PROJECT

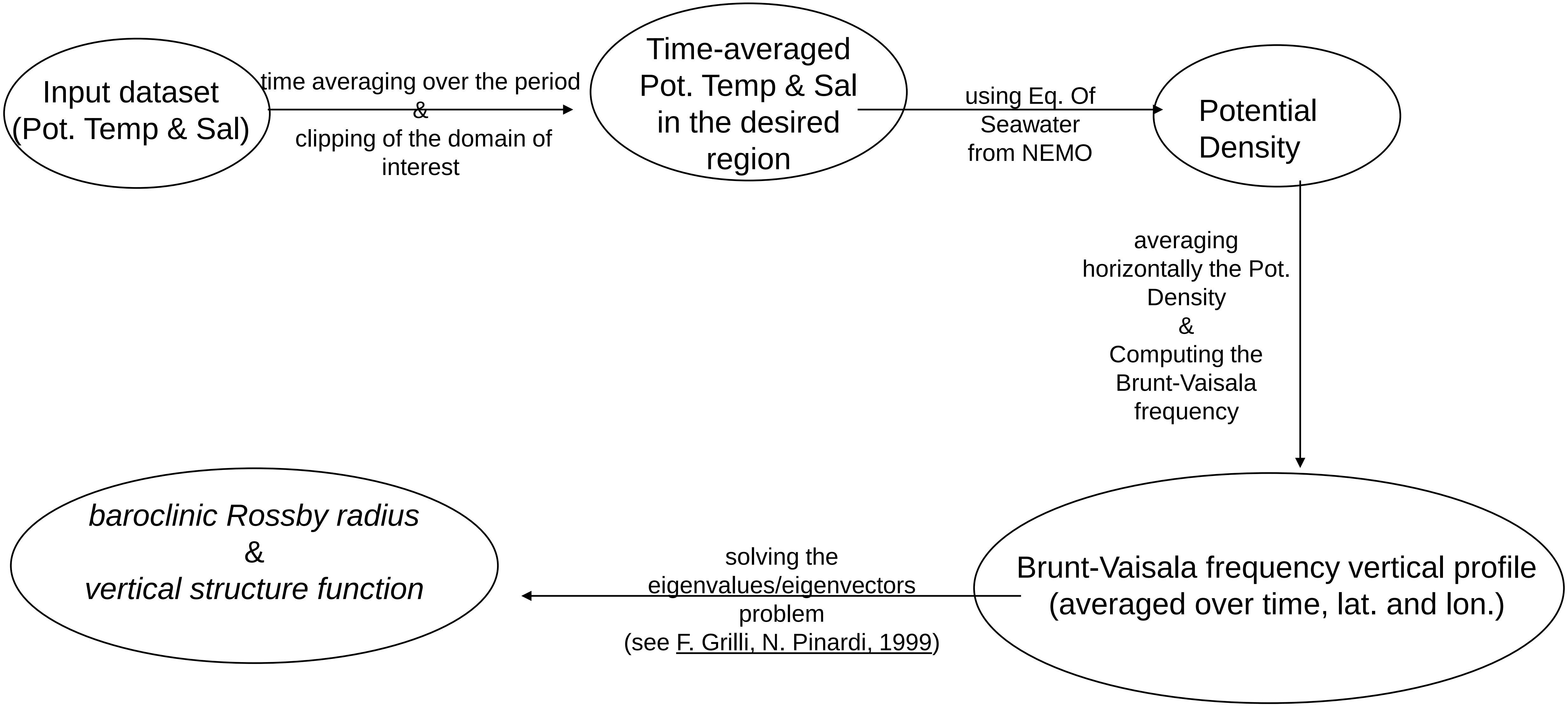
Ocean Baroclinic Modes 1.0 (OBM-1.0)

The Project: a downloadable repository on GitHub

- A «software» through which the user can compute the *baroclinic Rossby radius* and *baroclinic vertical structure function* for each mode of motion, in a region of interest.
- How?
    1. Save the input dataset, containing **Potential Temperature** and **Salinity** variables, in a specific directory. Save the **Bathimetry** dataset.
    2. Edit the configuration file.
    3. Run the «main.py» program

A directory is created, containing the results file.

Implementation Structure



Computing the baroclinic Rossby Radius & the vertical structure function [1]

- The Brunt-Vaisala frequency «N» is linearly interpolated on a new equally spaced depth grid with step equal to 1 m. Depth goes from 0 to the region mean depth.
- The eigenvalues/eigenvectors problem is solved:

$$\frac{d^2w}{dz^2} = -\lambda S w \quad (1) \qquad \text{where } w = \frac{1}{S}\frac{d\Phi}{dz}, \quad S = \frac{N^2H^2}{f_0^2L^2} \qquad \text{(BCs: w = 0 at z = 0,1)}$$

L = 100 km, H = 1 km, $f_0 = 10^{-4}$ 1/s; «Φ» vertical structure function.

A function from «scipy» is used for solving the discretized eigenvalues/eigenvectors problem:

$$\frac{1}{12dz^2}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ 12 & -24 & 12 & 0 & \dots & 0 & \dots & 0 \\ -1 & 16 & -30 & 16 & -1 & 0 & \dots & 0 \\ 0 & -1 & 16 & -30 & 16 & \dots & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \dots & \dots & \vdots \\ 0 & \dots & \dots & 0 & 0 & 12 & -24 & 12 \\ 0 & \dots & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ \vdots \\ \vdots \\ w_{n-1} \\ w_n \end{bmatrix} = -\lambda \begin{bmatrix} S_0 & 0 & 0 & 0 & \dots & \dots & 0 \\ 0 & S_1 & 0 & 0 & \dots & \dots & 0 \\ 0 & 0 & S_2 & 0 & \dots & \dots & 0 \\ 0 & 0 & 0 & S_3 & \dots & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & \dots & \dots & 0 & S_{n-1} & 0 \\ 0 & \dots & \dots & \dots & 0 & 0 & S_n \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ \vdots \\ \vdots \\ w_{n-1} \\ w_n \end{bmatrix}$$

[1] F. Grilli, N. Pinardi (1999) "Le Cause Dinamiche Della Stratificazione Verticale Nel Mediterraneo"

- The *baroclinic rossby radius* is computed from the eigenvalues «λ» for a number of modes of motion set by the user, as

$$R_n = \frac{L}{\lambda_n}$$

- The eigenvectors are computed integrating eq. (1) through Numerov's numerical method.

- The *vertical structure functions* are computed integrating the eigenvectors «w»:

$$\Phi_n(z) = \int_0^z S w_n dz + \Phi_0, \quad \text{with } \Phi_0 = \Phi(z=0)$$

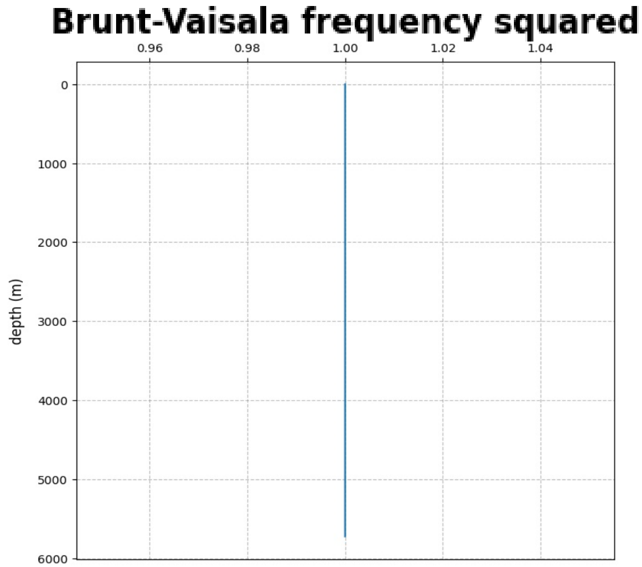
Here, $\phi_0 = 1$ is chose arbitrarily as amplitude of modes of motion.

Results for particular cases

Comparison with analytical results from *J. H. LaCasce*

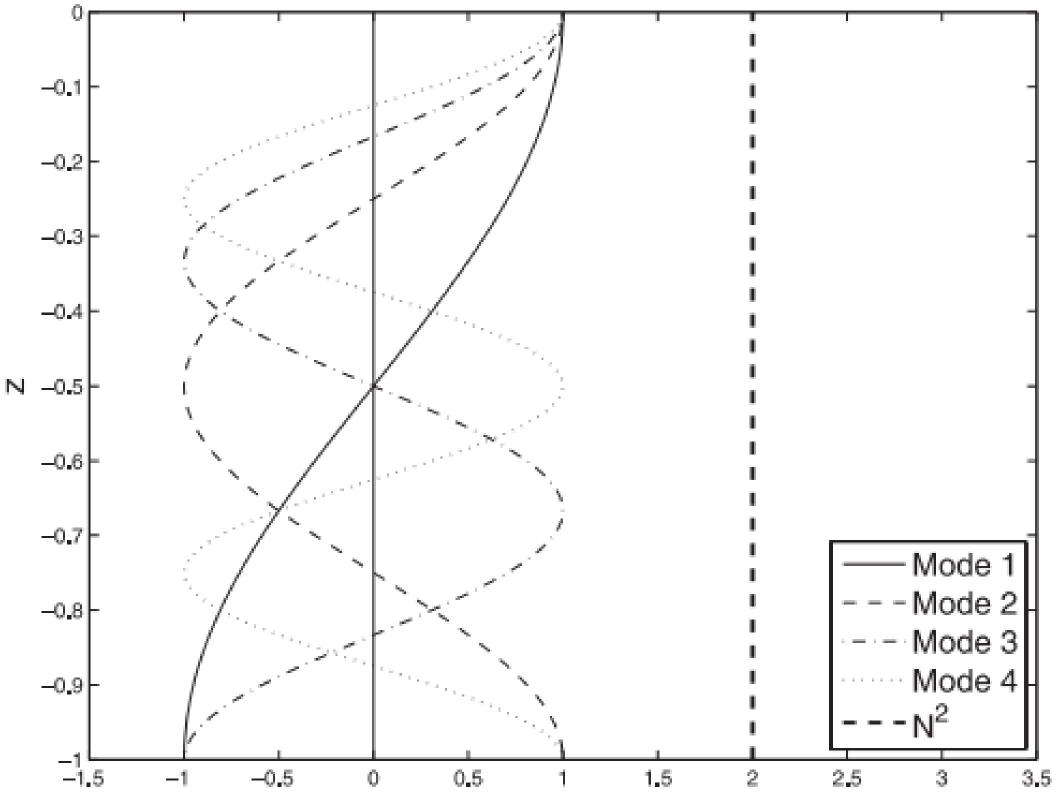
(«Surface Quasigeostrophic Solutions and Baroclinic Modes with Exponential Stratification», *2012)*

Constant Brunt-Vaisala Frequency

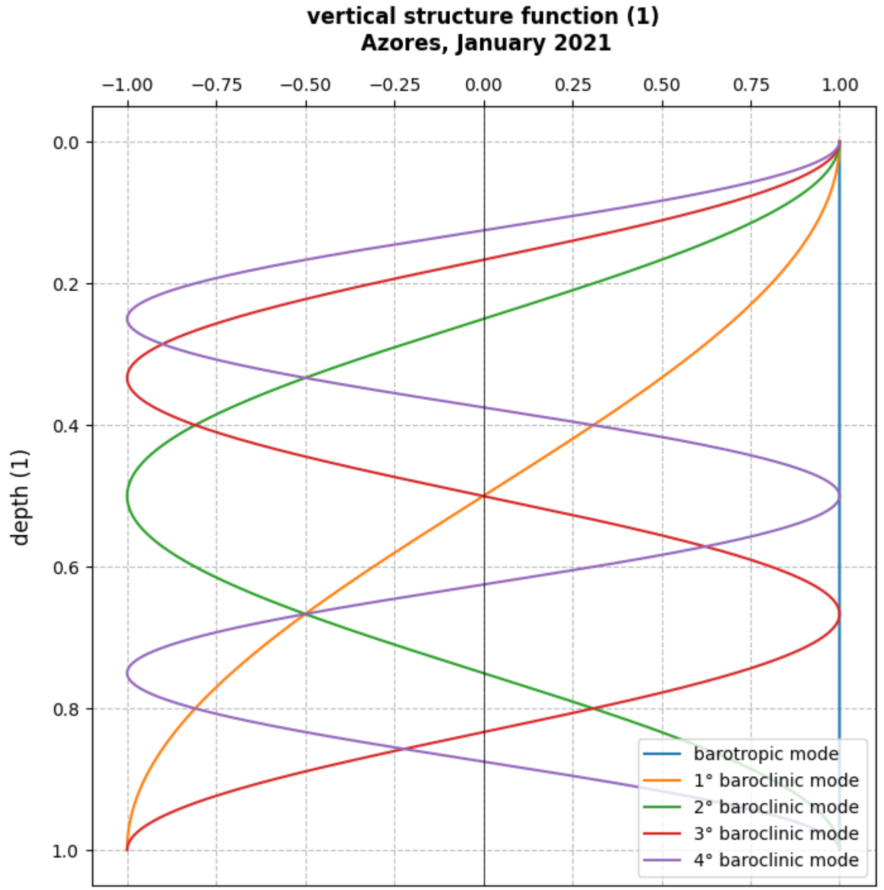


$$N^2(z) = \text{const}$$

Theoretical Solution (LaCasce, 2012)



Numerical Solution



Exponential Brunt-Vaisala Frequency

of type $N^2 = N_0^2 e^{-\alpha z}$

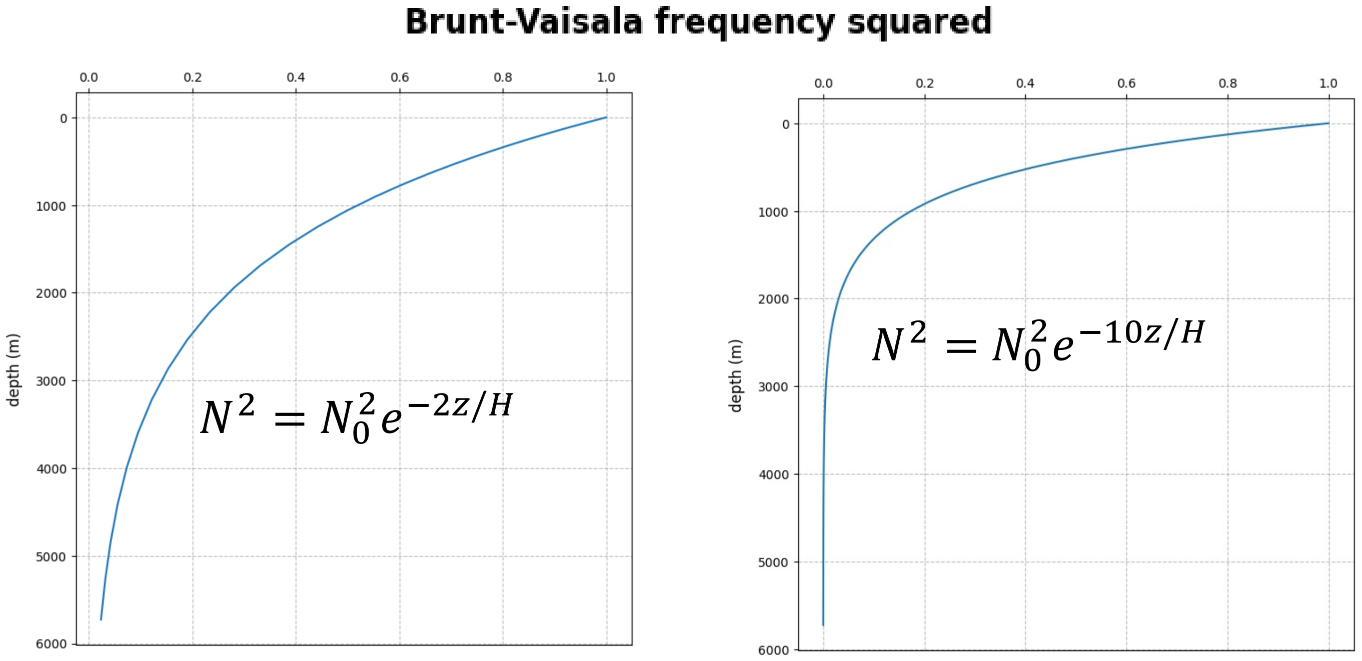
Theoretical Solution for the vertical structure function:

$$\phi = Ae^{\alpha z/2}[Y_0(2\gamma)J_1(2\gamma e^{\alpha z/2}) - J_0(2\gamma)Y_1(2\gamma e^{\alpha z/2})]$$

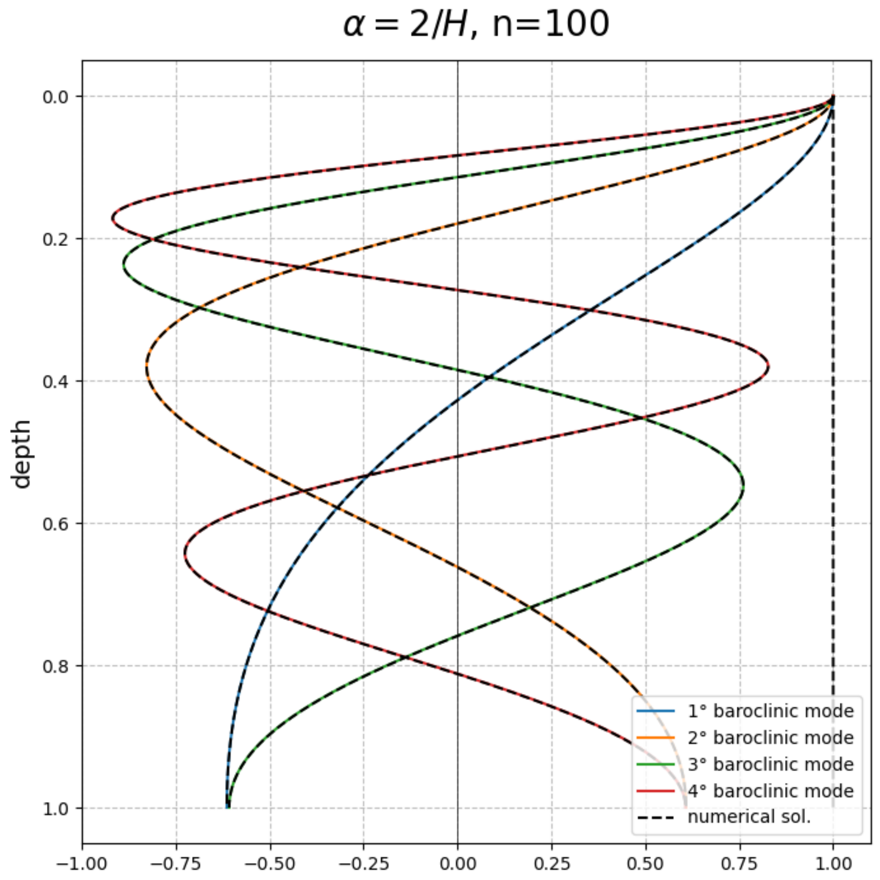
$$\gamma = N_0\lambda/(\alpha f_0)$$

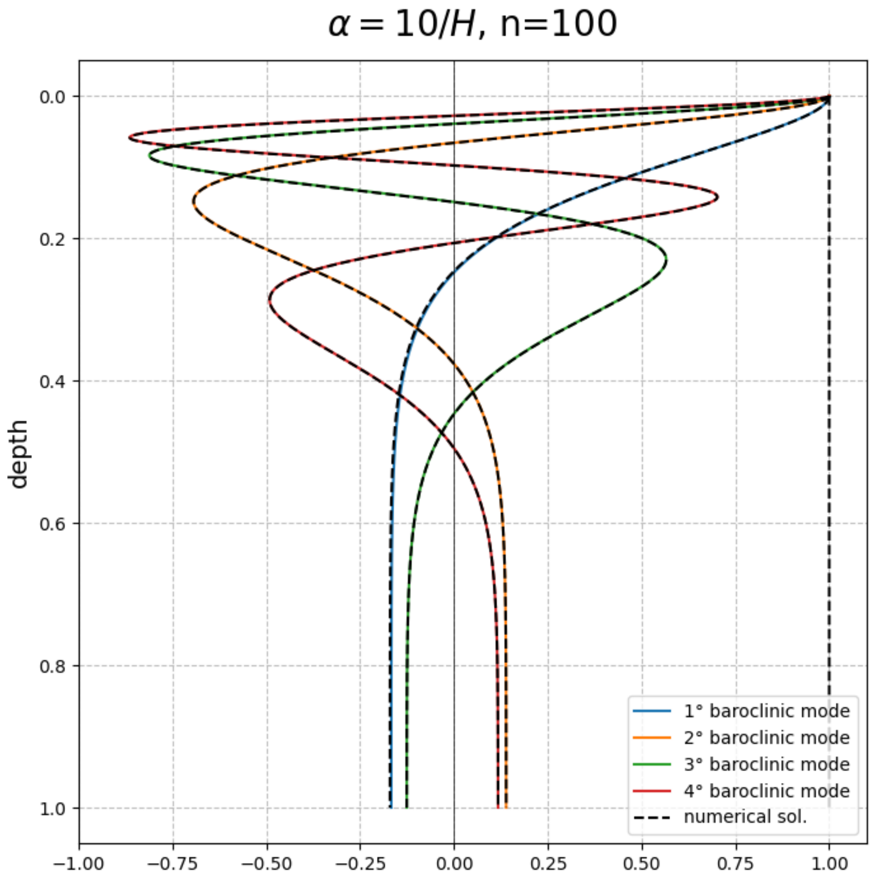
2γ values for the theoretical solutions

Mode	Exponential ($\alpha = 2/H$)	Exponential ($\alpha = 10/H$)
1	4.9107	2.7565
2	9.9072	5.9590
3	14.8875	9.1492
4	19.8628	12.334



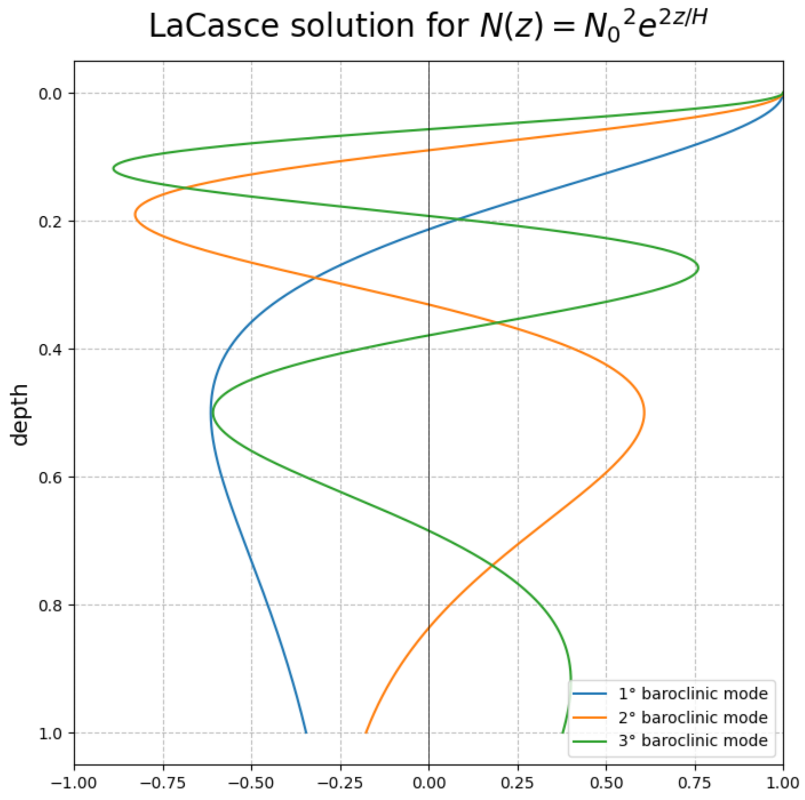
Theoretical (coloured) VS Numerical (black dashed) Solution: exponential cases

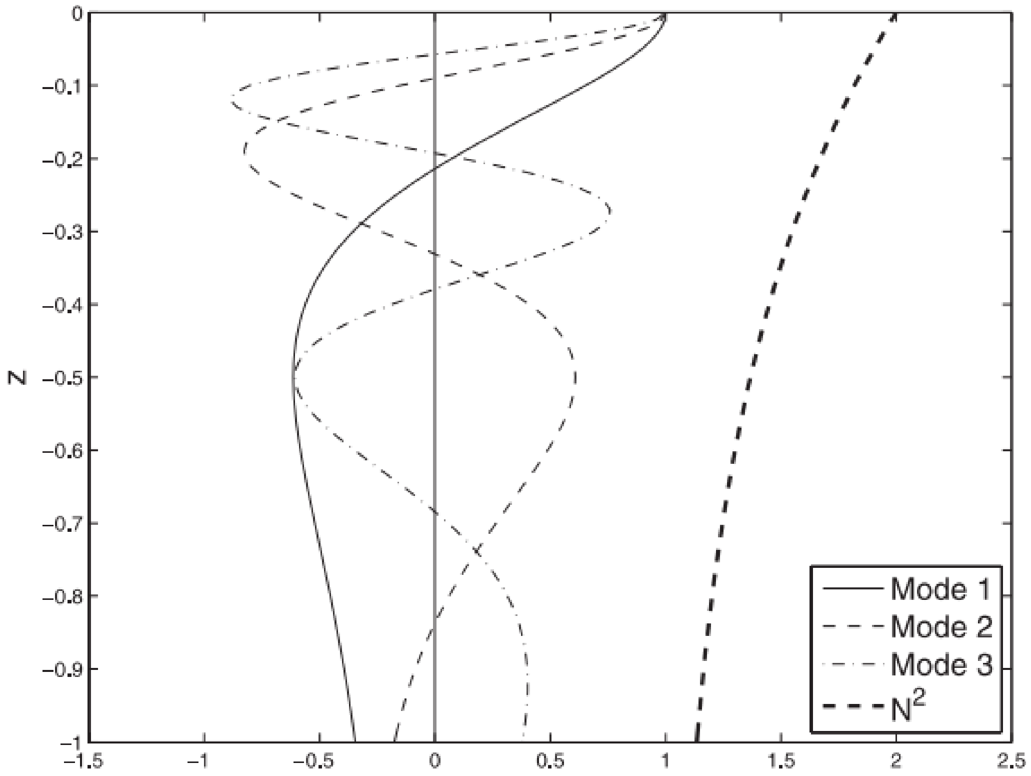




Errata: plots of baroclinic modes in LaCasce paper (Fig. 1)

$$\phi = Ae^{\alpha z/\cancel{2}}[Y_0(2\gamma)J_1(2\gamma e^{\alpha z/\cancel{2}}) - J_0(2\gamma)Y_1(2\gamma e^{\alpha z/\cancel{2}})]$$





½ factor missing in LaCasce plots

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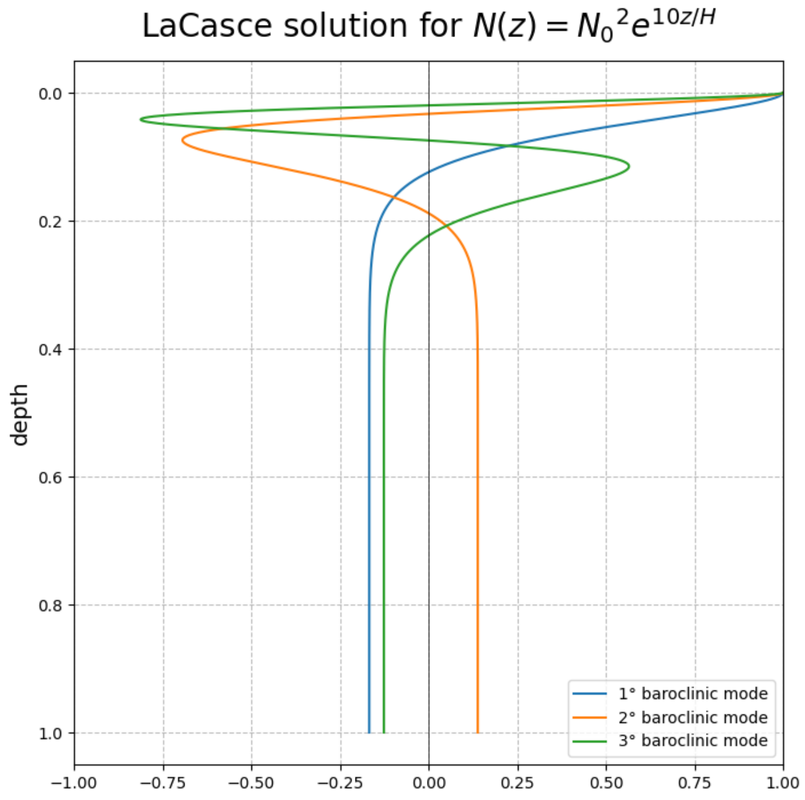
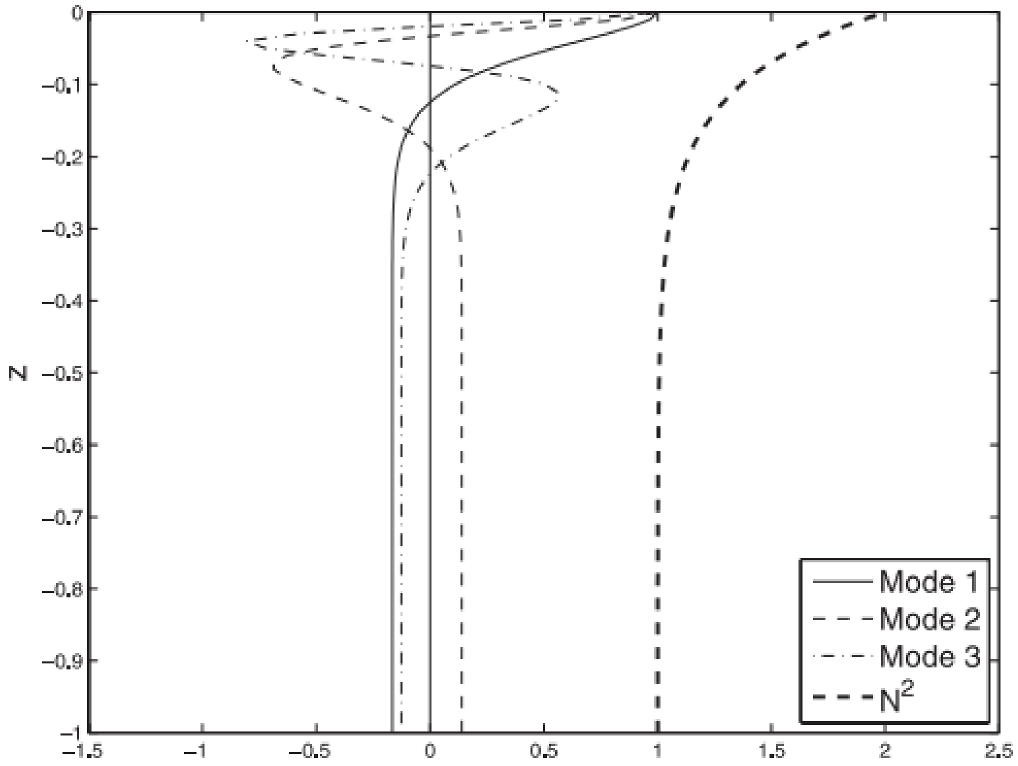
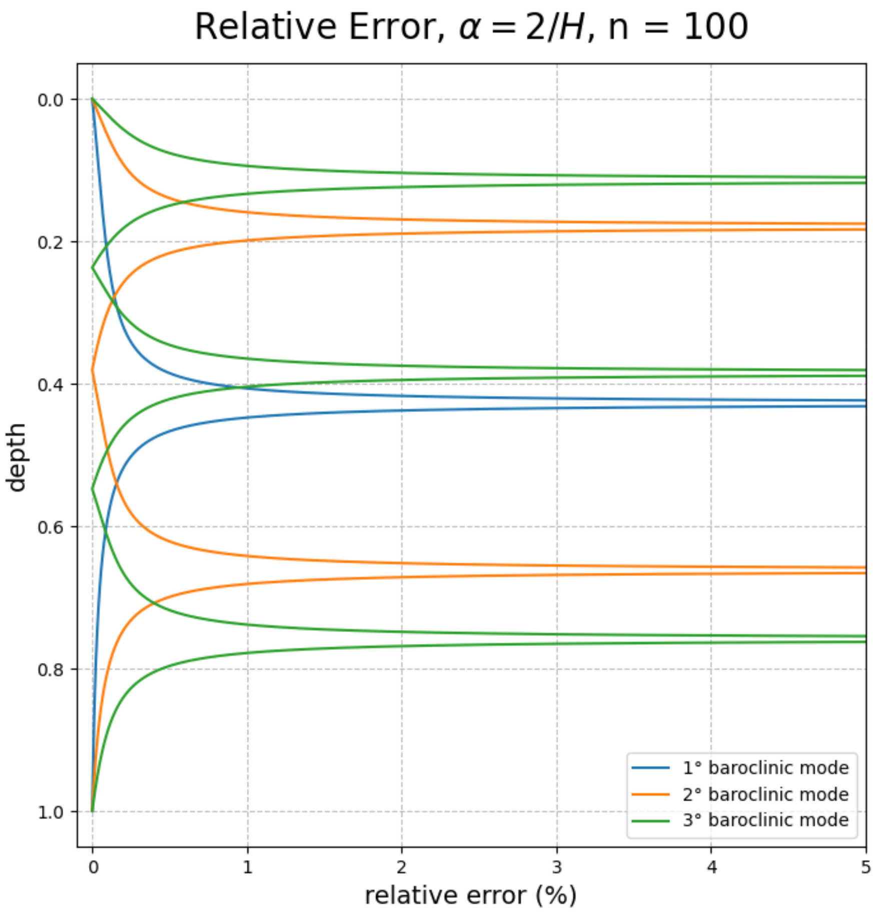


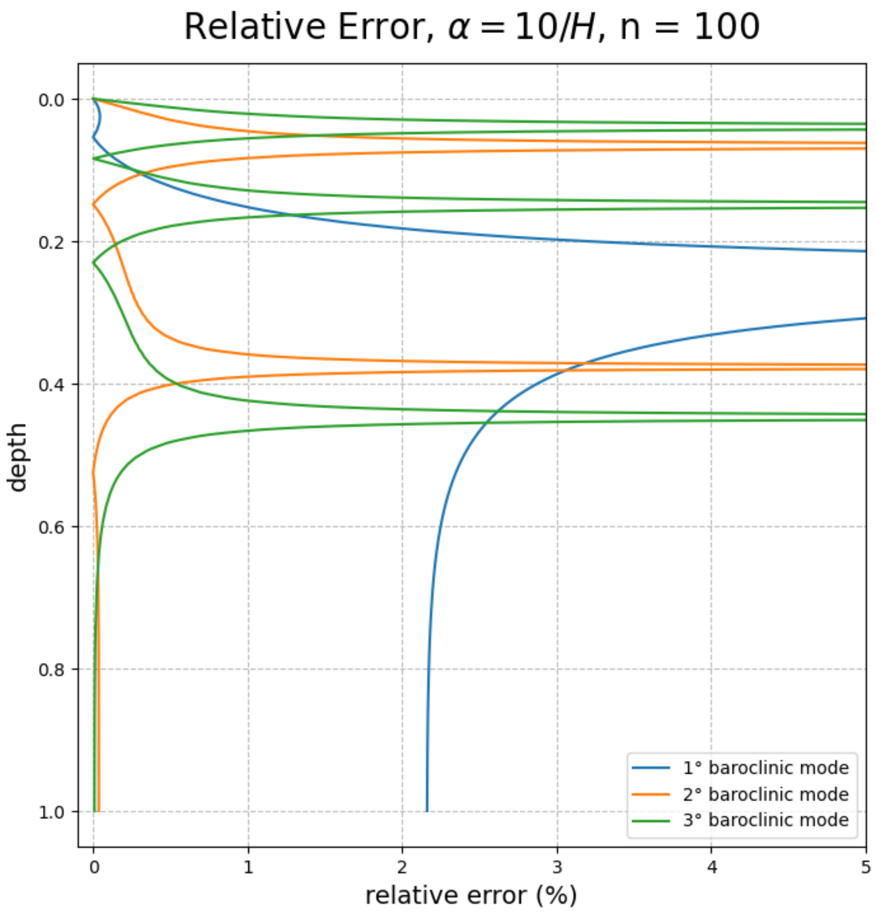


½ factor missing in LaCasce plots

Relative Errors: vertical structure function

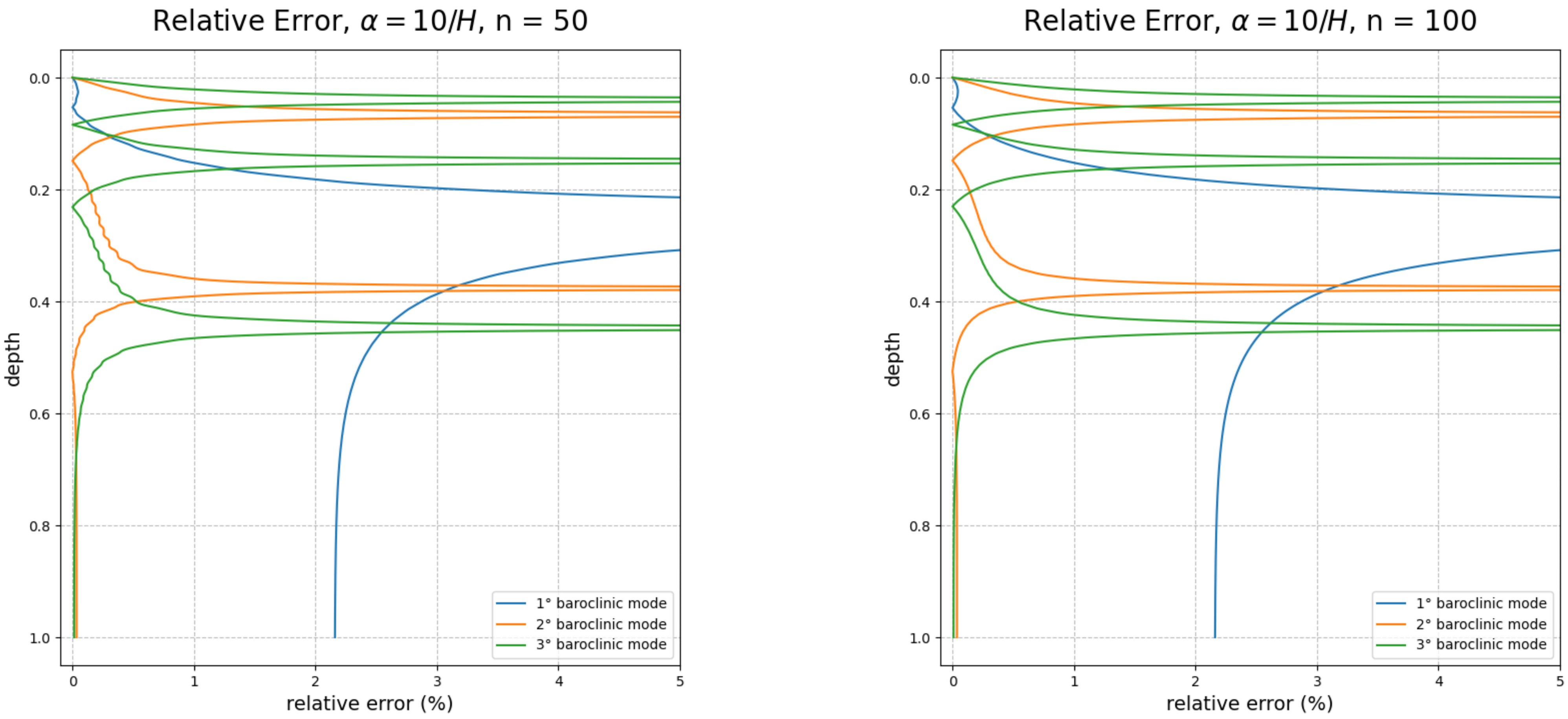
Relative error is always **<< 3%**, except near the function zeros





Relative Errors: vertical structure function

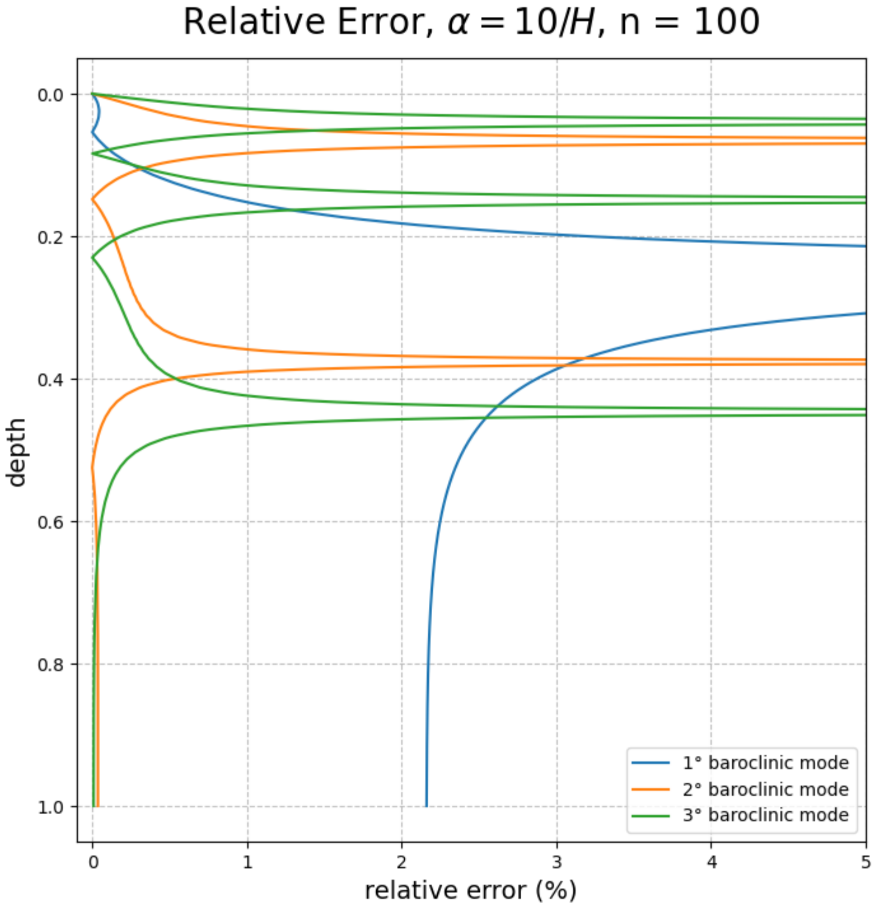
Relative error does not really seem to depend on **vertical resolution «n»** of the input BV frequency



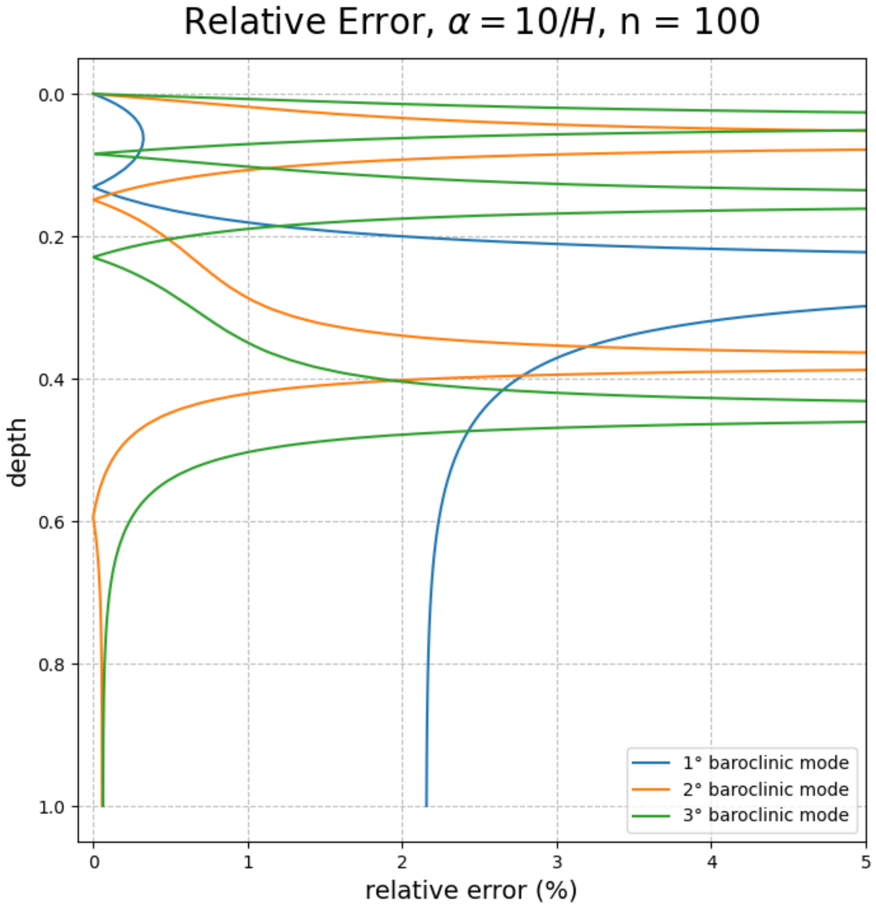
Relative Errors: vertical structure function

Relative error seems to depend on the region **mean depth**

Mean depth = 5 km



Mean depth = 1.5 km



NOTE:
H = 1 km in γ;
H = mean_depth in α

Relative error: Rossby radius

$$\gamma = N_0\lambda/(\alpha f_0)$$ (LaCasce) $\Rightarrow$ $$\gamma = \frac{N_0\sqrt{\lambda}H}{\alpha f_0 L}$$ (based on «S» parameter, where α is not divided by the mean depth here)

Relative λ error is obtained as relative γ error (no uncertainty on the other factors)
→ Rossby radius rel. error

Deformation radius relative error

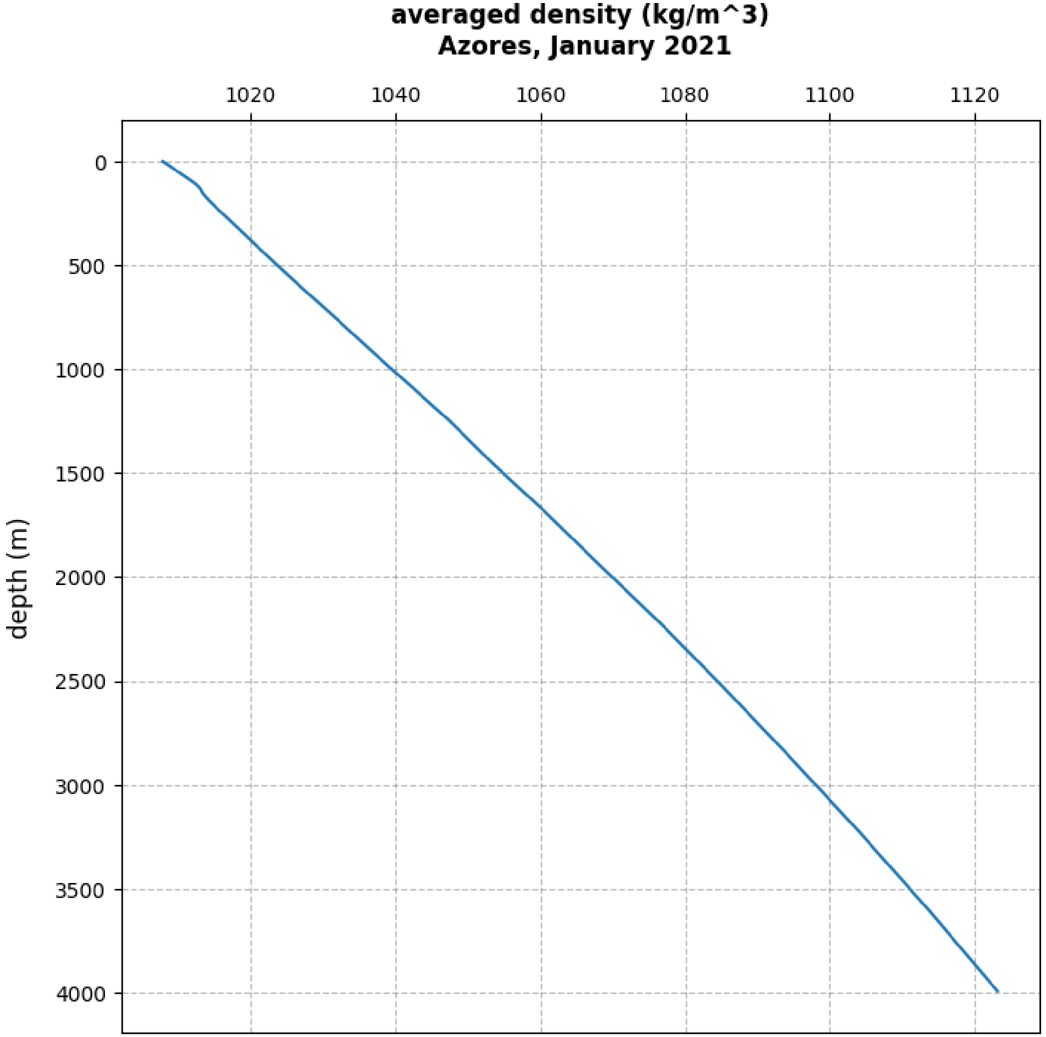
	α = 2/H	**α = 10/H**
1° mode	0.27 %	0.003 %
2° mode	0.04 %	0.0014 %
3° mode	0.04 %	0.0017 %
4°mode	0.04 %	0.0015 %

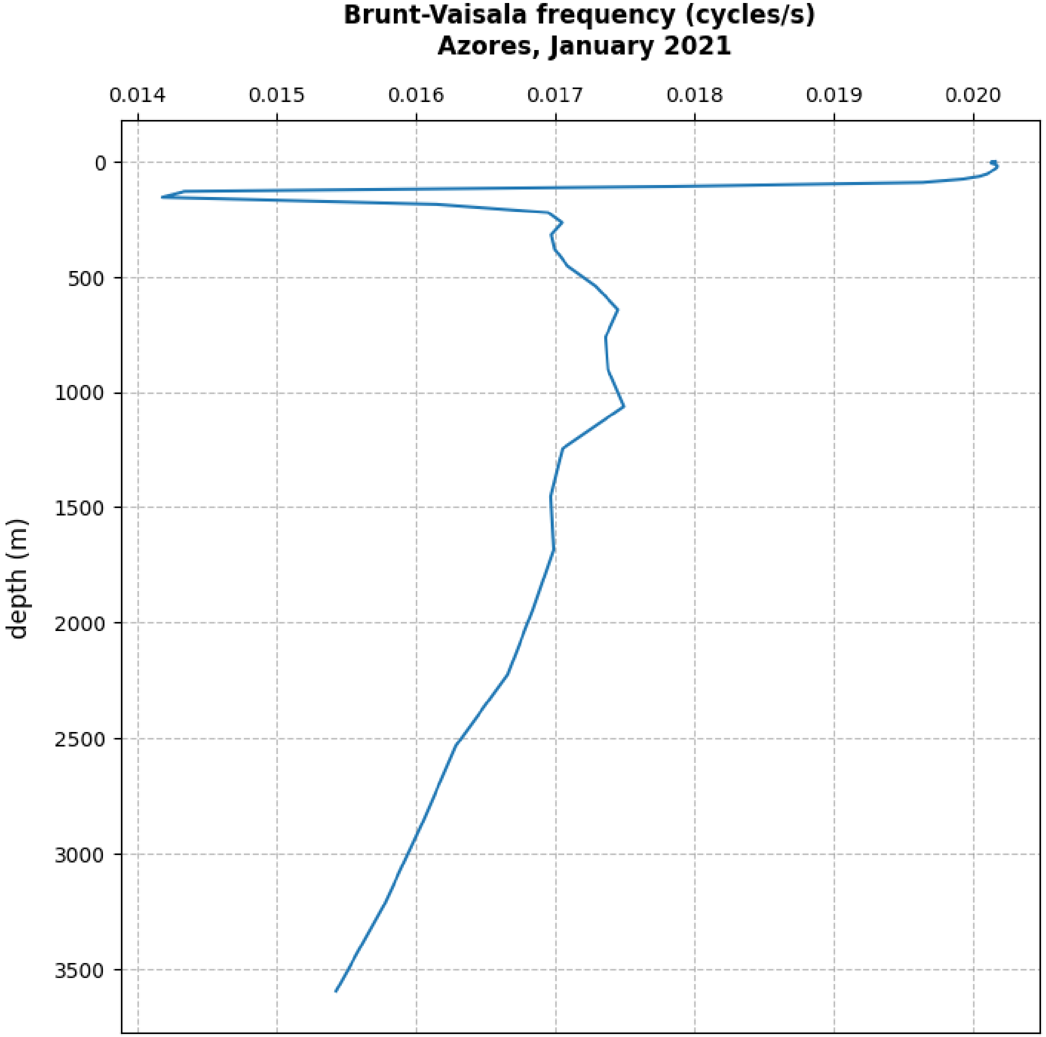
Relative error does not seem to depend on the region **mean depth or vertical resolution**

Test Case Results

(realistic Brunt-Vaisala frequency profile in the Azores region, Jan 2021)

Azores Region, January 2021
(latitude 33÷ 35 °N, longitude 28 ÷ 30°W)



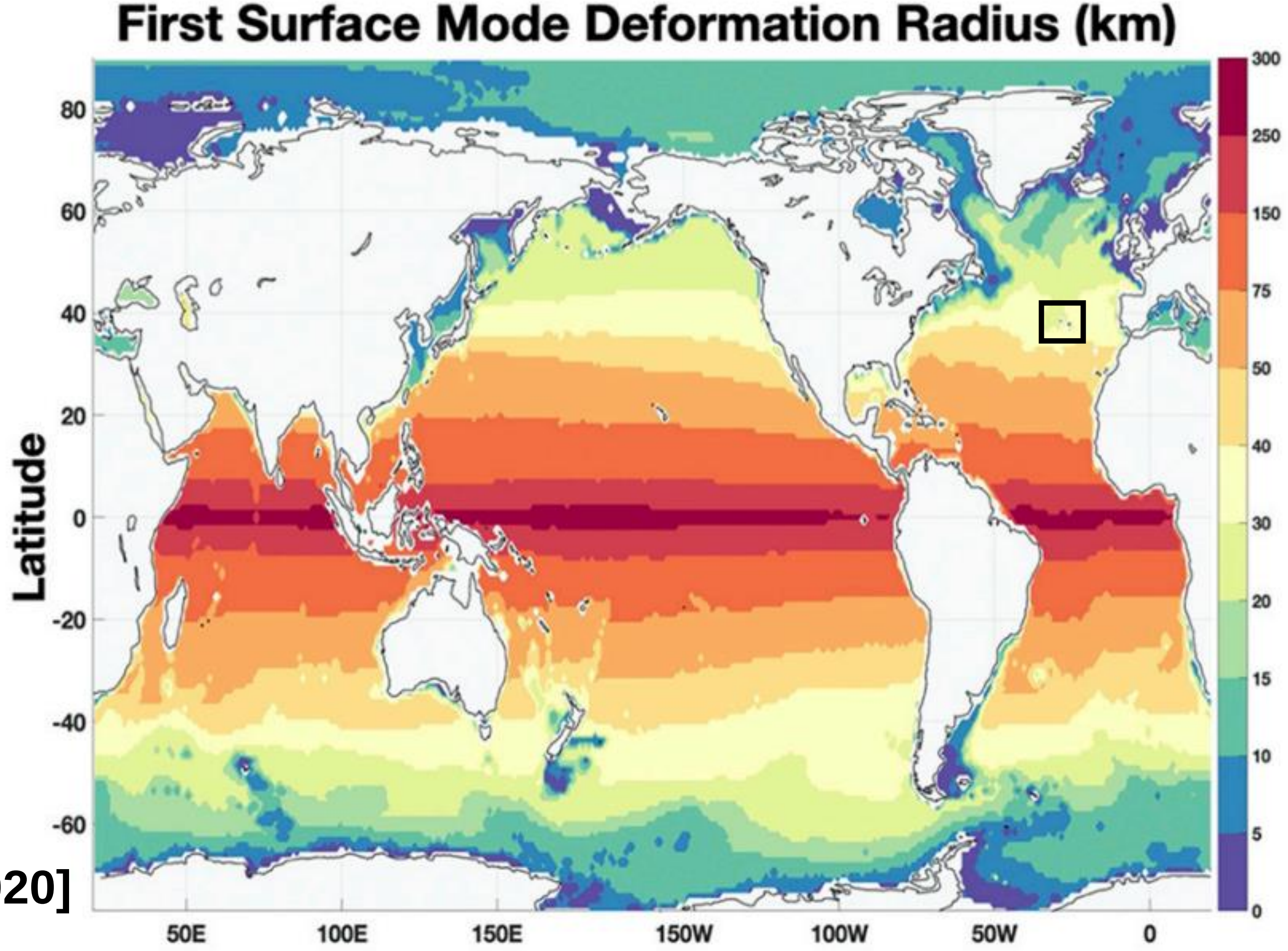


Azores Region, January 2021
(latitude 33÷ 35 °N, longitude 28 ÷ 30°W)

Rossby deformation radius (km)
Azores, January 2021

	Rossby deformation radius (km)
barotropic mode	nan
1° baroclinic mode	29.0
2° baroclinic mode	7.0
3° baroclinic mode	3.0
4° baroclinic mode	2.0

[numerical solution]



[From LaCasce, 2020]

Azores Region, January 2021
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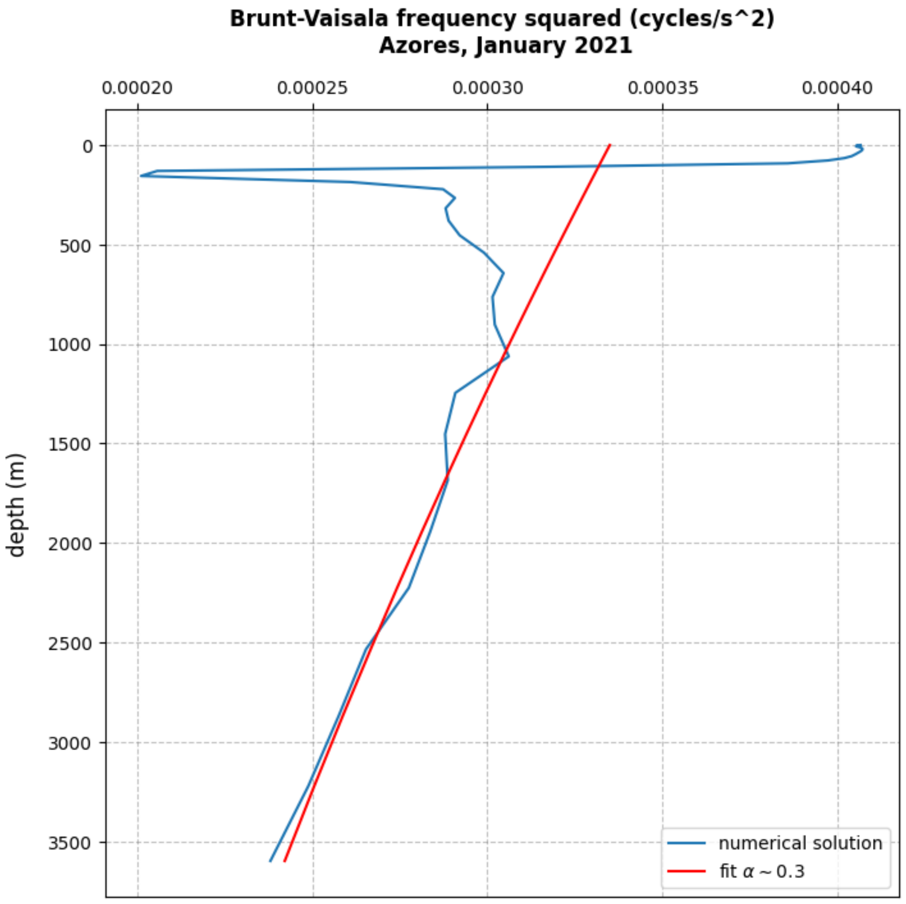
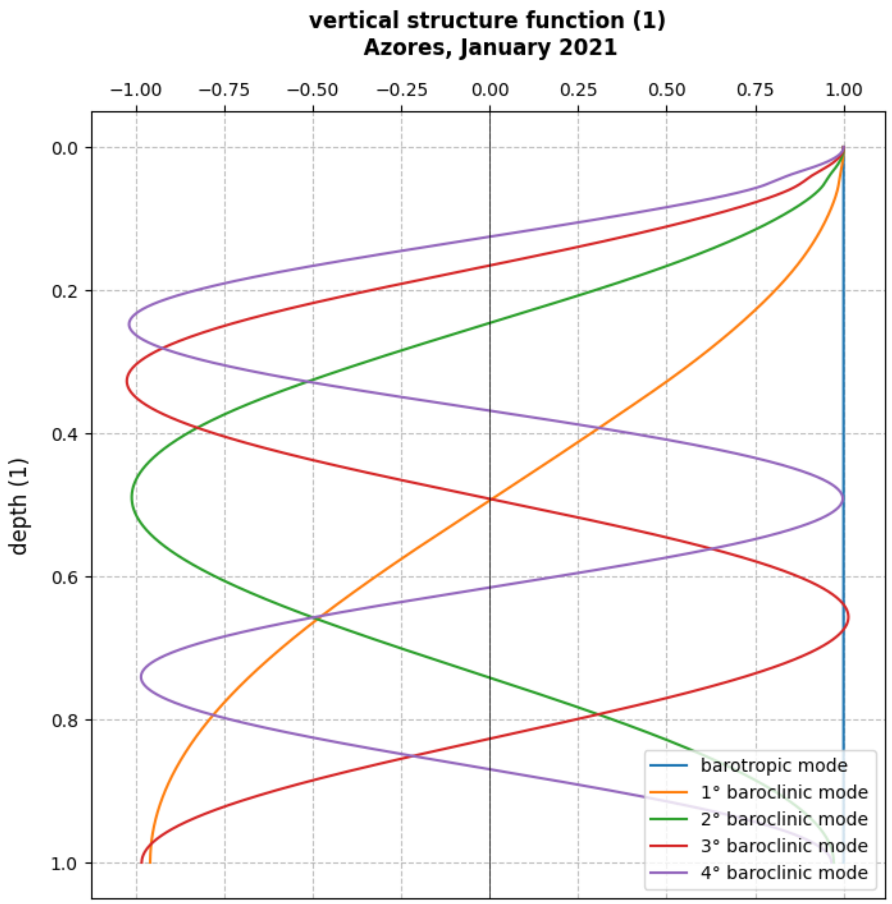


We expect the modes of motion to be **similar** (not equal) to the exponential case with α = 0.3

$$N^2 = \left(3.5e^{-\frac{0.3z}{H}} + 0.15\right) \times 10^{-4} 1/s^2$$

Azores Region, January 2021
(latitude 33÷ 35 °N, longitude 28 ÷ 30°W)

Real case numerical solution



Theoretical solution for α = 0.3