

Licenciatura em Engenharia Informática (LEI)

2024/2025

**Análise Matemática
(AMATA)**

CAPÍTULO 2

Integral Indefinido.

EXERCÍCIOS

INTEGRAIS IMEDIATOS OU QUASE IMEDIATOS

1. Calcule os seguintes integrais:

$$1.1 \int \frac{1}{x^2} dx$$

$$1.2 \int \sqrt[3]{x} dx$$

$$1.3 \int \sqrt{1+4x} dx$$

$$1.4 \int \frac{2x}{\sqrt[3]{1-x^2}} dx$$

$$1.5 \int \frac{\ln(5x)}{x} dx$$

$$1.6 \int \frac{dx}{1+4x}$$

$$1.7 \int \frac{\cos(x)}{1+\sin(x)} dx$$

$$1.8 \int \frac{x}{1+4x^2} dx$$

$$1.9 \int \frac{dx}{1+4x^2}$$

$$1.10 \int (3x+4)^4 dx$$

$$1.11 \int x(1-3x^2)^3 dx$$

$$1.12 \int e^{1-2x} dx$$

$$1.13 \int x^2 5^{x^3} dx$$

$$1.14 \int \frac{dx}{3+4x^2}$$

$$1.15 \int \frac{dx}{\sqrt{9-x^2}}$$

$$1.16 \int \frac{x}{\sqrt{9-x^2}} dx$$

$$1.17 \int \frac{dx}{(5-3x)^4}$$

$$1.18 \int \frac{x}{4+9x^2} dx$$

$$1.19 \int \frac{dx}{4+9x^2}$$

$$1.20 \int \frac{3}{\sqrt{5-x^2}} dx$$

$$1.21 \int 3x \sqrt{(5-x^2)^3} dx$$

$$1.22 \int x \cos(4x^2) dx$$

$$1.23 \int \frac{\operatorname{tg} \sqrt{x}}{\sqrt{x}} dx$$

$$1.24 \int \frac{x^3}{\sqrt{1-9x^4}} dx$$

$$1.25 \int \frac{x}{\sqrt{1-9x^4}} dx$$

$$1.26 \int \frac{e^{2x}}{1-e^{2x}} dx$$

$$1.27 \int \frac{e^x}{1+9e^{2x}} dx$$

$$1.28 \int \frac{e^x}{\sqrt{5-e^{2x}}} dx$$

$$1.29 \int \frac{dx}{x \sqrt[5]{3+2 \ln(x)}}$$

$$1.30 \int \frac{\sqrt[3]{2+\operatorname{tg}(x)}}{\cos^2(x)} dx$$

$$1.31 \int \frac{\operatorname{sen}(2x)}{1+\cos^4(x)} dx$$

$$1.32 \int \frac{1}{x(1+\ln^2(x))} dx$$

$$1.33 \int \operatorname{tg}\left(\frac{x}{2}\right) dx$$

$$1.34 \int \frac{5^{\sqrt{x}}}{\sqrt{x}} dx$$

$$1.35 \int \frac{1}{x \ln^3(x)} dx$$

$$1.36 \int \frac{\sec(2x) \operatorname{tg}(2x)}{(1+\sec(2x))^{\frac{3}{2}}} dx$$

$$1.37 \int \frac{\operatorname{sen}(2x)}{\sqrt{5-4 \operatorname{sen}^4(x)}} dx$$

$$1.38 \int \frac{\operatorname{tg}(x)}{1+\ln^2(\cos^2(x))} dx$$

2. Determinar uma função real de variável real $y = f(x)$, cuja derivada seja $f'(x) = -2x$ e cujo gráfico passe no ponto $(-1, 1)$.

MÉTODOS DE INTEGRAÇÃO POR DECOMPOSIÇÃO

3. Calcule os seguintes integrais:

$$3.1 \int \sin^2(2x) dx$$

$$3.2 \int \sin(2x) \cotg(x) dx$$

$$3.3 \int \frac{\sqrt{x} + \ln(x)}{x} dx$$

$$3.4 \int \frac{2x^3 + x}{4 + x^4} dx$$

$$3.5 \int \frac{x(2 + x^2\sqrt{9 + x^4})}{9 + x^4} dx$$

$$3.6 \int \frac{1 - \cotg(x)}{\sin^2(x)\sqrt{1 - \cotg^2(x)}} dx$$

$$3.7 \int \frac{\sin(x)\cos^3(x) + \sin(2x)}{\sqrt{1 - \cos^4(x)}} dx$$

$$3.8 \int \frac{6 + \ln(2x)}{x\sqrt{1 - \ln^2(2x)}} dx$$

$$3.9 \int \frac{x + x\sin(3 + e^{-x^2})}{e^{x^2}} dx$$

$$3.10 \int \frac{x^3(1 - \sqrt[3]{4 + x^4})}{4 + x^4} dx$$

$$3.11 \int \left(\frac{1}{\sqrt[3]{\sin(x)}} + \frac{\sin(x)}{\sqrt{1 - \cos^4(x)}} \right) \cos(x) dx$$

$$3.12 \int \frac{\sec^2(3x + 1) + \sin(3x + 1)}{\tg(3x + 1)} dx$$

MÉTODOS DE INTEGRAÇÃO POR PARTES

4. Calcule os seguintes integrais:

$$4.1 \int x e^{-3x} dx$$

$$4.2 \int x \operatorname{sen}(2x) dx$$

$$4.3 \int \ln(x^2 + 1) dx$$

$$4.4 \int (x^2 + 1)e^{-3x} dx$$

$$4.5 \int x^3 \sec^2(x^2 + 3) dx$$

$$4.6 \int x \ln^2(2x) dx$$

$$4.7 \int e^x \ln^2(e^x) dx$$

$$4.8 \int \sqrt[3]{x+5} \ln(x+5) dx$$

$$4.9 \int \frac{\ln(1+x^2)}{x^2} dx$$

$$4.10 \int \cos(2x) \ln(\operatorname{sen}(x)) dx$$

$$4.11 \int x^5 \operatorname{sen}(1-x^3) dx$$

$$4.12 \int \ln(x + \sqrt{1+x^2}) dx$$

$$4.13 \int \frac{x + e^{\sqrt{1+x}}}{\sqrt{1+x}} dx$$

$$4.14 \int (x^3 + 1)\cos(x) dx$$

$$4.15 \int e^x \operatorname{arctg}(e^x) dx$$

$$4.16 \int x \operatorname{arcsen}(x^2) dx$$

$$4.17 \int \operatorname{arctg}(2x) dx$$

MÉTODOS DE INTEGRAÇÃO POR SUBSTITUIÇÃO

5. Calcule os seguintes integrais, fazendo as substituições sugeridas:

$$5.1 \int \frac{dx}{\sqrt[4]{x} + \sqrt{x}} \quad (\sqrt[4]{x} = t)$$

$$5.2 \int \frac{x^2}{\sqrt{x-2}} dx \quad (\sqrt{x-2} = t)$$

$$5.3 \int \frac{\ln(x)}{\sqrt{x-1}} dx \quad (\sqrt{x-1} = t)$$

$$5.4 \int \frac{5^x(1+5^x)}{\sqrt{1-5^{2x}}} dx \quad (5^x = t)$$

$$5.5 \int \frac{1}{t\sqrt{t} + \sqrt{t}} dt \quad (\sqrt{t} = x)$$

$$5.6 \int \sqrt{x} e^{\sqrt{x}} dx \quad (\sqrt{x} = t)$$

$$5.7 \int \frac{\operatorname{sen}^3(\ln(x))}{x} dx \quad (\ln(x) = t)$$

$$5.8 \int \frac{1 + \sqrt[4]{x}}{\sqrt{x}(4 + \sqrt{x})} dx \quad (t = \sqrt[4]{x})$$

6. Calcule os seguintes integrais:

$$6.1 \int \frac{x \operatorname{arctg}(x)}{\sqrt{1+x^2}} dx \quad (x = \operatorname{tg}(t)), 0 < t < \frac{\pi}{2}$$

$$6.2 \int \frac{x}{\sqrt{4x^2+9}} dx \quad (2x = 3 \operatorname{tg}(t))$$

$$6.3 \int \frac{1}{x^3 \sqrt{x^2-25}} dx \quad (x = 5 \sec(t))$$

$$6.4 \int \frac{dx}{x \sqrt{3-x^2}} \quad (x = \sqrt{3} \operatorname{sen}(t))$$

$$6.5 \int \frac{x^2}{\sqrt{4-2x^2}} dx \quad (\sqrt{2}x = 2 \operatorname{sen}(t))$$

$$6.6 \int \frac{dx}{(x^2-1)^{\frac{3}{2}}} \quad (x = \sec(t))$$

$$6.7 \int \frac{x^3}{\sqrt{2-x^2}} dx \quad (x = \sqrt{2} \operatorname{sen}(t))$$

MISCELÂNEA DE INTEGRAIS

7. Sabendo que

$$f'(x) = \frac{\ln^2(x)}{x} 5^{\ln^3(x)}$$

determine a função f cujo gráfico passa no ponto $(1, 0)$.

8. Sabendo que

$$f'(x) = \frac{\ln(x)}{x} (1 + \sqrt{x^3})$$

determine a função f cujo gráfico passa no ponto $(1, 0)$.

9. Considere a função $f(x) = \frac{g(x)}{\ln(x^2 + 1)}$, onde g é uma função real de variável real.

9.1 Determine a função g de modo a que se verifique

$$\int f(x) dx = \ln(\ln^2(x^2 + 1)) + C, C \in \mathbb{R}$$

9.2 Fazendo $g(x) = x^{-2} \ln^2(x^2 + 1)$, resolva o seguinte integral $\int f(x) dx$, usando integração por partes.

10. Considere a função

$$f(x) = \frac{x^3}{\sqrt{5 - x^4}}$$

10.1 Determine a primitiva de f que passa no ponto $(1, \frac{1}{2})$.

10.2 Resolva o seguinte integral $\int x^4 f(x) dx$.

11. Considere a função

$$f(x) = \sec^2(x^4 - 1)$$

11.1 Determine a primitiva de $g(x) = x^3 f(x)$ que passa no ponto $(1, 1)$.

11.2 Usando integração por partes, resolva o seguinte integral $\int x^7 f(x) dx$.

12. Resolva o integral $\int \frac{\sqrt[4]{x} + 1}{\sqrt[4]{x^3} \sqrt{1 - \sqrt{x}}} dx$ fazendo a substituição $t = \sqrt[4]{x}$.

13. Usando integração por partes, resolva o seguinte integral $\int \frac{x^2}{\sqrt{1+x}} dx$.

14. Recorrendo à integração por partes, resolva o seguinte integral $\int x^5 \sin\left(-\frac{x^2}{2}\right) dx$.

15. Resolva o integral, $\int \frac{dx}{x^3 \sqrt{x^2 - 1}}$ fazendo a substituição $x = \sec(t)$.

16. Recorrendo à integração por partes, resolva o seguinte integral $\int (x-1) \ln(x-1) dx$.

17. Resolva o integral, $\int \frac{dx}{e^x + 1}$ fazendo a substituição $x = -\ln(t)$.
18. Aplicando integração por partes, resolva o seguinte integral $\int x^3 \sqrt{2x^2 + 1} dx$.
19. Resolva o integral $\int \frac{1}{x^2} \cos^2\left(\frac{3}{x}\right) dx$, fazendo a substituição $t = \frac{3}{x}$.
20. Usando integração por partes, resolva o seguinte integral $\int \frac{\ln^2(x)}{x^3} dx$.
21. Resolva o integral $\int \frac{dx}{e^x + e^{-x}}$, fazendo a substituição $e^x = t$.
22. Usando integração por partes, resolva o seguinte integral $\int (x+1)^2 \ln\left(\frac{1}{x+1}\right) dx$.
23. Resolva o integral $\int x^{-\frac{1}{2}} \sin^2(\sqrt{x} - 1) dx$, fazendo a substituição $u = \sqrt{x} - 1$.

24. Usando integração por partes, resolva o seguinte integral

$$\int \ln(9 + x^2) dx.$$

25. Resolva o integral $\int \frac{2^x(2 + 2^x)}{\sqrt{4 - 4^x}} dx$, fazendo a substituição $2^x = t$.

26. Resolva o integral $\int \frac{\sqrt{x}}{(1 - x)^{3/2}} dx$, fazendo a substituição $x = (\cos(t))^2$, $t \in \left[0, \frac{\pi}{2}\right]$.

SOLUÇÕES DOS EXERCÍCIOS PROPOSTOS

($C \in \mathbb{R}$)

- 1.1** $-\frac{1}{x} + C$; **1.2** $\frac{3}{4}x^{4/3} + C$; **1.3** $\frac{1}{6}(1+4x)^{3/2} + C$;
1.4 $-\frac{3}{2}(1-x^2)^{2/3} + C$; **1.5** $\frac{1}{2}\ln^2(5x) + C$;
1.6 $\frac{1}{4}\ln|1+4x| + C$; **1.7** $\ln|1+\operatorname{sen}(x)| + C$; **1.8** $\frac{1}{8}\ln(1+4x^2) + C$;
1.9 $\frac{1}{2}\operatorname{arctg}(2x) + C$; **1.10** $\frac{1}{15}(3x+4)^5 + C$; **1.11** $-\frac{1}{24}(1-3x^2)^4 + C$;
1.12 $-\frac{1}{2}e^{1-2x} + C$; **1.13** $\frac{5x^3}{3\ln 5} + C$; **1.14** $\frac{\sqrt{3}}{6}\operatorname{arctg}\left(\frac{2x}{\sqrt{3}}\right) + C$;
1.15 $\operatorname{arcsen}\left(\frac{x}{3}\right) + C$; **1.16** $-\sqrt{9-x^2} + C$;
1.17 $\frac{1}{9(5-3x)^3} + C$; **1.18** $\frac{1}{18}\ln(4+9x^2) + C$;
1.19 $\frac{1}{6}\operatorname{arctg}\left(\frac{3x}{2}\right) + C$; **1.20** $3\operatorname{arcsen}\left(\frac{x}{\sqrt{5}}\right) + C$;
1.21 $-\frac{3}{5}(5-x^2)^{5/2} + C$; **1.22** $\frac{1}{8}\operatorname{sen}(4x^2) + C$;
1.23 $-2\ln|\cos(\sqrt{x})| + C$; **1.24** $-\frac{1}{18}\sqrt{1-9x^4} + C$;
1.25 $\frac{1}{6}\operatorname{arcsen}(3x^2) + C$; **1.26** $-\frac{1}{2}\ln|1-e^{2x}| + C$;
1.27 $\frac{1}{3}\operatorname{arctg}(3e^x) + C$; **1.28** $\operatorname{arcsen}\left(\frac{e^x}{\sqrt{5}}\right) + C$;
1.29 $\frac{5}{8}(3+2\ln x)^{4/5} + C$; **1.30** $\frac{3}{4}\sqrt[3]{(2+\operatorname{tg} x)^4} + C$;
1.31 $-\operatorname{arctg}(\cos^2(x)) + C$; **1.32** $\operatorname{arctg}(\ln(x)) + C$;
1.33 $-2\ln\left|\cos\left(\frac{x}{2}\right)\right| + C$; **1.34** $\frac{2}{\ln(5)}5^{\sqrt{x}} + C$; **1.35** $-\frac{1}{2\ln^2(x)} + C$;
1.36 $-\frac{1}{\sqrt{1+\sec(2x)}} + C$; **1.37** $\frac{1}{2}\operatorname{arcsen}\left(\frac{2\operatorname{sen}^2 x}{\sqrt{5}}\right) + C$;
1.38 $-\frac{1}{2}\operatorname{arctg}(2\ln(\cos(x))) + C$; **2.** $f(x) = -x^2 + 2$;
3.1 $\frac{x}{2} - \frac{1}{8}\operatorname{sen}(4x) + C$; **3.2** $x + \frac{1}{2}\operatorname{sen}(2x) + C$;
3.3 $2\sqrt{x} + \frac{\ln^2(x)}{2} + C$; **3.4** $\frac{1}{2}\ln(4+x^4) + \frac{1}{4}\operatorname{arctg}\left(\frac{x^2}{2}\right) + C$;
3.5 $\frac{1}{3}\operatorname{arctg}\left(\frac{x^2}{3}\right) + \frac{1}{2}\sqrt{9+x^4} + C$;
3.6 $-\operatorname{arcsen}(\cotg(x)) - \sqrt{1-\cotg^2(x)} + C$;
3.7 $\frac{1}{2}\sqrt{1-\cos^4 x} - \operatorname{arcsen}(\cos^2 x) + C$;
3.8 $6\operatorname{arcsen}(\ln(2x)) - \sqrt{1-\ln^2(2x)} + C$;

- 3.9** $\frac{1}{2} \left(\cos \left(3 + e^{-x^2} \right) - e^{-x^2} \right) + C;$
3.10 $\frac{1}{4} \ln(4 + x^4) - \frac{3}{4} \sqrt[3]{4 + x^4} + C;$
3.11 $\frac{3}{2} (\operatorname{sen} x)^{2/3} - \frac{1}{2} \operatorname{arcsen} (\cos^2 x) + C;$
3.12 $\frac{1}{3} \ln |\operatorname{tg}(3x + 1)| + \frac{1}{3} \operatorname{sen}(3x + 1) + C;$ **4.1** $-\frac{3x+1}{9} e^{-3x} + C;$
4.2 $\frac{\operatorname{sen}(2x)}{4} - \frac{x \cos(2x)}{2} + C;$
4.3 $x \ln(x^2 + 1) - 2(x - \operatorname{arctg}(x)) + C;$
4.4 $-\frac{1}{27} (9x^2 + 6x + 11) e^{-3x} + C;$
4.5 $\frac{1}{2} x^2 \operatorname{tg}(x^2 + 3) + \frac{1}{2} \ln |\cos(x^2 + 3)| + C;$
4.6 $\frac{x^2}{4} (2 \ln^2(2x) - 2 \ln(2x) + 1) + C;$ **4.7** $(x^2 - 2x + 2) e^x + C;$
4.8 $\frac{3}{4} (x+5)^{4/3} \left(\ln(x+5) - \frac{3}{4} \right) + C;$ **4.9** $-\frac{\ln(1+x^2)}{x} + 2 \operatorname{arctg}(x) + C;$
4.10 $\frac{\operatorname{sen}(2x) \ln(\operatorname{sen}(x))}{2} - \frac{x}{2} - \frac{\operatorname{sen}(2x)}{4} + C;$
4.11 $\frac{1}{3} (x^3 \cos(1 - x^3) + \operatorname{sen}(1 - x^3)) + C;$
4.12 $x \ln(x + \sqrt{1 + x^2}) - \sqrt{1 + x^2} + C;$
4.13 $2x\sqrt{1+x} - \frac{4}{3}(1+x)^{\frac{3}{2}} + 2e^{\sqrt{1+x}} + C;$
4.14 $(x^3 - 6x + 1) \operatorname{sen}(x) + (3x^2 - 6) \cos(x) + C;$
4.15 $e^x \operatorname{arctg}(e^x) - \frac{1}{2} \ln(1 + e^{2x}) + C;$
4.16 $\frac{x^2}{2} \operatorname{arcsen}(x^2) + \frac{1}{2} \sqrt{1 - x^4} + C;$
4.17 $x \operatorname{arctg}(2x) - \frac{1}{4} \ln(1 + 4x^2) + C;$
5.1 $2\sqrt{x} - 4\sqrt[4]{x} + 4 \ln(\sqrt[4]{x} + 1) + C;$
5.2 $\frac{2}{5} (x-2)^{5/2} + \frac{8}{3} (x-2)^{3/2} + 8(x-2)^{1/2} + C;$
5.3 $\sqrt{x-1} (\ln x^2 - 4) + 4 \operatorname{arctg} \sqrt{x-1} + C;$
5.4 $\frac{1}{\ln 5} (\operatorname{arcsen}(5^x) - \sqrt{1 - 5^{2x}}) + C;$ **5.5** $2 \operatorname{arctg}(\sqrt{t}) + C;$
5.6 $2e^{\sqrt{x}} (x - 2\sqrt{x} + 2) + C.$ **5.7** $\frac{\cos^3(\ln(x))}{3} - \cos(\ln(x)) + C;$
5.8 $4\sqrt[4]{x} + 2 \ln(\sqrt{x} + 4) - 8 \operatorname{arctg} \left(\frac{\sqrt[4]{x}}{2} \right) + C;$
6.1 $\sqrt{1 + x^2} \operatorname{arctg}(x) - \ln |x + \sqrt{1 + x^2}| + C;$
6.2 $\frac{1}{4} \sqrt{4x^2 + 9} + C;$
6.3 $\frac{1}{250} \left(\arccos \left(\frac{5}{x} \right) + \frac{5\sqrt{x^2-25}}{x^2} \right) + C;$
6.4 $\frac{1}{\sqrt{3}} \ln \left| \frac{\sqrt{3}-\sqrt{3-x^2}}{x} \right| + C;$
6.5 $\frac{\sqrt{2}}{2} \left(\operatorname{arcsen} \left(\frac{\sqrt{2}}{2} x \right) - \frac{x}{2} \sqrt{2 - x^2} \right) + C;$

- 6.6** $-\frac{x}{\sqrt{x^2-1}} + C$; **6.7** $-\frac{\sqrt{2-x^2}}{3}(4+x^2) + C$;
7. $f(x) = \frac{1}{3\ln(5)} \left(5^{\ln^3(x)} - 1 \right)$;
8. $f(x) = \frac{\ln^2(x)}{2} + \frac{2}{3}x^{3/2}\ln(x) - \frac{4}{9}x^{3/2} + \frac{4}{9}$; **9.1** $g(x) = \frac{4x}{x^2+1}$;
9.2 $-\frac{1}{x}\ln(x^2+1) + 2\arctg(x) + C$;
10.1 $F(x) = -\frac{1}{2}\sqrt{5-x^4} + \frac{3}{2}$; **10.2** $-\frac{x^4}{2}\sqrt{5-x^4} - \frac{1}{3}(5-x^4)^{3/2} + C$;
11.1 $G(x) = \frac{1}{4}\text{tg}(x^4-1) + 1$;
11.2 $\frac{x^4}{4}\text{tg}(x^4-1) + \frac{1}{4}\ln|\cos(x^4-1)| + C$;
12. $-4\sqrt{1-\sqrt{x}} + 4\arcsen(\sqrt[4]{x}) + C$;
13. $2x^2\sqrt{1+x} - \frac{8x}{3}(1+x)^{3/2} + \frac{16}{15}(1+x)^{5/2} + C$;
14. $x^4\cos\left(-\frac{x^2}{2}\right) + 4x^2\text{sen}\left(-\frac{x^2}{2}\right) - 8\cos\left(-\frac{x^2}{2}\right) + C$;
15. $\frac{\sqrt{x^2-1}}{2x^2} + \frac{1}{2}\arccos\left(\frac{1}{x}\right) + C$; **16.** $\frac{(x-1)^2}{2}\ln(x-1) - \frac{1}{4}(x-1)^2 + C$;
17. $x - \ln|e^x + 1| + C$; **18.** $\frac{x^2}{6}(2x^2+1)^{3/2} - \frac{1}{30}(2x^2+1)^{5/2} + C$;
19. $-\frac{1}{2x} - \frac{1}{12}\text{sen}\left(\frac{6}{x}\right) + C$; **20.** $-\frac{(\ln(x))^2}{2x^2} - \frac{\ln(x)}{2x^2} - \frac{1}{4x^2} + C$;
21. $\arctg(e^x) + C$; **22.** $\frac{(x+1)^3}{3}\ln\left(\frac{1}{x+1}\right) + \frac{1}{9}(x+1)^3 + C$;
23. $(\sqrt{x}-1) - \frac{1}{2}\text{sen}(2\sqrt{x}-2) + C$;
24. $x\ln(9+x^2) - 2x + 6\arctg\left(\frac{x}{3}\right) + C$;
25. $\frac{2}{\ln 2}\arcsen(2^{x-1}) - \frac{1}{\ln(2)}\sqrt{4-4^x} + C$;
26. $\frac{2\sqrt{x}}{\sqrt{1-x}} + 2\arccos(\sqrt{x}) + C$.