THE ULTRA-INTUITIONISTIC CRITICISM AND THE ANTITRADITIONAL PROGRAM FOR FOUNDATIONS OF MATHEMATICS

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The aim of this program is to banish faith from the foundations of mathematics, faith being defines ad any violation of the law of sufficient reason.

I begin with a criticism of the following traditional assumptions underlying the body of modern mathematics.

T1. The uniqueness (up to isomorphism) of the natural number series;

T2. The existence of values of primitive recursive functions(prf) for every system of arguments for an arbitrary natural number series. If for each system X1, ..., Xn of arguments in a natural number series N the value of φ (X1, ….., Xn) exists in N, I shall call N closed with respect to φ.

T3. The principle of mathematical induction from n to n'.

T4. If the axioms of a formal system are true and the rules of inference conserve the truth then each theorem is true.

T5. The meaningfulness if the relations of identity and distinctness.

Feasible numbers: those up to which it is possible to count.

I call a natural number series N regular if it is closed with respect to An for each sequence An considered as a function of n. Of course, the assumption that every natural number series is regular is even stronger than T2: and so it is wrong.

Of course, with the help of T3 (for arbitrary induction properties) one can prove the regularity of every natural number series but this is an argument against the acceptance of T3, at least in its most general form. I call T4 the locality principle of proofs, because it asserts that the property of a tree-figure of being a proof depends only on the local properties of this figure – more strictly, on the properties of its summits and nodes the only "integral" property consisting of the fact that the local properties are satisfied by each summit and node. A similar principle may be formulated for deductions from arbitrary premises in terms of "truth relative to the premisses" (it is clear that T4 depends on T3).

The idea of establishment of convincingness is not completely strange even for traditional mathematical logic. Everyone understands that each tree-figure of proof requires a reference to T4.

I criticize this establishment of convincingness for its vicious circle (with T3 and T4). So I reject this traditional establishment of convincingness. I consider a special *genetic theory* dealing with these establishments.

Now I go further with my criticism of traditional assumptions. I shall consider T5. I recall the series F of feasible numbers. N denotes a natural number series.

Already Heraclitus pointed out that the notion of identity is not completely clear. But mathematicians prefer to proceed as if Heraclitus had not lived. I cannot continue in this way. The situation when an infinite process can be imbedded in a finite object is an ordinary one in investigations of distinct natural number series, and I shall need an apparatus for the explicit consideration of all identifications used in such cases. First of all I maintain that the relation of identity or distinctness has no other meaning than that two object have been identified or distinguished.

Generally, one can identify or distinguish any two objects in two arbitrary occurrences of them. But collations play in our arguments a role similar to that assumptions, and they must be made quite explicit or at least according to explicitly stated rules.

I refer to a *Zenonian* situation every case in which the events of an infinite process are to be identified with the parts of a finite object; and this finite object I call the field of a *Zenonian* situation. The described Zenonian situation is only an infinite imbedding of the infinite process F in the finite field. But if the identifications are not stated in an explicit manner it is impossible to notice the fact; and one talks about the field as if *this impossible structural identification were carried out.*

Now I describe briefly the structure of this antitraditional program: it contains a *central nucleus* consisting of two theories: the *ontological theory* and the *genetic theory.* The *ontological theory* deals with the substantiation of the possibility of natural number series and other discrete processes required for this consistency-proof; the *genetic theory* deals with the notion of proof for sentences related to the events of distinct processes.

Now I continue the list of traditional assumptions.

T6. The possibility of neglecting modalities and aim in foundations of mathematics.

T7. The possibility of neglecting tenses, voices and moods of verbs.

T8. The possibility of neglecting the rules of attention and neglecting.

T9. The hypothesis of potential feasibility.

T10. The division of theories into object theories and metatheories.

T11. The postulates of the intuitionistic predicate calculus.

The reasons for my criticism of T6 and T7 are connected with the previous analysis of traditional assumptions.

Principle of modal fulfilment:

The exact formulation of this principle requires a theory of modalities. The theory of modalities will be closely connected with *semiotics,* in semiotics I consider the act of *Indication* as central. A *language* is a method or system of indication, it is defined by means. A *character* or *tactic* is a system of *rules,* of propositions expressing permissions and demands. If a tactic is expressed in a language I call it a *method.* A *way* is a tactic or a method. To follow a tactic for following connections is to accept such sentences as "a is connected with b". If a sign is made by means of a language I call it the *name.* I call *direct* those occurrences of a sign in which the role of a sign is confined to its playing a part as a sign for its points, and to the participation in following those tactics of attention and collations which make it a sign.

There are the following three fundamental semiotical principles:

S1. Given a collection of indications {a0 -> a1, a1 -> a2, …, an-1 ->an} in a language L the indication a0 -> an also belongs to L. It is a rule for restricting languages. It enables us to follow the denotational connections. One uses this rule each time when the sense of word in a new occurrence is to be identified with its sense in a former one.

S2. If the connections f -> F, a1 -> A1, …, an->An belong to a language L, F being an operation applicable to the objects A1, …, An then the notation f(a1, …, an) -> (the result of applying F to A1, …, An) also belongs to L; some specifications of the role of the order "a1, …, an" may be introduced, and "f(a1, …, an)" may be replaced by another record which enables one to reconstruct it. (*The principle of structural parallelism.) Parameters* are indefinite indecomposable names.

S3. When a text is accepted the result of fixing some collection of parameters in it by their admissible values is also to be accepted; but this operation of fixation may require grammatical agreements in the text obtained by the fixation. The fixed values may in turn depend on parameters or even be parameters.

The theory of methods and ways is a generalization of the traditional theory of algorithms. There are at least four respect in which the former is more general than the latter:

(a) The primitive operations of the normal algorithms (the substitutions of words) are performed in a way which is not itself considered as an algorithm;

(b) The theory of algorithms depends on natural numbers, which are to be obtained in some way;
(c) The methods can be applied to objects of a completely general nature;
(d) The "work" of a method or way may be an undetermined process.

The principle of modal fulfilment (pmf). If the situation S is possible and the action or event A is possible in S then the situation S U {A} (obtained from S by adjoining A to it) is also possible.

The main principle of necessity. If a process D is described by a tactic X and for a situation S occurring in the course of D the rules of X demand an action or event A then for the continuation of D in S the fulfilment of A is necessary, in virtue of this principle that the definition are observed in the course of a development of a theory and it seems that this principle itself can be based essentially on the definition of the word "describe".

If an event e of a process D is over and done with the situation S of D and d precedes e in D then it is necessary that d is over in S. (*The principle of ordinal necessity,* p.o.n.).

The violation of the main principle of necessity is impossible in each possible situation. (*The principle of negative evidence, p.n.e.)*

If the aim T is not achieved in S, in order to achieve it in S it is necessary to apply (a) some sufficient means and (b) each of the necessary means (*The inversion principle)*.

Relevancy theory – it is a theory about how to take something into attention or to neglect it.

Hypothesis related to T9:

The central ontological hypothesis (c.o.h.). Let E be accomplished procedure. Let for each step *e* of E le be a finite procedure feasible if the *ld*'s are accomplished for all steps *d* preceding to *e* in E. Then a procedure El is feasible which consists in the reproduction of E with le following each step *e*. C.o.h. is a very strong hypothesis which practically implies all applications of T9. The acceptance of c.o.h. leads to a *third degree* of modalities and the strength of c.o.h. consists in the extension of the old rules to it.

In connection with T10 say that already the introduction of tactics of attentions and collations makes the structure of logical theories more complicated than is provided for by T10. Often it is inconvenient to consider the collations used in a text at the same level as the text, and one has to return from the collations to the collated signs.

As concern T11, I say here only that most of the intuitionistic postulates one can repeat a way of substantiation available in the traditional logic. There are several

methods, and the substantiation is always connected with the interpretation chosen for the ordinary logical operations.

So in order to show today's state of affairs in my program I can list these extreme directions. I can indicate today the following seven directions:

(1). *Ultra-ultra-intuitionism* (briefly: uduism) – the construction of the theory of disputes – the logic of confidence and the relevant parts of ethics;
(2). *Extra-ultra-intuitionism* (briefly: eduism) or relevantism – the substantiation of the hypothesis on obstacles, or its abolishment from our considerations;
(3). *Trans-ultra-intuitionism* (briefly: teduism) – the same for c.o.h.;
(4). *Pragma-ultra-intuitionism* (briefly: peduism);
(5). *Lega-ultra-intuitionism* (briefly: eluism) – the foundations of the primary permissions and the deontic relations between different extreme directions; the relevant parts of ethics;
(6). *Nega-ultra-intuitionism* (briefly: neguism) – the substantiation of the principle of the negative evidence;
(7). *Bi-nega-ultra-intuitionism* (briefly: bineguism) – the substantiation of the elimination of double negations in the foundations of prototheories.

ON THE LOGIC OF THE MORAL SCIENCES

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Originally published in *Social Problems* No. 12, 1972 (in Russian).

Logic is the science of standards of correct reasoning, the study of avoiding errors. In all fields of human activity where the risk of error is recognized as intolerable, the rigorous use of proof is demanded. I call each occasion when a judgment is recognized as true without proof *faith. "Why is this accepted?"* In the area of acceptance of judgments*, the law of sufficient reason* consists in considering *only* proven results as true. By proof of a judgment, I mean any honest method which makes the judgment incontestable. By honesty I mean the absence of coercion and fraud. The *law of sufficient reason* consists in the demand that the *understanding* of every rule be *grounded.* The right to doubt proven judgments is distinct from the right to criticize a proof, which in practice is more important.

"Faith" is often understood as preferring to accept certain judgments rather than reject them regardless of any rational arguments, irrespective of the presence or

absence of proofs, even holding to the preference with particular stubborness. Thus, any moral system, any legal system demanding even the smallest degree of faith limits freedom of thought, In that case I recommend the development of a special branch of logic which I call the *logic of confidence* and which I oppose to the "logic of proof." The logic of confidence claims to have full control over applications of faith.

In the important *theory of reasonings* having to do with criteria of "correctness" of judgments, rules are established characterizing one or another theoretical activity.

The following may serve as *grounds for the rejection of an act of confidence:*

a) Errors on the part of the source of confidence not considered possible at the time of the act;

b) Deterioration of conditions for verifying the testimony of the source compared to what is expected during the act of confidence;

c) Behavior on the part of the source of confidence which would promote such deterioration (in particular, violations of the principle of publicity *(glasnost)* by the courts, etc.);

d) A conscious lie authorized by the source of confidence in testimonies or sometimes even in judgments not related to his sphere of competence; e) Well-founded speculations about the deterioration of the source of confidence's capabilities or honesty;

f) Discovery of a better source of confidence; g) Increase in the demand for truthfulness, precision or authenticity in the testimony of the source of confidence; h) Resemblance to another source which has been denied confidence. This list of grounds is not exhaustive.

The search for such grounds is called *criticism of confidence* and evolves in accordance with the methodology of investigation in a given activity.

The *heuristic principle of confidence* plays a most significant role in human cognition.

Demands to refrain from an act are called *prohibitions* or *bans* of the act. The circumstances must be indicated in some way or other. The common way of indicating circumstances consists of describing them by means of some set or class of judgments.

The following *principle of modal fulfillment pmf* has a variation for each group of modalities: If situation S is possible, and A is Possible in S, then situation S + {A} is possible, resulting in the addition of judgment A to S. More precisely, if in pmf one discusses any possibilities other than epistemic and ontological ones, then A is the

name of an act or event. and, in the proposition A is possible in S, the given A must stand in the future tense or in the infinitive. But in S + {A}, A must stand in the past indicative. Most significant in *pmf* is the transformation of A from possible in S to actual in S + {A}. The *pmf* is the principle by means of which applications or realizations of possibilities are formalized. Unlike the rules of deductions, S + {A} is considered only possible. Situations are called *actual* if all their elements are accepted on the basis of perception; such actuality is called *real.* Actual situations are considered alethically possible. The grounding of *pmf* can be that only those situations which can be confirmed as possible on the basis of *pmf* are recognized as subsequently possible.

Deontic judgments, judgments about a rule's being accepted, are associated with rules. A *deontic judgment* often represents a permission, and *pmf* must be applied to it.

A method may be incomplete' in two fundamentally different respects. On the one hand, for a single situation S it may contain several fundamentally different allowances, for acts A, B..., without making any choice among them. I call such situations S *Buridanian.*

No single one of the processes can be counted as originally feasible in S until a way of choosing a variant has been shown.

The absence of any rule for A in S is another case of an incomplete method. In such cases completion usually occurs with the help of one of two preferences: either permission is preferred to prohibition, which constitutes the *principle of liberalism,* or, prohibition is preferred to permission, which constitutes the *principle of despotism.*

For any activity and any of its situations S, authorization of A in S, "contained" in the rules of the activity, is the *basis* for performing acts A in S. More precisely, this is *the primary basis.*

If process E is described by method M, the rules of which contain the requirement that act A be performed in situation S, then for the continuation of E in S it is organically necessary that A be performed in S. (Principle of deontic-organic necessity, *pd-on).*

If in situation S of process E event C has occurred, then in that situation each event prior to C must have occurred. (Principle of ordinal necessity).

In a possible situation S no violation of *pd-on* is possible. (Principle of negative evidence).

For alethic modalities the following principles are adopted:

P1. For a possible situation S, if A is obligatory in S, then A takes place in S (or will take place in a later situation).

P2. For a possible situation S, if A takes place in S (or happens in S), then A is possible in S.

For deontic modalities P1 and P2 can be violated, but this will violate the rules of the activity. P1 corresponds to the condition that in an activity which is going on any of its requirements is fulfilled.

In the theory of goals I adopt the following principles: To reach an unattained goal, sufficient means must be applied. To reach an unattained goal, every necessary means must be applied.

Three principles of sufficiency:

1. To reach an unattained goal, it is sufficient to apply sufficient means for this.

2. Every event is sufficient for the occurrence of any of its consequences

3. Means M is sufficient for the attainability of goal T if there is an applicable way to reach T with the help of M

Although a proven judgment is by definition incontestable, only the identification of a contestable judgment with an unproven judgment clearly corresponds to the meaning of the word ‘prove’, so that what is incontestable, is simply not-unproven.

No morality exists until there are clear terms for the categories of rights and obligations. Groundings for the acceptance of rules include two important groups — those sufficient for a goal and those necessary for it. I define *justice* as a following of the law of sufficient reason in adopting rules of conduct.

Therefore if a choice must be made to adopt one of two (or more) means, preference must be given to the one which will make it possible to adopt the other in time. This is the substance of the *principle of rational choke* among means.

This principle is frequently violated for the sake of another— the *principle of urgency—which* demands that the means of obvious usefulness in a given situation be selected. This preference for the interests of the present over those of the future can only be rationally grounded when violation of the interests of the present threatens the very continuation of a field of action. In the field of recognition of rights, this principle appears in preferences for authorizing the use of those rights which can aid in protecting other rights.

The right to the defence of every right must be recognized as the supreme right. The lawgiver must confine himself to prohibiting those acts which obstruct recognition or realization of the rights of individuals or lawful associations.