



# Chatter identification in end milling process based on EEMD and nonlinear dimensionless indicators



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## ABSTRACT

Vibration analysis is widely used to reveal the fundamental cutting mechanics in machining condition monitoring. In this work, vibration signals generated in different chatter conditions as well as stable cutting are studied to understand chatter characteristics. Considering the nonlinear and non-stationary properties of chatter vibration in milling process, a self-adaptive analysis method named ensemble empirical mode decomposition (EEMD) is adopted to analyze vibration signals and two nonlinear indices are extracted as chatter indicators. Firstly, the vibration signal is preprocessed with a comb filter to eliminate the interference of rotation frequency, tooth passing frequency and their harmonics. Secondly, EEMD is applied to decompose the filtered signal into a set of intrinsic mode functions (IMFs). Sensitive IMFs containing rich chatter information are selected. With the development of chatter, an accumulation phenomenon appears in the spectrum of sensitive IMFs and chatter frequencies are modulated by the rotation frequency and tooth passing frequency. Finally, two nonlinear dimensionless indices within the range of [0, 1], i.e.,  $C_0$  complexity and power spectral entropy, are extracted from the sensitive IMFs in both time domain and frequency domain. The proposed method is verified with well-designed cutting tests. It is found that, the stochastic noise dominates in the sensitive IMFs of stable cutting and both the  $C_0$  complexity value and power spectral entropy are the largest; with the increase of chatter severity level, the periodic chatter components dominate gradually and the proportion of stochastic noise decreases, and thus these two indicators decrease.

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## 1. Introduction

Chatter is a self-excited vibration accompanied by unstable, chaotic behavior and largely abnormal fluctuations of cutting tool. It arises in machining processes due to specific combinations of cutting parameters such as depth of cut and cutting speeds, imposing negative effects on the productivity because of poor surface quality, dimensional accuracy error, excessive noise, early tool wear, waste of materials and waste of energy [1]. To avoid chatter, conservative cutting parameters are usually selected, which of course becomes one of the main limitations of high productivity. Therefore, timely detection of the chatter onset is essential and crucial in machining process.

Early chatter detection allows operators to interfere in the machining process and avoid chatter damage. Extensive and significant research has been conducted on chatter detection in recent years [1–4]. Various sensor signals have been used to monitor chatter, such as acceleration signal [5–7], cutting force [8–10],

sound [11,12], motor current [13,14], torque signal [15]. It is worth noticing that Kuljanic et al. [16] designed a multi-sensor system composed of three or four sensors to detect chatter, achieving high levels of accuracy and of robustness against malfunctions. Signals acquired by these sensors are subjected to machining processing with the aim to generate certain features correlated (at least potentially) with machining conditions [4]. Different signal processing methodologies have been employed in time domain [6,17–20], frequency domain [9,21–23], and time-frequency domain [9,10,12,24–31], to extract relevant and sensitive features about chatter.

Chatter is a complex nonlinear and non-stationary phenomenon in machining process and the loss of stability in milling process results in different types of bifurcations, including sub-critical Hopf and period doubling bifurcations, as well as limit cycles, quasi-periodic and chaotic behavior [32,33]. Therefore, traditional spectral methods based on Fourier transform is hampered due to the nonlinear and non-stationary nature of chatter signals and noisy spectrum. In addition, both short-time Fourier transform and S-transform are based on Fourier transform theory, which are more suitable for quasi-stationary signal. As for wavelet transform, it is difficult to determine the suitable wavelet base

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functions and decomposition levels, which have a significant influence on the analysis results.

**Empirical mode decomposition (EMD)** is a self-adaptive analysis method for nonlinear and non-stationary signals. The EMD method is based on the local characteristic time scales of a signal and may decompose a complicated signal function into a set of complete and almost orthogonal components named **intrinsic mode functions (IMFs)** [34]. It has been widely used in fault diagnosis of rotating machinery in recent years, for example, rolling bearing fault diagnosis [35,36] and gear fault diagnosis [37]. Recently, EMD is being used in the area of machining process monitoring. Peng [38] adopted EMD based on time–frequency analysis for the detection of tool breakage in milling process. Raja et al. [39] used the HHT-based emitted sound signal analysis to monitor the tool flank wear. Li et al. [40] used EMD to extract the feature of chatter symptom in boring process. Liu et al. [13] decomposed the motor current signal into IMFs and extracted energy index and kurtosis index based on those IMFs for chatter detection. Cao et al. [41] combined the benefits of wavelet packet transform (WPT) and EMD to extract features according to the Hilbert–Huang spectrum for chatter detection. However, one of the major drawbacks of EMD is the mode mixing problem, which is defined as either a single IMF consisting of components of widely disparate scales, or a component of a similar scale residing in different IMFs [42]. To mitigate the problem of mode mixing in EMD, an improved method called **ensemble empirical mode decomposition (EEMD)** was presented recently [43]. EEMD is a noise-assisted data analysis method and by adding finite white noise to the analyzed signal, the EEMD method can eliminate the mode mixing problem automatically. Nevertheless, the application of EEMD for chatter detection/identification is rarely reported based on the authors' literature search.

Vibration analysis is widely used in machining condition monitoring to reveal the fundamental cutting mechanics. Due to the nonlinearity and non-stationary characteristic of milling process, EEMD should be a suitable tool to analyze the generated vibration signals during cutting. In this paper, an alternative method based on EEMD and nonlinear dimensionless indicators is proposed for chatter identification. With EEMD, the vibration signal is decomposed into a set of IMFs, and then IMFs containing rich chatter information are selected for feature extraction. Two nonlinear dimensionless indices, i.e.,  $C_0$  complexity and power spectral entropy, are calculated in both time domain and frequency domain which can be used as indicators to identify the existence of chatter in milling process.

The rest of paper is organized as follows. Brief introduction of empirical mode decomposition and ensemble empirical mode decomposition are given in Section 2. Section 3 introduces two feature extraction methods for chatter identification. Section 4 presents the chatter identification methods in this paper. In Section 5, the experimental setup is described. In Section 6, the results and discussions of the proposed chatter identification method are given. Finally, the conclusive remarks are laid out in Section 7.

## 2. Brief introduction of empirical mode decomposition and ensemble empirical mode decomposition

The EMD method is a novel and adaptive decomposition method proposed by Huang et al. [34]. Compared with wavelet transform, EMD is more suitable to analyze nonlinear and non-stationary data because its basis functions are determined by the data itself, while wavelet transform need to select optimal wavelet basis. By EMD, a complicated signal is decomposed into a set of complete and almost orthogonal components IMFs, and a residue. However, one of major shortcomings of EMD is that it is prone to

mode mixing, which may render the physical meaning of IMFs unclear. To alleviate the problem of mode mixing inherent in the use of EMD, an effective EEMD analysis was proposed by Wu and Huang [43]. The EEMD is a noise-assisted EMD procedure actually, utilizing the full advantage of the Gaussian white noise's statistical characteristic of uniform distribution to improve the distribution of extreme points in original signal, fairly well solving the mode mixing problem [44]. In this method, the mean of an ensemble of trials is treated as the true and more meaningful IMF components.

The following is the brief description of the EEMD algorithm

- (1) Initialize the number of ensemble  $M$  and the amplitude of the added white noise, with  $m=1$ .
- (2) Perform the  $m$ th trial on the signal added white noise.
  - (a) Add a white noise series with the given amplitude to the targeted signal to generate a new signal

$$x_m(t) = x(t) + n_m(t) \quad (1)$$

where  $n_m(t)$  represents the  $m$ th added white noise series,  $x(t)$  represents the investigated signal and  $x_m(t)$  indicates the noise-added signal used in EEMD of the  $m$ th trial.

- (b) Decompose the noise-added signal  $x_m(t)$  into a series of IMFs  $c_{i,m}(i = 1, 2, \dots, I)$  using the EMD method

$$x_m(t) = \sum_{i=1}^I c_{i,m} + r_n \quad (2)$$

where  $c_{i,m}$  denotes the  $i$ th IMF of the  $m$ th trial, and  $I$  is the number of IMFs of each trial.

- (c) If  $m < M$  then go to step (a) with  $m=m+1$ . Steps (a) and (b) are repeated with different white noise series each time until  $m = M$ .
- (3) Calculate the ensemble mean  $c_i$  of the  $M$  trials for corresponding each IMF in decompositions

$$c_i = \frac{1}{M} \sum_{m=1}^M c_{i,m}, \dots i = 1, 2, \dots, I, \quad m = 1, 2, \dots, M \quad (3)$$

- (d) Each mean  $c_i(i = 1, 2, \dots, I)$  of the  $I$  IMFs are the final IMFs, representing the simple oscillatory modes embedded in the investigated signal.

## 3. Feature extraction

After being processed with EEMD, feature extraction method is the most critical step to identify the milling states. The chatter indicator should show the highest correlation to the change of milling condition. Two nonlinear dimensionless features are introduced here, combining the analysis in time domain and frequency domain, respectively.

### 3.1. $C_0$ complexity

$C_0$  complexity is a new nonlinear analysis method proposed by Chen et al. [45]. In 2005, Shen et al. [46] proposed a modified version of this measure and proved some important properties rigorously. In 2009, Cai and Sun [47] proved the convergence of  $C_0$  complexity and verify that  $C_0$  complexity can converge even with short series.  $C_0$  complexity has been successfully applied in the bio-medical field as an effective chaotic dynamic index to measure the complexity of bio-medical signals [48–51]. Contrasted with traditional nonlinear analysis methods such as Lyapunov exponents, Lempel–Ziv complexity [52] or fractal dimensions,  **$C_0$  complexity has superiority in achieving a robust estimation even**

with short data and does not need coarse graining preprocessing [47].

Based on the assumption that complex time series can be divided into two parts (i.e., regular component and random component),  $C_0$  complexity is calculated as the energy ratio of random parts to the original signal by eliminating the regular components in signal and saving the irregular components. The larger the  $C_0$  complexity value is, the more complex the original signal is, namely the corresponding time series are closer to random series.

The algorithm of  $C_0$  complexity is shown as follows.

- (1) Suppose that  $\{y(k), k = 1, 2, \dots, N\}$  is a time series with a length of  $N$ , and the corresponding Fourier transformation is

$$F_N(j) = \frac{1}{N} \sum_{k=1}^N y(k) W_N^{-kj}, \quad j = 1, 2, \dots, N \quad (4)$$

where  $W_N = e^{2\pi i/N}$  and  $i = \sqrt{-1}$  denotes by the unit imaginary.

- (2) Assume that the mean square value of  $\{F_N(j), j = 1, 2, \dots, N\}$  is

$$G_N = \frac{1}{N} \sum_{j=1}^N |F_N(j)|^2 \quad (5)$$

Next, we introduce a parameter  $r$  so that  $C_0$  complexity can better reflect the dynamic characteristics of time series [53]. Keep spectrum components that are larger than  $rG_N$  be unchangeable, while replace all the other components with zero, i.e.

$$\tilde{F}_N(j) = \begin{cases} F_N(j) & \text{if } |F_N(j)|^2 > rG_N \\ 0 & \text{if } |F_N(j)|^2 \leq rG_N \end{cases} \quad (6)$$

where  $r(r > 1)$  is a given positive constant whose suitable range of values are 5–10 in practical application.

- (3) Calculate the inverse Fourier transform of  $\{\tilde{F}_N(j), j = 1, 2, \dots, N\}$  and obtain  $\tilde{y}(k) = \sum_{j=1}^N \tilde{F}_N(j) W_N^{kj}$ ,  $k = 1, 2, \dots, N$

Define  $C_0$  complexity as

$$C_0(r) = \frac{\sum_{k=1}^n |y(k) - \tilde{y}(k)|^2}{\sum_{k=1}^n |y(k)|^2} \quad (7)$$

According to the properties proved by Shen et al. [46],  $C_0$  complexity is a real number between 0 and 1, and it is 0 for a constant time series and the case  $C_0(r) \rightarrow 0$  corresponds to a periodic time series. It converges to 1 with a probability of 1 for a stochastic time series, when satisfying a certain condition. Hence,  $C_0$  complexity can be regarded as an indicator of random degree of the time series in some sense. We can also utilize the dimensionless  $C_0$  complexity as a quantitative index of complexity to distinguish and measure the regular and irregular nature of vibration signal in milling process. In addition,  $C_0$  complexity is computed very fast owing to the fact that the main steps involved in the algorithm is based on the Fast Fourier Transform (FFT).

### 3.2. Power spectral entropy

Shannon entropy is a quantitative uncertainty index, whose original meaning is related to the uncertainty level of information. The entropy is a function of the probability distribution function. The concept of power spectral entropy is the extension of Shannon entropy in frequency domain, which is linked to the distribution of frequency components. For a given signal, the power spectral entropy is obtained by following steps [54]:

- (1) Apply FFT to the sampled signal  $\{x(k), k = 1, 2, \dots, N\}$  and obtain its power spectrum

$$s(f) = \frac{1}{2\pi N} |X(w)|^2 \quad (8)$$

where  $N$  is the length of the time series,  $X(w)$  is the Fourier transformation of  $\{x(k), k = 1, 2, \dots, N\}$

- (2) The probability density function (PDF) for the spectrum can thus be estimated by normalization over all frequency components

$$P_i = s(f_i) / \sum_{k=1}^N s(f_k); \quad i = 1, 2, \dots, N \quad (9)$$

where  $s(f_i)$  is the spectral energy for the frequency component  $f_i$ ,  $P_i$  is the corresponding probability density, and  $N$  is the total number of frequency components in FFT.

- (3) The power spectral entropy is defined as

$$H = - \sum_{k=1}^N P_i \ln P_i \quad (10)$$

For the sake of convenient comparison in different working conditions, the result is usually normalized by the factor  $\ln N$ , i.e.,

$$E = \frac{H}{\ln N} = \frac{- \sum_{k=1}^N P_i \ln P_i}{\ln N} \quad (11)$$

Then the power spectral entropy is a non-dimensional indicator in the range of  $[0, 1]$ , where 1 corresponds to the spectrum whose distribution of frequency component is comparatively even and uncertain and 0 corresponds to the distribution uncertainty is the least.

## 4. The proposed chatter identification methodology

The vibration signal in milling process is composed of three parts: the periodic component due to the rotation of the cutter and intermittent milling of the tool, the chatter vibration component due to regenerative effect, and the stochastic perturbation component due to system noise, inhomogeneous material, etc. The key issue of chatter identification is to find out the occurrence of chatter component from the measured signal.

The flow chart of the proposed chatter identification scheme is illustrated in Fig. 1. Firstly, vibration signals generated during milling process are measured, and then the sampled signals pass through a comb filter to remove the rotation frequency, tooth passing frequency and their harmonics or multiples lying within the system's frequency range, thus highlighting the chatter components which are insensitive to the change of cutting parameters. Next, the EEMD decomposition method is applied to these filtered signals and sensitive IMFs containing rich chatter information are selected and integrated to form a new signal. Finally, two non-linear dimensionless indicators of the formed signal, i.e., the  $C_0$  complexity value and power spectral entropy, are calculated to describe the characteristics of the signal in both time domain and frequency domain. Based on these two chatter indicators, the milling state can be identified.

## 5. Experimental setup

Milling tests were performed on a CNC milling machine to verify the proposed methodology (Fig. 2). The workpiece material was a block of 7050 aluminum clamped on the worktable. The cutter used was a carbide end mill cutter with two flutes. The

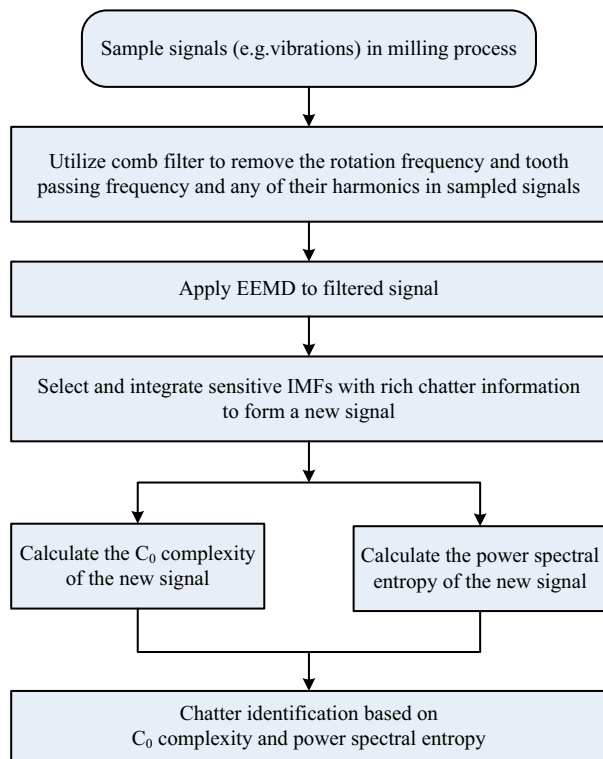


Fig. 1. Flow chart of the proposed methodology.

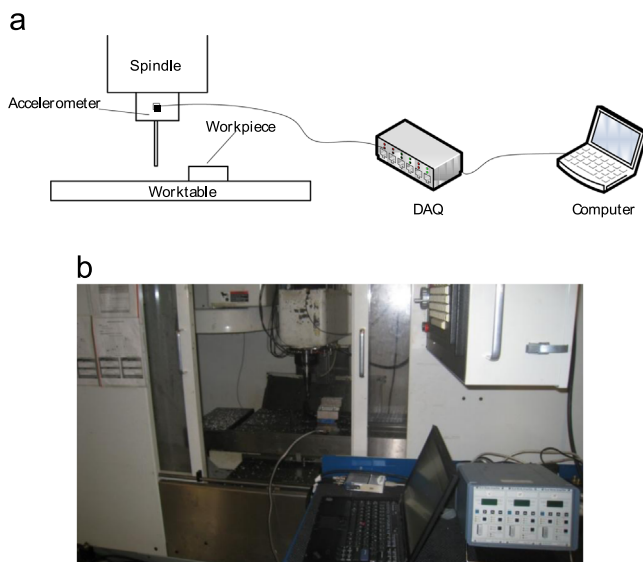


Fig. 2. Experimental setup: (a) schematic diagram, and (b) photo of the setup (Courtesy MAL-UBC).

measurements were conducted with Cutpro-MalDAQ<sup>®</sup>. Accelerometers were mounted on the spindle housing to measure the vibration signals during milling process. Vibration signals were sampled with a data acquisition card and then transmitted to a PC which was used for data storage and signal processing. The sampling frequency was set as 6400 Hz.

Since the occurrence of chatter is directly associated with the depth of cut, each test was performed at constant spindle speed (8500 r/min) and feed rate (1500 mm/min), with depth of cut at 1.0 mm, 2.0 mm, 3.0 mm and 5.0 mm, respectively. All the tests were slotting without cutting fluids. Four typical cases (i.e., stable, chatter level I, chatter level II and chatter level III) are included in

Table 1

The four cases with different chatter levels.

Cutting conditions	Depth of cut (mm)	Spindle speed (r/min)	Feedrate (mm/min)
Stable cutting	1	8500	1500
Chatter level I	2	8500	1500
Chatter level II	3	8500	1500
Chatter level III	5	8500	1500

this study, as listed in Table 1.

## 6. Results and discussions

The analysis of vibration signals measured in the milling process is carried out in this section.

In order to alleviate the interference of periodic components due to the rotation of the cutter and intermittent milling of the tool, a comb filter is used. Then, the EEMD method is applied to the filtered signal and sensitive IMFs are selected according to the relative energy ratio of each IMF. The two chatter indicators are calculated based on the sensitive IMFs.

### 6.1. Data analysis

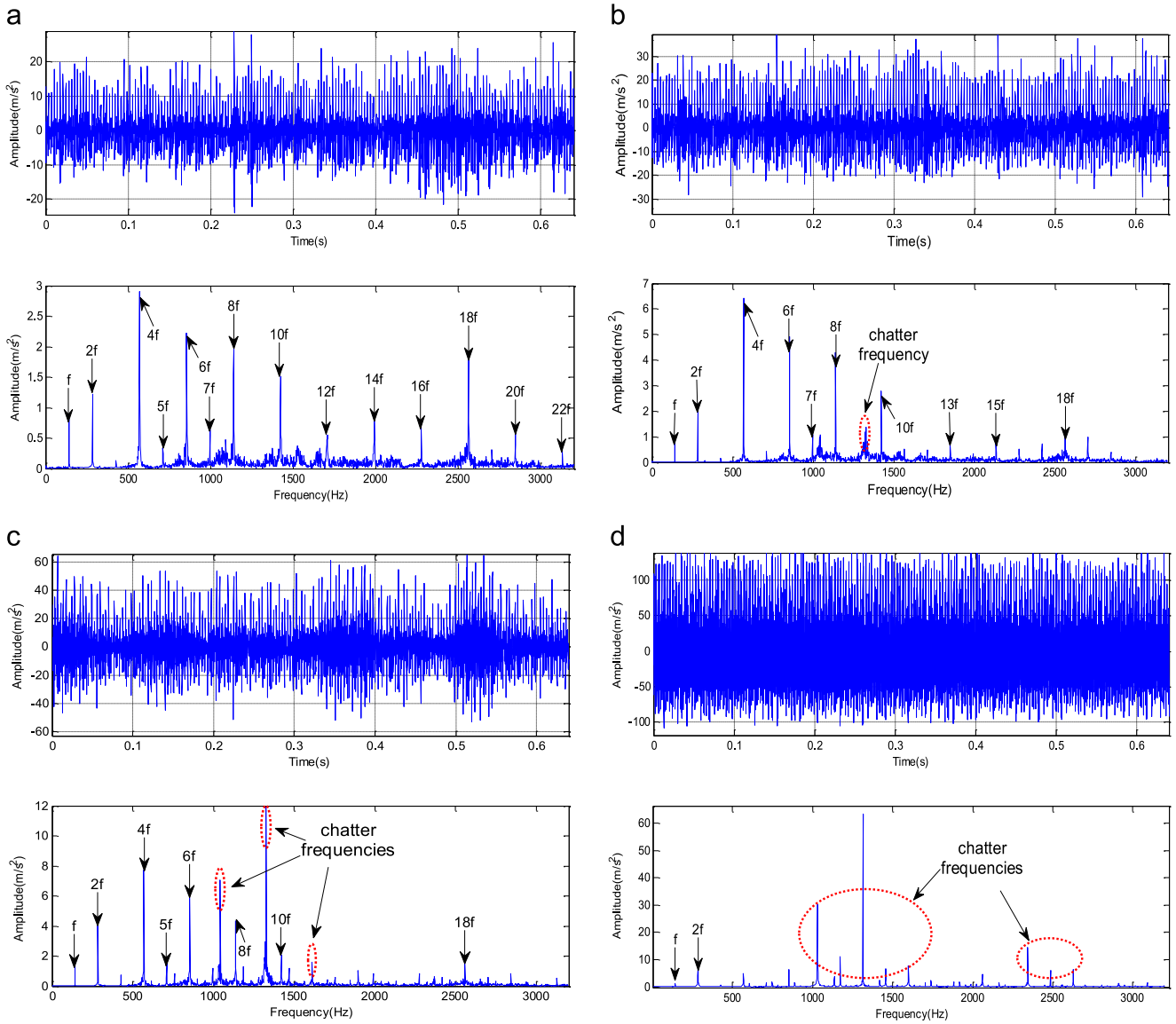
The analyses of vibration signals in time domain and frequency domain are shown in Fig. 3. In Fig. 3(a)–(d),  $f$  denotes the rotation frequency and  $2f$  denotes the tooth passing frequency.

In the stable condition of milling (Fig. 3a), the amplitude of vibration signal is relative small and the spectrum is dominated by rotation frequency and tooth passing frequency and their harmonics. When comb filter procedure is conducted, the remaining components are mainly stochastic noise, corresponding to homogenous spectrum components and even energy distribution. In the condition chatter level I (Fig. 3b), there is no obvious change of amplitude in time domain, while chatter symptom is evident in frequency domain due to the additional components corresponding to chatter frequencies that appeared in frequency spectrum, which demonstrated the necessity and advantage of frequency domain analysis. In this stage, however, no obvious chatter mark is visible on the workpiece yet. In the condition of chatter level II (Fig. 3c), the amplitude of vibration grows up with the development of chatter. The amplitude of chatter frequencies have increased dramatically, dominating the whole spectrum. In the condition of chatter level III (Fig. 3d), the amplitude of fluctuations is rather large. Moreover, almost all peaks of the spectrum are located at the chatter frequencies, indicating the major components in signal are chatter components.

In order to alleviate the interference of periodic components due to the rotation of the cutter and intermittent milling of the tool, a comb filter is used to remove the rotation frequency, tooth passing frequency and their harmonics or multiples lying within the system's frequency range, thus highlighting the chatter components which are insensitive to the change of cutting parameters. The filtered signals are then processed with EEMD and a set of IMFs are obtained. After analysis, it is found that the first eight IMFs (i.e., c1–c8) from high frequency to low frequency contain most of characteristic information. In Fig. 4, only the results of the stable cutting case are shown. The first eight IMFs of the filtered signal are displayed in Fig. 4(a), while the corresponding Fourier spectra are displayed in Fig. 4(b).

In order to select sensitive IMFs that containing chatter information, the relative energy ratio of the first eight IMFs to total energy are calculated. The results of all the four cases are displayed





**Fig. 3.** Vibration signal in time domain and frequency domain (spindle speed 8500 r/min, feed rate 1500 mm/min, slotting). (a) stable cutting, depth of cut: 1 mm. (b) chatter level I, depth of cut: 2 mm. (c) chatter level II, depth of cut: 3 mm. (d) chatter level III, depth of cut: 5 mm.

in Fig. 5. In the stable cutting case, the energy mainly distributes in the first three IMFs, while the energy of the first IMF is the largest. With the increase of chatter severity level, the distribution of energy concentrates on the first IMF c1 more and more. Thus, the first IMF c1 contains the richest information, which is selected as the sensitive IMF for subsequent analysis.

The first IMF c1 and its Fourier spectrum of each case are displayed in Fig. 6. In the stable cutting case (Fig. 6a), the vibration is mainly caused by the stochastic perturbation due to system noise, inhomogeneous material, etc. Most frequency components lie disorderly in the range of 500–3000 Hz. In the case of chatter level I (Fig. 6b), chatter frequencies occur ( $f_{c1} = 1042$  Hz,  $f_{c2} = 1327$  Hz) and the distribution of frequency components begins to concentrate around the chatter frequencies. In the cases of chatter level II ( $f_{c1} = 1042$  Hz,  $f_{c2} = 1327$  Hz) and III ( $f_{c1} = 1030$  Hz,  $f_{c2} = 1314$  Hz) (Fig. 6c and d), the Fourier spectra become very clean, which mainly consist of chatter frequencies.

Furthermore, it is also evident that peak values of the dominant chatter frequencies grows dramatically with the increase of chatter severity level, presenting an accumulation phenomenon and eventually locating near the system natural frequencies, which is a

typical regenerative chatter. In addition, the signal in stable state (Fig. 6a) presents the characteristic of random vibration, while in chatter condition (Fig. 6b–d), the periodic of signal is strengthened gradually and chatter frequencies are modulated by the rotation frequency and tooth passing frequency in the spectra.

## 6.2. Chatter identification

From above analysis, the chatter can be identified from the spectra of the sensitive IMF c1. However, it is not convenient for engineers to recognize chatter in the workshop. In order to identify chatter automatically,  $C_0$  complexity and power spectral entropy of the sensitive IMF c1 are calculated as chatter indicators which are independent of cutting parameters. The results of the four cases are listed in Table 2. In the stable cutting case, the  $C_0$  complexity and power spectral entropy are the largest that close to 1. With the increase of chatter level, both these two dimensionless indices decrease. In the case of chatter level III, where the most serious chatter occurs, the  $C_0$  complexity gets close to 0 and the power spectral entropy also becomes very small.

From Table 2, it can be found that the results were consistent

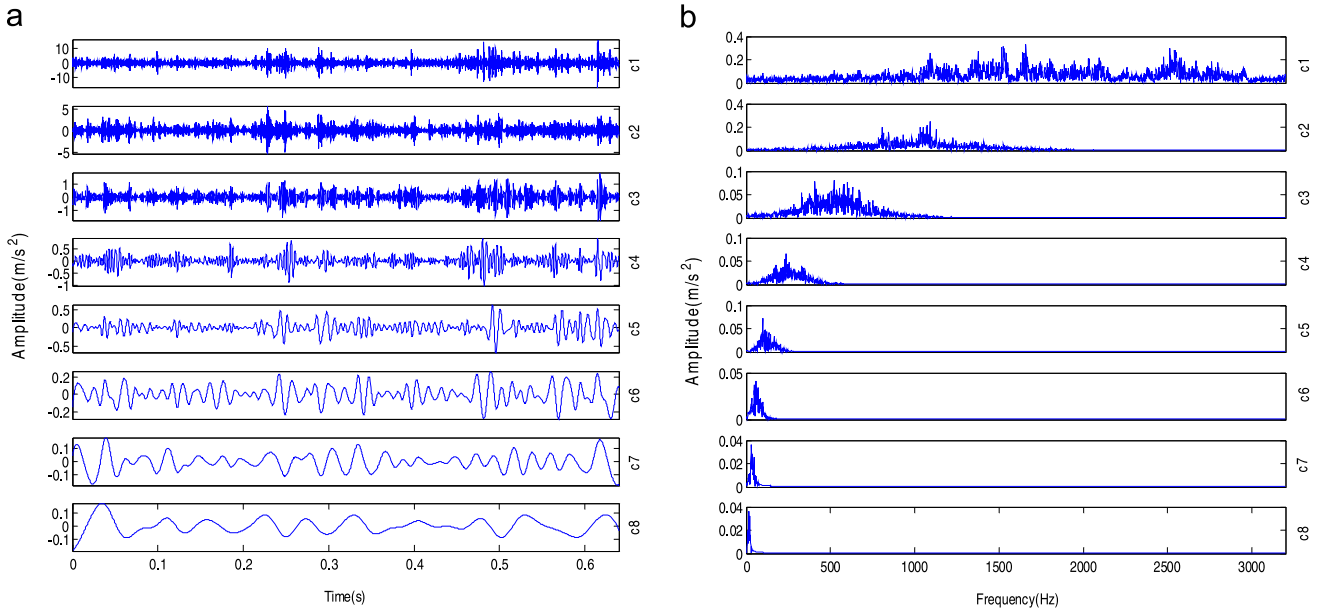


Fig. 4. The first eight IMFs of the filtered signal in time domain and frequency domain, stable cutting, depth of cut: 1 mm. (a) IMFs, and (b) Fourier spectra of IMFs.

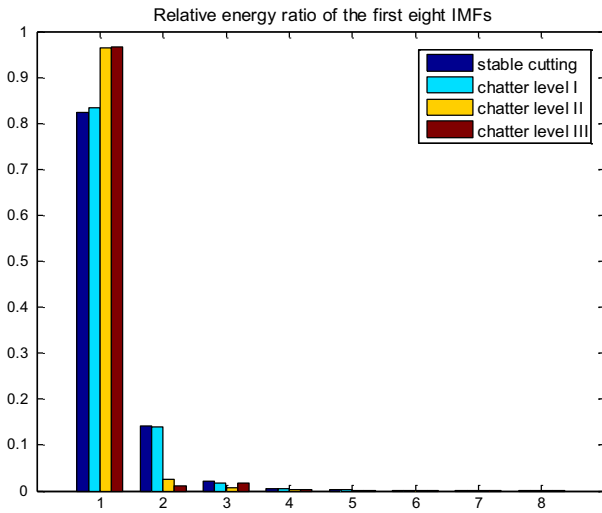


Fig. 5. Relative energy ratio of the first eight IMFs in the four cases.

well with the physical meaning of these two indices. The  $C_0$  complexity reflects the proportion of random components in signal. In the stable cutting case, when all periodic parts namely rotation frequency and tooth passing frequency and their harmonics are removed by the comb filter, leaving only stochastic noise in the signal, the  $C_0$  complexity value is sure to be the largest compared with other chatter conditions. As expected, with the development of chatter, the dominant components are gradually replaced by chatter components namely the periodic parts. In the same time, the proportion of stochastic noise declines and the  $C_0$  complexity value drops of nature. From the variation tendency of  $C_0$  complexity value, we could tell that  $C_0$  complexity is an excellent chatter indicator.

As for the power spectral entropy, it reflects the changing properties from the perspective of frequency domain. It was obvious that the frequency components of the sensitive IMF have a tendency of clustering from uniform distribution to gathering at chatter frequencies. This change of distribution characteristics could be measured by the power spectral entropy. In the stable cutting case, the frequency components are evenly distributed in

all frequency ranges, leading to the largest power spectral entropy. As the chatter severity level increases, the frequency components gradually gather at the location of chatter frequencies, which leads to the decrease of the power spectral entropy. This suggests that the power spectral entropy is a good chatter indicator as well. Combining the  $C_0$  complexity and power spectral entropy together, we could find the symptom of chatter occurrence in aspects of both time domain and frequency domain, which is more reliable. Furthermore, it can be seen that the chatter severity could be also estimated with the  $C_0$  complexity and power spectral entropy.

From above analysis, it can be seen that the proposed method can be used for chatter identification, and the two chatter indicators have the potential for online chatter detection in the milling process. However, due to the limitation of experiments, the thresholds of the two chatter indicators (i.e.,  $C_0$  complexity and power spectral entropy) are not determined in this work. In the future, more cutting tests will be designed, and the chatter thresholds of the two indicators will be determined with abundant data measured in the milling process.

## 7. Conclusion

In this paper, a new chatter identification method for the end milling process is presented. The vibration signals are decomposed with the EEMD method in a self-adaptive way, which avoids the interfering of operators. The sensitive IMFs that contain abundant chatter information can be selected according to the energy distribution of all IMFs. With the increase of chatter severity level, the frequency components present an accumulation phenomenon and eventually locating at near the system natural frequencies. Besides, the periodic of signal is strengthened gradually and chatter frequencies are modulated by the rotation frequency and tooth passing frequency in the spectra. Two nonlinear indices, i.e.,  $C_0$  complexity and power spectral entropy, are derived from the sensitive IMFs as chatter indicators. In the stable cutting, the stochastic noise dominates in the sensitive IMFs and the frequency components are evenly distributed in all frequency ranges, and thus both  $C_0$  complexity value and power spectral entropy are the largest. With the increase of chatter severity level, the dominant components are gradually replaced by the periodic chatter

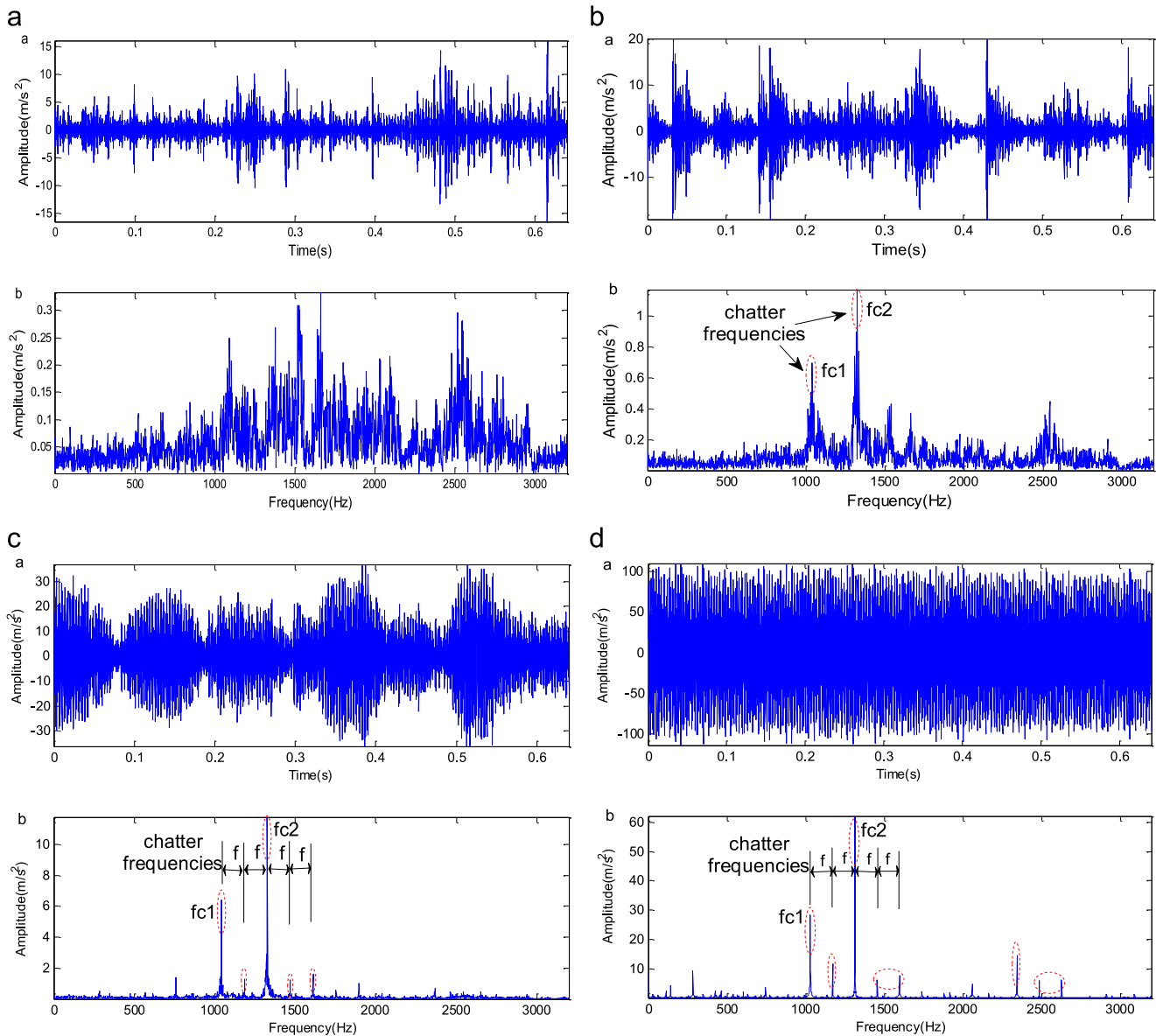


Fig. 6. The IMF c1 and its Fourier spectrum. (a) stable cutting, (b) chatter level I, (c) chatter level II, and (d) chatter level III.

**Table 2**  
 $C_0$  complexity and power spectral entropy of the sensitive IMFs in the four cases.

Cutting conditions	$C_0$ complexity	Power spectral entropy
Stable cutting	0.7157	0.8853
Chatter level I	0.5894	0.8527
Chatter level II	0.1795	0.4471
Chatter level III	0.0685	0.2982

components and the frequency components gradually gather at the location of chatter frequencies, leading to the decrease of  $C_0$  complexity value and power spectral entropy. These two nonlinear dimensionless indicators can reflect the chatter condition in both time and frequency domain, which provides an alternative solution for milling chatter identification.

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