A Theory of Public Good Provision with Heterogeneous Risk Preferences*

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Abstract

People with different attitudes to risk have different views on the extent to which society should invest in certain (risky) projects. This paper presents a theory of optimal provision of a (risky) public good when individuals have heterogeneous preferences for risk. The public good has an insurance purpose as it allows individuals to shift risk from private to public consumption. On the one hand, private provision of the public good is inefficient because people do not internalise the insurance gains of the other agents. On the other, public provision might fail to achieve the (ex-ante) first best outcome if agents cannot be targeted and compensated when the optimal policy does not reflect their specific risk preferences. When the government has full information, this paper argues that distorting intertemporal decisions by taxing safe capital income, on top of nonlinear labour earnings taxation, might be welfare-improving as it allows to increase insurance by changing individual savings behaviour while reducing labour-supply distortions.

JEL classification: H21, H41.

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1 Introduction

There is a growing literature documenting heterogeneous social preferences. Age, gender, economic background, educational attainment and other individual characteristics shape people's view concerning what the government should invest in and how progressive the tax system should be (Stantcheva, 2021). One area of government's policy that often polarises the public opinion is investment in infrastructures. Public projects are often perceived as too risky, either because they might take longer to be completed, or because the net benefits of the projects might fail to materialise. For instance, a survey conducted in 2018 on the high speed rail project "HS2", that is currently under construction in the UK, shows that 59% of those who oppose the project are worried that "costs are or potentially will be too high", possibly reflecting different preferences for risk associated with skyrocketing costs of public projects.

Another example in which different attitudes towards risk might play a role is climate change. While most people agree climate change is a problem and measures must be taken to mitigate or stop it,[1] different views on the scale of the actions to implement emerge. The 2021 Eurobarometer survey on climate change shows that 80% of respondents from Sweden agree that "reducing fossil fuel imports from outside the EU can increase energy security and benefit the EU economically", while only 59% in France think so. Moreover, the perceived costs of climate change are also heterogeneous. In Portugal, 52% of respondents totally agree with the statement that the cost of damage due to climate change is much higher than the cost of investment needed for a green transition, but only 28% does so in Poland and Finland. While differences across countries can be explained by different exposures to climate risks (Dechezleprêtre et al., 2022), differences within countries could partly reflect different attitudes to (climate change) risk.

As the examples presented above suggest, people with different attitudes to risk not only make different individual decisions throughout their lives, but also have different views on the extent to which the government should invest in certain (risky) public goods, like infrastructures or green investments to protect the environment. Despite its relevance, the normative issue of public provision of public goods in the context of heterogeneous attitudes to risk, and the optimal tax system needed to implement it, has not received much attention. This paper contributes to fill this gap by presenting a theory of optimal provision of a (risky) public good when individuals have heterogeneous preferences for risk, while facing aggregate risk in the economy.

I consider a two-period model in which each type of agent is characterised by their level of risk aversion and productivity (wage). Agents supply labour, and make intertemporal consumption and portfolio decisions, choosing between two types of assets: one is risk-free, while the other is subject to aggregate risk, but with a positive

[1] According to the 2021 Eurobarometer survey, 78% of the respondents consider climate change "a very serious problem", while 15% see it as "a fairly serious problem".

expected excess return.[2] First, I consider a simpler framework in which agents differ only by their risk preferences, and labour supply is exogeneous (sections 2-4). When the public good is offered on the basis of individual contributions, I consider the Nash equilibrium outcome (section 2). Then, I consider the solution of a social planner maximising a generic social welfare function (section 3-4). In section 4, the government raises revenues with two distinct proportional taxes on the safe return and the risky excess return (with symmetric loss offsets), along with a lump-sum tax, equal for all agents. In section 5, agents also choose their labour supply, and a non-linear tax schedule applies to labour earnings, along with linear taxation of safe normal returns and risky excess returns.

Tax revenues finance a public good that plays an insurance role, as risk is spread between public and private consumption, as in Christiansen (1993), Schindler (2008) and Boadway and Spiritus (2024). The public good is itself risky since tax revenues depend on the state of the economy. The tax parameters are chosen optimally ex-ante while the policy is implemented in the second period, after the realization of the state of the economy, such that the budget is balanced. This captures the idea that funds for a specific project might be limited in the short run, so if a negative shock occurs the project gets paused or reduced in scale. Alternatively, if we consider the environment as the public good, the risk in the economy is given by the climate risk associated with the range of possible future scenarios.

This paper delivers three main results. First, I show that the market would generally provide an inefficient probability distribution (and expected level) of the public good in the case of heterogeneous risk preferences. This is because agents do not internalise the fact that their contribution to the public good affects also the distribution of risk between private and public consumption for the other agents that have different risk preferences. Given a general social welfare function, the (ex-ante) First Best is achieved with (risk-aversion) type-specific lump-sum taxation.[3] Then, when different types cannot be targeted, the expected level of the public good will be generally suboptimal (second best outcome).

Second, I provide an application of the public good provision problem in which I characterise a tax system for labour earnings and capital income that is used to implement the second-best optimal policy. I show that the government sets the optimal variance of the public policy by balancing the volatility of public consumption with the average volatility of private consumption. The excess return tax plays a key role for this mechanism. Agents with different preferences for risk have different benefits from the tax/insurance policy at the margin, meaning their willingness to

[2] Following the optimal tax literature, I distinguish two different components of the rate of return on savings: the (riskless) normal return and the excess return. The normal return is the price for forgoing present-time-consumption. The excess return can reflect idionsyncratic characteristics and/or aggregate risk in the economy and drives returns heterogeneity.

[3] This differs from ex-post First Best that would require the type-specific lump-sum taxes to be state-contingent. From now on, First Best will simply indicate the ex-ante First Best.

shift risk from the individual to the societal level depends on their risk aversion. Hence, when agents with different risk preferences cannot be targeted, taxing only the excess return part of capital income fails to deliver the First Best allocation. This is because agents cannot be individually compensated when the public good distribution is not in line with their risk preferences.

Third, I argue that the taxation of the safe return can have different effects on individual labour supply and consumption choices, compared to labour earnings taxation. In the case of full information on the government side, taxation of safe capital income can be used along with labour earnings taxation to affect the individual savings behaviour while minimising labour supply distortions. Hence, the government might give up intertemporal efficiency to better target insurance to the different types of agents, while diminishing distortions on the number of hours worked.

This paper contributes to two main strands of literature. First, this paper is related to the literature that analyses public good provision and optimal taxes in a risky environment. For instance, Christiansen (1993) and Schindler (2008) examine the efficiency/insurance effects and the role of capital income taxes when the economy faces aggregate risk. The contribution of this paper is to develop a theory of public good provision in a risky environment in presence of heterogeneous agents that possess different attitudes to risk and possibly different productivities. Using a general social welfare function, the appropriate Samuelson rule is derived. Coeherently with Schindler (2008), the applications of the public good problem with capital income taxation show that the taxation of risky excess returns is key to set the risk-profile of the public policy. However, I argue that the different types of agents will benefit from the policy differently at the margin, such that other tax instruments are necessary to target agents with different risk preferences better.

Second, this paper is related to the growing literature on optimal taxation with heterogeneous returns. Previous papers have concentrated on the concepts of heterogeneous "investment ability" and/or scale effects (Boadway and Spiritus, 2024; Gerritsen et al., 2025; Gahvari and Micheletto, 2016; Kristjánsson, 2024) as drivers of heterogeneous returns. This paper, in line with new empirical evidence (Bach et al., 2020), considers different preferences for risk as an alternative driver of heterogeneous returns. This creates a connection between savings and returns heterogeneity, and preference heterogeneity, as returns stem endogenously from preferences for risk through type-specific portfolio choices. While most of the above contributions argue in favor of taxing the normal (safe) return on grounds of redistribution,[4] my application shows that it can also have a role, complementary to labour earnings taxes, in fostering insurance by affecting individual savings behaviour while minimising distortions on labour supply decisions.

The rest of the paper is organised as follows. Section 2 revisits the theory of

[4] In Boadway and Spiritus (2024); Gerritsen et al. (2025); Gahvari and Micheletto (2016), the taxation of safe capital income complements the redistributive role of earnings taxation.

private provision of public good in the case of heterogeneous risk preferences. Section 3 presents the first and second best allocations with a generic social welfare function, and how it is possible to improve on the second best allocation. Section 4 presents an application of the public good problem in which the excess and safe returns can be taxed, and agents differ by their risk aversion only. Section 5 extends section 4 by introducing heterogeneous productivities and allowing agents to choose their labour supply. Section 6 concludes.

2 Public Good Provision in a Risky Environment

From the textbook theory of public goods, we know that public goods will be undersupplied by the market when provided on the basis of individual voluntary contributions Stiglitz, 1980. In this section, I revisit the standard theory and show that the private provision of public goods is still inefficient in a risky environment when agents possess different risk preferences. Then, after presenting the Pareto optimal case – given by the appropriate Samuelson rule – I discuss the type of inefficiency that private provision produces in a risky environment.

Let us consider a set-up in which each agent j allocates a share of her endowment z to consumption in the first period c_0, and saves the residual $z - c_0$ investing in a risky portfolio with gross return $\tilde{R}_p$, given their level risk aversion θ_j. In the second period, each agent enjoys private consumption $\tilde{c}_1$ and public consumption $\tilde{G}$ that are both subject to risk. Without loss of generality, I impose no discounting of future consumption. Moreover, individuals evaluate private and public consumption with the same sub-utility function $u(\cdot)$, meaning that no specific difference in taste between private and public goods is modeled.

$$\begin{aligned}
\max_{c_0^j,\, g_j} \quad & U^j = u_j(c_0^j) + \mathbb{E}\left[u_j(\tilde{c}_1^j)\right] + \mathbb{E}\left[u_j\left(\tilde{G}\right)\right] \\
\text{s.t.} \quad & \tilde{c}_1 = \tilde{R}_p^j\left(z - c_0^j\right) - g_j, \\
& \tilde{G} = f\big(\textstyle\sum_j g_j, \tilde{x}\big)
\end{aligned}$$

Each individual j chooses how much to contribute (g_j) to the public good $\tilde{G}$ in the first period by maximising lifetime utility U taking the contributions of the other agents (g_{-j}) as given. In a no-risk situation, the level of the public good will be given by the sum of individual contributions. With an underlying source of risk in the economy $\tilde{x}$, then the public good is risky as it also depends on the state of the economy in the second period, i.e. $\tilde{G} = f\big(\sum_j g_j, \tilde{x}\big)$.[5] Hence, even if the marginal rate of transformation is one, meaning one unit of private consumption

[5] The underlying source of risk should be seen as a risk for the overall economy, affecting both private consumption through portfolio returns, as well as public good consumption through aggregate shocks to public finances.

buys one unit of public consumption in a riskless economy, aggregate risk can either increase or decrease the actual value of the public good relative to the value of private consumption that was initially sacrificed. The optimal private contribution rule g_j^* satisfies

$$g_j^* : MRS_{\tilde{G},\tilde{c}_1^j} = \frac{\mathbb{E}[u_j'(\tilde{G})]}{\mathbb{E}[u_j'(\tilde{c}_1^j)]} = 1 \ \forall j, \tag{1}$$

where the size of the public good $\tilde{G}$ depends on the sum of the individual contributions and aggregate risk in the economy $\tilde{x}$ i.e. $\tilde{G} = f\big(\sum_j g_j, \tilde{x}\big)$. Each agent j contributes until the risk-adjusted marginal rate of substitution between the public and private good $MRS_{\tilde{G},\tilde{c}_1^j}$ equates the marginal cost (exp. 1). At the Nash equilibrium, the set of individual contribution rules $\{g_j, g_{-j}\}$ will satisfy (2) for each type. Summing up (1) across agents $j = 1, \cdots, n$ gives $\sum_j MRS_{\tilde{G},\tilde{c}_1^j} = n$. By comparing the Nash equilibrium outcome with the Pareto efficient allocation,[6] given by the appropriate Samuelson rule

$$\sum_j MRS^j_{\tilde{G},\tilde{c}_1^j} = \sum_j \frac{\mathbb{E}[u_j'(\tilde{G})]}{\mathbb{E}[u_j'(\tilde{c}_1^j)]} = 1 < n, \tag{2}$$

we can state the following well-known result.

Lemma 1. *Public good provision in the case of private contributions is inefficient as agents do not internalise the external value of their individual contributions.*

In the context of aggregate risk in the economy, Lemma 1 has two specific implications. First, the public good will be underprovided in expectation. Second, private provision of public good generates an inefficient allocation of risk between private and public consumption, meaning that the probability distribution of the public good over different states of nature is suboptimal.

3 Samuelson Rule with a Social Welfare Function

The concept of Pareto optimality can be quite restricting when preferences are heterogeneous. Policy changes often benefits some agents while hurting others. Hence, we proceed by considering the optimal allocation that stems from the government's maximisation of a generic Social Welfare function

$$SW = \sum_j \phi_j \left(U_j\right),$$

[6] Each Pareto-efficient allocation can be achieved by private, decentralised optimisation when the public good is financed by (risk-aversion) type-specific lump-sum taxes such that (2) is satisfied.

where $\phi_j(U_j)$ is a weakly concave function of individual expected utility U_j that is able to accommodate different individual risk preferences.[7] Then, the optimal rules for public good provision are derived. Two benchmark cases are presented: 1) (Ex-ante) First Best: the government can levy type-specific lump-sum taxes that depend on individual risk aversion (section 3.1); 2) Second Best: the government sets one unique lump-sum tax for all types (section 3.2).

3.1 First Best: Type Specific Lump-sum Taxes

The government chooses t_i for each agent i, therefore perfectly targeting agents with different risk aversion. Expression 3 states that the optimal t_i equates the marginal private costs of agent i from contributing to the public good, expressed in terms of private consumption, to the social benefit of a marginal individual contribution:

$$\phi_i'\Big(U_i\Big)\,\mathbb{E}[u_i'(\tilde{c}_1^i)] = \sum_j \phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{G})] \quad \forall\, i. \tag{3}$$

At the optimum, expression (3) is satisfied for each agent i and gives the set of optimal lump-sum taxes t_j for all types of agent. The government sets t_i for each agent i such that the weighted expected marginal utility of private consumption is equalised across types to a certain level $\overline{k}$.

$$\{t_i, i = 1, \cdots, n\} \;\; \rightarrow \;\; \phi_i'\Big(U_i\Big)\,\mathbb{E}[u_i'(\tilde{c}_1^i)] = \overline{k} \quad \forall\, i = 1, \cdots, n. \tag{4}$$

By summing up condition (3) across types $i = 1, \cdots, n$, and using (4), we can obtain a modified Samuelson rule that is expressed in terms of the sum of socially-evaluated Marginal Rates of Substitutions (MRS^s).

Theorem 1 (First Best)**.** *At the First Best optimum, the sum of the MRS^s is equal to the marginal rate of transformation between the public and private good.*

$$\sum_j MRS_j^s = \sum_j \frac{\phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{G})]}{\phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{c}_1^j)]} = 1. \tag{5}$$

Condition (5) is the Samuelson rule that reflects the social preferences that are being maximised, where

$$MRS_j^s = \frac{\phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{G})]}{\phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{c}_1^j)]}$$

[7] The issue of choosing the appropriate functions ϕ_j for utility functions with different curvatures is widely debated in the welfare theory literature. For instance, Grant et al. (2010) shows how it is possible to accommodate concerns about different individuals' risk attitudes and concerns about fairness; Eden (2020) provides a practical method to perform welfare analysis.

is defined as the ratio between the marginal social welfare associated with an additional unit of the public good and the marginal social welfare associated with an additional unit of private consumption for an agent j.

While type specific taxation is a useful theoretical benchmark, it is unlikely to be feasible in practise. The next section considers an alternative case of public provision in which the lump-sum tax is set equal for all agents.

3.2 Second Best: Uniform Lump-sum Taxes

If the government cannot screen agents with different risk aversion, and therefore cannot set (risk-aversion) type specific lump-sum taxes, a uniform lump-sum tax t for all agents apply. At the second best optimum, marginal social costs, expressed in terms of private consumption, are balanced with the social benefits:

$$\sum_j \phi'_j\Big(U_j\Big)\,\mathbb{E}[u'_j(\tilde{c}^j_1)] = n\sum_j \phi'_j\Big(U_j\Big)\,\mathbb{E}[u'_j(\tilde{G})]. \tag{6}$$

Condition (6) can be rewritten in terms of $\widetilde{MRS}^s_j$, where

$$\widetilde{MRS}^s_j = \frac{\phi'_j\Big(U_j\Big)\,\mathbb{E}[u'_j(\tilde{G})]}{\frac{1}{n}\sum_j \phi'_j\Big(U_j\Big)\,\mathbb{E}[u'_j(\tilde{c}^j_1)]},$$

that is defined as the ratio between the marginal social welfare associated with an additional unit of the public good for an agent j and the average (across agents $j = 1, \cdots, n$) marginal social welfare associated with an additional unit of private consumption. This means that when type specific taxes are not available, the government evaluates the individual willingness to trade private with public consumption on the basis of the average sacrifice in terms of private consumption, e.g. $\widetilde{MRS}^s_j$ rather than MRS^s_j . Hence, condition (7) is the modified Samuelson rule in the case of heterogeneous risk preferences when the government does not discriminate the different risk-aversion-types.

Theorem 2 (Second Best)**.** *At the second best optimum, the sum of $\widetilde{MRS}^s_j$ equates the marginal rate of transformation.*

$$\sum_j \widetilde{MRS}^s_j = \sum_j \left(\frac{\phi'_j\Big(U_j\Big)\,\mathbb{E}[u'_j(\tilde{G})]}{\frac{1}{n}\sum_j \phi'_j\Big(U_j\Big)\,\mathbb{E}[u'_j(\tilde{c}^j_1)]}\right) = 1. \tag{7}$$

The Second Best outcome will differ from the First Best benchmark, both in terms of public good provision and private consumption/savings, and therefore social welfare, when $\sum \widetilde{MRS}^s_j \neq \sum MRS^s_j$, namely when condition (7) differs from (5). We can now state Proposition 1.

Proposition 1. *The inability of the government to discriminate different risk-aversion-types in the economy leads to a suboptimal provision of the public good.*

Proof. Suppose Proposition 1 is false, and $\tilde{G}^{SB} \equiv \tilde{G}^{FB}$, then it must be that (5) coincides with (7) and that

$$\frac{1}{n}\sum_j \phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{c}_1^{SB})] = \phi_i'\Big(U_i\Big)\,\mathbb{E}[u_i'(\tilde{c}_1^{FB})]\;\;\forall i = 1, \cdots, n.$$

In order for this to be true, it should follow that $t_i^{FB} = t^{SB}$ for $i = 1, \cdots, n$, meaning the individual private costs of raising a unit of consumption are equal across different types to begin with. Unless specific welfare weights are chosen to obtain this result or the equality happens to hold given the utility functions being used in the first place, proposition 1 will be true in the case with heterogeneous preferences. □

While risk-preference-specific lump-sum taxes might be unfeasible, let alone lump-sum taxes, we know that agents with different risk preferences will make different consumption, savings, portfolio and labour supply choices. These differences can therefore be exploited to increase social welfare.

In the next section (section 4), I study an application of this principle with regard to savings and portfolio choices, in which the optimal (public) provision of public goods is studied when the government taxes the safe normal return and the risky excess return. Then, in section 5, I investigate the role of safe capital income taxation, alongside nonlinear taxation of earnings, when agents also choose their labour supply.

4 Application: Taxation of Excess and Safe Return

I consider a two-period consumption/savings and portfolio model where agents have different risk preferences and the government finances the public good by raising equal lump-sum contributions and by taxing the riskless return and risky excess return separately.

4.1 Agent's Problem

$\Theta = \{(\theta_1), \cdots, (\theta_j), \cdots, (\theta_n)\}$ is a discrete set of agent-types with relative risk aversion parameters θ_j, $j = 1, \ldots, n$, from the sample space $\mathbf{S}$, and each type has equal weight in the population. Given exogenous earnings z, agents choose first period consumption, c_0, and how to invest the residual $(z - c_0)$. Agents choose the portfolio share of risky assets s with the risky return $\tilde{r}_e$ being Normally distributed: $\tilde{r}_e \sim \mathcal{N}(\overline{r}_e - \sigma_r^2/2, \sigma_r^2)$. The resulting portfolio return $\tilde{r}_p$ will also be Normally distributed: $\tilde{r}_p \sim \mathcal{N}(\overline{r}_p - \sigma_p^2/2, \sigma_p^2)$. The (risky) excess return is defined as the difference between the risky return and the riskless return: $\tilde{r}^{exc} := \tilde{r}^e - r$. Without loss of generality, I impose no discounting: $\beta = 1$. Agent's utility is separable over time as well as between private and public consumption. The problem for agent-type j is thus:

$$\begin{aligned}
\max_{c_0^j,\, s_j} \quad & U_j = u_j(c_0^j) + \beta\, \mathbb{E}\left[u_j(\tilde{c}_1^j) + u_j(\tilde{G})\right] \\
\text{s.t.} \quad & \tilde{c}_1 = \tilde{R}_p^j \left(z - c_0^j\right) - t, \\
& \tilde{R}_p^j = \left[1 + r\left(1 - \tau_r\right) + s_j\left(\tilde{r}^{exc}\right)\left(1 - \tau_k\right)\right]
\end{aligned}$$

where $\tilde{R}_p$ is the gross portfolio return; t is a lump-sum tax, equal for all types; τ_r and τ_k are the tax rates on the riskless and excess return respectively. The FOCs for first period consumption c_0, and share of risky assets s are:

$$\begin{aligned}
c_0^* : \quad & u'(c_0) = \mathbb{E}\left[u'(\tilde{c}_1)\tilde{R}_p\right] \\
s^* : \quad & 0 = \mathbb{E}\left[u'(\tilde{c}_1)\tilde{r}^{exc}\right]
\end{aligned} \tag{8}$$

Notice that, while taxation of the riskless return changes the resource allocation (first period consumption, savings), taxing the excess returns with loss offsets does not. Agents will adjust their portfolio shares to get the same pre-tax expected portfolio return (Domar and Musgrave, 1944). After applying the covariance identity to (8), we can derive the expression for the risk premium with respect to the expected excess return.

$$\mathbb{E}[R^e - R] = -\frac{\operatorname{cov}\left[u_j'(\tilde{c}_1^j), \tilde{r}^{exc}\right]}{\mathbb{E}\left[u_j'(\tilde{c}_1^j)\right]}. \tag{9}$$

For each agent j with relative risk aversion parameter θ_j, expression (9) maps the covariance term to the expected marginal utility of second-period consumption $\tilde{c}_1$.

4.2 The Government

The government provides a public good $\tilde{G}$ that enters the utility function separately from consumption. The policy is financed by taxing risky excess return at rate τ_k, the safe return at rate τ_r, and levying a lump-sum tax t, equal for all agents. The public good is itself risky as it depends on risky tax revenues from the excess return. Tax rates are set by the government in the first period anticipating agents' optimal behaviour. After the state of the economy is realised, the government implements the policy and balances the budget. As a result, the provision of the public good is stochastic and depends on the state of the economy in the second period.

The government's objective is to maximise social welfare (SW) that is defined as a weighted sum of agents' expected utilities, where U is a Von Neumann-Morgenstern utility function.

$$\max_{\tau_k,\tau_r,t} SW = \sum_j \phi_j\Big(U_j\left(c_0^{j*},\, \tilde{R}_p^j\left(z-c_0^{j*}\right)-t,\, \tilde{G}\right)\Big)$$
$$\text{s.t. } \tilde{G} = \sum_j \Big[\left(\tau_k\, s_j^*\, \tilde{r}^{exc} + \tau_r r\right)\left(z-c_0^{j*}\right) + t\Big]$$

with $\phi_j'\Big(U_j\Big) = \eta_j$ being the marginal social welfare weight associated to expected utility U_j. I substitute the expression for the public good directly in the social welfare function, as in Schindler (2008). It ensures that the budget is balanced for any state of the world.[8]

4.3 Optimality Conditions

In this section, I show the optimality conditions for the linear taxes on the excess and riskless return when a pure public good is provided. Moreover, I assume that the government cannot screen agents with different risk preferences, meaning the government can only use a uniform lump-sum tax, along with capital income taxes.

4.3.1 Excess Return Tax

$$\mathbb{E}[\tilde{r}^{exc}] = -\frac{\sum_j \eta_j \operatorname{cov}\Big[u_j'(\tilde{G}), \tilde{r}^{exc}\Big]}{\sum_j \eta_j\, \mathbb{E}[u_j'(\tilde{G})]} \tag{10}$$

The optimal excess return tax rate τ_k^* has to satisfy condition (10). Given the risk preferences of all agents in society, the government chooses τ_k^* to reach the optimal allocation of risk between private and public consumption, and sets the optimal variance of the public good policy. The key novelty in this setting is that the "benefits" of the public good differ among agents because of heterogeneous attitudes to risk. Since the public good is itself risky, the welfare gain from increasing the tax rate τ_k varies across agents. Hence, the government aims to balance the different "preferences" for the public good, providing insurance against aggregate risk.

Two interpretations can be developed that focus on the issues of tax revenue collection and the public good respectively. The first one is that the government is choosing what share of the budget (tax revenues) should be risky.[9] As individuals make portfolio choices on the basis of their risk preferences according to (9), similarly the government decides the distribution of tax revenues financing the public good over states of nature on the basis of (10), such that the risk preferences of all agents are taken into account.

[8]This is a stricter requirement than balancing the budget in expectation, that would instead imply transferring resources from good states to bad states of the world.

[9]The realisation of the excess return depends on the state of the economy.

The second interpretation relates to the use of tax revenues. The (risky) revenues collected from taxing the excess return will generate a certain probability distribution of the public good. This distribution entails a certain allocation of risk between private and public consumption: the public good has an insurance role.[10] The government's objective is to choose the social-welfare-maximising public good distribution that achieves the optimal allocation of risk in the economy. In doing so, the government takes into account the agents' willingness to shift risk to the societal level. Condition (11) reformulates (10) and better represents this concept. Private and public consumption volatility are represented by the covariance between marginal utility of private and public consumption respectively with the risky excess return and jointly govern the individual willingness to pay for the public good with an extra euro of risky excess capital income.

Theorem 3. *When agents with different risk preferences cannot be discriminated, τ_k^* equalizes public consumption volatility with (average) private consumption volatility.*

$$\sum_j \frac{\eta_j \operatorname{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right)}{n^{-1} \sum_j \eta_j \operatorname{cov}\left(u_j'\left(\tilde{c}_1\right), \tilde{r}^{exc}\right)} = 1 \tag{11}$$

Proof. See Appendix A.

Theorem 3 says that τ_k^* is chosen on the basis of a weighted average of private consumption volatility, i.e. $n^{-1} \sum_j \eta_j \operatorname{cov}\left(u_j'\left(\tilde{c}_1\right), \tilde{r}^{exc}\right)$. This is because the government cannot target individual risk-aversion types. This creates an inefficiency: an agent with relatively (more) risky private consumption c_1, compared to other agents, would be better-off by shifting more risk to the public good. For that to be the case, a higher tax rate τ_k should be implemented, so that private consumption volatility is traded with public consumption volatility. Thus, the outcome produced by (11) could be improved if the government were able to target different types, and compensate agents that would prefer a different distribution of the public good.

Corollary 1. *When the government can target different risk aversion types (11) simplifies to*

$$\sum_j \frac{\eta_j \operatorname{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right)}{\eta_j \operatorname{cov}\left(u_j'\left(\tilde{c}_1\right), \tilde{r}^{exc}\right)} = 1 \tag{12}$$

and the allocation of risk between private and public consumption relates to individual willingness to shift risk from private to public consumption $\frac{\eta_j \operatorname{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right)}{\eta_j \operatorname{cov}\left(u_j'(\tilde{c}_1), \tilde{r}^{exc}\right)}$.

[10] When a bad (good) state of economy realises in the second period, losses (gains) due to negative (positive) excess returns will be spread over private and public consumption, so that the utility loss (gain) is minimised (maximised).

Proof. See Appendix B.

To sum up, when agents have heterogeneous risk preferences, the optimal tax τ_k is not just about balancing the volatility of private and public consumption, as in Boadway and Spiritus (2024) and Schindler (2008), but also aims to balance the different "preferences" for the public good, providing insurance against aggregate risk (Theorem 3). As the optimal tax rate τ_k^* implies a specific probability distribution of the public good, taxing the excess return only may be suboptimal. Corollary 1 shows that welfare improvements are possible when different types can be targeted. If type-specific lump-sum taxation is not available, other tax instruments that have differential impacts on different types could be used. In the next section, I argue that the taxation of the safe return has this feature. Finally, it is possible to show that the optimal excess return tax rate is positive.

4.3.2 Safe Return Tax

The optimal safe return tax rate τ_r^* satisfies:

$$\underbrace{-\sum_j \eta_j \mathbb{E}\left[u_j'\left(\tilde{c}_1^j\right)\right]\left(z-c_0^j\right)r}_{\text{private welfare effect } = \Delta U} + \underbrace{\sum_j \eta_j \mathbb{E}\left[u_j'\left(\tilde{G}\right)\right]\sum_j\left(z-c_0^j\right)r}_{\text{mechanical welfare effect } = \Delta M}$$
$$\underbrace{-\tau_r r \sum_j \eta_j \mathbb{E}\left[u_j'\left(\tilde{G}\right)\right]\sum_j \frac{\partial c_0^j}{\partial \tau_r}}_{\text{behavioural effect } = \Delta B} = 0. \tag{13}$$

A marginal change in τ_r determines a welfare loss for agents as second period consumption is lowered: this is the private welfare effect (ΔU). On the other hand, the additional tax revenues finance the public good which increases agents' utility: this is the mechanical welfare effect (ΔM). However, varying the tax rate affects agents' savings too (via change in first period consumption c_0) through substitution and income effects: this is the behavioural effect (ΔB), which affects the tax base and therefore tax revenues.

At the optimum, private welfare losses are balanced with the welfare gains from public good provision, net of behavioural effects: $\Delta U + \Delta M + \Delta B = 0$. Unless the mechanical and private welfare effects sum up to zero, the optimality condition is satisfied only with $\tau_r \neq 0$, provided that $\sum_j (\partial c_0^j / \partial \tau_r) \neq 0$.

Proposition 2. *a) With heterogeneous risk preferences, the mechanical and private welfare do not sum up to zero: $\Delta U + \Delta M \neq 0$ or equivalently $|\Delta U| \neq |\Delta M|$. b) Then, $\tau_r^* \neq 0$ solves the optimal tax condition (14).*

Proof. (Utilitarian case, i.e. $\phi_j(U) = U$ for all $j = 1, \cdots, n$.)

a) We can use (6) to rewrite the mechanical term of expression (13) as follows

$$\Delta M = \frac{1}{n} \sum_j \mathbb{E}\left[u_j'(\tilde{c}_1^j)\right] \sum_j (z - c_0^j) r$$

Define $\mathbb{E}\left[u_j'\left(\tilde{c}_1^j\right)\right] = a_j$, and $\left(z - c_0^j\right) r = b_j$. Notice that as $a_H \neq a_L$ and $b_H \neq b_L$ for any agents H, L with $\theta_H > \theta_L$, then $|\Delta U| = \sum_j a_j b_j \neq n^{-1} \sum_j a_j \sum_j b_j = |\Delta M|$.
b) Consider $\sum_j (\partial c_0^j / \partial \tau_r) \neq 0$. If $|\Delta U| \neq |\Delta M|$, then the optimality condition is satisfied when $\tau_r^* \neq 0$, as we have $\Delta B \neq 0 \iff \tau_r^* \neq 0$. □

Hence, it can be optimal to tax the safe return when agents have heterogeneous risk preferences. With CRRA utility, different relative risk aversion parameters imply different tastes for risk (portfolio decisions), different slopes of the consumption path as well as different responses of savings decisions responses to taxation of returns. Proposition 2 tells us that exploiting these differences by taxing (positively or negatively) the riskless part of the return can increase social welfare. Therefore, a trade-off between insurance and intertemporal efficiency can arise.

Hence, the taxation of the safe return acts as an (imperfect) substitute for (risk-aversion) type-specific taxation. Indeed, when the government can impose type-specific lump-sum taxes, the safe return becomes redundant.

Corollary 2. *When the government can target agents with different risk preferences with type-specific lump-sum taxes, (13) is always satisfied by* $\tau_r^* = 0$.

The results highlighted by Proposition 2 and Corollary 2 are consistent with the fact that the Atkinson and Stiglitz (1976) theorem does not hold under preference heterogeneity, unless each type can be targeted with no efficiency costs.

5 Heterogeneous Productivity and Risk Aversion

In this section, I consider an environment in which each type of agent is characterised by their productivity and risk aversion preferences, and the government taxes labour earnings non linearly and capital income linearly, distinguishing the type of returns from investment, i.e. safe normal return versus risky excess return. Agents with different risk preferences choose different amount of hours worked, conditional on their productivity (wage). Then, the government sets type-specific earnings tax rates τ_l^{ij} for each combination of risk aversion and productivity levels. With a high number of types and individual preferences, this parametrization generates a non-linear tax schedule on labour earnings $T(z)$.

The question that this section aims to investigate is whether the taxation of safe capital income can still play a role for the optimal public good policy when earnings are taxed non-linearly. In other terms, we will investigate to what extent the Atkinson and Stiglitz (1976) theorem applies in this setting.

5.1 Additional Notation

The set of agent-types is a $m \times n$ matrix Θ, where each element is given by the pair (w_i, θ_j) that represents an individual-type ij who has productivity (wage) w_i and relative risk aversion θ_j. F is the $m \times n$ frequency matrix of types in the population, with each type having frequency f_{ij} in the population.

$$\Theta_{m\times n} = \begin{pmatrix} w_1, \theta_1 & \cdots & w_1, \theta_n \\ \vdots & \ddots & \vdots \\ w_m, \theta_1 & \cdots & w_m, \theta_n \end{pmatrix}, \quad F_{m\times n} = \begin{pmatrix} f_{11} & \cdots & f_{1n} \\ \vdots & \ddots & \vdots \\ f_{m1} & \cdots & f_{mn} \end{pmatrix}.$$

As in the previous section, each component of capital income is taxed linearly: τ_k is the excess return tax, τ_r is the tax rate on the safe return. Labour earnings are subject to a non-linear schedule $T(z)$. The after-tax labour earnings y are defined as $y := z(1-\tau_l)$. Savings are defined as $a = z(1-\tau_l) - c_0$.

5.2 Uncompensated elasticities

Define the uncompensated elasticities of labour earnings z^{ij} and savings a^{ij} with respect to net-of-tax rate of labour earnings, $\epsilon^{ij}_{z,l}$, $\epsilon^{ij}_{a,l}$, and with respect to net-of-tax rate of safe capital income, $\epsilon^{ij}_{z,r}$, $\epsilon^{ij}_{a,r}$ as follows:

$$\epsilon^{ij}_{z,l} = \frac{\partial z^{ij}}{z^{ij}} \cdot \frac{1-\tau^{ij}_l}{\partial(1-\tau^{ij}_l)}; \qquad \epsilon^{ij}_{z,r} = \frac{\partial z^{ij}}{z^{ij}} \cdot \frac{1-\tau_r}{\partial(1-\tau_r)};$$

$$\epsilon^{ij}_{a,l} = \frac{\partial a^{ij}}{a^{ij}} \cdot \frac{1-\tau^{ij}_l}{\partial(1-\tau^{ij}_l)}; \qquad \epsilon^{ij}_{a,r} = \frac{\partial a^{ij}}{a^{ij}} \cdot \frac{1-\tau_r}{\partial(1-\tau_r)}.$$

Notice that under CRRA utility, with relative risk aversion parameters θ_{ij} greater than 1 for all agents, it is possible to infer the signs of these elasticities, as the income effect is stronger than the substitution effect. In particular, $\epsilon^{ij}_{z,l} < 0$; $\epsilon^{ij}_{a,l} > 0$ and $\epsilon^{ij}_{z,r} < 0$; $\epsilon^{ij}_{a,r} < 0$.

5.3 Endogeneous Labour Supply with Observable Types

In this section, I analyse non-linear taxation of labour earnings and linear taxation of safe and risky capital income in the case in which agents also choose their labour supply, and the transfer t and the risky capital income tax τ_k on excess returns are already set optimally.[11] The government has complete information on the different

[11]For the transfer t and the risky capital income tax on excess returns τ_k, the intuition remain similar to the framework discussed in section 4. The whole set of optimality conditions is provided in the appendix C.

types of agent in the economy, meaning that individual productivities, labour supply decisions, the types (risky versus safe) and amount of capital income are known.

For an agent type (w_a, θ_b), with productivity w_a and risk aversion parameter θ_b, the labour income tax condition satisfies condition (14). Private welfare effects from a small reform $\Delta\tau_l^{ab} > 0$ are accompanied by a mechanical increase in labour earnings tax revenues and multiple behavioural responses on the government budget: (i) changes to tax revenues from earnings taxation due to changes to labour earnings; (ii) changes to tax revenues from safe capital income due to responses of savings. Under CRRA utility, and with relative risk aversion parameters θ_{ij} greater than 1 for all agents, effect (i) increases pre-tax labour earnings, and therefore the tax base for earnings taxation, while effect (ii) will decrease savings, therefore decreasing revenues from capital income taxation as a result.

$$\begin{aligned}\tau_l^{ab*} : f_{ab}\eta_{ab}\, \mathbb{E}\left[u_b'(\tilde{c}_1^{ab})\right] z_{ab}p_0 = & \sum_{i,j} f_{ij}\eta_{ij}\, \mathbb{E}\left[u_{ij}'(\tilde{G})\right] \\ & \times f_{ab}\left[z_{ab}p_0 - \frac{\tau_r r}{(1-\tau_l^{ab})}\cdot\left(\epsilon_{a,l}^{ab}a^{ab}\right) - \frac{\tau_l^{ab}}{1-\tau_l^{ab}}\epsilon_{z,l}^{ab} z_{ab}p_0\right]\end{aligned} \tag{14}$$

Similarly, besides the private and mechanical welfare effects, a marginal change in the tax rate on safe capital income $\Delta\tau_r > 0$ will generate some behavioural effects on tax revenues from labour earnings and safe capital income (expression 15). In particular, under CRRA utility and with relative risk aversion parameters θ_{ij} greater than 1 for all agents, both labour earnings and savings will increase, therefore increasing tax revenues from labour earnings and capital income.

$$\begin{aligned}\tau_r^* : \sum_{i,j} f_{ij}\eta_{ij}\, \mathbb{E}\left[u_{ij}'(\tilde{c}_1^i)\right] a^{ij} r = & \sum_{i,j} f_{ij}\eta_{ij}\, \mathbb{E}\left[u_{ij}'(\tilde{G})\right] \\ & \times \sum_{i,j} f_{ij}\left[a^{ij} r - \frac{\tau_r\cdot r}{(1-\tau_r)}\left(\epsilon_{a,r}^{ij}a^{ij}\right) - \frac{\tau_l^{ij}}{1-\tau_r} z^{ij} p_0 \epsilon_{z,r}^{ij}\right]\end{aligned} \tag{15}$$

Hence, under certain conditions (CRRA preferences and strong income effects), labour earnings and safe capital income taxes have different effects on individual consumption, labour supply and savings decisions. These differences can be exploited to achieve the optimal public good policy while reducing distortions on labour supply.

Let's consider an example with regard to an agent-type (w_a, θ_b) who is relative less risk averse than the average agent in the economy. The variability of the public good policy is chosen optimally by τ_k. However, since agent (w_a, θ_b) is relatively less risk averse than average, this individual is investing more than optimally from the government's perspective. Hence, condition (15) tells us that this individual would face a positive labour income tax rate, when safe capital income remains untaxed. A positive labour income tax would induce agent (w_a, θ_b) to decrease savings while working more. However, the government could also adopt a negative capital income tax on the safe return which would contribute to reduce the individual's savings while correcting the distortion on working hours. This example suggests that the taxation of the safe return on investment might not be redundant when a non-linear tax schedule for earnings is available, and that the Atkinson and Stiglitz (1976) theorem does not apply in this setting, even when (risk-aversion-type-specific) nonlinear taxation of earnings apply.

One case in which a non-zero tax on safe capital income is always optimal is when the government is restricted to choose weakly positive tax rates on individual labour earnings, independently from risk preferences.

Proposition 3. *If the government cannot set negative tax rates on labour earnings for any agent in the economy, taxing the safe capital income is always optimal.*

In the following proof by contradiction, I show that positive labour earnings taxes make $\tau_r = 0$ suboptimal: the optimality conditions for labour earnings and safe capital income are not satisfied after imposing $\tau_r = 0$.

Proof. Suppose that $\tau_r = 0$ is optimal. By exploiting the optimal conditions for transfer t and excess return tax τ_k, we can rewrite conditions (16) as follows

$$\tau_r^* : \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}\left[u'_{ij}(\tilde{c}_1^i)\right]a^{ij} = \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}\left[u'_{ij}(\tilde{c}_1^i)\right]\sum_{i,j} f_{ij}a^{ij} - \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}\left[u'_{ij}(\tilde{c}_1^i)\right]\frac{\tau_l^{ij}}{1-\tau_r}z^{ij}\epsilon_{z,r}^{ij}$$

The mechanical effect (first term RHS) is always larger than the private welfare effect (LHS). With $\epsilon_{z,r}^{ij} < 0$, and positive tax rates on labour earnings, the optimality condition is not satisfied. On the other hand, the condition on labour earnings (16) is also not satisfied. For an agent-type (w_a, θ_b), after substituting the condition for the transfer t and excess return τ_k:

$$\tau_l^{ab*} : f_{ab}\eta_{ab}\,\mathbb{E}\left[u'_{ab}(\tilde{c}_1^{ab})\right] = f_{ab}\sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}\left[u'_{ij}(\tilde{c}_1^i)\right]\left[1 - \frac{\tau_l^{ab}}{1-\tau_l^{ab}}\epsilon_{z,l}^{ab}\right] \qquad (16)$$

According to condition (16), an agent type with lower weighted expected marginal utility of second period consumption compared to the average must have a negative tax rate labour earnings (subsidy). This would allow to increase the agent's savings to match the variance of the public good policy, which is set by τ_k. □

6 Concluding Remarks

People with different attitudes to risk have different views on the extent to which society should invest in certain (risky) projects. This paper presents a theory of optimal provision of a (risky) public good when individuals have heterogeneous preferences for risk and face aggregate risk in the economy. In an environment in which the public good is a tool to shift risk from private to public consumption, this paper shows that the inefficiency of private provision of public goods comes from agents failing to internalize the insurance effects of the public good for the other agents. Given a social welfare function, I characterise the (ex-ante) First Best allocation, which is achieved with (risk-aversion) type-specific lump-sum taxation. Discriminating the different types of agent allows the government to compensate them when the distribution of the public good is not in the line with their risk preferences.

Then, I characterize the second best allocation. In an application with capital income taxation, I show that the excess return tax is key to match the individual savings and portfolio decisions with the variance of the public insurance policy. Moreover, the taxation of safe normal returns might play a role. In a framework where individuals differ by their risk preferences and producivitiy, and make labour supply along with saving and portfolio decisions, taxing the safe return has a non-redundant role even when earnings are taxed non-linearly. I showed that even when the government has full information agents' characteristics, giving up intertemporal efficiency can be useful to provide better-targeted insurance to agents. This is obtained by using safe capital income taxation to influence savings behaviour, alongside labour earnings taxes, while minimising distortions on labour supply decisions.

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Appendix A - Theorem 3

The optimality condition for the excess return tax τ_k reads as follows.

$$\sum_j \phi_j'(U_j)\mathbb{E}\Big[u_j'(\tilde{G})\sum_j s_j\left(z^j - c_0^j\right)\tilde{r}^{exc}\Big] + \tau_k \sum_j \phi_j'(U_j)\mathbb{E}\left[u_j'(\tilde{G})\sum_j \frac{\partial s_j}{\partial \tau_k}\left(z^j - c_0^j\right)\tilde{r}^{exc}\right] = 0.$$

Using the fact that $\frac{\partial s_j}{\partial \tau_k} = \frac{s_j}{(1-\tau_k)}$, we can collect terms.

$$\sum_j \phi_j'(U_j)\mathbb{E}\Big[u_j'(\tilde{G})\sum_j s_j\left(z^j - c_0^j\right)\tilde{r}^{exc}\Big]\left(1 + \frac{\tau_k}{1-\tau_k}\right) = 0.$$

Then, as $\sum_j s_j\left(z^j - c_0^j\right)\left(1 + \frac{\tau_k}{1-\tau_k}\right) \neq 0$, we get the following expression

$$\sum_j \phi_j'(U_j)\mathbb{E}\left[u_j'(\tilde{G})\tilde{r}^{exc}\right] = 0.$$

By further manipulating the above expression using the covariance identity, and conditions (7) and (9), we can rewrite the optimality condition for τ_k as follows

$$\frac{1}{n}\sum_j \phi_j'(U_j)\operatorname{cov}\left(u_j'\left(\tilde{c}_1^j\right), \tilde{r}^{exc}\right) = \sum_j \phi_j'(U_j)\operatorname{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right).$$

or

$$\sum_j \frac{\phi_j'(U_j)\operatorname{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right)}{\frac{1}{n}\sum_j \phi_j'(U_j)\operatorname{cov}\left(u_j'\left(\tilde{c}_1^j\right), \tilde{r}^{exc}\right)} = 1.$$

$\square$

Appendix B - Corollary 1

Condition (5), plus the covariance identity and condition (9) imply that the term

$$\phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1^j\right), \tilde{r}^{exc}\right)$$

is equal across types j. Hence, expression 12 can be reformulated as follows

$$\begin{aligned}
&\sum_j \frac{\phi_j'(U_j)\,\mathrm{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right)}{\frac{1}{n}\sum_j \phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1^j\right), \tilde{r}^{exc}\right)} \\
=&\sum_j \frac{\phi_j'(U_j)\,\mathrm{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right)}{\phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1^j\right), \tilde{r}^{exc}\right)} = 1.
\end{aligned}$$

□

Appendix C Heterogeneous Productivity and Risk Aversion

The individual maximisation problem for agent-type (w^i, θ_j) is:

$$\begin{aligned}
\max_{c_0^{ij},\, s_j,\, l^{ij}} \quad & U_{ij} = u_j(c_0^{ij}) + \beta\, \mathbb{E}\left[u_j(\tilde{c}_1^{ij}) + u_j(\tilde{G})\right] - v(z^{ij}/w_i) \\
\text{s.t.} \quad & \tilde{c}_{ij} = \tilde{R}_p^j \left(z^{ij}(1-\tau_l^{ij}) - c_0^{ij}\right) - t, \\
& \tilde{R}_p^j = \left[1 + r\,(1-\tau_r) + s_j\,(\tilde{r}^{exc})\,(1-\tau_k)\right]
\end{aligned}$$

where $\tilde{R}_p$ is the gross portfolio return; t is a lump-sum tax or transfer, equal for all types; τ_r and τ_k are the tax rates on the riskless and excess return respectively. As each agent is infinitesimal compared to the size of the economy, the effects of individual choices on the level of the public good are not taken into account.

C.1 First order conditions for individual maximisation problem

The FOCs for first period consumption c_0, share of risky assets s, and labour supply l read:

$$\begin{aligned}
c_0^* : \quad & u'_{ij}(c_0^{ij}) = \mathbb{E}\left[u'_{ij}(\tilde{c}_1^{ij})\tilde{R}_p\right] \\
s^* : \quad & 0 = \mathbb{E}\left[u'_{ij}(\tilde{c}_1^{ij})\tilde{r}^{exc}\right] \\
l^* : \quad & 0 = \mathbb{E}\left[u'_{ij}(\tilde{c}_1^{ij})\tilde{R}_p\right] w_i(1-\tau_l^{ij}) - v'(z^{ij}/w_i)
\end{aligned}$$

C.2 Endogeneous Labour Supply with Observable Types

First, the conditions for transfer t and excess return tax τ_k:

$$t^* : \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}[u'_{ij}(\tilde{c}_1^{ij})] = \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}[u'_{ij}(\tilde{G})] \tag{17}$$

$$\tau_k^* : \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}\left[u'_{ij}(\tilde{G})r^{exc}\right] = 0 \tag{18}$$

Then, under optimally set taxes/transfer t^* and τ_k^* the conditions for a labour income tax on each ability and risk aversion type, and the safe return tax equal for all types are:

$$\begin{aligned}\tau_l^{ab*} : f_{ab}\eta_{ab}\,\mathbb{E}\left[u'_b(\tilde{c}_1^{ab})\right] z_{ab}p_0 &= \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}\left[u'_{ij}(\tilde{G})\right] \\ &\quad\times f_{ab}\left[z_{ab}p_0 - \frac{\tau_r r}{(1-\tau_l^{ab})}\cdot\left(\epsilon_{a,l}^{ab}a^{ab}\right) - \frac{\tau_l^{ab}}{1-\tau_l^{ab}}\epsilon_{z,l}^{ab}z_{ab}p_0\right];\end{aligned} \tag{19}$$

$$\begin{aligned}\tau_r^* : \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}\left[u'_{ij}(\tilde{c}_1^{i})\right] a^{ij} r &= \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}\left[u'_{ij}(\tilde{G})\right] \\ &\quad\times \sum_{i,j} f_{ij}\left[a^{ij}r - \frac{\tau_r\cdot r}{(1-\tau_r)}\left(\epsilon_{a,r}^{ij}a^{ij}\right) - \frac{\tau_l^{ij}}{1-\tau_r}z^{ij}p_0\epsilon_{z,r}^{ij}\right].\end{aligned} \tag{20}$$