Modelling the Ship

Forces & Motion

To describe the motion of the ship, we define a Cartesian coordinate system with origin at its centre of mass (⊕). This coordinate system is shown in Figure 3. It is referred to as the *ego* coordinate system as it aligns to a first person view of the ship onto the world. The term ego is commonly used in vehicle dynamics, and we will adopt this term to differentiate the ego and fixed world coordinate systems.

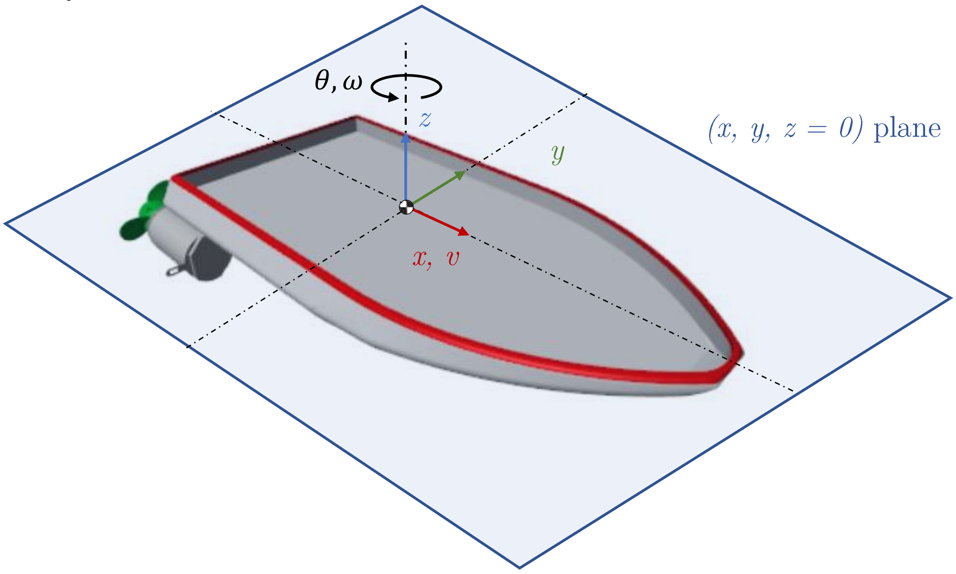

Figure 3: A right-handed coordinate system is used to describe the motion of the ship.

The ship moves forward along the x-axis at a linear velocity, v, and rotates about the z-axis by an angle θ, referred to as the *yaw* angle. The angular velocity, ω, at which the ship rotates about the z-axis is referred to as the yaw rate.

The **kinematics of the ship** are described by the unicycle model, discussed in week 23. This model is given by

$$v_x = v\,\cos\theta \tag{1}$$

$$v_y = v\,\sin\theta \tag{2}$$

$$\frac{d\theta}{dt} = \omega \tag{3}$$

The linear and angular velocities of the ship can be related to the forces and moments acting on the ship to describe the **dynamics** of the system. The forces and moments acting on the ship are illustrated in Figure 4 in the form of a free body diagram.

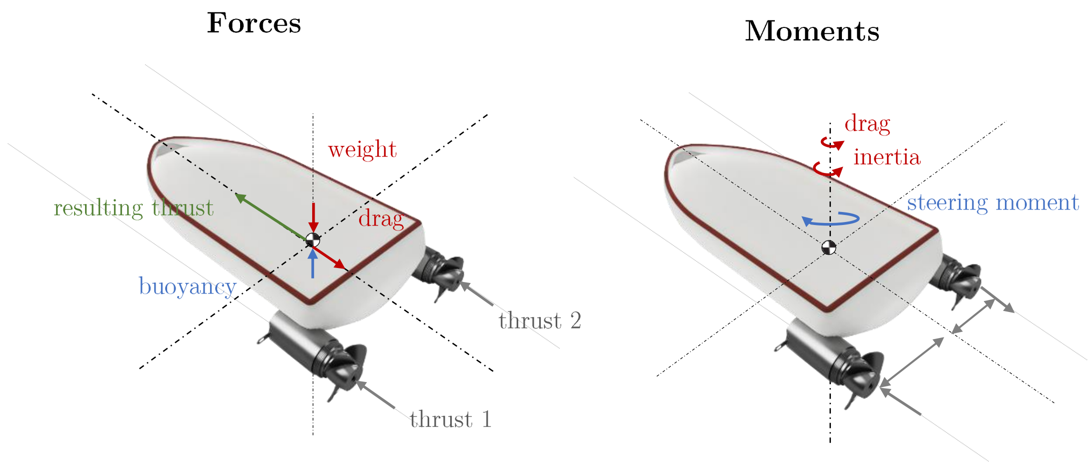

Figure 4: Free body diagrams of forces and moments acting on the ship.

Vertical forces: The *weight* of the ship, W, is given by the product of the mass, m, and the gravitational acceleration, g,

$$W = m\,g. \tag{4}$$

A *buoyancy force* opposes the weight and allows the ship to float. The buoyancy force B is given by:

$$B = \rho\,V\,g, \tag{5}$$

where $\rho = 1$ g/cm^3 is the density of the water displaced and V is the submerged volume of the ship.

The ship will also experience the *vertical component of the propeller thrust* if these are inclined upwards and a *lift force* as it accelerates through the water, given by:

$$F_L = \frac{1}{2} C_L \rho A_s v\,|v|, \tag{6}$$

The lift force is closely coupled with the drag force, which will be discussed in further detail soon.

Vertical forces are neglected in this analysis. The simulation assumes that vertical forces are in balance and constrains the motion of the ship to the (x,y)-plane intersecting $z = 0$ (see figure 3 for reference)**.**

Horizontal forces: The ship is propelled by two submerged, rotating propellers. The rotation of the propeller sheds vertical structures away from the ship and causes an equal and opposite thrust force (see Figure 5). The thrust of the propellers is assumed to act along the axis of rotation of the propeller.

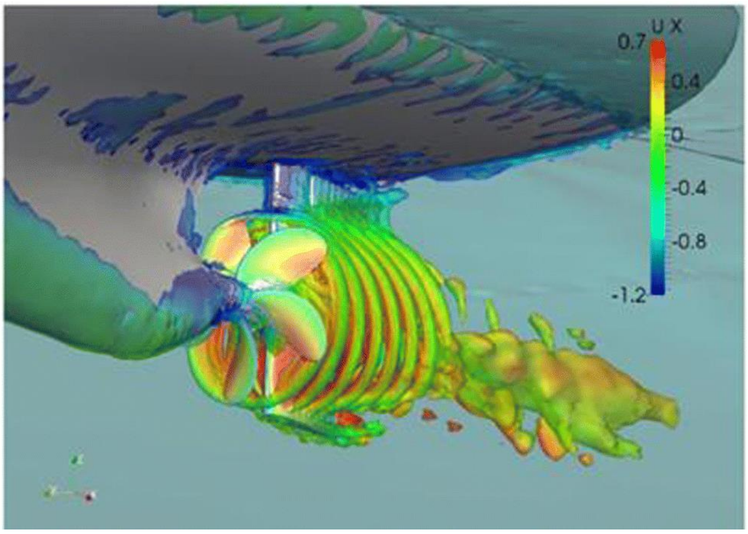

Figure 5: The image shows a computational simulation of the flow around the propeller, that demonstrates the vortical structures that are shed from it.
Source: [Application Progress of Computational Fluid Dynamic Techniques for Complex Viscous Flows in Ship and Ocean Engineering]

Forces act through the centre of mass of the ship. A resulting thrust can be computed by adding the propeller thrust vectors (Figure 4).

The resulting thrust is opposed by a drag force. Drag forces are the result of the viscous fluid forces that act on the ship. Using dimensional analysis, a steady drag force can be deduced:

$$F_d = \frac{1}{2} C_d \rho A_s v |v|, \tag{7}$$

where A_s is the projected area of the ship and C_d is a dimensionless drag coefficient. Likewise, the rotational torque drag on the ship can be defined as:

$$Q_d = \frac{1}{2} C_\omega \rho A_s^4 P_s \omega |\omega|, \tag{8}$$

where P_s is the waterline perimeter of the ship and C_ω is a dimensionless rotational drag coefficient. The drag coefficients C_d and C_ω depend on flow conditions and geometry of the ship, and techniques to define these coefficients exceed the scope of the course.

For reference, there are two types of drag forces that affect the aerodynamics of the ship:

- **Skin friction drag** occurs due to the deceleration of the fluid in the proximity of the body, in a region known as a *boundary layer*. Skin friction drag is shear force that acts tangential to the surface. The force is reduced for smoother surfaces.
- **Form drag** arises due to the separation of the boundary layer from the body due to adverse pressure gradients. The separation of the flow creates a pressure difference between the front of the ship, known as the *stagnation point*, and a turbulent region formed in the lee of the ship, known as the *wake* (see Figure 6). Form drag can be reduced by limiting flow separation, and as a result wake size. Separation is less prevalent in thin, *streamlined* structures compared to wide, *bluff* bodies.

In the simulation, the drag force is assumed to be proportional to velocity $(F \propto v)$ of the ship, which approximates the real physical behaviour, in which drag is proportional to velocity squared $(F \propto v^2)$.

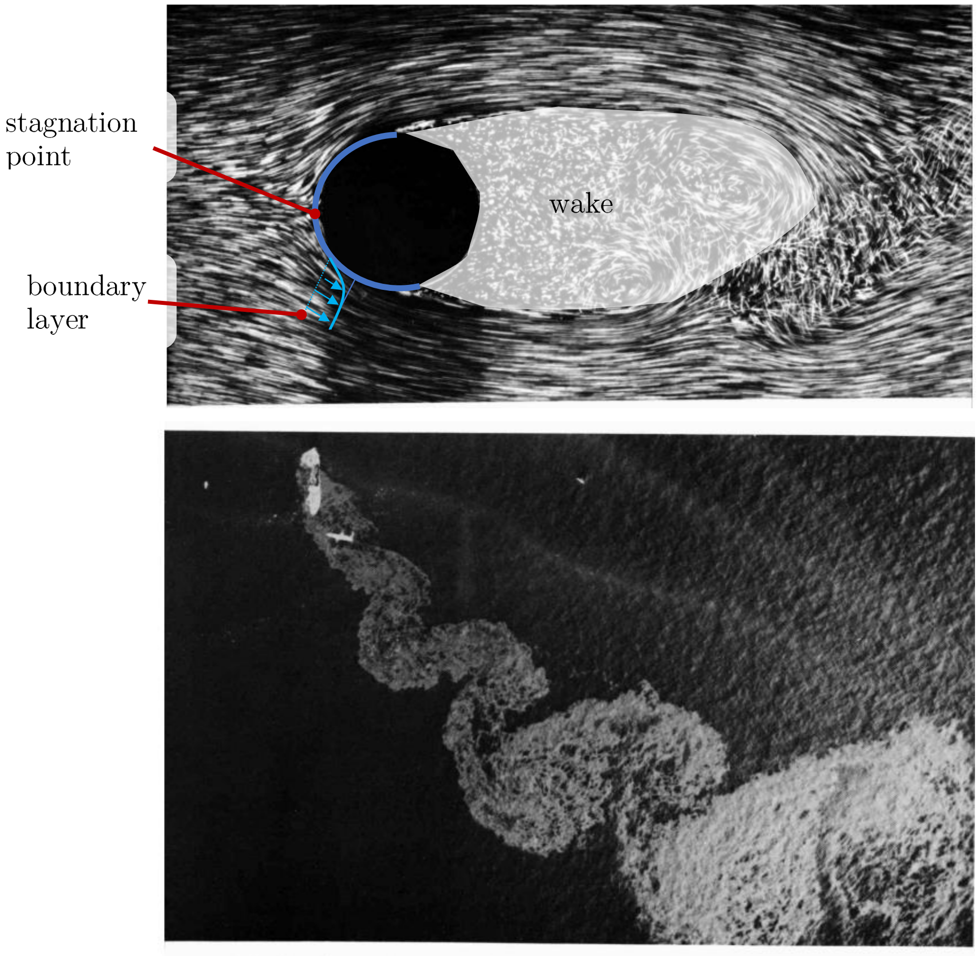

Figure 6: The flow around the ship causes friction and form drag. An example flow around a bluff body (top) and a remarkable wake formed in lee of a grounded ship (bottom).
Source: An Album of Fluid Motion

Ships can also be affecting by *external force disturbances* such as water currents, wind shear and wave loading. These are also neglected in the simulation.

Thrust model

The linear and angular velocity can be mapped to the thrust generated by the propellers (see Figure 7), by considering the forces acting on the ship:

$$m\dot{v} = (T_1 + T_2) - F_d \tag{9}$$

and

$$J\dot{\omega} = \frac{L}{2}(T_1 - T_2) - Q_d. \tag{10}$$

The distance $L/2$ is the moment arm between the propeller axis and the centre of mass.

Steering

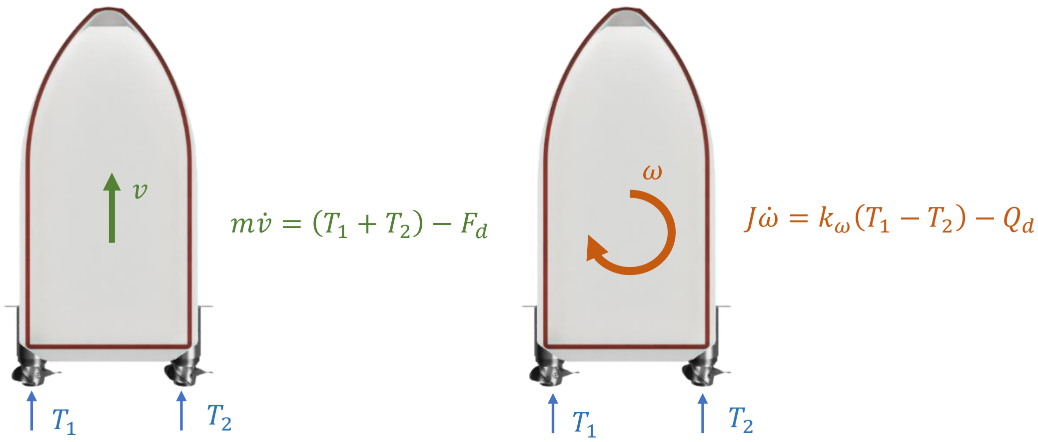

Figure 7: Mapping thrust to linear and angular velocity.

The thrust generated by the propeller is characterised by the equation

$$T = k_t \rho \Omega_p^2 D_p^4 \tag{11}$$

where k_t is a dimensionless thrust coefficient, Ω_p is the rotational velocity of the propeller, measured in rad/s, and D_p is the nominal propeller diameter.

Motor model

The spin of the propeller is driven by an electric DC motor, illustrated in Figure 8. You will control the motor input by sending a PWM signal to modulate the voltage of the circuit between -$V_{\max}$ and $V_{\max}$. The circuit dynamics of the DC motor (assuming quick response) are given by:

$$V_{\max} \times \mathrm{PWM} = I R + k_m \Omega_s \tag{12}$$

where I is the current running through the motor, measured in Amps, and R is the motor resistance, measured in Ohms. The angular velocity of the motor shaft is denoted Ω_s and k_m is the motor constant which characterises the back EMF of the motor.

The circuit equation is coupled with the dynamics of the rotor, where the motor torque is assumed to be proportional to the current running through the motor, such that the balance of torques on the shaft gives:

$$J \frac{d\Omega_s}{dt} = k_m I \tag{13}$$

where J is the moment of inertia of the rotor.

A gearbox model is used to increase the speed of the propeller relative to that of the rotor. Assuming conservation of rotational power,

$$\Omega_s T_s = \Omega_p T_p \tag{14}$$

the speed increase from the gearbox can be deduced from the gear ratio N:

$$\frac{\Omega_p}{\Omega_s} = \frac{1}{N} = \frac{T_s}{T_p}. \tag{15}$$

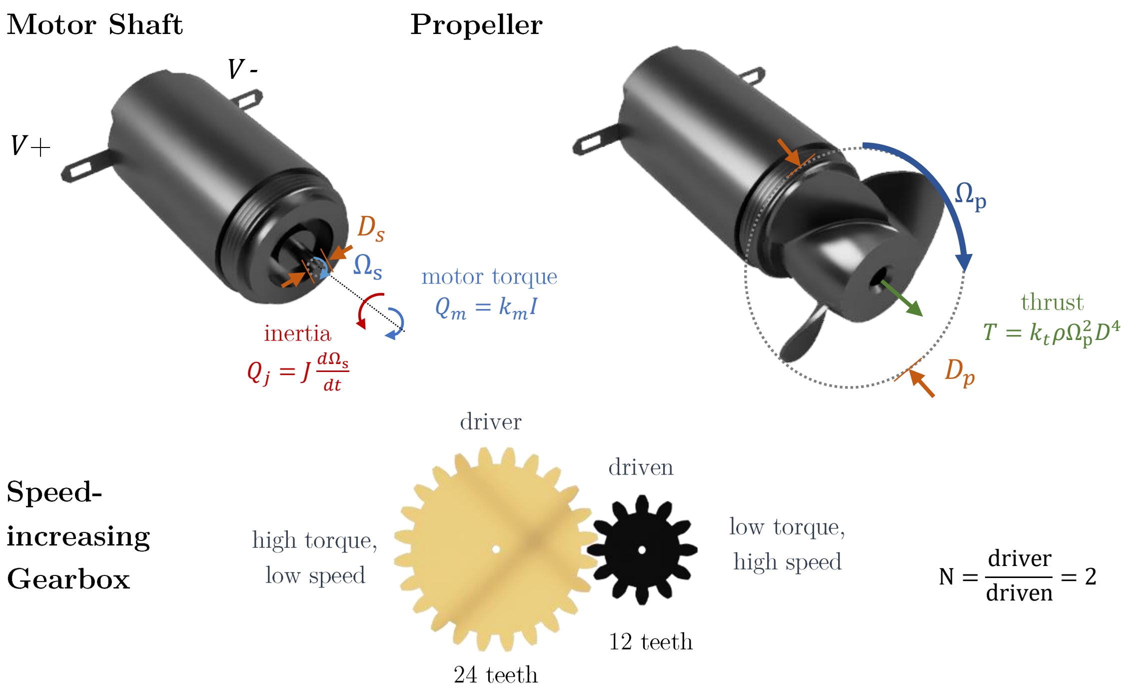

Figure 8: DC motor and propeller thrust model.