

Quiz 2 - Statistical Inference

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Question 1

What is the variance of the distribution of the average an IID draw of n observations from a population with mean μ and variance σ^2 ?

- $\frac{\sigma^2}{n^2}$
- σ^2
- $2 \frac{\sigma}{\sqrt{n}}$
- $\frac{\sigma}{n}$

Answer : $\frac{\sigma^2}{n}$.

Question 2

Suppose that diastolic blood pressures (DBPs) from men aged 35-44 are normally distributed with a mean of 80mmHg and a standard deviation of 10 mmHg. About what is the probability that a random 35-44 year old has a DBP less than 70 ?

- 22%
- 32%
- 8%
- 16%

Answer : We need to calculate $P(X \leq 70)$, knowing that $X \sim \mathcal{N}(80, 100)$.

```
pnorm(70,mean=80,sd=10,lower.tail = TRUE)
```

```
## [1] 0.1586553
```

The searched probability is around 16%.

Question 3

Brain volume for adult women is normally distributed with a mean of about 1,100 cc for women with a standard deviation of 75 cc. What brain volume represents the 95th percentile ?

- approximately 1247
- approximately 1223
- approximately 1175
- approximately 977

Answer : We just need to calculate the quantile corresponding to a probability of 0.95, knowing that the brain volume follows a normal law $\mathcal{N}(1100, 75^2)$.

```
qnorm(0.95,mean=1100,sd=75,lower.tail = TRUE)
```

```
## [1] 1123.364
```

So the 95th quantile for this distribution is approximately 1123.

Question 4

Refer to the previous question. Brain volume for adult women is normally distributed with a mean of about 1,100 cc for women with a standard deviation of 75 cc. Consider the sample mean of 100 random adult women from this population. What is the 95th percentile of the distribution of the sample mean ?

- approximately 1112
- approximately 1088
- approximately 1110
- approximately 1115

Answer : As the number of people is large enough, we can consider that the sample mean follows a normal distribution $\mathcal{N}(\mu, \sigma^2/n)$, where $\mu = 1100$, $\sigma = 75$ and $n = 100$.

```
qnorm(0.95,mean=1100,sd=75/10,lower.tail = TRUE)
```

```
## [1] 1112.336
```

So the 95th quantile for this distribution mean is approximately 1113.

Question 5

You flip a fair coin 5 times. About what is the probability of getting 4 or 5 heads ?

- 6%
- 19%
- 3%
- 12%

Answer : As the coin is fair, the probability of getting 1 head at each flip is 0.5. The probability of getting at least 4 heads after 5 flips can be computed using the binomial law :

$$p = C_5^4 \cdot (0.5)^4 (1 - 0.5) + C_5^5 \cdot (0.5)^5$$

```
pbinom(3,size=5,prob=0.5, lower.tail = FALSE)
```

```
## [1] 0.1875
```

So the probability to get at least 4 heads after 5 flips with a fair coin is approximately 19%.

Question 6

The respiratory disturbance index (RDI), a measure of sleep disturbance, for a specific population has a mean of 15 (sleep events per hour) and a standard deviation of 10. They are not normally distributed. Give your best estimate of the probability that a sample mean RDI of 100 people is between 14 and 16 events per hour.

- 68%
- 95%
- 34%
- 47.5%

Answer : The standard error of the mean is $\frac{\sigma}{\sqrt{n}}$, where $\sigma = 10$ and $n = 100$. So the value is 1. **We then want to measure the probability of the RDI being inside 1 standard deviation around the mean. Thus it should be around 68%.**

Question 7

Consider a standard uniform density. The mean for this density is 0.5 and the variance is 1/12. You sample 1000 observations from this distribution and take the sample mean, what value would you expect it to be near ?

- 0.75
- 0.25
- 0.10
- 0.5

Answer : Using the LLN, the value should be approximately 0.5.

Question 8

The number of people showing up at a bus stop is assumed to be Poisson with a mean of 5 people per hour. You watch the bus stop for 3 hours. About what's the probability of viewing 10 or fewer people ?

- 0.08
- 0.12
- 0.03
- 0.06

```
ppois(10, lambda=15)
```

```
## [1] 0.1184644
```

Answer : The probability is approximately 12%.