A Theory of Public Good Provision with Heterogeneous Risk Preferences

Francesco Alosa*

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Abstract

People with different attitudes to risk have different views on the extent to which society should invest in certain (risky) projects. This paper presents a theory of optimal provision of a (risky) public good when individuals have heterogeneous preferences for risk. The public good has an insurance purpose as it allows individuals to shift risk from private to public consumption. On the one hand, private provision of the public good is inefficient because people do not internalise the insurance gains of the other agents. On the other, public provision might fail to achieve the (ex-ante) first best outcome if agents cannot be targeted and compensated when the policy does not reflect their specific risk preferences. With an application on capital income and endowment taxation, this paper shows it is possible to improve welfare by exploiting the different choices of the agents with different risk preferences.

JEL classification: H21, H41.

Keywords: risky public good provision, optimal capital income taxation, insurance, intertemporal efficiency, aggregate risk.

*Department of Economics, University of Paris Dauphine - PSL, France; email: francesco.alosa@dauphine.psl.eu

1 Introduction

There is a growing literature documenting heterogeneous social preferences. Age, gender, economic background, educational attainment and other individual characteristics shape people's view concerning what the government should invest in and how progressive the tax system should be (Stantcheva, 2021). One area of government's policy that often polarises the public opinion is investment in infrastructures. Public projects are often perceived as too risky, either because they might take longer to be completed, or because the net benefits of the projects might fail to materialise. For instance, a survey conducted in 2018 on the high speed rail project "HS2", that is currently under construction in the UK, shows that 59% of those who oppose the project are worried that "costs are or potentially will be too high", possibly reflecting different preferences for risk associated with skyrocketing costs of public projects.

Another example in which different attitudes towards risk might play a role is climate change. While most people agree climate change is a problem and measures must be taken to mitigate or stop it,[1] different views on the scale of the actions to implement emerge. The 2021 Eurobarometer survey on climate change shows that 80% of respondents from Sweden agree that "reducing fossil fuel imports from outside the EU can increase energy security and benefit the EU economically", while only 59% in France think so. Moreover, the perceived costs of climate change are also heterogeneous. In Portugal, 52% of respondents totally agree with the statement that the cost of damage due to climate change is much higher than the cost of investment needed for a green transition, but only 28% does so in Poland and Finland. While differences across countries can be explained by different exposures to climate risks (Dechezleprêtre et al., 2023), differences within countries could partly reflect different attitudes to (climate change) risk.

As the examples presented above suggest, people with different attitudes to risk not only make different individual decisions throughout their lives, but also have different views on the extent to which the government should invest in certain (risky) public goods, like infrastructures or green investments to protect the environment. Despite its relevance, the normative issue of public provision of public goods in the context of heterogeneous attitudes to risk, and the optimal tax system needed to implement it, has not received much attention. This paper fills this gap by presenting a simple theory of optimal provision of a (risky) public good when individuals have heterogeneous preferences for risk and face aggregate risk in the economy.

I consider a two-period model in which each type of agent is characterised by their level of risk aversion and endowment. Agents make intertemporal consumption and portfolio decisions, choosing between two types of assets: one is risk-free, while the other is subject to aggregate risk, but with a positive expected excess return.[2]

[1] According to the 2021 Eurobarometer survey, 78% of the respondents consider climate change "a very serious problem", while 15% see it as "a fairly serious problem".

[2] Following the optimal tax literature, we distinguish two different components of the rate of

When the public good is offered on the basis of individual contributions, we consider the Nash equilibrium outcome. Then, we consider the solution of a social planner maximising a generic social welfare function. The government can raise revenues with two distinct proportional taxes for the safe return and the risky excess return (with symmetric loss offsets), and by taxing the endowments. Tax revenues finance a public good that plays an insurance role, as risk is spread between public and private consumption. The public good is itself risky since tax revenues depend on the state of the economy. The tax parameters are chosen optimally ex-ante while the policy is implemented in the second period, after the realization of the state of the economy, such that the budget is balanced. This captures the idea that funds for a specific project might be limited in the short run, so if a negative shock occurs the project gets paused or reduced in scale. Alternatively, if we consider the environment as the public good, the risk in the economy is given by the climate risk associated with the range of possible future scenarios.

This paper delivers three main results. First, I show that the market would generally provide an inefficient probability distribution (and expected level) of the public good in the case of heterogeneous risk preferences. This is because agents do not internalise the fact that their contribution to the public good affects also the distribution of risk between private and public consumption for the other agents that have different risk preferences. Given a general social welfare function, the (ex-ante) First Best is achieved with (risk-aversion) type-specific lump-sum taxation.[3] Then, we consider the case in which different types cannot be targeted: the expected level of the public good will be generally suboptimal (second best outcome).

Second, I provide an application of the public good provision problem in which I characterise a simple tax system for endowments and capital income that is used to implement the second-best optimal policy. I show that the government sets the optimal variance of the public policy by balancing the volatility of public consumption with the average volatility of private consumption. The excess return tax plays a key role for this mechanism. Agents with different preferences for risk have different benefits from the tax/insurance policy at the margin, meaning their willingness to shift risk from the individual to the societal level depends on their risk aversion. Hence, when agents with different risk preferences cannot be targeted, taxing only the excess return part of capital income fails to deliver the First Best allocation. This is because agents cannot be individually compensated when the public good distribution is not in line with their risk preferences.

Third, I show that if endowments are not perfectly correlated with risk preferences, taxing the safe return can be optimal: the government gives up intertemporal

return on savings: the (riskless) normal return and the excess return. The normal return is the price for forgoing present-time-consumption. The excess return reflects idionsyncratic characteristics and/or aggregate risk in the economy and drives returns heterogeneity.

[3] This differs from ex-post First Best that would require the type-specific lump-sum taxes to be state-contingent. From now on, First Best will simply indicate the ex-ante First Best.

efficiency to better target insurance to the different types of agents. It is possible to increase social welfare by exploiting the different individual portfolio and savings decisions as well as their different responses to changes to the endowment tax schedule.

This paper contributes to two main strands of literature. First, this paper is related to the literature that analyses public good provision and optimal taxes in a risky environment. For instance, Christiansen (1993) and Schindler (2008) examine the efficiency/insurance effects and the role of capital income taxes when the economy faces aggregate risk. The contribution of this paper is to develop a theory of public good provision in a risky environment in presence of heterogeneous agents that possess different attitudes to risk and have potentially different endowments. Using a general social welfare function, the appropriate Samuelson rule is derived. Moreover, differently from Schindler (2008), distorting intertemporal decisions by taxing the safe return from investments can be a useful tool to achieve the optimal allocation of risk in the economy, as this allows a more accurate targetting of agents with different risk preferences better.

Second, this paper is related to the growing literature on optimal taxation with heterogeneous returns. Previous papers have concentrated on the concepts of heterogeneous "investment ability" and/or scale effects (Boadway and Spiritus, 2021; Jacobs et al., 2020; Gahvari and Micheletto, 2016; Kristjánsson, 2016) as drivers of heterogeneous returns. This paper, in line with new empirical evidence (Bach et al 2020), considers different preferences for risk as an alternative driver of heterogeneous returns. This creates a connection between return heterogeneity and preference heterogeneity, as returns stem endogenously from preferences for risk through type-specific portfolio choice. While most of the above contributions argue in favor of taxing the normal (safe) return on grounds of redistribution,[4] our application shows that it can also have a role, alongside excess return taxation, in fostering insurance.

The rest of the paper is organised as follows. Section 2 revisits the theory of private provision of public good in the case of heterogeneous risk preferences. Section 3 presents the first and second best allocations with a generic social welfare function, and how it is possible to improve on the second best allocation. Section 4 presents an application of the public good problem in which the excess and safe returns can be taxed, and agents differ by their risk aversion. Section 5 extends section 4 by considering heterogeneous endowments. Section 6 concludes.

2 Public Good Provision in a Risky Environment

From the textbook theory of public goods, we know that public goods will be under-supplied by the market when provided on the basis of individual voluntary contri-

[4] In Boadway and Spiritus (2021), Jacobs et al. (2020), Gahvari and Micheletto (2016), the taxation of safe capital income complements the redistributive role of earnings taxation.

butions (Atkinson and Stiglitz, 1980). In this section, I revisit the standard theory and show that the private provision of public goods is still inefficient in a risky environment when agents possess different risk preferences. Then, after presenting the Pareto optimal case – given by the appropriate Samuelson rule – I discuss the type of inefficiency that private provision produces in a risky environment.

Let us consider a set-up in which each agent j allocates a share of her endowment z to consumption in the first period c_0, and saves the residual $z - c_0$ investing in a risky portfolio with gross return $\tilde{R}_p$, given their level risk aversion θ_j. In the second period, each agent enjoys private consumption $\tilde{c}_1$ and public consumption $\tilde{G}$ that are both subject to risk. Without loss of generality, I impose no discounting of future consumption. Moreover, individuals evaluate private and public consumption with the same sub-utility function $u(\cdot)$, meaning that no specific difference in taste between private and public goods is modeled.

$$\begin{aligned} \max_{c_0^j, g_j} \quad & U^j = u_j(c_0^j) + \mathbb{E}\left[u_j(\tilde{c}_1^j)\right] + \mathbb{E}\left[u_j\left(\tilde{G}\right)\right] \\ \text{s.t.} \quad & \tilde{c}_1 = \tilde{R}_p^j\left(z - c_0^j\right) - g_j, \\ & \tilde{G} = f\big(\textstyle\sum_j g_j, \tilde{x}\big) \end{aligned}$$

Each individual j chooses how much to contribute (g_j) to the public good $\tilde{G}$ in the first period by maximising lifetime utility U taking the contributions of the other agents (g_{-j}) as given. In a no-risk situation, the level of the public good will be given by the sum of individual contributions. With an underlying source of risk in the economy $\tilde{x}$, then the public good is risky as it also depends on the state of the economy in the second period, i.e. $\tilde{G} = f\big(\sum_j g_j, \tilde{x}\big)$.[5] Hence, even if the marginal rate of transformation is one, meaning one unit of private consumption buys one unit of public consumption in a riskless economy, aggregate risk can either increase or decrease the actual value of the public good relative to the value of private consumption that was initially sacrificed. The optimal private contribution rule g_j^* satisfies

$$g_j^* : MRS_{\tilde{G},\tilde{c}_1^j} = \frac{\mathbb{E}[u_j'(\tilde{G})]}{\mathbb{E}[u_j'(\tilde{c}_1^j)]} = 1 \ \ \forall j, \tag{1}$$

where the size of the public good $\tilde{G}$ depends on the sum of the individual contributions and aggregate risk in the economy $\tilde{x}$ i.e. $\tilde{G} = f\big(\sum_j g_j, \tilde{x}\big)$. Each agent j contributes until the risk-adjusted marginal rate of substitution between the public and private good $MRS_{\tilde{G},\tilde{c}_1^j}$ equates the marginal cost (exp. 2.1). At the Nash

[5] The underlying source of risk should be seen as a risk for the overall economy, affecting both private consumption through portfolio returns, as well as public good consumption through aggregate shocks to public finances.

equilibrium, the set of individual contribution rules $\{g_j, g_{-j}\}$ will satisfy (2.2) for each type. Summing up (2.1) across agents $j = 1, \cdots, n$ gives $\sum_j MRS_{\tilde{G},\tilde{c}_1^j} = n$. By comparing the Nash equilibrium outcome with the Pareto efficient allocation,[6] given by the appropriate Samuelson rule

$$\sum_j MRS^j_{\tilde{G},\tilde{c}_1^j} = \sum_j \frac{\mathbb{E}[u_j'(\tilde{G})]}{\mathbb{E}[u_j'(\tilde{c}_1^j)]} = 1 < n, \tag{2}$$

we can state the following well-known result.

Lemma 1. *Public good provision in the case of private contributions is inefficient as agents do not internalise the external value of their individual contributions.*

In the context of aggregate risk in the economy, Lemma 1 has two specific implications. First, the public good will be underprovided in expectation. Second, private provision of public good generates an inefficient allocation of risk between private and public consumption, meaning that the probability distribution of the public good over different states of nature is suboptimal.

3 Samuelson Rule with a Social Welfare Function

The concept of Pareto optimality can be quite restricting when preferences are heterogeneous. Policy changes often benefits some agents while hurting others. Hence, we proceed by considering the optimal allocation that stems from the government's maximisation of a generic Social Welfare function

$$SW = \sum_j \phi_j (U_j),$$

where $\phi_j (U_j)$ is a weakly concave function of individual expected utility U_j that is able to accommodate different individual risk preferences.[7] Then, the optimal rules for public good provision are derived. Two possible cases are presented: 1) (Ex-ante) First Best: the government can levy type-specific lump-sum taxes that depend on individual risk aversion (section 2.3.1); 2) Second Best: the government sets one unique lump-sum tax for all types (section 2.3.2).

[6] Each Pareto-efficient allocation can be achieved by private, decentralised optimisation when the public good is financed by (risk-aversion) type-specific lump-sum taxes such that (2.2) is satisfied.

[7] The issue of choosing the appropriate functions ϕ_j for utility functions with different curvatures is tackled separately in the third chapter of this thesis. The issue is widely debated in the welfare theory literature. For instance, Grant et al. (2010) shows how it is possible to accommodate concerns about different individuals' risk attitudes and concerns about fairness; Eden (2020) provides a practical method to perform welfare analysis.

3.1 First Best: Type Specific Lump-sum Taxes

The government chooses t_i for each agent i, therefore perfectly targeting agents with different risk aversion. Expression 2.3 states that the optimal t_i equates the marginal private costs of agent i from contributing to the public good, expressed in terms of private consumption, to the social benefit of a marginal individual contribution:

$$\phi_i'\Big(U_i\Big)\,\mathbb{E}[u_i'(\tilde{c}_1^i)] = \sum_j \phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{G})] \quad \forall\, i. \tag{3}$$

At the optimum, expression (2.3) is satisfied for each agent i and gives the set of optimal lump-sum taxes t_j for all types of agent. The government sets t_i for each agent i such that the weighted expected marginal utility of private consumption is equalised across types to a certain level $\overline{k}$.

$$\{t_i, i = 1, \cdots, n\} \;\; \rightarrow \;\; \phi_i'\Big(U_i\Big)\,\mathbb{E}[u_i'(\tilde{c}_1^i)] = \overline{k} \quad \forall\, i = 1, \cdots, n. \tag{4}$$

By summing up condition (2.3) across types $i = 1, \cdots, n$, and using (2.4), we can obtain a modified Samuelson rule that is expressed in terms of the sum of socially-evaluated Marginal Rates of Substitutions (MRS^s), where

$$MRS_j^s = \frac{\phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{G})]}{\phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{c}_1^j)]}.$$

The MRS^s is defined as the ratio between the marginal social welfare associated with an additional unit of the public good and the marginal social welfare associated with an additional unit of private consumption for an agent j.

Theorem 1 (First Best)**.** *At the First Best optimum, the sum of the MRS^s is equal to the marginal rate of transformation between the public and private good.*

$$\sum_j MRS_j^s = \sum_j \frac{\phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{G})]}{\phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{c}_1^j)]} = 1. \tag{5}$$

Condition (2.5) is the Samuelson rule that reflects the social preferences that are being maximised. While type specific taxation is a useful theoretical benchmark, it is unlikely to be feasible in practise. The next section therefore considers an alternative case of public provision in which the lump-sum tax is set equal for all agents.

3.2 Second Best: Uniform Lump-sum Taxes

If the government cannot screen agents with different risk aversion, and therefore cannot set (risk-aversion) type specific lump-sum taxes, a uniform lump-sum tax t for all agents apply. At the second best optimum, marginal social costs, expressed in terms of private consumption, are balanced with the social benefits:

$$\sum_j \phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{c}_1^j)] = n\sum_j \phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{G})]. \tag{6}$$

Condition (2.6) can be rewritten in terms of $\widetilde{MRS}_j^s$, where

$$\widetilde{MRS}_j^s = \frac{\phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{G})]}{\frac{1}{n}\sum_j \phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{c}_1^j)]},$$

that is defined as the ratio between the marginal social welfare associated with an additional unit of the public good for an agent j and the average (across agents $j = 1, \cdots, n$) marginal social welfare associated with an additional unit of private consumption. This means that when type specific taxes are not available, the government evaluates the individual willingness to trade private with public consumption on the basis of the average sacrifice in terms of private consumption, e.g. $\widetilde{MRS}_j^s$ rather than MRS_j^s . Hence, condition (2.7) is the modified Samuelson rule in the case of heterogeneous risk preferences when the government does not discriminate the different risk-aversion-types.

Theorem 2 (Second Best)**.** *At the second best optimum, the sum of $\widetilde{MRS}_j^s$ equates the marginal rate of transformation.*

$$\sum_j \widetilde{MRS}_j^s = \sum_j \left(\frac{\phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{G})]}{\frac{1}{n}\sum_j \phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{c}_1^j)]}\right) = 1. \tag{7}$$

The Second Best outcome will differ from the First Best benchmark, both in terms public good provision and private consumption, and therefore social welfare, when $\sum \widetilde{MRS}_j^s \neq \sum MRS_j^s$, namely when condition (2.7) differs from (2.5). We can now state Proposition 1.

Proposition 1. *The inability of the government to discriminate different risk-aversion-types in the economy leads to a suboptimal provision of the public good.*

Proof. Suppose Proposition 1 is false, and $\tilde{G}^{SB} \equiv \tilde{G}^{FB}$, then it must be that (2.5) coincides with (2.7) and that

$$\frac{1}{n}\sum_j \phi_j'\Big(U_j\Big)\,\mathbb{E}[u_j'(\tilde{c}_1^{SB})] = \phi_i'\Big(U_i\Big)\,\mathbb{E}[u_i'(\tilde{c}_1^{FB})] \;\; \forall i = 1, \cdots, n.$$

In order for this to be true, it should follow that $t_i^{FB} = t^{SB}$ for $i = 1, \cdots, n$, meaning the individual private costs of raising a unit of consumption are equal across different types to begin with. Unless specific welfare weights are chosen to obtain this result or the equality happens to hold given the utility functions being used in the first place, proposition 1 will be true in the case with heterogeneous preferences. □

While risk-preference-specific lump-sum taxes might be unfeasible, let alone lump-sum taxes, we know that agents with different risk preferences will make different consumption, savings, portfolio and labour supply choices. These differences can therefore be exploited to get closer to the first-best optimum.

In the next two sections, I study an application of this principle with regard to savings and portfolio choices: the optimal (public) provision of public goods is characterised when the government taxes capital income.

4 Application: Taxation of Excess and Safe Return

I consider a simple two-period consumption/savings and portfolio model where agents have different risk preferences and the government finances the public good by raising equal lump-sum contributions and by taxing the riskless return and risky excess return separately.

4.1 Agent's Problem

$\Theta = \{(\theta_1), \cdots, (\theta_j), \cdots, (\theta_n)\}$ is a discrete set of agent-types with relative risk aversion parameters θ_j, $j = 1, \ldots, n$, from the sample space $\mathbf{S}$, and each type has equal weight in the population. Given an endowment z, agents choose first period consumption, c_0, and how to invest the residual $(z - c_0)$. Agents choose the portfolio share of risky assets s with the risky return $\tilde{r}_e$ being Normally distributed: $\tilde{r}_e \sim \mathcal{N}(\overline{r}_e - \sigma_r^2/2, \sigma_r^2)$. The resulting portfolio return $\tilde{r}_p$ will also be Normally distributed: $\tilde{r}_p \sim \mathcal{N}(\overline{r}_p - \sigma_p^2/2, \sigma_p^2)$. The (risky) excess return is defined as the difference between the risky return and the riskless return: $\tilde{r}^{exc} := \tilde{r}^e - r$. Without loss of generality, I impose no discounting: $\beta = 1$. Agent's utility is separable over time as well as between private and public consumption. The problem for agent-type j is thus:

$$\begin{aligned}
\max_{c_0^j,\, s_j} \quad & U_j = u_j(c_0^j) + \beta\, \mathbb{E}\left[u_j(\tilde{c}_1^j) + u_j(\tilde{G})\right] \\
\text{s.t.} \quad & \tilde{c}_1 = \tilde{R}_p^j \left(z - c_0^j\right) - t, \\
& \tilde{R}_p^j = \left[1 + r\,(1 - \tau_r) + s_j\,(\tilde{r}^{exc})\,(1 - \tau_k)\right]
\end{aligned}$$

where $\tilde{R}_p$ is the gross portfolio return; t is a lump-sum tax, equal for all types; τ_r and τ_k are the tax rates on the riskless and excess return respectively. The FOCs for first period consumption c_0, and share of risky assets s are:

$$\begin{aligned}
c_0^* : \quad & u'(c_0) = \mathbb{E}\left[u'(\tilde{c}_1)\tilde{R}_p\right] \\
s^* : \quad & 0 = \mathbb{E}\left[u'(\tilde{c}_1)\tilde{r}^{exc}\right] \qquad (8)
\end{aligned}$$

Notice that, while taxation of the riskless return changes the resource allocation (first period consumption, savings), taxing the excess returns with loss offsets does not. Agents will adjust their portfolio shares to get the same pre-tax expected portfolio return (Domar and Musgrave, 1944). After applying the covariance identity to (2.8), we can derive the expression for the risk premium with respect to the expected excess return.

$$\mathbb{E}[R^e - R] = -\frac{\operatorname{cov}\left[u_j'(\tilde{c}_1^j), \tilde{r}^{exc}\right]}{\mathbb{E}\left[u_j'(\tilde{c}_1^j)\right]}. \tag{9}$$

For each agent j with relative risk aversion parameter θ_j, expression (2.9) maps the covariance term to the expected marginal utility of second-period consumption $\tilde{c}_1$.

4.2 The Government

The government provides a public good $\tilde{G}$ that enters the utility function separately from consumption. The policy is financed by taxing risky excess return at rate τ_k, the safe return at rate τ_r, and levying a lump-sum tax t, equal for all agents. The public good is itself risky as it depends on risky tax revenues from the excess return. Tax rates are set by the government in the first period anticipating agents' optimal behaviour. After the state of the economy is realised, the government implements the policy and balances the budget. As a result, the provision of the public good is stochastic and depends on the state of the economy in the second period.

The government's objective is to maximise Social welfare (SW) that is defined as a weighted sum of agents' expected utilities, where U is a Von Neumann-Morgenstern utility function.

$$\begin{aligned}\max_{\tau_k,\tau_r,t} SW &= \sum_j \phi_j\Big(U_j\Big)U_j\left(c_0^{j*},\, \tilde{R}_p^j\left(z - c_0^{j*}\right) - t,\, \tilde{G}\right)\\ \text{s.t. } \tilde{G} &= \sum_j \left[\left(\tau_k\, s_j^*\, \tilde{r}^{exc} + \tau_r r\right)\left(z - c_0^{j*}\right) + t\right]\end{aligned}$$

I substitute the expression for the public good directly in the social welfare function, as in Schindler (2008). It ensures that the budget is balanced for any state of the world.[8]

4.3 Optimality Conditions

In this section, I show the optimality conditions for the linear taxes on the excess and riskless return when a pure public good is provided. Moreover, I assume that the

[8]This is a stricter requirement than balancing the budget in expectation, that would instead imply transferring resources from good states to bad states of the world.

government cannot screen agents with different risk preferences, meaning the government uses a uniform lump-sum tax, and endowments are equal in the population.

4.3.1 Excess Return Tax

$$\mathbb{E}[\tilde{r}^{exc}] = -\frac{\sum_j \phi_j'(U_j)\,\mathrm{cov}\left[u_j'(\tilde{G}), \tilde{r}^{exc}\right]}{\sum_j \phi_j'(U_j)\,\mathbb{E}[u_j'(\tilde{G})]} \tag{10}$$

The optimal excess return tax rate τ_k^* has to satisfy condition (2.10). Given the risk preferences of all agents in society, the government chooses τ_k^* to reach the optimal allocation of risk between private and public consumption, and sets the optimal variance of the public good policy. The key novelty in this setting is that the "benefits" of the public good differ among agents because of heterogeneous attitudes to risk. Since the public good is itself risky, the welfare gain from increasing the tax rate τ_k varies across agents. Hence, the government aims to balance the different "preferences" for the public good, providing insurance against aggregate risk.

Two interpretations can be developed that focus on the issues of tax revenue collection and the public good respectively. The first one is that the government is choosing what share of the budget (tax revenues) should be risky.[9] As individuals make portfolio choices on the basis of their risk preferences according to (2.9), similarly the government decides the distribution of tax revenues financing the public good over states of nature on the basis of (2.10), such that the risk preferences of all agents are taken into account.

The second interpretation relates to the use of tax revenues. The (risky) revenues collected from taxing the excess return will generate a certain probability distribution of the public good. This distribution entails a certain allocation of risk between private and public consumption: the public good has an insurance role.[10] The government's objective is to choose the social-welfare-maximising public good distribution that achieves the optimal allocation of risk in the economy. In doing so, the government takes into account the agents' willingness to shift risk to the societal level. Condition (2.11) reformulates (2.10) and better represents this concept. Private and public consumption volatility are represented by the covariance between marginal utility of private and public consumption respectively with the risky excess return and jointly govern the individual willingness to pay for the public good with an extra euro of risky excess capital income.

[9] The realisation of the excess return depends on the state of the economy.

[10] When a bad (good) state of economy realises in the second period, losses (gains) due to negative (positive) excess returns will be spread over private and public consumption, so that the utility loss (gain) is minimised (maximised).

Theorem 3. *When agents with different risk preferences cannot be discriminated, τ_k^* equalizes public consumption volatility with (average) private consumption volatility.*

$$\sum_j \frac{\phi_j'(U_j)\,\mathrm{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right)}{n^{-1}\sum_j \phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1\right), \tilde{r}^{exc}\right)} = 1 \tag{11}$$

Proof. See Appendix A.

Theorem 3 says that τ_k^* is chosen on the basis of a weighted average of private consumption volatility, i.e. $n^{-1}\sum_j \phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1\right), \tilde{r}^{exc}\right)$. This is because the government cannot target individual risk-aversion types. This creates an inefficiency: an agent with relatively (more) risky private consumption c_1, compared to other agents, would be better-off by shifting more risk to the public good. For that to be the case, a higher tax rate τ_k should be implemented, so that private consumption volatility is traded with public consumption volatility. Thus, the outcome produced by (2.11) could be improved if the government were able to target different types, and compensate agents that would prefer a different distribution of the public good.

Corollary 1. *When the government can target different risk aversion types (2.11) simplifies to*

$$\sum_j \frac{\phi_j'(U_j)\,\mathrm{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right)}{\phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1\right), \tilde{r}^{exc}\right)} = 1 \tag{12}$$

and the allocation of risk between private and public consumption relates to individual willingness to shift risk from private to public consumption $\frac{\phi_j'(U_j)\,\mathrm{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right)}{\phi_j'(U_j)\,\mathrm{cov}\left(u_j'(\tilde{c}_1), \tilde{r}^{exc}\right)}$.

Proof. See Appendix B.

To sum up, when agents have heterogeneous risk preferences, the optimal tax τ_k is not just about balancing the volatility of private and public consumption, as in Boadway and Spiritus (2021) and Schindler (2008), but also aims to balance the different "preferences" for the public good, providing insurance against aggregate risk (Theorem 3). As the optimal tax rate τ_k^* implies a specific probability distribution of the public good, taxing the excess return only may be suboptimal. Corollary 1 shows that welfare improvements are possible when different types can be targeted. If type-specific lump-sum taxation is not available, other tax instruments that have differential impacts on different types could be used. In the next section, I argue that the taxation of the safe return has this feature. Finally, it is possible to show that the optimal excess return tax rate is positive.

4.3.2 Safe Return Tax

The optimal safe return tax rate τ_r^* satisfies:

$$\underbrace{-\sum_j \phi_j'(U_j)\mathbb{E}\left[u_j'\left(\tilde{c}_1^j\right)\right]\left(z - c_0^j\right) r}_{\text{private welfare effect } = \Delta U} + \underbrace{\sum_j \phi_j'(U_j)\mathbb{E}\left[u_j'\left(\tilde{G}\right)\right]\sum_j\left(z - c_0^j\right) r}_{\text{mechanical welfare effect } = \Delta M}$$
$$\underbrace{-\tau_r r\sum_j \phi_j'(U_j)\mathbb{E}\left[u_j'\left(\tilde{G}\right)\right]\sum_j \frac{\partial c_0^j}{\partial \tau_r}}_{\text{behavioural effect } = \Delta B} = 0. \tag{13}$$

A marginal change in τ_r determines a welfare loss for agents as second period consumption is lowered: this is the private welfare effect (ΔU). On the other hand, the additional tax revenues finance the public good which increases agents' utility: this is the mechanical welfare effect (ΔM). However, varying the tax rate affects agents' savings too (via change in first period consumption c_0) through substitution and income effects: this is the behavioural effect (ΔB), which affects the tax base and therefore tax revenues.

At the optimum, private welfare losses are balanced with the welfare gains from public good provision, net of behavioural effects: $\Delta U + \Delta M + \Delta B = 0$. Unless the mechanical and private welfare effects sum up to zero, the optimality condition is satisfied only with $\tau_r \neq 0$, provided that $\sum_j(\partial c_0^j/\partial \tau_r) \neq 0$.

Theorem 4. *a) With heterogeneous risk preferences, the mechanical and private welfare do not sum up to zero: $\Delta U + \Delta M \neq 0$ or equivalently $|\Delta U| \neq |\Delta M|$. b) Then, $\tau_r^* \neq 0$ solves the optimal tax condition (14).*

Proof. (Utilitarian case, i.e. $\phi_j(U) = U$ for all $j = 1, \cdots, n$.)

a) We can use (2.6) to rewrite the mechanical term as follows

$$\Delta M = \frac{1}{n}\sum_j \mathbb{E}\left[u_j'(\tilde{c}_1^j)\right]\sum_j (z - c_0^j) r$$

Define $\mathbb{E}\left[u_j'\left(\tilde{c}_1^j\right)\right] = a_j$, and $\left(z - c_0^j\right) r = b_j$. Notice that as $a_H \neq a_L$ and $b_H \neq b_L$ for any agents H, L with $\theta_H > \theta_L$, then $|\Delta U| = \sum_j a_j b_j \neq n^{-1}\sum_j a_j \sum_j b_j = |\Delta M|$.
b) Consider $\sum_j(\partial c_0^j/\partial \tau_r) \neq 0$. If $|\Delta U| \neq |\Delta M|$, then the optimality condition is satisfied when $\tau_r^* \neq 0$, as we have $\Delta B \neq 0 \iff \tau_r^* \neq 0$. □

Hence, it can be optimal to tax the safe return when agents have heterogeneous risk preferences. With CRRA utility, different relative risk aversion parameters imply different tastes for risk (portfolio decisions), different slopes of the consumption path as well as different responses of savings decisions responses to taxation of returns. Theorem 4 tells us that exploiting these differences by taxing (positively or negatively) the riskless part of the return can increase social welfare. Therefore, a trade-off between insurance and intertemporal efficiency can arise.

Finally, we consider the case in which the government can impose type-specific lump-sum taxes.

Corollary 2. *When the government can target agents with different risk preferences with type-specific lump-sum taxes, (2.13) is always satisfied by* $\tau_r^* = 0$.

Hence, the taxation of the safe return acts as an (imperfect) substitute for (risk-aversion) type-specific taxation.

5 Heterogeneous Endowments and Risk Aversion

In this section, I revisit the results of section 2.4 by considering an environment in which each type of agent is characterised by their endowment and risk aversion, and the government taxes endowments and capital income. The question that this section aims to answer is whether taxing the safe return is still optimal when endowments are taxed non-linearly and the level of endowment is correlated with the individual's risk aversion.

5.1 Additional Notation

The set of agent-types is a $m \times n$ matrix Θ, where each element is given by the pair (z_i, θ_j) that represents an individual-type ij who has endowment z_i and relative risk aversion θ_j. F is the frequency matrix of types in the population, with each type having frequency f_{ij} in the population. We also define an endowment group i as the i-th row of Θ: $(z_i, -) := \{(z_i, \theta_1), \cdots, (z_i, \theta_n)\}$. Thus, $(z_i, -)$ is the group of agents that have the same endowment z_i, but differ by their risk aversion $\theta_j, j = 1, \cdots, n$.

$$\Theta_{m\times n} = \begin{pmatrix} z_1, \theta_1 & \cdots & z_1, \theta_n \\ \vdots & \ddots & \vdots \\ z_m, \theta_1 & \cdots & z_m, \theta_n \end{pmatrix}, \quad F = \begin{pmatrix} f_{11} & \cdots & f_{1n} \\ \vdots & \ddots & \vdots \\ f_{m1} & \cdots & f_{mn} \end{pmatrix}.$$

As in the previous section, each component of capital income is taxed linearly: τ_k is the excess return tax, τ_r is the tax rate on the safe return. Endowments are subject to a piece-wise linear schedule $T(z)$, where τ_e^i is the average tax rate on endowment z_i given the tax schedule $T(z)$ that applies on the endowment group $(z_i, -)$. With many levels of endowment z_i, $T(z)$ will approximate a non-linear function of z_i. y_i is the net-of-tax endowment: $y_i := z_i(1 - \tau_e^i)$.

5.2 Taxation of Endowments and Safe Capital Income

Conditions (2.14) and (2.15) are the optimality conditions for the endowment tax τ_e^i and the safe return tax τ_r respectively. First, we analyse the optimality condition

(2.14) for the optimal average tax rate τ_e^i on each endowment group $(z_i, -)$, $i = 1, \cdots, m$.

$$\tau_e^{i*}: \frac{\sum_j f_{ij}\phi_j'(U_{ij})\,\mathbb{E}\left[u_{ij}'(\tilde{c}_1^i)\right]z_i(1+r(1-\tau_r))}{\sum_j f_{ij}\left[z_i(1+r(1-\tau_r)) - \tau_r r\cdot\frac{\partial c_0^{ij}}{\partial \tau_e^i}\right]} = \sum_{i,j} f_{ij}\phi_j'(U_{ij})\,\mathbb{E}\left[u_{ij}'(\tilde{G})\right] \tag{14}$$

$$\tau_r^*: \frac{\sum_{i,j} f_{ij}\phi_j'(U_{ij})\,\mathbb{E}\left[u_{ij}'(\tilde{c}_1^i)\right]\left[y_i - c_0^{ij}\right]}{\sum_{i,j} f_{ij}\left[y_i - c_0^{ij} - \tau_r\frac{\partial c_0^{ij}}{\partial \tau_r}\right]} = \sum_{i,j} f_{ij}\phi_j'(U_{ij})\,\mathbb{E}\left[u_{ij}'(\tilde{G})\right] \tag{15}$$

At the optimum, condition (2.14) establishes that the marginal private welfare costs from raising τ_e^i for endowment group $(z_i, -)$, expressed in terms of revenues being raised, is equalised to the (population-wide) marginal social value of the public good. This should, in turn, equalise the marginal private welfare costs from raising the safe return tax τ_r, in terms of revenues being raised, so that (2.15) is satisfied too.

A fully optimised endowment tax schedule $T(z)$ requires (2.14) to hold for each endowment group, meaning that marginal private welfare costs from raising the endowment tax rate are equalised across endowment groups. Moreover, taxing the endowment will generate heterogeneous consumption responses that will affect also tax revenues from safe capital income taxation, i.e. the term $-\tau_r r\cdot\frac{\partial c_0^{ij}}{\partial \tau_e^i}$ in (2.14). These differential effects across types can therefore be exploited to make sure (2.14) is indeed satisfied across endowment groups. When there is imperfect correlation (or no correlation at all) between endowment and risk aversion in the economy, it is not obvious that (2.14) will be holding across endowment groups without taxing the safe return. For each endowment group with a specific distribution of attitudes to risk within it, the welfare costs from raising the endowment tax might differ (LHS of condition 2.14), that is why taxing the safe return might improve welfare.

When the level of the endowment is not highly correlated with the level of individual risk aversion, endowment taxation is not an effective way to discriminate across agents with different risk aversion. With its heterogeneous effects on agents with different risk attitudes, taxing the safe return can therefore help to achieve a better allocation of aggregate risk in the economy between private and public consumption and across agents. Of course, if there is perfect correlation between the level of the endowment and the risk aversion parameter, and endowments are taxed according to $T(z)$, the first-best public good distribution can be achieved without taxing the safe return. In this case the piece-wise linear tax schedule $T(z)$ would be equivalent to a type-specific lump-sum tax system, as each endowment level in the type-set Θ is attached to one specific risk aversion parameter, and taxing the safe return becomes redundant - see a short proof in the appendix.

6 Concluding Remarks

People with different attitudes to risk have different views on the extent to which society should invest in certain (risky) projects. This paper presents a theory of optimal provision of a (risky) public good when individuals have heterogeneous preferences for risk and face aggregate risk in the economy. In an environment in which the public good is a tool to shift risk from private to public consumption, this paper shows that the inefficiency of private provision of public goods comes from agents failing to internalize the insurance effects of the public good for the other agents. Given a social welfare function, I characterise the (ex-ante) First Best allocation, which is achieved with (risk-aversion) type-specific lump-sum taxation. Discriminating the different types of agent allows the government to compensate them when the distribution of the public good is not in the line with their risk preferences.

Then, I characterize the second best allocation, which is the optimum that can be achieved under the constraint that type-specific taxes and transfer are not available. In an application with capital income taxation, I show that the excess return tax is key to match the optimal variance of the public insurance policy. Moreover, it is possible to justify a positive tax on the safe return: it is optimal for the government to give up intertemporal efficiency by taxing the safe return to provide better-targeted insurance to agents. This complements the excess return tax: the government exploits agents' different portfolio and savings decisions as well as different responses to tax changes to increase social welfare.

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Appendix A - Theorem 3

The optimality condition for the excess return tax τ_k reads as follows.

$$\sum_j \phi_j'(U_j)\mathbb{E}\Big[u_j'(\tilde{G})\sum_j s_j\left(z^j - c_0^j\right)\tilde{r}^{exc}\Big] + \tau_k\sum_j \phi_j'(U_j)\mathbb{E}\left[u_j'(\tilde{G})\sum_j \frac{\partial s_j}{\partial \tau_k}\left(z^j - c_0^j\right)\tilde{r}^{exc}\right] = 0.$$

Using the fact that $\frac{\partial s_j}{\partial \tau_k} = \frac{s_j}{(1-\tau_k)}$, we can collect terms.

$$\sum_j \phi_j'(U_j)\mathbb{E}\Big[u_j'(\tilde{G})\sum_j s_j\left(z^j - c_0^j\right)\tilde{r}^{exc}\Big]\left(1 + \frac{\tau_k}{1-\tau_k}\right) = 0.$$

Then, as $\sum_j s_j\left(z^j - c_0^j\right)\left(1 + \frac{\tau_k}{1-\tau_k}\right) \neq 0$, we get the following expression

$$\sum_j \phi_j'(U_j)\mathbb{E}\left[u_j'(\tilde{G})\tilde{r}^{exc}\right] = 0.$$

By further manipulating the above expression using the covariance identity, and conditions (7) and (9), we can rewrite the optimality condition for τ_k as follows

$$\frac{1}{n}\sum_j \phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1^j\right), \tilde{r}^{exc}\right) = \sum_j \phi_j'(U_j)\,\mathrm{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right).$$

or

$$\sum_j \frac{\phi_j'(U_j)\,\mathrm{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right)}{\frac{1}{n}\sum_j \phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1^j\right), \tilde{r}^{exc}\right)} = 1.$$

$\square$

Appendix B - Corollary 1

Condition (5), plus the covariance identity and condition (9) imply that the term

$$\phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1^j\right), \tilde{r}^{exc}\right)$$

is equal across types j. Hence, expression 12 can be reformulated as follows

$$\begin{aligned}
&\sum_j \frac{\phi_j'(U_j)\,\mathrm{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right)}{\frac{1}{n}\sum_j \phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1^j\right), \tilde{r}^{exc}\right)} \\
=&\sum_j \frac{\phi_j'(U_j)\,\mathrm{cov}\left(u_j'(\tilde{G}), \tilde{r}^{exc}\right)}{\phi_j'(U_j)\,\mathrm{cov}\left(u_j'\left(\tilde{c}_1^j\right), \tilde{r}^{exc}\right)} = 1.
\end{aligned}$$

$\square$

Appendix C Perfect Correlation Between Risk Aversion and Endowments

Short Proof. Consider the following set of types Θ and the corresponding frequency matrix of types in the population F:

$$\Theta = \begin{pmatrix} z_A, \theta_A & z_A, \theta_B \\ z_B, \theta_A & z_B, \theta_B \end{pmatrix}, \quad F = \begin{pmatrix} f_{AA} & f_{AB} \\ f_{BA} & f_{BB} \end{pmatrix}.$$

With perfect correlation between risk aversion and endowment level, only types on either of the two diagonals of Θ have positive frequencies. Suppose that (z_A, θ_A); (z_B, θ_B) have positive frequencies in the population, that are $f_{AA}, f_{BB} > 0$ respectively. Suppose the optimal safe return tax rate is zero: $\tau_r^* = 0$. Then, condition (2.14) must be valid for both agents with $\tau_r = 0$. The following relationship can be derived as a result:

$$\phi_A'(U_{AA})\, \mathbb{E}\left[u_{AA}'(\tilde{c}_1)\right] = \phi_B'(U_{BB})\, \mathbb{E}\left[u_{BB}'(\tilde{c}_1)\right]. \tag{16}$$

Exploiting (16) makes the condition for τ_r^* (2.15) satisfied, meaning that our initial guess $\tau_r = 0$ is indeed a solution of the optimality condition for the safe return tax (2.15). Moreover, as (16) is equivalent to (2.4), the endowment tax schedule $T(z)$ is equivalent to a type-specific lump-sum tax system in this example. Hence, the first best allocation is achieved by simply taxing endowments and the risky excess return. $\square$