# A Theory of Public Good Provision with Heterogeneous Risk Preferences*

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**Abstract**

People with different attitudes to risk have different views on the extent to which society should invest in certain (risky) projects. This paper presents a theory of optimal provision of a risky public good when individuals have heterogeneous preferences for risk and labour productivities. The public good has an insurance purpose as it allows individuals to shift risk from private to public consumption. When it is privately provided, the risk allocation in the economy is inefficient because people do not internalise the insurance gains of the public good for the other agents who have different preferences for risk. When the public good is publicly provided, and it is financed by a piecewise linear tax schedule on labour earnings and linear taxes on safe and risky (excess) returns on savings, a trade-off between insurance and equity arises. The progressivity of the labour income tax schedule and the desirability of imposing a tax on the safe return from savings crucially depend on whether the government prioritises redistribution towards low-income individuals or high risk averse individuals.

**JEL classification**: H21, H41.

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# 1 Introduction

There is a growing literature documenting heterogeneous social preferences. Age, gender, economic background, educational attainment and other individual characteristics shape people's view concerning what the government should invest in and how progressive the tax system should be (Stantcheva, 2021). One area of government's policy that often polarises the public opinion is investment in infrastructures. Public projects are often perceived as too risky, either because they might take longer to be completed, or because the net benefits of the projects might fail to materialise. For instance, a survey conducted in 2018 on the high speed rail project "HS2", that is currently under construction in the UK, shows that 59% of those who oppose the project are worried that "costs are or potentially will be too high", possibly reflecting different preferences for risk associated with skyrocketing costs of public projects.

Another example in which different attitudes towards risk might play a role is climate change. While most people agree climate change is a problem and measures must be taken to mitigate or stop it,[1] different views on the scale of the actions to implement emerge. The 2021 Eurobarometer survey on climate change shows that 80% of respondents from Sweden agree that "reducing fossil fuel imports from outside the EU can increase energy security and benefit the EU economically", while only 59% in France think so. Moreover, the perceived costs of climate change are also heterogeneous. In Portugal, 52% of respondents totally agree with the statement that the cost of damage due to climate change is much higher than the cost of investment needed for a green transition, but only 28% does so in Poland and Finland. While differences across countries can be explained by different exposures to climate risks (Dechezleprêtre et al., 2022), differences within countries could partly reflect different attitudes to (climate change) risk.

As the examples presented above suggest, people with different attitudes to risk not only make different individual decisions throughout their lives, but also have different views on the extent to which the government should invest in certain (risky) public goods, like infrastructures or green investments to protect the environment. Despite its relevance, the normative issue of public provision of public goods in the context of heterogeneous attitudes to risk, and the optimal tax system needed to implement it, has not received much attention. This paper contributes to fill this gap by presenting a theory of optimal provision of a (risky) public good when individuals have heterogeneous preferences for risk, while facing aggregate risk in the economy.

In the first part of the paper (sections 2-3), I consider a two-period framework in which agents differ only by their risk preferences, and labour supply is exogeneous. I consider the Nash equilibrium outcome when the public good is offered on the basis of individual contributions (section 2), and the solution of a social planner maximising

[1] According to the 2021 Eurobarometer survey, 78% of the respondents consider climate change "a very serious problem", while 15% see it as "a fairly serious problem".

a generic social welfare function (section 3) to derive the Samuelson rule. Then, I study the role of labour and capital income taxes to finance a public good. Agent-types are characterised by their level of risk aversion and productivity (wage), and make decisions on labour supply, consumption and savings, with constant relative risk aversion utility. They also make portfolio decisions, choosing between two types of assets: one is risk-free, while the other is subject to aggregate risk, but with a positive expected excess return.[2] The government raises revenues with a piece-wise linear tax schedule on labour earnings and with two distinct proportional taxes on the safe return and the risky excess return (with symmetric loss offsets), along with a lump-sum tax that is equal for all agents.

The public good plays an insurance role, as risk is spread between public and private consumption, as in Christiansen (1993), Schindler (2008) and Boadway and Spiritus (2024). The public good is itself risky since tax revenues depend on the state of the economy. The tax parameters are chosen optimally ex-ante while the policy is implemented in the second period, after the realization of the state of the economy, such that the budget is balanced. This captures the idea that funds for a specific project might be limited in the short run, so if a negative shock occurs the project gets paused or reduced in scale. Alternatively, if we consider the environment as the public good, the risk in the economy is given by the climate risk associated with the range of possible future scenarios.

This paper presents three main findings. First, I argue that the market would generally provide an inefficient probability distribution of the public good in the case of heterogeneous risk preferences. This is because agents do not internalise the fact that their contribution to the public good affects the distribution of risk between private and public consumption also for the other agents who might have different risk preferences. Hence, agents underestimate the insurance effect of the public good. Given a general social welfare function, the (ex-ante) First Best allocation is achieved with (risk-aversion) type-specific lump-sum taxation.[3] Then, when different types cannot be targeted without efficient costs, the expected level of the public good will be generally suboptimal (second best outcome).

Second, I consider the full model and study the second-best policy in the case in which the public good is financed by a piece-wise linear tax schedule on labour earnings and proportional taxes on risky and safe capital income. I study the optimal tax system under two alternative welfare criteria that either prioritises redistribution towards low-income agents (equity-driven redistribution) or high-risk-aversion agents (insurance-driven redistribution). Beside the standard equity-efficiency trade-off, I

[2] Following the optimal tax literature, I distinguish two different components of the rate of return on savings: the (riskless) normal return and the excess return. The normal return is the price for forgoing present-time-consumption. The excess return can reflect idionsyncratic characteristics and/or aggregate risk in the economy and drives returns heterogeneity.

[3] This differs from ex-post First Best that would require the type-specific lump-sum taxes to be state-contingent. From now on, First Best will simply indicate the ex-ante First Best.

argue that there is a tension between the equity and insurance motives that affects the optimal tax system. Under a more equity-oriented social welfare criterion, the labour income tax system is progressive and positive taxation of safe capital income can be optimal. On the contrary, under an insurance-oriented welfare criterion, the optimal degree of progressivity of the labour income tax schedule declines, and safe capital income taxation can potentially become redundant.

Third, I investigate the optimal allocation of risk between private and public consumption. Individuals with different preferences for risk have different benefits from the tax/insurance policy at the margin, meaning their willingness to shift risk from the individual to the societal level depends on their risk aversion. I find that the excess return tax plays a key role to find the optimal balance, setting the volatility of public consumption with the average volatility of private consumption across different agent-types. While the taxation of the risky excess return is key to set the risk profile of the public good, it cannot be directly used to redistribute across agents with different productivity and/or risk preferences.

This paper contributes to two main strands of literature. First, this paper is related to the literature that analyses public good provision and optimal taxes in a risky environment. For instance, Christiansen (1993), Schindler (2008) and Boadway and Spiritus (2024) examine the role of capital income taxes when the economy faces aggregate risk. The contribution of this paper is to develop a theory of public good provision in a risky environment in presence of heterogeneous agents that possess different attitudes to risk and different productivities. Using a generic social welfare function, the appropriate Samuelson rule is derived. Coherently with Schindler (2008) and Boadway and Spiritus (2024), I show that the taxation of risky excess returns is key to set the risk-profile of the public good policy. The key novelty with respect to this literature is that the marginal benefit from the public good policy differs across types due to different preferences for risk. Hence, the optimal policy requires not just balancing the volatility of the public good with that of private consumption, as in the cases of Schindler (2008) and Boadway and Spiritus (2024), but also requires to balance the different "risk preferences" for the public good.

Second, this paper is related to the growing literature on optimal taxation with heterogeneous returns. Previous papers have concentrated on the concepts of heterogeneous “investment ability” and/or scale effects (Boadway and Spiritus, 2024; Gerritsen et al., 2025; Gahvari and Micheletto, 2016; Kristjánsson, 2024) as drivers of heterogeneous returns. This paper, in line with new empirical evidence (Bach et al., 2020), considers different preferences for risk as an alternative driver of heterogeneous returns. This creates a connection between savings and returns heterogeneity, and preference heterogeneity, as returns stem endogenously from preferences for risk through type-specific portfolio choices. Most of the above contributions argue in favor of taxing the normal (safe) return on grounds of redistribution.[4] This paper shows

[4]In Boadway and Spiritus (2024); Gerritsen et al. (2025); Gahvari and Micheletto (2016), the

that it can still play a role, complementary to labour earnings taxes, under heterogeneous risk preferences. However, as the government gives more weight to insurance concerns towards the high risk averse agents, the role of safe capital income taxation becomes uncertain and less important. This shows that under heterogeneous risk preferences and productivities, a tension between the equity and insurance concerns arises.

The rest of the paper is organised as follows. Section 2 revisits the theory of private provision of public good in the case of heterogeneous risk preferences. Section 3 presents the first and second best allocations with a generic social welfare function, and how it is possible to improve on the second best allocation. Section 4 presents an application of the public good problem in which agents have heterogeneous productivities and risk preferences, and the government funds the public good with the taxation of labour and capital incomes. Section 5 concludes.

## 2 Public Good Provision in a Risky Environment

From the textbook theory of public goods, we know that public goods will be undersupplied by the market when provided on the basis of individual voluntary contributions Stiglitz, 1980. In this section, I revisit the standard theory and show that the private provision of public goods is still inefficient in a risky environment when agents possess different risk preferences. Then, after presenting the Pareto optimal case – given by the appropriate Samuelson rule – I discuss the type of inefficiency that private provision produces in a risky environment.

Let us consider a set-up in which each agent $j$ allocates a share of her endowment $z$ to consumption in the first period $c_0$, and saves the residual $z - c_0$ investing in a risky portfolio with gross return $\tilde{R}_p$, given their level risk aversion $\theta_j$. In the second period, each agent enjoys private consumption $\tilde{c}_1$ and public consumption $\tilde{G}$ that are both subject to risk. Without loss of generality, I impose no discounting of future consumption. Moreover, individuals evaluate private and public consumption with the same sub-utility function $u(\cdot)$, meaning that no specific difference in taste between private and public goods is modeled.

$$
\begin{aligned}
\max_{c_0^j,\, g_j} \quad & U^j = u_j(c_0^j) + \mathbb{E}\left[u_j(\tilde{c}_1^j)\right] + \mathbb{E}\left[u_j\left(\tilde{G}\right)\right] \\
\text{s.t.} \quad & \tilde{c}_1 = \tilde{R}_p^j \left(z - c_0^j\right) - g_j, \\
& \tilde{G} = f\big(\textstyle\sum_j g_j, \tilde{x}\big)
\end{aligned}
$$

Each individual $j$ chooses how much to contribute $(g_j)$ to the public good $\tilde{G}$ in the first period by maximising lifetime utility $U$ taking the contributions of the other agents $(g_{-j})$ as given. In a no-risk situation, the level of the public good

taxation of safe capital income complements the redistributive role of earnings taxation.

will be given by the sum of individual contributions. With an underlying source of risk in the economy $\tilde{x}$, the public good is risky as it also depends on the state of the economy in the second period, i.e. $\tilde{G} = f\big(\sum_j g_j, \tilde{x}\big)$.[5] Hence, even if the marginal rate of transformation is one, meaning one unit of private consumption buys one unit of public consumption in a riskless economy, aggregate risk can either increase or decrease the actual value of the public good relative to the value of private consumption that was initially sacrificed. The optimal private contribution rule $g_j^*$ satisfies

$$g_j^* : MRS_{\tilde{G},\tilde{c}_1^j} = \frac{\mathbb{E}[u_j'(\tilde{G})]}{\mathbb{E}[u_j'(\tilde{c}_1^j)]} = 1 \ \forall j, \tag{1}$$

where the size of the public good $\tilde{G}$ depends on the sum of the individual contributions and aggregate risk in the economy $\tilde{x}$ i.e. $\tilde{G} = f\big(\sum_j g_j, \tilde{x}\big)$. Each agent $j$ contributes until the risk-adjusted marginal rate of substitution between the public and private good $MRS_{\tilde{G},\tilde{c}_1^j}$ equates the marginal cost (exp. 1). At the Nash equilibrium, the set of individual contribution rules $\{g_j, g_{-j}\}$ will satisfy (1) for each type. Summing up (1) across agents $j = 1, \cdots, n$ gives $\sum_j MRS_{\tilde{G},\tilde{c}_1^j} = n$. By comparing the Nash equilibrium outcome with the Pareto efficient allocation,[6] given by the appropriate Samuelson rule

$$\sum_j MRS^j_{\tilde{G},\tilde{c}_1^j} = \sum_j \frac{\mathbb{E}[u_j'(\tilde{G})]}{\mathbb{E}[u_j'(\tilde{c}_1^j)]} = 1 < n, \tag{2}$$

we can state the following well-known result.

**Lemma 1.** *Public good provision in the case of private contributions is inefficient as agents do not internalise the external value of their individual contributions.*

In the context of aggregate risk in the economy, Lemma 1 has two specific implications. First, the public good will be underprovided in expectation. Second, private provision of public good generates an inefficient allocation of risk between private and public consumption, meaning that the probability distribution of the public good over different states of nature is suboptimal.

# 3 Samuelson Rule with a Social Welfare Function

The concept of Pareto optimality can be quite restricting when preferences are heterogeneous. Policy changes often benefits some agents while hurting others. Hence,

[5]The underlying source of risk should be seen as a risk for the overall economy, affecting both private consumption through portfolio returns, as well as public good consumption through aggregate shocks to public finances.

[6]Each Pareto-efficient allocation can be achieved by private, decentralised optimisation when the public good is financed by (risk-aversion) type-specific lump-sum taxes such that (2) is satisfied.

we proceed by considering the optimal allocation that stems from the government's maximisation of a generic Social Welfare function

$$SW = \sum_j \phi_j(U_j),$$

where $\phi_j(U_j)$ is a weakly concave function of individual expected utility $U_j$ that is able to accommodate different individual risk preferences and $\eta_j = \phi_j'(U_j)$ is the corresponding individual welfare weight.[7] Then, the optimal rules for public good provision are derived. Two benchmark cases are presented: 1) (Ex-ante) First Best: the government can levy type-specific lump-sum taxes that depend on individual risk aversion (section 3.1); 2) Second Best: the government sets one unique lump-sum tax for all types (section 3.2).

### 3.1 First Best: Type Specific Lump-sum Taxes

The government chooses $t_i$ for each agent $i$, therefore perfectly targeting agents with different risk aversion. Expression 3 states that the optimal $t_i$ equates the marginal private costs of agent $i$ from contributing to the public good, expressed in terms of private consumption, to the social benefit of a marginal individual contribution:

$$\eta_i\,\mathbb{E}[u_i'(\tilde{c}_1^i)] = \sum_j \eta_j\,\mathbb{E}[u_j'(\tilde{G})] \quad \forall\, i. \tag{3}$$

At the optimum, expression (3) is satisfied for each agent $i$ and gives the set of optimal lump-sum taxes $t_j$ for all types of agent. The government sets $t_i$ for each agent $i$ such that the weighted expected marginal utility of private consumption is equalised across types to a certain level $\overline{k}$.

$$\{t_i, i = 1, \cdots, n\} \;\; \rightarrow \;\; \eta_i\,\mathbb{E}[u_i'(\tilde{c}_1^i)] = \overline{k} \quad \forall\, i = 1, \cdots, n. \tag{4}$$

By summing up condition (3) across types $i = 1, \cdots, n$, and using (4), we can obtain a modified Samuelson rule that is expressed in terms of the sum of socially-evaluated Marginal Rates of Substitutions ($MRS^s$).

**Proposition 1** (First Best)**.** *At the First Best optimum, the sum of the $MRS^s$ is equal to the marginal rate of transformation between the public and private good.*

$$\sum_j MRS_j^s = \sum_j \frac{\eta_j\,\mathbb{E}[u_j'(\tilde{G})]}{\eta_j\,\mathbb{E}[u_j'(\tilde{c}_1^j)]} = 1. \tag{5}$$

[7] The issue of choosing the appropriate functions $\phi_j$ for utility functions with different curvatures is widely debated in the welfare theory literature. For instance, Grant et al. (2010) shows how it is possible to accommodate concerns about different individuals' risk attitudes and concerns about fairness; Eden (2020) provides a practical method to perform welfare analysis.

Condition (5) is the Samuelson rule that reflects the social preferences that are being maximised, where

$$MRS_j^s = \frac{\eta_j \, \mathbb{E}[u_j'(\tilde{G})]}{\eta_j \, \mathbb{E}[u_j'(\tilde{c}_1^j)]}$$

is defined as the ratio between the marginal social welfare associated with an additional unit of the public good and the marginal social welfare associated with an additional unit of private consumption for an agent $j$.

While type specific taxation is a useful theoretical benchmark, it is unlikely to be feasible in practise. The next section considers an alternative case of public provision in which the lump-sum tax is set equal for all agents.

### 3.2 Second Best: Uniform Lump-sum Taxes

If the government cannot screen agents with different risk aversion, and therefore cannot set (risk-aversion) type specific lump-sum taxes, a uniform lump-sum tax $t$ for all agents apply. At the second best optimum, marginal social costs, expressed in terms of private consumption, are balanced with the social benefits:

$$\sum_j \eta_j \, \mathbb{E}[u_j'(\tilde{c}_1^j)] = n \sum_j \eta_j \, \mathbb{E}[u_j'(\tilde{G})]. \tag{6}$$

Condition (6) can be rewritten in terms of $\widetilde{MRS}_j^s$, where

$$\widetilde{MRS}_j^s = \frac{\eta_j \, \mathbb{E}[u_j'(\tilde{G})]}{\frac{1}{n} \sum_j \eta_j \, \mathbb{E}[u_j'(\tilde{c}_1^j)]},$$

that is defined as the ratio between the marginal social welfare associated with an additional unit of the public good for an agent $j$ and the average (across agents $j = 1, \cdots, n$) marginal social welfare associated with an additional unit of private consumption. This means that when type specific taxes are not available, the government evaluates the individual willingness to trade private with public consumption on the basis of the average sacrifice in terms of private consumption, e.g. $\widetilde{MRS}_j^s$ rather than $MRS_j^s$ . Hence, condition (7) is the modified Samuelson rule in the case of heterogeneous risk preferences when the government does not discriminate the different risk-aversion-types.

**Proposition 2** (Second Best)**.** *At the second best optimum, the sum of* $\widetilde{MRS}_j^s$ *equates the marginal rate of transformation.*

$$\sum_j \widetilde{MRS}_j^s = \sum_j \left( \frac{\eta_j \, \mathbb{E}[u_j'(\tilde{G})]}{\frac{1}{n} \sum_j \eta_j \, \mathbb{E}[u_j'(\tilde{c}_1^j)]} \right) = 1. \tag{7}$$

The Second Best outcome will differ from the First Best benchmark, both in terms of public good provision and private consumption/savings, and therefore social welfare, when $\sum \widetilde{MRS}_j^s \neq \sum MRS_j^s$, namely when condition (7) differs from (5). We can now state proposition 3.

**Proposition 3.** *The inability of the government to discriminate different risk-aversion-types in the economy leads to a suboptimal provision of the public good.*

*Proof.* Suppose proposition 3 is false, and $\tilde{G}^{SB} \equiv \tilde{G}^{FB}$, then it must be that (5) coincides with (7) and that

$$\frac{1}{n}\sum_j \eta_j \, \mathbb{E}[u_j'(\tilde{c}_1^{SB})] = \eta_i \, \mathbb{E}[u_i'(\tilde{c}_1^{FB})] \;\; \forall i = 1, \cdots, n.$$

In order for this to be true, it should follow that $t_i^{FB} = t^{SB}$ for $i = 1, \cdots, n$, meaning the individual private costs of raising a unit of consumption are equal across different types to begin with. Unless specific welfare weights are chosen to obtain this result or the equality happens to hold given the utility functions being used in the first place, proposition 3 will be true in the case with heterogeneous preferences. □

While risk-preference-specific lump-sum taxes might be unfeasible, let alone lump-sum taxes, we know that agents with different risk preferences will make different consumption, savings, portfolio and labour supply choices. These differences can therefore be exploited to increase social welfare. In the next section, I study an optimal tax system for labour and capital income financing the public good within the second-best framework.

# 4 Heterogeneous Productivity and Risk Aversion

I consider an environment in which each type of agent is characterised by their productivity and risk aversion, and the government taxes labour earnings with a piecewise linear schedule and capital income linearly, distinguishing the types of return from investment, i.e. safe normal return versus risky excess return. Agents with different risk preferences choose different amount of hours worked, conditional on their productivity (wage).

This section aims to investigate the roles of the different tax instruments in this setting and the interplay between the insurance mechanism and the standard equity-efficiency trade-off. Finally, I study the role of commodity taxation (taxation of safe capital income) when earnings are taxed non-linearly and risk preferences are not observed by the government.

## 4.1 Set-up

The set of agent-types is a $m \times n$ matrix $\Theta$, where each element is given by the pair $(w_i, \theta_j)$ that represents an individual-type who has productivity (wage) $w_i$ and

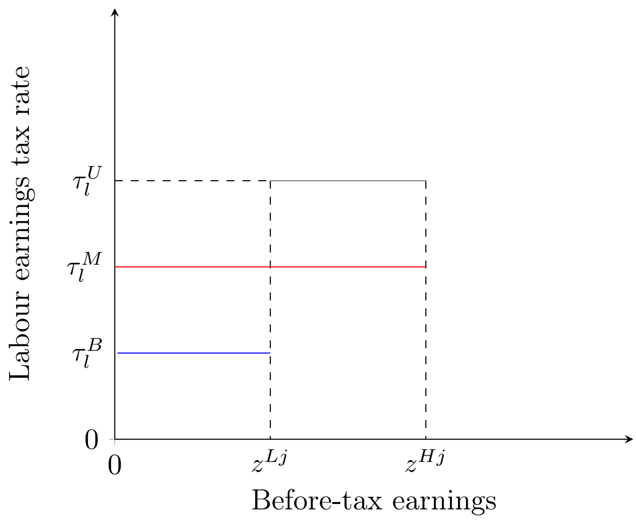


**Figure 1:** Stylized step-wise linear tax schedule of labour earnings for agent-types with different productivities but equal risk aversion $\theta_j$.

relative risk aversion $\theta_j$. $F$ is the $m \times n$ frequency matrix of types in the population, with each type having frequency $f_{ij}$ in the population. I also define a risk aversion group $j$ as the j-th column of $\Theta$: $(\cdot, \theta_j) := \{(w_1, \theta_j), \cdots, (w_m, \theta_j)\}$, and a productivity group $i$ as the i-th row of $\Theta$: $(w_i, \cdot) := \{(w_i, \theta_1), \cdots, (w_i, \theta_n)\}$. Thus, $(\cdot, \theta_j)$ is the group of agents that have different productivities $w_1, \cdots, w_m$, but equal risk aversion parameter $\theta_j$, and $(w_i, \cdot)$ is the group of agents with productivity $w_i$ and different risk aversion parameters $\theta_1, \cdots, \theta_n$.

$$\Theta_{m\times n} = \begin{pmatrix} w_1, \theta_1 & \cdots & w_1, \theta_n \\ \vdots & \ddots & \vdots \\ w_m, \theta_1 & \cdots & w_m, \theta_n \end{pmatrix}, \quad F_{m\times n} = \begin{pmatrix} f_{11} & \cdots & f_{1n} \\ \vdots & \ddots & \vdots \\ f_{m1} & \cdots & f_{mn} \end{pmatrix}.$$

Agents choose their labour supply $l$, first period consumption $c_0$, and the portfolio composition of their savings. Agents invest their savings $a = z - c_0$ choosing the portfolio share of risky assets $s$ with the risky return $\tilde{r}_e$ being Normally distributed: $\tilde{r}_e \sim \mathcal{N}(\overline{r}_e - \sigma_r^2/2, \sigma_r^2)$. The resulting portfolio return $\tilde{r}_p$ will also be Normally distributed: $\tilde{r}_p \sim \mathcal{N}(\overline{r}_p - \sigma_p^2/2, \sigma_p^2)$. The (risky) excess return is defined as the difference between the risky return and the riskless return: $\tilde{r}^{exc} := \tilde{r}^e - r$. Without loss of generality, I impose no time discounting of future consumption.

Each component of capital income is taxed linearly: $\tau_k$ is the tax on the excess return, $\tau_r$ is the tax rate on the safe return. Labour earnings $z$ are subject to a piece-wise linear schedule $\tau_l(z^{ij})$. With a high number of types and individual preferences, this parametrization generates a non-linear tax schedule on labour earnings $T(z)$, as in figure 1. The social marginal welfare weight of an agent-type, normalised with respect to the population, is defined as: $g_{ij} = \frac{f_{ij}\eta_{ij}U_c^{ij}}{\sum_{i,j} f_{ij}\eta_{ij}U_c^{ij}}$. Each agent-type will "count more" in the SFW, the more frequent its type is in the population (higher $f_{ij}$), the higher the weight $\eta_{ij}$ which depends on the concavity of the SWF, the higher the expected marginal utility of second-period consumption $U_c^{ij}$.

Then, define the uncompensated elasticities of labour earnings $z^{ij}$ and savings $a^{ij}$ with respect to net-of-tax rate on labour earnings, $\epsilon_{z,l}^{ij}$, $\epsilon_{a,l}^{ij}$ , and with respect to net-of-tax rate on safe capital income, $\epsilon_{z,r}^{ij}$, $\epsilon_{a,r}^{ij}$ as follows:

$$\epsilon_{z,l}^{ij} = \frac{\partial z^{ij}}{z^{ij}} \cdot \frac{1-\tau_l^{ij}}{\partial(1-\tau_l^{ij})}; \qquad \epsilon_{z,r}^{ij} = \frac{\partial z^{ij}}{z^{ij}} \cdot \frac{1-\tau_r}{\partial(1-\tau_r)};$$

$$\epsilon_{a,l}^{ij} = \frac{\partial a^{ij}}{a^{ij}} \cdot \frac{1-\tau_l^{ij}}{\partial(1-\tau_l^{ij})}; \qquad \epsilon_{a,r}^{ij} = \frac{\partial a^{ij}}{a^{ij}} \cdot \frac{1-\tau_r}{\partial(1-\tau_r)}.$$

These elasticities are all positive when the substitution effect outweights the income effect: $\epsilon_{z,l}^{ij}; \epsilon_{a,l}^{ij}; \epsilon_{z,r}^{ij}; \epsilon_{a,r}^{ij} > 0$.[8] I will assume this is the case throughout the next sections.

## 4.2 Agent's Problem

Agent's utility is separable over time, and separable between private and public consumption. The problem for agent-type $(w_i, \theta_j)$ is thus:

$$\begin{aligned}
\max_{c_0^{ij}, s_j, l^{ij}} \quad & U_{ij} = u_{ij}(c_0^{ij}) + \mathbb{E}\left[u_{ij}(\tilde{c}_1^{ij}) + u_{ij}(\tilde{G})\right] - v(z^{ij}/w_i) \\
\text{s.t.} \quad & \tilde{c}_{ij} = \tilde{R}_p^j a^{ij} - t, \\
& \tilde{R}_p^j = \left[1 + r\,(1-\tau_r) + s_j\,(\tilde{r}^{exc})\,(1-\tau_k)\right]
\end{aligned}$$

where $\tilde{R}_p$ is the gross portfolio return; $a^{ij} = z^{ij}(1-\tau_l^{ij}) - c_0^{ij}$ is the value of assets; $t$ is a lump-sum tax or transfer, equal for all types. As each agent is infinitesimal compared to the size of the economy, the effects of individual choices on the level of the public good are not taken into account.

$$c_0^* : \qquad u'_{ij}(c_0^{ij}) = \mathbb{E}\left[u'_{ij}(\tilde{c}_1^{ij})\tilde{R}_p\right] \tag{8}$$

$$l^* : \qquad 0 = \mathbb{E}\left[u'_{ij}(\tilde{c}_1^{ij})\tilde{R}_p\right] w_i(1-\tau_l(z^{ij})) - v'(z^{ij}/w_i) \tag{9}$$

$$s^* : \qquad 0 = \mathbb{E}\left[u'_{ij}(\tilde{c}_1^{ij})\tilde{r}^{exc}\right] \tag{10}$$

Conditions (8)-(9) are the standard first order conditions (FOCs) for first-period consumption $c_0$, and labour supply $l$. Expression (10) is the FOC for the share of risky assets $s$, determining the portfolio allocation with risky and riskless assets.

[8] This is the case, for instance, when we consider CRRA utility with relative risk aversion parameters $\theta_{ij}$ lower than 1 for all agents.

After applying the covariance identity to condition (10), we can derive the expression for the risk premium with respect to the expected excess return.

$$\mathbb{E}[R^e - R] = -\frac{\text{cov}\left[u'_{ij}(\tilde{c}_1^{ij}), \tilde{r}^{exc}\right]}{\mathbb{E}\left[u'_{ij}(\tilde{c}_1^{ij})\right]}. \tag{11}$$

For each agent with productivity $w_i$ and relative risk aversion parameter $\theta_j$, expression (11) maps the covariance term to the expected marginal utility of second-period consumption $\tilde{c}_1$. Under constant relative risk aversion utility, a close form solution for the share of risky assets can also be derived:

$$s_j^* = \frac{\bar{r}^e - r}{\theta_j \sigma_r^2} \cdot \frac{(R - r\tau_r)}{R(1 - \tau_k)} \tag{12}$$

While the first term relates to the original formula by Merton (1969) and Samuelson (1969),[9] the second term captures the effect of the different taxes on capital income. In particular, increasing the tax on safe capital income reduces the optimal share of risky assets: $\frac{\partial s^*}{\partial \tau_r} < 0$. On the contrary, increasing the excess return tax, while keeping the loss offsets, increases $s_j^*$: $\frac{\partial s^*}{\partial \tau_k} > 0$ (Domar and Musgrave, 1944). Moreover, while taxation of the riskless return changes the resource allocation (first period consumption, savings), taxing the excess returns with loss offsets does not. Agents will adjust their portfolio shares to get the same pre-tax expected portfolio return (Boadway and Spiritus, 2024; Domar and Musgrave, 1944).

### 4.3 The Government

The government's objective is to maximise social welfare (SW) that is defined as a weighted sum of agents' expected utilities, where $U$ is a Von Neumann-Morgenstern utility function, and $\eta_{ij}$ is the individual welfare weight:

$$\max_{\tau_k, \tau_r, \tau_l(z^{ij}), t} SW = \sum_{ij} f_{ij}\eta_{ij}\left(U_{ij}\left(c_0^{ij*}, \underbrace{a^{ij}\tilde{R}_p^j - t}_{\tilde{c}_1^{j*}(s_j^*, l^{ij*})}, l^{ij*}, \tilde{G}\right)\right).$$

The government provides a public good $\tilde{G}$ that enters the utility function separately from consumption. The public good is financed by taxation of labour income through the schedule $\tau_l(z^{ij})$, capital income taxation on the excess return at rate $\tau_k$ and on the safe return at rate $\tau_r$ and a uniform lump-sum tax $t$. The public good is given by the following expression.

[9] Merton (1969) and Samuelson (1969) study the optimal portfolio choice of a consumer with CRRA utility with a multi-period horizon and multiple risky assets.

$$\tilde{G} = \sum_j \left[ \left( \tau_k s_j^* \left( \tilde{r}^{exs} \right) + \tau_r r \right) \left( z^{ij*} (1 - \tau_l(z^{ij}) - c_0^{ij*} \right) + \tau_l(z^{ij}) z^{ij*} + t \right] \tag{13}$$

Tax rates are set by the government in the first period anticipating agents' optimal behaviour. After the state of the economy is realised, the government implements the public good policy and balances the budget. As a result, the provision of the public good is stochastic and depends on the state of the economy in the second period that determines the value of revenues from the taxation of excess returns on savings. I substitute the expression for the public good directly in the social welfare function, as in Schindler (2008). It ensures that the budget is balanced for any state of the world.[10]

### 4.4 Mechanisms

In this economy, the provision of the public good and its funding through taxation has two possible redistributive objectives: (i) insurance across risk-aversion agent-types; (ii) redistribution across productivity agent-types motivated by equity concerns. Figure 2 represents these two mechanisms at play in a 4-types version of the economy in which each agent-type is characterised by either high or low productivity $\{w_h; w_l\}$, and high or low risk aversion $\{\theta_h; \theta_l\}$: $(w_l, \theta_l); (w_l, \theta_h)$; $(w_h, \theta_l); (w_h, \theta_h)$. The red-colored arrows represent transfers that are motivated by equity concerns, which benefits some productivity (income) people at the expense of others, e.g. redistribution from high productivity to low productivity types. Then, the blue-colored arrows represent transfers that are motivated by insurance purposes which may benefit either high or low risk aversion people, depending on the specific social preferences. Finally, the gray-coloured arrows represent transfers between agent-types that differ by both characteristics – productivity and risk aversion – and are motivated by a mix of equity and insurance concerns. The relative importance of these two channels will determine which agent-types will benefit the most from the public good policy.

Hence, the optimal tax mix to fund the public good will depend on social preferences, agent-types' characteristics, their distribution in the economy, and on the ability of the government to observe them. Given these factors, a tension between insurance and equity arises, as some policies targeted to high risk averse individuals might benefit high income people as well. In the following subsections, we explore how the the tax system would look like under two scenarios: i) observable types; ii) unobservable risk preferences. In each of these two scenarios, we keep fixed the distribution of types in the economy, and explore how the equity and the insurance motives affect the tax system under alternative welfare criteria.

[10]This is a stricter requirement than balancing the budget in expectation, that would instead imply transferring resources from good states to bad states of the world.

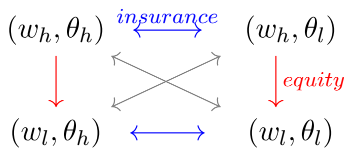


**Figure 2:** Mechanisms at play in a 4-types economy: insurance (blue arrows); redistribution (red arrows); mix of insurance and redistribution (gray arrows). Each agent-type is characterised by either high or low productivity $\{w_h; w_l\}$, and high or low risk aversion $\{\theta_h; \theta_l\}$. The set of agent-types is therefore: $(w_l, \theta_l); (w_l, \theta_h); (w_h, \theta_l); (w_h, \theta_h)$.

I will consider two alternative criteria. The first one, named insurance-driven redistribution, gives more weight to individuals with high risk aversion, and to low income people conditional on risk preferences.

**Criterion 1** (Insurance-driven redistribution)**.** *High risk averse agents count more than low risk averse agents, but low income people count more than high income people conditional on risk preference:* $g_{lh} > g_{hh} > g_{ll} > g_{hl}$.

Alternatively, the government could prioritise equity to the insurance motive through the equity-driven redistribution criterion.

**Criterion 2** (Equity-driven redistribution)**.** *Low income people count more than high income people, but high risk averse people more than low risk averse people conditional on productivity:* $g_{lh} > g_{ll} > g_{hh} > g_{hl}$.

Hence, there is a trade-off between the insurance and the equity concerns that will have an impact on the optimal tax system.

## 4.5 Optimality Conditions: Observable Types

In this section, I analyse the optimality conditions for a piecewise linear tax schedule on labour earnings and linear taxation of riskless and risky capital income when the government has complete information on the different types of agent in the economy. Hence, individual productivities, labour supply decisions, the risk type (risky versus safe) and amount of capital income are known. Moreover, the transfer $t$ is already set optimally, and the intuition remain similar to the framework discussed in section 3.2. The whole set of optimality conditions is provided in the appendix C.

Under observable types, the government is able to set agent-type-specific tax rates on labour income. However, capital income taxes remain uniform across types of agent. Hence, any redistribution on the grounds of equity or insurance can be made through the labour income tax schedule. Therefore, consistently with the Atkinson and Stiglitz (1976) theorem, there is no need for distorting the intertemporal consumption decisions by taxing the safe rate of return. Hence, we can analyse optimal agent-type-specific labour income taxes and the risky capital income tax $\tau_k$ on excess returns, under zero tax on safe capital income.

**Proposition 4.** *Under observable types, the optimal tax system on labour and risky capital income* $\{\tau_l^{ij}, \tau_k\}$ *for each agent-type* $(w_i, \theta_j)$ *is given by the following optimality conditions:*

1. *taxation of labour income*

$$\tau_l^{ij} = \frac{1 - g_{ij}}{1 - g_{ij} + \epsilon_{z,l}^{ij}}, \tag{14}$$

2. *taxation of risky capital income*

$$\tau_k : \mathbb{E}[\tilde{r}^{exc}] = -\frac{\sum_{i,j} f_{ij}\eta_{ij} \operatorname{cov}\left[u'_{ij}(\tilde{G}), \tilde{r}^{exc}\right]}{\sum_{i,j} f_{ij}\eta_{ij}\, \mathbb{E}[u'_{ij}(\tilde{G})]} \tag{15}$$

*where* $g_{ij} = \frac{\eta_{ij} U_c^{ij}}{\sum_{i,j} f_{ij}\eta_{ij} U_c^{ij}}$ *is the social marginal welfare weight of an agent-type* $(w_i, \theta_j)$*, normalised with respect to the population.*

Conditions (14) is an implicit tax formula capturing the standard equity-efficiency trade-off, as well as the new insurance channel. While the efficiency channel is represented by the elasticity of $\epsilon_{z,l}^{ij}$, the social marginal welfare weight $g_{ij}$ captures both the insurance and the equity concerns. This can be seen easily by considering that $g_{ij}$ can be rewritten in terms of covariances $g_{ab} = \frac{f_{ab}\eta_{ab} \operatorname{cov}(U_c^{ab}, \tilde{r}^{exc})}{\sum_{i,j} f_{ij}\eta_{ij} \operatorname{cov}(U_c^{ij}, \tilde{r}^{exc})}$. Hence, the higher the covariance term, the more an agent-type will matter in the social welfare function, therefore lowering the optimal tax rate on labour earnings.

For each agent-type, the sign of the labour income tax rate depends on the value of the welfare weight $g_{ij}$. For instance, under insurance-based redistribution (criterion 1), agents with low risk aversion will face higher tax rates than high risk aversion agents conditional on income. This will ensure that resources are redistributed from low risk aversion agents towards high risk aversion agents. However, the resulting tax system would not be progressive as some high income people would be subsidised, while some low income people would be taxed at a positive rate. This is because criterion 1 prioritises the insurance channel over the equity channel.

Alternatively, under equity-based redistribution (criterion 2), low income individuals are subsidized while high income agents face a positive tax rate. While the resulting tax schedule is progressive, some of the high income individuals facing a positive tax rate have high risk aversion, while some low risk aversion agents are subsidised. This is because criterion 2 prioritises the equity channel over the insurance channel. These two examples show that there is a tension between the insurance and the equity channel when individuals differ by their productivity and risk preferences.

Then, the optimal tax rate on excess returns $\tau_k$ is set by condition (15). The revenues collected from taxing the excess return are risky as the realisation of the excess return depends on the state of the economy. Hence, the public good will have a certain probability distribution which entails a certain allocation of risk between private and public consumption. Condition (15) determines the variance of the public good policy and the allocation of risk between private and public consumption that maximises social welfare.[11] In other terms, the government is choosing the optimal risky share of the budget (tax revenues), given agents' preferences and social welfare weights.

The key novelty of this setting with respect to the literature, specifically the work by Schindler (2008) and Boadway and Spiritus (2024), is that the "benefits" of the public good differ among agents because of heterogeneous attitudes to risk. Since the public good is itself risky, the welfare gain from increasing the tax rate $\tau_k$ varies across different types of agent. Hence, the government aims to balance the different "preferences" for the public good, providing insurance against aggregate risk. As individuals make portfolio choices on the basis of their risk preferences according to condition (11), similarly the government decides the distribution of tax revenues financing the public good over states of nature on the basis of (15). In doing so, the government takes into account the agents' willingness to shift risk to the societal level. Condition (16) in corollary 1 reformulates (15) and better represents this concept. Private and public consumption volatility are represented by the covariance between marginal utility of private and public consumption respectively with the risky excess return and jointly govern the individual willingness to pay for the public good with an extra euro of risky capital income.

**Corollary 1.** *When agents with different risk preferences cannot be discriminated with zero efficiency costs (second best), $\tau_k^*$ equalizes public consumption volatility with (average) private consumption volatility. The optimal tax rate is positive but strictly lower than 100%.*

$$\sum_{ij} \frac{f_{ij}\eta_{ij} \operatorname{cov}\left(u'_{ij}(\tilde{G}), \tilde{r}^{exc}\right)}{\sum_{ij} f_{ij}\eta_{ij} \operatorname{cov}\left(u'_{ij}\left(\tilde{c}_1\right), \tilde{r}^{exc}\right)} = 1 \tag{16}$$

*Proof.* See Appendix A.

Corollary 1 says that $\tau_k^*$ is chosen on the basis of a weighted average of private consumption volatility, i.e. $\sum_{ij} f_{ij}\eta_{ij} \operatorname{cov}\left(u'_{ij}\left(\tilde{c}_1\right), \tilde{r}^{exc}\right)$. This is because the government cannot target individual risk-aversion types at no efficiency cost. This creates an inefficiency: an agent with relatively (more) risky private consumption $c_1$, compared to other agents, would be better-off by shifting more risk to the public good. For that to be the case, a higher tax rate $\tau_k$ should be implemented, so that private consumption volatility is traded with public consumption volatility.[12] More-

[11] When a bad (good) state of economy realises in the second period, losses (gains) due to negative (positive) excess returns will be spread over private and public consumption, so that the utility loss (gain) is minimised (maximised).

[12] If the government were able to target different types at no efficiency costs, then it would be

over, it can be shown that the optimal tax rate is positive but strictly lower than 100%.

Another aspect to investigate is whether the taxation of excess returns can have any redistributive role across types of agent. This is clarified by corollary 2.

**Corollary 2.** *The optimal taxation of the risky excess returns does not serve a redistributive role across different types of agent, either across different levels of productivity or different levels of risk aversion.*

*Sketched Proof.* Suppose it is possible for the government to condition $\tau_k$ on agents' characteristics by setting a set of taxes $\tau_k^{ij}$, one for each $(w_i, \theta_j)$. Then, it is possible to show that the optimality conditions would be equal across types. Hence, we would have $\tau_k^{ij} = \tau_k$ for any $i, j$.

Hence, the role of the risky excess return is solely to set the variability of tax revenues and the public good on the aggregate level. This extends the finding by Boadway and Spiritus (2024) that such tax cannot enhance redistribution on equity grounds when agents differ by their labour and investment productivity.

### 4.6 Optimality Conditions: Unobservable Risk Preferences

When the government does not observe risk preferences, labour income taxes can only depend on the productivity level, while capital income tax rates still apply uniformly to all agents. Formula (17) gives the optimal tax rate on labour earnings for agents with productivity level $w_i$ and any risk preferences:

$$\tau_l^i = \frac{1 - \overline{g}_z^i - \tau_r \cdot \overline{\epsilon_{a,l}^i}/p_0}{1 - \overline{g}_z^i + \overline{\epsilon_{z,l}^i}}. \tag{17}$$

The redistributive motives due to the equity and insurance channels are captured by $\overline{g}_z^i = \sum_j f_{ij} g_{ij} z_{ij} / Z^i$ that is the ratio between the average pre-tax income within productivity group $(w_i, \cdot)$ weighted by welfare weights $g_{ij}$, and average pre-tax earnings $Z^i$. The efficiency channel is represented by two elasticity terms: (i) the normalised weighted average of earnings elasticities $\overline{\epsilon_{z,l}^i} = \sum_j f_{ij}(\epsilon_{z,l}^{ij} z_{ij})/Z^i$; (ii) the normalised weighted average of savings elasticities $\overline{\epsilon_{a,l}^i} = \sum_j f_{ij}(\epsilon_{a,l}^{ij} a_{ij})/Z^i$. The first term relates to the direct efficiency costs of taxation due to its discouraging effect on labour supply. The second term relates to the indirect effect on revenues from the taxation of safe capital income: discouraging labour supply has a negative impact on savings that in turn lowers revenues from capital income taxation.

When the government has strong equity concerns (equity-driven redistribution criterion), meaning that low income people count more than high income people,

possible to compensate agents that would prefer a different distribution of the public good from the optimal one given by (16). For a short proof, see appendix B.

formula (17) will entail a progressive tax schedule. However, if the government priorities the insurance concerns (insurance-driven redistribution criterion), giving more weight to some high-income agents relative to lower income agents given their risk preferences, then the degree of progressivity of the optimal labour income tax schedule will decline. The reason is that those high-income agents who have high risk aversion count more in the social welfare function than low-income people with low risk aversion, which makes the optimal policy less redistributive in terms of equity.

Another factor influencing the tax schedule and its progressivity is the frequency distribution of agent-types. Suppose there is some correlation between risk preferences and productivity. If low risk aversion agents also tend to be more productive in the population and earn a higher income, then the tension between the insurance and the equity channels will be weaker. Therefore, redistribution on the grounds of equity and insurance will complement each other. On the contrary, if low risk aversion agents tend to be less productive in the population, the clash between the insurance and equity motives will be stronger. This is because, in this case, redistributing towards the high risk averse agents would require in many cases to redistribute from the poorer to the richer individuals.

Regarding capital income taxes, condition 15 for the risky excess return and its intuition remain valid. Then, for safe capital income, it can be shown that introducing a tax or subsidy is welfare enhancing when the following condition is satisfied:

$$\frac{\partial SW}{\partial \tau_r} = 1 - \overline{g}_a - \frac{R}{A} \cdot \left[ \sum_{i,j} f_{ij} \epsilon_{z,r}^{ij} z^{ij} \tau_l^i \right] \neq 0. \tag{18}$$

Condition (18) is the social welfare effect of introducing $\tau_r$, where the term $\overline{g}_a = \frac{\sum_{ij} f_{ij} g_{ij} a_{ij}}{A}$ captures the social concerns for equity and insurance, and it is defined as the ratio between average savings within productivity group $(w_i, \cdot)$ weighted by individual welfare weights $g_{ij}$, and average savings $A = \sum_{i,j} f_{ij} a^{ij}$. The last term relates to the indirect effect of capital income taxation on revenues from labour income taxes. When condition (18) is satisfied, taxing (or subsdising) the safe return complements the role of labour income taxation.

When the government has strong equity concerns (equity-driven redistribution criterion), condition (18) is positive, provided that the indirect effect of capital income taxation on revenues on labour income tax revenues is small enough. This would imply that $\tau_r > 0$ is welfare improving. On the contrary, if the government prioritises insurance concerns (insurance-driven redistribution criterion), then the sign becomes uncertain. Hence, the desirability of positive taxation of safe capital income will also depend on the importance of equity relative to the insurance motive.

## 5 Concluding Remarks

People with different attitudes to risk have different views on the extent to which society should invest in certain (risky) projects. This paper presents a theory of optimal provision and funding of a (risky) public good when individuals have heterogeneous labour productivities and preferences for risk and face aggregate risk in the economy. In this framework, the public good is a tool to shift risk from private to public consumption, and private provision is inefficient as agents fail to internalize the insurance effects of the public good for the other agents.

I characterize the second best allocation by studying the optimal tax system with piece-wise linear taxation of labour earnings and proportional taxation of risky and safe capital income. I show that the excess return tax is key to balance the individual savings and portfolio decisions with the variance of the public good policy. By setting this tax optimally, the government finds a balance among the different preferences for risk of the different agents. However, this the excess return tax cannot be used to directly pursue redistribution across different productivity and/or risk-aversion types of agent.

Then, the taxation of the safe (normal) return might play a role along with labour earnings taxation when risk preferences are unobserved by the government and equity concerns are relatively important. However, when the government prioritises redistribution towards the high risk averse agents, the labour income tax schedule is less progressive and safe capital income taxation plays a less important role. Unless aversion to risk is negatively related to labour productivity and income, a tension between equity and insurance arises.

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# Appendix A Proof Corollary 1

The optimality condition for the excess return tax $\tau_k$ reads as follows.

$$\sum_{ij} f_{ij}\eta_{ij}\mathbb{E}\Big[u'_{ij}(\tilde{G})\sum_{ij} f_{ij}s_j\left(z^{ij}-c_0^{ij}\right)\tilde{r}^{exc}\Big]+\tau_k\sum_{ij} f_{ij}\eta_{ij}\mathbb{E}\left[u'_{ij}(\tilde{G})\sum_{ij} f_{ij}\frac{\partial s_j}{\partial \tau_k}\left(z^{ij}-c_0^{ij}\right)\tilde{r}^{exc}\right]=0.$$

Using the fact that $\frac{\partial s_j}{\partial \tau_k}=\frac{s_j}{(1-\tau_k)}$, we can collect terms.

$$\sum_{ij} f_{ij}\eta_{ij}\mathbb{E}\Big[u'_{ij}(\tilde{G})\sum_{ij} f_{ij}s_j\left(z^{ij}-c_0^{ij}\right)\tilde{r}^{exc}\Big]\left(1+\frac{\tau_k}{1-\tau_k}\right)=0.$$

Then, as $\sum_{ij} f_{ij}s_j\left(z^{ij}-c_0^{ij}\right)\left(1+\frac{\tau_k}{1-\tau_k}\right)\neq 0$, we get the following expression

$$\sum_{ij} f_{ij}\eta_{ij}\mathbb{E}\left[u'_{ij}(\tilde{G})\tilde{r}^{exc}\right]=0.$$

By further manipulating the above expression using the covariance identity, and conditions (7) and (11), we can rewrite the optimality condition for $\tau_k$ as follows

$$\sum_{ij} f_{ij}\eta_{ij}\,\mathrm{cov}\left(u'_{ij}\left(\tilde{c}_1^{ij}\right),\tilde{r}^{exc}\right)=\sum_{ij} f_{ij}\eta_{ij}\,\mathrm{cov}\left(u'_{ij}(\tilde{G}),\tilde{r}^{exc}\right).$$

or

$$\sum_{ij}\frac{f_{ij}\eta_{ij}\,\mathrm{cov}\left(u'_{ij}(\tilde{G}),\tilde{r}^{exc}\right)}{\sum_{ij} f_{ij}\eta_{ij}\,\mathrm{cov}\left(u'_{ij}\left(\tilde{c}_1\right),\tilde{r}^{exc}\right)}=1. \tag{19}$$

Moreover, the optimal tax rate is positive but strictly lower than 100%. Only in such a case both the numerator and the denominator are non-zero terms with the same sign so that the condition can be indeed satisfied.

# Appendix B

Suppose the government can target different risk aversion types at no efficiency costs using type-specific lump-sum taxes, then condition 19 simplifies to

$$\sum_{ij} \frac{f_{ij}\eta_{ij} \operatorname{cov}\left(u'_{ij}(\tilde{G}), \tilde{r}^{exc}\right)}{f_{ij}\eta_{ij} \operatorname{cov}\left(u'_{ij}\left(\tilde{c}_1\right), \tilde{r}^{exc}\right)} = 1 \tag{20}$$

and the allocation of risk between private and public consumption relates to individual willingness to shift risk from private to public consumption $\frac{f_{ij}\eta_{ij} \operatorname{cov}\left(u'_{ij}(\tilde{G}),\tilde{r}^{exc}\right)}{f_{ij}\eta_{ij} \operatorname{cov}\left(u'_{ij}(\tilde{c}_1),\tilde{r}^{exc}\right)}$.

This formula is obtained by noticing that the conditions (5), (11), and the covariance identity imply that the term

$$f_{ij}\eta_{ij} \operatorname{cov}\left(u'_{ij}\left(\tilde{c}_1^{ij}\right), \tilde{r}^{exc}\right)$$

is equal across agent-types. Hence, expression (16) can be reformulated as (20).

$\square$

# Appendix C

## C.1 Individual Maximisation Problem and First Order Conditions

The individual maximisation problem for agent-type $(w^i, \theta_j)$ is:

$$\begin{aligned}
\max_{c_0^{ij}, s_j, l^{ij}} \quad & U_{ij} = u_j(c_0^{ij}) + \beta\,\mathbb{E}\left[u_j(\tilde{c}_1^{ij}) + u_j(\tilde{G})\right] - v(z^{ij}/w_i) \\
\text{s.t.} \quad & \tilde{c}_{ij} = \tilde{R}_p^j \left(z^{ij}(1 - \tau_l(z^{ij})) - c_0^{ij}\right) - t, \\
& \tilde{R}_p^j = \left[1 + r\,(1 - \tau_r) + s_j\,(\tilde{r}^{exc})\,(1 - \tau_k)\right]
\end{aligned}$$

where $\tilde{R}_p$ is the gross portfolio return; $t$ is a lump-sum tax or transfer, equal for all types; $\tau_r$ and $\tau_k$ are the tax rates on the riskless and excess return respectively. As each agent is infinitesimal compared to the size of the economy, the effects of individual choices on the level of the public good are not taken into account.

The FOCs for first period consumption $c_0$, share of risky assets $s$, and labour supply $l$ read:

$$\begin{aligned}
c_0^* : \quad & u'_{ij}(c_0^{ij}) = \mathbb{E}\left[u'_{ij}(\tilde{c}_1^{ij})\tilde{R}_p\right] \\
s^* : \quad & 0 = \mathbb{E}\left[u'_{ij}(\tilde{c}_1^{ij})\tilde{r}^{exc}\right] \\
l^* : \quad & 0 = \mathbb{E}\left[u'_{ij}(\tilde{c}_1^{ij})\tilde{R}_p\right] w_i(1 - \tau_l(z^{ij})) - v'(z^{ij}/w_i)
\end{aligned}$$

## C.2 The Government's problem

$$\begin{aligned}
\max_{\tau_k, \tau_r, \tau_l^{ij}, t} SW = & \sum_{ij} \phi_{ij} \Big(U_{ij}\Big(c_0^{ij*}, \underbrace{a^{ij}\tilde{R}_p^j - t}_{\tilde{c}_1^{j*}(s_j^*, l^{ij*})}, l^{ij*}, \tilde{G}\Big)\Big) \\
\text{s.t. } \tilde{G} = & \sum_j \left[\left(\tau_k s_j^*\,(\tilde{r}^{exs}) + \tau_r r\right)\left(z^{ij*}(1 - \tau_l(z^{ij})) - c_0^{ij*}\right) + \tau^{ij} z^{ij*} + t\right] \\
\text{with } \tilde{R}_p^j = & \Big[\underbrace{1 + r(1 - \tau_r)}_{\substack{\text{net-of-tax} \\ \text{safe return}}} + s_j^*\,\underbrace{(\tilde{r}^{exs})\,(1 - \tau_k)}_{\substack{\text{net-of-tax} \\ \text{excess return}}}\Big]
\end{aligned}$$

## C.3 Endogeneous Labour Supply with Observable Types

First, the conditions for transfer $t$ and excess return tax $\tau_k$:

$$t^* : \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}[u'_{ij}(\tilde{c}_1^{ij})] = \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}[u'_{ij}(\tilde{G})] \tag{21}$$

$$\tau_k^* : \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}\left[u'_{ij}(\tilde{G})r^{exc}\right] = 0 \tag{22}$$

Then, under optimally set taxes/transfer $t^*$ and $\tau_k^*$ the conditions for a labour income tax on each ability and risk aversion type, and the safe return tax equal for all types are:

$$\begin{aligned}
\tau_l^{ab*} : f_{ab}\eta_{ab}\,\mathbb{E}\left[u'_b(\tilde{c}_1^{ab})\right] z_{ab}p_0 &= \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}\left[u'_{ij}(\tilde{G})\right] && (23)\\
&\times f_{ab}\left[z_{ab}p_0 - \frac{\tau_r r}{(1-\tau_l^{ab})}\cdot\left(\epsilon_{a,l}^{ab}a^{ab}\right) - \frac{\tau_l^{ab}}{1-\tau_l^{ab}}\epsilon_{z,l}^{ab}z_{ab}p_0\right];\\
\tau_r^* : \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}\left[u'_{ij}(\tilde{c}_1^{i})\right]a^{ij}r &= \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}\left[u'_{ij}(\tilde{G})\right] && (24)\\
&\times \sum_{i,j} f_{ij}\left[a^{ij}r - \frac{\tau_r\cdot r}{(1-\tau_r)}\left(\epsilon_{a,r}^{ij}a^{ij}\right) - \frac{\tau_l^{ij}}{1-\tau_r}z^{ij}p_0\epsilon_{z,r}^{ij}\right].
\end{aligned}$$

To derive formula 14 from condition 23, define $g_{ij} = \frac{\eta_{ij}U_c^{ij}}{\sum_{i,j} f_{ij}\eta_{ij}U_c^{ij}}$ as the social marginal welfare weight of an agent-type, normalised with respect to the population. Set $\tau_r = 0$, and rework the condition as

$$(1-g_{ij})(1-\tau_l^{ij}) - \tau_l^{ij}\epsilon_{z,l}^{ij} = 0.$$

Finally, solve for $\tau_l^{ij}$.

## C.4 Endogeneous Labour Supply with Unobservable Risk Aversion

Suppose that risk aversion is unobservable such that a risk-aversion-specific earnings tax schedule is not feasible. The government optimally chooses $\tau_l^i$ for each productivity group $(w_i, \cdot)$, composed by agents with productivity $(w_i)$ across different levels of risk aversion.. The condition for a productivity-specific labour earnings tax reads:

$$\tau_l^{i*} : \sum_j f_{ij}\eta_{ij}\,\mathbb{E}\left[u_j'(\tilde{c}_1^{ij})\right] z_{ij}p_0 = \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}\left[u_{ij}'(\tilde{G})\right] \times \sum_j f_{ij}\left[z_{ij}p_0 - \frac{\tau_r r}{(1-\tau_l^i)} \cdot \left(\epsilon_{a,l}^{ij} a^{ij}\right) - \frac{\tau_l^i}{1-\tau_l^i}\epsilon_{z,l}^{ij} z_{ij}p_0\right] \tag{25}$$

Rewrite the condition using the definition of the relative marginal social welfare weight of type $ij$: $g_{ij} = \frac{\eta_{ij}U_c^{ij}}{\sum_{i,j} f_{ij}\eta_{ij}U_c^{ij}}$ and aggregate earnings for productivity level $i$: $Z^i = \sum_j f_{ij}z_{ij}$:

$$\sum_j f_{aj}g_{aj}z_{aj} - \left(\sum_j f_{aj}z_{aj} - \frac{\tau_r/p_0}{1-\tau_l^a}\sum_j f_{aj}\left(\epsilon_{a,l}^{aj}a_{aj}\right) - \frac{\tau_l^a}{1-\tau_l^a}\sum_j f_{aj}\left(\epsilon_{z,l}^{aj}z_{aj}\right)\right) = 0. \tag{26}$$

Then, define $\overline{g}_z^i = \sum_j f_{ij}g_{ij}z_{ij}/Z^i$ as the average pre-tax income within productivity group $(w_i, \cdot)$ weighted by individual welfare weights $g_{ij}$, and average pre-tax earnings $Z^i$; $\epsilon_{a,l}^i = \sum_j f_{ij}(\epsilon_{a,l}^{ij}a_{ij})/Z^i$, namely the normalised weighted average of savings elasticities; and $\overline{\epsilon_{z,l}^i} = \sum_j f_{ij}(\epsilon_{z,l}^{ij}z_{ij})/Z^i$: normalised weighted average of earnings elasticities. Now, the condition can be rewritten as

$$\overline{g}_z^i - \left(1 - \frac{\tau_r/p_0}{1-\tau_l^i}\overline{\epsilon_{a,l}^i} - \frac{\tau_l^i}{1-\tau_l^i}\overline{\epsilon_{z,l}^i}\right) = 0. \tag{27}$$

Solving for the tax rate gives:

$$\tau_l^i = \frac{1 - \overline{g}_z^i - \tau_r \cdot \overline{\epsilon_{a,l}^i}/p_0}{1 - \overline{g}_z^i + \overline{\epsilon_{z,l}^i}}.$$

Then, the safe capital income tax condition reads as follows:

$$\tau_r^* : \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}\left[u_{ij}'(\tilde{c}_1^i)\right] a^{ij}r = \sum_{i,j} f_{ij}\eta_{ij}\,\mathbb{E}\left[u_{ij}'(\tilde{G})\right] \times \sum_{i,j} f_{ij}\left[a^{ij}r - \frac{\tau_r \cdot r}{(1-\tau_r)}\left(\epsilon_{a,r}^{ij}a^{ij}\right) - \frac{\tau_l^i}{1-\tau_r}z^{ij}p_0\epsilon_{z,r}^{ij}\right]. \tag{28}$$

To derive the condition 18, rewrite 28 using $\overline{g}_a = \frac{\sum_{ij} f_{ij} g_{ij} a_{ij}}{A}$, and $A = \sum_{i,j} f_{ij} a^{ij}$ ( aggregate savings).

$$1 - \overline{g}_a - \frac{1}{A} \cdot \left[ \frac{\tau_r \cdot r}{(1-\tau_r)} \left( \sum_{i,j} f_{ij} \epsilon^{ij}_{a,r} a^{ij} \right) + \frac{p_0}{(1-\tau_r)} \left( \sum_{i,j} f_{ij} \epsilon^{ij}_{z,r} z^{ij} \tau^i_l \right) \right] = 0$$

This is the welfare effect of perturbating $\tau_r = 0$. By evaluating it at $\tau_r = 0$, we obtain condition 18.