Exploration librairie swirl : Binary Outcomes

```
> swirl()

 Welcome to swirl! Please sign in. If you've been here before, use the same name as you did
then. If you are new, call yourself something unique.

What shall I call you? jlbellier

 Would you like to continue with one of these lessons?

1: Regression Models Least Squares Estimation
2: No. Let me start something new.

Selection: 2

 Please choose a course, or type 0 to exit swirl.

1: Regression Models
2: Statistical Inference
3: Take me to the swirl course repository!

Selection: 1

 Please choose a lesson, or type 0 to return to course menu.

 1: Introduction                                  2: Residuals
 3: Least Squares Estimation                      4: Residual Variation
 5: Introduction to Multivariable Regression      6: MultiVar Examples
 7: MultiVar Examples2                            8: MultiVar Examples3
 9: Residuals Diagnostics and Variation          10: Variance Inflation Factors
11: Overfitting and Underfitting                 12: Binary Outcomes
13: Count Outcomes

Selection: 12
                                                                                          0%

 Binary Outcomes. (Slides for this and other Data Science courses may be found at  github
https://github.com/DataScienceSpecialization/courses. If you care to use  them, they must be
downloaded as a zip file and viewed locally. This lesson  corresponds to
Regression_Models/03_02_binaryOutcomes.)
                                                   ===                                    4%
 Frequently we care about outcomes that have two values such as alive or dead, win or  lose,
success or failure. Such outcomes are called binary, Bernoulli, or 0/1. A  collection of
```

exchangeable binary outcomes for the same covariate data are called binomial outcomes. (Outcomes are exchangeable if their order doesn't matter.)

====== 8%

In this unit we will use glm() to model a process with a binary outcome and a continuous predictor. We will also learn how to interpret glm coefficients, and how to find confidence intervals. But first, let's discuss odds.

========= 12%

The Baltimore Ravens are a team in the American Football League. In post season (championship) play they win about 2/3 of their games. In other words, they win about twice as often as they lose. If I wanted to bet on them, I would have to offer 2-to-1 odds--if they lost I would pay you $2, but if they won you would pay me only $1. That way, in the long run over many bets, we'd both expect to win as much money as we'd lost.

============ 15%

During the regular season the Ravens win about 55% of their games. What odds would I have to offer in the regular season?

1: 1.22222 to 1
2: Any of these
3: 11 to 9
4: 55 to 45

Selection: 2

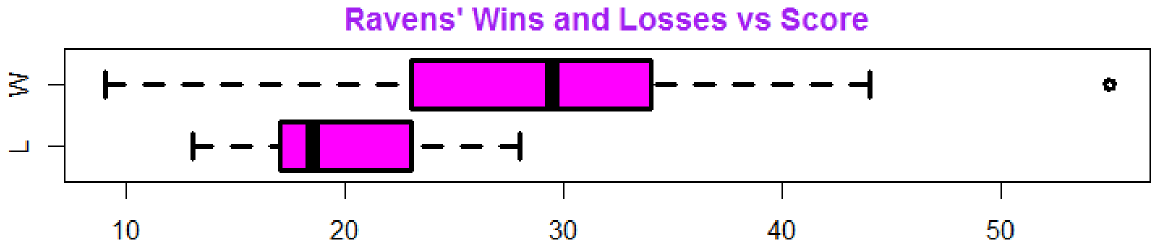
Keep working like that and you'll get there!

=============== 19%

All of the answers are correct because they all represent the same ratio. If p is the probability of an event, the associated odds are p/(1-p).

================== 23%

Now suppose we want to see how the Ravens' odds depends on their offense. In other words, we want to model how p, or some function of it, depends on how many points the Ravens are able to score. Of course, we can't observe p, we can only observe wins, losses, and the associated scores. Here is a Box plot of one season's worth of such observations.



===================== 27%

We can see that the Ravens tend to win more when they score more points. In fact, about 3/4 of their losses are at or below a certain score and about 3/4 of their wins are at or above it. What score am I talking about? (Remember that the purple boxes represent 50% of the samples, and the "T's" 25%.)

1: 18
2: 40
3: 30

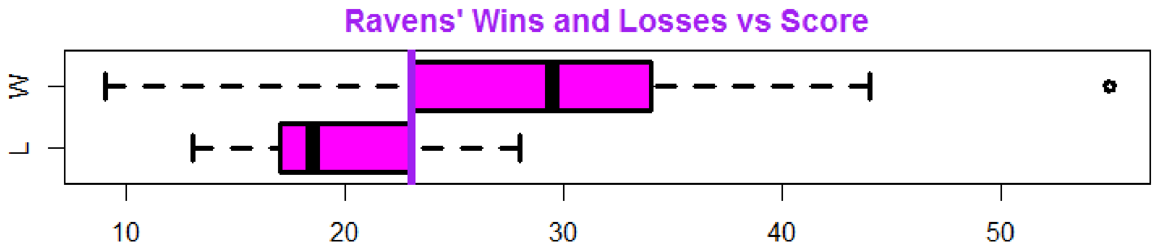
4: 23

Selection: ***4***

That's a job well done!

========================   31%

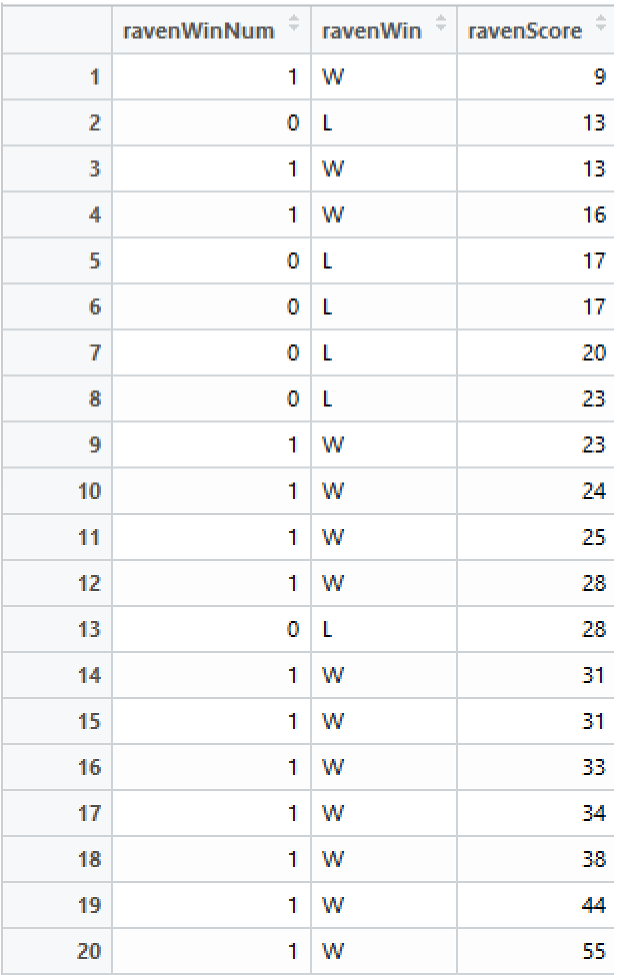
There were 9 games in which the Ravens scored 23 points or less. They won 4 of these games, so we might guess their probability of winning, given that they score 23 points or less, is about 1/2.



===========================   35%

There were 11 games in which the Ravens scored 24 points or more. They won all but one of these. Verify this by checking the data yourself. It is in a data frame called ravenData. Look at it by typing either ravenData or View(ravenData).

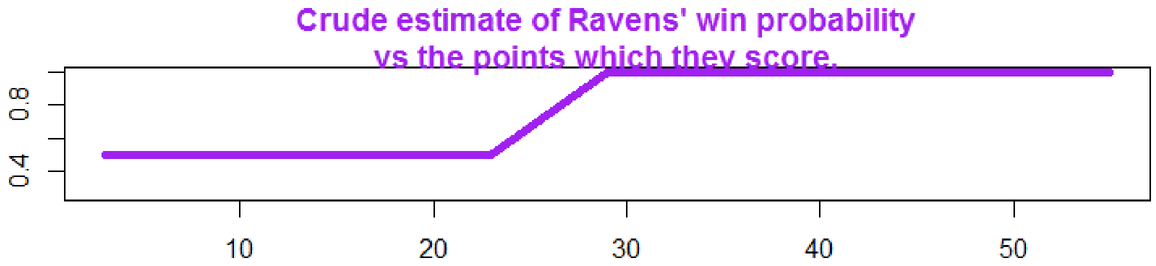
> ***View(ravenData)***



	ravenWinNum	ravenWin	ravenScore
1	1	W	9
2	0	L	13
3	1	W	13
4	1	W	16
5	0	L	17
6	0	L	17
7	0	L	20
8	0	L	23
9	1	W	23
10	1	W	24
11	1	W	25
12	1	W	28
13	0	L	28
14	1	W	31
15	1	W	31
16	1	W	33
17	1	W	34
18	1	W	38
19	1	W	44
20	1	W	55

Keep working like that and you'll get there!

============================== 38%


Crude estimate of Ravens' win probability vs the points which they score.

We see a fairly rapid transition in the Ravens' win/loss record between 23 and 28 points. At 23 points and below they win about half their games, between 24 and 28 points they win 3 of 4, and above 28 points they win them all. From this, we get a very crude idea of the correspondence between points scored and the probability of a win. We get an S shaped curve, a graffiti S anyway.

================================= 42%

*Of course, we would expect a real curve to be smoother. We would not, for instance, expect the Ravens to win half the games in which they scored zero points, nor to win all the games in which they scored more than 28. A generalized linear model which has these properties supposes that the log odds of a win depend linearly on the score. That is, log(p/(1-p)) = b0 + b1*score. The link function, log(p/(1-p)), is called the logit, and the process of finding the best b0 and b1, is called logistic regression.*

==================================== 46%

The "best" b0 and b1 are those which maximize the likelihood of the actual win/loss record. Based on the score of a game, b0 and b1 give us a log odds, which we can convert to a probability, p, of a win. We would like p to be high for the scores of winning games, and low for the scores of losses.

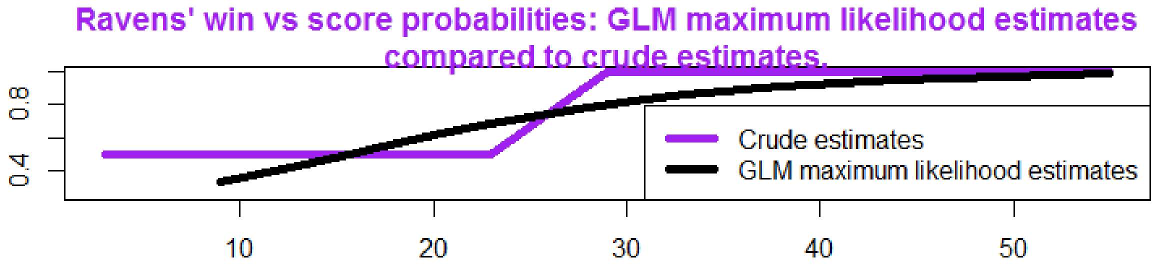
====================================== 50%

We can use R's glm() function to find the b0 and b1 which maximize the likelihood of our observations. Referring back to the data frame, we want to predict the binary outcomes, ravenWinNum, from the points scored, ravenScore. This corresponds to the formula, ravenWinNum ~ ravenScore, which is the first argument to glm. The second argument, family, describes the outcomes, which in our case are binomial. The third argument is the data, ravenData. Call glm with these parameters and store the result in a variable named mdl.

```
> mdl <- glm(ravenWin~ravenScore,"binomial",ravenData)
```

All that practice is paying off!

=== 54%


Ravens' win vs score probabilities: GLM maximum likelihood estimates compared to crude estimates.

The probabilities estimated by logistic regression using glm() are represented by *the black curve. It is more reasonable than our crude estimate in several respects: It increases smoothly with score, it estimates that 15 points give the Ravens a 50% chance of winning, that 28 points give them an 80% chance, and that 55 points make a win very likely (98%) but not absolutely certain.*

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          ================================================                  58%
```

The model is less credible at scores lower than 9. Of course, there is no data in that region; the Ravens scored at least 9 points in every game. The model gives them a 33% chance of winning if they score 9 points, which may be reasonable, but it also gives them a 16% chance of winning even if they score no points! We can use R's predict() function to see the model's estimates for lower scores. The function will take mdl and a data frame of scores as arguments and will return log odds for the give scores. Call predict(mdl, data.frame(ravenScore=c(0, 3, 6))) and store the result in a variable called lodds.

```
> lodds <- predict(mdl,data.frame(ravenScore=c(0,3,6)))
```

You got it right!

```
        ===================================================                 62%
```

Since predict() gives us log odds, we will have to convert to probabilities. To convert log odds to probabilities use exp(lodds)/(1+exp(lodds)). Don't bother to store the result in a variable. We won't need it.

```
> exp(lodds)/(1+exp(lodds))
        1         2         3 
0.1570943 0.2041977 0.2610505 
```

All that hard work is paying off!

```
      ======================================================                65%
```

As you can see, a person could make a lot of money betting against this model. When the Ravens score no points, the model might like 16 to 84 odds. As it turns out, though, the model is not that sure of itself. Typing summary(mdl) you can see the estimated coefficients are both within 2 standard errors of zero. Check out the summary now.

```
> summary(mdl)

Call:
glm(formula = ravenWin ~ ravenScore, family = "binomial", data = ravenData)

Deviance Residuals: 
    Min       1Q   Median       3Q      Max  
-1.7575  -1.0999   0.5305   0.8060   1.4947  

Coefficients:
            Estimate Std. Error z value Pr(>z)
(Intercept) -1.68001    1.55412  -1.081     0.28
ravenScore   0.10658    0.06674   1.597     0.11

(Dispersion parameter for binomial family taken to be 1)
```

```
    Null deviance: 24.435  on 19  degrees of freedom
Residual deviance: 20.895  on 18  degrees of freedom
AIC: 24.895

Number of Fisher Scoring iterations: 5
```

That's correct!

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  |=====================================================================                               |  69%
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The coefficients estimate log odds as a linear function of points scored. They have a natural interpretation in terms of odds because, if b0 + b1*score estimates log odds, then exp(b0 + b1*score)=exp(b0)exp(b1*score) estimates odds. Thus exp(b0) is the odds of winning with a score of 0 (in our case 16/84,) and exp(b1) is the factor by which the odds of winning increase with every point scored. In our case exp(b1) = exp(0.10658) = 1.11. In other words, the odds of winning increase by 11% for each point scored.

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  |=========================================================================                           |  73%
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However, the coefficients have relatively large standard errors. A 95% confidence interval is roughly 2 standard errors either side of a coefficient. R's function confint() will find the exact lower and upper bounds to the 95% confidence intervals for the coefficients b0 and b1. To get the corresponding intervals for exp(b0) and exp(b1) we would just exponentiate the output of confint(mdl). Do this now.

```
> exp(confint(mdl))
Waiting for profiling to be done...
                  2.5 %   97.5 %
(Intercept) 0.005674966 3.106384
ravenScore  0.996229662 1.303304
```

Great job!

```
  |=============================================================================                       |  77%
```

What is the 2.5% confidence bound on the odds of winning with a score of 0 points?

1: 0.005674966
2: 2.5%
3: 0.996229662

Selection: **1**

Great job!

```
  |=================================================================================                   |  81%
```

The lower confidence bound on the odds of winning with a score of 0 is near zero, which seems much more realistic than the 16/84 figure of the maximum likelihood model. Now look at the lower bound on exp(b1), the exponentiated coefficient of ravenScore. How does it suggest the odds of winning will be affected by each additional point scored?

1: They will increase slightly
2: They will increase by 30%

```
3: They will decrease slightly

Selection: 3
```

Excellent work!

```
  ==================================================================          85%
```

The lower confidence bound on exp(b1) suggests that the odds of winning would decrease slightly with every additional point scored. This is obviously unrealistic. Of course, confidence intervals are based on large sample assumptions and our sample consists of only 20 games. In fact, the GLM version of analysis of variance will show that if we ignore scores altogether, we don't do much worse.

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  =====================================================================       88%
```

Linear regression minimizes the squared difference between predicted and actual observations, i.e., minimizes the variance of the residual. If an additional predictor significantly reduces the residual's variance, the predictor is deemed important. Deviance extends this idea to generalized linear regression, using (negative) log likelihoods in place of variance. (For a detailed explanation, see the slides and lectures.) To see the analysis of deviance for our model, type anova(mdl).

```
> anova(mdl)
Analysis of Deviance Table

Model: binomial, link: logit

Response: ravenWin

Terms added sequentially (first to last)


          Df Deviance Resid. Df Resid. Dev
NULL                         19     24.435
ravenScore  1   3.5398       18     20.895
```

Nice work!

```
   =====================================================================      92%
```

The value, 3.5398, labeled as the deviance of ravenScore, is actually the difference between the deviance of our model, which includes a slope, and that of a model which includes only an intercept, b0. This value is centrally chi-square distributed (for large samples) with 1 degree of freedom (2 parameters minus 1 parameter, or equivalently 19-18.) The null hypothesis is that the coefficient of ravenScore is zero. To confidently reject this hypothesis, we would want 3.5398 to be larger than the 95th percentile of chi-square distribution with one degree of freedom. Use qchisq(0.95, 1) to compute the threshold of this percentile.

```
> qchisq(0.95,1)
[1] 3.841459
```

All that hard work is paying off!

=== 96%

As you can see, 3.5398 is close to but less than the 95th percentile threshold, 3.841459, hence would be regarded as consistent with the null hypothesis at the conventional 5% level. In other words, ravenScore adds very little to a model which just guesses that the Ravens win with probability 70% (their actual record that season) or odds 7 to 3 is almost as good. If you like, you can verify this using mdl0 <- glm(ravenWinNum ~ 1, binomial, ravenData), but this concludes the Binary Outcomes example. Thank you.

===100%