elimination

$\neg(\neg p \vee q) \vee \neg r$ De Morgan

$(p \wedge \neg q) \vee \neg r$ De Morgan

$(\neg r \vee p) \wedge (\neg r \vee \neg q)$ distributivity of $\vee$ over $\wedge$

c) $((p \vee q) \vee (r \wedge (\neg(q \Rightarrow r))))$

$((p \vee q) \vee (r \wedge (\neg(\neg q \vee r))))$ implication elimination

$((p \vee q) \vee (r \wedge q \wedge \neg r))$ De Morgan

$(p \vee q)$ $r \wedge \neg r \equiv F$

d) Yes. By definition, a formula is in CNF if it is a conjunction of one or more disjunctive clauses. The solution to c) has one clause, which is a disjunction of literals, therefore is a CNF.

2.2 Inference in propositional logic

CONVERSION TO CNF

a) $(p \Rightarrow q) \Rightarrow q$

$(\neg p \vee q) \Rightarrow q$ implication elimination

$\neg(\neg p \vee q) \vee q$ — implication elimination

$(p \wedge \neg q) \vee q$ — De Morgan

$(p \vee q) \wedge (\neg q \vee q)$ — distributivity of $\vee$ over $\wedge$

$(p \vee q)$ — $\neg q \vee q \equiv T$

b) $p \Rightarrow r$

$(\neg p \vee r)$ — implication elimination

c) $(r \Rightarrow s) \Rightarrow (\neg(s \Rightarrow q))$

$\neg(r \Rightarrow s) \vee (\neg(s \Rightarrow q))$ — implication elimination

$\neg(\neg r \vee s) \vee (\neg(\neg s \vee q))$ — implication elimination

$(r \wedge \neg s) \vee (s \wedge \neg q)$ — De Morgan

$((s \wedge \neg q) \vee r) \wedge ((s \wedge \neg q) \vee \neg s)$ — distributivity of $\vee$ over $\wedge$

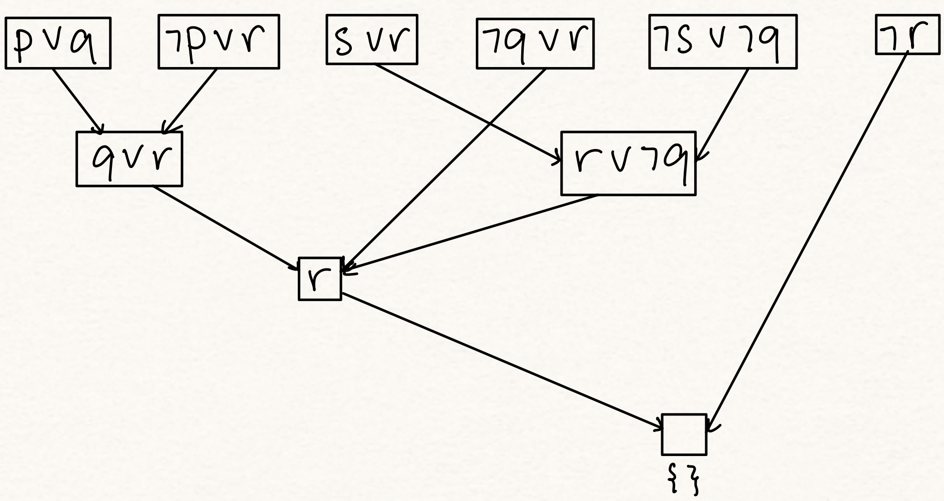
$((s \vee r) \wedge (\neg q \vee r)) \wedge ((s \vee \neg s) \wedge (\neg s \vee \neg q))$ — distributivity of $\vee$ over $\wedge$

$(s \vee r) \wedge (\neg q \vee r) \wedge (\neg s \vee \neg q)$ — $s \vee \neg s \equiv T$

KB: $p \vee q$, $\neg p \vee r$, $s \vee r$, $\neg q \vee r$, $\neg s \vee \neg q$

does KB $\models r$?

disproving KB $\wedge \neg r$ proves KB $\models r$



Therefore KB $\models r$

3. Representation in First-Order Logic (FOL)

a) Samson is a pitcher

Pitcher(Samson)

b) Jones is not a friend of Robinson

$\neg$friend(Jones, Robinson)

4. Resolution in FOL

a) Argument A

CONVERSION TO CNF

- $\forall x : [\text{flies}(x, \text{Robinson}) \Rightarrow \text{Pitcher}(x)]$

  $\forall x : [\neg \text{flies}(x, \text{Robinson}) \vee \text{Pitcher}(x)]$ eliminate implication

  $\neg \text{flies}(x, \text{Robinson}) \vee \text{Pitcher}(x)$ drop universal quantif.

- $\text{scones}(\text{Crabb}) \Rightarrow (\text{flies}(\text{Samson}, \text{Robinson}) \wedge \text{Centerfielder}(\text{Robinson}))$

  $\neg \text{scones}(\text{Crabb}) \vee (\text{flies}(\text{Samson}, \text{Robinson}) \wedge \text{Centerfielder}(\text{Robinson}))$ eliminate implication

  $(\neg \text{scones}(\text{Crabb}) \vee \text{flies}(\text{Samson}, \text{Robinson})) \wedge (\neg \text{scones}(\text{Crabb}) \vee \text{Centerfielder}(\text{Rob.}))$ distribute $\vee$ over $\wedge$

- $\text{scones}(\text{Crabb})$

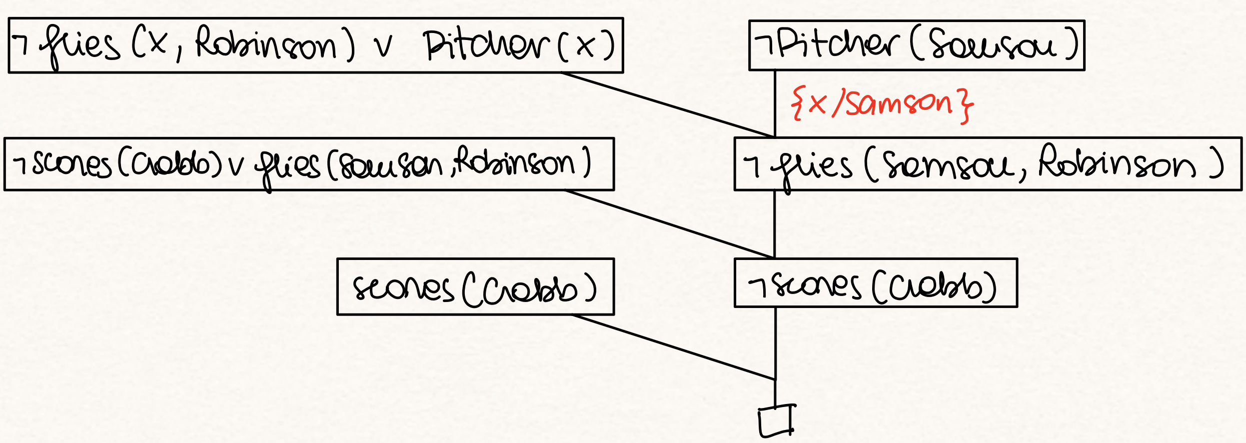
KNOWLEDGE BASE

1. $\neg \text{flies}(x, \text{Robinson}) \vee \text{Pitcher}(x)$
2. $\neg \text{scones}(\text{Crabb}) \vee \text{flies}(\text{Samson}, \text{Robinson})$
3. $\neg \text{Scones}(\text{Crabb}) \vee \text{Centerfielder}(\text{Robinson})$

Facts: $\text{scones}(\text{Crabb})$

Add: $\neg \text{Pitcher}(\text{Samson})$

RESOLUTION



b) Argument B

CONVERSION TO CNF

- $\forall x: [(Centerfielder(x) \land \neg scores(x)) \Rightarrow \neg \exists y: [friend(y,x)]]$

  $\forall x: [\neg(Centerfielder(x) \land \neg scores(x)) \lor \neg \exists y: [friend(y,x)]]$ eliminate implication

  $\forall x: [\neg Centerfielder(x) \lor scores(x) \lor \forall y: \neg friend(y,x)]$ De Morgan

  $\neg Centerfielder(x) \lor scores(x) \lor \neg friend(y,x)$ drop universal quantifiers

- $Centerfielder(Robinson) \land Centerfielder(Jones)$

- $\forall x: [(Centerfielder(x) \land flies(x, Jones)) \Rightarrow \neg scores(x)]$

  $\forall x: [\neg(Centerfielder(x) \land flies(x, Jones)) \lor \neg scores(x)]$ eliminate implication

  $\forall x: [\neg Centerfielder(x) \lor \neg flies(x, Jones) \lor \neg scores(x)]$ De Morgan

  $\neg Centerfielder(x) \lor \neg flies(x, Jones) \lor \neg scores(x)$ drop universal quantifiers

- $flies(Robinson, Jones)$

KNOWLEDGE BASE

1. $\neg Centerfielder(x) \lor scores(x) \lor \neg friend(y,x)$
2. $\neg Centerfielder(x) \lor \neg flies(x, Jones) \lor \neg scores(x)$

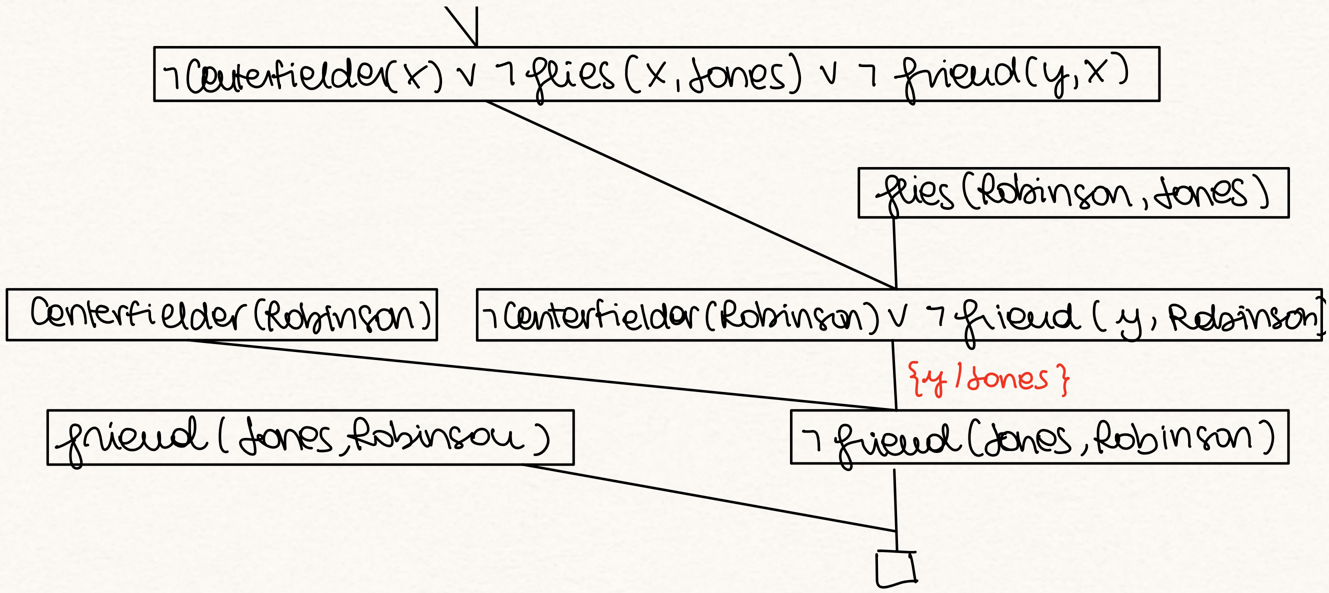
Facts: $Centerfielder(Robinson)$
$Centerfielder(Jones)$
$flies(Robinson, Jones)$

Add: $friend(Jones, Robinson)$

RESOLUTION

$\boxed{\neg Centerfielder(x) \lor \neg flies(x, Jones) \lor \neg scores(x)}$

$\boxed{\neg Centerfielder(x) \lor scores(x) \lor \neg friend(y,x)}$



ASSIGNMENT 5
TDT4136 - Introduction to Artificial Intelligence

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