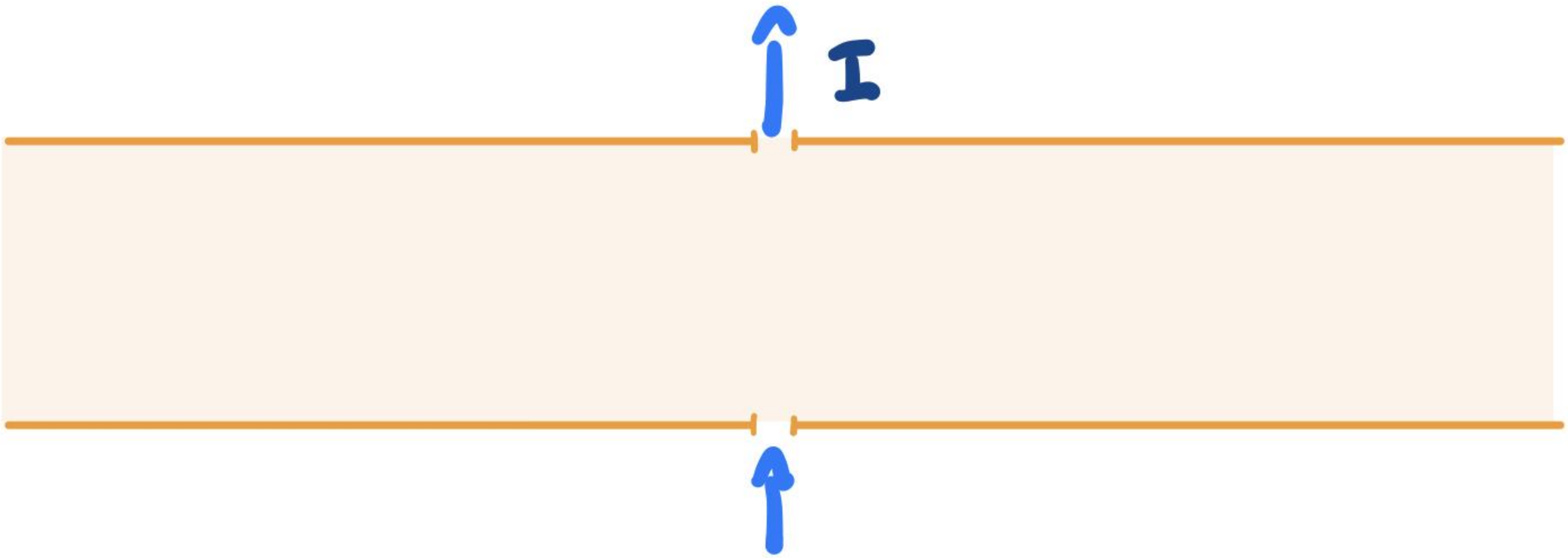
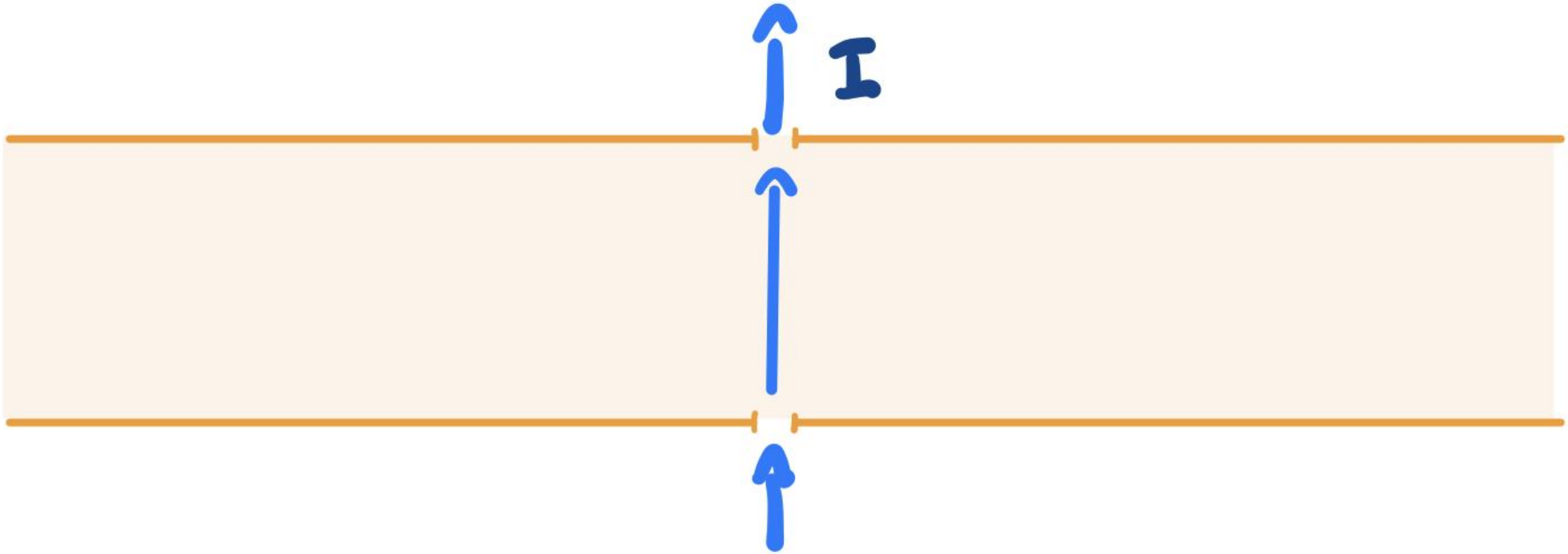
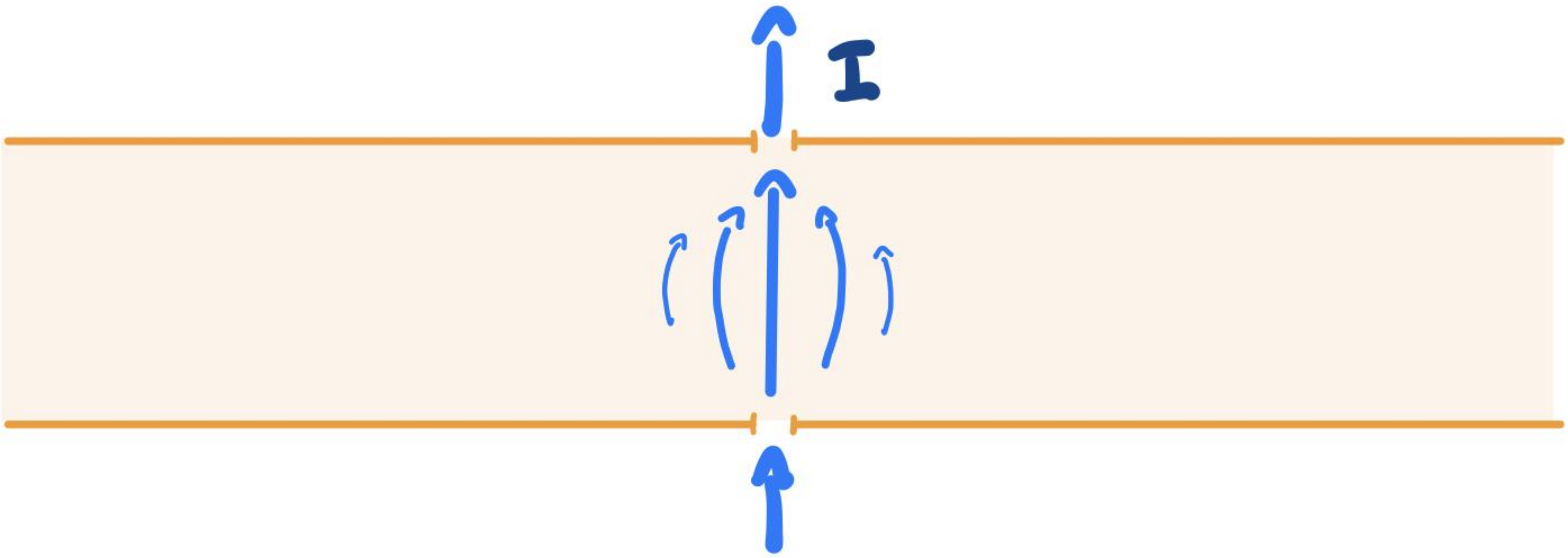
Trasporto non-locale di valle nel grafene a doppio strato

Relatore:
Marco Polini

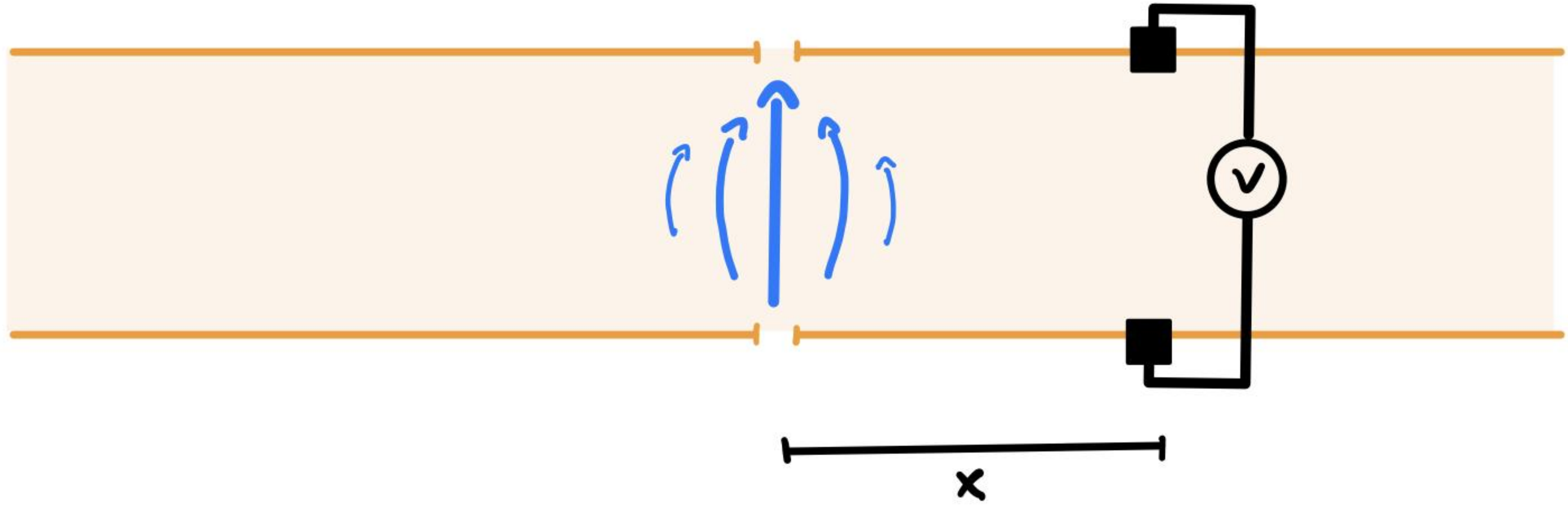
Candidato:
Francesco Sacco

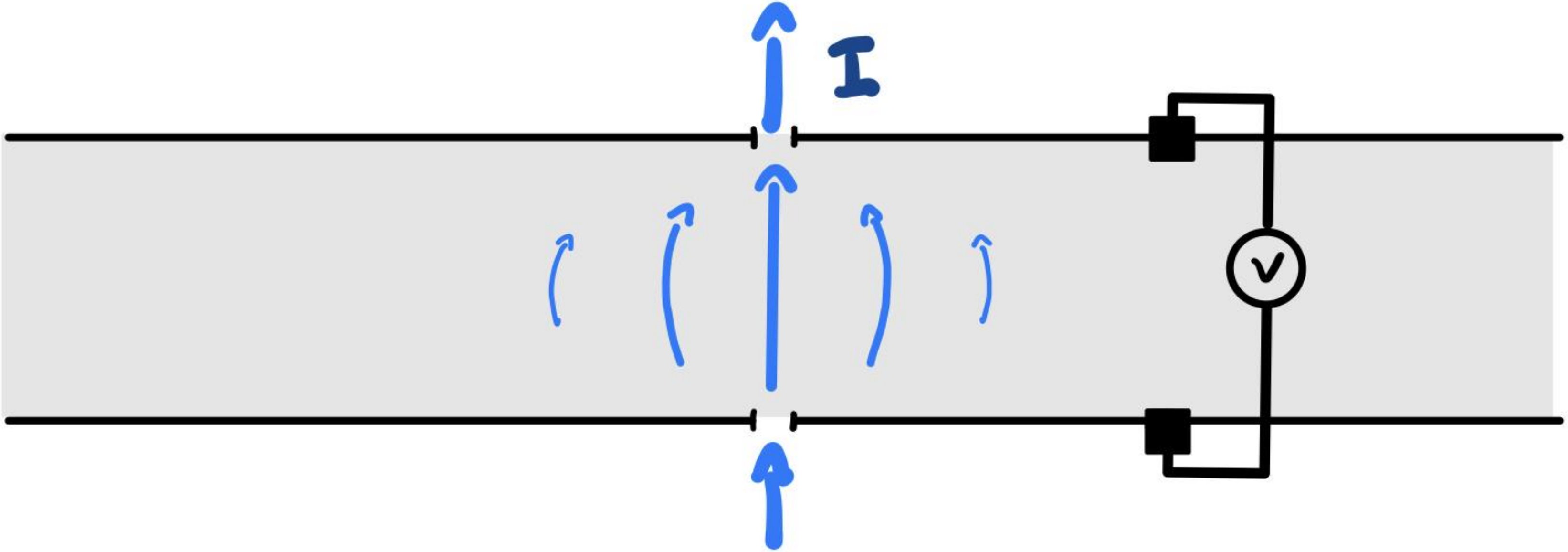

I


I

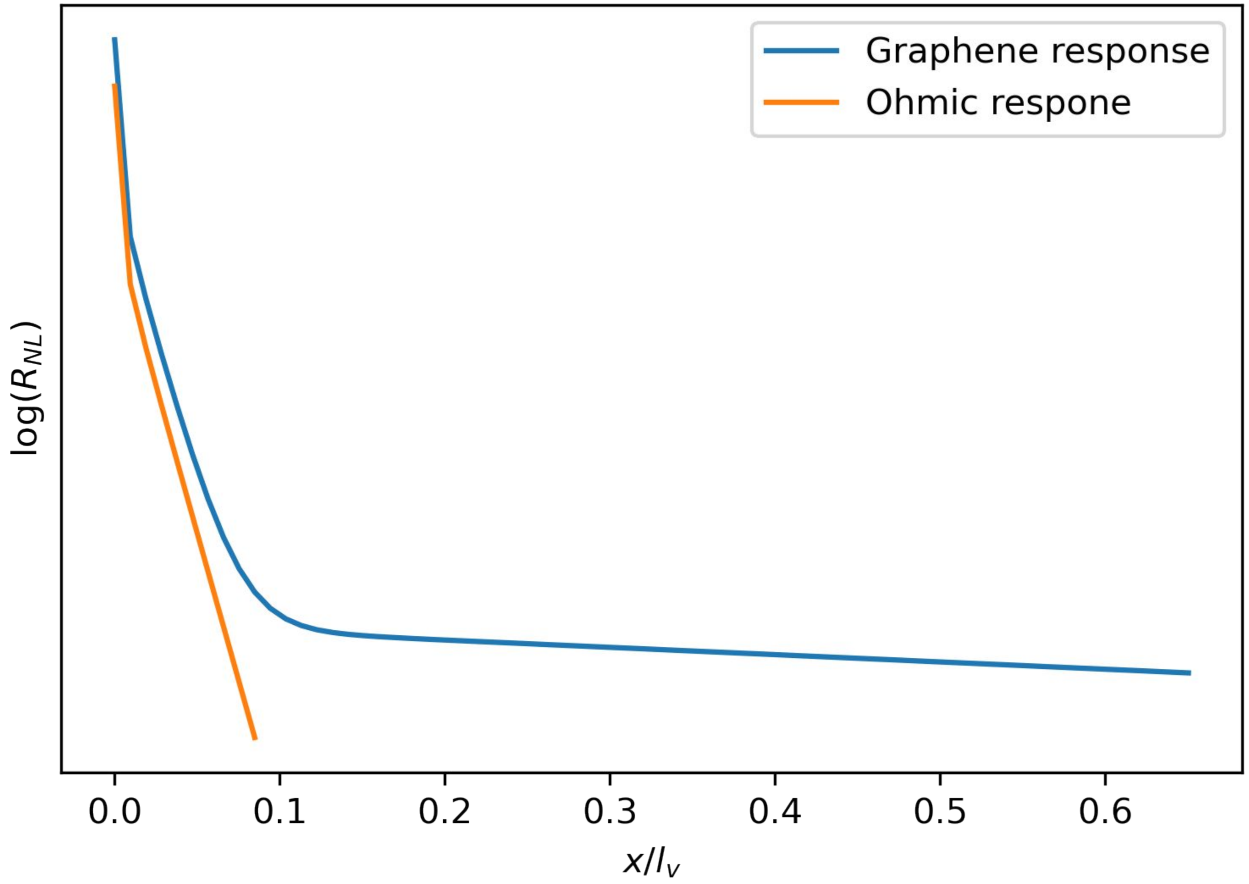

I

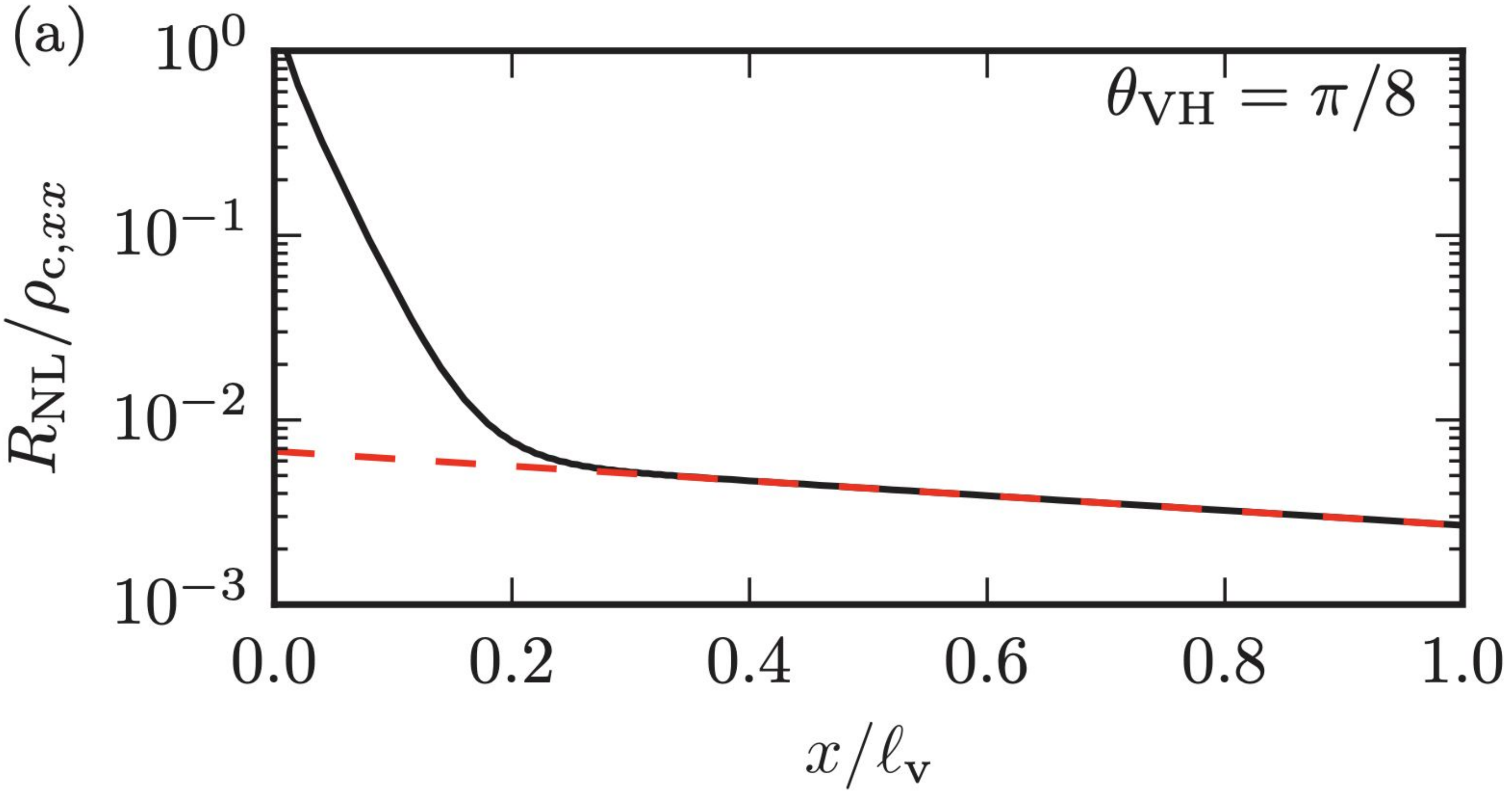
$$\frac{V(x)}{I} \equiv R_{\mathrm{NL}}(x) = \frac{2\rho}{\pi} \ln \left| \coth\left(\frac{\pi x}{2W}\right) \right| \approx \frac{4\rho}{\pi} e^{-\pi|x|/W}$$



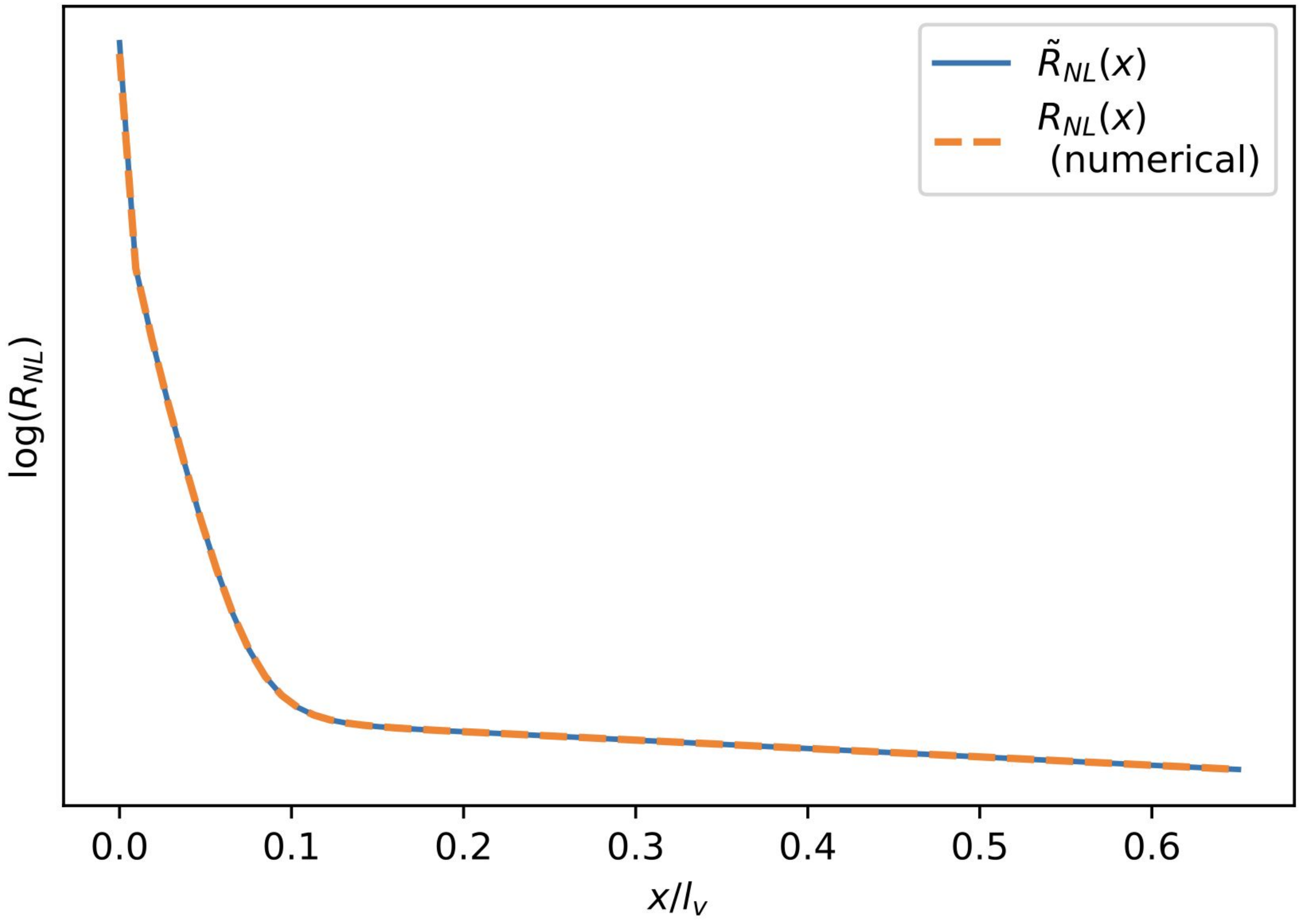


$$\frac{R_{xx}}{2L_{\mathrm{v}}}e^{-|x|/L_{\mathrm{v}}}\sin^2(\theta_{\mathrm{VH}})-\frac{2R_{xx}}{\pi W}\ln\left|\tanh\left(\frac{\pi x}{2W}\right)\right|\cos^2(\theta_{\mathrm{VH}})$$

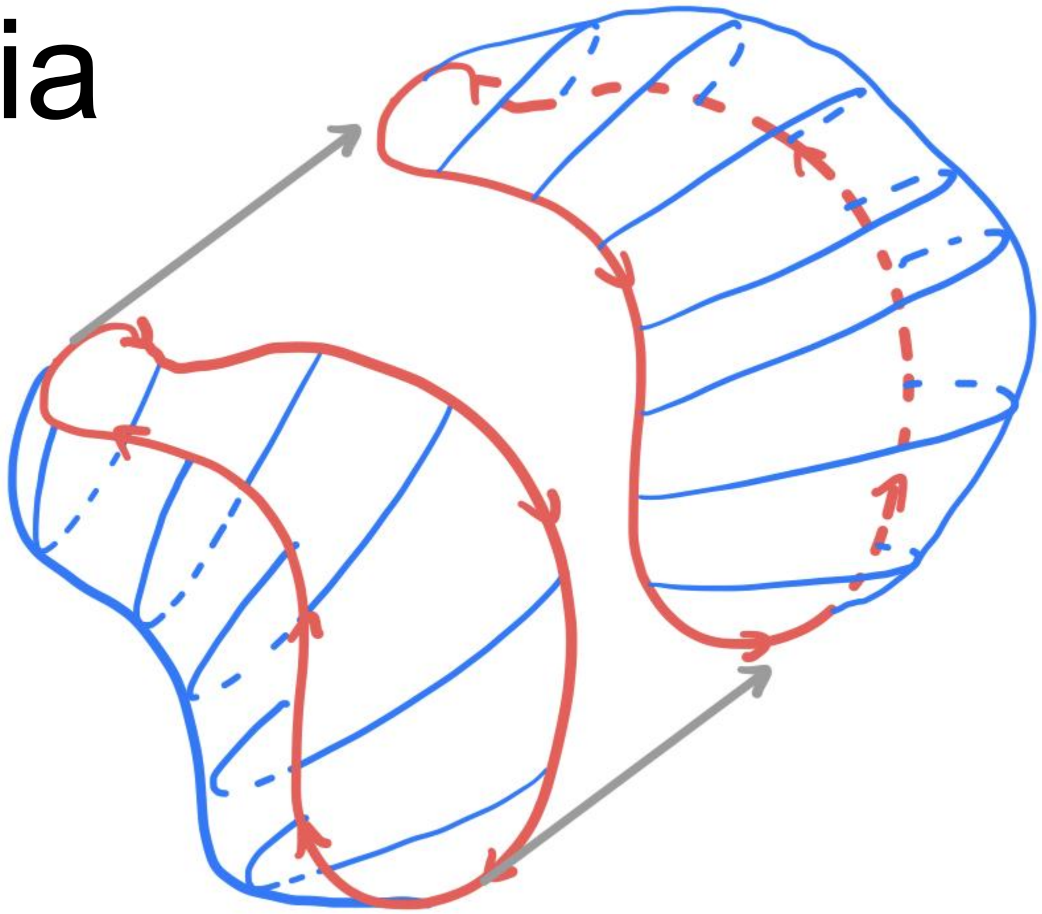

Graphene response
Ohmic respone
$\log(R_{NL})$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
x/l_v



M. Beconcini, F. Taddei, and M. Polini, "Nonlocal topological valley transport at large valley hall angles," *Physical Review B*, vol. **94**, no. 12, p. 121 408, 2016.


$\tilde{R}_{NL}(x)$
$R_{NL}(x)$ (numerical)
$\log(R_{NL})$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
x/l_v

Berryologia

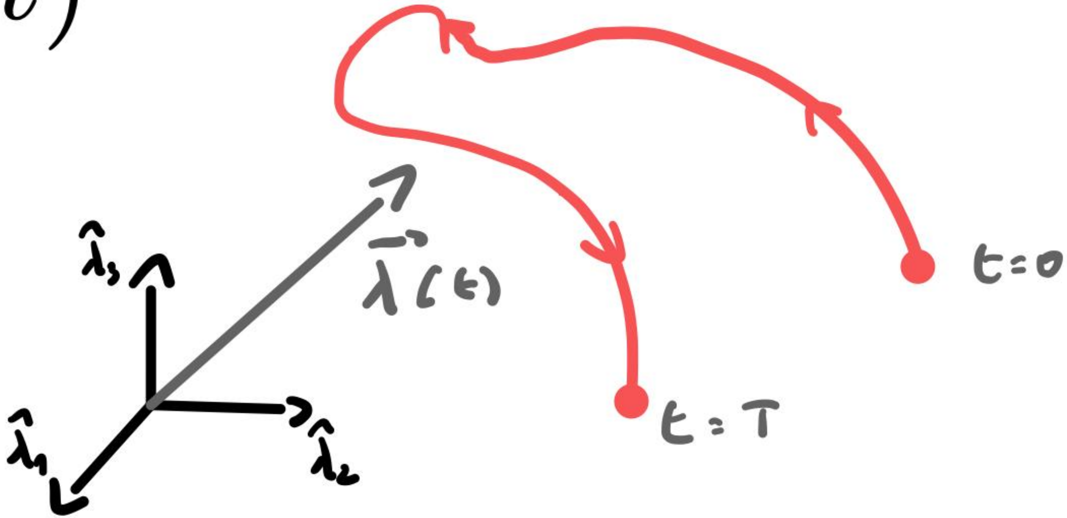


$$H(\boldsymbol{\lambda})\,|n,\boldsymbol{\lambda}\rangle = E_n(\boldsymbol{\lambda})\,|n,\boldsymbol{\lambda}\rangle$$

$$\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots)$$

$$|n,\boldsymbol{\lambda}\rangle \to \underbrace{e^{i\gamma_n(\boldsymbol{\lambda})}}_{\text{Berry phase}}\,|n,\boldsymbol{\lambda}\rangle$$

$$\boldsymbol{\lambda} = \boldsymbol{\lambda}(t)$$



$$\boldsymbol{\lambda} = \boldsymbol{\lambda}(t) \quad |n, \boldsymbol{\lambda}\rangle \to e^{i\gamma_n(\boldsymbol{\lambda}(t))} |n, \boldsymbol{\lambda}\rangle$$

$$|\psi_n(t)\rangle = \underbrace{e^{i\gamma_n(\boldsymbol{\lambda}(t))}}_{\text{Berry phase}} \cdot \underbrace{e^{-\frac{i}{\hbar}\int_0^t E_n(\boldsymbol{\lambda}(t'))dt'}}_{\text{dynamical phase}} |n, \boldsymbol{\lambda}(t)\rangle \, ,$$

$$\dot{\gamma}_n(t) = i \langle n, t | \partial_t | n, t \rangle$$

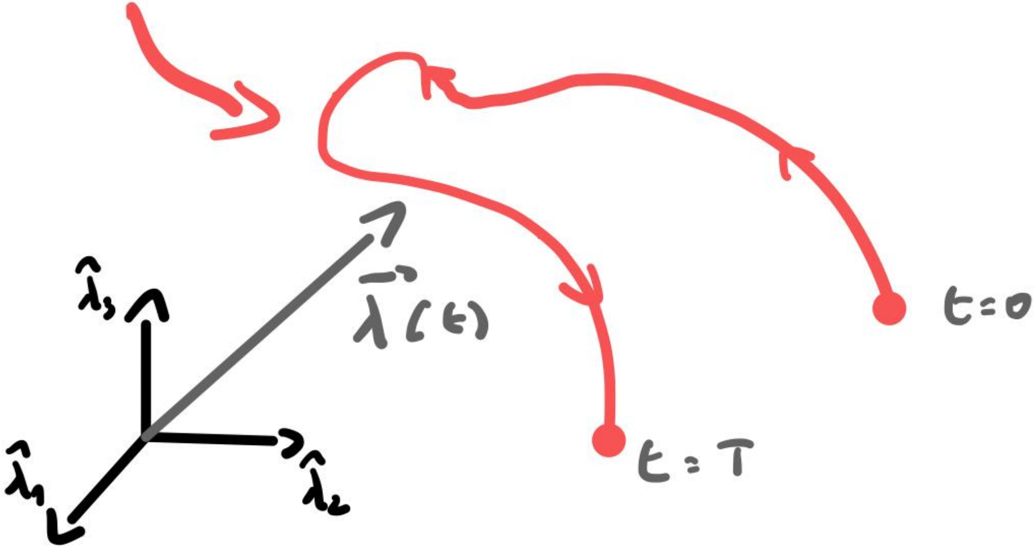
$$\dot{\gamma}_n(t) = \dot{\boldsymbol{\lambda}} \cdot \boldsymbol{A}_n(\boldsymbol{\lambda})$$

$$\boldsymbol{A}_n(\boldsymbol{\lambda}) = i \langle n, t | \partial_{\boldsymbol{\lambda}} | n, t \rangle$$

$$\dot{\gamma}_n(t) = \dot{\boldsymbol{\lambda}} \cdot \boldsymbol{A}_n(\boldsymbol{\lambda})$$

$$\gamma_n = \int_{\mathcal{P}} \mathbf{A}_n(\boldsymbol{\lambda}) \cdot d\boldsymbol{\lambda}$$

$$\gamma_n = \int_P \vec{A}_n(\vec{\lambda}) \cdot d\vec{\lambda}$$

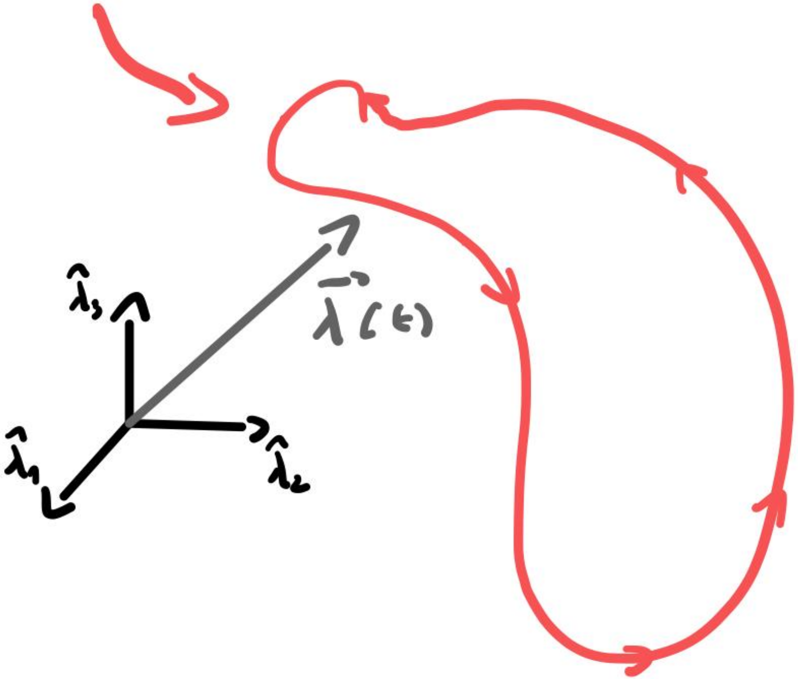


$$|n, \boldsymbol{\lambda}\rangle \rightarrow e^{if_n(\boldsymbol{\lambda})}\, |n, \boldsymbol{\lambda}\rangle$$

$$\mathbf{A}_n \rightarrow \mathbf{A}_n - \partial_{\boldsymbol{\lambda}} f_n$$

$$\gamma_n = \int_{\mathcal{P}} \boldsymbol{A}_n(\boldsymbol{\lambda}) \cdot d\boldsymbol{\lambda}$$

$$\gamma_n = \oint_P \vec{A}_n(\vec{\lambda}) \cdot d\vec{\lambda}$$



$$\gamma_n = \oint_{\mathcal{P}} \mathbf{A}_n(\boldsymbol{\lambda}) \cdot d\boldsymbol{\lambda} + 2n\pi$$

$$\Omega^n_{\mu\nu} = \partial_\mu A^n_\nu(\boldsymbol{\lambda}) - \partial_\nu A^n_\mu(\boldsymbol{\lambda})$$

Berry Curvature

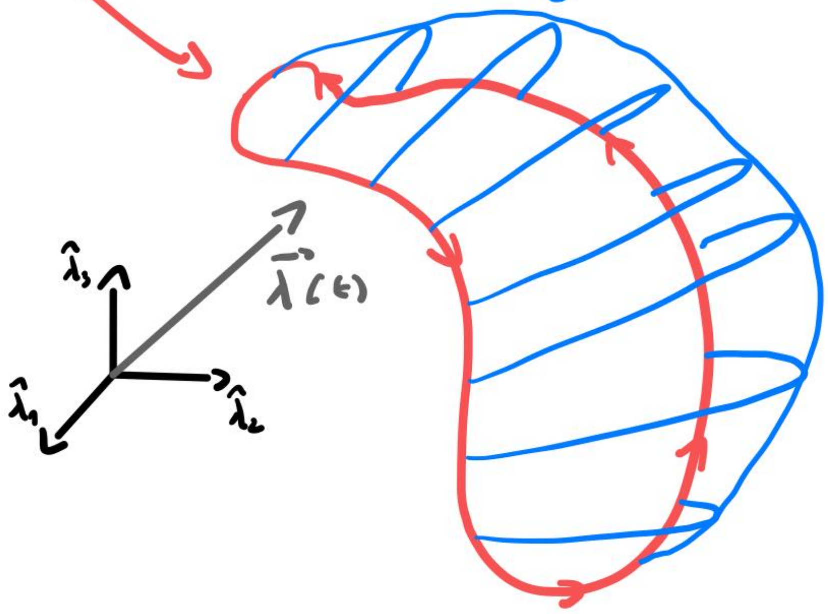
$$\Omega^n_{\mu\nu} = \partial_\mu A^n_\nu(\boldsymbol{\lambda}) - \partial_\nu A^n_\mu(\boldsymbol{\lambda})$$

$$\Omega^n_{\mu\nu} = i \sum_{n' \neq n} \frac{\langle n| \, \partial_\mu H \, |n'\rangle \, \langle n'| \, \partial_\nu H \, |n\rangle}{(E_{n'} - E_n)^2} - (\mu \leftrightarrow \nu)$$

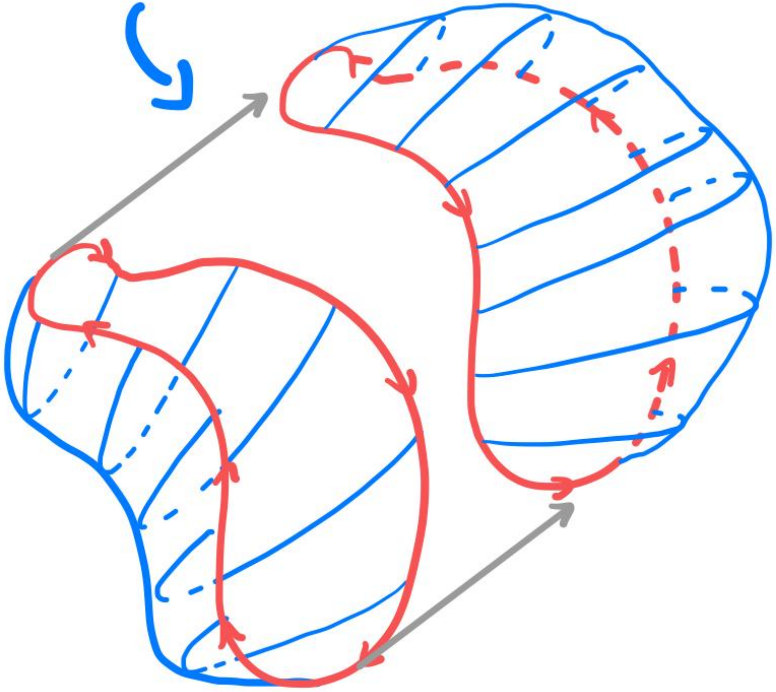
$$\gamma_n = \oint_{\partial} \vec{A}_n(\vec{\lambda}) \cdot d\vec{\lambda} = \frac{1}{2} \int_S \Omega^{n}_{\mu\nu} \, d\lambda^{\mu} \wedge d\lambda^{\nu}$$

Curva di integrazion

Superfice di integrazione



$$\oint_S \Omega^n_{\mu\nu}\, d\lambda^\mu \wedge d\lambda^\nu$$



$$\oint_S \Omega^n_{\mu\nu} d\lambda^\mu \wedge d\lambda^\nu = 2\pi C_n \qquad C_n \in \mathbb{Z}$$

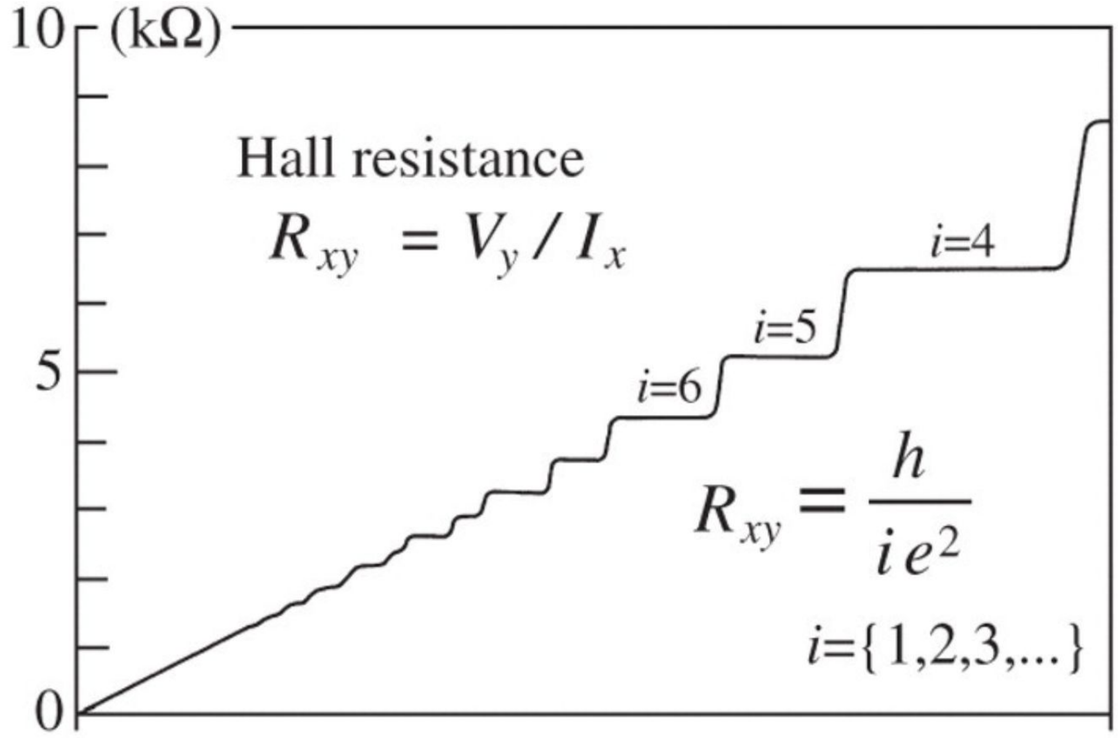
$$\sigma_{xy} = i\frac{e^2}{\hbar} \sum_{m, E_n < E_F} \int_{\mathcal{B}} \frac{\langle n, \boldsymbol{k} | \partial_y H | m, \boldsymbol{k} \rangle \langle m, \boldsymbol{k} | \partial_x H | n, \boldsymbol{k} \rangle - (n \leftrightarrow m)}{E_m(\boldsymbol{k}) - E_n(\boldsymbol{k})} \frac{d^2\boldsymbol{k}}{(2\pi)^2}$$

$$\sigma_{xy} = \frac{e^2}{\hbar} \sum_{E_n < E_F} \int_{\mathcal{B}} \Omega^n_{k_x k_y} \frac{d^2\boldsymbol{k}}{(2\pi)^2}$$

D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs, "Quantized hall conductance in a two-dimensional periodic potential," *Physical review letters*, vol. **49**, no. 6, p. 405, 1982.

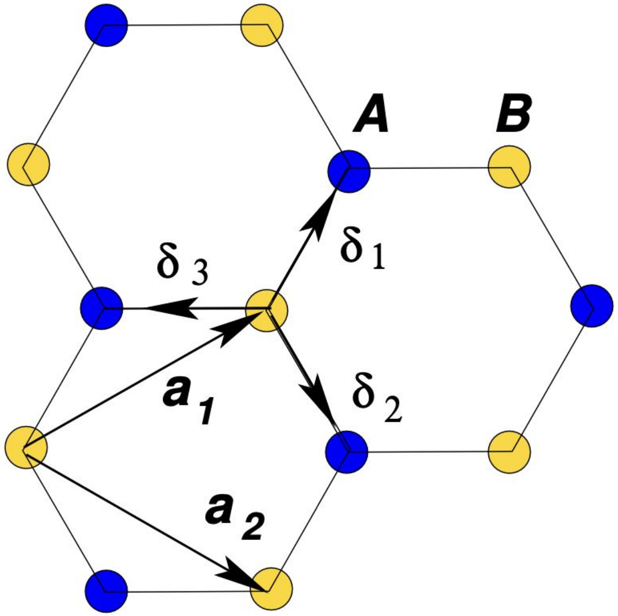
$$\sigma_{xy} = \frac{e^2}{2\pi\hbar}\sum_{n}^{N} C_n$$

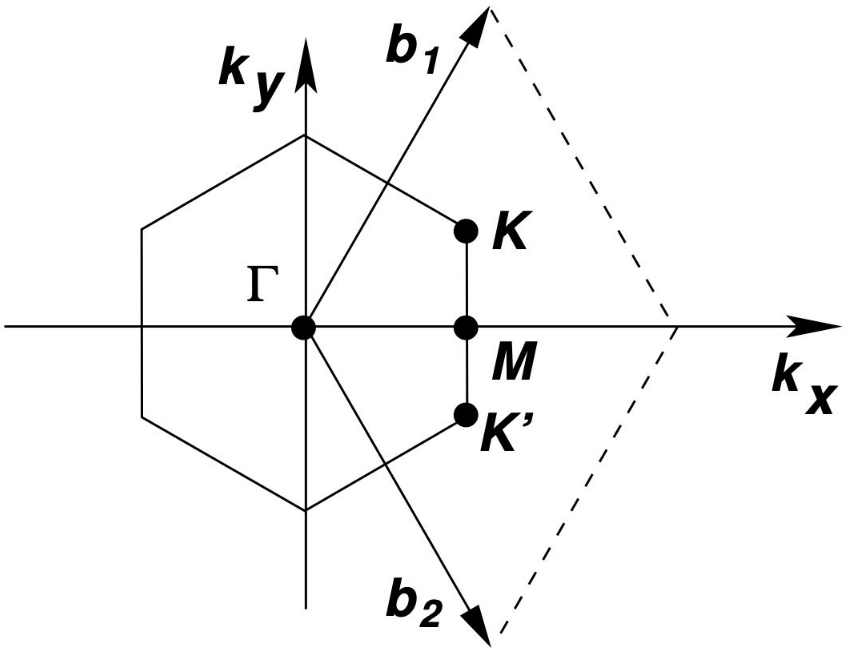
Quantized conductivity

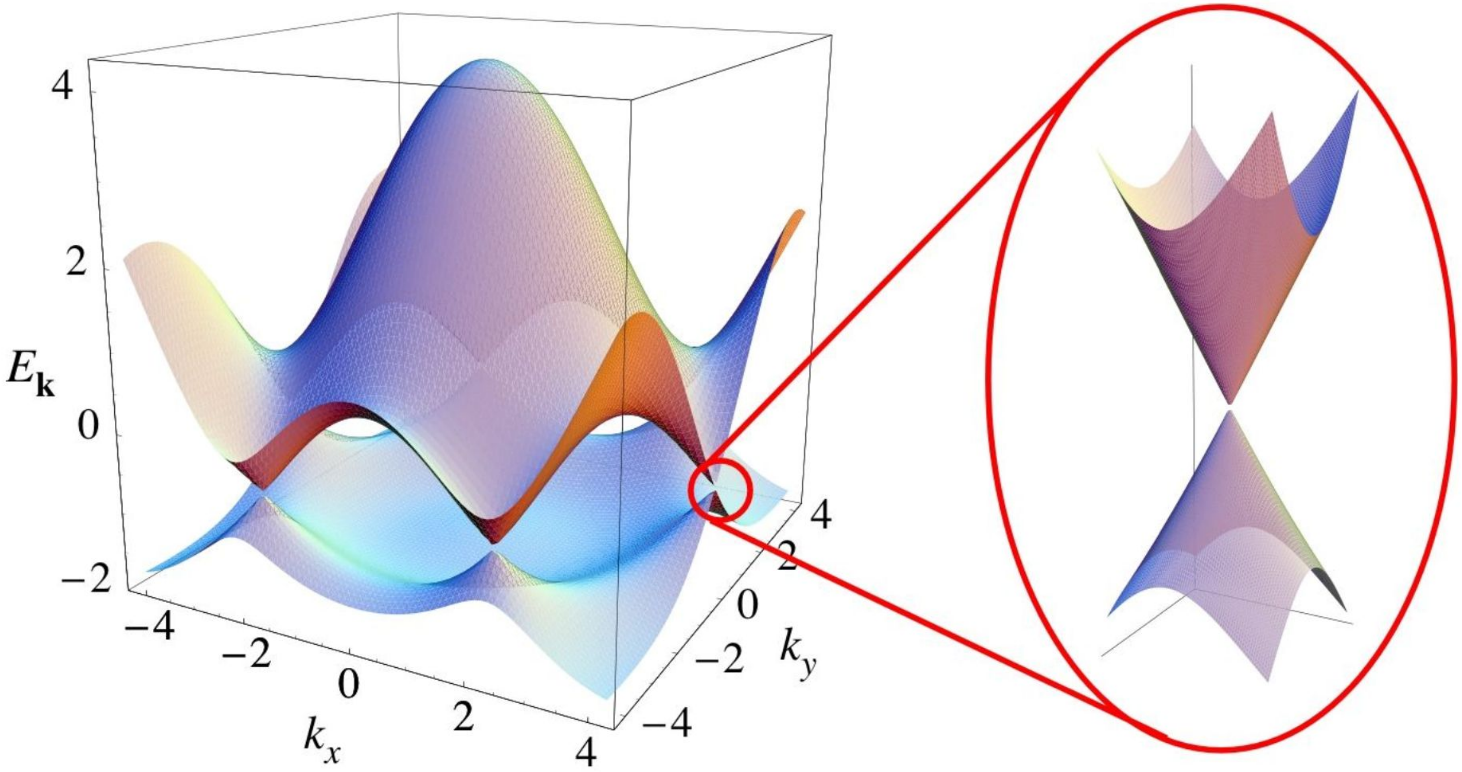


$$t \to -t \quad \Omega(\boldsymbol{k}) \to -\Omega(-\boldsymbol{k})$$
$$\boldsymbol{r} \to -\boldsymbol{r} \quad \Omega(\boldsymbol{k}) \to \Omega(-\boldsymbol{k})$$

Grafene







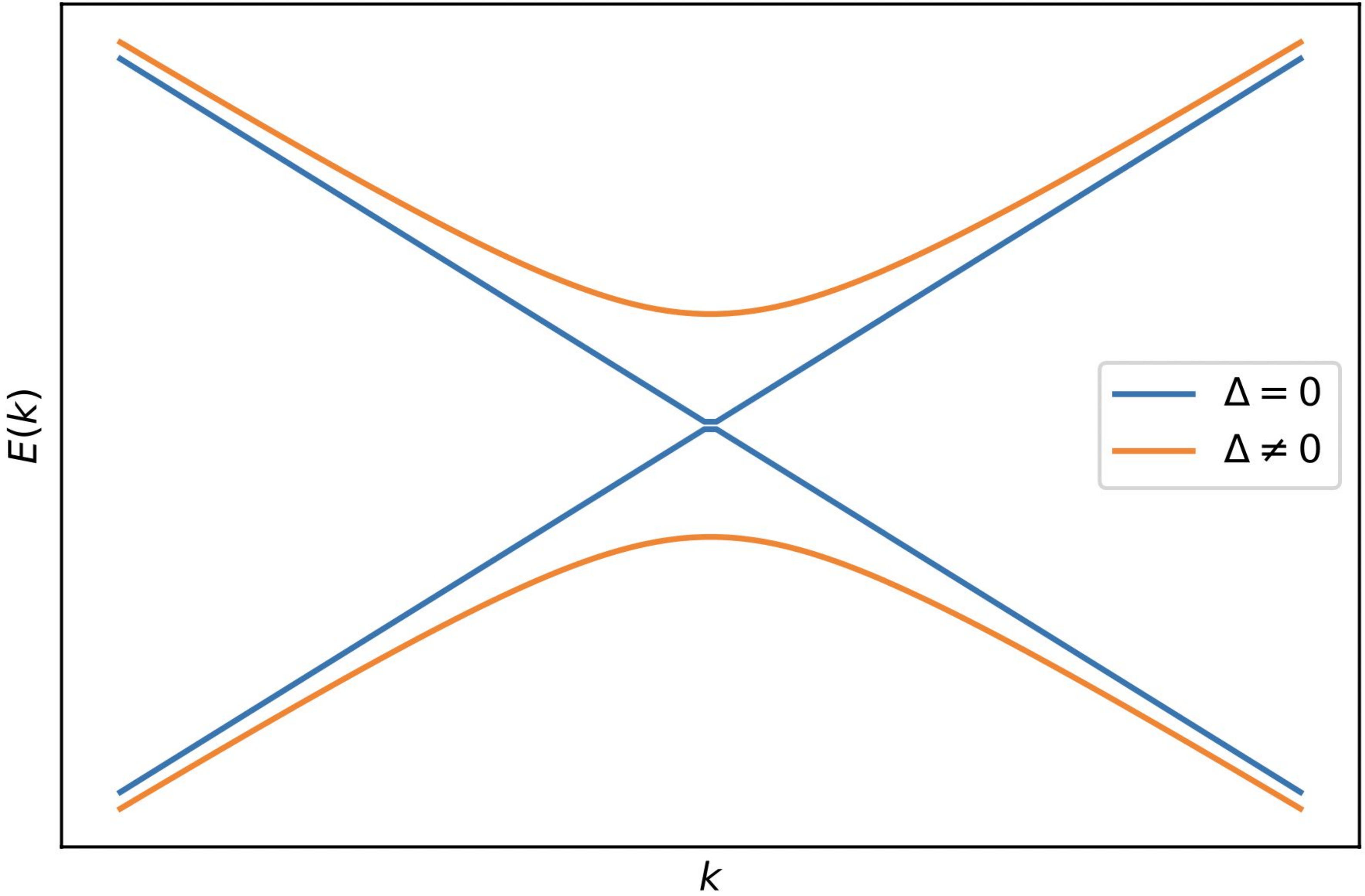
A. C. Neto, F. Guinea, N. M. Peres, K. S. Novoselov, and A. K. Geim, "The electronic properties of graphene," *Reviews of modern physics*, vol. **81**, no. 1, p. 109, 2009.

$$H_{\boldsymbol{K}_0}(\boldsymbol{k}) = -H^*_{\boldsymbol{K}_1}(\boldsymbol{k}) = \begin{bmatrix} 0 & v_F\hbar(k_x - ik_y) \\ v_F\hbar(k_x + ik_y) & 0 \end{bmatrix}$$

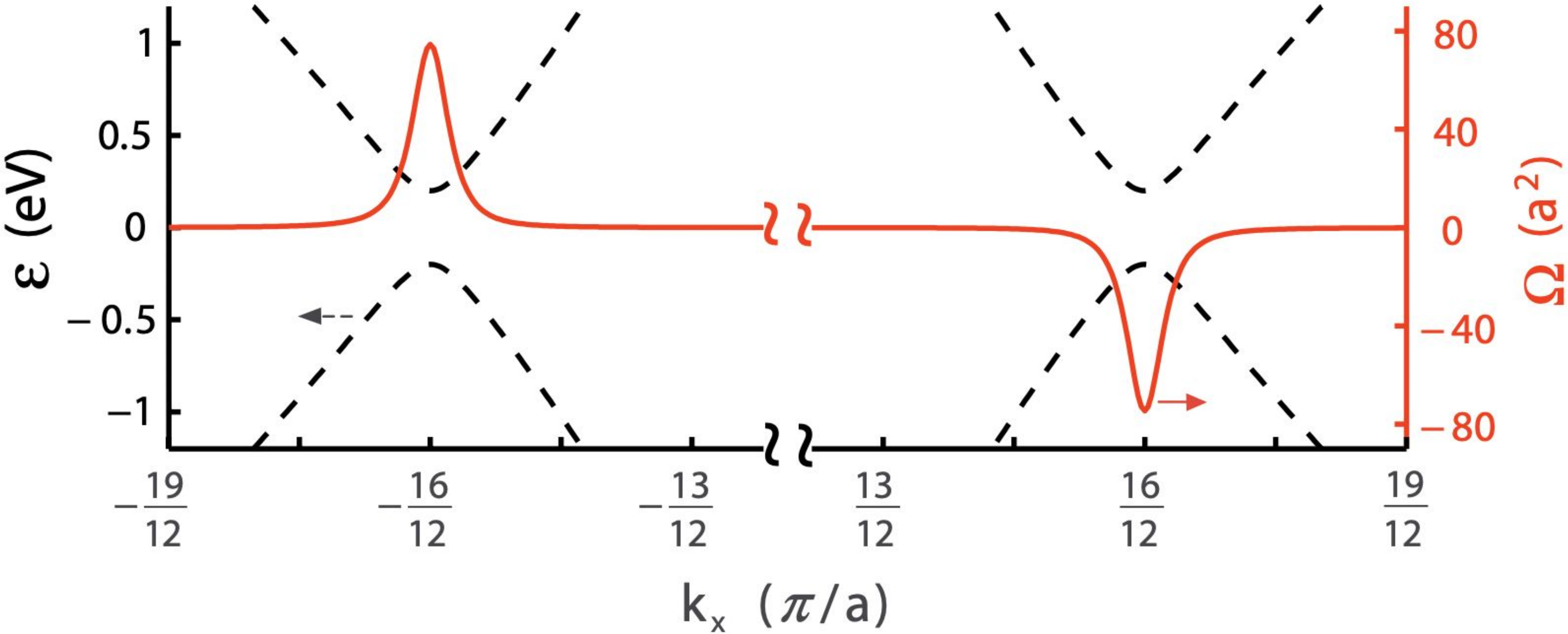
$$H(\boldsymbol{k}) = \hbar v_{\mathrm{F}}(\sigma_x \tau_z k_x + \sigma_y k_y)$$

$$H(\boldsymbol{k}) = \begin{bmatrix} \Delta & v_F\hbar(\tau_z k_x - ik_y) \\ v_F\hbar(\tau_z k_x + ik_y) & -\Delta \end{bmatrix}$$

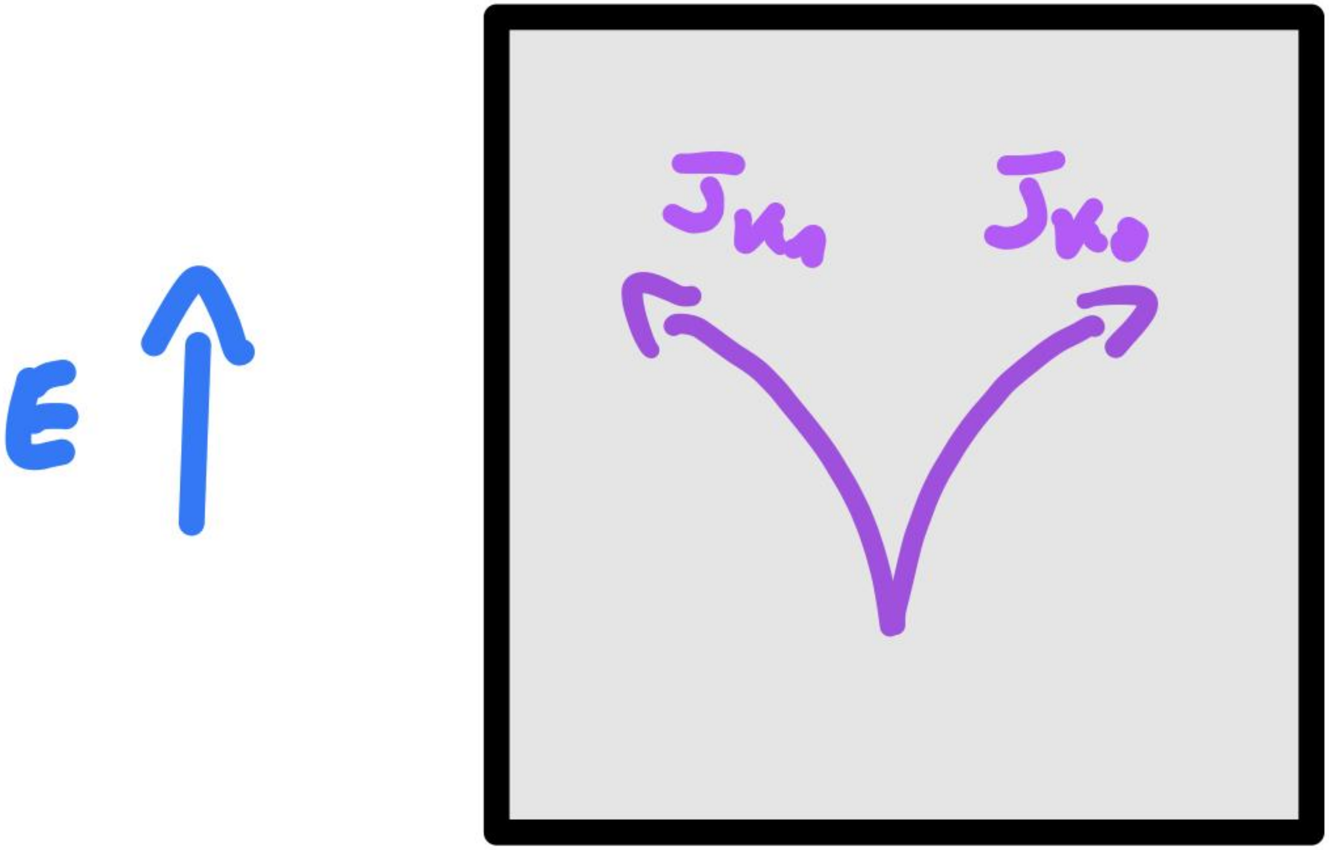
$$H(\boldsymbol{k}) = \hbar v_{\mathrm{F}}(\sigma_x \tau_z k_x + \sigma_y k_y) + \Delta\sigma_z$$

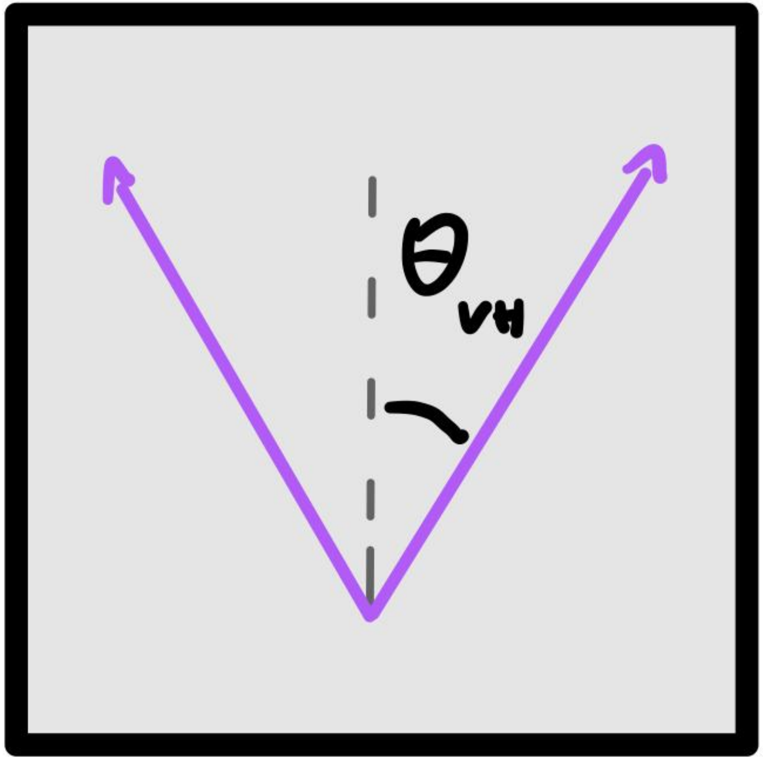


$E(k)$
k
$\Delta = 0$
$\Delta \neq 0$



ε (eV)
1
0.5
0
−0.5
−1
Ω (a²)
80
40
0
−40
−80
−19/12
−16/12
−13/12
13/12
16/12
19/12
k_x (π/a)


E
$\bar{J}_{K1}$
$\bar{J}_{K0}$



θ_{VH}

$$\nabla\psi_{K_\alpha}(\mathbf{r}) = \nabla V(\mathbf{r}) - \frac{1}{e}\frac{\partial}{\partial n_{K_\alpha}}\mu_\alpha(n_{K_\alpha}(\mathbf{r}))\nabla n_{K_\alpha}$$

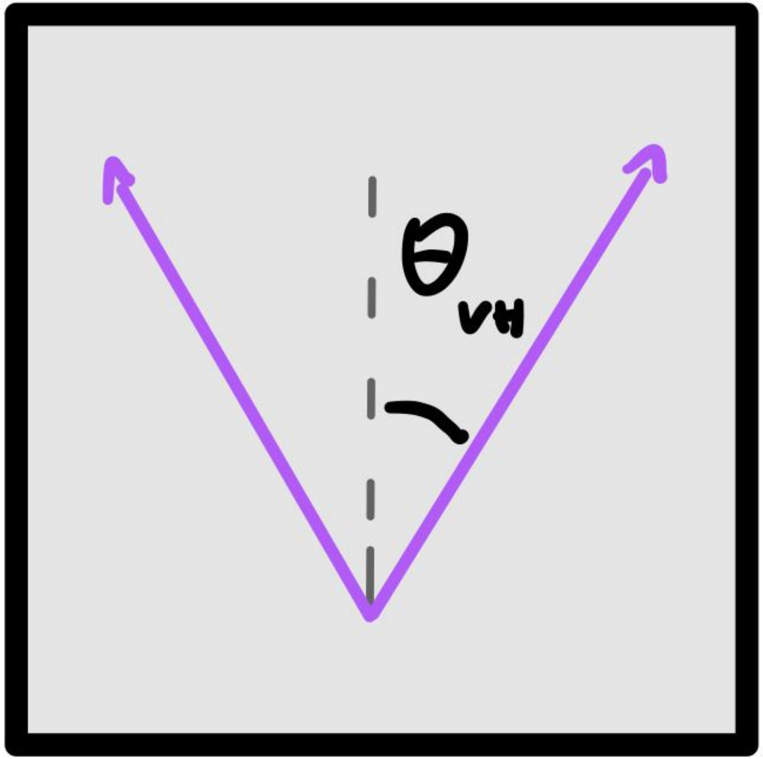
$$-e\mathbf{J}_{K_\alpha}(\mathbf{r}) = \sigma_{K_\alpha}(\mathbf{r})\nabla\psi_{K_\alpha}(\mathbf{r})$$

$$\sigma_{\boldsymbol{K}_0,ij} = \sigma_{xx} \begin{bmatrix} 1 & \tan(\theta_{\mathrm{VH}}) \\ -\tan(\theta_{\mathrm{VH}}) & 1 \end{bmatrix}$$

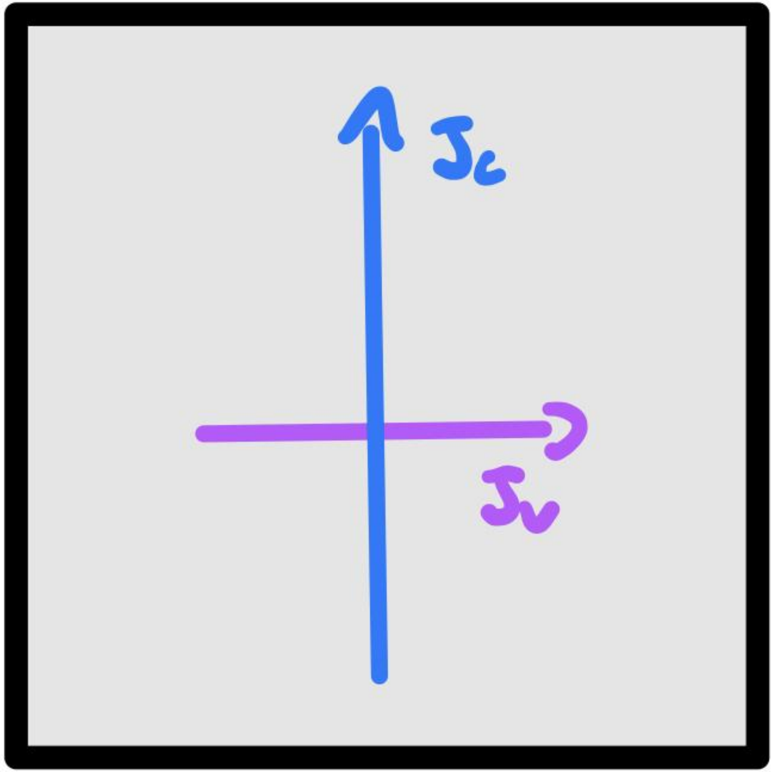
$$\sigma_{\boldsymbol{K}_1,ij} = \sigma_{xx} \begin{bmatrix} 1 & -\tan(\theta_{\mathrm{VH}}) \\ \tan(\theta_{\mathrm{VH}}) & 1 \end{bmatrix}$$

$$-eJ_{K_\alpha,i}(\mathbf{r}) = \sigma_{K_\alpha,ij}E_j(\mathbf{r}) - eD_{K_\alpha,ij}\partial_j n_{K_\alpha}(\mathbf{r})$$

$$e^2 D_{K_\alpha,ij} = \sigma_{K_\alpha,ij}\frac{\partial \mu_\alpha}{\partial n_{K_\alpha}}[n_{K_\alpha}(\mathbf{r})]$$



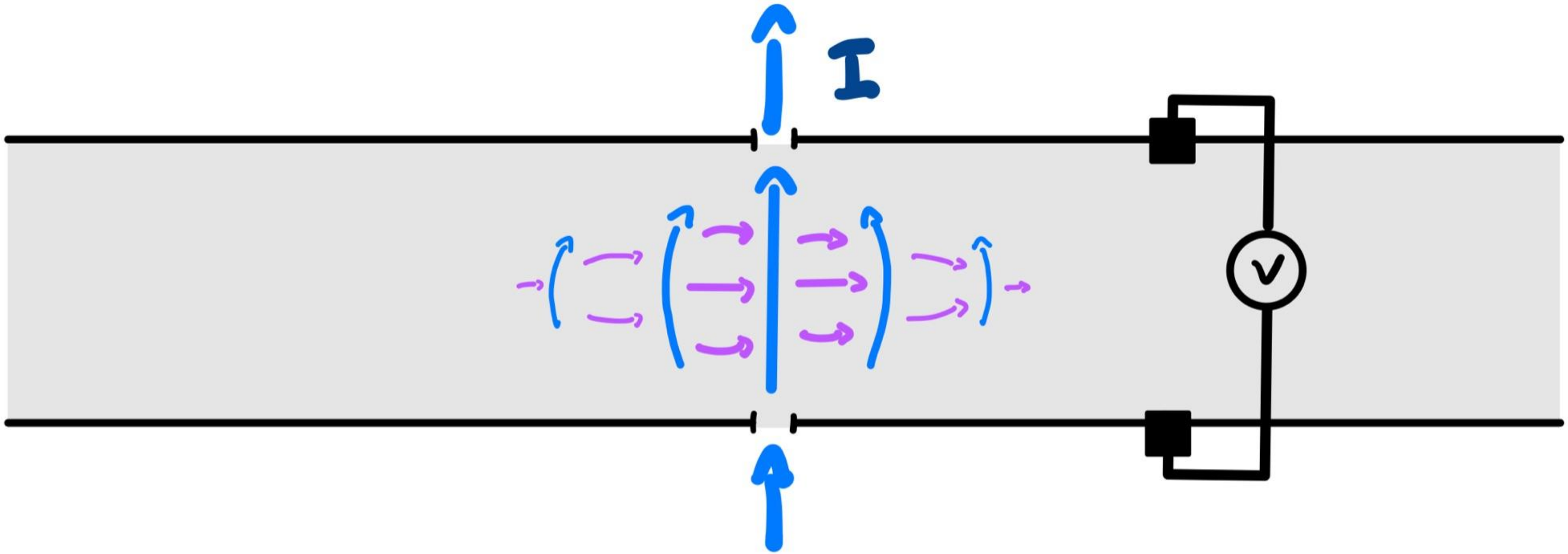
θ_{VH}



J_c
J_v

$$J_{c,i}(\boldsymbol{r}) = \sum_j \sigma_{c,xx}\delta_{ij}E_i(\boldsymbol{r}) + eD_{cv,xy}\epsilon_{ij}\partial_j n_v(\boldsymbol{r})$$

$$J_{v,i}(\mathbf{r}) = \sum_j \sigma_{c,xy}\epsilon_{ij}E_j(\mathbf{r}) + eD_{v,xx}\delta_{ij}\partial_j n_v(\mathbf{r})$$

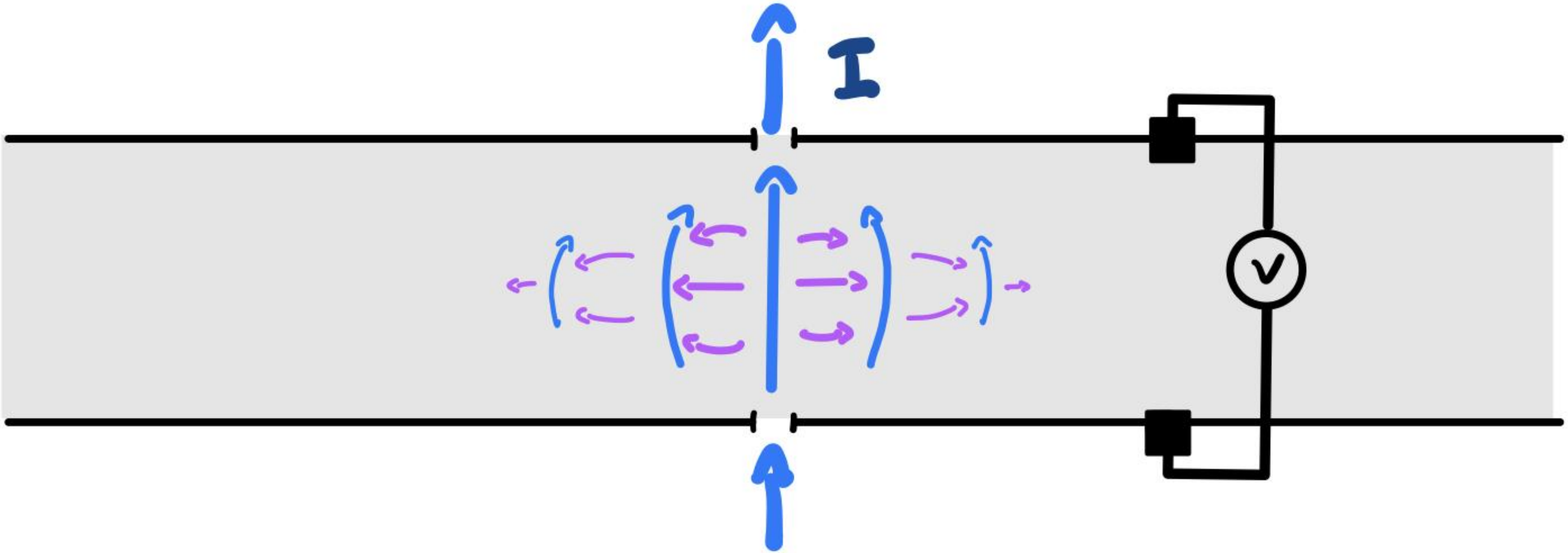

I
V

$$\nabla^2 \boldsymbol{J}_{\mathrm{c}} = 0$$

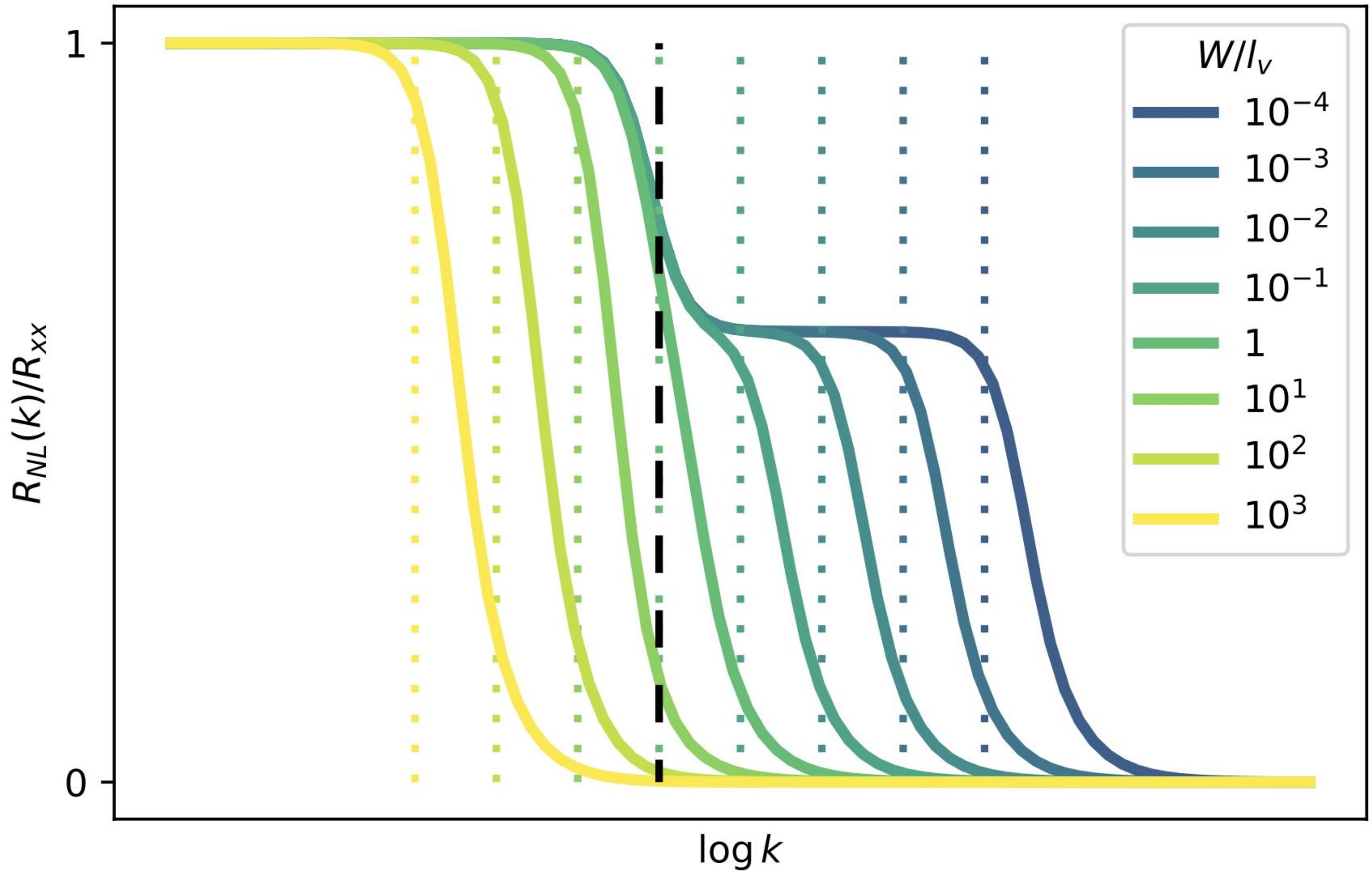
$$\nabla^2 \boldsymbol{J}_{\mathrm{v}} = \frac{\partial n_{\mathrm{v}}}{\partial t}$$

$$\nabla^2 V(x, y) = 0$$

$$(\nabla^2 - l_{\mathrm{v}}^{-2}) n_v(\boldsymbol{r}) = 0$$


I
V

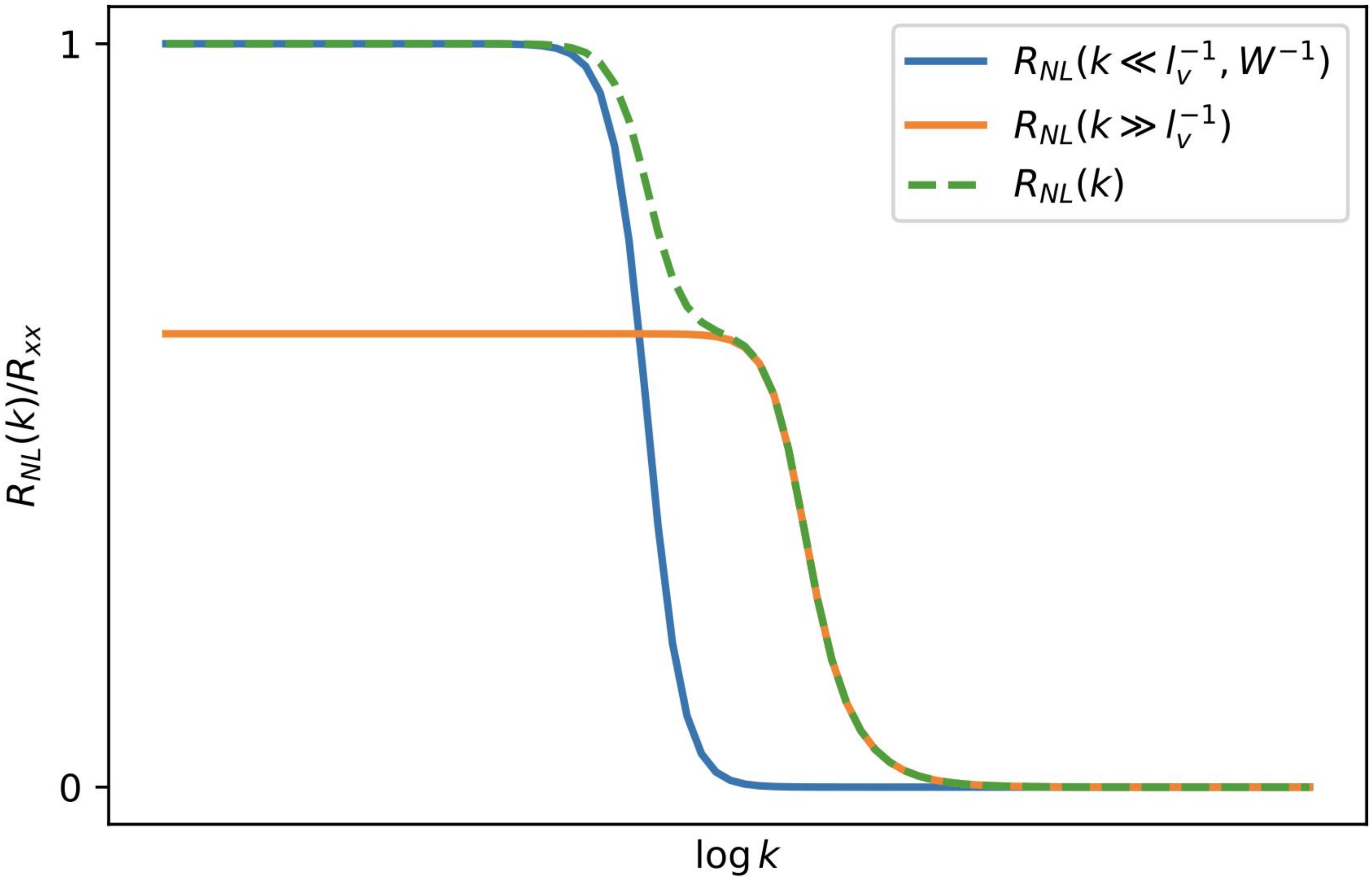
$$R_{\mathrm{NL}}(k) = \frac{2\omega(k)}{k\sigma_c}\left\{\frac{\omega(k)}{\tanh(kW/2)} + \frac{k\tan^2(\theta_{\mathrm{VH}})}{\tanh[\omega(k)W/2]}\right\}^{-1}$$



1
0
$R_{NL}(k)/R_{xx}$
$\log k$
W/l_v
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10^1
10^2
10^3

$$R_{\mathrm{NL}}(k \gg l_{\mathrm{v}}^{-1}) \approx \cos^2(\theta_{\mathrm{VH}}) R_{\mathrm{NL}}^{(0)}(k)$$

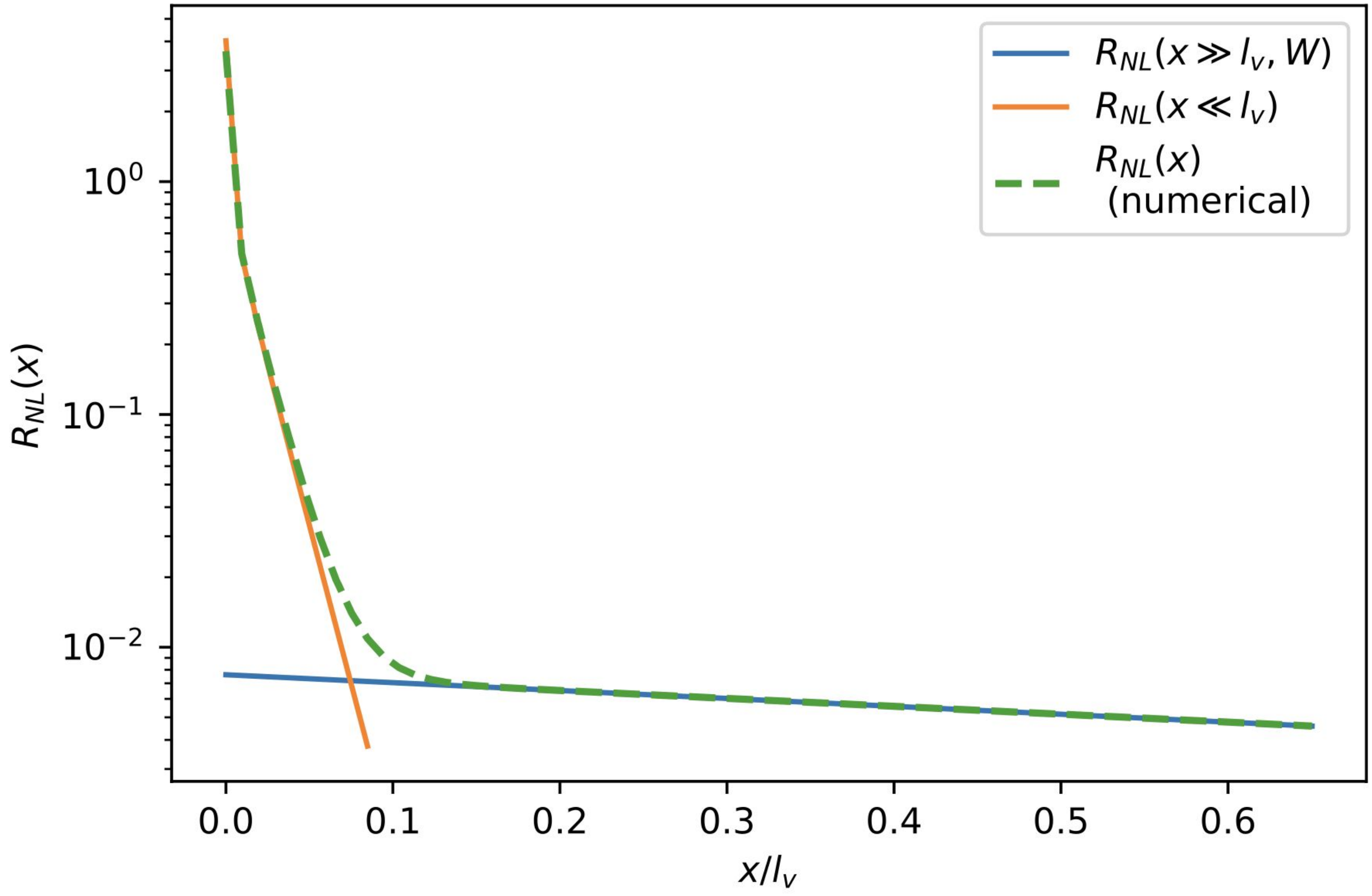
$$R_{\mathrm{NL}}(k \ll l_{\mathrm{v}}^{-1}, W^{-1}) \approx R_{\mathrm{NL}}^{\mathrm{T}}(k) = \frac{R_{xx}}{1 + L_{\mathrm{v}}^2 k^2}$$

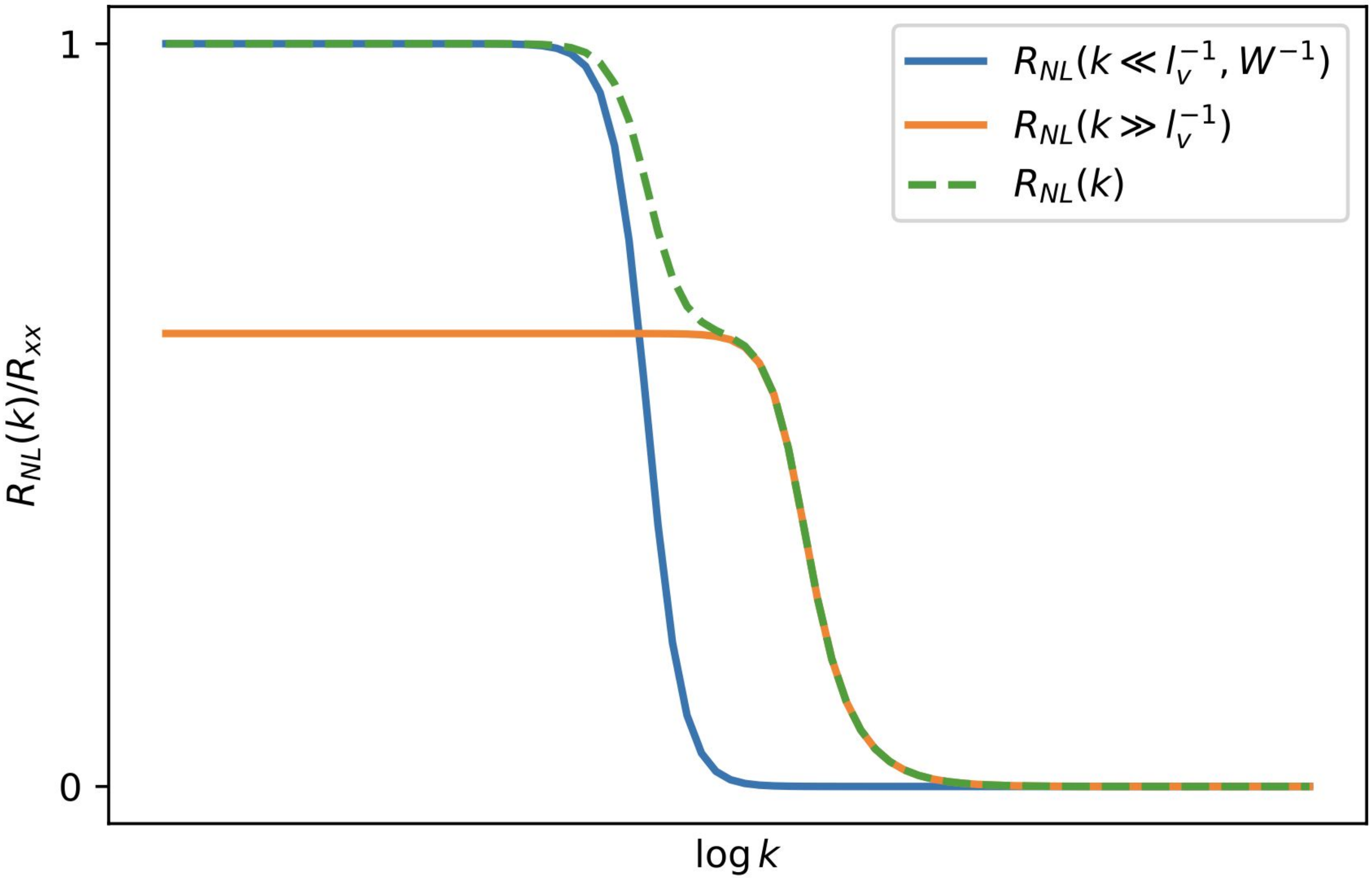


1
0
$R_{NL}(k)/R_{xx}$
$\log k$
$R_{NL}(k \ll l_v^{-1}, W^{-1})$
$R_{NL}(k \gg l_v^{-1})$
$R_{NL}(k)$

$$\cos^2(\theta_{\mathrm{VH}})R^0_{\mathrm{NL}}(x) = -\cos^2(\theta_{\mathrm{VH}})\frac{2\rho}{\pi}\ln\left|\tanh\left(\frac{\pi x}{2W}\right)\right|$$

$$R^T_{\mathrm{NL}}(x) = R_{xx}\int_{-\infty}^{+\infty}\frac{e^{-ikx}}{1+L_{\mathrm{v}}^2k^2}\frac{dk}{2\pi} = \frac{R_{xx}}{2L_{\mathrm{v}}}e^{-\frac{|x|}{L_{\mathrm{v}}}}$$

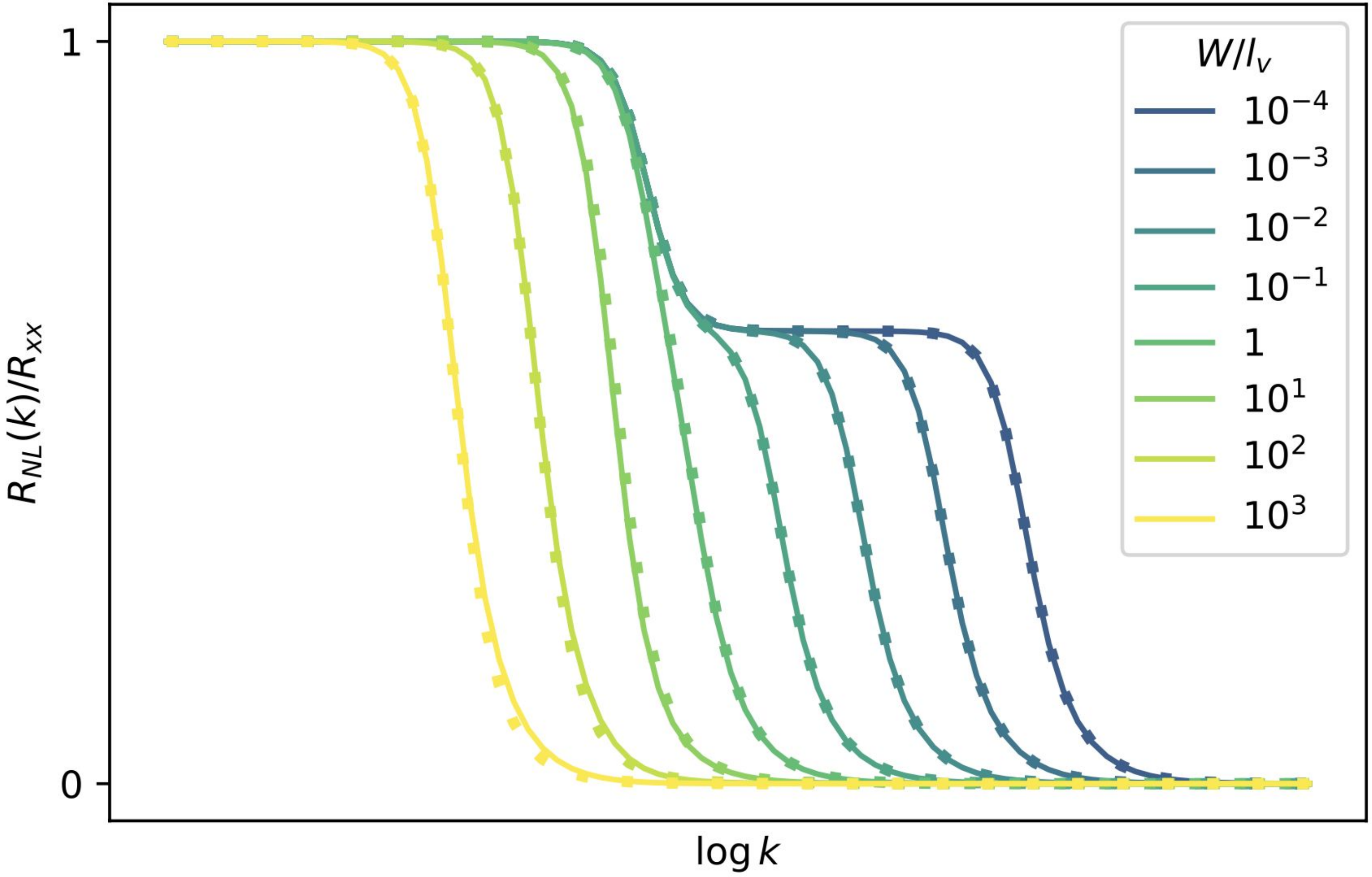

$R_{NL}(x \gg l_v, W)$
$R_{NL}(x \ll l_v)$
$R_{NL}(x)$
(numerical)
10^0
10^{-1}
10^{-2}
$R_{NL}(x)$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
x/l_v



1
0
$R_{NL}(k)/R_{xx}$
$\log k$
$R_{NL}(k \ll l_v^{-1}, W^{-1})$
$R_{NL}(k \gg l_v^{-1})$
$R_{NL}(k)$

$$R_{\mathrm{NL}}(k) \approx \alpha R_{\mathrm{NL}}(k \ll l_{\mathrm{v}}^{-1}, W^{-1}) + \beta R_{\mathrm{NL}}(k \gg l_{\mathrm{v}}^{-1})$$

$$\tilde{R}_{\mathrm{NL}}(k) \equiv \sin^2(\theta_{\mathrm{VH}}) R_{\mathrm{NL}}^{T}(k) + \cos^2(\theta_{\mathrm{VH}}) R_{\mathrm{NL}}^{(0)}(k)$$

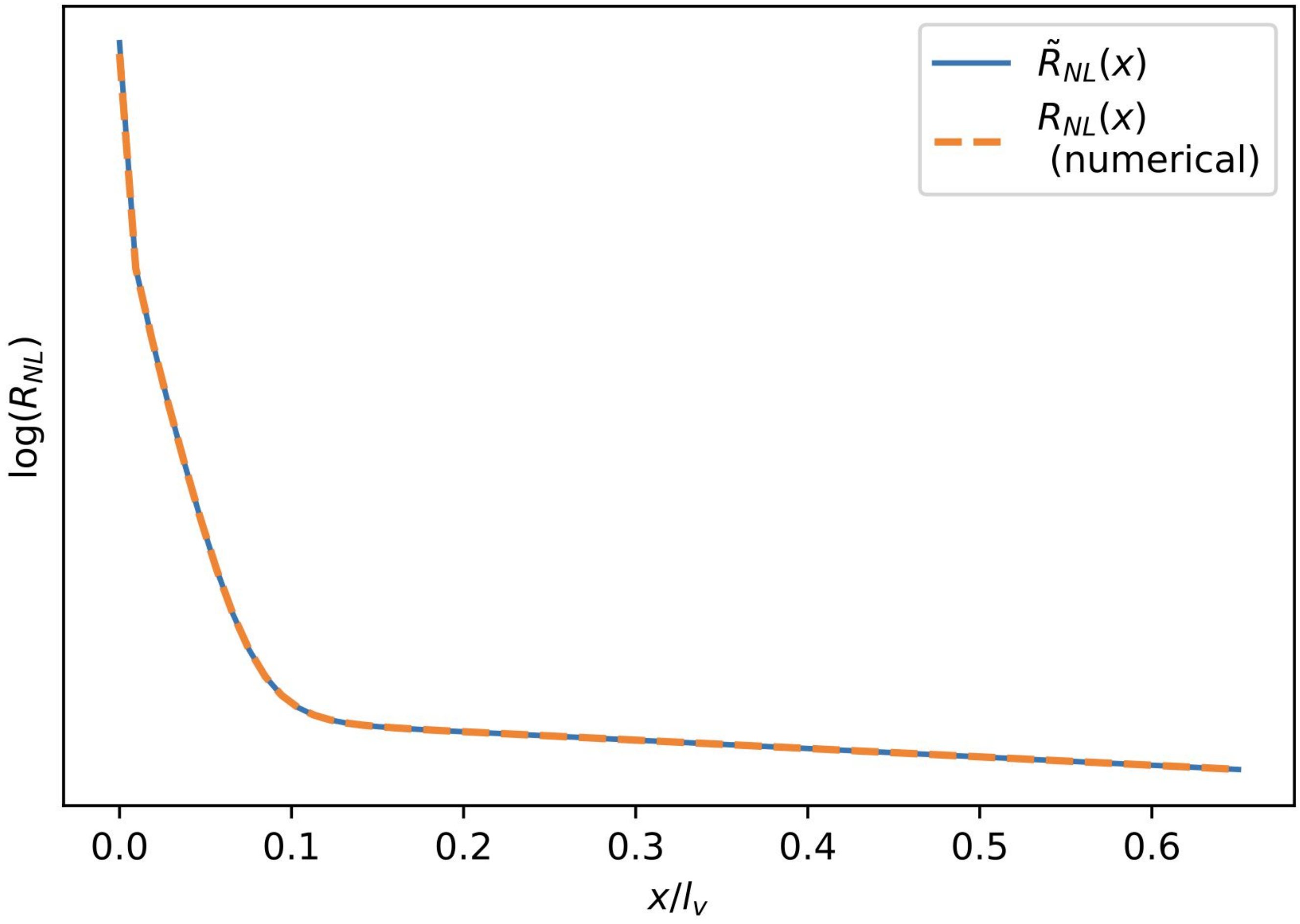


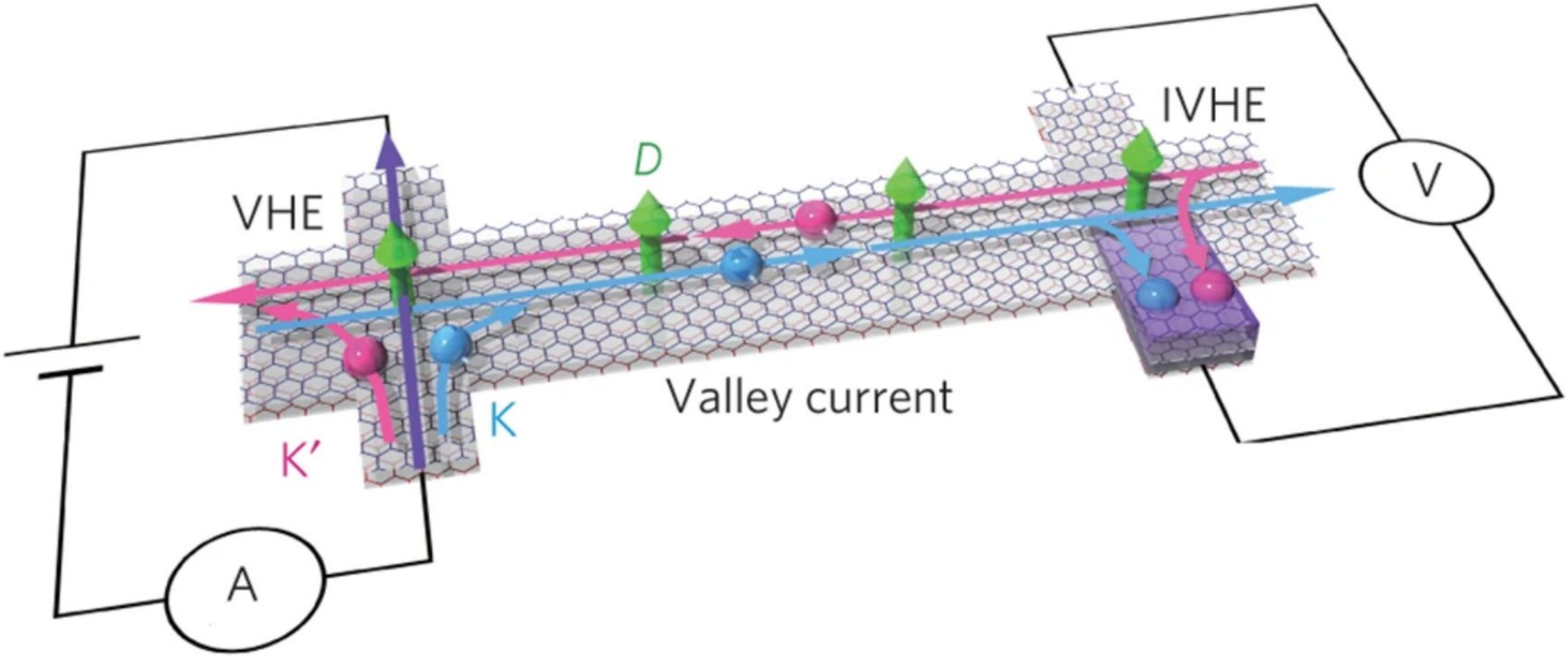
1
0
$R_{NL}(k)/R_{xx}$
$\log k$
W/l_v
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10^1
10^2
10^3

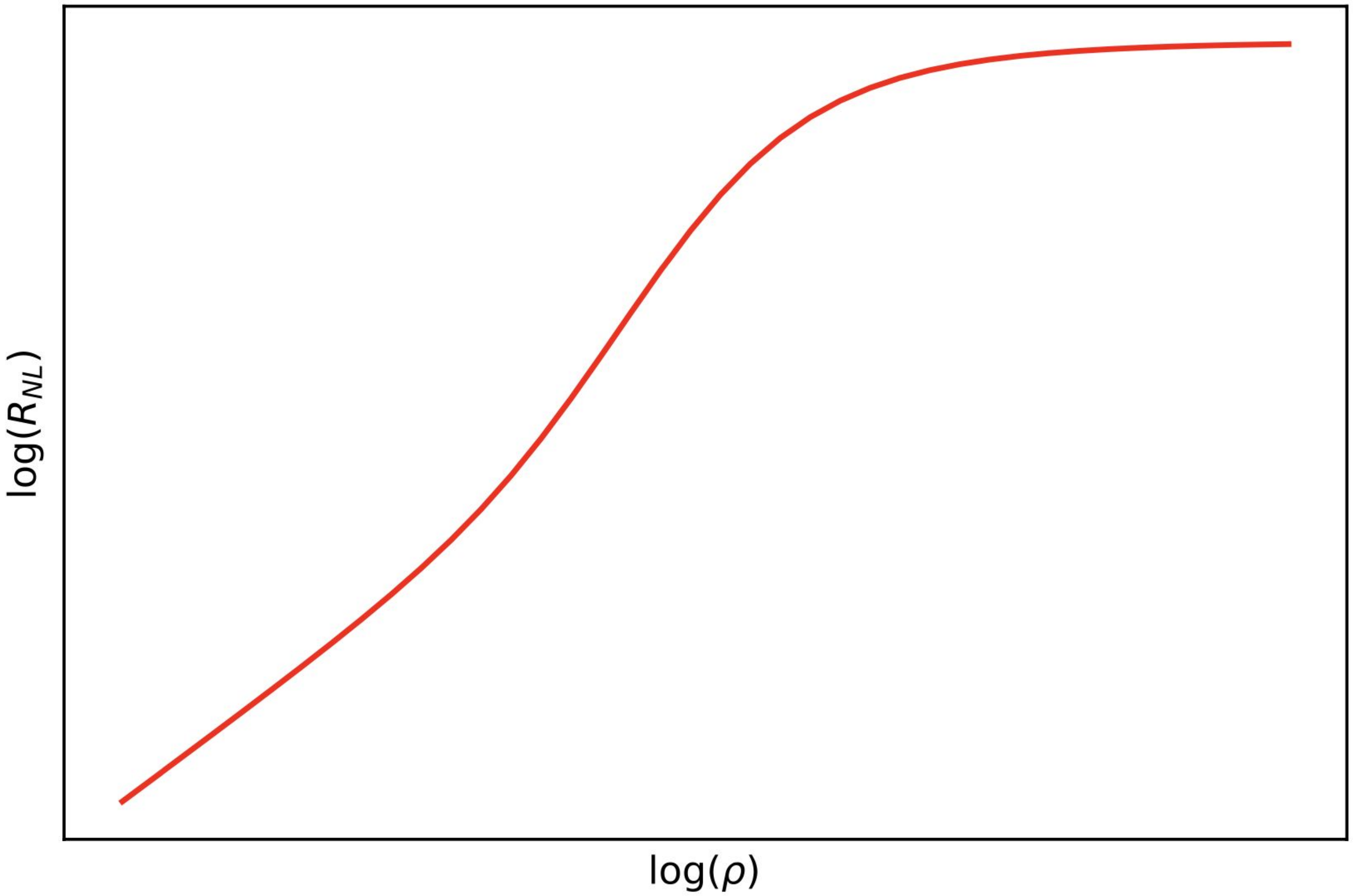
$$\tilde{R}_{\mathrm{NL}}(k) \equiv \sin^2(\theta_{\mathrm{VH}}) R_{\mathrm{NL}}^{T}(k) + \cos^2(\theta_{\mathrm{VH}}) R_{\mathrm{NL}}^{(0)}(k)$$

$$\tilde{R}_{\mathrm{NL}}(x) = \sin^2(\theta_{\mathrm{VH}}) R_{\mathrm{NL}}^{T}(x) + \cos^2(\theta_{\mathrm{VH}}) R_{\mathrm{NL}}^{(0)}(x)$$

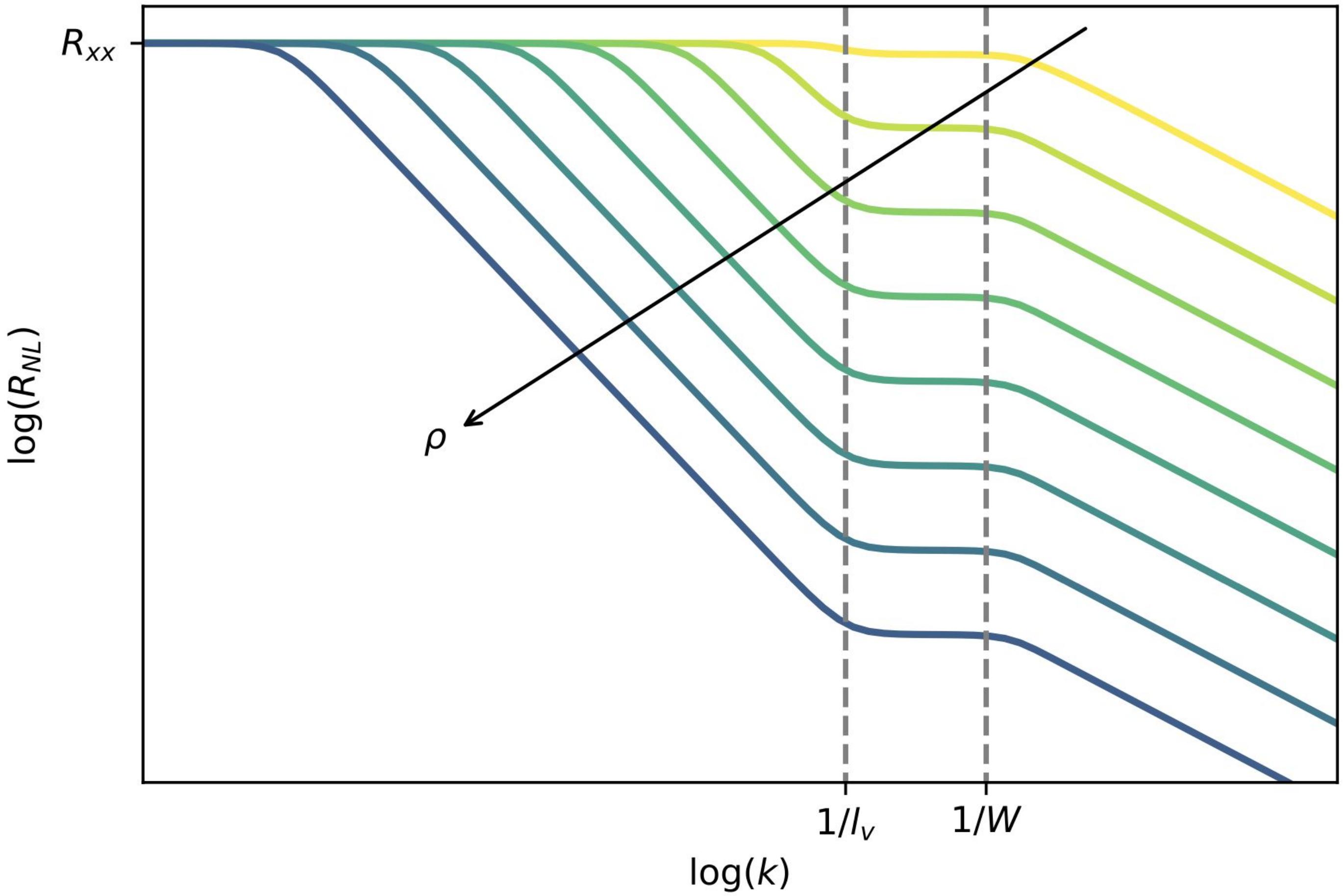
$$\tilde{R}_{\mathrm{NL}}(x) = \frac{R_{xx}}{2L_{\mathrm{v}}} e^{-|x|/L_{\mathrm{v}}} \sin^2(\theta_{\mathrm{VH}}) - \frac{2R_{xx}}{\pi W} \ln \left| \tanh\left(\frac{\pi x}{2W}\right) \right| \cos^2(\theta_{\mathrm{VH}})$$


$\tilde{R}_{NL}(x)$
$R_{NL}(x)$ (numerical)
$\log(R_{NL})$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
x/l_v

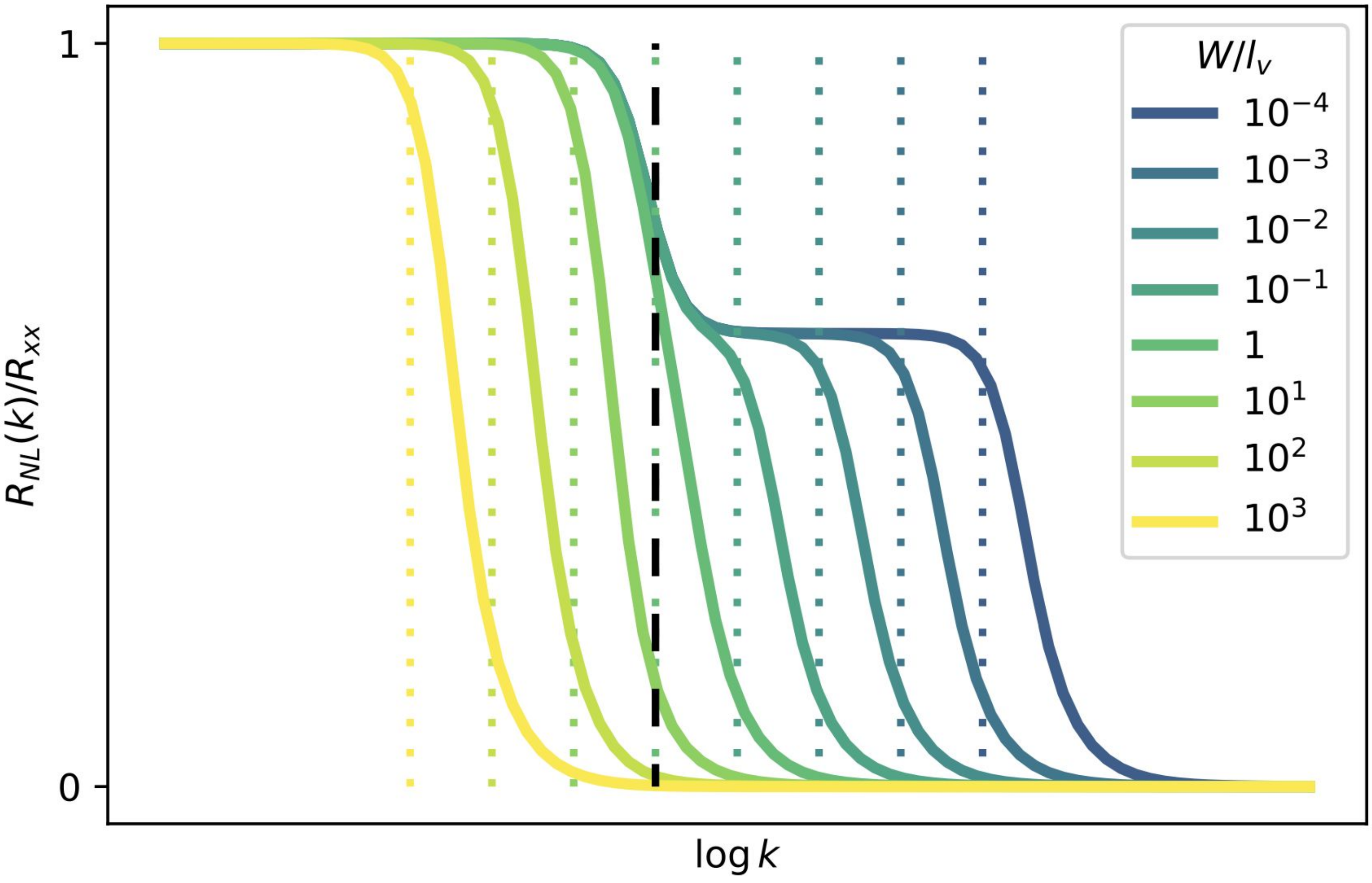

VHE
D
IVHE
V
K'
K
Valley current
A



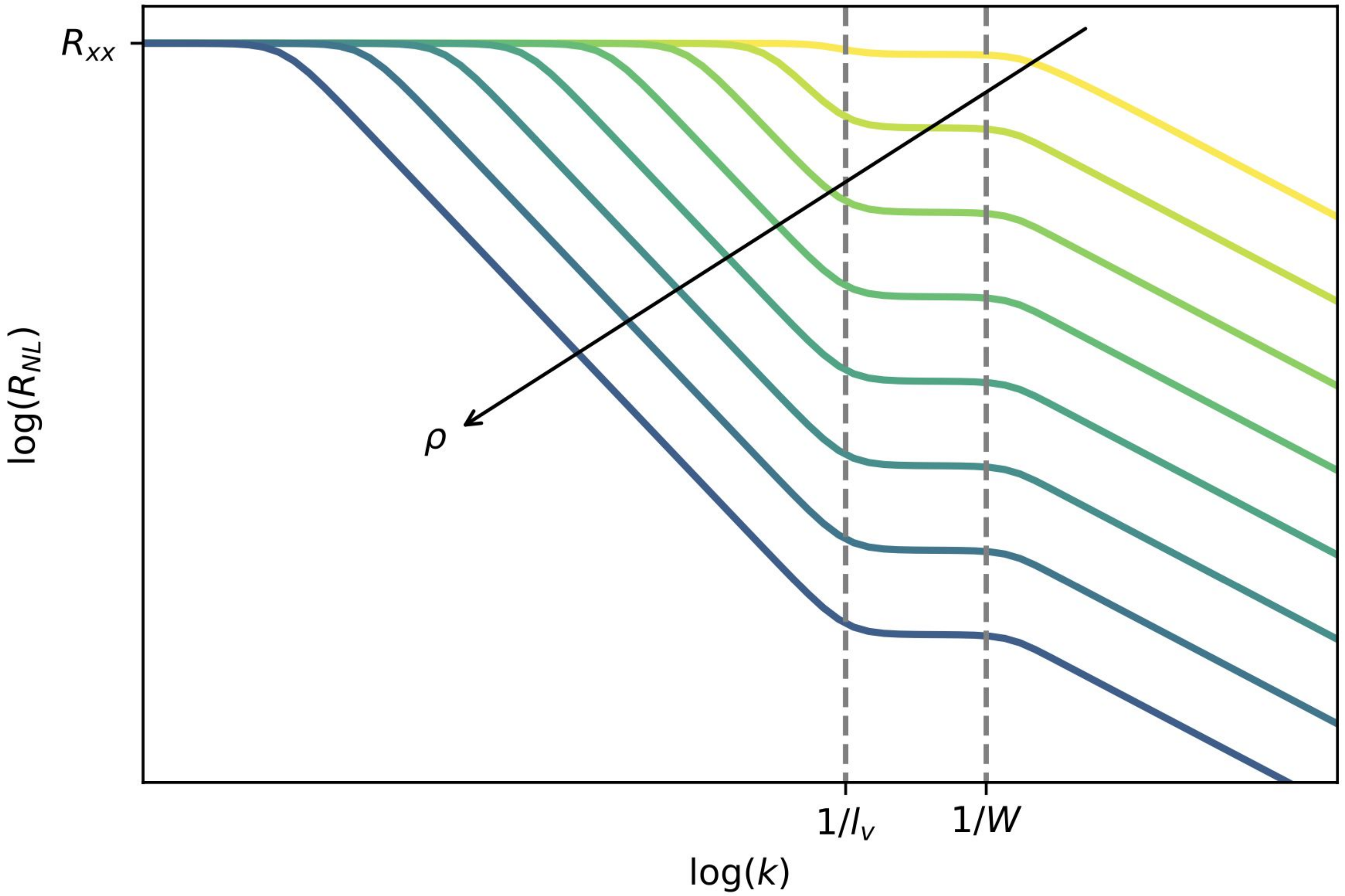
log(R_{NL})
log(ρ)



R_{xx}
$\log(R_{NL})$
ρ
$1/l_v$
$1/W$
$\log(k)$

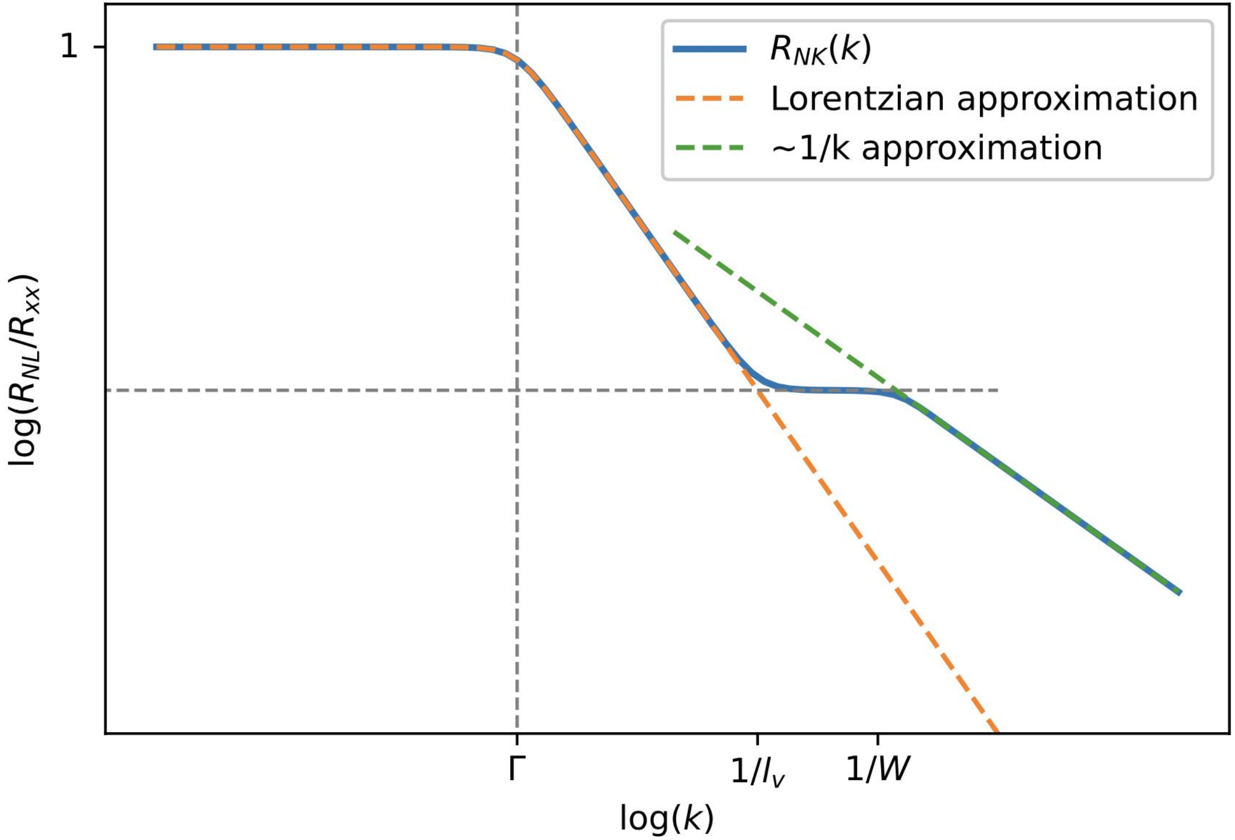


1
0
$R_{NL}(k)/R_{xx}$
$\log k$
W/l_v
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
10^{1}
10^{2}
10^{3}


R_{xx}
$\log(R_{NL})$
ρ
$1/l_v$
$1/W$
$\log(k)$

$$R_{\mathrm{NL}}(k) = \frac{2\omega(k)}{k\sigma_c}\left\{\frac{\omega(k)}{\tanh(kW/2)} + \frac{k\tan^2(\theta_{\mathrm{VH}})}{\tanh[\omega(k)W/2]}\right\}^{-1}$$

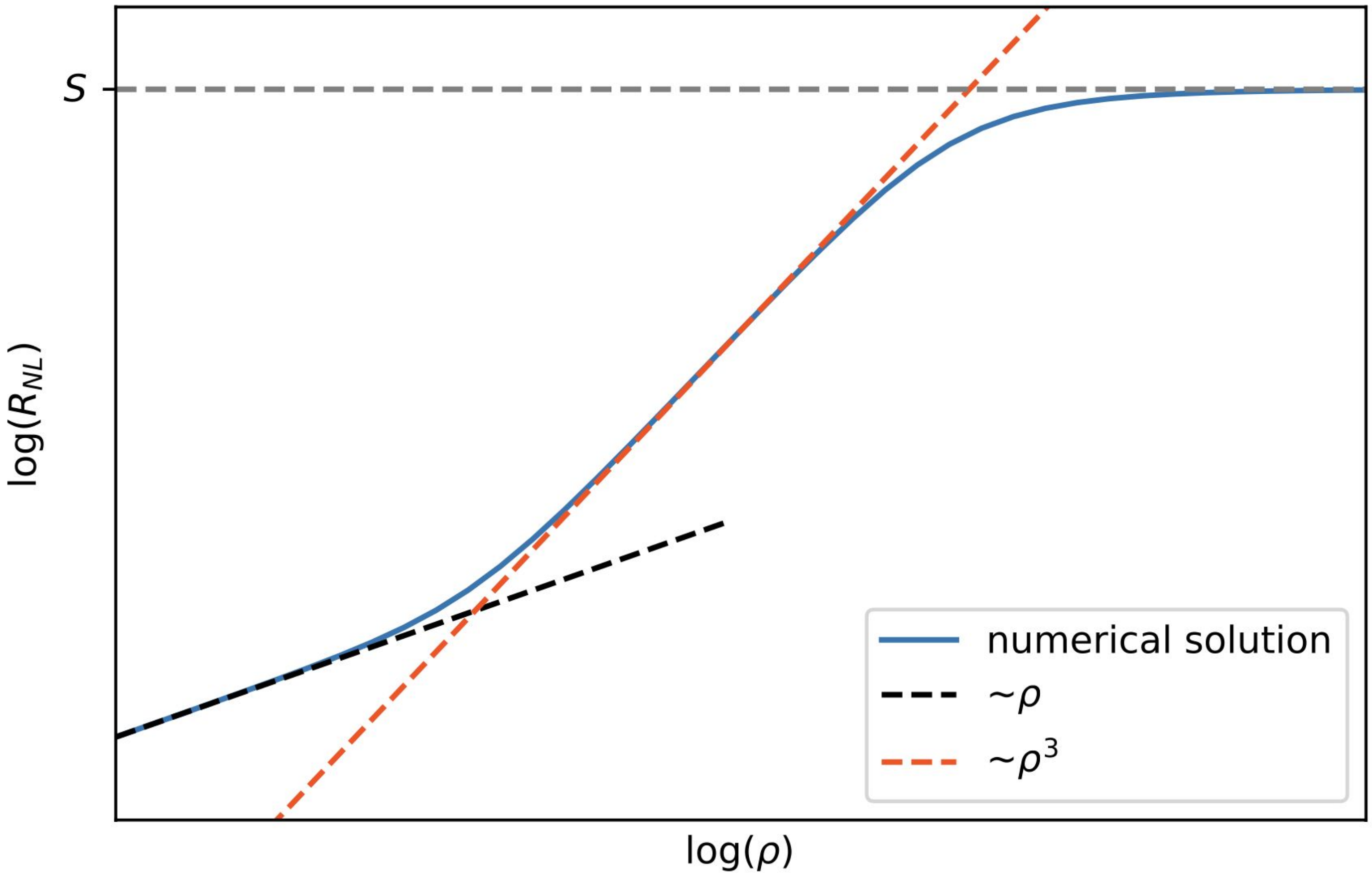
$$\lim_{\rho\to 0} R_{\mathrm{NL}}(x) = \frac{2\rho}{\pi}\ln\left|\coth\left(\frac{\pi x}{2W}\right)\right| + \rho^3 F(x) + o(\rho^5)$$



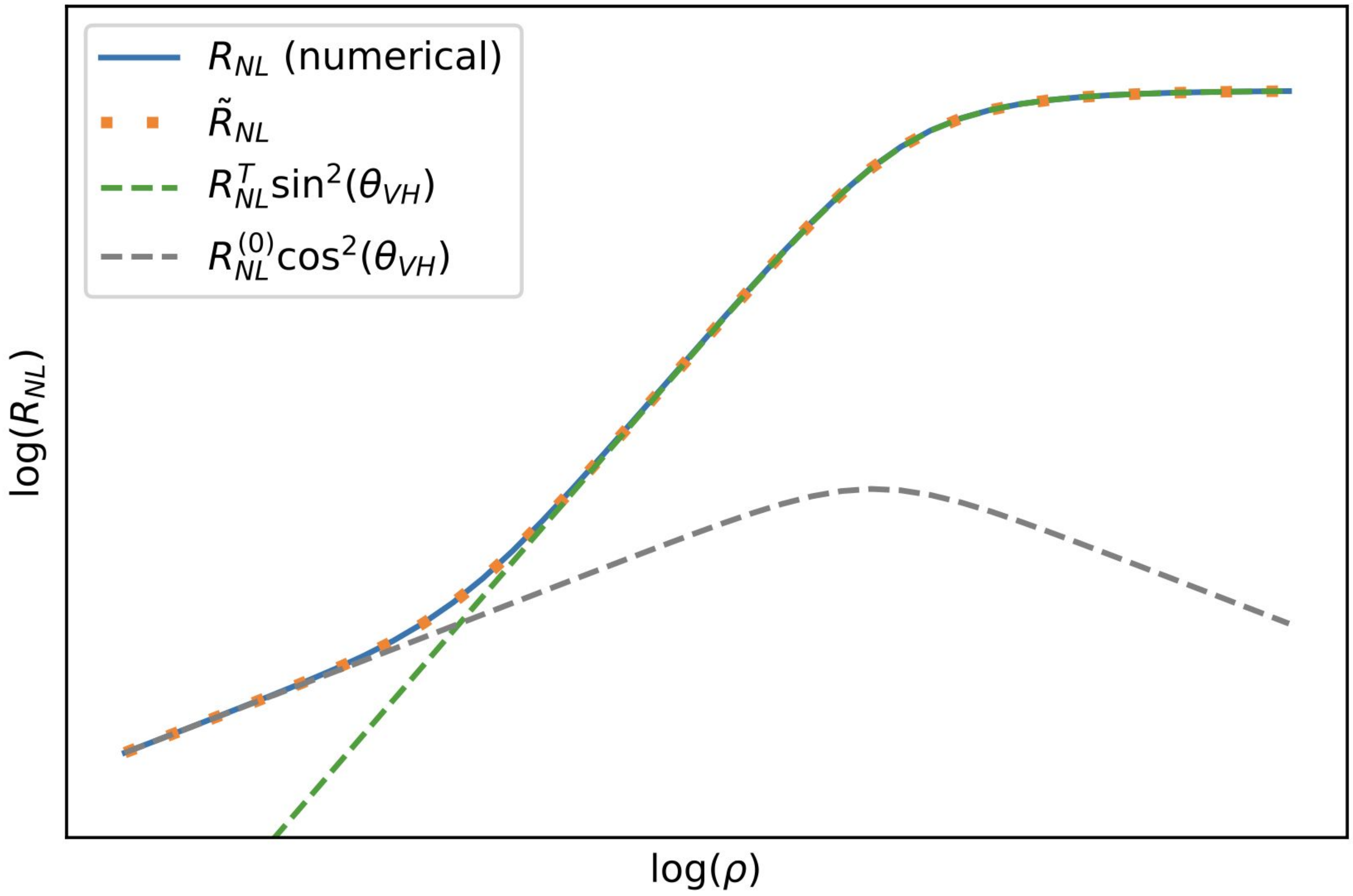
$$\lim_{\rho\to\infty} R_{\mathrm{NL}}(k) = \frac{\rho W}{1+(k\rho/\Gamma)^2} + \frac{1}{\rho}G(k)$$

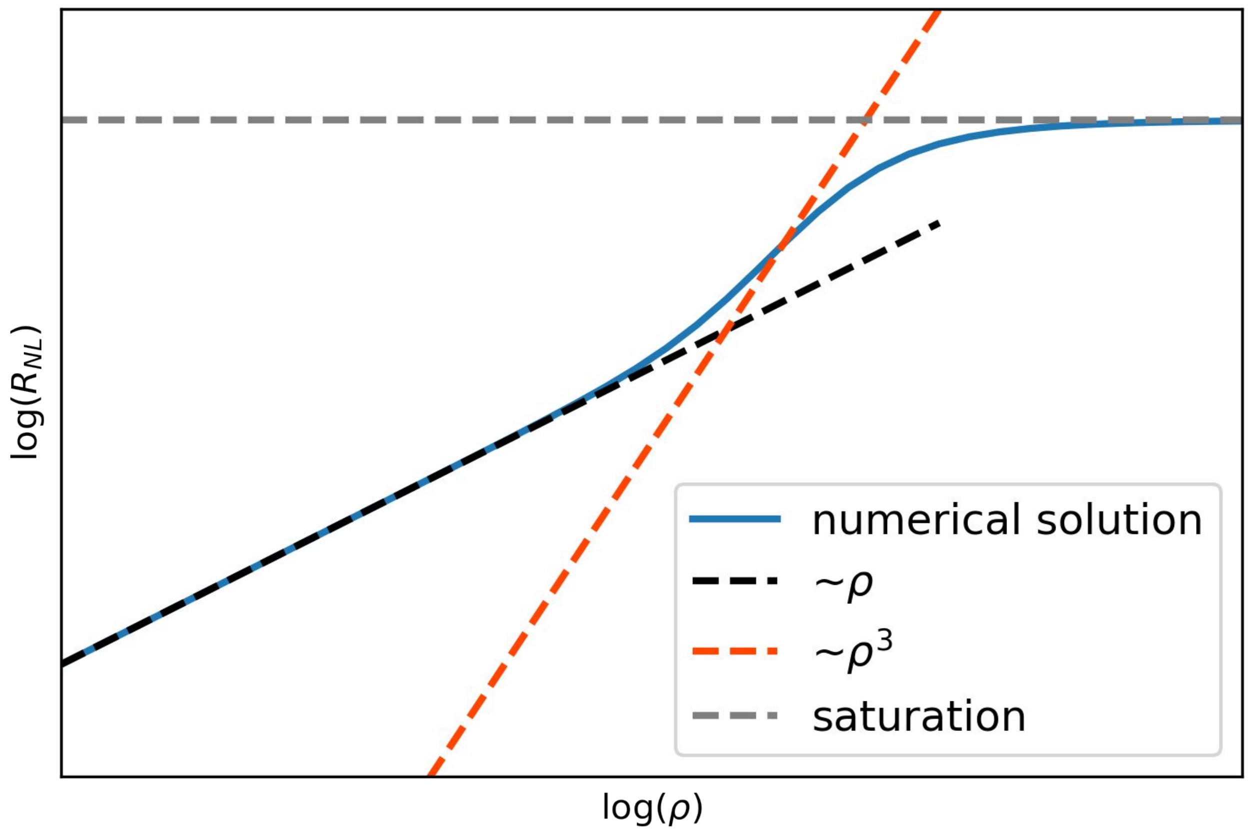
$$\lim_{\rho\to\infty} R_{\mathrm{NL}}(k) = \frac{\rho W}{1+(k\rho/\Gamma)^2} + \frac{1}{\rho}G(k)$$

$$\lim_{\rho\to\infty} R_{\mathrm{NL}}(x) = \frac{1}{\sigma_{\mathrm{v}}}\sqrt{\frac{2W}{l_{\mathrm{v}}}\tanh\left(\frac{W}{2l_{\mathrm{v}}}\right)}$$

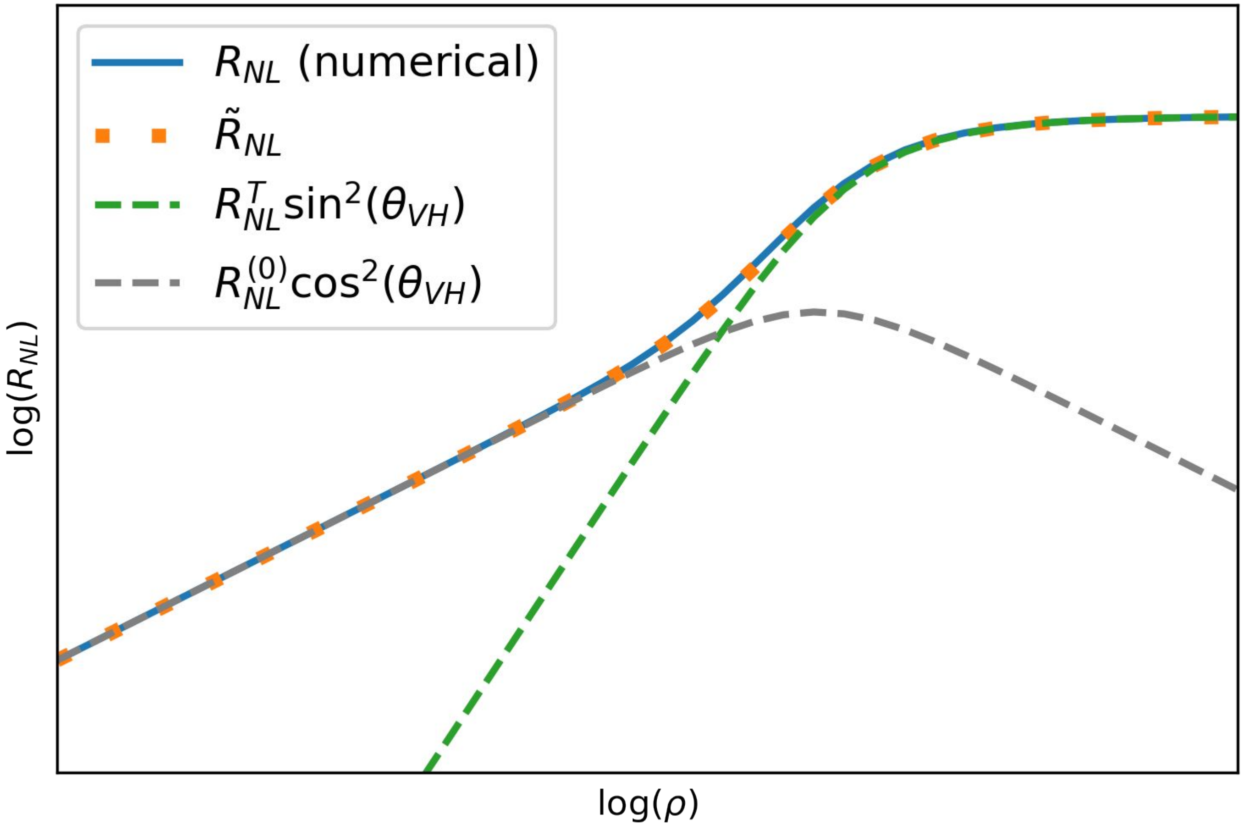


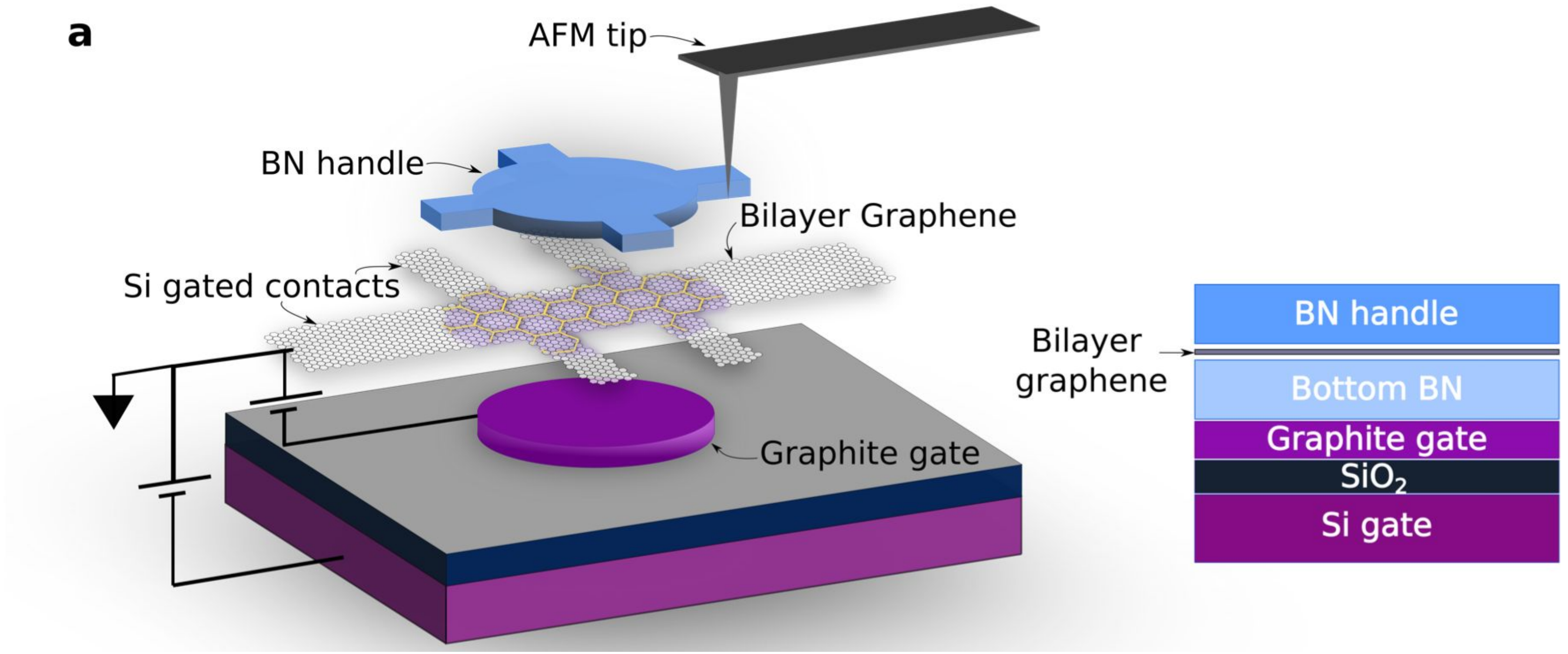
S
$\log(R_{NL})$
$\log(\rho)$
numerical solution
$\sim\rho$
$\sim\rho^3$


R_{NL} (numerical)
$\tilde{R}_{NL}$
$R_{NL}^{T}\sin^2(\theta_{VH})$
$R_{NL}^{(0)}\cos^2(\theta_{VH})$
$\log(R_{NL})$
$\log(\rho)$

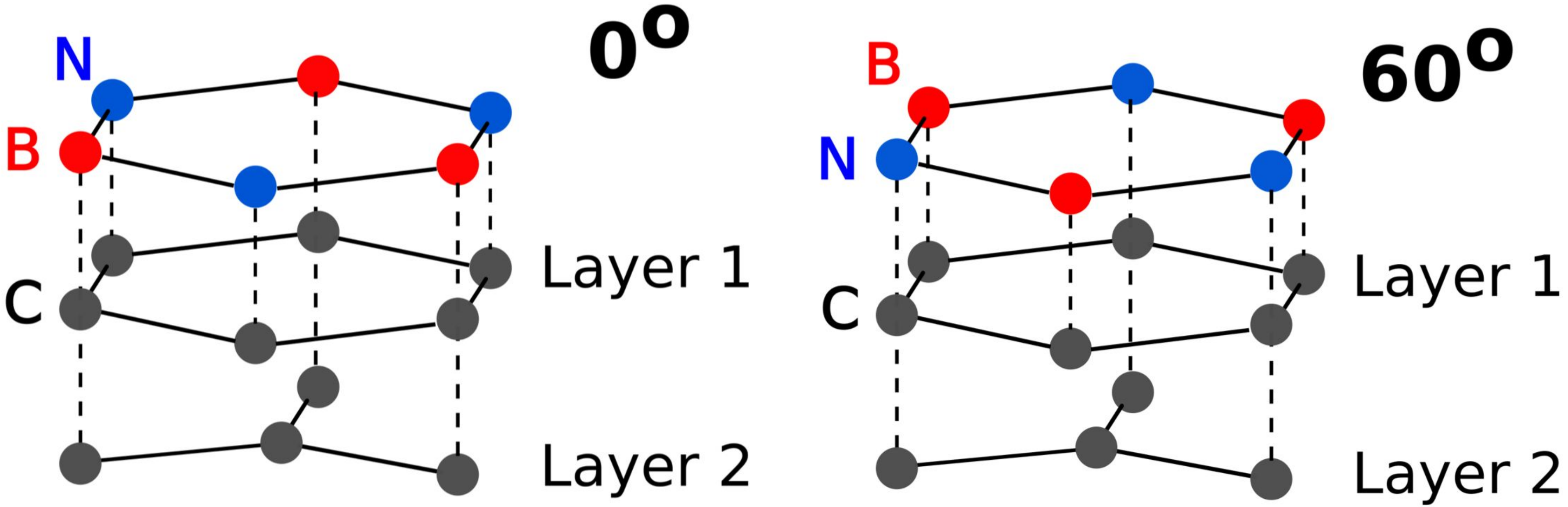


log(R_{NL})
log(ρ)
numerical solution
$\sim\rho$
$\sim\rho^3$
saturation

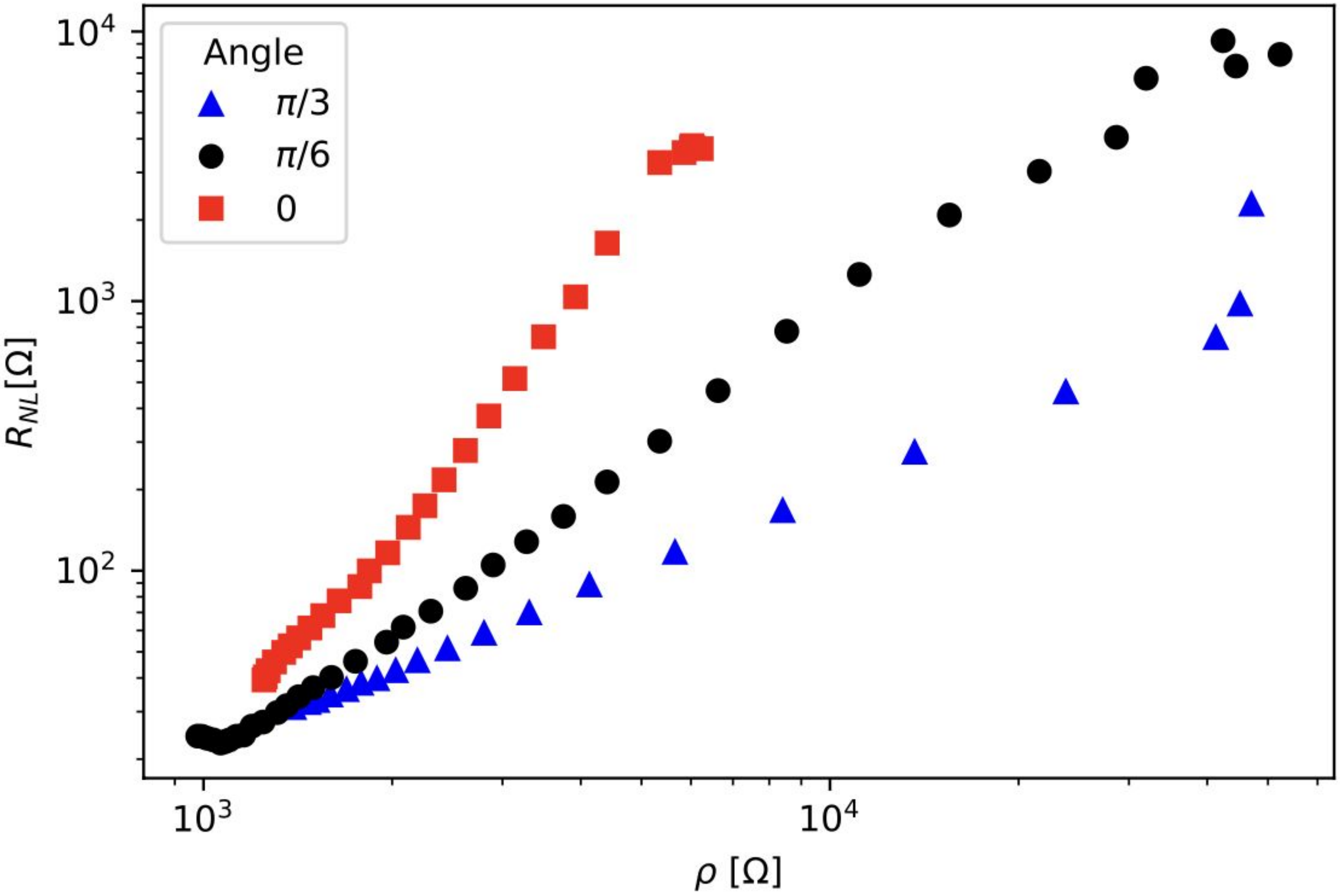

R_{NL} (numerical)
$\tilde{R}_{NL}$
$R_{NL}^{T}\sin^2(\theta_{VH})$
$R_{NL}^{(0)}\cos^2(\theta_{VH})$
$\log(R_{NL})$
$\log(\rho)$



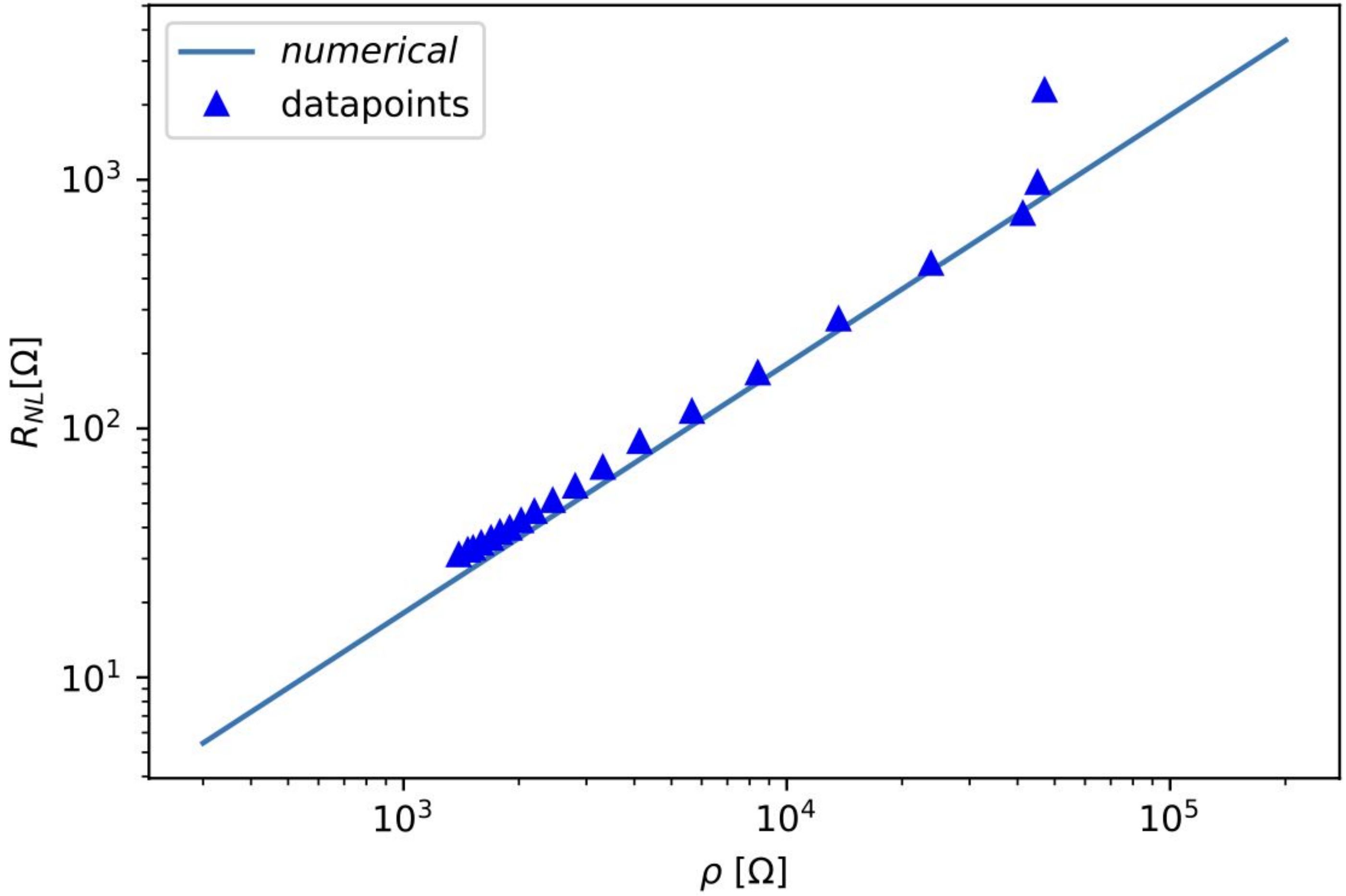
E Arrighi *et al.*, "Non-identical moiré twins in bilayer graphene,"

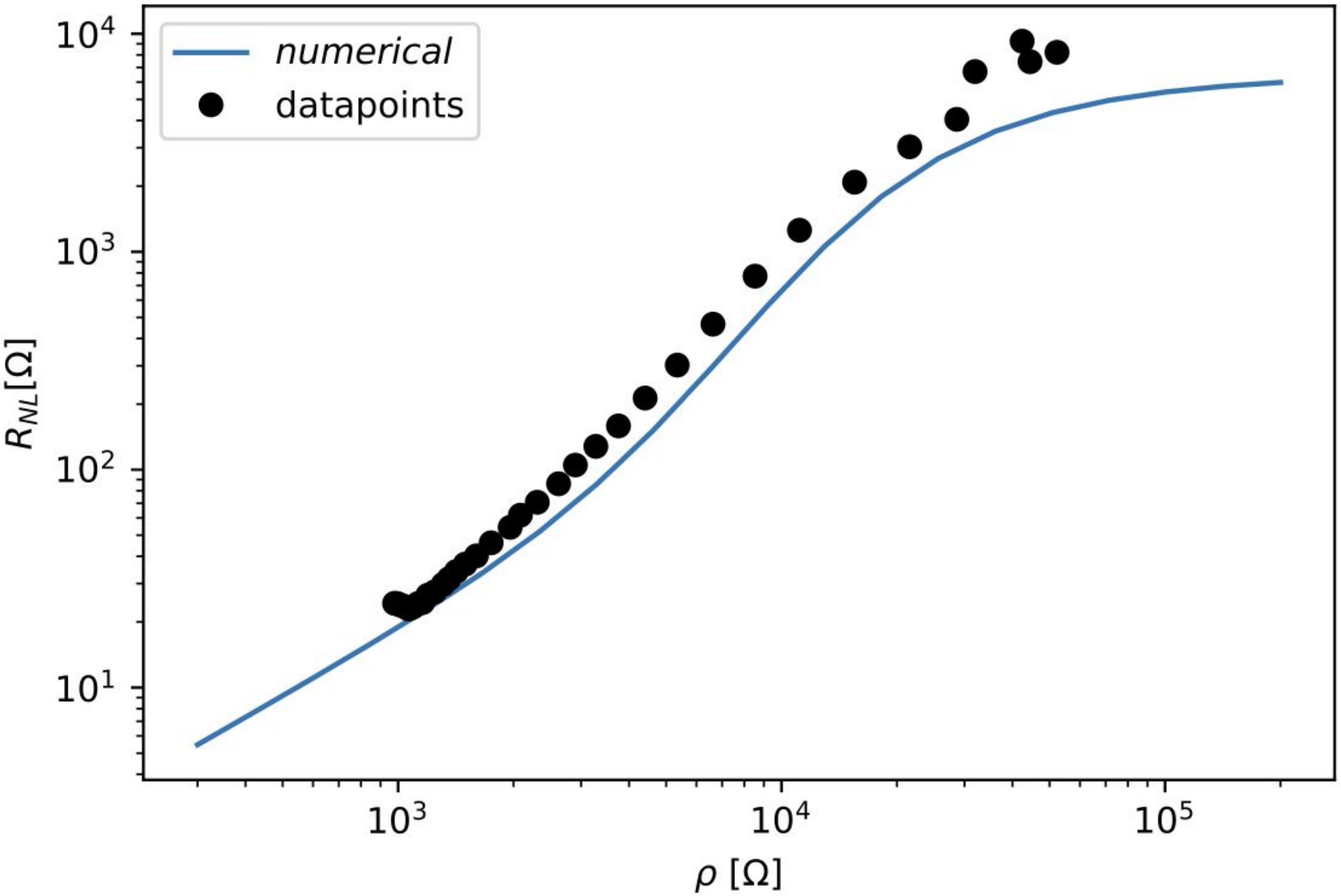


E Arrighi *et al.*, "Non-identical moiré twins in bilayer graphene,"

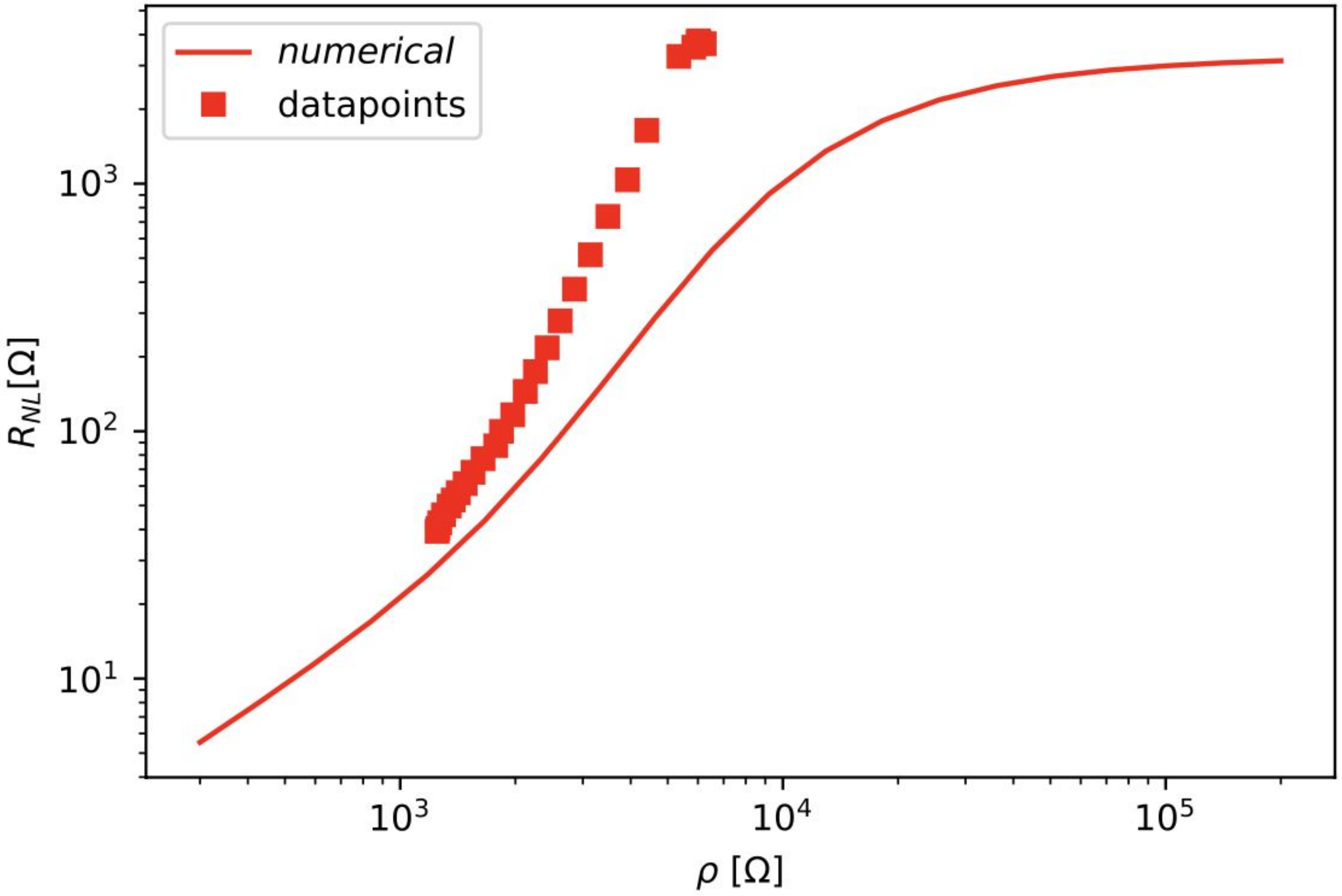


E Arrighi *et al.*, “Non-identical moiré twins in bilayer graphene,”


numerical
datapoints
$R_{NL} [\Omega]$
10^1
10^2
10^3
10^3
10^4
10^5
ρ [Ω]



numerical
datapoints
10^4
10^3
10^2
10^1
$R_{NL}[\Omega]$
10^3
10^4
10^5
$\rho\ [\Omega]$



numerical
datapoints
$R_{NL}[\Omega]$
10^1
10^2
10^3
10^3
10^4
10^5
$\rho\ [\Omega]$

Domande?