Licenciatura em Engenharia Informática (LEI)

2024/2025

Análise Matemática (AMATA)

CAPÍTULO 1

Cálculo Diferencial em $\mathbb{R}$ e em $\mathbb{R}^n$.

EXERCÍCIOS

EXERCÍCIOS DE REVISÃO: FUNÇÃO EXPONENCIAL E LOGARÍTMICA

1. Considere a função $f(x) = 1 + 2^{1-x}$:

   1.1 Determine o domínio e o contradomínio da função f.

   1.2 Determine a expressão analítica da função inversa $f^{-1}(x)$.

   1.3 Determine o conjunto solução da equação

   $$f(-1) + f^{-1}(1-x) = \frac{1}{2}\log_2(25) + 6.$$

2. Considere a função $f(x) = \log_3 \sqrt{\frac{2}{x}}$:

   2.1 Determine o domínio e o contradomínio da função f.

   2.2 Determine a expressão analítica da função inversa $f^{-1}(x)$.

   2.3 Determine o conjunto solução da equação

   $$f\left(\frac{2}{81}\right) + f^{-1}(-1) = |x|.$$

3. Considere a função,

   $$y = f(x) = 1 - e^{(1-x)\ln(2)}.$$

   3.1 Indique o domínio e o contradomínio da função $h(x) = 3 + |f(x)|$.

   3.2 Determine a expressão da função inversa $f^{-1}(x)$.

   3.3 Resolva a seguinte equação:

   $$e^{2\ln(x)-2\ln(1)} + 9\,x\,5^{-2\log_5(3)} + \log_2(1-f(x)) - \sec\left(\frac{\pi}{3}\right) = 0.$$

4. Considere a função,

$$y = f(x) = 3 + \frac{1}{2}\log_7(2x-1).$$

   4.1 Indique o domínio e o contradomínio da função $f(x)$.

   4.2 Escreva a equação da reta normal à curva de f no ponto de abcissa $x = f(25)$.

5. Considere a função $f(x) = 1 - \log_3(5-2x)$. Calcule o valor da expressão $A = \sec(f(1)) + \operatorname{cotg}\left(\dfrac{\pi/2}{f^{-1}(-1)}\right)$.

6. Considere a seguinte função $f(x) = -2 + \log_2(6-x)$.

   6.1 Determine o domínio da função f.

   6.2 Caracterize a função inversa de f.

   6.3 Resolva a seguinte equação:

$$f^{-1}(x) = \sqrt{3}\operatorname{cosec}\left(\frac{7\pi}{3}\right) - \log_2|f(-2)| + 3\,e^{\ln(2)-\ln(3)}.$$

   6.4 Escreva uma equação da reta normal à curva da função f no ponto $(4,-1)$.

7. Considere a seguinte função real de variável real $f(x) = 2 - e^{1-\frac{1}{x}}$.

   7.1 Determine o domínio da função f.

   7.2 Caracterize a função inversa de f (indicando o seu domínio e contradomínio).

7.3 Resolva a seguinte equação:

$$f^{-1}\left(\operatorname{tg}\left(\frac{5\pi}{4}\right)\right)+\log_3\left(\frac{1}{9}\right)^{\sqrt{x}}+\left(\frac{1}{e}\right)^{\ln\left(\frac{1}{3}\right)}=0.$$

7.4 Escreva uma equação da reta normal à curva da função f no ponto $(1,1)$.

FUNÇÕES TRIGONOMÉTRICAS INVERSAS E SUAS DERIVADAS

8. Determine o domínio e o contradomínio das seguintes funções:

   8.1 $f(x)=\dfrac{\pi}{6}-4\operatorname{arcsen}\left(\dfrac{x}{3}\right)$.

   8.2 $f(x)=-\dfrac{\pi}{2}-3\operatorname{arccotg}(2x)$.

   8.3 $f(x)=\pi-\dfrac{5}{3}\left|\operatorname{arctg}(1-4x)\right|$.

9. Usando a definição de funções trigonométricas inversas, calcule:

   9.1 $\operatorname{arcsen}\left(-\dfrac{\sqrt{2}}{2}\right)$.

   9.2 $2\arccos(-1)$.

   9.3 $\operatorname{arctg}(-\sqrt{3})$.

   9.4 $2\operatorname{arccotg}(-1)$.

9.5 $\operatorname{sen}\left(\arccos\left(\frac{\sqrt{3}}{2}\right)\right)$.

9.6 $\operatorname{cotg}\left(\operatorname{arctg}\left(\sqrt{3}\right)\right)$.

9.7 $\sec\left(\operatorname{arctg}\left(\frac{3}{5}\right)\right)$.

9.8 $\operatorname{cosec}\left(\arccos\left(-\frac{5}{12}\right)\right)$.

10. Resolva as seguintes equações:

10.1 $\operatorname{arcsen}(3x-\pi)=\frac{\pi}{2}$.

10.2 $\arccos(2x)=\frac{\pi}{6}$.

10.3 $1+3\operatorname{arctg}(2x)=\pi\left(1+\frac{1}{\pi}\right)$.

10.4 $\operatorname{arcsen}\left(\sqrt{2}\,x\right)=\operatorname{arccotg}\left(\operatorname{cotg}\left(\frac{9\pi}{4}\right)\right)$.

10.5 $e^{3\ln(y)}=2\sec\left(-\frac{\pi}{3}\right)+\sec\left(\arccos\left(\frac{1}{4}\right)\right)$.

11. Considere a função,

$$y=f(x)=\pi-\frac{1}{2}\operatorname{arcsen}(1-2x).$$

11.1 Indique o domínio e o contradomínio da função f.

11.2 Resolva a equação $f(x)=\pi$.

12. Considere a função,

$$y=g(x)=\operatorname{arctg}(e^{2x-1}).$$

12.1 Indique o domínio e o contradomínio da função g.

12.2 Determine a expressão da função inversa $g^{-1}(x)$.

13. Determine umas equações das retas tangente e normal ao gráfico representativo da função $y = \operatorname{arcsen}(x-1)$ no ponto $\left(\dfrac{3}{2}, \dfrac{\pi}{6}\right)$.

14. Considere a função real de variável real, definida por:

$$y = f(x) = -2\arccos(\ln(x)).$$

14.1 Determine o domínio e o contradomínio de f.

14.2 Calcule $\left\{x \in \mathbb{R} : f(x) + 2\pi = \operatorname{arccotg}\left(\dfrac{\sqrt{3}}{3}\log_2\left(\operatorname{sen}\left(\dfrac{\pi}{6}\right)\right)\right)\right\}$.

15. Considere a função real de variável real $f(x) = 1+2\cos\left(x - \dfrac{\pi}{3}\right)$.

15.1 Determine o domínio e o contradomínio da função.

15.2 Calcule $A = f\left(\frac{7\pi}{2}\right) - f\left(\frac{7\pi}{6}\right)$.

15.3 Determine a expressão da função inversa e caracterize-a.

16. Considere a função $y = f(x) = \pi - \operatorname{arcsen}(2x-1)$.

16.1 Determine a expressão da função inversa $f^{-1}(x)$.

16.2 Determine

$$S = \left\{x \in \mathbb{R} : f(x) = \operatorname{tg}(-\pi) + \operatorname{arccotg}\left(10^{2\log_{10}\left(\sqrt{\sqrt{3}}\right)}\right) + \arccos\left(\cos\left(\frac{4\pi}{3}\right)\right)\right\}.$$

17. Considere as seguintes funções $f(x) = 4 - 3^{\sqrt{x}}$ e $g(x) = -2\,\text{arcsen}\left(\dfrac{x-3}{2}\right)$.

    17.1 Determine o domínio e o contradomínio de função f e caracterize a sua função inversa.

    17.2 Determine o domínio e o contradomínio da função g.

    17.3 Resolva a seguinte equação
    $$\frac{\pi}{2} f\left((\log_3(2))^2\right) = \text{arctg}\left(10^{2\log_{10}(\sqrt[4]{3})}\right) + g(x).$$

    17.4 Escreva uma equação da reta tangente à curva da função f no ponto $\left(g(3)+1, \dfrac{g^{-1}(0)}{3}\right)$.

DERIVADAS PARCIAIS DE FUNÇÕES REAIS DE VARIÁVEIS REAIS

18. Considere a função $f(x,y) = x^3 + x^2y^3 - 2y^2$. Calcule $f'_x(2,1)$ e $f'_y(2,1)$.

19. Sendo $f(x,y,z) = \sqrt{x^2+y^2+z^2}$. Calcule $f'_x(1,2,0)$, $f'_y(1,2,0)$ e $f'_z(1,2,0)$.

20. Sendo $z = xy + xe^{\frac{y}{x}}$, prove que $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = xy + z$.

21. Se $u = \dfrac{x}{y} + \dfrac{y}{z} + \dfrac{z}{x}$, prove que $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$.

22. Determine as derivadas parciais de 1^a ordem das seguintes funções:

    22.1 $z = x\cos(y) - y\cos(x)$.

    22.2 $z = x\ln(y)$.

    22.3 $z = e^y \text{sen}(xy)$.

    22.4 $w = \ln\left(x^2 + xyz + y^2 z\right)$.

    22.5 $f(x, y) = xy\cos(x - 2y)$.

    22.6 $g(x, y) = \dfrac{x^2 y e^y}{x^4 + 4y^2}$.

    22.7 $h(x, y) = e^{x^2 y} + \text{arctg}(x + \sqrt{y})$.

    22.8 $w = y^{y-z} + 5^{\text{tg}(\ln(x+1))}$.

    22.9 $f(x, y, z) = xy^4 z^3 + 2xy$.

    22.10 $u(x, t) = t^{-\frac{x}{2}} e^{-\frac{x^2}{4t}}$.

    22.11 $u = x^{\frac{y}{z}}$.

    22.12 $u = y^{xz} + z\sqrt{x}\cos(y) + \text{arctg}(x^2)\ln(y + z)$.

23. Se $z = \sqrt{x^2 + y^2}$ mostre que $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = z$.

24. Calcule as derivadas parciais indicadas:

24.1 $f(x,y) = \ln\left(x + \sqrt{x^2+y^2}\right)$; $f'_x(3,4)$.

24.2 $f(x,y,z) = \dfrac{y}{x+y+z}$; $f'_y(2,1,-1)$.

24.3 $f(x,y,z) = \sqrt{\operatorname{sen}^2(x) + \operatorname{sen}^2(y) + \operatorname{sen}^2(z)}$; $f'_z(0,0,\pi/4)$.

25. Calcule todas as derivadas de 2^a ordem da função,
$v = x^2 + \dfrac{e^{zy}}{z} + y^2$.

26. Calcule as derivadas parciais indicadas:

26.1 $z = \operatorname{sen}(xy)$; $\dfrac{\partial^2 z}{\partial x \partial y}$.

26.2 $z = \ln(e^x + e^y)$; $\dfrac{\partial^2 z}{\partial x^2}$ e $\dfrac{\partial^2 z}{\partial y \partial x}$.

26.3 $z = e^x \ln(y) + \operatorname{sen}(y)\ln(x)$; $\dfrac{\partial^2 z}{\partial y^2}$ e $\dfrac{\partial^3 z}{\partial x^2 \partial y}$.

26.4 $u = e^{x^2+y^2+z^2}$; $\dfrac{\partial^3 u}{\partial x \partial y^2}$.

27. Considere a função definida por $f(x,y,z) = x^2 e^{yz} + y\ln(z)$, determine $\dfrac{\partial^2 f}{\partial x^2}$ e $\dfrac{\partial^2 f}{\partial y \partial x}$.

28. Se $z = \dfrac{xy}{x-y}$, mostre que $x^2\dfrac{\partial^2 z}{\partial x^2} + 2xy\dfrac{\partial^2 z}{\partial x \partial y} + y^2\dfrac{\partial^2 z}{\partial y^2} = 0$.

29. Considere a função $h(x,y) = (1-2x)^2 e^{x^2y^3}$. Calcule $\dfrac{\partial^2 h}{\partial y \partial x}|_{(1,1)}$.

30. Considere a função $h(x,y) = \dfrac{x^y}{y} + 3y\sqrt{x}$. Calcule $\dfrac{\partial^2 h}{\partial x \partial y}|_{(1,1)}$.

31. Considere a função $h(x,y,z) = x^{z+y^2} + y\ln(x^y)$.

31.1 Calcule $\dfrac{\partial^2 h}{\partial z \partial x}|_{(1,1,1)}$.

31.2 Prove que $(\ln(x))\dfrac{\partial^2 h}{\partial z \partial y} = \dfrac{\partial^3 h}{\partial y \partial z^2}$.

32. Calcule as derivadas parciais indicadas:

32.1 $f(x,y) = y^3 x^{\cos(x)}$; $\quad \dfrac{\partial^2 f}{\partial x \partial y}|_{(\pi,1)}$.

32.2 $g(x,y,z) = e^{y^2} x^{\mathrm{tg}(z)}$; $\quad \dfrac{\partial^3 g}{\partial z \partial x \partial y}$.

32.3 $g(x,y,z) = y^2\sqrt{z}\ln(2x-y)$; $\quad \dfrac{\partial^3 g}{\partial y \partial z \partial x}$.

32.4 $f(x,y) = \ln^2\left(\dfrac{y}{x}\right)$; $\quad \dfrac{\partial^3 f}{\partial x \partial y^2}$.

32.5 $f(x,y,z) = xe^{xy} + \mathrm{sen}\left(z - x^3y^2\right)$; $\quad \dfrac{\partial^3 f}{\partial x \partial z^2}$.

32.6 $u = y\ln\left(\dfrac{x+2}{y+3}\right) - 2^{1-x+3y}$; $\quad \dfrac{\partial^3 u}{\partial x^2 \partial y}$.

DIFERENCIAL
CÁLCULO DE VALORES APROXIMADOS

33. Determine a expressão do diferencial das funções seguintes:

33.1 $f(x) = \dfrac{2x^2 - 1}{x\sqrt{1+x^2}}$. 33.2 $f(x) = \dfrac{\sec(x)}{1+\operatorname{tg}(x)}$.

34. Calcule o diferencial das funções seguintes, nos pontos apresentados:

34.1 $y = x + \operatorname{cotg}(x)$ no ponto $x = \frac{\pi}{6}$.

34.2 $y = \big(1 + \operatorname{arcsen}(x^2 - 1)\big)^3$ no ponto $x = 1$.

35. Aplicando o conceito de diferencial, determine o valor aproximado de $f(-0.03)$ sendo $f(x) = \operatorname{tg}(\sqrt{x + \pi^2})$.

36. Aplicando o conceito de diferencial, determine um valor aproximado dos valores seguintes:

36.1 $(1.999)^4$ 36.2 $e^{-0.015}$ 36.3 $\sqrt[3]{1001}$

37. Determine os diferenciais das funções seguintes:

37.1 $z = x^3 \ln(y^2)$.

37.2 $z = \ln(x^2 + y^2) + x\operatorname{tg}(y)$ no ponto $(0, \pi/4)$.

37.3 $f(x, y, z) = \ln(xy + z)^y$ no ponto $(1, 2, 2)$.

37.4 $z = 2^{\ln(x)^y} - x^y e^{x^2}$ no ponto $(1, 1)$.

38. Considere a função $f(x,y,z) = e^{x^2y}\text{arctg}(zx) + \ln(z^y)$.

    38.1 Calcule $df(1,0,1)$.

    38.2 Prove que $x\dfrac{\partial f}{\partial x} - z\dfrac{\partial f}{\partial z} - \dfrac{2y}{x^2}\dfrac{\partial^2 f}{\partial y^2} = -y$.

39. Aplicando o conceito de diferencial total, determine um valor aproximado de $h(-0.98, 2.01)$, sendo
$h(x,y) = \ln\left(e^2 - x^2 + \sqrt{9-y^3}\right)$.

40. Considere a função $f(x,y) = (xy+7)^{\frac{y}{3}}$. Aplicando o conceito de diferencial total, calcule o valor aproximado de $f(0.99, 1.01)$.

41. Aplicando o conceito de diferencial total, calcule um valor aproximado das expressões seguintes:

    41.1 $\sqrt{25.1 - 15.8}$

    41.2 $\log_2\left(5 - \dfrac{\sqrt{0.98}}{1.01}\right)$

42. Considere a função $h(x,y,z) = x^{2x+y^2} + y\ln\left(\dfrac{z}{x}\right)$.

    42.1 Mostre que $\dfrac{\partial^3 h}{\partial y \partial z^2} + \left(\dfrac{\partial^2 h}{\partial z \partial y}\right)^2 = 0$.

    42.2 Aplicando o conceito de diferencial total, calcule o valor aproximado de $h(0.99, 0.01, 1.98)$.

DERIVADAS DE FUNÇÕES COMPOSTAS

43. Calcule $\dfrac{dz}{dt}$ se $z = \dfrac{x}{y}$ com $x = e^t$ e $y = \ln(t)$.

44. Calcule $\dfrac{du}{dt}$ se $u = xyz$ com $x = t^2+1$, $y = \ln(t)$ e $z = \operatorname{tg}(t)$.

45. Calcule $\dfrac{\partial u}{\partial x}$ e $\dfrac{du}{dx}$ se $u = \operatorname{arcsen}\left(\dfrac{x}{z}\right)$ e $z = \sqrt{x^2+1}$.

46. Se $x = t\cos(t)$, $y = t\operatorname{sen}(t)$ e $z = e^{xy^2}$. Calcular $\dfrac{dz}{dt}$ para $t = \frac{\pi}{2}$.

47. Seja $w = x^2y + 2t^2z$ em que $x = u^2 - v^2$, $y = v^2 - 2s$, $z = us$ e $t = s^3$. Determine $\dfrac{\partial w}{\partial u}$, $\dfrac{\partial w}{\partial v}$ e $\dfrac{\partial w}{\partial s}$.

48. Considere a função $f(x,y,z) = xe^{x+y+1} + e^{z^2+y}$.

    48.1 Determine $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial f}{\partial y}$ e $\dfrac{\partial f}{\partial z}$.

    48.2 Calcule $df(1,1,1)$.

    48.3 Sendo $x = 4 - z^3$ e $y = \operatorname{arctg}(z)$, calcule $\dfrac{df}{dz}$ quando $y = 0$.

DERIVADAS DE FUNÇÕES DEFINIDAS IMPLICITAMENTE

49. Calcule $\dfrac{\partial z}{\partial x}$ e $\dfrac{\partial z}{\partial y}$ se $x\cos(y) + y\cos(z) + z\cos(x) = 1$.

50. Calcule dz no ponto $(1,1,2)$ da função $\ln\left(z^x - y^z\right) = x^{z-1}$.

51. Considere a função definida por $e^{z\ln(y)} + \sqrt{xz} = 1$. Mostre que:
$$\frac{x}{z}\frac{\partial z}{\partial x} + \frac{y\ln(y)}{z}\frac{\partial z}{\partial y} = -1.$$

52. Considere a função definida por $x^{yz} + \ln\left(x^2+z\right) + 4xy = 9$ e calcule $\dfrac{\partial z}{\partial x}$ e $\dfrac{\partial z}{\partial y}$ no ponto $(1,2,0)$.

53. Considerando a função $z = y^{xy} + \operatorname{arctg}\left(\ln(x+1)\right)$, calcule $\dfrac{dz}{dt}$ se $t^2 + \operatorname{sen}(x) = 1$ e $y = e^t$ para $t = 1$ e $x \in \left[0, \frac{\pi}{2}\right]$.

SOLUÇÕES DOS EXERCÍCIOS PROPOSTOS

1.1 $D_f = \mathbb{R},\ D'_f =]1, +\infty[$; **1.2** $f^{-1}(x) = 1 - \log_2(x-1)$;
1.3 C.S.= $\left\{-\dfrac{1}{5}\right\}$ **2.1** $D_f = \mathbb{R}^+,\ D'_f = \mathbb{R}$; **2.2** $f^{-1}(x) = \dfrac{2}{3^{2x}}$;
2.3 C.S.= $\{-20, 20\}$; **3.1** $D_h = \mathbb{R}$, $D'_h = [3, +\infty[$;
3.2 $f^{-1}(x) = 1 - \log_2(1-x)$; **3.3** $x = 1$;
4.1 $D_f = \left]\frac{1}{2}, +\infty\right[$, $D'_f = \mathbb{R}$; **4.2** $y - \frac{7}{2} = -7\ln 7(x-4)$;
5. $A = 0$; **6.1** $D_f =]-\infty, 6[$;
6.2 $D_{f^{-1}} = \mathbb{R}$, $D'_{f^{-1}} = D_f =]-\infty, 6[$, $f^{-1}(x) = 6 - 2^{x+2}$
6.3 $x = -1$; **6.4** $y + 1 = \ln 4(x-4)$; **7.1** $D_f = \mathbb{R} \setminus \{0\}$;
7.2 $D_{f^{-1}} =]-\infty, 2[\setminus\{2-e\}$, $D'_{f^{-1}} = D_f = \mathbb{R} \setminus \{0\}$,
$f^{-1}(x) = \dfrac{1}{1 - \ln(2-x)}$; **7.3** $x = 4$; **7.4** $y = x$; **8.1** $D = [-3, 3]$,
$D' = \left[-\frac{11\pi}{6}, \frac{13\pi}{6}\right]$;
8.2 $D = \mathbb{R}$, $D' = \left]-\frac{7\pi}{2}, -\frac{\pi}{2}\right[$;
8.3 $D = \mathbb{R}$, $D' = \left]\frac{\pi}{6}, \pi\right]$; **9.1** $-\dfrac{\pi}{4}$; **9.2** 2π; **9.3** $-\dfrac{\pi}{3}$; **9.4** $\dfrac{3\pi}{2}$;
9.5 $\dfrac{1}{2}$; **9.6** $\dfrac{\sqrt{3}}{3}$; **9.7** $\dfrac{\sqrt{34}}{5}$; **9.8** $\dfrac{12\sqrt{119}}{119}$;
10.1 $x = \dfrac{1+\pi}{3}$; **10.2** $x = \dfrac{\sqrt{3}}{4}$; **10.3** $x = \dfrac{\sqrt{3}}{2}$; **10.4** $x = \frac{1}{2}$;
10.5 $y = 2$; **11.1** $D_f = [0, 1]$, $D' = \left[\dfrac{3\pi}{4}, \dfrac{5\pi}{4}\right]$; **11.2** $x = \frac{1}{2}$;
12.1 $D_g = \mathbb{R}$, $D'_g = \left]0, \frac{\pi}{2}\right[$; **12.2** $g^{-1}(x) = \dfrac{\ln(\operatorname{tg} x) + 1}{2}$;
13. tangente: $y - \dfrac{\pi}{6} = \dfrac{2\sqrt{3}}{3}\left(x - \dfrac{3}{2}\right)$,
normal: $y - \dfrac{\pi}{6} = -\dfrac{\sqrt{3}}{2}\left(x - \dfrac{3}{2}\right)$;
14.1 $D = \left[\frac{1}{e}, e\right]$; $D' = [-2\pi, 0]$; **14.2** $x = \frac{1}{\sqrt{e}}$;
15.1 $D_f = \mathbb{R}$; $D'_f = [-1, 3]$; **15.2** $A = 0$;
15.3 $D_{f^{-1}} = [-1, 3]$; $D'_{f^{-1}} = \left[\frac{\pi}{3}, \frac{4\pi}{3}\right]$; $f^{-1}(x) = \frac{\pi}{3} + \arccos\left(\frac{x-1}{2}\right)$;
16.1 $f^{-1}(x) = \frac{1}{2}\left(\operatorname{sen}(\pi - x) + 1\right)$;

16.2 $S=\left\{\frac{3}{4}\right\}$. **17.1** $D_f=[0,+\infty[=D'_{f^{-1}}$, $D'_f=]-\infty,3]=D_{f^{-1}}$, $f^{-1}(x)=\log_3^2(4-x)$; **17.2** $D_g=[1,5]$, $D'_g=[-\pi,\pi]$; **17.3** $x=3-\sqrt{3}$; **17.4** $y-1=-\frac{3\ln 3}{2}(x-1)$;
18. $f'_x(2,1)=16$, $f'_y(2,1)=8$; **19**. $\frac{\sqrt{5}}{5}$, $\frac{2\sqrt{5}}{5}$, 0;
22.1 $\frac{\partial z}{\partial x}=\cos(y)+y\mathrm{sen}(x)$, $\frac{\partial z}{\partial y}=-x\mathrm{sen}(y)-\cos(x)$;
22.2 $\frac{\partial z}{\partial x}=\ln y$, $\frac{\partial z}{\partial y}=\frac{x}{y}$;
22.3 $\frac{\partial z}{\partial x}=ye^y\cos(xy)$, $\frac{\partial z}{\partial y}=e^y\mathrm{sen}(xy)+xe^y\cos(xy)$;
22.4 $\frac{\partial w}{\partial x}=\frac{2x+yz}{x^2+xyz+y^2z}$, $\frac{\partial w}{\partial y}=\frac{xz+2yz}{x^2+xyz+y^2z}$, $\frac{\partial w}{\partial z}=\frac{xy+y^2}{x^2+xyz+y^2z}$;
22.5 $\frac{\partial f}{\partial x}=-xy\mathrm{sen}(x-2y)+y\cos(x-2y)$, $\frac{\partial f}{\partial y}=x\cos(x-2y)+2xy\mathrm{sen}(x-2y)$; **22.6** $\frac{\partial g}{\partial x}=\frac{(8xy^3-2x^5y)e^y}{16y^4+8x^4y^2+x^8}$, $\frac{\partial g}{\partial y}=\frac{(4x^2y^3-4x^2y^2+x^6y+x^6)e^y}{16y^4+8x^4y^2+x^8}$;
22.7 $\frac{\partial h}{\partial x}=2xye^{x^2y}+\frac{1}{(\sqrt{y}+x)^2+1}$,
$\frac{\partial h}{\partial y}=x^2e^{x^2y}+\frac{1}{2\sqrt{y}((\sqrt{y}+x)^2+1)}$ **22.8** $\frac{\partial w}{\partial x}=\frac{5^{\mathrm{tg}(\ln(x+1))}\sec^2(\ln(x+1))\ln(5)}{x+1}$,
$\frac{\partial w}{\partial y}=y^{y-z}(\frac{y-z}{y}+\ln(y))$, $\frac{\partial w}{\partial z}=-y^{y-z}\ln(y)$; **22.9** $\frac{\partial f}{\partial x}=y^4z^3+2y$,
$\frac{\partial f}{\partial y}=4xy^3z^3+2x$, $\frac{\partial f}{\partial z}=3xy^4z^2$; **22.10** $\frac{\partial u}{\partial x}=-\frac{1}{2}\left(\ln t+\frac{x}{t}\right)t^{-\frac{x}{2}}e^{-\frac{x^2}{4t}}$,
$\frac{\partial u}{\partial t}=-\frac{x}{2t}\left(1-\frac{x}{2t}\right)t^{-\frac{x}{2}}e^{-\frac{x^2}{4t}}$; **22.11** $\frac{\partial u}{\partial x}=\frac{y}{z}x^{\frac{y}{z}-1}$, $\frac{\partial u}{\partial y}=x^{\frac{y}{z}}\frac{\ln(x)}{z}$,
$\frac{\partial u}{\partial z}=-\frac{y}{z^2}x^{\frac{y}{z}}\ln(x)$;
22.12 $\frac{\partial u}{\partial x}=y^{xz}z\ln(y)+\frac{z}{2\sqrt{x}}\cos(y)+\frac{2x}{1+x^4}\ln(y+z)$,
$\frac{\partial u}{\partial y}=xzy^{xz-1}-z\sqrt{x}\mathrm{sen}(y)+\mathrm{arctg}(x^2)\frac{1}{y+z}$,
$\frac{\partial u}{\partial z}=y^{xz}x\ln(y)+\sqrt{x}\cos(y)+\mathrm{arctg}(x^2)\frac{1}{y+z}$;
24.1 $f'_x(3,4)=\frac{1}{5}$;
24.2 $f'_y(2,1,-1)=\frac{1}{4}$; **24.3** $f'_z(0,0,\pi/4)=\frac{\sqrt{2}}{2}$;
25. $\dfrac{\partial^2 v}{\partial x^2}=2$, $\dfrac{\partial^2 v}{\partial y^2}=ze^{yz}+2$, $\dfrac{\partial^2 v}{\partial z^2}=\dfrac{(y^2z^2-2yz+2)e^{yz}}{z^3}$,
$\dfrac{\partial^2 v}{\partial x\partial y}=\dfrac{\partial^2 v}{\partial y\partial x}=0$, $\dfrac{\partial^2 v}{\partial x\partial z}=\dfrac{\partial^2 v}{\partial z\partial x}=0$, $\dfrac{\partial^2 v}{\partial y\partial z}=\dfrac{\partial^2 v}{\partial z\partial y}=ye^{yz}$;
26.1 $\frac{\partial^2 z}{\partial x\partial y}=\cos(xy)-xy\mathrm{sen}(xy)$; **26.2** $\frac{\partial^2 z}{\partial x^2}=\frac{e^{x+y}}{(e^x+e^y)^2}$,
$\frac{\partial^2 z}{\partial y\partial x}=-\frac{e^{x+y}}{(e^x+e^y)^2}$;
26.3 $\frac{\partial^2 z}{\partial y^2}=-\frac{e^x}{y^2}-\ln(x)\mathrm{sen}(y)$, $\frac{\partial^3 z}{\partial x^2\partial y}=\frac{e^x}{y}-\frac{\cos(y)}{x^2}$;

26.4 $\frac{\partial^3 u}{\partial x \partial y^2} = 4xe^{x^2+y^2+z^2}(1+2y^2)$;

27. $\dfrac{\partial^2 f}{\partial x^2} = 2e^{yz}$, $\dfrac{\partial^2 f}{\partial y \partial x} = 2xze^{yz}$;

29. $\dfrac{\partial^2 h}{\partial y \partial x}|_{(1,1)} = 24e$; **30.** $\dfrac{\partial^2 h}{\partial x \partial y}|_{(1,1)} = \dfrac{3}{2}$;

31.1 $\dfrac{\partial^2 h}{\partial z \partial x}|_{(1,1,1)} = 1$; **32.1** $\dfrac{\partial^2 f}{\partial x \partial y}|_{(\pi,1)} = -\dfrac{3}{\pi^2}$;

32.2 $\dfrac{\partial^3 g}{\partial z \partial x \partial y} = 2ye^{y^2}x^{\text{tg}(z)-1}\sec^2(z)(\text{tg}(z)\ln(x)+1)$;

32.3 $\dfrac{\partial^3 g}{\partial y \partial z \partial x} = \dfrac{4xy-y^2}{\sqrt{z}(2x-y)^2}$;

32.4 $\dfrac{\partial^3 f}{\partial x \partial y^2} = \dfrac{2}{xy^2}$;

32.5 $\dfrac{\partial^3 f}{\partial x \partial z^2} = 3x^2y^2\cos(z-x^3y^2)$;

32.6 $\dfrac{\partial^3 u}{\partial x^2 \partial y} = -\dfrac{1}{(x+2)^2} - 3\,(\ln(2))^3\, 2^{1-x+3y}$;

33.1 $df = \frac{4x^2+1}{x^2(1+x^2)^{\frac{3}{2}}}\,dx$; **33.2** $df = \dfrac{\sec(x)(\text{tg}(x)-1)}{(1+\text{tg}(x))^2}\,dx$;

34.1 $dy(\frac{\pi}{6}) = -3\,dx$; **34.2** $dy(1) = 6\,dx$; **35.** $f(-0.03) \approx -\frac{3}{200\pi}$;

36.1 $(1.999)^4 \approx 2^4 - \frac{32}{1000}$; **36.2** $e^{-0.015} \approx 1 - \frac{15}{1000}$;

36.3 $\sqrt[3]{1001} \approx 10 + \frac{1}{300}$; **37.1** $dz = 6x^2\ln(y)\,dx + \frac{2x^3}{y}\,dy$;

37.2 $dz\left(0, \frac{\pi}{4}\right) = 1\,dx + \frac{8}{\pi}\,dy$;

37.3 $df(1,2,2) = dx + (\ln(4) + \frac{1}{2})dy + \frac{1}{2}dz$;

37.4 $dz = (\ln(2) - 3e)dx$;

38.1 $df|_{(1,0,1)} = \frac{1}{2}\,dx + \frac{\pi}{4}\,dy + \frac{1}{2}\,dz$; **39.** $h(-0.98, 2.01) \approx 2 - \frac{1}{50e^2}$;

40. $f(0.99, 1.01) \approx 2 + \frac{\ln(2)}{50}$; **41.1** $\sqrt{25.1 - 15.8} \approx 3 + \frac{1}{20}$;

41.2 $\log_2\left(5 - \dfrac{\sqrt{0.98}}{1.01}\right) \approx 2 + \frac{1}{200\ln(2)}$;

42.2 $h(0.99, 0.01, 1.98) \approx 1 + \frac{\ln(2)-2}{100}$;

43. $\frac{dz}{dt} = \frac{(t\ln t - 1)e^t}{t\ln^2 t}$;

44. $\frac{du}{dt} = 2t\ln(t)\text{tg}(t) + \frac{(t^2+1)\text{tg}(t)}{t} + (t^2+1)\ln(t)\sec^2(t)$;

45. $\frac{\partial u}{\partial x} = \frac{1}{\sqrt{z^2-x^2}}$, $\frac{du}{dx} = \frac{1}{x^2+1}$;
46. $\dfrac{dz}{dt}|_{t=\frac{\pi}{2}} = -\dfrac{\pi^3}{8}$;
47. $\dfrac{\partial w}{\partial u} = 4u(u^2-v^2)(v^2-2s)+2s^7$,
$\dfrac{\partial w}{\partial v} = 2v(u^2-v^2)^2-4v(u^2-v^2)(v^2-2s)$, $\dfrac{\partial w}{\partial s} = 14s^6u-2(u^2-v^2)^2$;
48.1 $\dfrac{\partial f}{\partial x} = xe^{y+x+1}+e^{y+x+1}$,
$\dfrac{\partial f}{\partial y} = e^{z^2+y}+xe^{y+x+1}$, $\dfrac{\partial f}{\partial z} = 2ze^{z^2+y}$;
48.2 $df(1,1,1) = 2e^3\,dx + (e^3+e^2)\,dy + 2e^2\,dz$;
48.3 $\dfrac{df}{dz}|_{z=0} = 4e^5+1$;
49. $\frac{\partial z}{\partial x} = \frac{z\text{sen}(x)-\cos(y)}{\cos(x)-y\,\text{sen}\,z}$, $\frac{\partial z}{\partial y} = \frac{x\text{sen}(y)-\cos(z)}{\cos(x)-y\text{sen}(z)}$;
50. $dz(1,1,2) = (1-2\ln 2)\,dx + 2\,dy$;
52. $\frac{\partial z(1,2,0)}{\partial x} = -10$, $\frac{\partial z(1,2,0)}{\partial y} = -4$; **53**. $\dfrac{dz}{dt}|_{t=1} = -2e-2$.