Boolean algebra – extra exercises

October 2020 (v1.0)

1. Simplify each of following boolean sentences.
   (a) $X \cdot (X \cdot \overline{Y})$
   (b) $(\overline{X} \cdot Y) + X$
   (c) $(X \cdot Y) + (X \cdot \overline{Y})$
   (d) $X \cdot (X + \overline{Y})$
   (e) $\overline{(\overline{X} \cdot Y)}$
   (f) $\overline{X} \cdot (X \cdot Y)$
   (g) $(\overline{X} \cdot Y) + \overline{Y}$
   (h) $(X \cdot Y) + (X \cdot \overline{Y})$
   (i) $\overline{(X \cdot Y) \cdot \overline{(X + Y)}}$
   (j) $\left[X \cdot (\overline{Y} + Z) \cdot \overline{(X \cdot Z)} \right] \cdot \overline{Y}$

Solution:

   (a) $X \cdot \overline{Y}$

   (b) $Y + X$

   (c) X

   (d) X

   (e) $X + \overline{Y}$

   (f) 0

   (g) $\overline{X} + \overline{Y}$

   (h) X

   (i) 1

   (j) $(\overline{X} + Z) \cdot \overline{Y}$

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2. Prove algebraically the following equivalence relations.
   (a) $\overline{[A \cdot (A + B)]} \cdot A = 0$
   (b) $(P \cdot Q) + (P \cdot \overline{Q}) + (\overline{P} \cdot Q) = P + Q$
   (c) $(P \cdot Q) + (\overline{P} \cdot Q) + (\overline{P} \cdot \overline{Q}) = \overline{P} + Q$

3. Assume that the 16-bit variable `control_lights` codes the current status of the lights in the rooms of one floor, such that (i) 0 means lights are off and (ii) 1 means lights are on. Bit 0 holds the status of lights in room B300, bit 1 holds the status in room B301 and so on until bit 15 for room B315. Assume that the initial value of `control_lights` is $A814_{(16)}$

(a) Which rooms have the lights on?

(b) Make sure that rooms B310 to B315 are switched off. What is the resulting value of `control_lights`?

(c) Toggle the light status of rooms B314 and B303. What is the resulting value of `control_lights`?

(d) Turn on the lights of all rooms. What is the resulting value of `control_lights`?