Quiz 4 - Statistical Inference

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Question 1

A pharmatical company is interested in testing a potential blood pressure lowering medication. Their first examination considers only subjects that received the medication at baseline then two weeks later. The data are as follows (SBP in mmHg) :

```
## Warning: package 'knitr' was built under R version 3.3.2
```

subject	baseline	Week2
1	140	132
2	138	135
3	150	151
4	148	146
5	135	130

Consider testing the hypothesis that there was a mean reduction in blood pressure. Give the P-value for the associated two-sided T test. Hint : consider that the observations are paired.

Answer : We want to test the hypothesis $H_0 : \mu_0 = 0$ against $H_a : \mu_0 \neq 0$, where μ_0 is the difference of the means between the baseline and the measures after two weeks.

- 0.087
- 0.05
- 0.10
- 0.043

```r
baseline <- c(140,138,150,148,135)
Week2 <- c(132,135,151,146,130)

t.test(Week2,baseline,alternative="two.sided",paired=TRUE)
```

```
##
##  Paired t-test
##
## data:  Week2 and baseline
## t = -2.2616, df = 4, p-value = 0.08652
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -7.5739122  0.7739122
## sample estimates:
## mean of the differences
##                    -3.4
```

The p-value is 0.087.

Question 2

A sample of 9 men yielded a sample average brain volume of 1,100cc and standard deviation of 30cc. What is the complete set of values of μ_0 that a test of $H_0 : \mu = \mu_0$ would fail to reject the null hypothesis in a two-sided 5% Students t-test ?

- 1031 to 1169
- 1080 to 1120
- 1081 to 1119
- 1077 to 1123

Answer : We want to determine the 95% confidence interval for this Student's t-test.

```
CI <- 1100 + c(-1,1)*qt(0.975,8)*30/sqrt(9)
CI
```

```
## [1] 1076.94 1123.06
```

The complete set of values is 1077 to 1123.

Question 3

Researchers conducted a blind taste test of Coke versus Pepsi. Each of four people was asked which of two blinded drinks given in random order that they preferred. The data was such that 3 of the 4 people chose Coke. Assuming that this sample is representative, report a P-value for a test of the hypothesis that Coke is preferred to Pepsi using a one-sided exact test.

- 0.62
- 0.005
- 0.10
- 0.31

Answer : Let us call p the proportion of people who prefer Coke than Pepsi. Then we want to test the hypothesis $H_0 : p = 0.5$ versus $H_a : p > 0.5$. It can be done usin the khi squared independance test :

```
chisq.test(c(3,1),p=c(0.5,0.5))
```

```
## Warning in chisq.test(c(3, 1), p = c(0.5, 0.5)): Chi-squared approximation
## may be incorrect
```

```
##
##  Chi-squared test for given probabilities
##
## data:  c(3, 1)
## X-squared = 1, df = 1, p-value = 0.3173
```

So the p-value for a test of the hypothesis that Coke is preferred to Pepsi using a one-sided exact test is ***0.31***.

Question 4

Infection rates at a hospital above 1 infection per 100 persons days at risk are believed to be too high ad are used as a benchmark. A hospital that had previously been above the benchmark recently had 10 infections over the last 1787 person days at risk. About what is the one-sided P-value for the relevant test of whether the hospital is "below" the standard ?

- 0.52
- 0.22
- 0.11
- 0.03

Answer : We want to test the hypothesis $H_0 : \lambda = 0.01$ versus $H_a : \lambda < 0.01$. We have $X = 10$, $t = 1787$ and we assume that $X_{H_0} \sim Poisson(\lambda.t)$.

```
lambda <- 0.01
t <- 1787
ppois(10,lambda*t)
```

```
## [1] 0.03237153
```

The one-sided P-value is then 0.03.

Question 5

Suppose that 18 obese subjects were randomized, 9 each to a new diet pill and a placebo. Subjects' body mass indice (BMIs) were measured at a baseline and after having received the treatment or placebo for four weeks. The average difference from followup to the baseline (followup - baseline) was $-3kg/m^2$ for the treated group and $1kg/m^2$ for the placebo group. The corresponding standard deviations of the differences was $1.5kg/m^2$ for the treatment group and $1.8kg/m^2$ for the placebo group. Does the change in BMI appear to differ between the treated and the placebo groups ? Assuming normality of the underlying data and a common population variance, give a p-value for a two-sided t-test.

- Less than 0.1 but larger than 0.05
- Less than 0.05 but larger than 0.01
- Less than 0.01
- Larger than 0.1

Answer : Let us call $\mu_{diff,treated}$ and $\mu_{diff,placebo}$ the mean values of the difference (followup-baseline) for the treated group and the placebo group. The hypothesis H_0 is then :

$$H_0 : \mu_{diff,treated} = \mu_{diff,placebo}$$

```
n_plac <- 9
n_treat <- 9
mudiff_treat <- -3
mudiff_plac <- 1
sddiff_plac <- 1.8
sddiff_treat <- 1.5

s <- sqrt(((n_plac-1)*sddiff_plac^2 + (n_treat-1)*sddiff_treat^2)/(n_plac + n_treat -2))
t <- (mudiff_treat-mudiff_plac)/(s*sqrt(1/n_plac + 1/n_treat))
2*pt(t,n_plac + n_treat-2)
```

```
## [1] 0.0001025174
```

The p-value is less than 0.01.

Question 6

Brain volumes for 9 men yielded a 90% confidence interval of 1077 cc to 1123 cc. Would you reject in a two sided 5% hypothesis test of $H_0 : \mu = 1078$?

- It's impossible to tell
- Where does Brian come up with these questions ?
- No you would not reject
- Yes you would reject

Answer : The value 1078 is in the 90% confidence interval. As the 95% is wider than the 90% interval, ***you would not reject***.

Question 7

Researchers would like to conduct a study on 100 healthy adults to detect a four year mean brain volume loss of 0.01 mm^3. Assume that the standard deviation of four year volume loss in this population is 0.04 mm^3. About what would be the power of the study for a 5% one sided test versus a null hypothesis of no volume loss ?

- 0.80
- 0.70
- 0.60
- 0.50

Answer :
Let us call μ_{diff} the mean of the difference of loss (Four weeks - baseline). Then we want to test $H_0 : \mu_{diff} = 0$ versus $H_a : \mu_{diff} < 0$.

The test statistic is $t = \frac{\bar{X}}{\sigma}.\sqrt{n}$. The hypothesis is rejected if $t > Z_{0.95} = 1.645$.

$P\left(t > 1.645 \,\middle|\, \mu_{diff} = 0.01\right) = P\left(\frac{\bar{X}-0.01}{\sigma}.\sqrt{n} > 1.645 - \frac{0.01}{\sigma}.\sqrt{n} \,\middle|\, \mu_{diff} = 0.01\right) = P\left(Z > -0.855\right) = 0.80$.

Question 8

Researchers would like to conduct a study of n healthy adults to detect a four year mean brain loss of $0.1mm^3$. Assume that the standard deviation of four year voumle loss is $0.4mm^3$. About what would be the value of n needed for 90% power of type one error rate of 5% one-sided test versus a numm hypothesis of no volume loss ?

- 160
- 120
- 180
- 140

Answer : We want to test the null hypothesis $H_0 : \mu_{diff} = 0$ against $H_a : \mu_{diff} \neq 0$, where μ_{diff} is the brain volume loss.

From the previous question, we want to have

$P\left(Z > 1.645 - \frac{\sqrt{n}}{4}\right) = 0.90$,i.e.

$1.645 - \frac{\sqrt{n}}{4} > Z_{0.1} = -1.282$

So $n = (4(1.645 + 1.282))^2 = 137.07$

So the right answer is ***n=140***.

Question 9

As yu increase the type of error rate, α, what happens to the power ? * It's impossible to tell given the information in the problem * You will get a larger power * You will get a smaller power * No, for real, where does Brian come up with these problems ?

Answer : When α increases, you get less evidence to reject, ***so the power increases***.