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Review

Information cascades in complex networks

MAHDI JALILI†
School of Engineering, RMIT University, Swanston St., Melbourne, 3001 VIC, Australia
†Corresponding author. Email: mahdi.jalili@rmit.edu.au

MATJAŽ PERC
Faculty of Natural Sciences and Mathematics, University of Maribor, Koroška cesta 160, SI-2000 Maribor, Slovenia and CAMTP - Center for Applied Mathematics and Theoretical Physics, University of Maribor, Mladinska 3, SI-2000 Maribor, Slovenia
matjaz.perc@uni-mb.si

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Information cascades are important dynamical processes in complex networks. An information cascade can describe the spreading dynamics of rumour, disease, memes, or marketing campaigns, which initially start from a node or a set of nodes in the network. If conditions are right, information cascades rapidly encompass large parts of the network, thus leading to epidemics or epidemic spreading. Certain network topologies are particularly conducive to epidemics, while others decelerate and even prohibit rapid information spreading. Here we review models that describe information cascades in complex networks, with an emphasis on the role and consequences of node centrality. In particular, we present simulation results on sample networks that reveal just how relevant the centrality of initiator nodes is on the latter development of an information cascade, and we define the spreading influence of a node as the fraction of nodes that is activated as a result of the initial activation of that node. A systemic review of existing results shows that some centrality measures, such as the degree and betweenness, are positively correlated with the spreading influence, while other centrality measures, such as eccentricity and the information index, have negative correlation. A positive correlation implies that choosing a node with the highest centrality value will activate the largest number of nodes, while a negative correlation implies that the node with the lowest centrality value will have the same effect. We discuss possible applications of these results, and we emphasize how information cascades can help us identify nodes with the highest spreading capability in complex networks.

Keywords: complex network; social dynamics; centrality measure; information spreading; influence maximization; epidemics; information cascade; online popularity; memes.

1. Introduction

The last two decades have been witnessed with a birth of new movement in science, and an inter-disciplinary field has emerged under the name of *Network Science* [1–5]. Many natural phenomena can be modelled as networked structures, where a number of individual entities are connected through interaction links. Examples include the Internet, power grids, the human brain, World Wide Web, online social networks, water distribution and transportation networks. Graph theory combined with data mining led to a dramatic progress in this field, attracting scholars from different fields, ranging from physics to mathematics, computer science, biology, ecology, finance and social sciences. Early studies in network science focused on understanding structural properties of real complex networks and introducing proper models to construct synthetic networks mimicking their properties. Networks, extracted from real-world data [6], have been shown to share a number of common structural properties such as a scale-free degree distribution, small-worldness [7], community structure [8] densification and shrinking diameter [9]. However, real networks might have different properties as well, and in [10] a method was proposed to compare properties of different complex networks. Dynamical processes have also been studied on networked structures. For example, interacting dynamical systems may show collective behaviour such as synchronization and consensus [11–15]

Many real networks have a scale-free degree distribution, meaning that the nodes have different roles in the structure of the network, and consequently on its function [16–19]. Indeed, nodes have different importance (vitality) in networks, and the degree is only one, albeit an important, representation of this vitality. Node degree plays a vital role in many processes occurring on or related to complex networks, such as controllability [20, 21], opinion formation [22], preventing catastrophic cascading failures [23–26], identifying high-performing research scholars [27, 28], fostering cooperation [29–33] and synchronization [14, 34, 35]. However, identifying vital nodes that are significant for all network functions is not an easy task, as degree is not the only property indicating vitality [36, 37]. Some network functional properties are better described by other centrality measures such as betweenness, closeness or coreness [38]. Indeed, no single centrality measure indicates nodes' vitality for all network functions. The other challenge is a need to identify a set of vital nodes instead of a single vital node (or ranking the nodes based on their vitality) in many cases. Putting the most vital nodes together does not often guarantee to find the set of vital nodes. Indeed, such a set often includes nodes that are not individually vital, but have significant influence when considered with a group of other nodes. This has been shown for network functions such as synchronization [39], controllability [40–43], communicability [44, 45] and information spreading [46–50]. There has been much attention to identification of vital nodes within the community of network science in recent years. Here we provide a review on how conventional centrality metrics for structural vitality of nodes perform in information cascade models.

Information diffusion (also known as information transmission, dissemination, cascade or spread) has attracted tremendous attention within the community of network science due to its potential applications in various disciplines. For example, a company may give a certain product to a selected number of *influential* individuals for free with a hope that they recommend the product to their friends if they are satisfied with that product [51–56]. It has been argued that for products with positive effects on the users, the sellers often can take the advantage of *positive externalities* to make the crowd more likely to buy the product with such strategies [54]. Sellers can also choose to offer discounts based on the influence of individuals where the revenue can even be further maximized [57]. Target immunization is another example where a small subset of influential nodes is selected and immunized [58–61]. Efficient immunization strategies have also been proposed under limited budget [62]. Network components may undergo a cascade of failures, where a failure in one component might trigger cascaded failures and substantially break down the whole network [24, 63–67]. It has been shown that failure in nodes with high centrality values has significant effect on failure propagation.

Finding the most influential spreaders has always been a hot research topic within the community of network science [46, 47, 68–74]. The *influence maximization problem* tries to find a small subset of individuals of which triggering causes the largest information spread across the network. This manuscript provides a mini-review on the relation between nodes' centrality values and their influence in information cascade. Section 2 provided overview of information spread and cascade models. Two general graph-based models including *linear threshold model* and *independent cascade model* are briefly reviewed in this section. Then, Section 3 summarizes structural centrality measured covered in this manuscript and Section 4 briefly reviews state-of-the-art in finding the most influential spreaders. In Section 5, we present some results on the relation of nodes' centrality and their spreading influence on some model and real networks. Finally, the manuscript is concluded and outlook is given in Section 6.

2. Information spreading and cascade models

Recently, there has been increasing literature on information cascade (also known as information diffusion, dissemination, spread or transmission) in various disciplines ranging from biology to social sciences, mathematics, physics and computer science [75]. With ever-increasing importance of online social networks, in particular, studying information spread in social communities have been recently subject to heaving investigations within computer science disciplines [76–78]. Information propagation through online social networks has proved to be a powerful tool in many situations, like importance of Twitter 2009 US presidential election [79] and influence of Facebook in 2010 Arab Spring [80]. Another important research in this field is epidemics and virus spread, which has attracted many scholars in ecology and biology disciplines [81, 82]

Information cascade has various applications in computational sciences. Viral marketing is one of the most important applications [51–53, 57]. Viral and world-of-mouth marketing (and advertising) is new form of product marketing that try to maximally use network-based effects (e.g. through various social networks) to increase awareness of specific product and achieve marketing goals. It comes in various forms including images, videos, emails, text messages, twits, games and blogs. All these forms can be sent from one person to another. It has frequently happened that contents posted on social media platforms have gone viral and become a hot topic [83]. Compared to traditional marketing tools, marketing through social networking is rather easy to implement, has much lower cost, better potential for long-term influence and exponential growth. Viral marketing is now a major practice for many campaigns. The information is shared with a number of influential players that play the role of spreaders in the network. Sometime, these key players receive some bonus (e.g. free product or a substantial discount) to encourage discrimination.

Diffusion of innovation is another topic that has been frequently studied over network [84, 85]. New ideas, technologies and ways of doing things can quickly propagate through a network. It has been shown that people tend to adopt a technology (or product) with increasing likelihood, depending on their friends and neighbours (that is determined based on their connections in the network) to whom they trust. The innovation can be a novel social practice [86], a new form of employment contract [87], or a technological advance such as a new software package [88]. There are various factors determining how efficient (and fast) innovations diffuse across the network, including network topology [74, 89] and payoff gains [90]. The Internet and instantaneous availability of information have dramatically changed how technology and innovation spreads. For example, it took 50 years for electricity to have 50 million users, decades for fax and telephone, but only a few years (even in some cases less than a year) for social networks such as LinkedIn, Facebook, Twitter and MySpace. Indeed, nowadays networks and in particular social networking platforms are essential tools to spread innovations.

Networks have also major role in opinion formation among individuals. Individuals, each having an opinion, exchange their ideas and opinions through their friendship links in networks, and influence one another's opinions [22, 91–96]. A number of works modelled the opinion formation process in complex networks. For example, in bounded confidence model [22, 94, 97, 98], the agents have a continuous-valued opinion vector. At each step, a node is chosen randomly and is influenced by one of its randomly selected neighbouring node. If they are negotiable, that is, their opinion values are less than a certain threshold, they update their opinion values such that their opinions get closer. Otherwise, no update process happens. It has been shown that if certain conditions are met, clusters of nodes with the same opinion values will appear. For certain cases, all nodes will reach a consensus and have the same opinions.

2.1 *Cascade models*

Predictive cascade models can be generally divided into two classes: graph-based models and non-graph-based ones. The non-graph-based models are epidemic models that mathematically study diffusion using population-based dynamics [99–101]. Well-known models in this context are susceptible-infected-removed (SIR) [102] and susceptible-infected-susceptible (SIS) [103]. In SIR model, the population is divided into three classes: susceptible, infected and removed. The susceptible portion of the population is those that have not been infected yet (i.e. unaware of the information) and might be infected (with a certain probability) in contact with an infected person. When a person is infected, he/she is removed (becomes immune or dies depending on the nature of infection) after a period of time. The removed agents do not influence others. In SIS model, when a person is infected, he/she becomes susceptible again after being recovered. In these methods, one often would like to study the time-evolution of the ratio of susceptible, infected and removed nodes. These epidemic models have been frequently used to study how a virus spreads within a population as well as on other contagion processes such as innovation diffusion, information diffusion, rumour spreading and spread of political movements.

In graph-based predictive cascade models, the dynamics is often studied in the level of individual nodes A kind of information starts from an initial set of nodes and spread through the network based on a cascade model. There are two well-known graph-based models: linear threshold [104–107] and independent cascade [108, 109]

2.1.1 *Linear threshold cascade model*

The linear threshold model has been proposed to describe binary decision making, and has been frequently used to model such a process in economy and sociology. The model has been proposed to mimic herding-like behaviour, where an individual makes a decision based on his/her neighbours' actions [110, 111]. In this model, there are a number of states that each node can adopt. Often, the nodes can have two states: *active* or *silent*. In the start of the process all nodes are assumed to be in the silence mode. Then, a number of nodes are selected as early adopters for which the state is switched to active mode. These early adopters may trigger some of their neighbours, if the threshold condition is met. In general, a silent node i becomes active if at least t_i fraction of its neighbours are active. This threshold can be uniform across all nodes (which is often the case) or can be drawn from a certain distribution to have node-specific threshold values. The iterative diffusion process continues until the steady state solution is obtained that is, there are no further changes in nodes' state and no further node is activated. If certain conditions are met, one might have full cascade over the networks where all nodes become active. The linear threshold model is a simple model that has been frequently used in many studies.

2.1.2 *Independent cascade model*

Another commonly used cascade model is independent cascade model [108, 109]. Similar to the linear threshold model, in independent cascade model the nodes can be either *active* or *silent*. Often, two assumptions are made for the independent cascade model:

1. Any active node i has only a single chance to activate its silent neighbour j, and if the activation process is not successful, there will be no influence on node j from node i;
2. The probability that a silent node j is influenced by an active node i (which is located in its neighbourhood) is independent of the influence of other active nodes on node j

The process of the independent cascade model is as follow. Similar to the linear threshold mode, first, all nodes are assumed to be silent at the start of the process. Then, a (usually small) fraction of nodes become active. Let's denote the nodes that become active in step t of the process by $A_C(t)$. At any time t of the algorithm, each of the nodes that have become active in the previous step ($A_C(t-1)$) activate one of their silent nodes with a certain probability. Let's denote the probability that an active node i activating a silent neighbour j by Q_{ij}. This edge-specific probability is also known as diffusion probability in the literature. These probability values should be given before the start of the numerical simulation process. If these probabilities are unknown, one can use a maximum likelihood method to estimate their values from observations on cascade sequences [112]. If a silent node j has more than one neighbour that has been activated in the previous step, the active neighbours tend to influence node j in an arbitrary order. This procedure is repeated until the steadystate solution is obtained. Finally, the number (or ratio) of activated nodes by initially activating a (set) of node(s) is denoted as its spreading influence (or influence range). The higher is the spreading influence of a node, the more vital is that node for information spread.

Figure 1shows a sample network with 15 nodes and a number of edges. The cascade process starts by initially activating node 1. In this example, the diffusion probability is set to 0.5 for all edges. Node 1 has

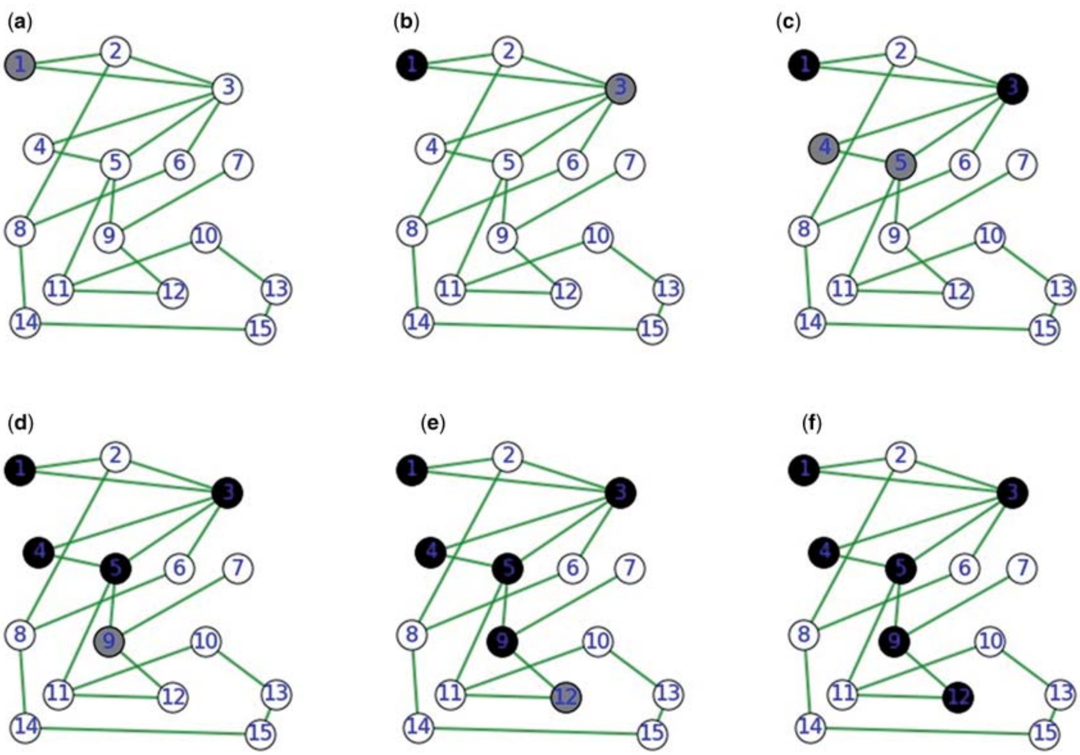

FIG. 1. Schematic illustration of an information cascade in a network with 15 nodes and 17 edges with a uniform diffusion probability of 0.5. The diffusion starts by initially activating node 1 (a). In the second step one of neighbours of node 1 (node 3) is activated (b). In the third step, node 3 that has been activated in the last step, activates two of its neighbours nodes 4 and 5 (c). Node 4 does not have any silent neighbours, but node 5 activates node 9 in the fourth step (d), which itself activates one of its neighbours in the next step (d). Finally, as no node is activated in step 6, the process stops leading to activation of 5 nodes as initially activating nodes 1 (f).

two neighbours (nodes 2 and 3), of which only node 3 becomes active in the first step of the simulation process. Note that node 2 will not be influenced by node 1 in the future steps. Out of three neighbours of newly activated node 3, two of them (nodes 4 and 5) are activated in the second step of the process. Finally, the cascade process stops and six nodes are activated as a result of initially activating node 1.

3. Node centrality measures

The influence of a node in a complex network largely depends on its structural position in the network, and the most of introduced node centrality measures are based on only structural information on networks [113]. Indeed, the concept of centrality was first introduced by Freeman to distinguish the nodes by their structural centrality. In order to quantify their centrality in the network, each node is assigned a real value, which makes it possible to compare the node for its centrality values. Node centrality measures play a significant role in studying properties of real systems. For example, Mantzaris *et al.* used dynamic centrality measures to characterize aggregate activity in different brain regions, as well as to reproduce learning-related information in the brain [114] In this section, we briefly review well-known structural centrality measures.

Let's consider an undirected and unweighted network $G(VE)$ where V is the set of nodes and E is the set of edges. Although in this work we focus on undirected and unweighted networks, most of the concepts can be easily extended to directed and weighted networks. The network is fully described by its adjacency matrix $A = [a_{ij}]$, where $a_{ij} = 1$ when there is a link between nodes i and j, and $a_{ij} = 0$, otherwise. Let's also denote the edge between nodes i and j by e_{ij}. The simplest centrality measure of a node is its degree (in- and out-degree for directed networks). Degree of a node is the total number of links that node has with others. Degree k_i of node i is calculated as

$$k_i = \sum_j a_{ij}. \tag{1}$$

Degree is a local measure, meaning that a node only requires being aware of its immediate neighbours. Degree is an important centrality measure, and many dynamical processes have been linked to node degree [20, 115]. Degree takes into account only the number of neighbours, however sometimes the importance of a particular neighbour may also matter. For example, connecting to an important individual can bring more importance (or prestige) for a node than being connected to more non-important neighbours. To account for this, Hirsch index (known as H-index) has been extended for node centrality. H-index, proposed by Hirsch as an index to quantify an individual's research impact [116] H-index of an individual is the number of his/her publications that have received at least h citations. The same concept has been extended to networks [27, 28, 117, 118]. To make it mathematical, one can define an operator H applying which on a set of real-valued variables $[y_1, \ldots, y_n]$ returns the maximum integer h such that among the members of this set there are at least h members with a value no less than h. With such a definition for H, H-index of node i is defined as

$$h_i = H\left(k_j\right), j \in N_i. \tag{2}$$

where N_i is the set of neighbours of node i. Indeed, H-index of a node is the maximum value h such that its neighbours have at least degree h. In order to compute the H-index, each node is required to have its degree and those of its neighbours.

Many dynamical processes depend not only on local properties, but also on global properties of networks. A number of centrality measures have been proposed in the literature that takes into account global structure of the networks. A class of these measures is those based on (usually shortest) path length. Here we consider three path-based centralities including eccentricity, betweenness and closeness centrality. Betweenness centrality, as introduced by Freeman [119, 120], takes into account centrality of nodes in navigation through the network. Betweenness centrality B_i of node i in a graph, which shows the number of shortest paths making use of node i (except those between the i-th node with the other nodes), is computed as

$$B_i = \sum_{j \neq i \neq k} \frac{\Gamma_{jk}(i)}{\Gamma_{jk}}, \tag{3}$$

where Γ_{jk} is the number of shortest paths between nodes j and k and $\Gamma_{jk}(i)$ is the number of these shortest paths making use of the node i. From the above equation, it is easy to see that the betweenness centrality of a leaf node is zero, as it is not on any shortest paths. A number of variants to the betweenness centrality have been proposed in the literature such as connection graph stability [121, 122], random walk based method [123, 124] and communicability [44].

Eccentricity of a node is other path-based centrality measure [125]. Let's define the shortest path between nodes i and j by p_{ij}. Based on eccentricity centrality index, the shorter is the distance of a node from other nodes, the higher is the importance of that node. Eccentricity of node i is the maximum distance between i and other nodes, and is calculated as [125]

$$E_i = \max_j(p_{ij}). \tag{4}$$

Under the above definition, a node with smaller eccentricity will have more central position in the network. Closeness is another centrality measure that measures the mean distance from a node to other nodes. Closeness centrality of node i is calculated as

$$C_i = \frac{1}{N-1} \sum_j \frac{1}{p_{ij}}. \tag{5}$$

The above quantity takes lower value for the nodes that have shorter distance from others, that is, nodes that are closer to the centre of the network. Such nodes might have better access to the information at other nodes and provide better direct influence on them.

An alternative approach to calculate vitality of a node is to account for its importance in the amount of information contained in all possible paths between pairs of nodes in the network [126–128]. Although one can define the term 'information' in various forms, it is defined based on path information in this approach. It is supposed that the longer the path is, the more the information loss. The information index is indeed a different from of closeness centrality where a different way to consider the contribution of each path is considered. Mathematically, the information index for node i is calculated as [126–128]

$$I_i = \left[\frac{1}{N} \sum_j \frac{1}{r_{ij}}\right]^{-1}; \; Q = (q_{ij}) = (K - A + \mathbf{1})^{-1}; \; r_{ij} = \left(r_{ii} +_{jj} -2r_{ij}\right)^{-1}, \tag{6}$$

where K is a diagonal matrix with node degrees in the diagonal elements and $\mathbf{1}$ is a matrix with all elements equal to 1.

Here we also consider eigenvector centrality, which considers that not only the number of neighbours of a node is importance in determining its vitality, but also the importance of the neighbours [129, 130]. Eigenvector centrality of a node is the corresponding entry in the (left) eigenvector corresponding to the largest eigenvalue of the connectivity matrix A. However, Eigen-decomposition of large-scale matrices is complex (time complexity of $O(N^3)$), which might not be practical in many cases. Alternatively, once can use iterative power methods to compute the eigenvector centrality of nodes [130, 131]. A simple iterative algorithm to compute the eigenvector centrality is a follows. Let's denote the eigenvector centrality of node i as v_i, where $v_i(t)$ indicates its value at step t of the algorithm. First, for all i's we take $v_i(t) = 1$. Then, at each step we compute $v_i(t+1) = \sum_j \frac{a_{ij}v_j(t)}{k_j}$, and make appropriate normalization. The algorithm continues until a steadystate solution is obtained. There are a number of variants to the eigenvector centrality, including the well-known PageRank [132, 133], Katz centrality [134] and LeaderRank [46, 135].

4. Influential spreaders

Not surprisingly, nodes of a network have different spreading capabilities; some nodes are in a strategic position and can spread the information more influentially than others. There have been heavy investigations on the problem of *influence maximization* and finding the super spreaders in complex networks [68, 69, 71]. The influence maximization problem was first formally defined by Kempe *et al.* [71]. Let's define the influence of a set of nodes S, denoted by $I(S)$, to be defined the expected number of activated nodes as a result of initially activating the nodes in S. One can formally define the influence maximization problem as follows [71]. Given a fixed parameter k, the influence maximization problem finds a k-node set S whose influence $I(S)$ is maximum, that is, $I(S)$ has the largest number of nodes. It was shown that this problem is NP-hard for many cascade models including the linear threshold and independent cascade. The same formulation can also be used to study viral marketing [51, 136] and revenue maximization problems [54, 57]. In the revenue maximization problems, some samples of products are given for free (or with a certain discount) for a selected number of nodes with a hope that they positively influence the crowd and help maximizing the revenue. The influence maximization problem has been extended from different perspectives. For example, Wang *et al.* introduced *positive influence domination set* problem under the linear threshold model [137]. In this problem, one finds a set of nodes S such that every node in the network has at least half of its neighbouring nodes in S. He *et al.* introduced the *minimum-sized influential node set* problem, which is to find the set of influential nodes with the minimum size such that every node in the network can be influenced by this set no less than a certain threshold [109]

It has been shown that for submodular cascade functions [138], one can efficiently construct a hill-climbing algorithm to find an approximate solution for the standard influence maximization problem [54, 71, 139]. The approximate algorithm guarantees providing a solution within 63% of the optimal solution, and is as follows. First, the best node is numerically determined by computing the influence of all nodes one-by-one and obtaining the optimal one; this node is put into set S. Then, this node is removed from the network, and the optimal node for the new network is obtained in a similar way. The new optimal node is added to S. This procedure continues until the size of set S reaches k. It was shown that the problem can be solved in a linear time in certain graphs. For example, Chen *et al.* showed that this is the case for directed cyclic graphs for which a scalable algorithm can be found [70]. However, real social networks might not have such a structure, which makes it difficult to apply such algorithms to them.

Nodes with high centrality values (e.g. those with high degree, betweenness or closeness centrality) are candidates for influential spreaders. Kitsak *et al.* showed that this is not the case for many real networks [72]. They showed that the strategic location of a node, known as its coreness, is more important than conventional centrality measures such as degree and betweenness, in evaluating its influence on information spreading. Indeed, the best spreaders are those located in the core side of a network, which can be identified by k-sell decomposition algorithms [140, 141]. Coreness of a network is obtained as follows. Given an undirected and unweighted network, initially the coreness of all isolated nodes is defined as zero. Such nodes are removed from the network. Then, in the k-sell decomposition algorithm, all nodes with degree $k = 1$ are removed from the network, resulting in the reduction of network edges. The nodes removed in this step will have coreness of 1. Then, the nodes with the degree less or equal than 1 in the remaining graph are identified and further removed from the network. These nodes will have coreness of 2. The process continues until the nodes with the maximum coreness are identified. The coreness can be efficiently computed for large-scale networks; however, it cannot be applied to many model networks such as tree and scale-free networks constructed using Barabasi-Albert algorithm [115], for which the coreness of all nodes have close values such that one cannot distinguish them. Alternative approaches have been proposed to solve this problem including mixed degree [142] and generalized degree discount [73].

5. Centrality of nodes and its role in information spreading

5.1 *Centrality values and their influence on information spread*

There are various research reports showing that a network's structure has a major role in determining its capability for information spread; however, this also depends on the dynamical process under study. For example, a node might be a key spreader in information dynamics, but not in epidemics dynamics. In spite of that, some general centrality measures have been shown to be rather vital for many spreading dynamics [38]. de Arruda *et al.* [143] studied how different centrality measures are correlated with their epidemic spreading capabilities. They found that degree and k-core show the highest correlation for epidemic spreading in non-spatial networks, whereas closeness and average neighbourhood degree are the most related ones to rumour dynamics. Identifying nodes with central role in information spread will have many potential applications in various fields. For example, by determining the most influential spreaders in criminal groups, the relevant investigation body can better manage crime and implement preventive strategies [144].

5.2 *Numerical simulations*

In this section we provide results on numerical simulations in a number of synthetic and real networks. For simulations, we consider a uniform diffusion probability for all edge as $Q_{ij} = Q$. Under a certain Q, for any node i we obtain its spreading influence range S_i as the ratio of the nodes that are activated by initially activating node i. Higher values of S_i indicates higher spreading capability for i. These spreading influence values are then correlated with the centrality values.

5.2.1 *Model networks* As model networks, we consider well-known preferential attachment scale-free, random and small-world networks. The random networks are constructed by the model proposed by Erdos and Renyi [145]. In Erdos–Renyi networks there is a link between any pair of nodes with probability P. Small-world networks are constructed by the algorithm proposed by Watts and Strogatz in their seminal work [7]. To construct Watts–Strogatz networks, first a ring graph with N nodes each connected to its m-nearest neighbours are considered. Then, the links, one-by-one, are rewired with probability P. For different values of the rewiring probability, one obtains networks from regular (for $P = 0$) to complete disorder (for $P = 1$). Erdos–Renyi and Watts–Strogatz networks have almost homogeneous degree distribution; however, many real networks have been shown to have heterogeneous node degrees. Barabasi-Albert model [115] results in such networks. The model is as follows. First, an all-to-all connected network with m nodes is considered. Then, as each proceeding step, a new node is added to network and creates m links with old nodes. The probability of connecting the newly added node to an old node is proportional to the degree of the old node; the higher the degree of an old node, the higher the probability of tipping to the new node.

5.2.2 *Real networks* Although studying model networks can provide useful information on how real networked systems behave, models cannot however mimic all properties of real networks. Therefore, we also consider a number of real networks and study the relations between their nodes' centrality values and spreading influence. The real networks considered in this work include [146] co-authorship network of scientists active in the field of Network Science (Net Sci), social networks between PhD students of Computer science (CS PhD), Facebook-like, a network of Online Dictionary of Library and Information Science (ODLIS), US Airports, the Internet as in the level of Autonomous System (AS), US electric grid, EU electric grid.

5.2.3 *Simulation results* Figures 2 and 3 show how the spreading influence of nodes is correlated with their centrality scores in scale-free networks with varying average degree and heterogeneity (indicated by B), respectively. These results show mean values over 20 independent runs. Spearman rank correlation values are obtained between the spreading influence and centrality values including degree, betweenness, closeness, eccentricity, H-index, information index and eigenvalue centrality. Positive (or direct) correlation indicates that as nodes take higher centrality values, their spreading influence is higher, which means that more nodes will be activated by initially activating them. Negative (or indirect) correlation indicates the opposite. It is seen that, as the networks become denser, the absolute values of the correlation between the spreading influence and centrality values decreases (Fig. 2). While degree, betweenness, closeness, H-index and eigenvalue centralities are positively correlated with the spreading influence of nodes, information index and eccentricity show negative correlation values. The information index has the highest negative correlation, while degree often has the most significant positive correlation. This indicates that choosing the highest-degree nodes likely leads to activation of a larger set of silent nodes, whereas choosing those with the highest information index will result in the lowest number of activated nodes. The profile of the correlation patterns is almost independent of the heterogeneity level of the network, as expressed by B (i.e. the higher the B, the lower the heterogeneity level), and the correlations vanish for large values of the threshold parameter Q (Fig. 3).

Scale-free networks are typical examples of networks with heterogeneous node degrees. As examples for networks with almost homogeneous distribution of degrees, we consider Watts–Strogatz small-world and Erdos–Renyi networks. Figure 4 shows the correlation of spreading influence with the centrality measures of the nodes for Watts–Strogatz networks with $N = 500$, $P = 0.2$ and different average degree (the networks have average degree equal to $2m$). One can observe almost similar pattern as for scale-free networks (Fig. 2), where by increasing the density of the network (i.e. increasing m), the absolute value of the correlations decreases. Again, information index and eccentricity show negative correlation, with the information index being the one with the strongest indirect correlation. Other centrality measures have positive correlation with the spreading influence with degree being the one with the strongest direct correlation. Figure 5 shows the correlation values for Watts–Strogatz networks with varying rewiring

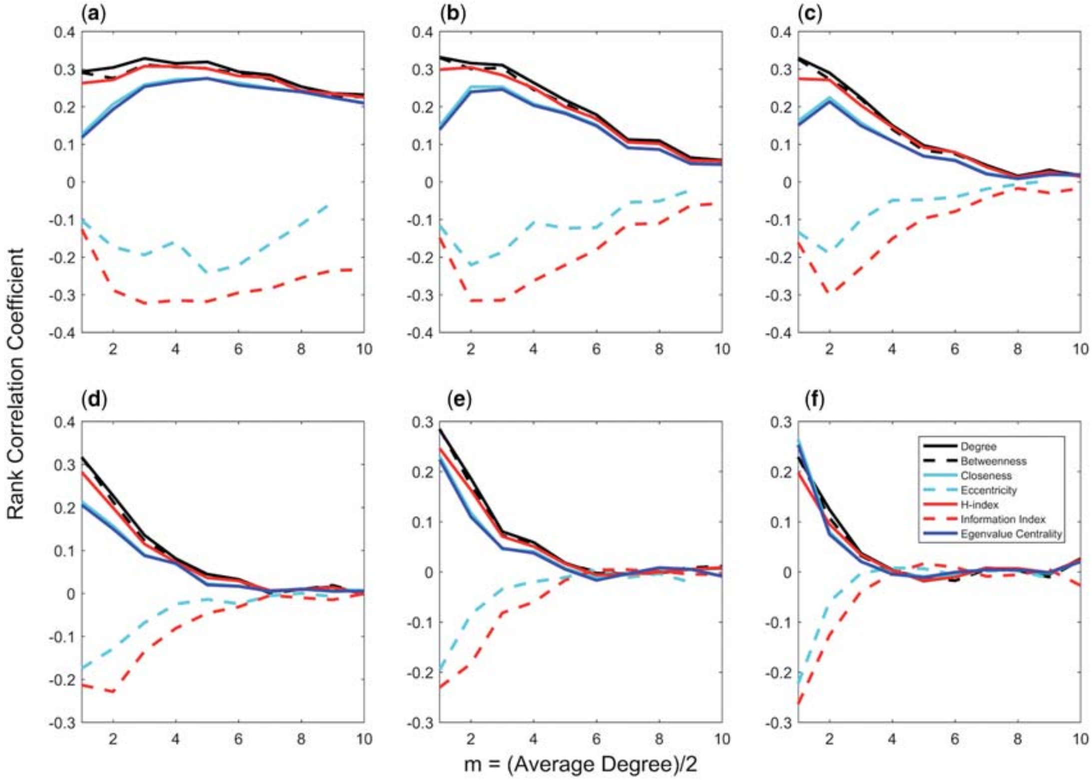

FIG. 2. Spearman rank correlation values between the centrality of nodes and their spreading influence, as a function of average degree ($2m$) in scale-free networks with size $N = 500$ and $B = 0$ (see text for explanation of this parameter). The threshold for the independent cascade model is fixed at 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6, respectively for (a)–(f). The spreading influence of nodes, measured by obtaining the ratio of activated nodes as initially activating a particular node, is correlated with centrality scores including degree, betweenness, closeness, eccentricity, H-index, information index and eigenvalue centrality (see text for complete explanations of these measures). Data represent mean values over 20 independent runs.

probability P. The correlations are not significant for large values of the threshold Q. For small values of Q and as the rewiring probability increases, the strength of the correlation (almost linearly) increases. The information index has by far the strongest indirect correlation. Again, degree has higher direct correlation almost in all cases. Figure 6 shows the correlation values in Erdos–Renyi networks with $N = 500$ and varying connection probability P. As P increases, the networks become denser, the correlations vanishes similar to the other cases. Again, information index and eccentricity have indirect correlation, while other centrality measures show direct correlations with the spreading influence.

To further study the pattern of correlations, we consider a number of real networks ranging from social networks to transportation and technological ones. Figure 7 shows these results for different values of the threshold parameter. One can observe significant differences between these results and those obtained for synthetic networks. Unlike synthetic networks, there are quite substantial differences between the measures showing direct correlations. In social networks (i.e. Net Sci, CS PhD and Facebook-like), H-index shows the strongest positive correlation with the spreading influence, followed by degree in one of them (Net Sci) and betweenness in the other two. Similar to the synthetic networks, the information

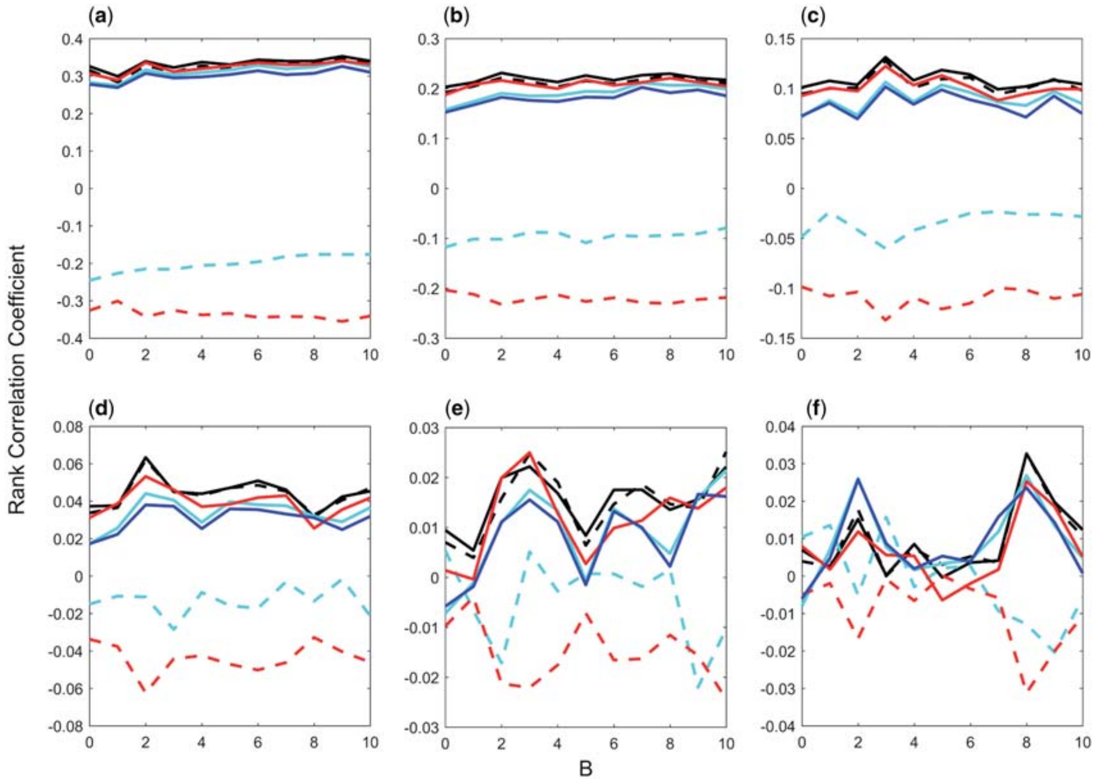

FIG. 3. Spearman rank correlation values between the centrality of nodes and their spreading influence, as a function of B is scale-free networks with $N = 500$ and average degree of 10. B indicates the parameter controlling heterogeneity of the network, such as the higher B is, the less the heterogeneity of the network. The threshold values for different panels and the legend for different lines styles is the same as in Fig. 2.

index shows negative correlation in real networks, but for Net Sci where the correlation is almost zero. In technological networks including the Internet in the level of AS, US Electric Grid and EU Electric Grid, degree shows the strongest correlation, followed by H-index. The other observation is the profile of correlation values for different threshold values. While in some networks (Net Sci and CS PhD) the direct/indirect correlations become more significant as the threshold increases, they become less significant in some other networks such as US Airport and Facebook-like. The absolute value of the direct correlations is almost independent of the threshold in technological networks, while the negative correlations become stronger as the threshold increases in these networks. The results indicate that real networks have distinct properties and even network of the same class (e.g. social networks) might exhibit different properties.

5.3 *Applications*

Identifying nodes with high levels of spreading capabilities has many potential applications. In the following we review some of the significant applications where fundamental theories developed in this field can be effectively used.

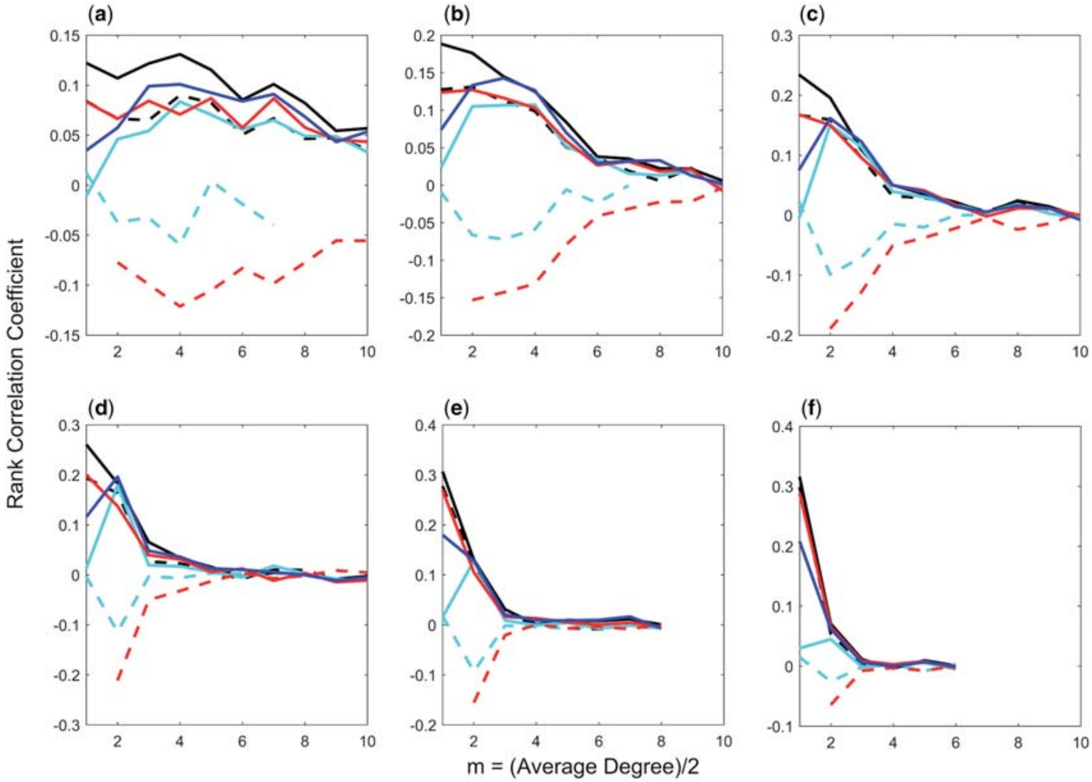

FIG. 4. Spearman rank correlation values between the centrality of nodes and their spreading influence, as a function of average degree ($2m$) is Watts–Strogatz small-world networks with $N = 500$ and rewiring probability $P = 0.2$. The threshold values for different panels and the legend for different lines styles is the same as in Fig. 2.

5.3.1 *Mitigating contagious disease and preventing epidemics* It has been frequently shown that networks with hub elements, for example scale-free networks with high-degree nodes, are prone for viruses and contagious diseases [82, 100, 147–149]. Scale-free networks do not have an epidemic threshold, meaning that a disease can spread superfast in such networks, and therefore a large-fraction of the nodes would be at risk of contagious. Computer viruses have also similar spreading pattern and can spread fast on the network due to scale-free topology of the Internet [150]. In networks with mobile agents, for example social networks, the nodes move around and make new connections. This is often the main driver behind global epidemics such as 2009 H1N1 influenza pandemics and 2003 SARS epidemics [151, 152]. It has been shown that one can find an effective distance predicting the approximate arrival time of the disease [153]. By analysing large-scale mobility patterns and individual-based traffic data, Eubank *et al.* found that connections are strongly small-world with a well-defined scale-free degree distribution [154]. This allows efficient outbreak detection by placing the sensors in the hub locations, where many high-degree individuals are also involved.

Central (or hub) nodes have significant role in facilitating or blocking the contagious. Christley *et al.* showed that a simple centrality measure such as degree can be effectively used to identify the risk of infection in populations exposed to a virus [155]. If influential spreaders are infected by the diseases,

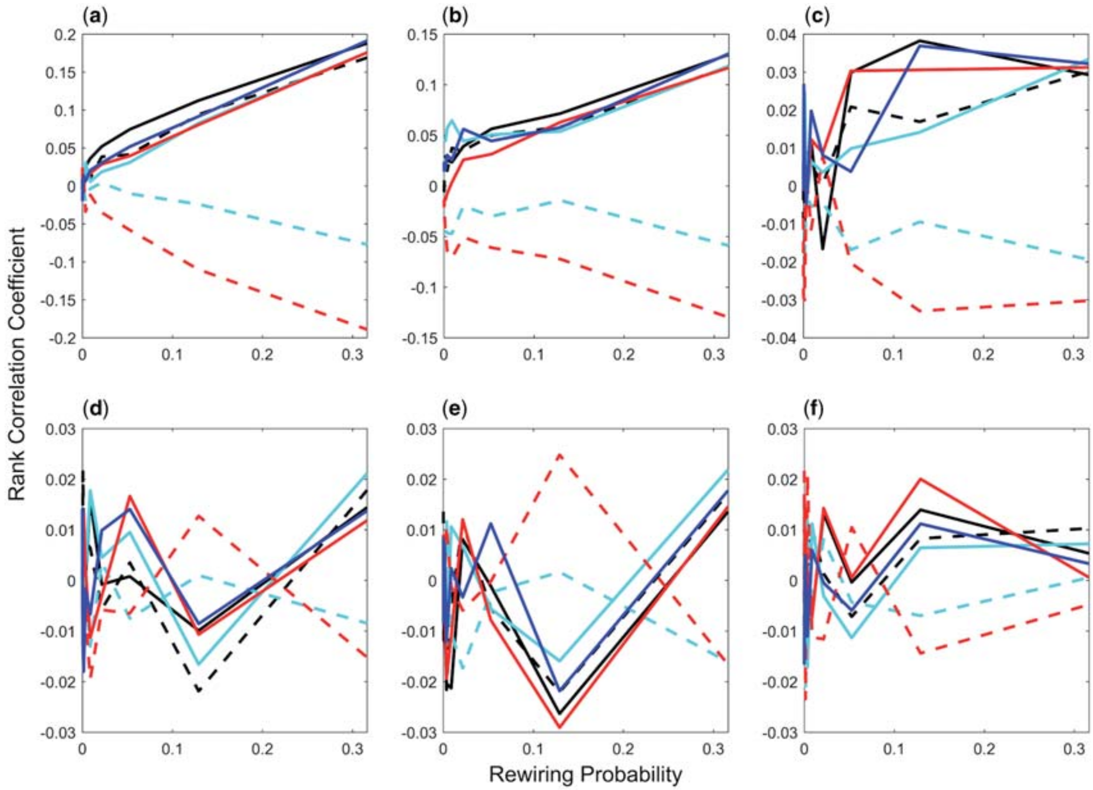

FIG. 5. Spearman rank correlation values between the centrality of nodes and their spreading influence, as a function of rewiring probability is Watts–Strogatz small-world networks with $N = 500$ and average degree of 10. The threshold values for different panels and the legend for different lines styles is the same as in Fig. 2.

they are likely to disseminate it fast because of their central location in the contact network. On the other hand, by identifying such influential spreaders and blocking them by means of vaccination or temporarily isolating them until the spread is controlled and the risk of epidemics disappears. It has been frequently shown that targeted vaccination of complex networks is much more effective than random vaccination [58–60]. Salathe *et al.* collected social networks of close proximity interactions and analysed the relations between the spread of disease and the structure of the network [156]. They also showed that one can accurately predict the real influenza cases by efficiently analysing the close proximity network structure. The information of such a network can be used to design effective targeted immunization strategies. One can also design more effective immunization strategies by optimizing the vaccination cost. Mirzasoleiman *et al.* proposed a simple algorithm to design immunization strategy under limited budget [62]. They showed that under limited budget conditions, one can use a simple discount strategy to decide the amount of discount to give to the central nodes, where the amount of discount (to receive the vaccination) depends on the centrality values.

5.3.2 *Viral marketing* It has been frequently shown by means of both experimenting on real groups and studying on model systems that structure of social networks have a major role in diffusion of behaviours

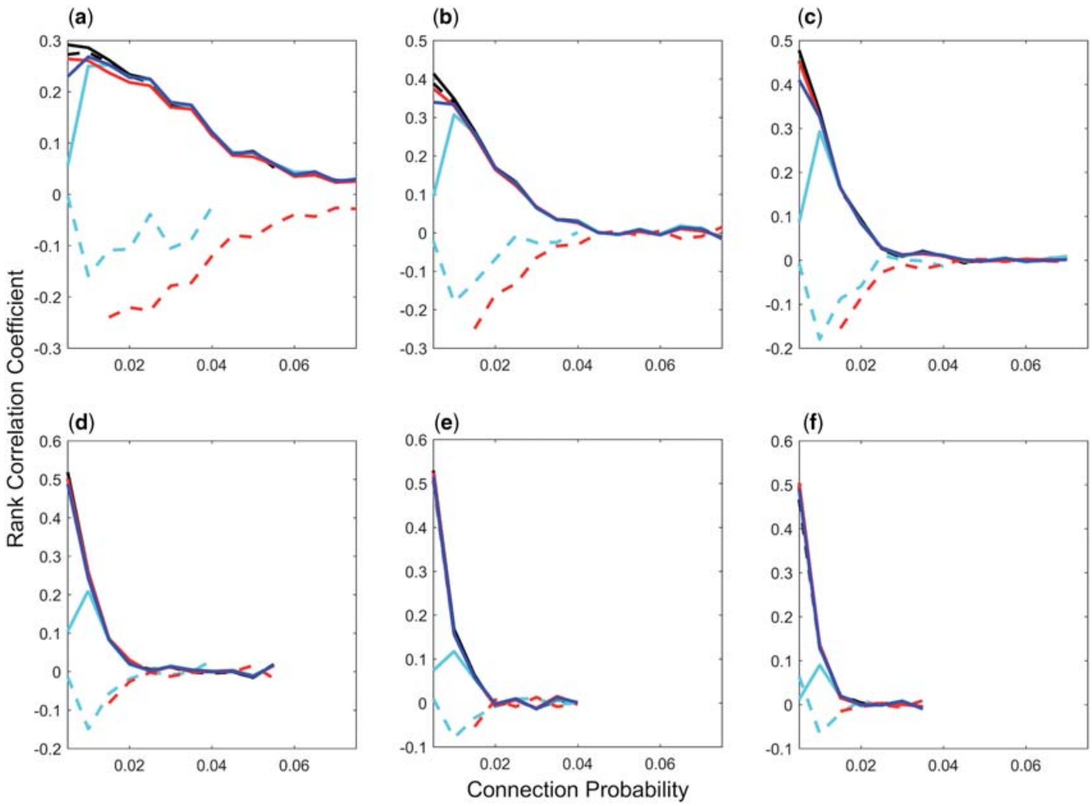

FIG. 6. Spearman rank correlation values between the centrality of nodes and their spreading influence, as a function of connection probability in Erdos–Renyi random networks with $N = 500$. The threshold values for different panels and the legend for different lines styles is the same as in Fig. 2.

and innovations among the individuals [89, 107, 157]. In marketing, 'word-of-mouth' is a well-known strategy [158, 159]. It is informal communication behaviour between the customers about their positive/negative experiences with specific products, services and/or providers. This phenomenon is a basic strategy used in 'viral marketing' [51, 52, 160, 161]. Viral marketing is a modern marketing strategy where social networks are used to increase brand awareness, spread of technology adoption and maximizing the influence. In such strategies, peers pass their views about a particular product, service or provider to their neighbours. As individuals often trust their neighbouring friends in their social networks, and therefore view their feedbacks positively. This strategy is a powerful marketing tool. The structure of social networks has vital role on the efficient spread of a viral message, and one should take this into account to optimize campaign performance [161, 162]. De Bruyn and Lilien carried out an experiment and asked a number of individuals to forward unsolicited emails to their friends inviting them to participate in a survey, and also spread the work about it [163]. They found that the strength of the social tie facilitated awareness, demographic similarity had a negative influence on the spread and perceptual affinity activated recipients' interest on the topic.

An important issue in viral marketing through social networks is 'influence maximization', which is defined as: given a fixed social network topology and a cascade model, what is the set with k nodes

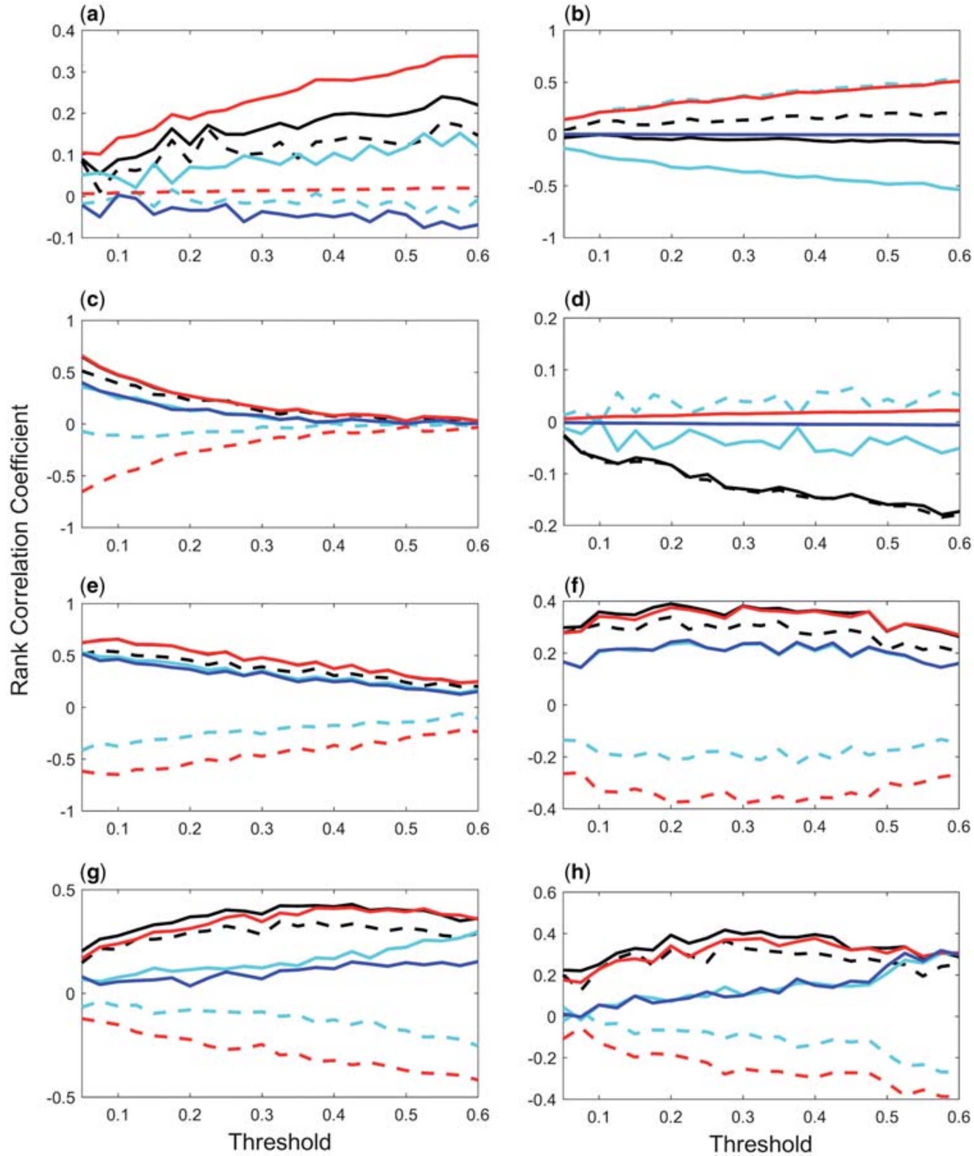

FIG. 7. Spearman rank correlation values between the centrality of nodes and their spreading influence, as a function of threshold of the independent cascade models in a number of real networks including (a) co-authorship network of scientists active in the field of Network Science (Net Sci), (b) social networks between PhD students of Computer science (CS PhD), (c) Facebook-like, (d) a network of Online Dictionary of Library and Information Science (ODLIS), (e) US Airports, the Internet as in the level of Autonomous System (AS), (f) US electric grid and (g) EU electric grid. The legend for different lines styles is the same as in Fig. 2.

such that initially activating them results in the largest cascade effect, that is the largest number of finally activated nodes [71]? The top-k nodes are those that are given the product for free (or with the highest discount rate) in marketing campaigns, with the hope that they can trigger others towards that product, and thus maximize the revenue [57, 164]. The influence maximization problem is an NP-hard problem [57]. For certain cases however, for example, sub-modular influence models, one can find approximate algorithms guaranteeing the performance up to a certain optimality level [71].

Shakarian and Paulo introduced a scalable method to efficiently find a set of initial adaptors that guarantee spreading to all nodes [160]. They also find that dense community structure throughout the network and highly clustered neighbourhoods suppress the information spread across the network, an observation that has also been reported by others [107]. Borgs *et al.* introduced a fast algorithm obtaining the new-optimal solution that is runtime-optimal up to a logarithmic factor [165]. Although some other fast algorithms have been proposed, they are still much slower than topology-based methods that are based on obtaining central nodes and using them as the initial adopters [166]. Most of the top-k most influential nodes are often among those with high centrality values. Optimal consensus and information spreading is also related to the dismantling problem where a minimal set of nodes have to be found such that by removing them the network is broken into connected components of subextensive size [167, 168]. Such nodes are likely among those that play an important role in information spreading.

5.3.3 *Opinion spreading and consensus* Another potential application for the identification of vital nodes for information spread is opinion spread and consensus phenomenon in complex networks. Individuals interacting on a networked structure may influence their neighbours and/or be influence from them, and consequently change their opinion values [94, 96, 169–171]. A number of models have been proposed to study the evolution of opinions on complex networks, which can be generally categorized into two broad classes: models with continuous-time opinions [95, 97, 98, 172, 173] and those with discrete-time opinion values [96, 174–176]. In most of the opinion formation studies, first the agents are assigned with a certain opinion value, for example randomly selected from a range, and then, they adapt their opinion values considering that of their neighbouring nodes who can influence them. For example, in the well-known bounded confidence model [97, 98, 172, 173], one first chooses a random edge in each iteration of the evolution process. Then, the end-nodes of the selected edge change their opinions (and make them closer) if their opinions are close enough, that is they are negotiable. This procedure continues until steady-state solution is obtained and no further opinion change happens. If certain conditions are met, all (or most) of the agents reach a consensus in their opinion values [97].

In real social networks, different nodes have different influence levels on their friends. For example, society leaders often have more significant role in shaping opinion of crowd than normal individuals. At the same time, the leaders might be less affected by others than normal individuals. Indeed, individuals have different levels of social power [177]. Jalili studied the role of the social power on opinion formation process, where agents with higher centrality have higher social power to others [22]. He showed that introducing social power in the opinion formation process often facilitates the consensus. Afshar and Asadpour showed that a small number of informed agents with a strategic position in the network can successfully shape the opinion of the whole society [178]. Askari Sichani and Jalili showed that by connecting the informed agents to the agents with rather small degrees but with high degree neighbours, one can maximize the influence [68]. Although general centrality measures have been successfully used in facilitating opinion formation in complex networks, constructing centrality measures specific to the properties of opinion formation process is still a hot topic in this area.

5.3.4 *Preventing cascading failures* Real-world networked structures might be subject to failures in their components (nodes and edges) [179, 180]. Component failure can be generally in two forms: random failure often denoted as *error* or intentional (targeted) failure often denoted by *attack*. Although many networks show surprising resiliency against errors, they are fragile against intentional attacks. In some cases, a component failure can have more drastic consequences, where it might trigger a cascade of failures in other components [23, 181, 182]. In real networks, when a component fails, the load (or traffic) passing through that component is often redistributed to other components. This might cause some components to go beyond their capacity, and consequently fail. This process might continue until a large fraction of nodes fails.

Studying cascaded failures and understanding their behaviour has significant role in safeguarding critical networked infrastructures. For example, it has been shown that component failures may often trigger a cascade of failures in water distribution networks [183]. Power networks are also prone to cascaded failure, and such a process has been responsible for some of large-scale blackouts [25, 184, 185]. Cascaded failures have been heavily studied for power grids, due to its significance applications in the proper functioning of the grid. Power grids are spatial network and show particular behaviour against cascaded failures [63]. Yan *et al.* proposed an extended betweenness centrality combining network structure and electrical characteristics of power network and studied the vulnerability of the networks against different attack strategies [186]. Ghanbari *et al.* considered a measure based on the maximum power flow and investigated random and targeted failures in a number of benchmark power networks [187].

The location of the initial failure has significant role in the cascade outcome, that is the number of failed components. Ghanbari and Jalili studied how nodal centrality indices are correlated with their cascade depth, that is the number of failed nodes as a result of an initial failure in a node [188]. Surprisingly, they found that for many model and real networks, degree is negatively correlated with the cascade depth, indicating that initial failure in high-degree nodes often results in non-significant cascading effect. They also identified centrality measures with positive correlation with the cascade depth. Mirzasoleiman *et al.* proposed a strategy based on node betweenness centrality in order to maximize resiliency of weighted networks against cascaded failures [24].

5.3.5 *Discovering institutional financial risks* Financial institutes often have intense interconnections, borrowing/lending money from one another. They form a network where they influence each other's functionality, and one might minimize systematic financial risks by properly analysing topology of such networks [189]. Similar to cascaded failures, a risk in a certain financial institute might spread to others, which can lead to drastic financial outcomes. Gai and Kapadia studied the contagion in financial networks, and showed the 'robust yet fragile' phenomena in these networks [190], similar to previous findings on other types of networks [180]. While the probability of contagion may be low in financial networks, the consequence can be drastic and widespread when problems happen. If a financial institution has problems (e.g. cannot collect the repayments from its customers) and defaults, its creditor may face a loss. If the losses that a financial institution collectively faces are more than its equity, that institution will also default. This process may result in many institutions to default, which may lead to dramatic financial situations [191]. A framework has been developed to find out whether an initially defaulted financial institution causes a cascade that extends to a large fraction of the network [192]. The cascade condition computed in this way can be considered as a systematic risk inherent in the financial network structure.

Structure of the financial network has significant role on how risk develop in its participating financial institutes [193, 194]. Amini *et al.* obtained asymptotic results for the magnitude of contagion in financial networks, and introduced a resiliency criterion to quantify how contagion intensify small shocks [195]. By analysing the cross-border banking flows for 184 countries and global banking network, Minoiu and Reyes found that network density in 2007 was not significantly different from earlier peaks [196]. This indicates that factors other than connectedness of financial networks, such as the strategic location of the initial shock, have major contributions to the severity of the financial crisis [196]. Demange proposed the aggregate debt repayment model, which is obtained based on each individual institute's characteristics and its connections with others [197]. This model can be used to measure the spillover effect. Centrality measures have significant applications in studying contagion in financial networks. Some scholars have also proposed specific centrality measures for this purpose. Battiston *et al.* proposed DebtRank, to measure the systematic risk of nodes in financial networks [198]. SinkRank was proposed to quantify the disruption caused by the failure of a financial institution in the payment system and identify those affected more [199].

5.3.6 *Preventing abnormal behaviour and terroristic relations* Although social networks have various applications and daily life might be disrupted without them, they can propagate anomalies and illegal activities. Detection of anomalies in social networks is often used to identify spammers, sexual predators, malicious users, terroristic activities, money laundering and online fraudsters [200–202]. Abnormal behaviour is defined as something that is significantly different from networks' normal behaviour. For example, spurious nodes often have significantly different behaviour (and features) than normal nodes. Spurious links are those that should not exist if the network naturally evolves. It is not always an easy task to find criminals in the network as the individuals intending to do a criminal activity over the networks are smart, and often find new ways to stay unrecognized. Although the leaders of illegal activities have strategic position in the network, in many cases they do not have many connections, but only a few critical ones. Money laundering activates often involve a number of collaborative individuals, and the evidence may only be discovered when only collective behaviour of these individuals is considered [203]. In order to have effective detection of money laundering activates, one can efficiently use network analysis and supervised learning [203].

Although there has been many works on predicting forthcoming links in social networks, there has been little efforts to predict abnormal links, which is partly due to lack of reliable datasets. Magalingam *et al.* proposed a method based on shortest paths to find a small subset of nodes suspected for criminal activities [204]. Yasami and Safaei proposed a multi-step method for anomaly detection in social networks [205]. In their proposed method, the first step is to identify the normal behaviour of the network, and the abnormalities are identified in the next step. Das and Sinha used nodal centrality measures to detect malicious users in social networks [206]. Kaur *et al.* used centrality measures to identify criminal users and showed that by combining a number of centrality measures one can better identify such users as compared to the case of using the centrality measures independently [207].

6. Conclusions and outlook

Information cascade is an important topic in network science with significant applications in various fields such as viral marketing, disease spread, rumour spreading in social communities, cascaded failures in infrastructure networks and contagion in financial networks. In this paper, we provided a brief review on information cascaded models. We provided a detailed explanation of independent cascade model, a frequently used model to theoretically study information cascade in complex networked systems. In the independent cascade model, the cascade starts from an initial set of nodes, spreads through the network and stops when no agent becomes activated anymore. An important issue in the studies related to information cascade is how to choose the initially activated nodes. This is also known as *influence maximization* problem in the related literature. In this work we studied the role of centrality measures in the influence maximization problem. For some sample synthetic and real networks, a number of centrality measures were calculated and correlated with their spreading influence. The spreading influence of a node can be obtained by finding the portion of the network that is activated when that node is initially activated. We identified centrality measures such as degree and betweenness, showing positive correlation with the spreading influence and measures such as eccentricity and information index often negatively correlated with that.

Although great progress has been achieved in recent years on how dynamical processes, such as information cascade, evolve over complex networks, there are still many open questions in this field. Viral Marketing for example, is how to efficiently use social networking to maximize the revenue, and the heart of viral marketing is how to identify a 'good' initial set of influential users [208]. Although a number of algorithms have been proposed to find the most influential nodes, and thus maximize the influence, development of practical approaches is still in its infancy. Therefore, future research in this field would be put the algorithms into practical applications and fine tune them on real data.

Vital nodes are often detected by statistical tests through computing various centrality measures in networks. This requires mapping the whole network, which is difficult (if not impossible) in many real networks such as online social communities. There have been some efforts to estimate centrality of nodes in the case when only partial information is available on the network connectivity. For example, Christakis and Fowler proposed a method based on monitoring the friends of randomly selected individuals, which does not require ascertainment of the whole network structure [209]. They verified their proposed model on early detection of flu outbreak within students of a university. There are also some works in the literature to construct network statistical parameters, including centrality measures, from partial network measurement, for example when fraction of nodes and/or edges are missing [210, 211]

Recently, the information propagation phenomenon has also been studied in multilayer networks, realistic network models for some networked systems such as online social networks, transportation and biological networks [212–214]. Some real systems should be modelled as multilayer networks, where the nodes develop connections in different layers [215, 216]. Examples of such networks include online social networks where the individuals might have friendship connections in different social networking platforms [217], or transportation networks where the same cities might be connected through air, rail and road networks [218]. Multilayer networks show properties that are different from single-layers networks in some aspects [219, 220] Targeted immunization strategy in a layer is not often effective to stop epidemics in other layers [221, 222]. In other words, when the central nodes are vaccinated in only one of the layers, although it is effective in immunizing that layer, it does not have that much influence on the other layers. Therefore, more effective immunization strategies are required for networks with multiple layers.

Many real networks show surprising recovery after failure of their components. For example, economic systems are often highly resilient to small shocks, and quickly recover their normal functioning state. The brain is another example where it recovers after a seizure. This indicates that real networks have intrinsic mechanisms to stop the spread and damp sudden changes to the network. Majdandzic *et al.* proposed a model to study spontaneous recovery of complex networks [26], which was then extended to interdependent networks [223]. This however requires further studies.

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