Natural, Mathematical & Physical Sciences

24/1/2022

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Department of Engineering

4CCE1MCP: Design, Making a Connection

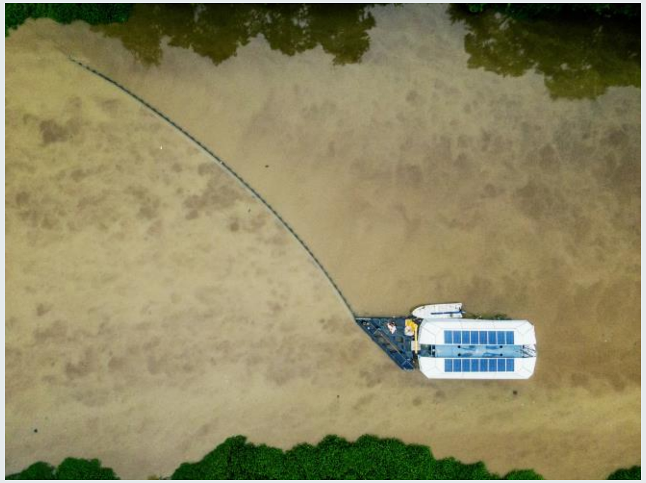
Week 23

Modelling Engineering Systems

Learning Outcomes

- Deduce relationships between physical quantities by applying dimensional analysis
- Draw a free body diagram for a mechanical component and identify the forces acting on it.
- Define inertia, damping and stiffness and critique how these mechanical properties affect system response.
- Explain how and why to use mechanical mechanisms (load transfer, motion control, operational efficiency, etc.).
- Interface with the simulation template that you will use for the individual coursework project

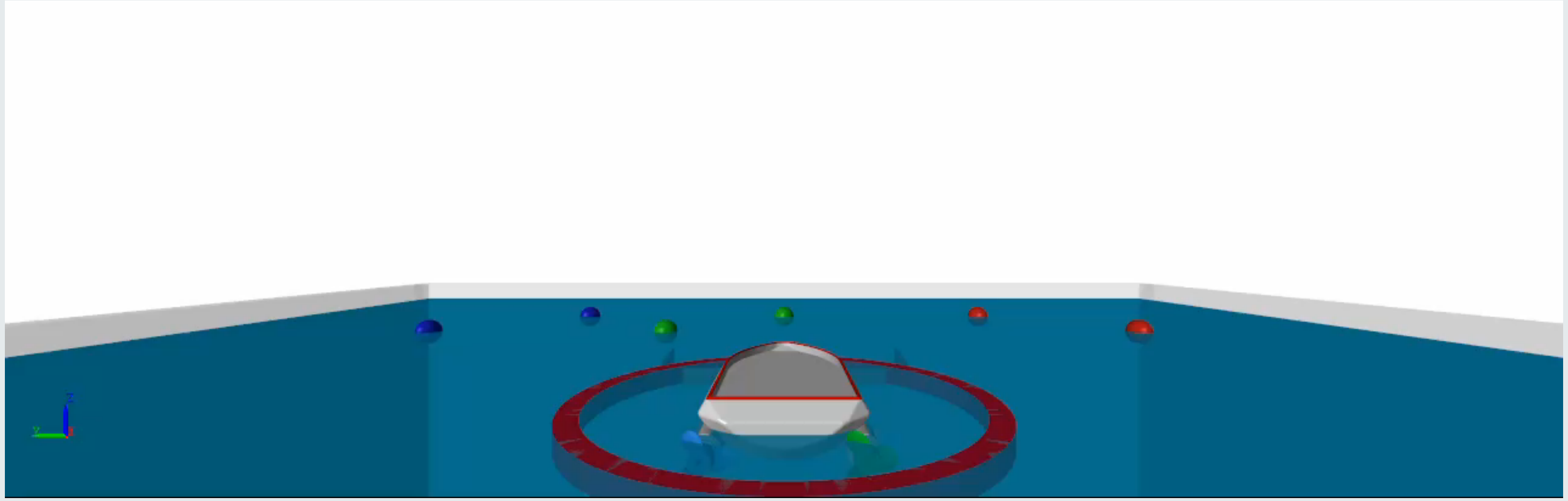
Autonomous Marine Debris Clean-up Project

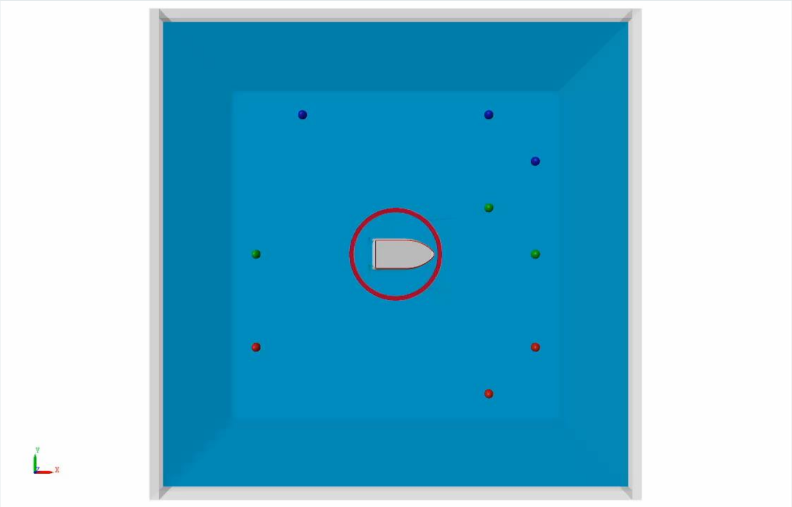


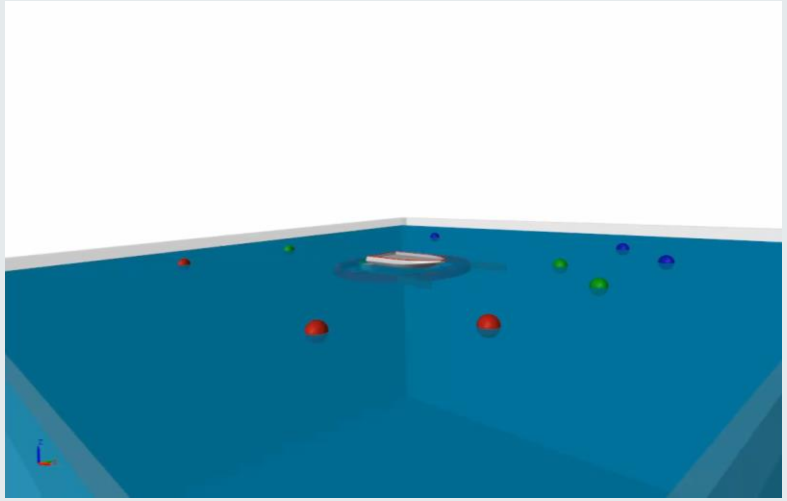
By the end of the session…

Implement models of ship components

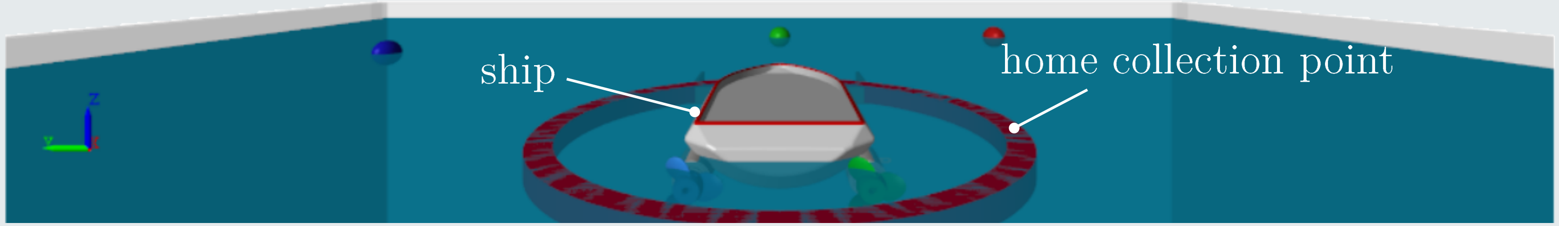
Operate the simulation template for the individual coursework

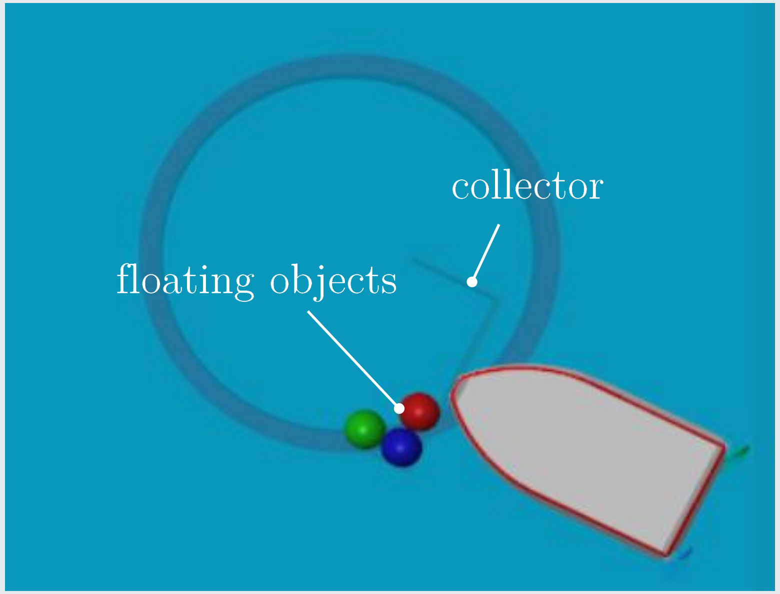


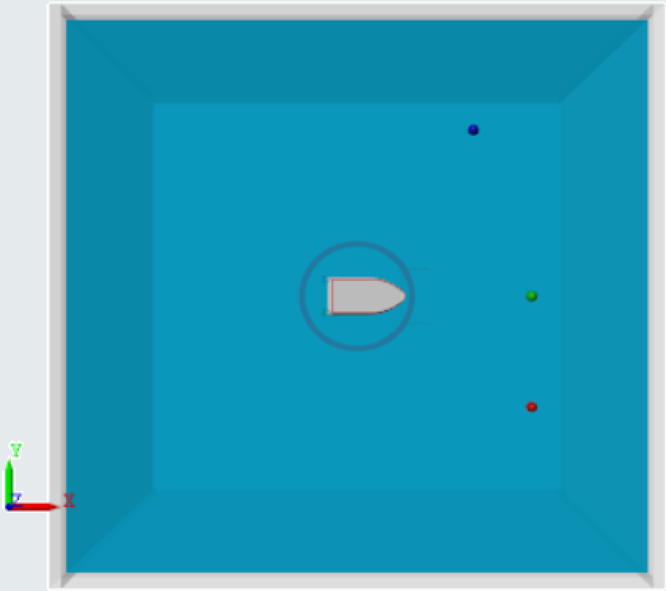




By the end of the session…







Agenda

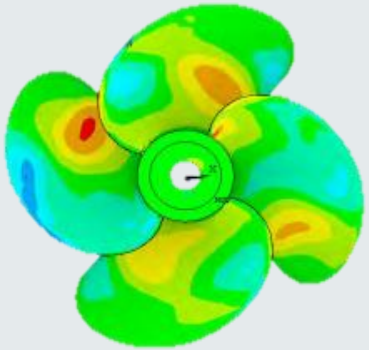
Modelling

- Forces acting on ship
- Drag and Control surface models
- Vehicle models
- Propeller models

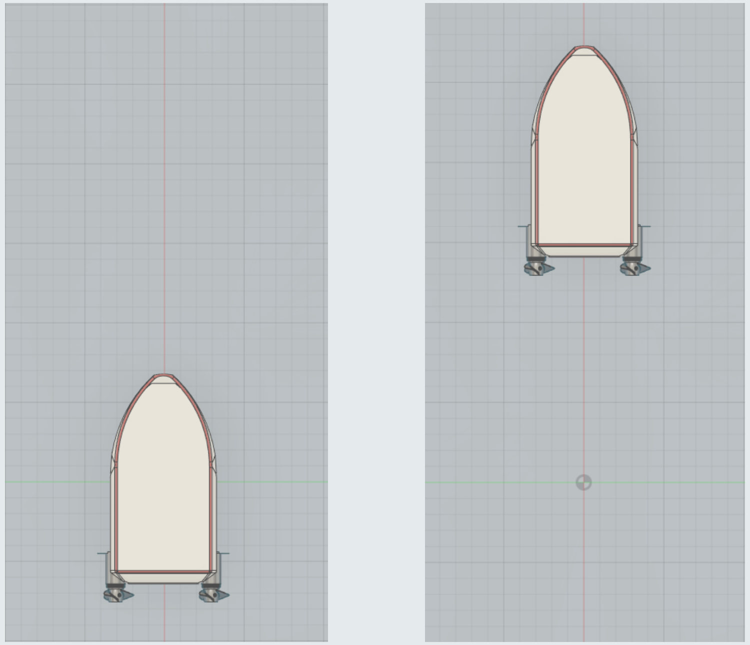
Mechanisms

- Rotational vs Translational motion
- Rotational mechanisms: gears (spur, worm, bevel), cables and belts, shafts, pulleys
- Translational mechanisms: sliders / rails, pistons
- Converter mechanisms: Wheel and axle, crank and slider, rack and pinion

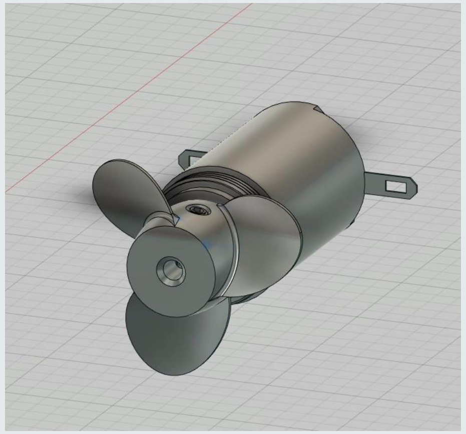
Modelling the ship



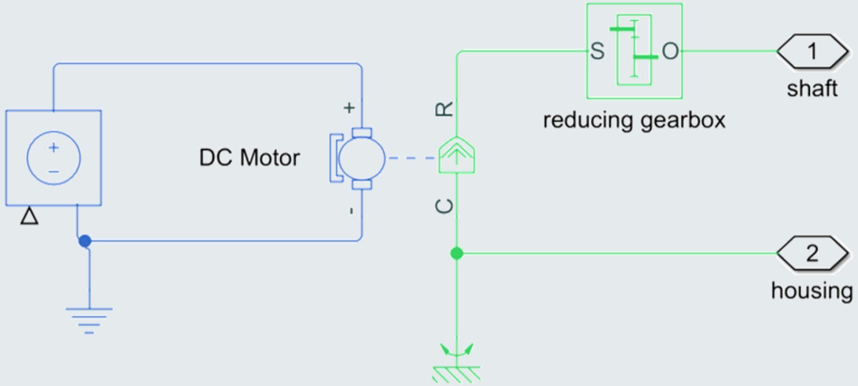
material stresses



ship motion



propeller thrust



DC motor circuit

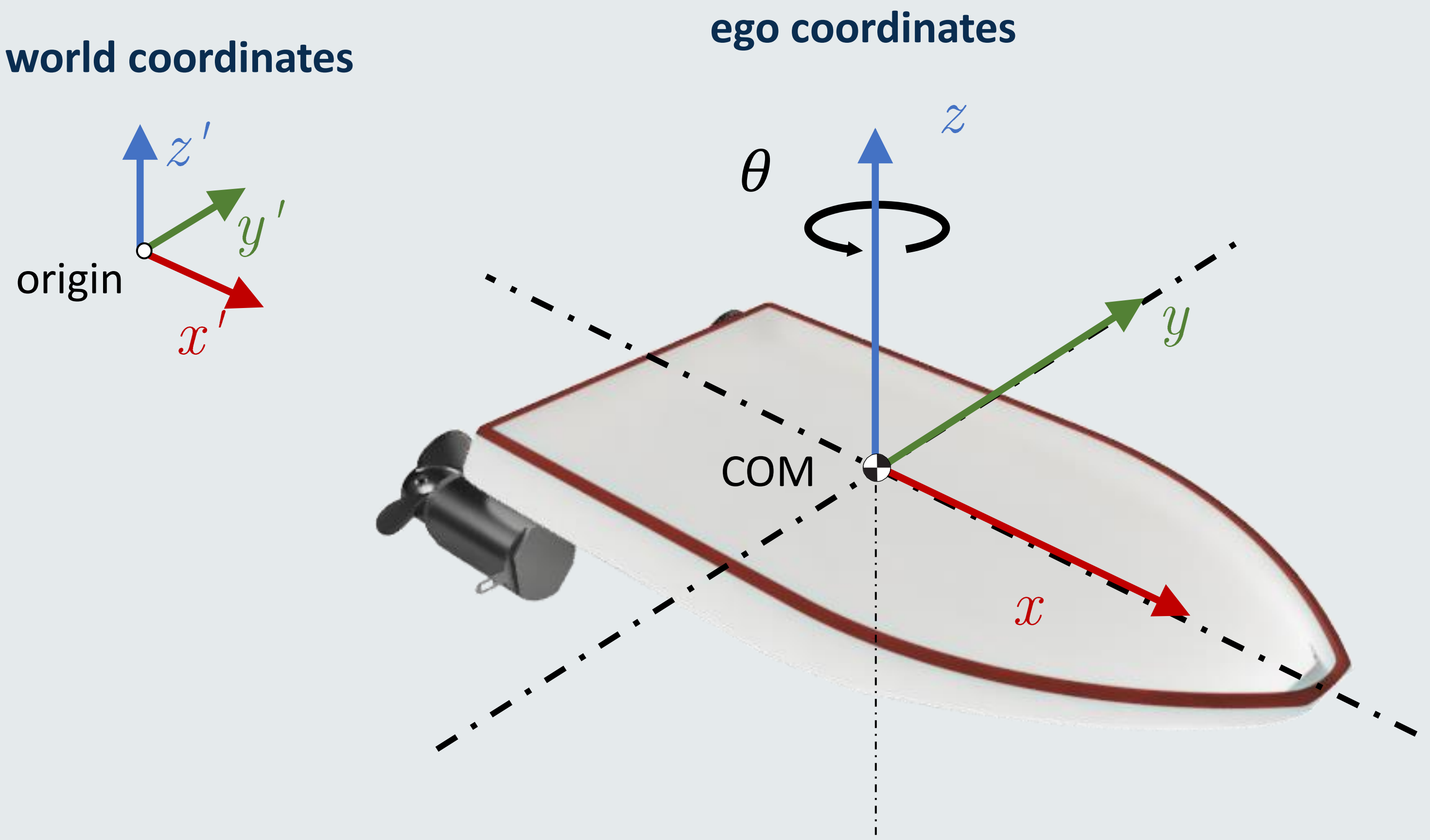
ship stability

Ship motion

Define **coordinate systems** (CS) to describe motion

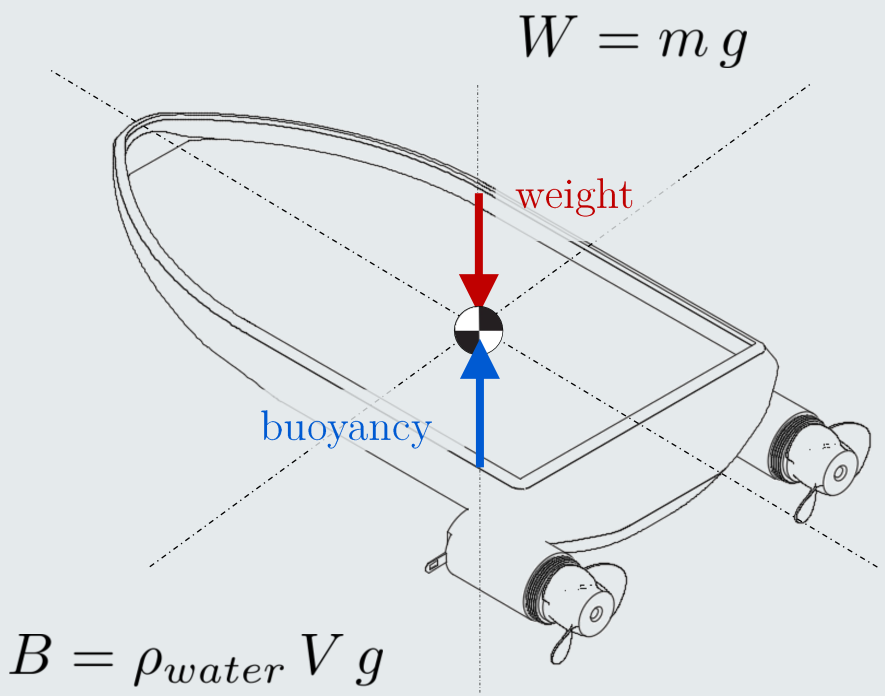
World coordinates describes motion with respect to a fixed origin

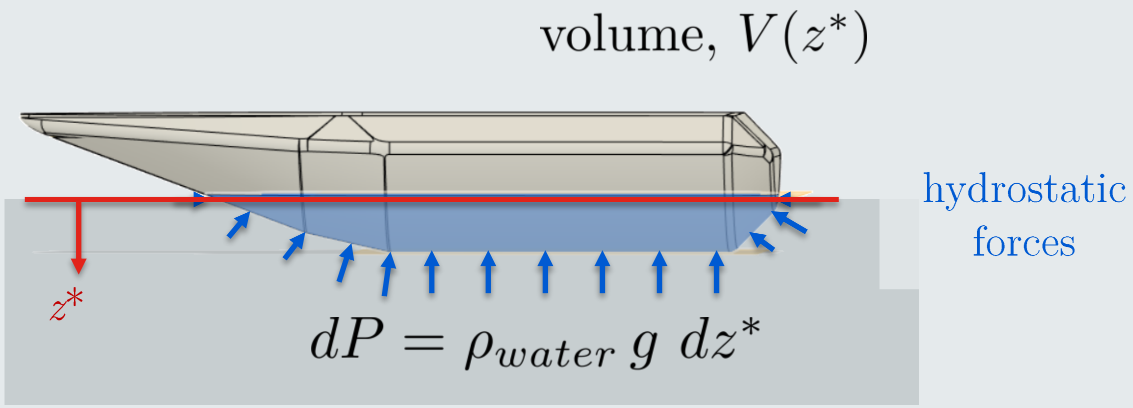
Ego coordinates describe motion with respect to ship centre of mass (COM ◑)



Flotation analysis - Will your ship sink?

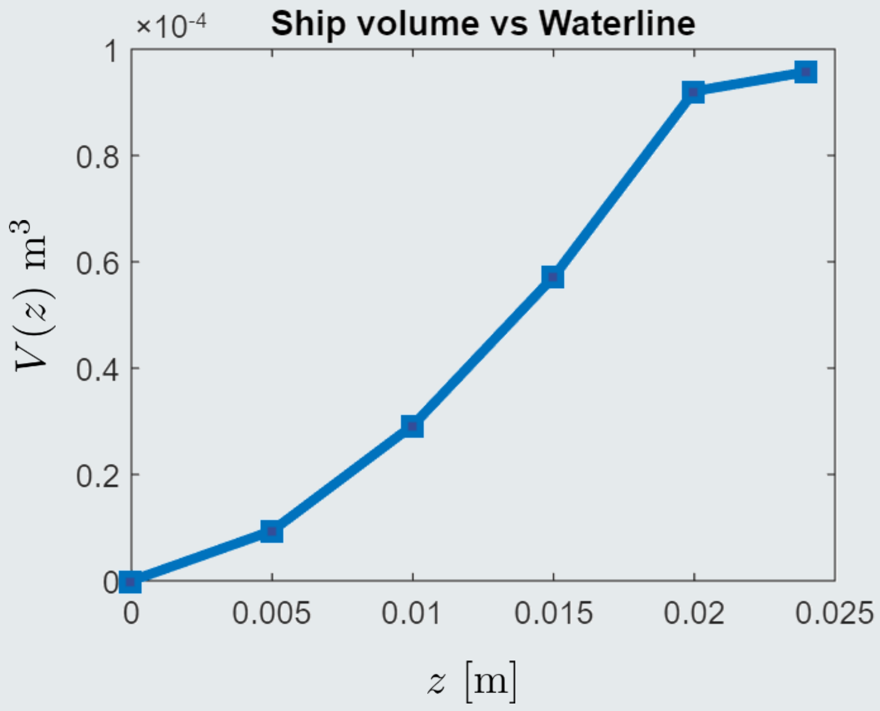
Freebody diagrams – Vertical forces

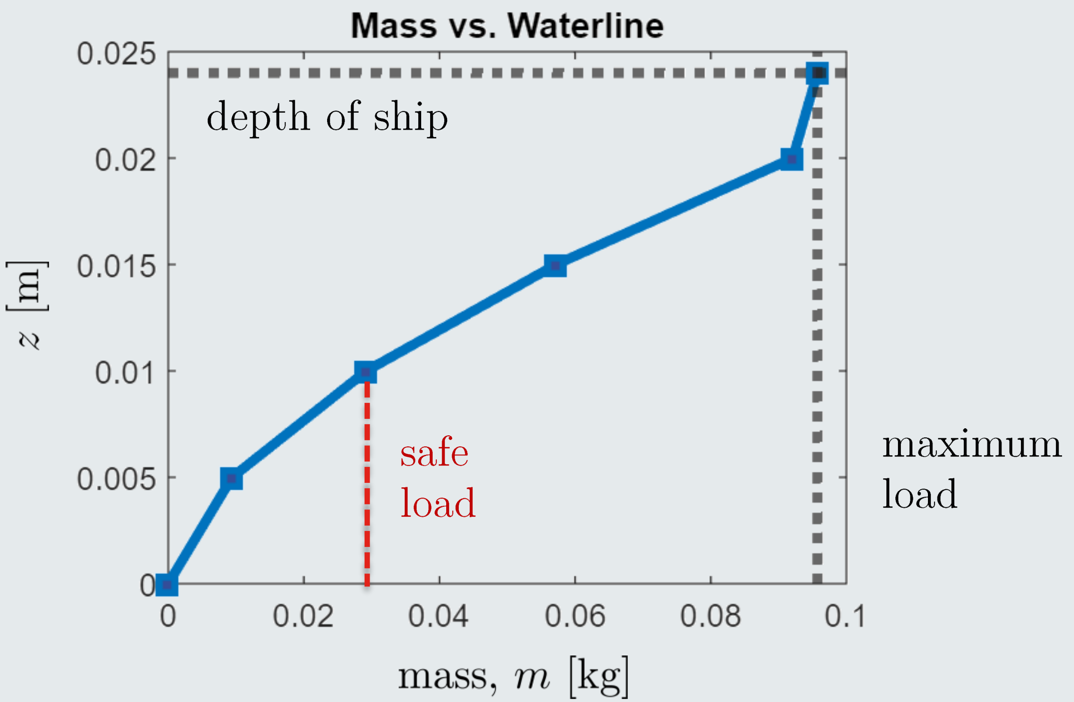


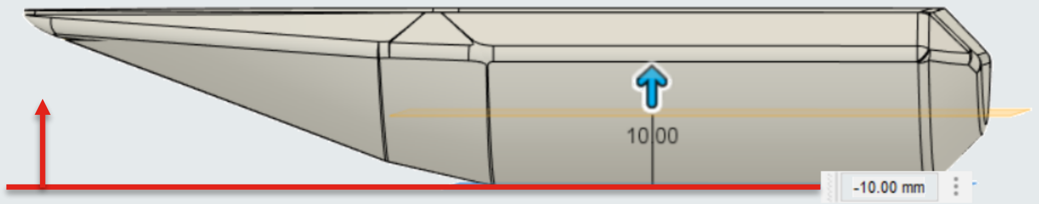


integrate pressure over area
to get buoyancy force

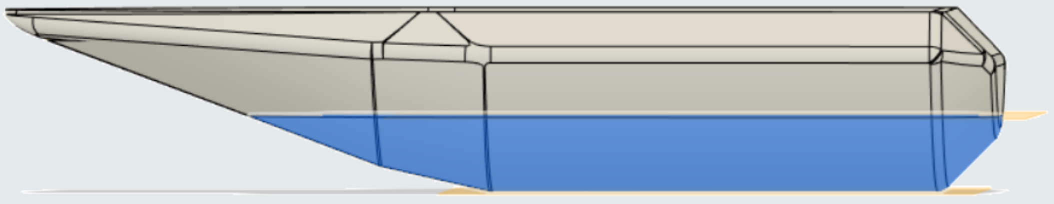
Waterline Level (draught) & Maximum Payload





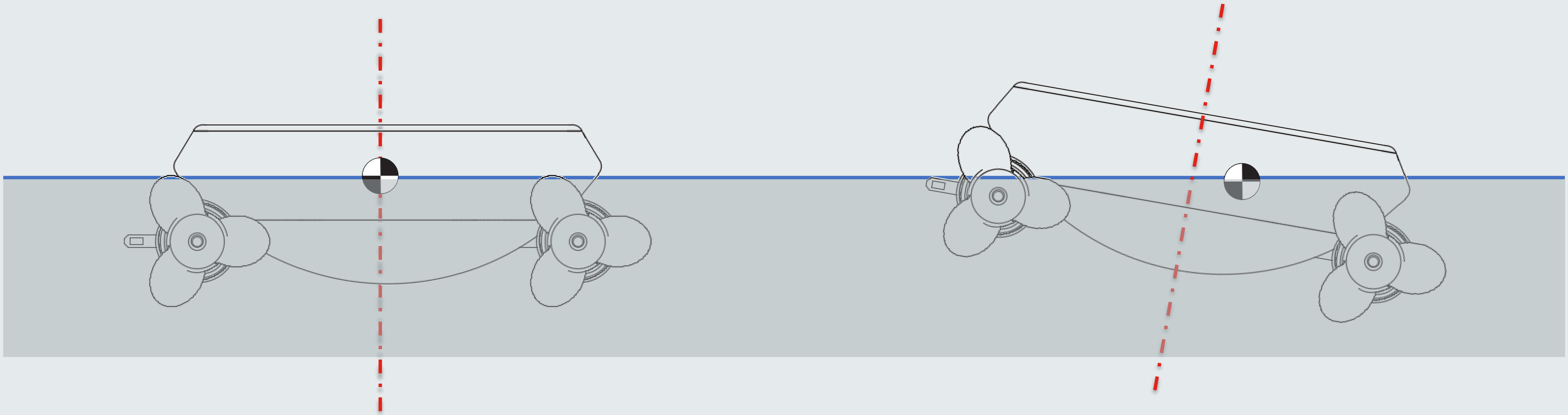


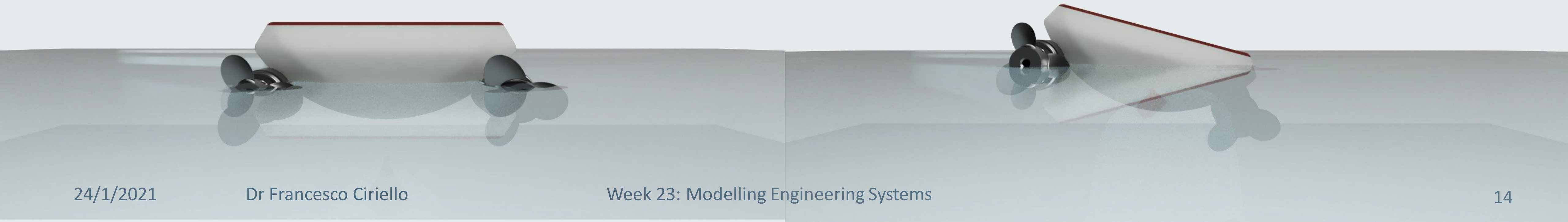
waterline, z



volume, $V(z)$

Will your ship cap size? Trim & Heel Analysis





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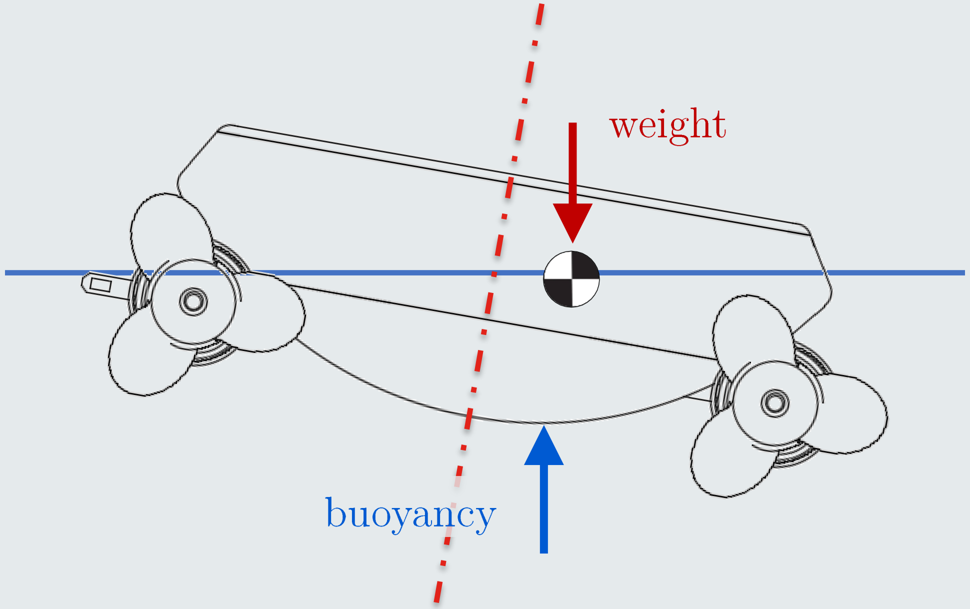
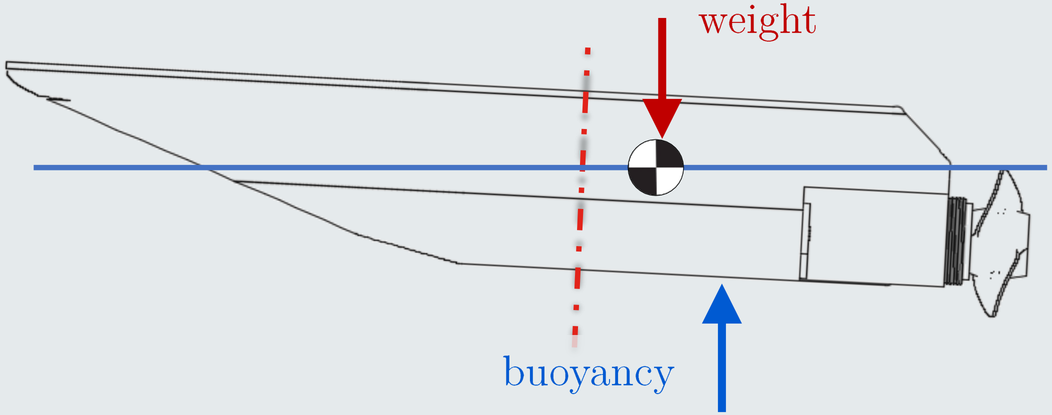




Heel & trim stability estimated using **curves of form** or **experimentation**

$$COM = \int \int \int dx\, dy\, dz$$

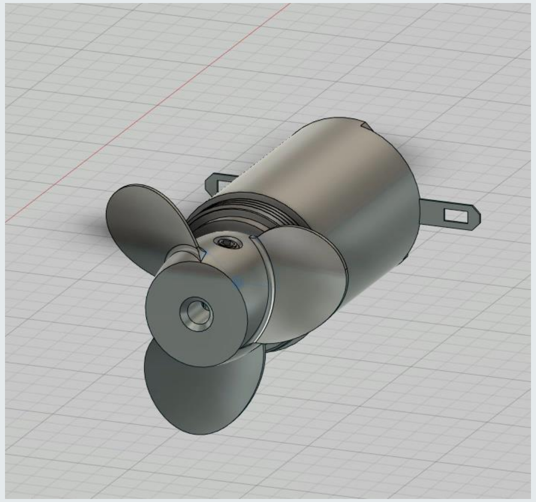
$$M_W = W \times COM$$

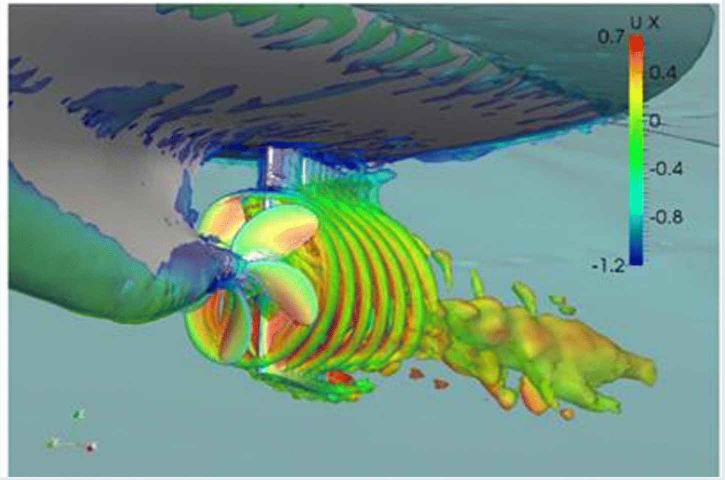
$$B = \int \int \rho g\, dz\, dA$$

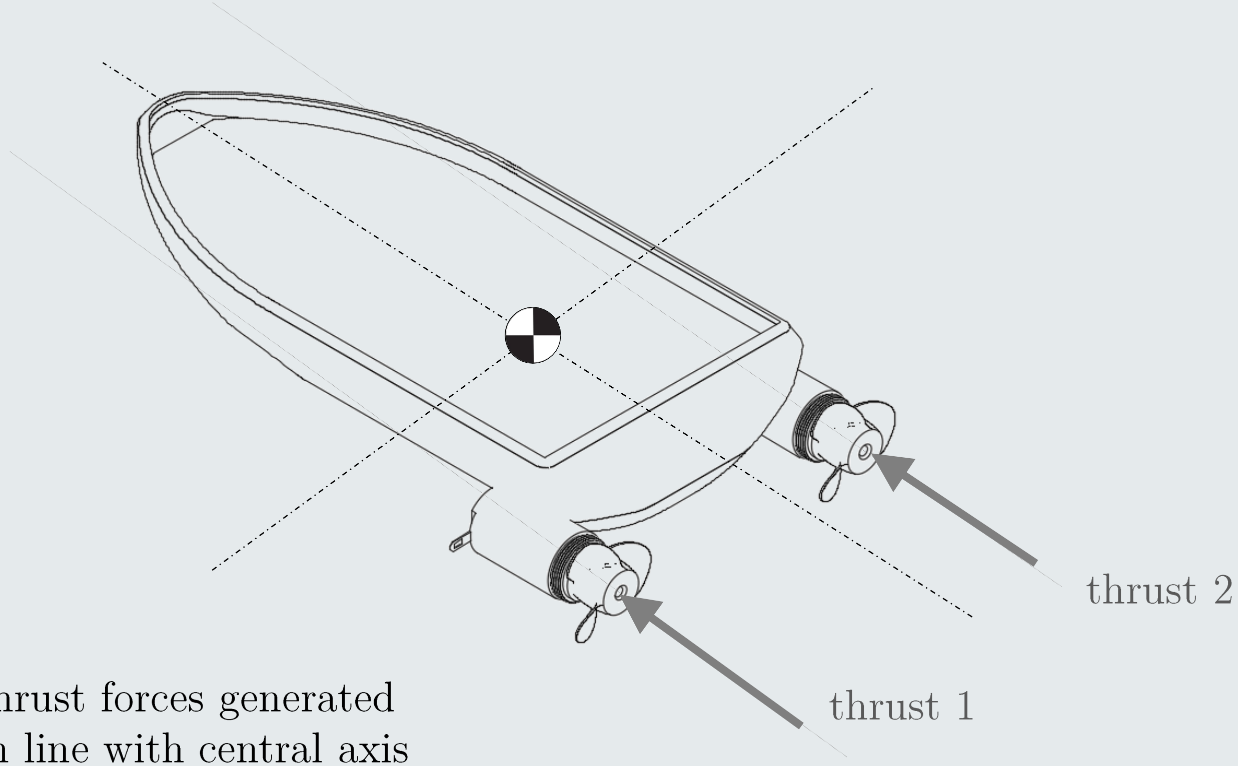
$$M_B = B \times COB$$

Good practice to keep COM as close to as possible to the geometric centre of ship

Freebody diagrams – Horizontal Forces



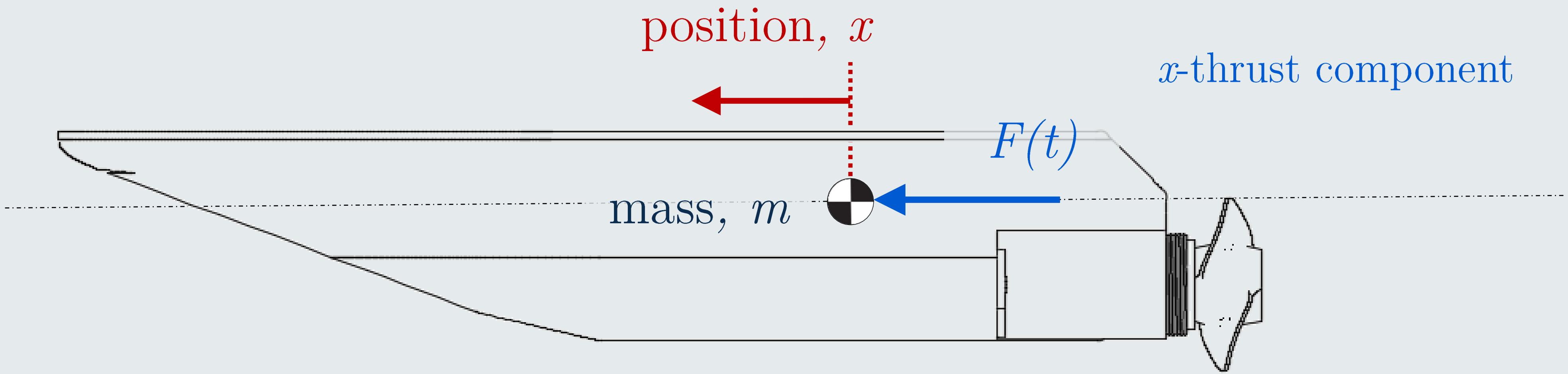




thrust forces generated
in line with central axis
of propeller

Particle mass models

Apply Newton's 2nd Law of motion, $$m\ddot{x}(t) = F(t)$$



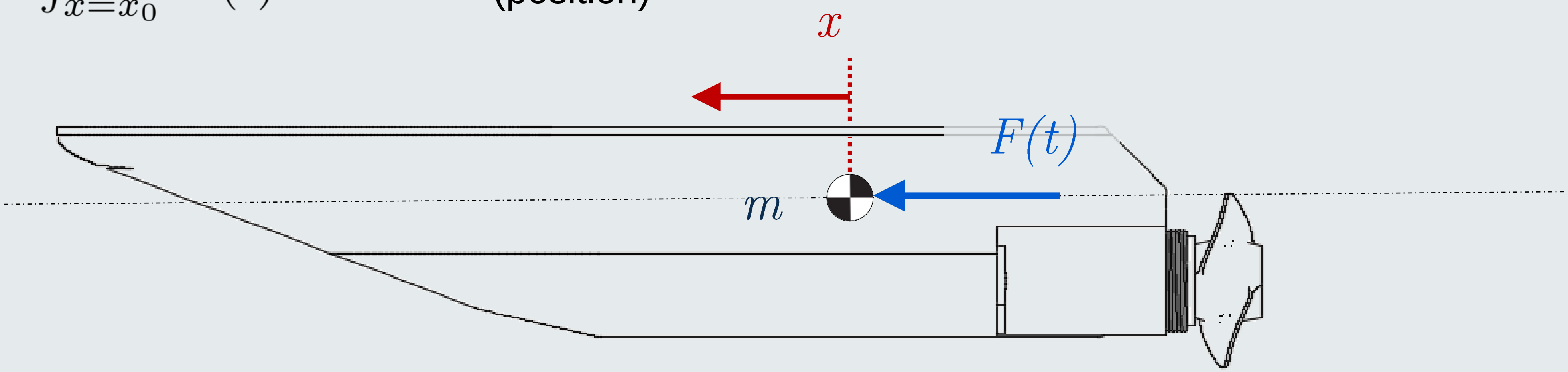
Particle mass models

Integrate equation twice with respect to time to obtain position

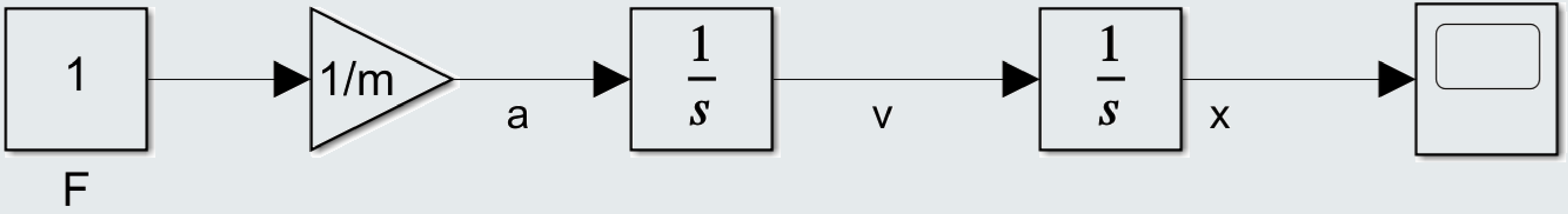
$$\ddot{x}(t) = F(t)/m$$ (acceleration)

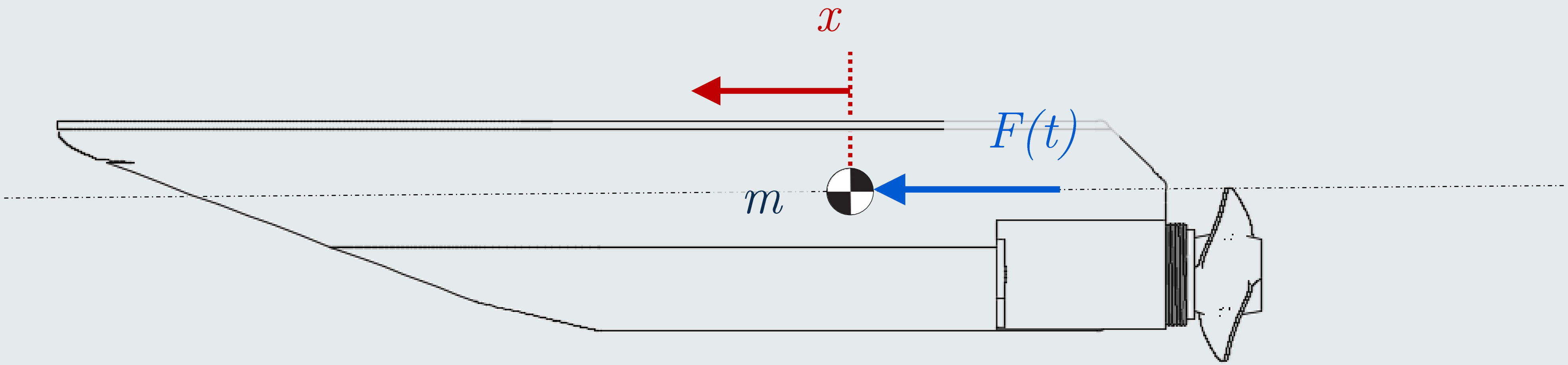
$$\dot{x} = \int_{\dot{x}=\dot{x}_0}^{\dot{x}=\dot{x}_1} \ddot{x}(t)\, dt$$ (velocity)

$$x = \int_{x=x_0}^{x=x_1} \dot{x}(t)\, dt$$ (position)

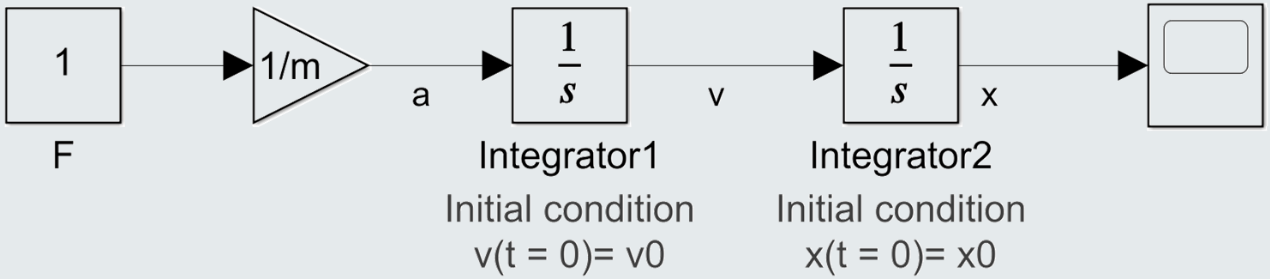


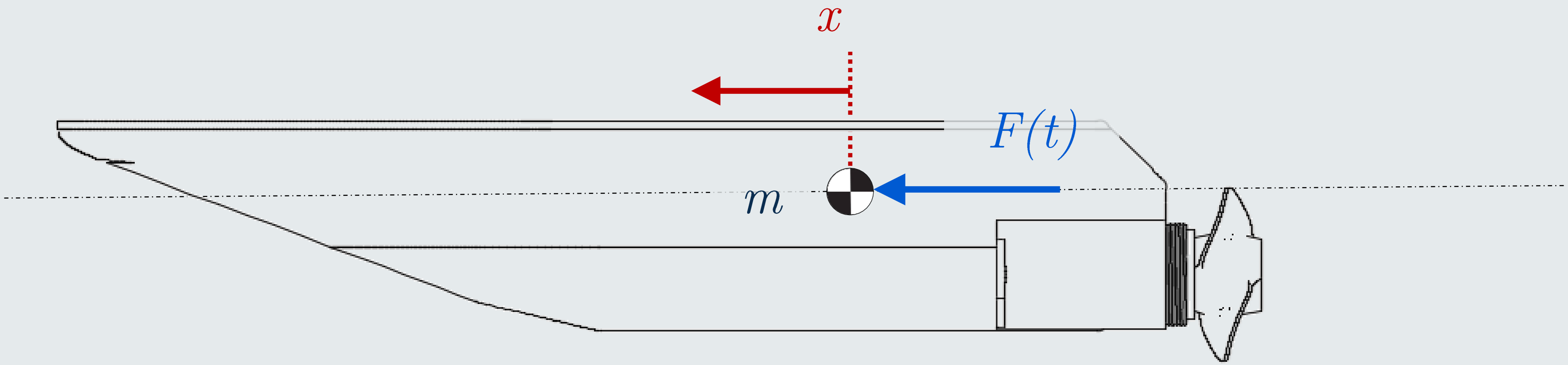
Particle mass models





Particle mass models





Drag

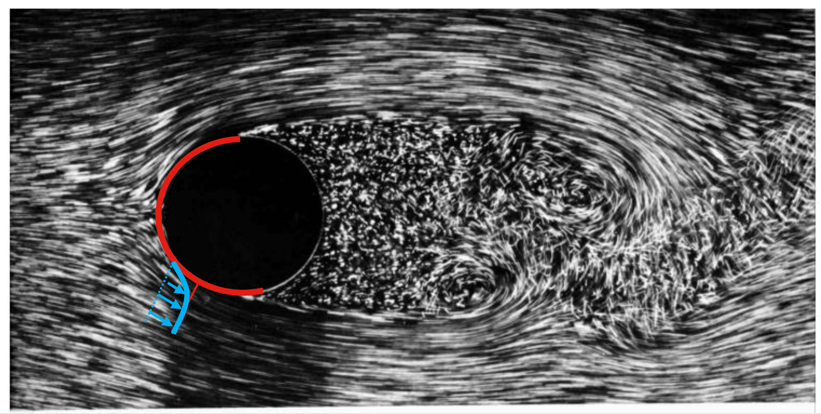
Drag forces act against the direction of motion

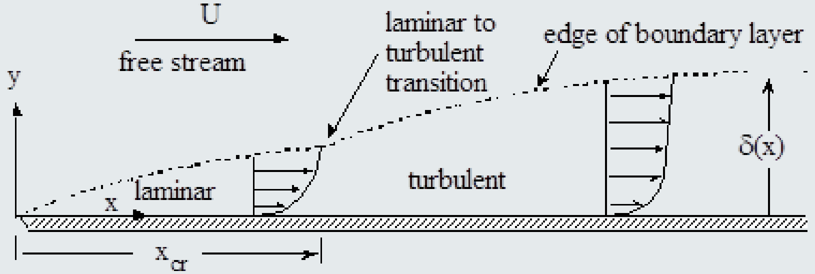
They arise due to the **viscous fluid forces** acting on the ship

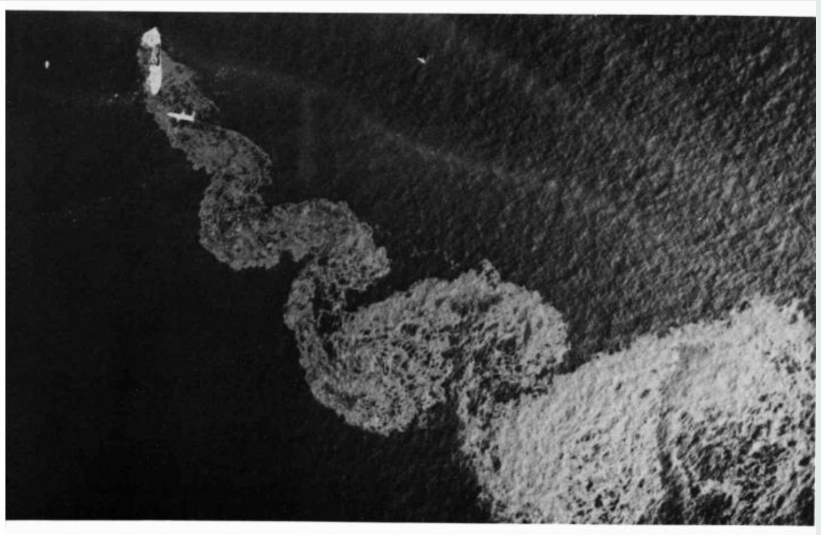
The **drag force** is given by

$$F_d = \frac{1}{2} C_d \rho A_s v|v|$$

where a **drag coefficient** C_d characterizes the dynamics of the viscous forces, which depend on flow conditions and geometry



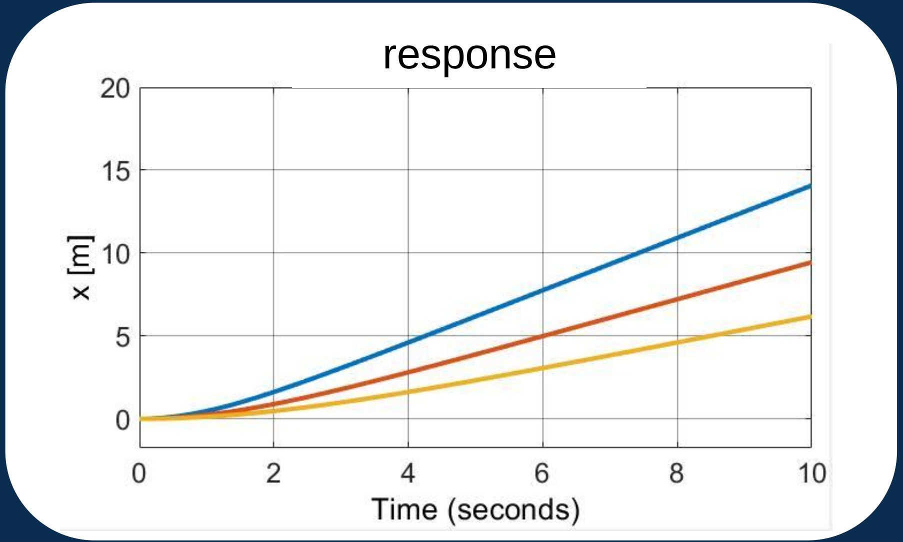




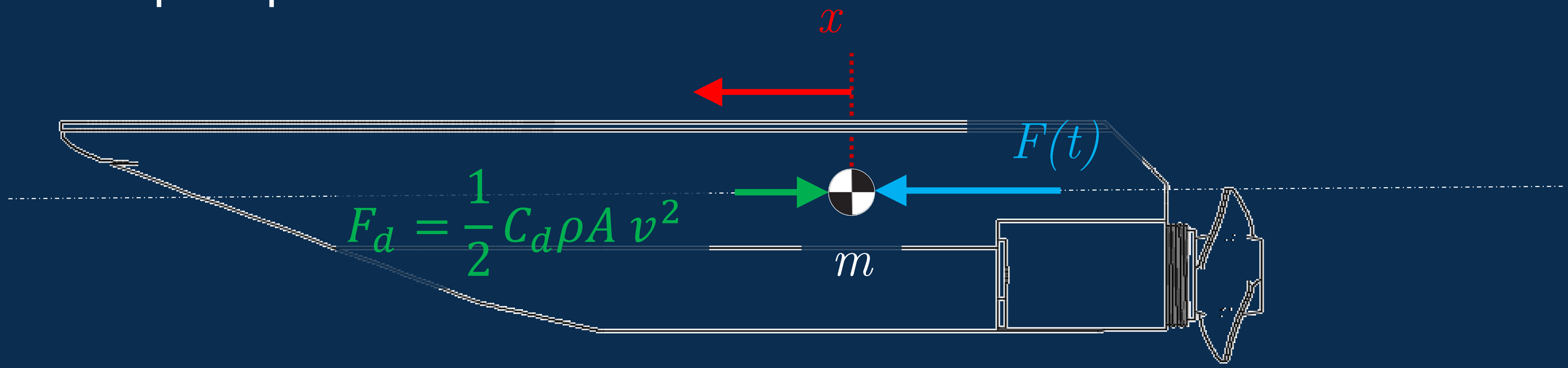
Thrust Response

Example: Simulink demonstration

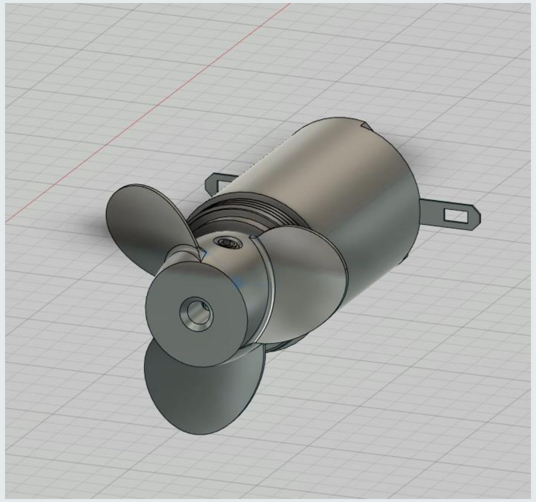
$$m\,\ddot{x} = F(t) - b_d\,\dot{x}\,|\dot{x}|$$

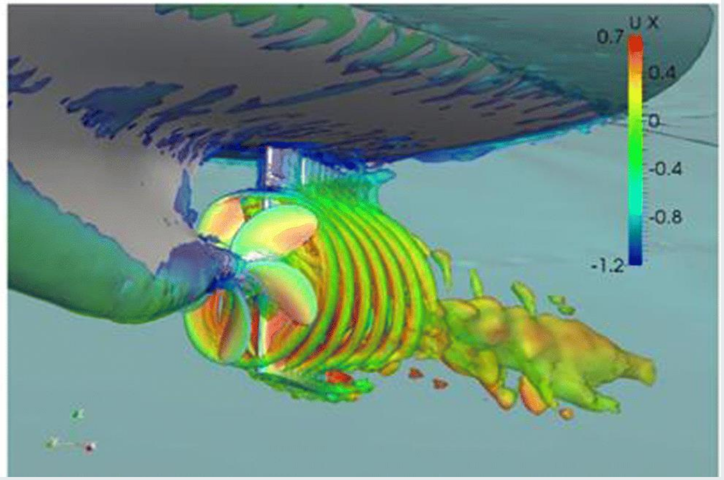


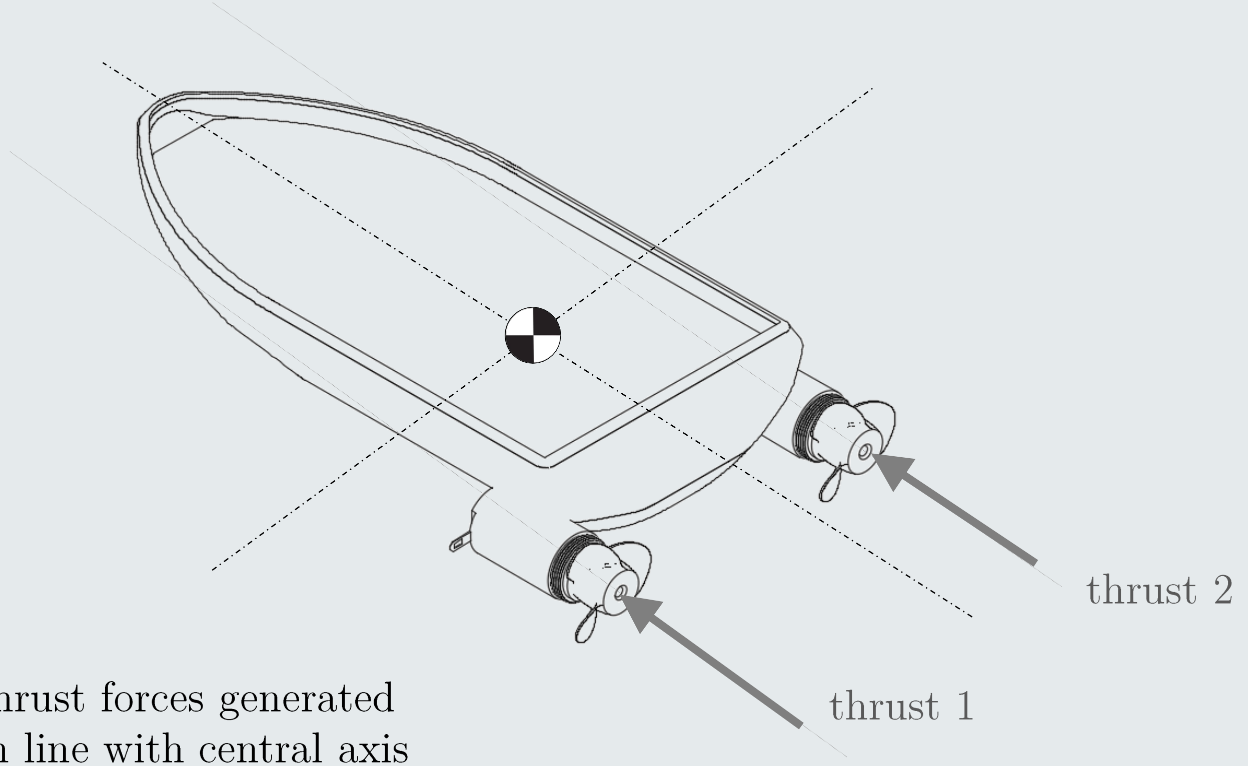
Exercise: Vary thrust, mass and drag and inspect ship response



Freebody diagrams – Horizontal Forces

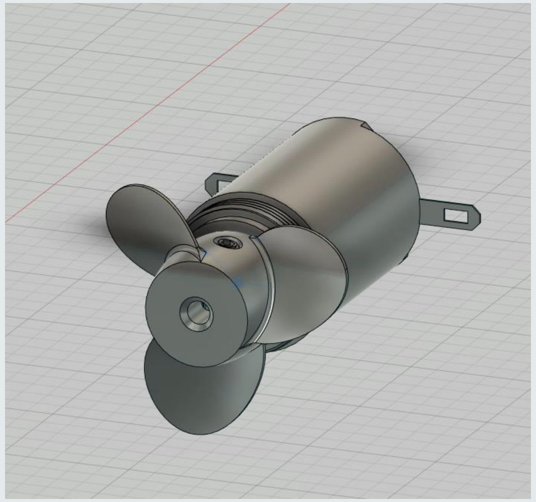


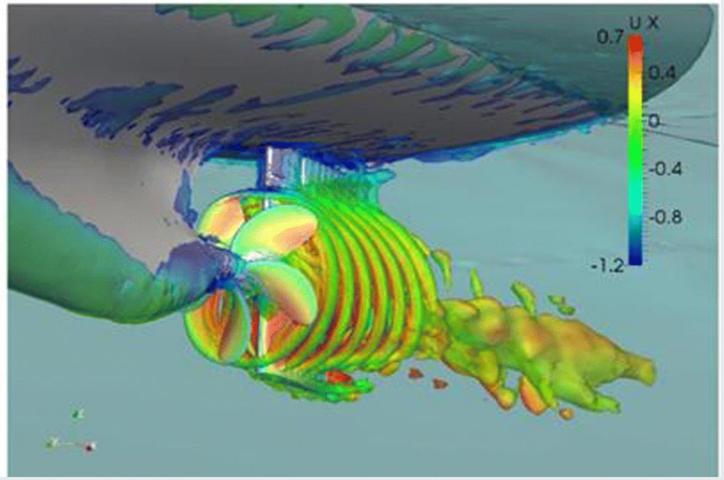


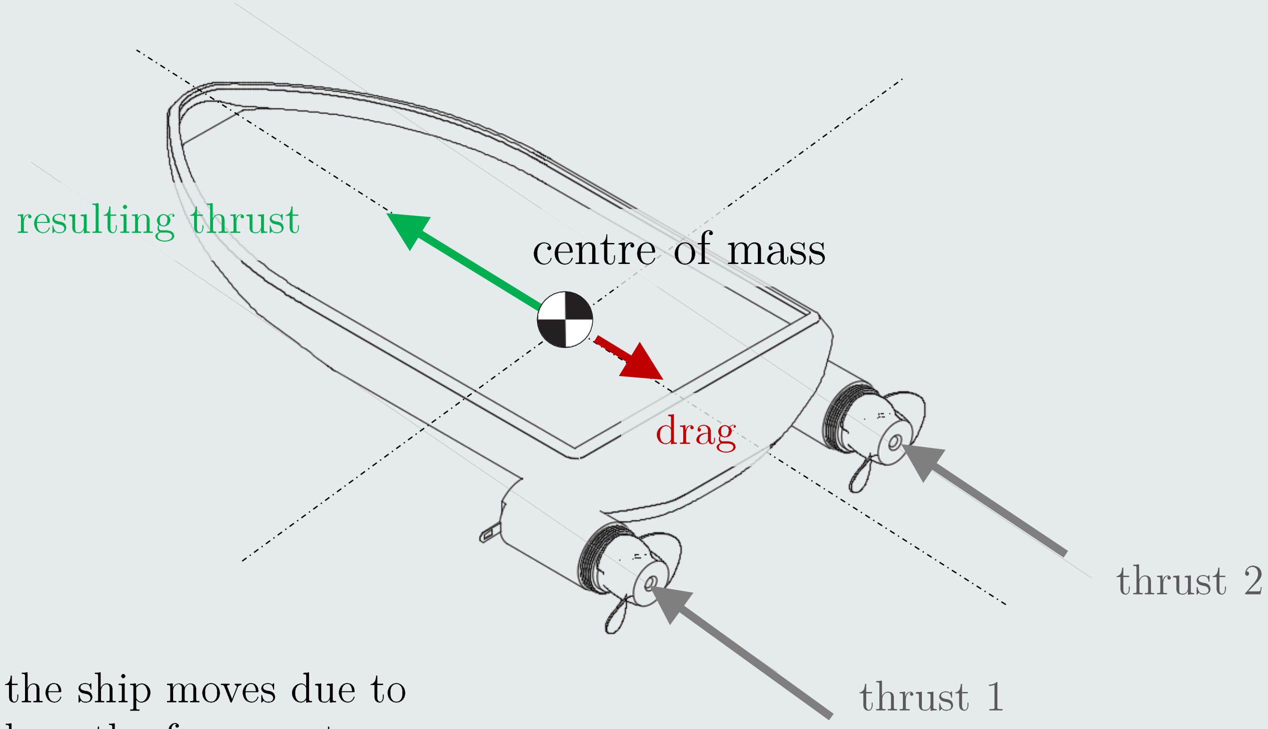


thrust forces generated
in line with central axis
of propeller

Freebody diagrams – Horizontal Forces

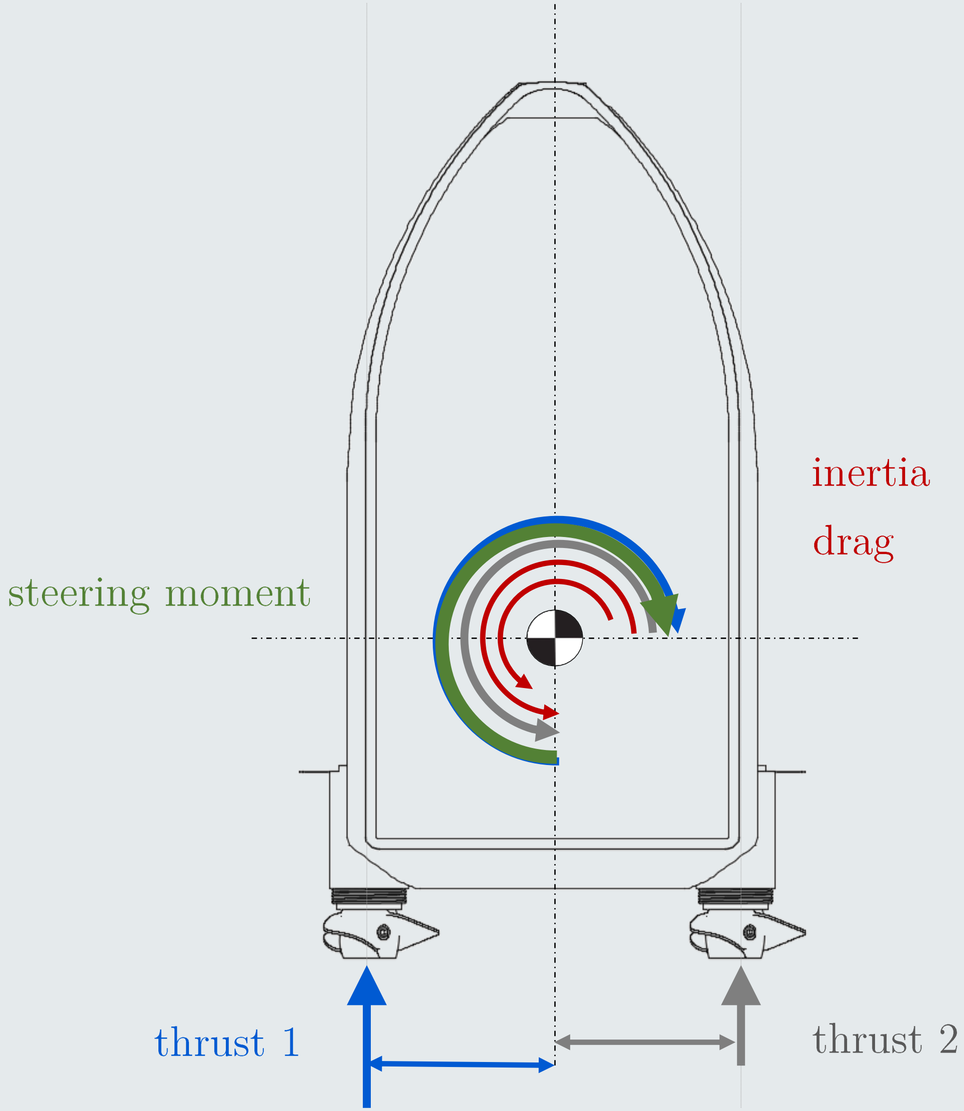




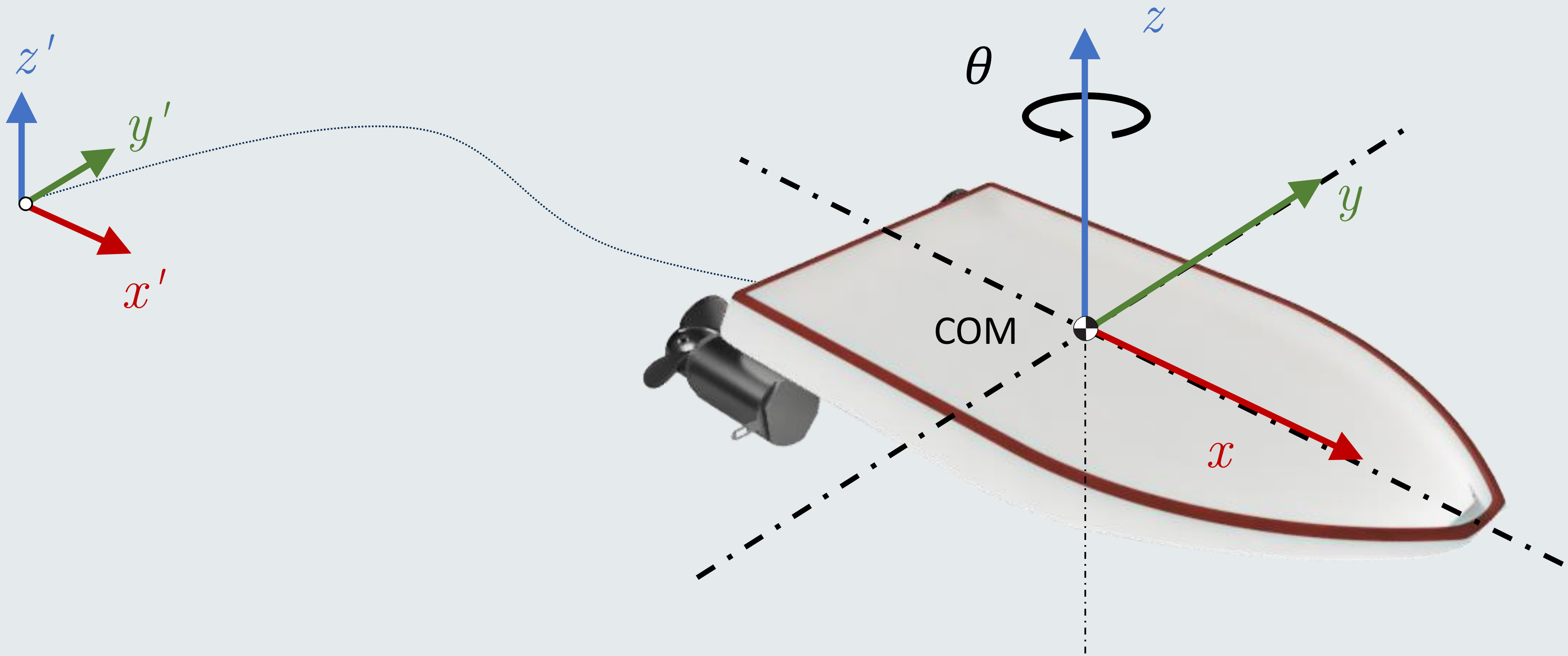


the ship moves due to how the forces get transferred through its centre of mass

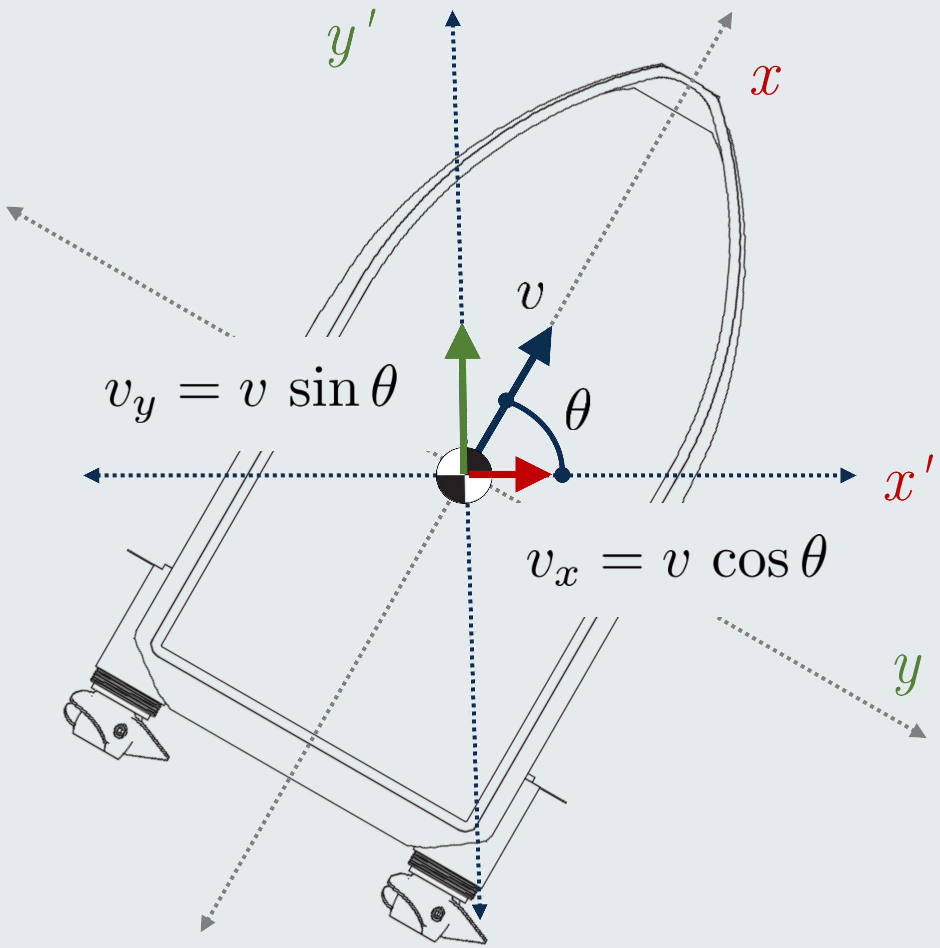
Freebody diagrams – Moments



Kinematic Equations



The Unicycle model

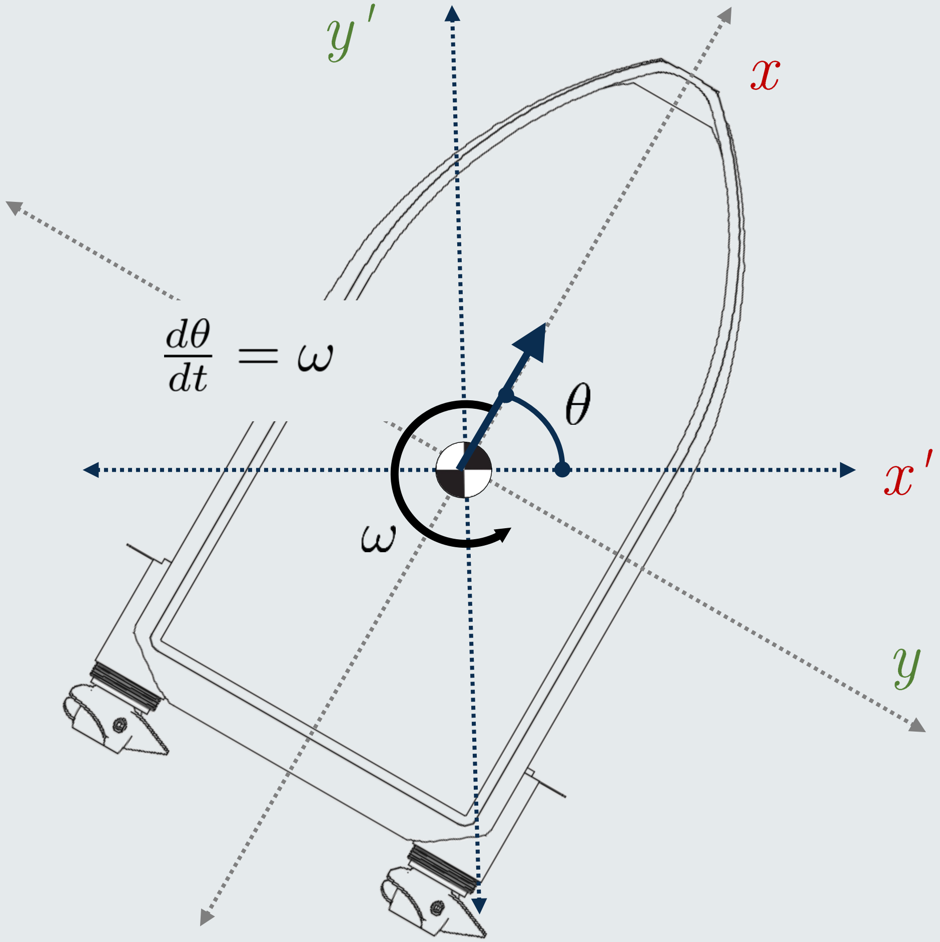


Transform ego coordinates to world coordinates

$$v_x = v \cos\theta$$

$$v_y = v \sin\theta$$

The Unicycle model



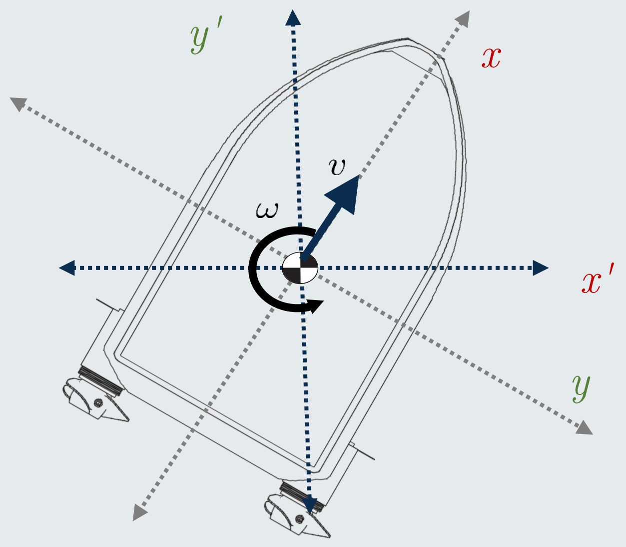
Transform **ego** coordinates to **world** coordinates

$$v_x = v \cos\theta$$

$$v_y = v \sin\theta$$

$$\frac{d\theta}{dt} = \omega$$

Vehicle pose



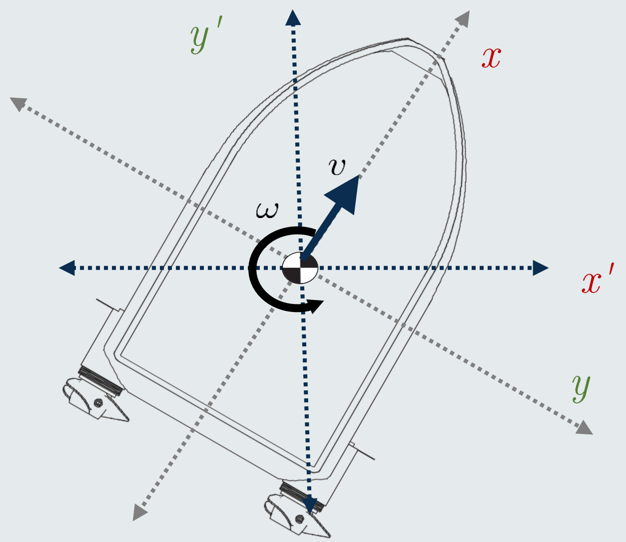
Integrate velocities with respect to time to obtain **position** and **heading angle**

$$x = \int v(t)\, \cos\theta(t)\, dt$$

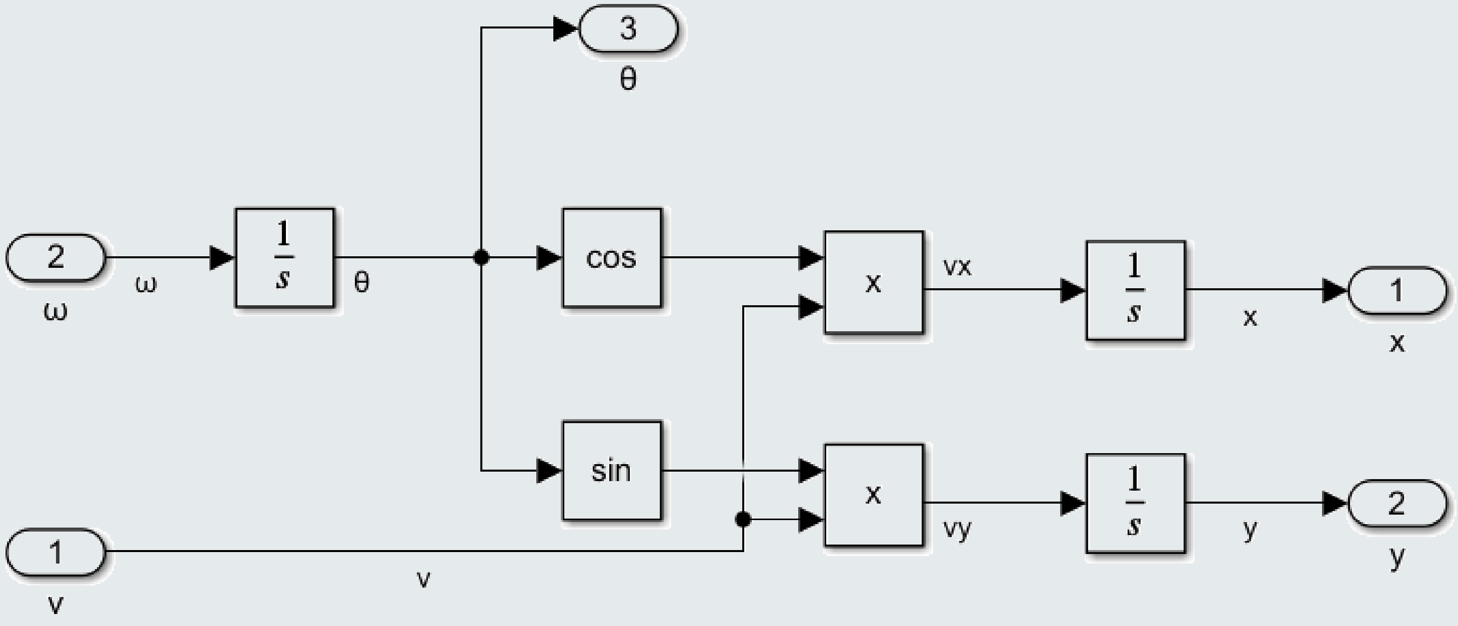
$$y = \int v(t)\, \sin\theta(t)\, dt$$

$$\theta = \int \omega(t)\, dt$$

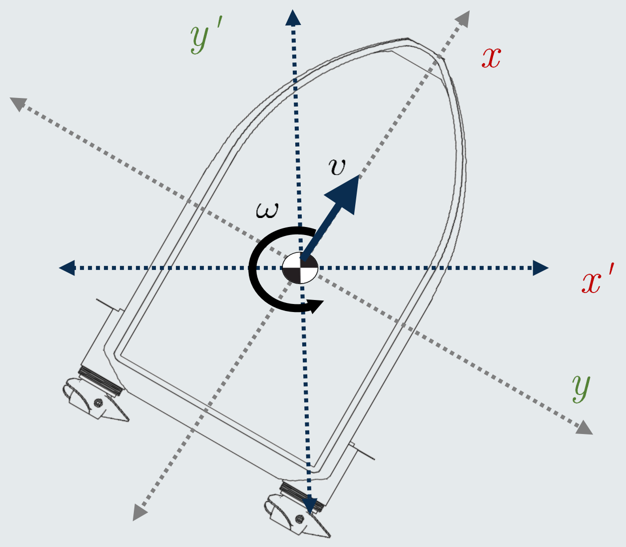
Vehicle pose



Use **integrator blocks** to solve coupled system of equations



Vehicle pose

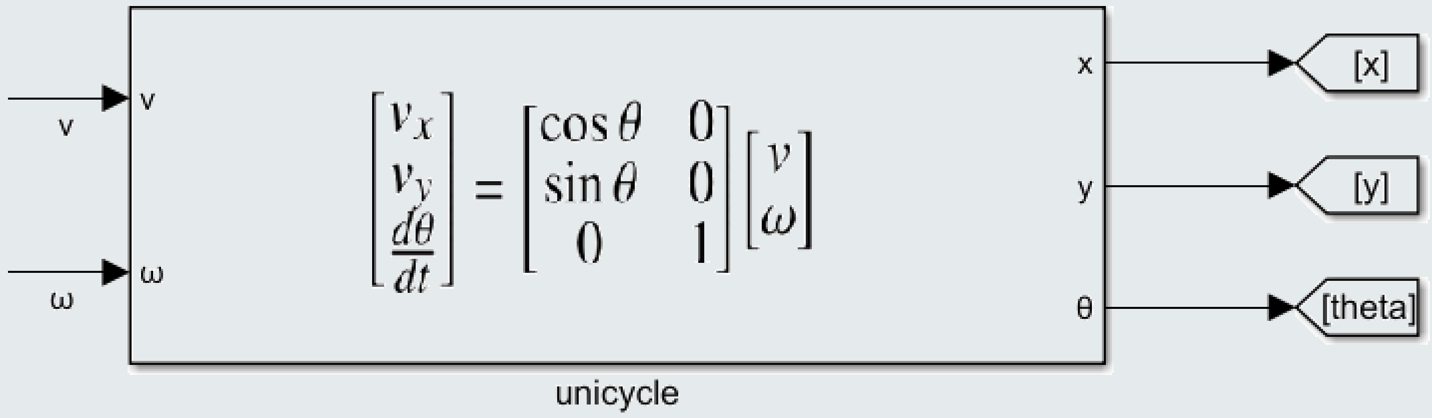


The solution to the vehicle model with **inputs**:

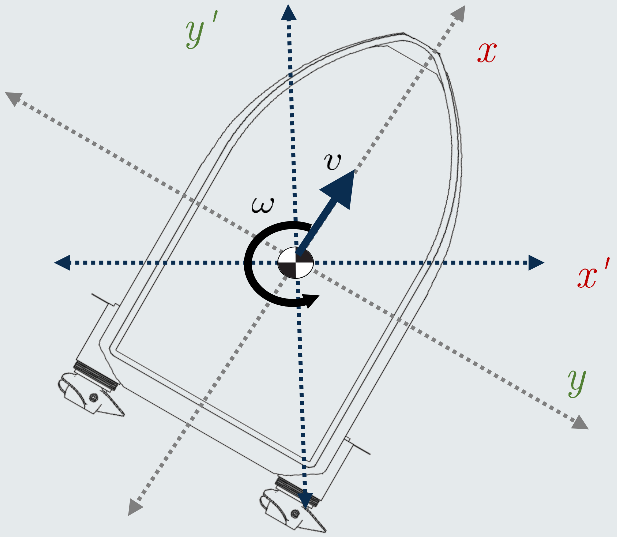
v – linear velocity

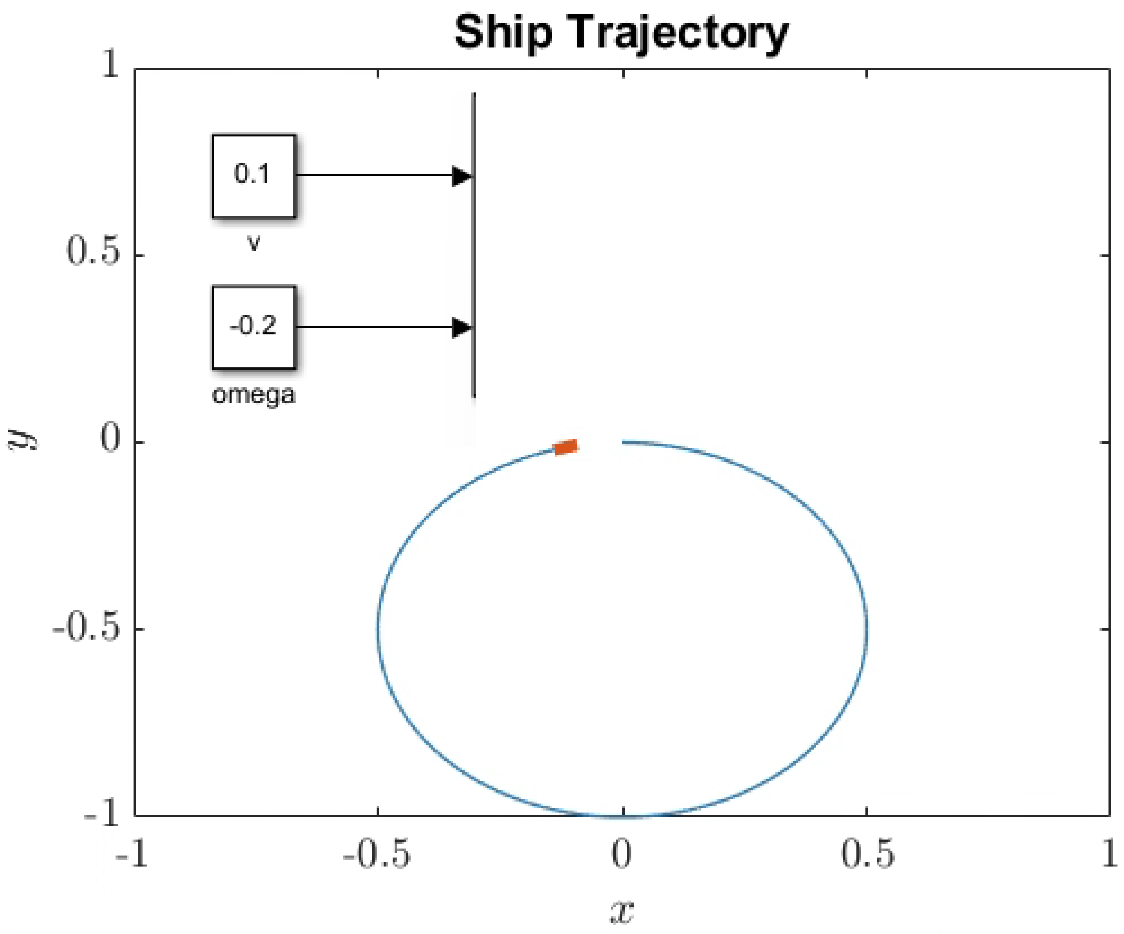
ω – angular velocity

are the **outputs**, x, y, θ, referred to as the **pose** of the vehicle



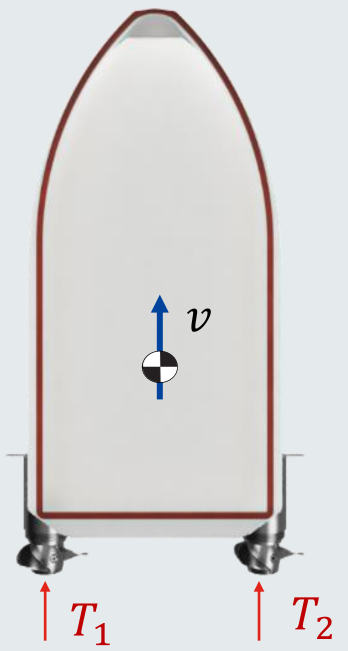
Kinematic Response





Dynamic equations

Apply Newton's 2nd law of motion to deduce how the **linear velocity** v changes over time



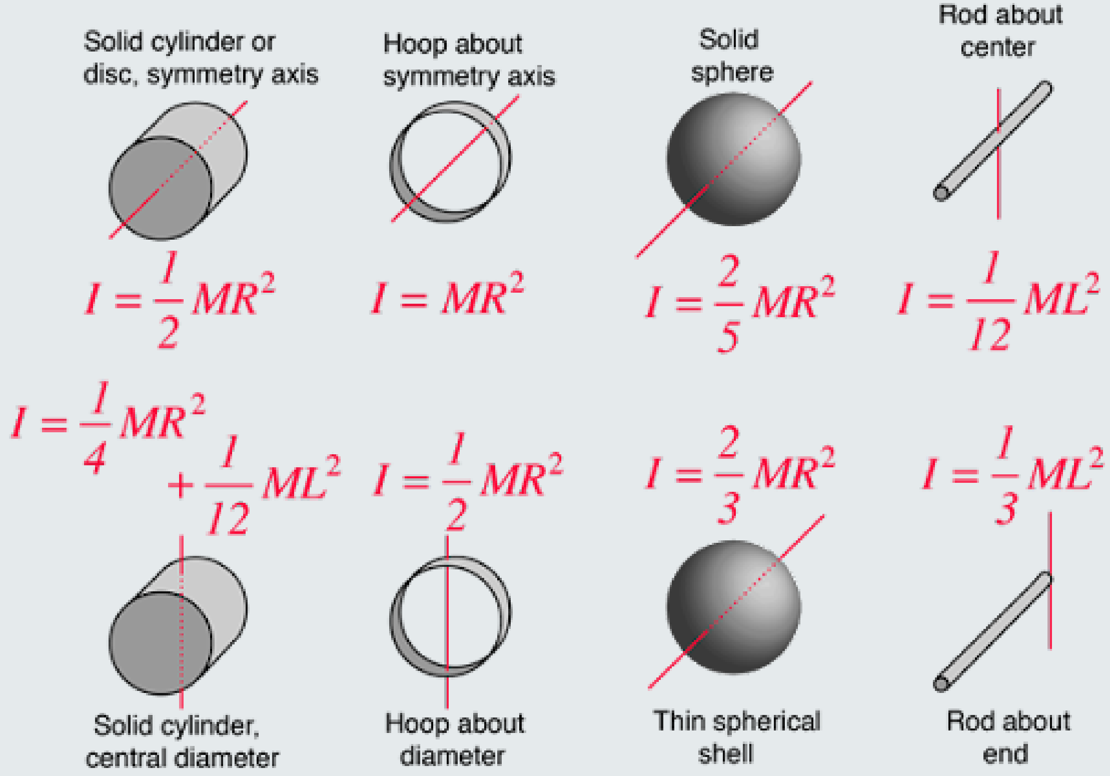
$$m\dot{v} = (T_1 + T_2) - F_d$$

Dynamic equations

Apply Newton's 2nd law of motion to deduce how the **angular velocity** ω changes over time

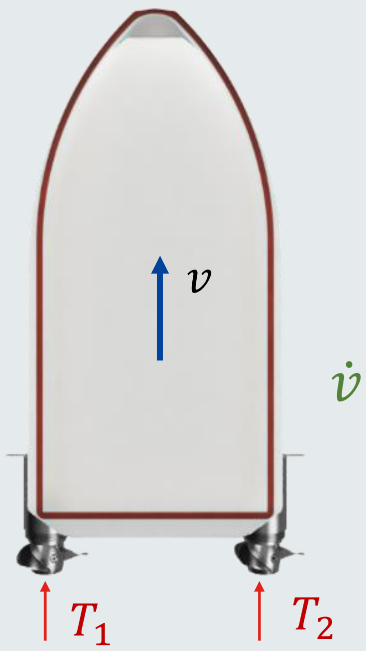
$$J\dot{\omega} = \frac{L(T_1 - T_2)}{2} - Q_d$$

moment of inertia, J [kgm^2]



Dynamic equations

Integrate accelerations $\dot{v}$ and $\dot{\omega}$ with respect to time to get velocities



$$\dot{v} = \frac{(T_1 + T_2) - F_d}{m}$$

$$v = \int \dot{v}\, dt$$

$$\dot{\omega} = \frac{L(T_1 - T_2) - Q_d}{2J}$$

$$\omega = \int \dot{\omega}\, dt$$

Steering Response

Example: Simulink demonstration

$$m\,\dot{v} = T_1(t) + T_2(t) - b_d\, v\, |v|$$

$$J\,\dot{\omega} = \frac{L(T_1(t) - T_2(t))}{2} - b_{d\omega}\, \omega\, |\omega|$$

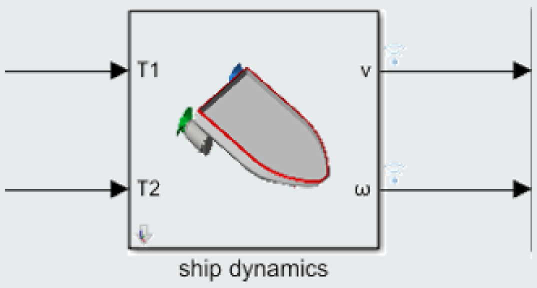
$$v_x = v\, \cos\theta$$

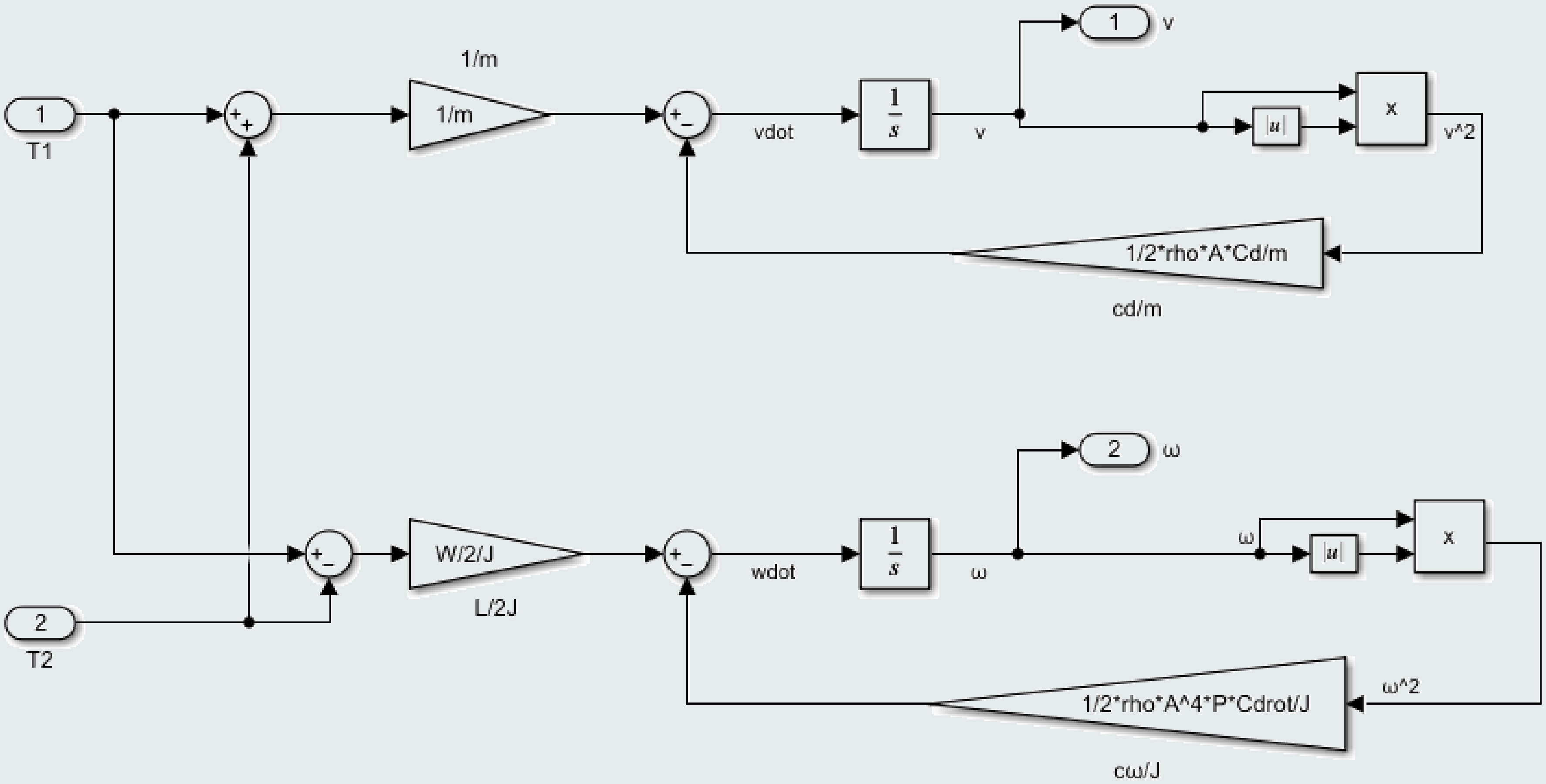
$$v_y = v\, \sin\theta$$

$$\frac{d\theta}{dt} = \omega$$

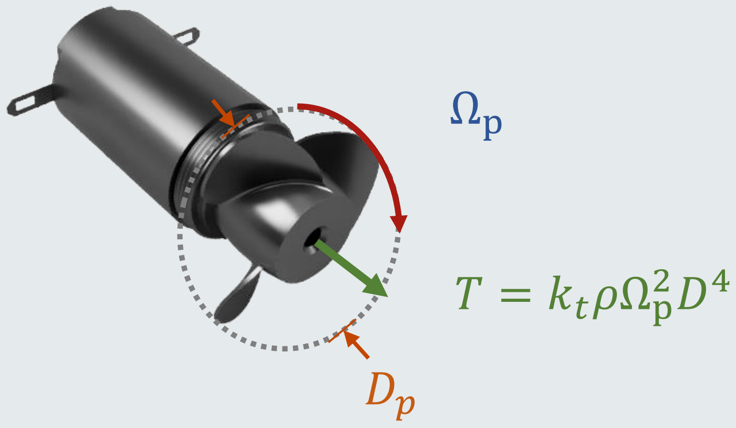
Exercise: Vary thrust, mass, inertia and drag and inspect ship response

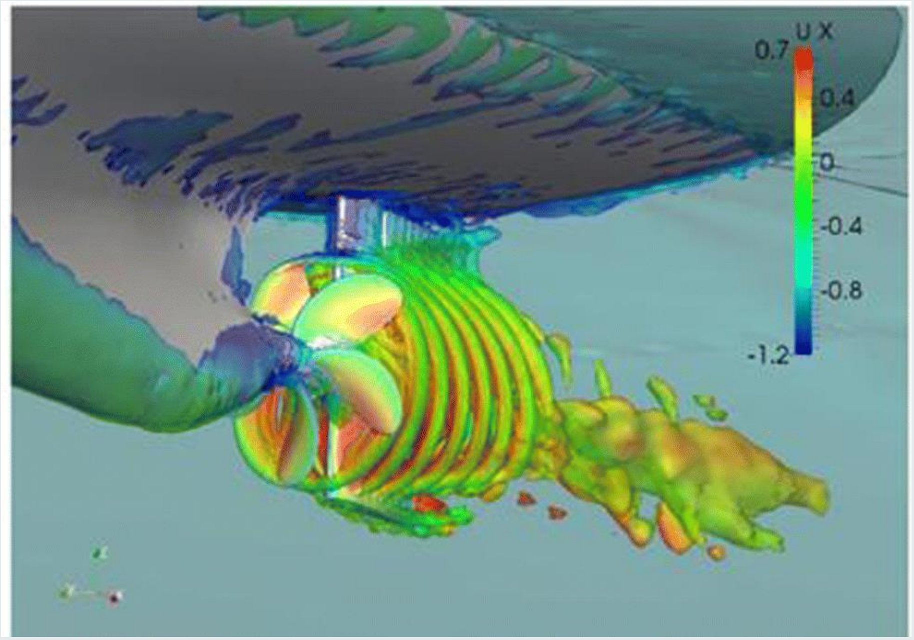
Solution: Dynamic equations of Ship



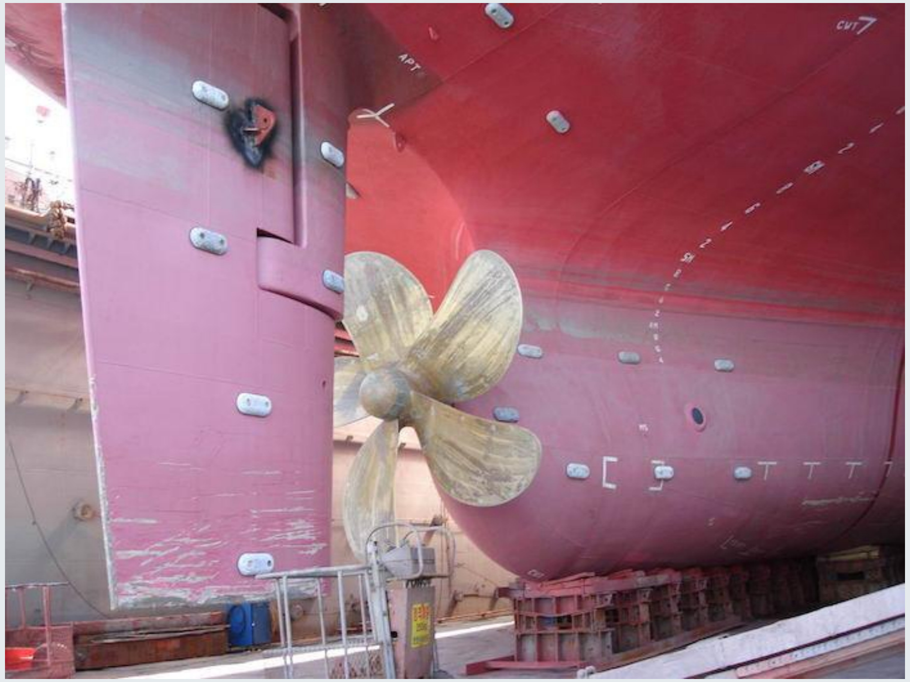


Propeller

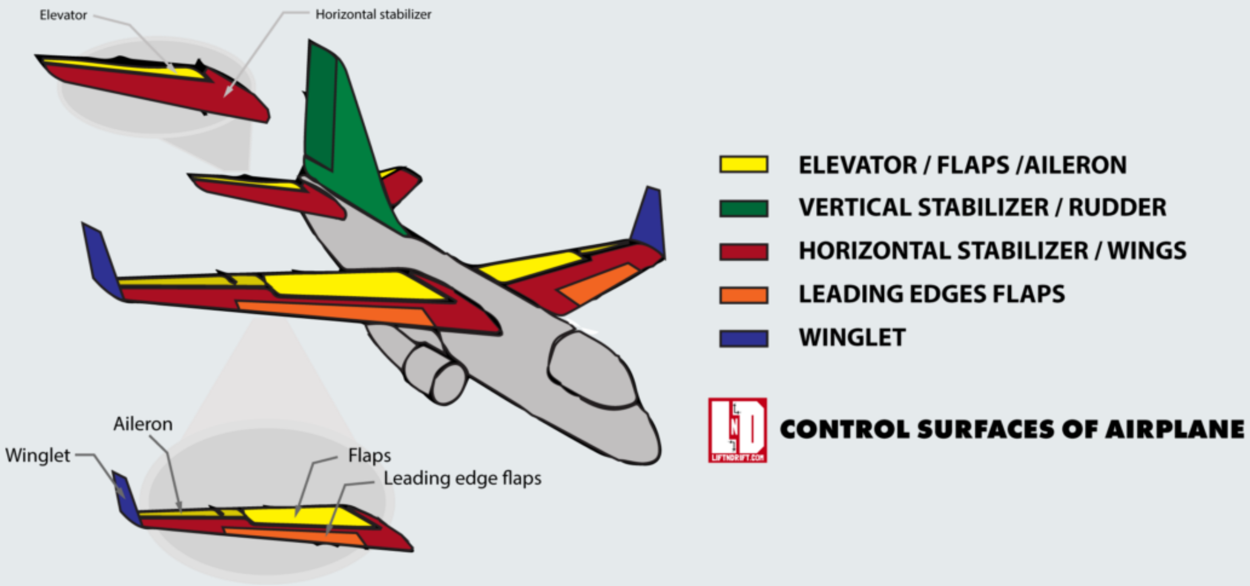




Control surfaces



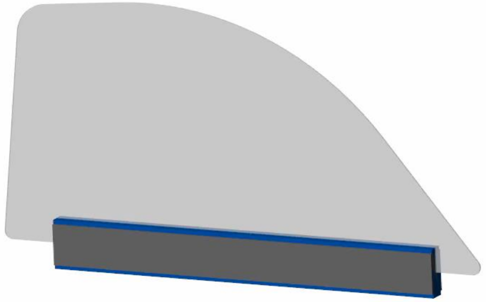
Rudders



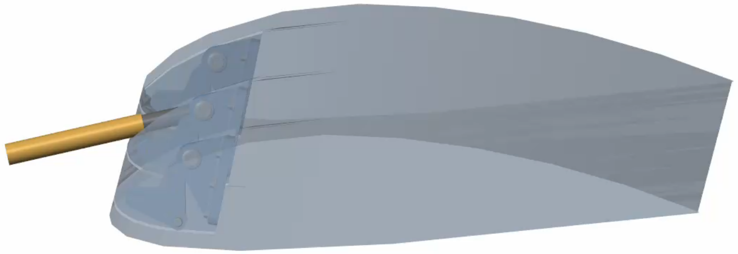
Aileron & Elevators

Rotational vs Translational Motion

Engineers design **mechanisms** to control the motion of components



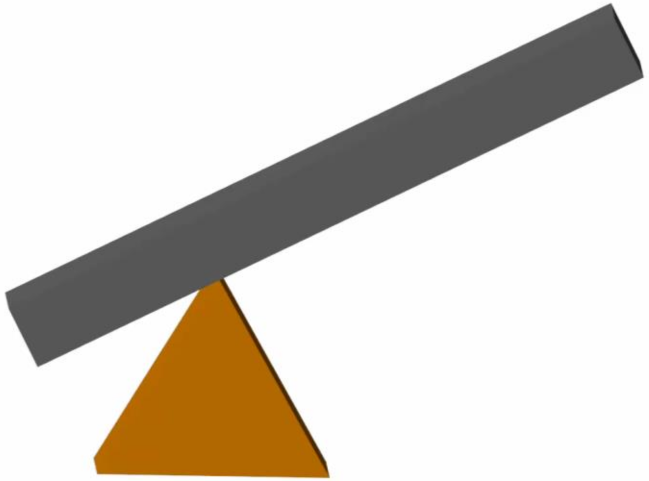
car window



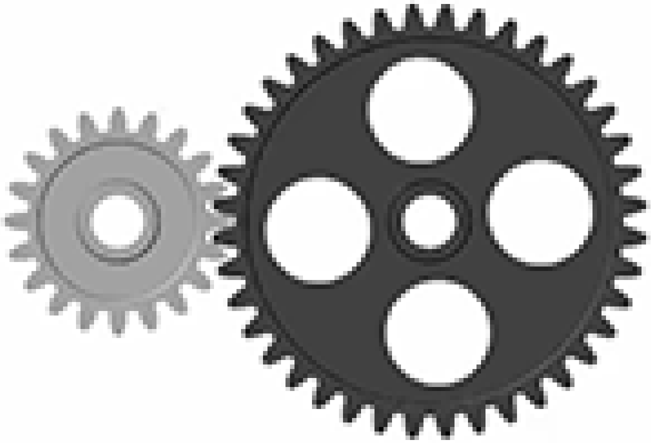
aileron

Mechanical advantage

Power = Force × speed *vs.* Torque × rotational speed



lever



spur gear

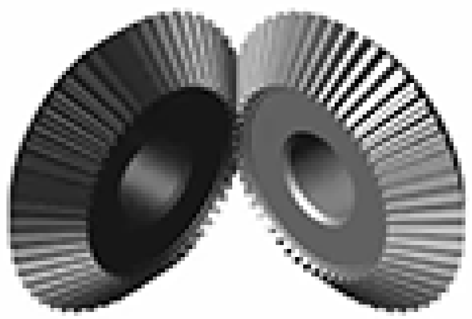
Load transfer

Control how forces are distributed in components

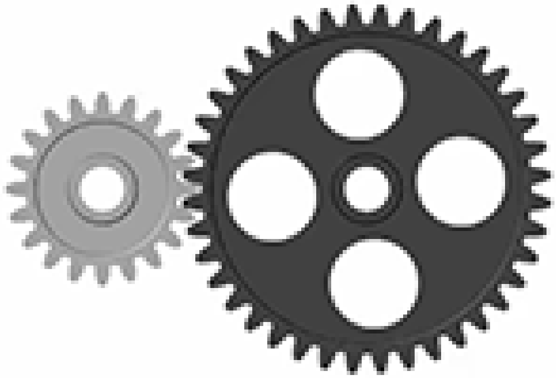
shafts

belts & pulleys

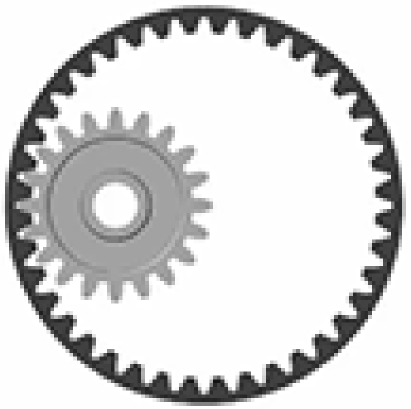
Rotational Mechanisms



Bevel Gear



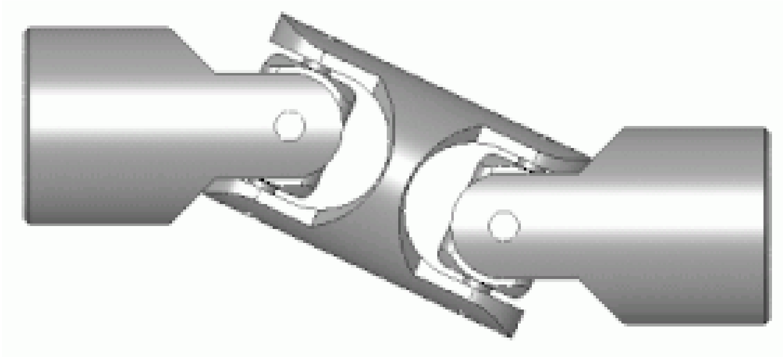
External Spur Gear



Internal Spur Gear

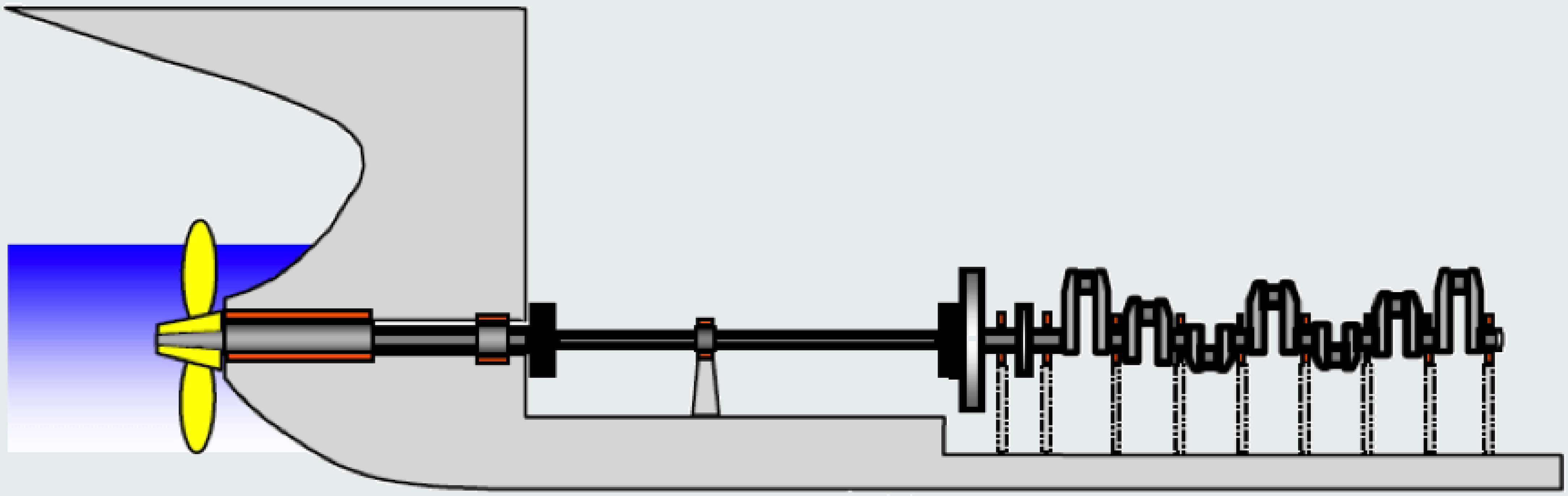
Rotational Mechanisms – Drivetrains & Shafts

Propeller shaft with bearings

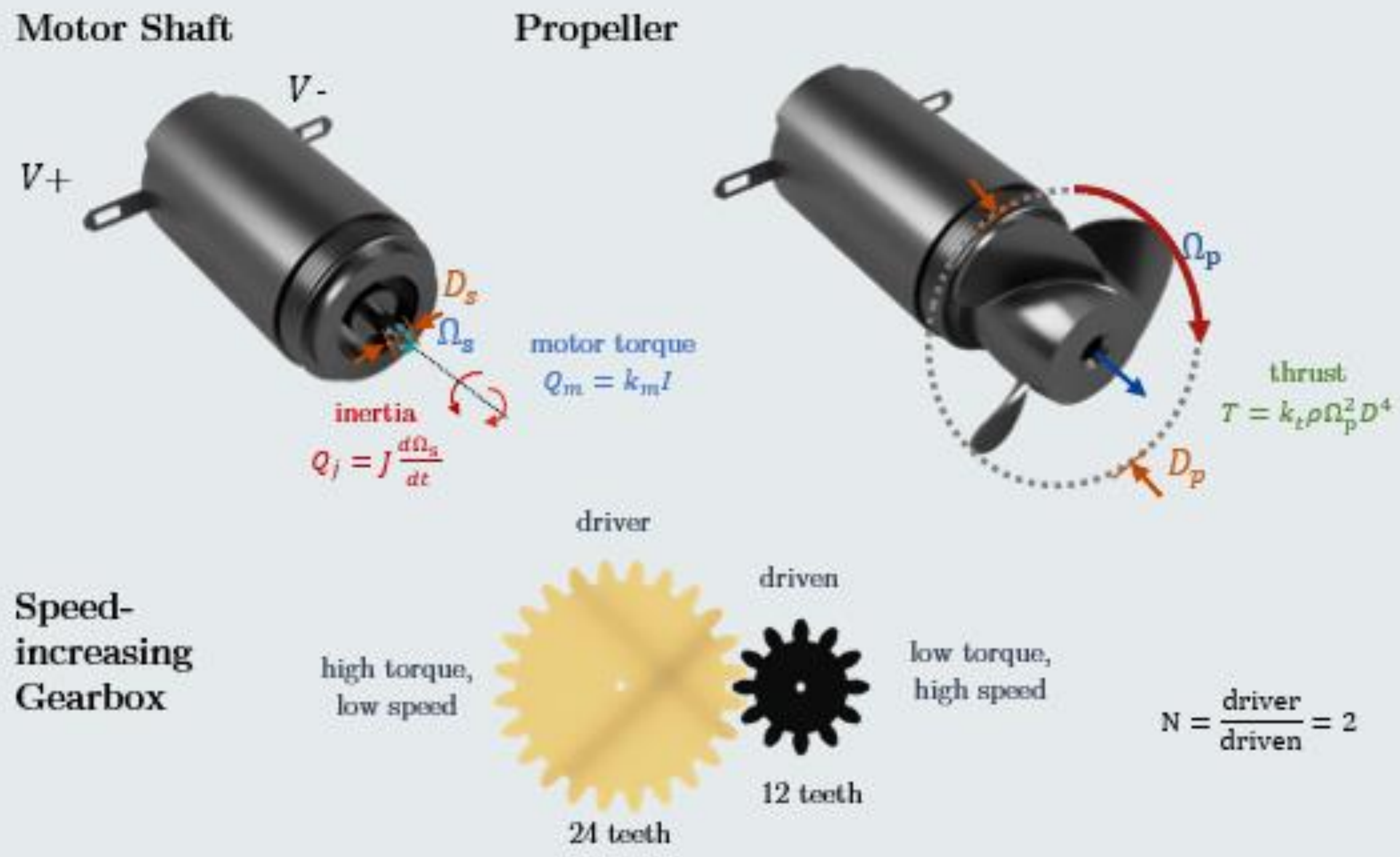


Propeller Shafts with Universal Joints

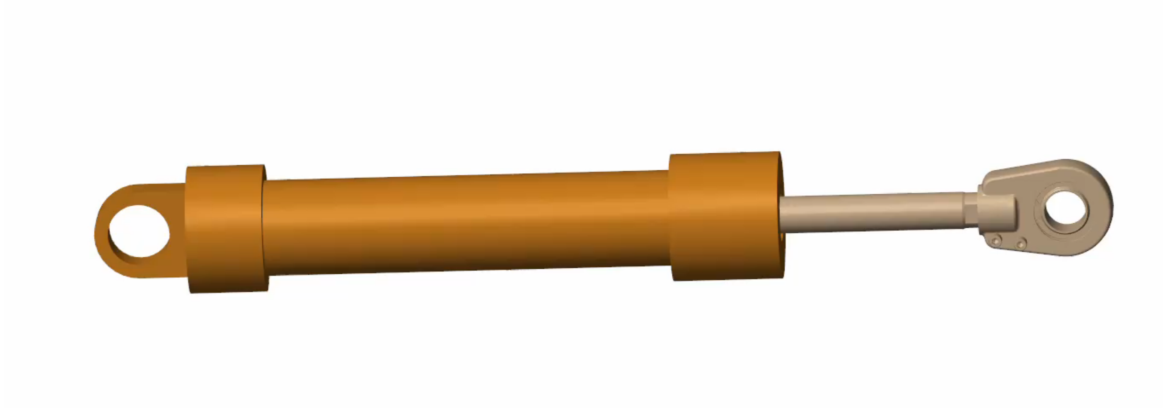
Rotational Mechanisms – Drivetrains & Shafts



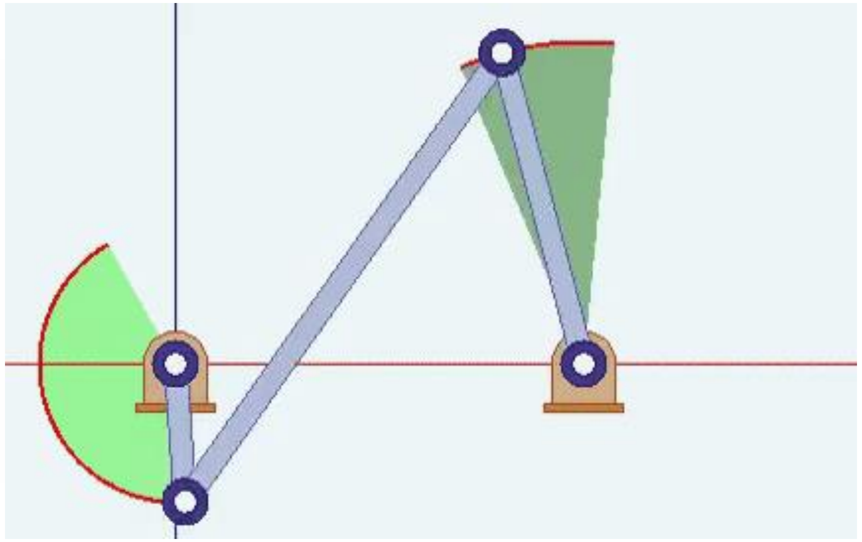
Ship Gearbox



Translational Mechanisms

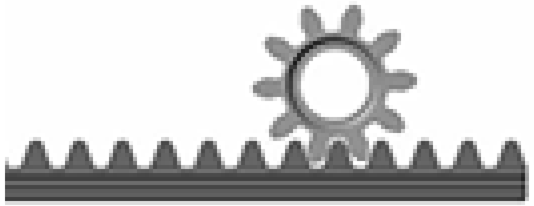


Pistons

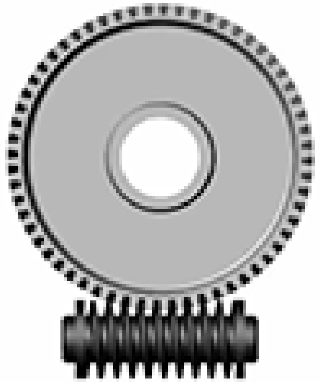


crank-rocker

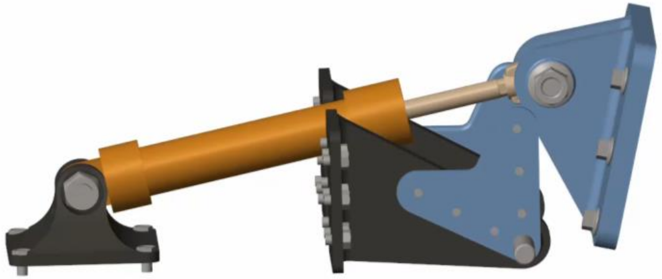
Converter Mechanisms

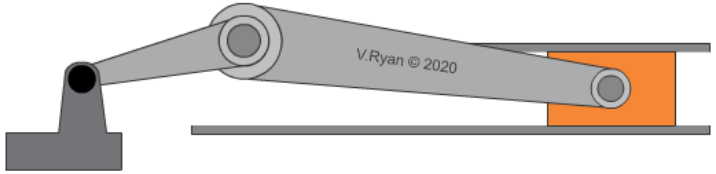


rack and pinion



worm gear



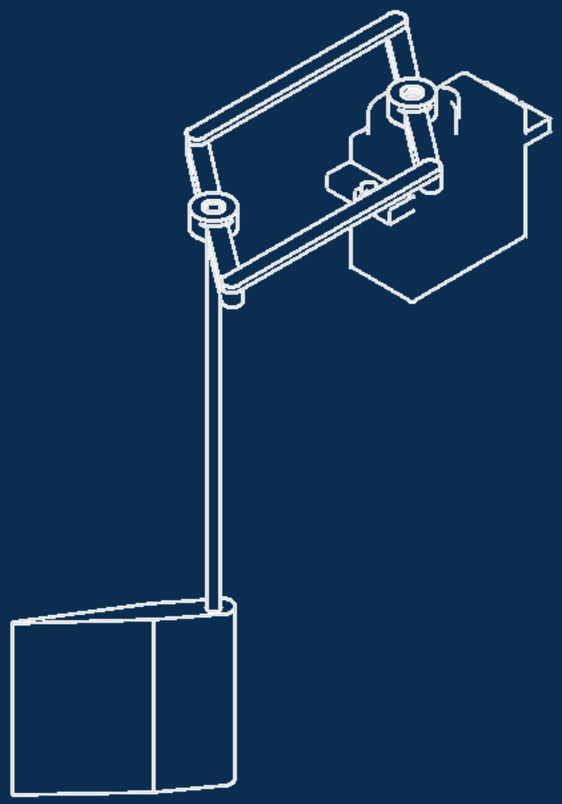


crank and slider

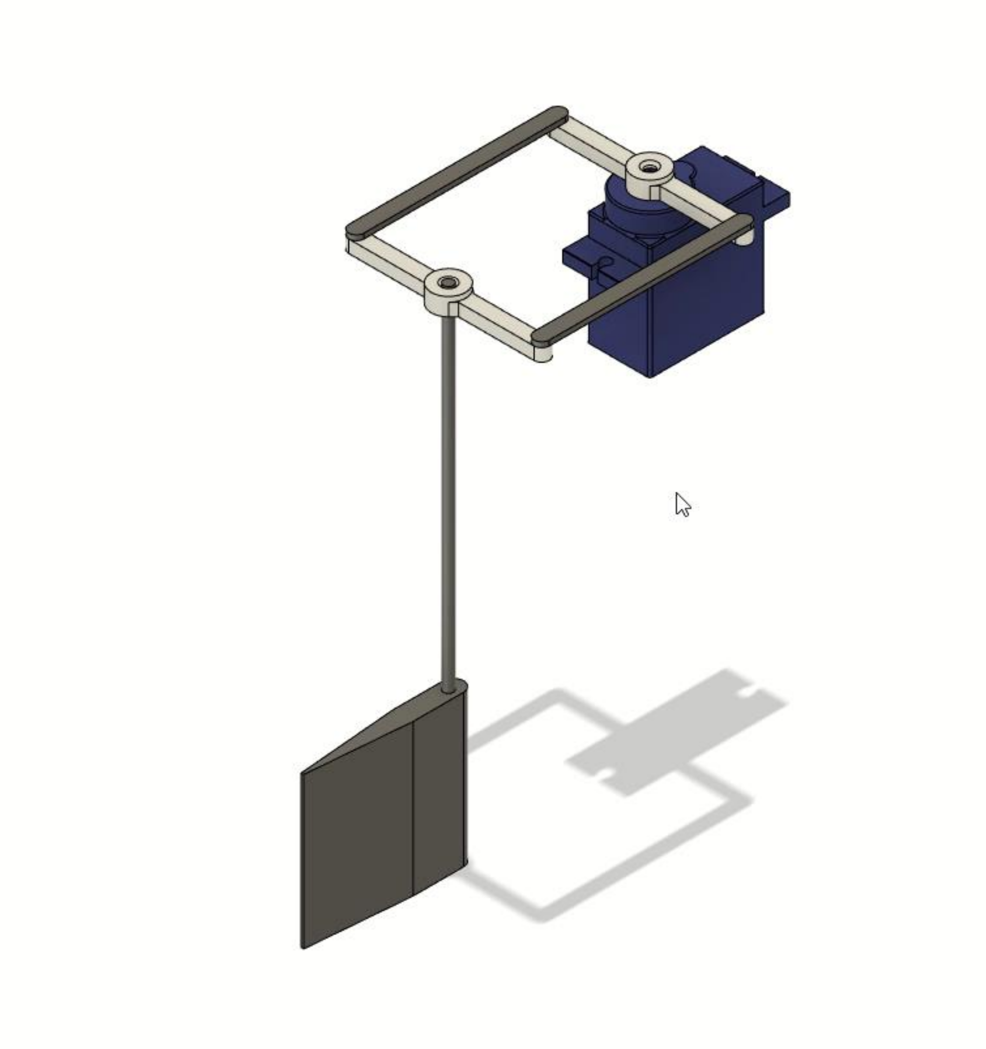
Rudder design

Individual exercise - sketch a rudder mechanism
(3 min)

Share drawing in small group - breakout rooms of 3



Solution: Rudder Mechanism



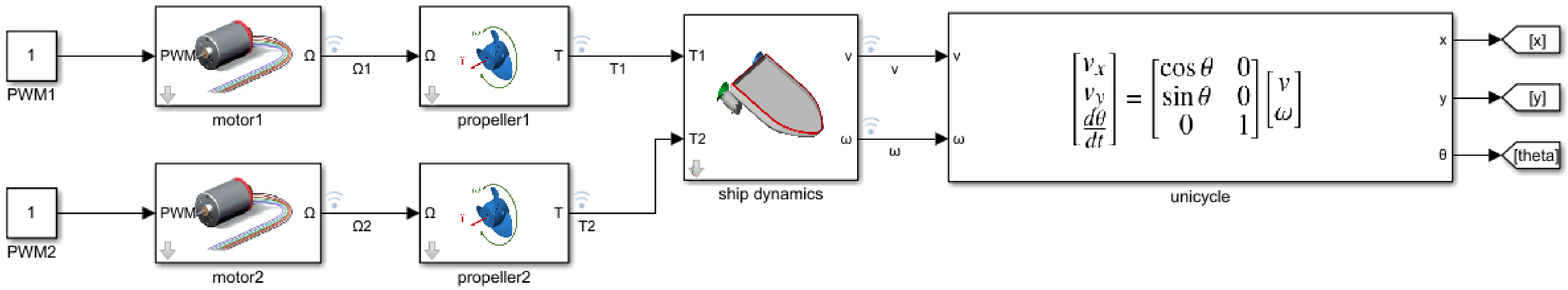
Conclusion

In this session we covered how to…

- Deduce relationships between physical quantities by applying dimensional analysis
- Draw a free body diagram for a mechanical component and identify the forces acting on it.
- Define inertia, drag and stiffness and critique how these mechanical properties affect system response.
- Explain how and why to use mechanical mechanisms (load transfer, motion control, operational efficiency, etc.).
- Interface with the simulation template that you will use for the individual coursework project

Lab Preview

In the second lab, you will use Simulink to model ship components. The assembly of all the components forms the ship model you will use in the individual coursework.



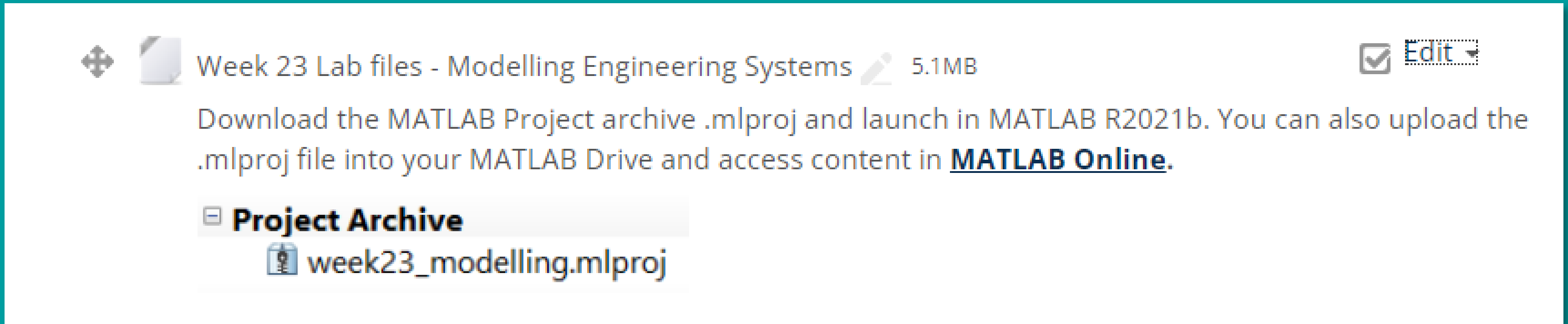
$$\begin{bmatrix} v_x \\ v_y \\ \frac{d\theta}{dt} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

During Computer Lab – Week 23

Labs held in Bush House (S)7.01/2 (Lab)

Recommended: Bring your own laptop to run the tutorial on MATLAB Desktop

Files available for download from KEATS


Week 23 Lab files - Modelling Engineering Systems 5.1MB Edit

Download the MATLAB Project archive .mlproj and launch in MATLAB R2021b. You can also upload the .mlproj file into your MATLAB Drive and access content in **MATLAB Online**.

Project Archive

week23_modelling.mlproj