Advanced Environmental System Analysis – Mod. 1

Academic year 2022-2023

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Exercise resolution – ex. 11 Lake Tufts

Question

A municipality on the shores of Lake Tufts wants to guarantee a certain water quality in the water body and that it has selected as indicator the average peak phytoplankton concentration.
Since the nutrient load into the lake is mainly due to inorganic phosphorous, the problem is to determine which is the maximum concentration of inorganic phosphorous in the inflow (assumed constant through the year and equal to 2 m^3/s) that can guarantees a peak total phytoplankton concentration (measured in terms of chlorophyll) below **12 µg/L**. It is required to solve the problem developing a surrogate model that links the phosphorous load to the peak phytoplankton value, and then check the solution we got on the original *Lake2K* implementation. Finally, it is necessary to check if the solution would be different if the indicator was the concentration in the epilimnion instead of the lake average.

Introduction

The *Excel* file *Lake Tufts_64bit.xls* has been provided, it contains the *Lake2K* implementation with the characteristics of Lake Tufts, and it has been used for further processing. *Lake2K* is a physically based model used for water quality simulations in lakes susceptible to stratification. The latter's difficulty in re-oxygenating the water at certain times of the year and the large release of nutrients could lead to a strong increase in phytoplankton concentration and, in the worst cases, to the eutrophication of the lake. For this reason, it is important to monitor water quality, to study the relationship between nutrients and phytoplankton growth, and consequently, to control nutrient discharges into the lake.

Resolution

Firstly, the **Water Quality Index** (**WQI**) must be determined to set the resolution. As suggested by the delivery, we considered two cases where, respectively, the **WQI** is:

- **a.** the maximum average value of phytoplankton concentration, in one year, among the three layers of the lake (epilimnion, metalimnion, hypolimnion);
- **b.** the maximum value of phytoplankton concentration, in one year, only in the epilimnion.

Having two different **WQIs**, two different surrogate models must be devised. A surrogate model is a simplified model that can reproduce the relationship between two desired variables, for a certain range of values, but without reproducing the physical phenomena. In this case, the relationships between the concentration of inorganic phosphorus in the inflow and the **WQIs** must be found. In addition, the surrogate model is used to make as many simulations as possible without always using the physically-based model *Lake2K*, which requires much more computational effort.

Subsequently, enough values of inorganic phosphorus concentrations and the corresponding peak phytoplankton concentrations were acquired, so that robust models could be developed. For this reason, several runs of Lake2K were made by gradually increasing the value of the inorganic phosphorus concentration as input. The other variables (e.g., nutrients in input such as nitrogen and silicon) on the other hand were kept unchanged, and also the default values were used for the parameters (e.g., solar radiation and temperature). The only further change was to set the inflow rate constant at 2 m^3/s, as specified by the text. *Figures 1* and *2* show the data obtained from the simulations with the two different **WQIs**.

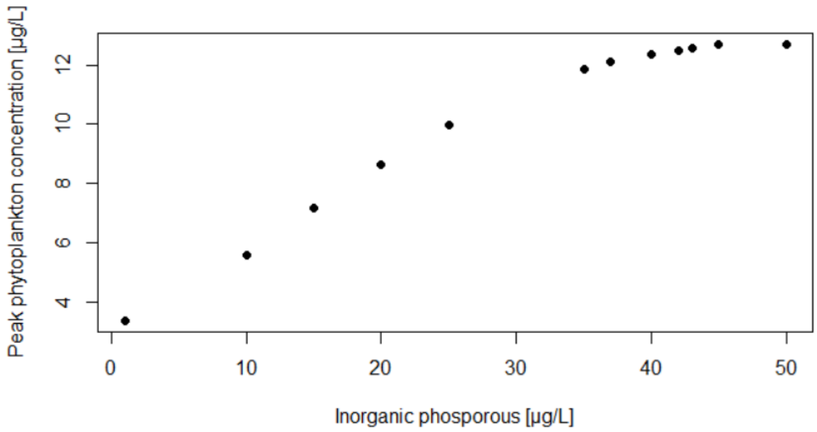

Figure 1- Values of inorganic phosphorus concentrations and the corresponding peak phytoplankton concentrations for case **a.**

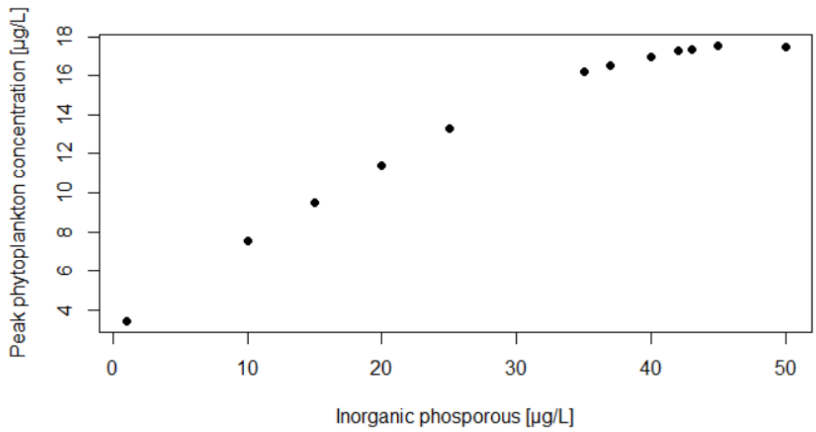

Figure 2- Values of inorganic phosphorus concentrations and the corresponding peak phytoplankton concentrations for case **b***.*

Analysis of limiting factors

As can be seen from *Figures 1* and *2*, as the inorganic phosphorus input concentration continues to increase, the phytoplankton concentration reaches a plateau and then stabilises. This phenomenon occurs because limiting factors may be present in the growth of the phytoplankton. The main factors that promote his growth, apart from light and temperature, are phosphorus, nitrogen and silicon. Not all these nutrients, however, are needed in the same quantities and depending on the availability of each one, they may limit or promote growth. To verify the actual presence of limiting factors, it was decided to conduct an analysis of the input nutrients, taking the data from case ***a*** as a reference. To carry out the input analysis, the following steps were performed[1]:

1. apply the Michaelis-Menten formula **(1)** to establish which nutrient is the limiting factor, i.e., the one with the lowest nutrient attenuation factor ϕ_{Np};
2. make new simulations by gradually increasing the input concentration of the limiting factor until a new plateau is reached;
3. if a new plateau is reached, repeat the previous steps until the increase in phytoplankton concentration is no longer relevant.

$$\phi_{Np} = min\left(\frac{n_a+n_n}{K_{sN}+n_a+n_n}, \frac{p_i}{K_{sP}+p_i}, \frac{s_i}{K_{sS}+s_i}\right) \quad \textbf{(1)}$$

The inorganic phosphorus is confirmed to be the initial and major limiting factor, anyways it has been verified that further limiting factors were present when the amount of nutrients in input had been increased.

[1] The numerical calculations are shown in the Excel file *Fattori_Limitanti.xls*.

Surrogate models

Once collected the values of inorganic phosphorus concentrations and the corresponding peak phytoplankton concentration, the surrogate models can be developed. These were constructed by applying regressions to the data series of *Figure 1* and *2*. The inorganic phosphorus was set as the independent variable *X*, while the peak of phytoplankton concentration was placed as the dependent variable *Y*. To obtain models capable of explaining the datasets as far as possible, different types of regression were tested using the software *RStudio*[2], starting with the simplest possible model: the linear model. The regression obtained for case ***a*** is shown in *Table 1*.

	Formula	R^2
Linear Regression	Y=4.082 + 0.199X	0.953

Table 1- Formula and R^2 of the linear regression for case **a.**

Although linear regression already represents the model significantly (R^2 is almost 1), we proceeded by testing a slightly more complicated model, such as the polynomial one, to see if there were further improvements (*Table 2*).

	Formula	R^2
Polynomial Regression	$Y = 2.634 + 0.371X - 0.003X^2$	0.994

Table 2- Formula and R^2 of the polynomial regression for case **a.**

The polynomial model shows an even higher R^2 and a better fitting to the data as visible in *Figure 3*.

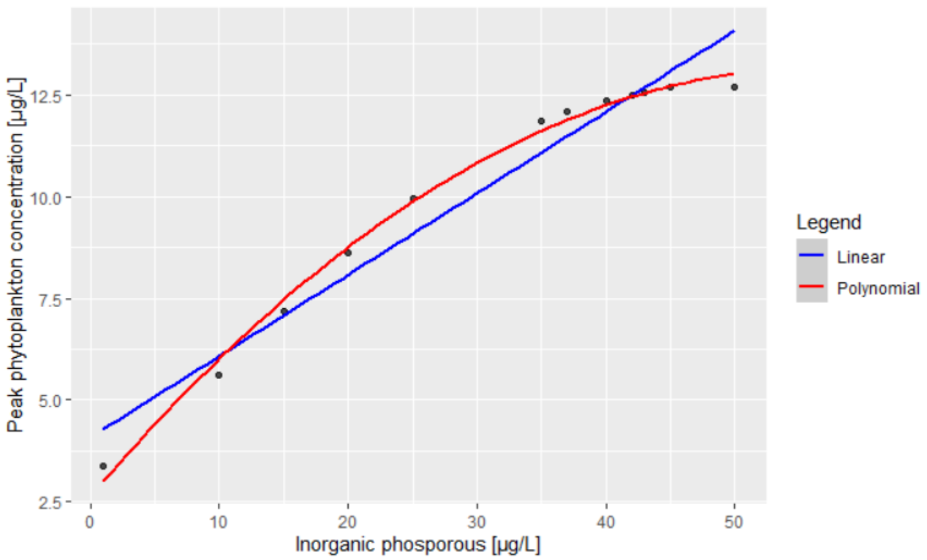

Figure 3- Comparison between the linear regression and the polynomial one for case ***a***.

The case ***b*** was then evaluated using the same procedure. The regression models, their coefficients (*Table 3*) and their comparison (*Figure 4*) are shown below. Once again, the polynomial regression was found to be the most representative with a higher R^2.

	Formula	R^2
Linear Regression	Y= 4.783 +0.294X	0.954
Polynomial Regresssion	$Y = 2.634 + 0.371X - 0.003X^2$	0.997

Table 3- Formulas and R^2 of the linear regression and the polynomial one for case ***b.***

[2] The processing and the R codes are shown in the files *script_LakeTufts.R* and *epilimnio_script.R*.

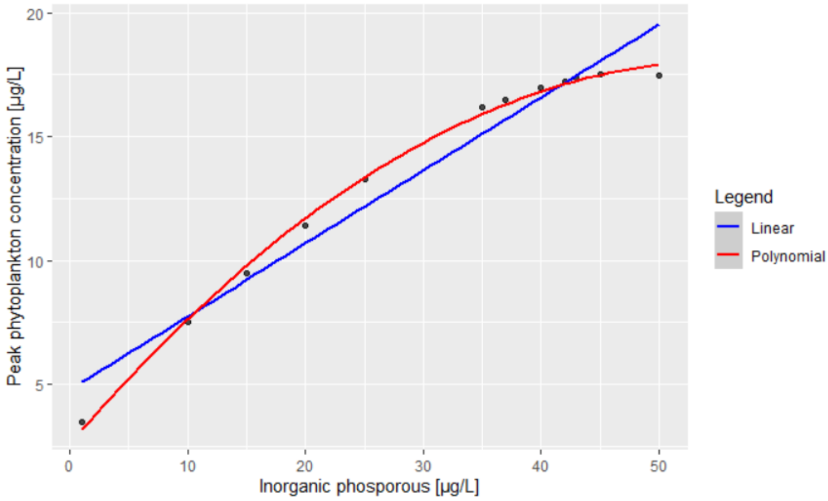

*Figure 4- Comparison between the linear regression and the polynomial one for case **b**.*

In order to calculate the maximum concentration of inorganic phosphorous in the inflow that can guarantees a peak total phytoplankton concentration below the threshold of **12 µg/L**, the equations of the two chosen surrogate models, i.e., the polynomial regressions, were inverted and solved as a function of *X*, substituting the value **12 µg/L** in place of *Y*. The resulting values of concentration of inorganic phosphorous loads are shown in *Table 4*.

	Case *a*	**Case *b***
Inorganic phosphorus load [µg/L]	37.91	20.89

*Table 4- Concentration of inorganic phosphorous in the inflow that can guarantees a peak total phytoplankton concentration of 12 µg/L for case **a** and **b.***

Validation

Finally, to be certain that the results of *Table 4* are reliable, a validation must be carried out; the phosphorus loads (*Table 4*) were used as input to perform new simulations with *Lake2K*. The peak phytoplankton concentrations resulting from these simulations were then compared with the threshold (*Table 5)*. It can be observed that the values obtained with the simulations are very similar to **12 µg/L** and therefore the surrogate models can be considered a good approximation of the original model *Lake2K* for phytoplankton growth.

		Surrogate model	**Lake2K**
Phytoplankton peak [µg/L]	**Case *a***	12	12.19
	Case *b*	12	11.76

Table 5- Comparison between the values of peak of phytoplancton concentration obtained with Lake2K, using in input the phosphorus loads previously calculated, and the threshold.

In conclusion, the amount of inorganic phosphorous in the inflow must be lower than **37.91 µg/L** for case ***a*** and lower than **20.89 µg/L** for case ***b*** to guarantee a peak total phytoplankton concentration below **12 µg/L.** In case ***a***, the permissible phosphorus load is higher because the concentration of phytoplankton tends to decrease as the depth increases, so the average concentration in the three layers is lower than the concentration in the epilimnion alone. In fact, in the superficial layer there are the most favourable conditions for the growth of phytoplankton because the solar radiation and consequently the temperature are higher and for these reasons, in case ***a*** it is possible to discharge more inorganic phosphorus into the lake before the threshold is reached. Whereas case ***b*** is more restrictive and therefore with less phosphorous, the threshold is reached more easily. So, depending on the **WQI**, the results may vary greatly and it is important to choose a suitable indicator according to your objectives.