Advanced Automotive Engineering

PIPE VIBRATION

Mechanical Vibration

Academic Year: 2020/2021

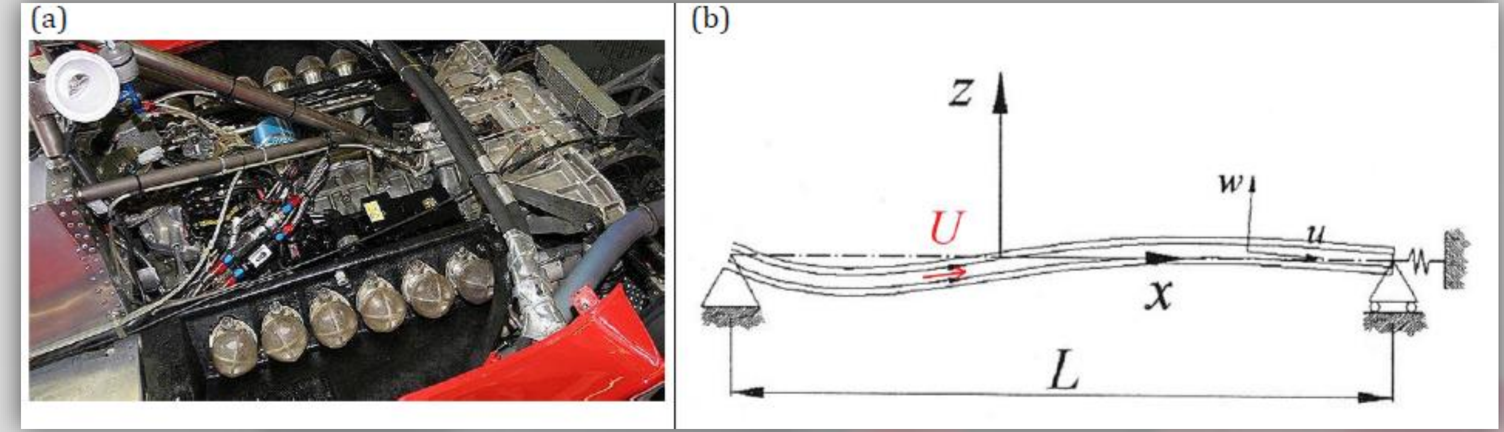
Prof. F. Pellicano

Prof. A. Zippo

Ass. G. Iarriccio

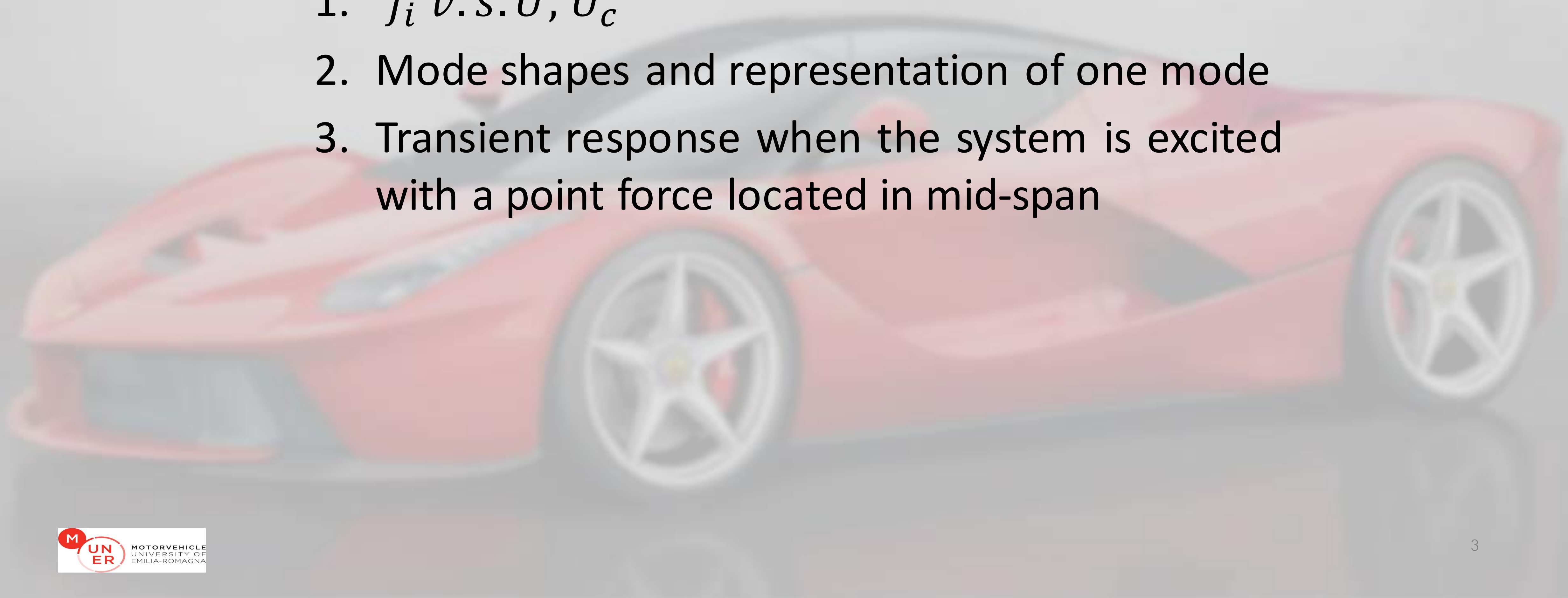
Group D:
Juan Sebastian Cepeda Munevar
Giacomo Chinni
Giovanni Coppola
Francesco Cuius Iuculano
Marco Cunaccia
Giacomo Davanzo
Edoardo d'Amicis
Denis Tonini

INTRODUCTION



- Pipe with a flowing fluid $U = const$.
- Transversal oscillation w

TARGETS

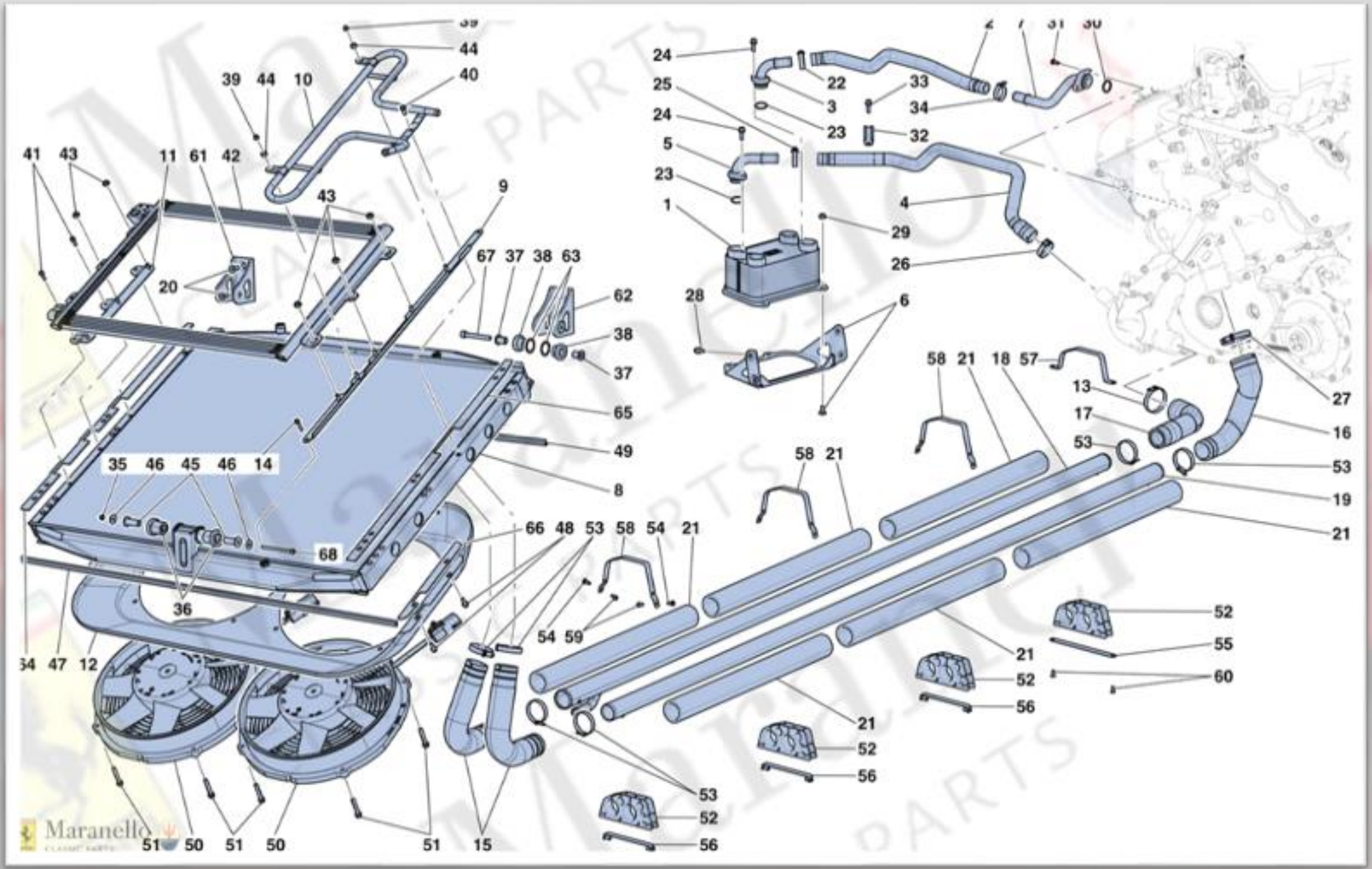


1. $f_i\ v.s.U, U_c$
2. Mode shapes and representation of one mode
3. Transient response when the system is excited with a point force located in mid-span

MOTORVEHICLE UNIVERSITY OF EMILIA-ROMAGNA

CASE OF STUDY

Ferrari LaFerrari: pipes from front radiator to rear engine



MODEL HYPOTHESIS

1. Euler-Bernoulli beam equation
2. Both supported ends
3. Small section compared to length (slender pipe)
4. Uncompressible fluid
5. Coriolis and centrifugal forces
6. Axial displacements negligible compared to the transversal ones

SYSTEM PARAMETERS

Rubber reinforced pipe with flowing water	
Fluid density (water) $[\rho_f]$	$1077\ kg/m^3$
Flow rate at max rpm $[Q]$	$4\ l/s$
Length $[L]$	$2\ m$
External diameter $[D_o]$	$19\ mm$
Thickness $[t]$	$3{,}9\ mm$
Rubber density $[\rho_p]$	$2300\ kg/m^3$
Young modulus $[E]$	$100\ Mpa$
Poisson coefficient $[\nu]$	$0{,}49$
Viscosità cinematica $[\eta]$	$3{,}98 \cdot 10^{-6}\ m^2/s$

SYSTEM PARAMETERS

- Internal diameter: $D_i = D_o - 2t = 11{,}2\ [\mathrm{mm}]$
- Mean velocity: $U = \frac{Q}{2A} = 20{,}3 \left[\frac{m}{s}\right];\ NOTICE!\ 2\ pipes.$
- Flexural inertia moment: $I = \frac{\pi({D_o}^4 - {D_i}^4)}{64} = 5{,}62\ \cdot 10^{-9}\ [m^4]$
- $\bar{p} = 2 \cdot 10^5\ [Pa]$
- $\bar{T} = 100\ [\mathrm{N}]$
- Friction effect: $Re = \frac{UD_i}{\eta}$

$$Re < 2300 \qquad \lambda = \frac{75}{Re}$$

$$Re \geq 2300 \qquad \lambda = 0{,}3164/Re^{1/4}$$

In our case, when $U = \text{const.}$, $Re \geq 2300, \lambda = 0.205.$

MUNER MOTORVEHICLE UNIVERSITY OF EMILIA-ROMAGNA

EOM, PDE

From:

$$EI\frac{\partial^4 w}{\partial x^4} + \left\{MU^2\left(\frac{\lambda L}{4D_i}+1\right) - \bar{T} + \bar{p}A(1-2v)\right\}\frac{\partial^2 w}{\partial x^2} + 2MU\frac{\partial^2 w}{\partial x \partial t} + (M+m)\frac{\partial^2 w}{\partial t^2} = 0$$

Simplifications:

$$\bar{T} = 0;\ \bar{p} = 0; \lambda = 0$$

Hence:

$$EI\frac{\partial^4 w}{\partial x^4} + MU^2\frac{\partial^2 w}{\partial x^2} + 2MU\frac{\partial^2 w}{\partial x \partial t} + (M+m)\frac{\partial^2 w}{\partial t^2} = 0$$

GALERKIN METHOD

EOM, PDE, compact form:

$$\ddot{w}(x,t) + \mathcal{L}\big(w(x,t)\big) = 0$$

$$where$$

$$\mathcal{L}(\cdot) = \frac{EI}{M+m}\frac{\partial^4 w}{\partial x^4} + \frac{1}{M+m}\left\{MU^2\left(\frac{\lambda L}{4D_i}+1\right) - \bar{T} + \bar{p}A(1-2v)\right\}\frac{\partial^2 w}{\partial x^2} + \frac{2MU}{M+m}\frac{\partial^2 w}{\partial x \partial t}$$

Displacement, Galerkin form:

$$w(x,t) = \sum_{j=1}^{N} q_j(t)\phi_j(x)$$

Substituting and projecting in Hilbert space (functions internal product thus integration):

$$\int_0^L [(M+m)\phi_j\phi_n]dx\,\ddot{q}_n + \int_0^L [2MU\phi_j^I\phi_n]dx\,\dot{q}_n + \int_0^L \left[\left[EI\phi_j^{IV} + \left(MU^2\left(\frac{\lambda L}{4D_i}+1\right) - \bar{T} + \bar{p}A(1-2v)\right)\phi_j^{II}\right]\phi_n\right]dx q_n = 0$$

i.e. $\quad \boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{K}\boldsymbol{q} = \boldsymbol{0}$

GALERKIN METHOD, DISPLACEMENT

$$w(x,t) = \sum_{j=1}^{N} q_j(t)\phi_j(x)$$

Trial function ϕ_j must respect boundary conditions.

Hinged trial function: $\phi_j(x) = \sin\left(\frac{j\pi x}{L}\right)$ $with\, j = 1,..,7$

CRITICAL VELOCITY

- Simple model, hinged pipe
- Dimensionless velocity: $u = \sqrt{\frac{M}{EI}}UL$
- When $u \to \pi \;\Rightarrow f_1 \to 0$ and buckling of beam
- $U_c = \frac{\pi}{L}\sqrt{\frac{EI}{M}}$

IMPULSE POINT LOAD

- Creation impulse located in the middle.

$$F(x,t) = F_1(t)\delta(x - x_F) \quad where \quad x \in (0, L) \quad t > 0 \; and \; x_F = \frac{L}{2}$$

- Galerkin's projection

$$\int_0^L F_1(t)\delta\left(x - \frac{L}{2}\right)\phi_j dx = F_1(t)\phi_j\left(\frac{L}{2}\right)$$

WHAT WE HAVE DONE

STANDARD PROJECT		
Point 1.1	simple model	$U_c < U$
Point 1.2	$\bar{p}$	$U_c < U$
Point 1.3	λ	$U_c < U$
Point 1.4	$\bar{T}$	$U_c > U$
Point 1.5	$\bar{p}, \bar{T}, \lambda$	
Point 2.1	$\bar{p}, \bar{T}, \lambda$	
Point 2.2	$\bar{p}, \bar{T}, \lambda$	$U = 0$
Point 3.1	$\bar{p}, \bar{T}, \lambda$	

FURTHER DEVELOPMENT	
Point 2	$\frac{dU}{dt}, \bar{p}, \bar{T}, \lambda$
Point 3	$\overline{\frac{dU}{dt}}, p, \bar{T}, \lambda$
Point 1	clamped, $\bar{p}, \bar{T}, \lambda$
Point 2	clamped, $\bar{p}, \bar{T}, \lambda, \frac{dU}{dt}$
Point 3	clamped, $\bar{p}, \bar{T}, \lambda, \frac{dU}{dt}$

All points matrices $\boldsymbol{M}, \boldsymbol{C}, \boldsymbol{K}$ calculated NUMERICALLY:

- x discretized with 10000 points
- t discretized every $1\ ms$, window of 10 s

Points 1.5, 2.1, 2.2, 3.1 checked calculating matrices $\boldsymbol{M}, \boldsymbol{C}, \boldsymbol{K}$ SIMBOLICALLY.

- NOTICE! Point 1.5 λ friction coefficient approximated turbolent for every U

ODEs from PDEs solved always simbolically with ode45 MATLAB function

POINT 1, CODE

- Definition of parameters
- Definition of trial function $\phi_j(x)$
- Matrices $\boldsymbol{M}, \boldsymbol{C}, \boldsymbol{K}$ calculation as explained before (Galerkin)
- $\lambda_i = \omega_i^2 = eigva_i\left(\frac{\boldsymbol{K}}{\boldsymbol{M}}\right)$
- Where $\boldsymbol{K} = \boldsymbol{K}(U)$ and $0\ m/s < U < 50\ m/s$
- Plot of $f_1(U)$
- When $U \rightarrow U_c$ (too high) buckling of the pipe, $f_1 \rightarrow 0$

POINT 2, CODE

- Definition of parameters
- Definition of trial function $\phi_j(x)$
- Matrices $\boldsymbol{M}, \boldsymbol{C}, \boldsymbol{K}$ calculation as explained before (Galerkin)
- $\boldsymbol{M\ddot{q}} + \boldsymbol{C\dot{q}} + \boldsymbol{Kq} = \boldsymbol{0}$

$$\ddot{\boldsymbol{q}} = -\frac{\boldsymbol{C}}{\boldsymbol{M}}\dot{\boldsymbol{q}} - \frac{\boldsymbol{K}}{\boldsymbol{M}}\boldsymbol{q}, \boldsymbol{p} = [\dot{\boldsymbol{q}}, \boldsymbol{q}]^T, \ \dot{\boldsymbol{p}} = [\ddot{\boldsymbol{q}}, \dot{\boldsymbol{q}}]^T$$

$$\dot{\boldsymbol{p}} = \begin{bmatrix} \ddot{\boldsymbol{q}} \\ \dot{\boldsymbol{q}} \end{bmatrix} = \begin{bmatrix} -\frac{\boldsymbol{C}}{\boldsymbol{M}} & -\frac{\boldsymbol{K}}{\boldsymbol{M}} \\ \boldsymbol{I} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{q}} \\ \boldsymbol{q} \end{bmatrix} = \begin{bmatrix} -\frac{\boldsymbol{C}}{\boldsymbol{M}} & -\frac{\boldsymbol{K}}{\boldsymbol{M}} \\ \boldsymbol{I} & \boldsymbol{0} \end{bmatrix} \boldsymbol{p}$$

$$\boldsymbol{A} = \begin{bmatrix} -\frac{\boldsymbol{C}}{\boldsymbol{M}} & -\frac{\boldsymbol{K}}{\boldsymbol{M}} \\ \boldsymbol{I} & \boldsymbol{0} \end{bmatrix}$$

- Eigenvectors of $\boldsymbol{A}$.
- $\dot{\boldsymbol{p}} = \boldsymbol{A} \cdot \boldsymbol{p}$, symbolical resolution, ode45, initial conditions: eigenvectors 1st mode.
- $w(x,t) = \sum_{j=1}^{N} q_j(t)\phi_j(x)$
- Plot of $9s$, every $1s$.

MOTORVEHICLE UNIVERSITY OF EMILIA-ROMAGNA

POINT 3, CODE

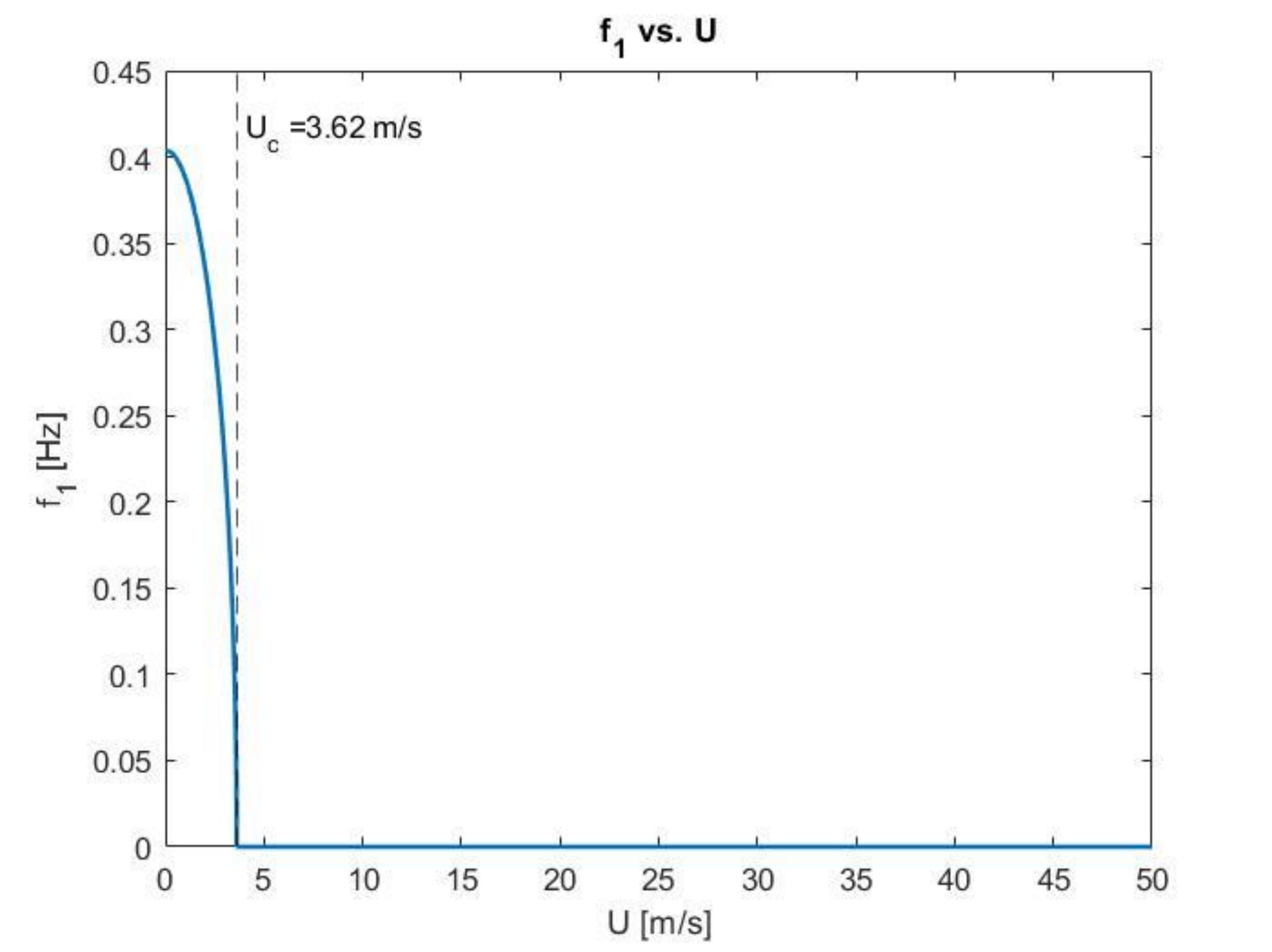
- Definition of parameters
- Definition of trial function $\phi_j(x)$
- Matrices $\boldsymbol{M}, \boldsymbol{C}, \boldsymbol{K}$ calculation as explained before (Galerkin)
- $\boldsymbol{A}$ like point 2
- Creation sinbump:

$$(duration[W], amplitude[A], time[P]) = (0{,}01s, 10N, 1s)$$

$$f(t) = A\left[0{,}5 + 0{,}5\cos\left(2\pi\frac{t-P}{W}\right)\right] [\mathrm{N}] \quad t \in \left(P - \frac{W}{2}, P + \frac{W}{2}\right)$$

- $F_i = -f(t) * \phi_j\left(\frac{L}{2}\right) \qquad j = 1, \dots, 7$
- $\dot{\boldsymbol{p}} = \boldsymbol{A} \cdot \boldsymbol{p} + \boldsymbol{F}$, symbolical resolution, ode45, initial conditions: $\boldsymbol{0}$
- $w(x,t) = \sum_{j=1}^{N} q_j(t)\phi_j(x)$
- Animation $10s$, sampled every 5 ms

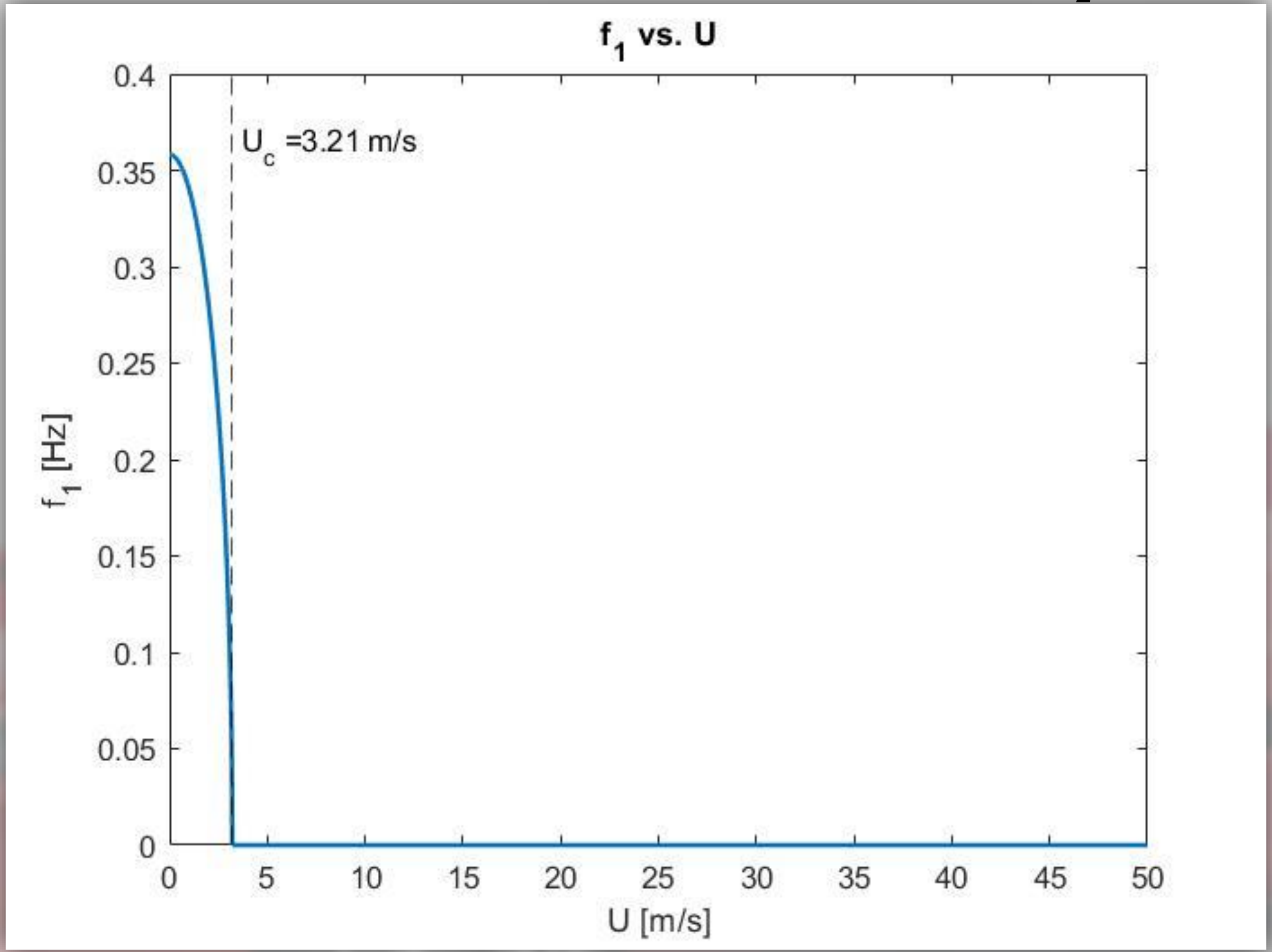
RESULT: POINT 1.1, SIMPLE MODEL



Theoretical calculation corresponds to numerical result:

$$\frac{\pi}{L}\sqrt{\frac{EI}{M}} = \frac{\pi}{2[m]}\sqrt{\frac{100[MPa]*5.62*10^{-9}[m^4]}{1077[kg/m^3]*\pi*5.6^2*10^{-6}[mm^2]}} = 3.62\left[\frac{\mathrm{m}}{s}\right] = U_c < U = 20.3\left[\frac{\mathrm{m}}{s}\right]$$

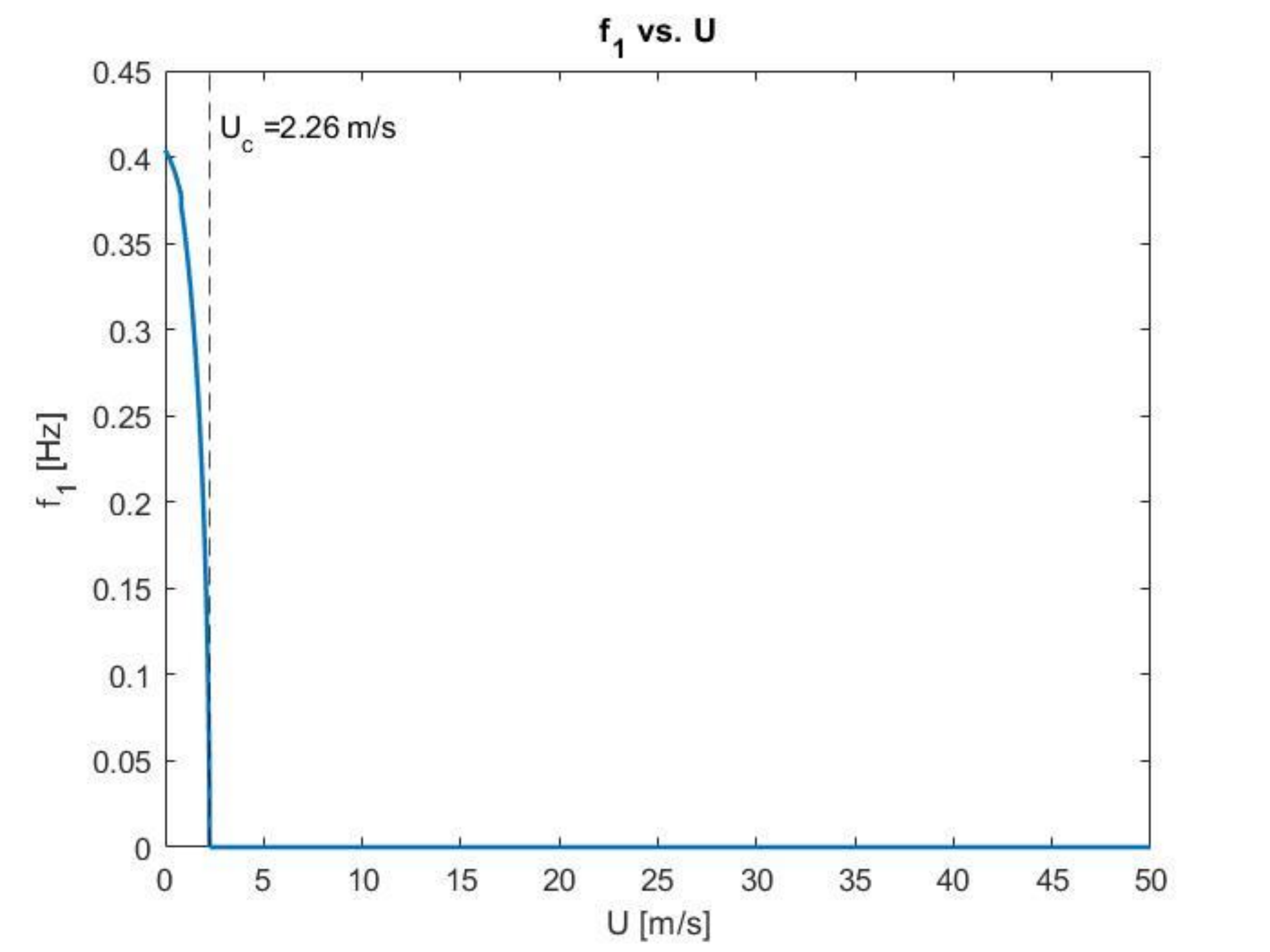
RESULT: POINT 1.2, ONLY $\bar{p}$



Real $\bar{p} = 2 * 10^5[Pa]$ but U_c<0 ⇒ Fake $\bar{p} = 2 * 10^4[Pa]$, for didactic purpose.

$$0 < U_c < U = 20.3\left[\frac{\mathrm{m}}{\mathrm{s}}\right]$$

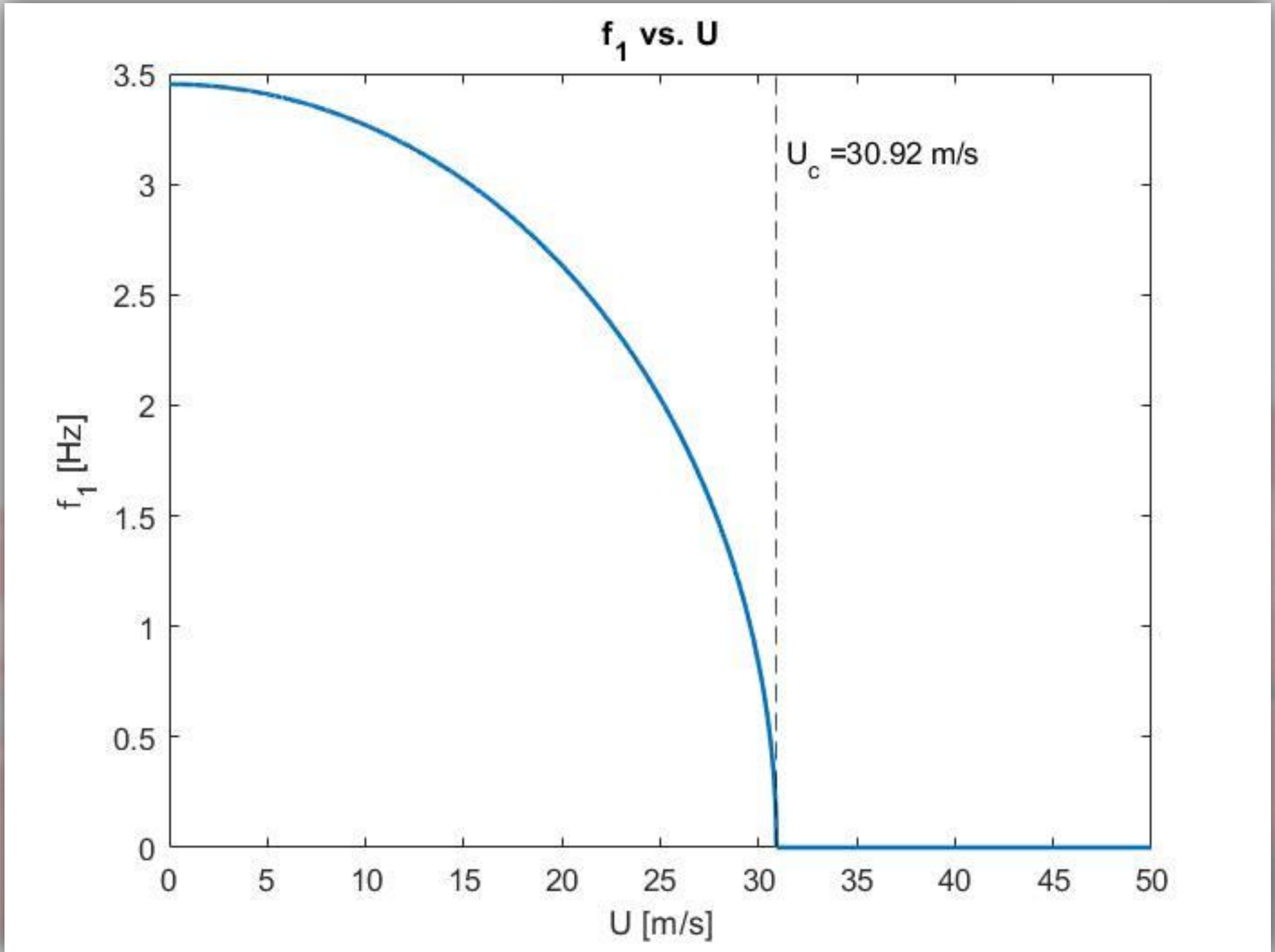
RESULT: POINT 1.3, ONLY λ



$$U_c < U$$

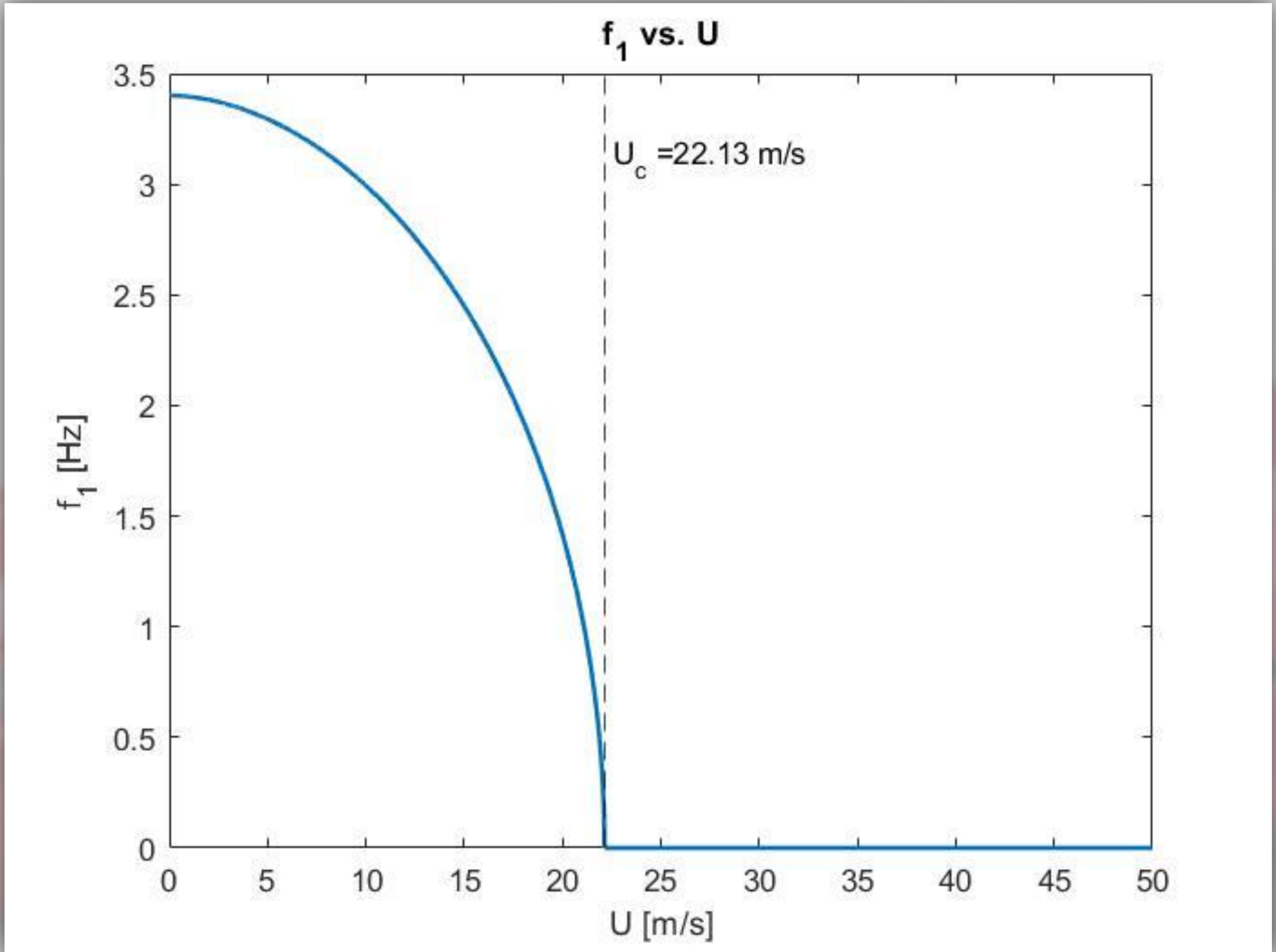
MOTORVEHICLE UNIVERSITY OF EMILIA-ROMAGNA

RESULT: POINT 1.4, ONLY $\bar{T}$



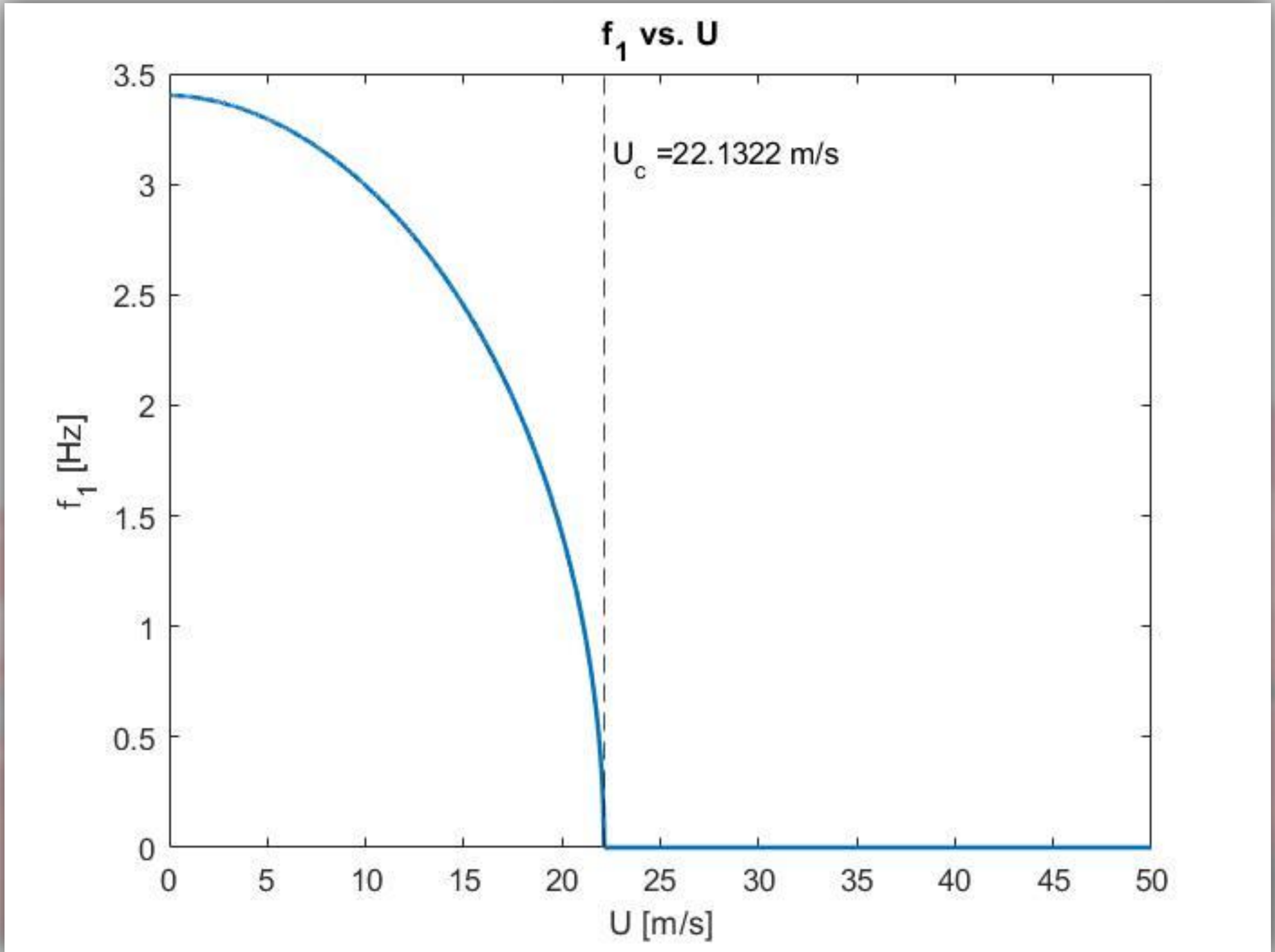
$$U_c > U$$

RESULT: POINT 1.5, COMPLETE MODEL $\bar{p}, \bar{T}, \lambda$



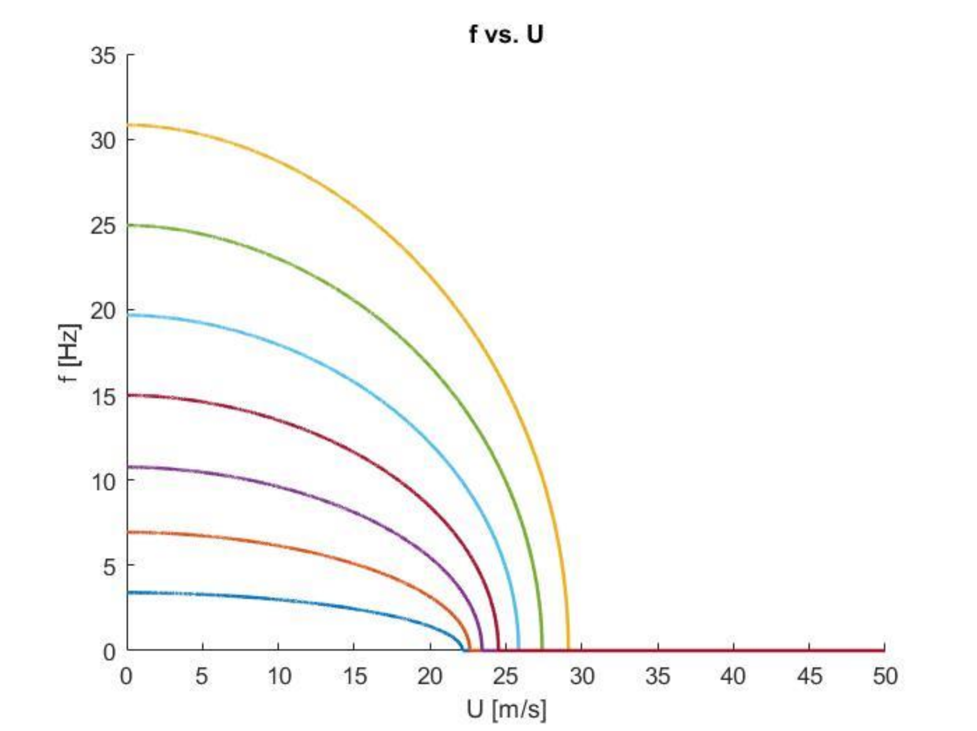
Numerical approach, $U_c > U$

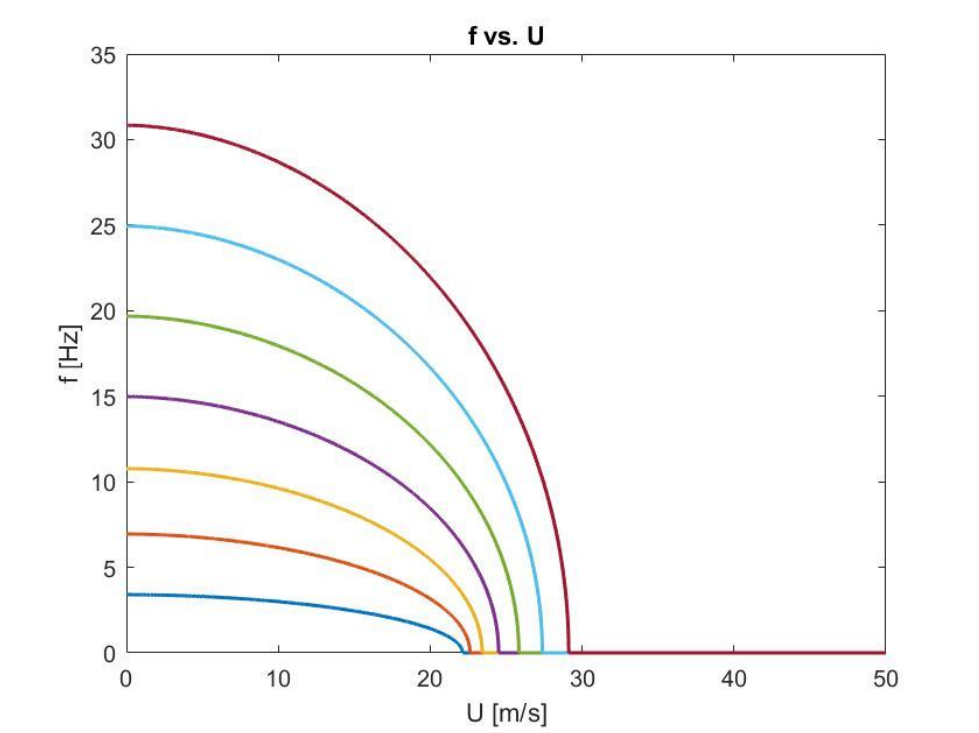
RESULT: POINT 1.5, COMPLETE MODEL $\bar{p}, \bar{T}, \lambda$



Symbolic approach, $U_c > U$

RESULT: POINT 1.5, COMPLETE MODEL $\bar{p}, \bar{T}, \lambda$





Symbolic approach

Numerical approach

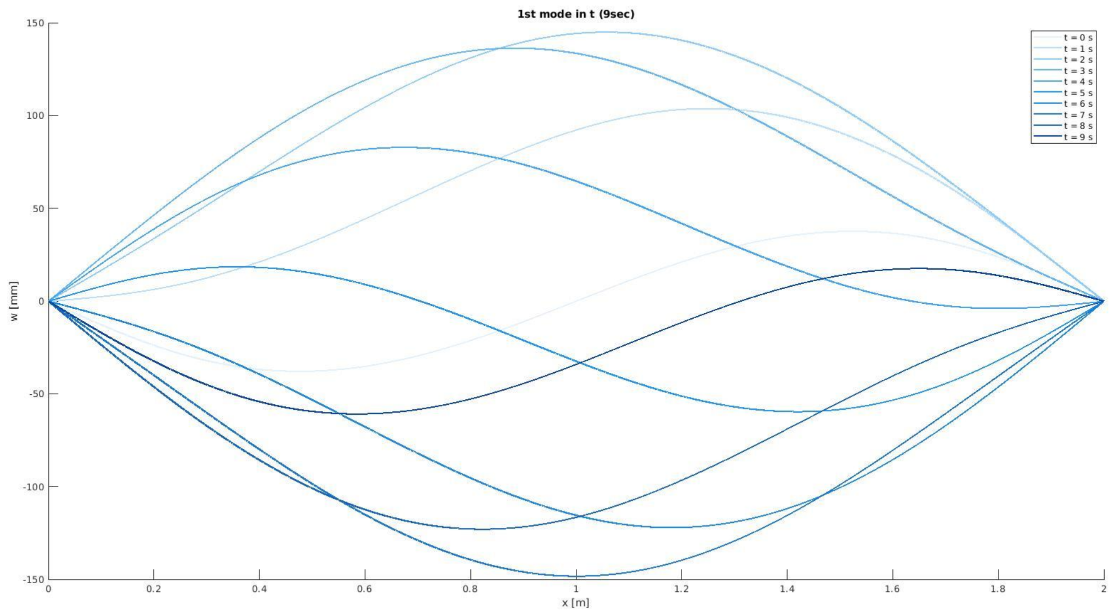
Natural frequencies vs. U

RESULT: POINT 1.5, COMPLETE MODEL $\bar{p}, \bar{T}, \lambda$

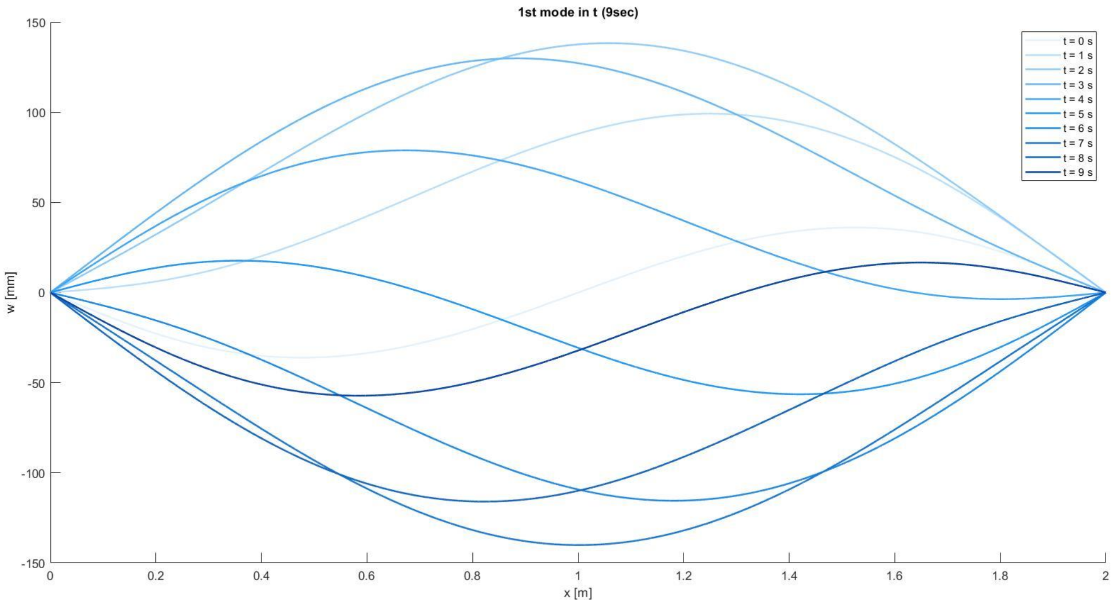
Natural frequencies for $U = 20.3 \left[\frac{m}{s}\right]$

i	$f_i[Hz]$
1	1.3170
2	2.9825
3	5.2304
4	8.1798
5	11.8851
6	16.3713
7	21.6502

RESULT: POINT 2.1, $U = 20.3[m/s]$

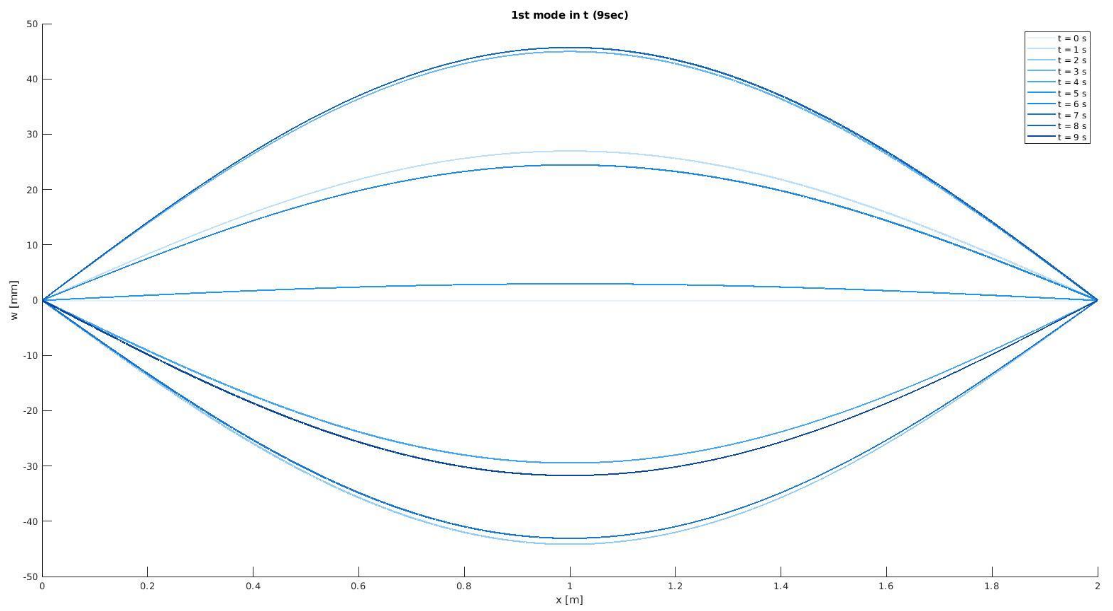


Symbolic approach

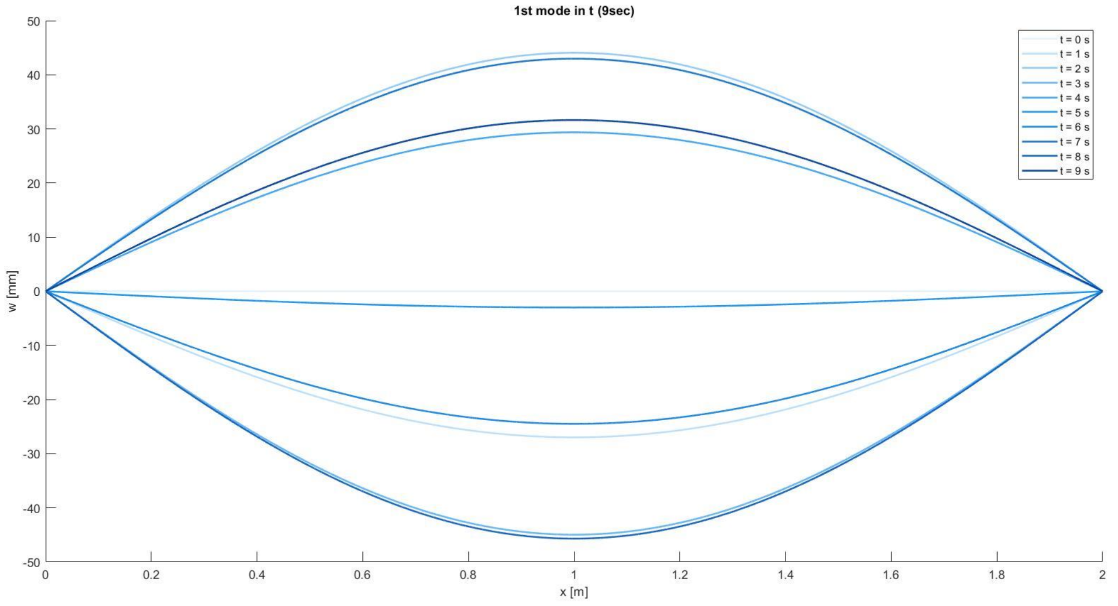


Numerical approach

RESULT: POINT 2.2, $U = 0[m/s]$

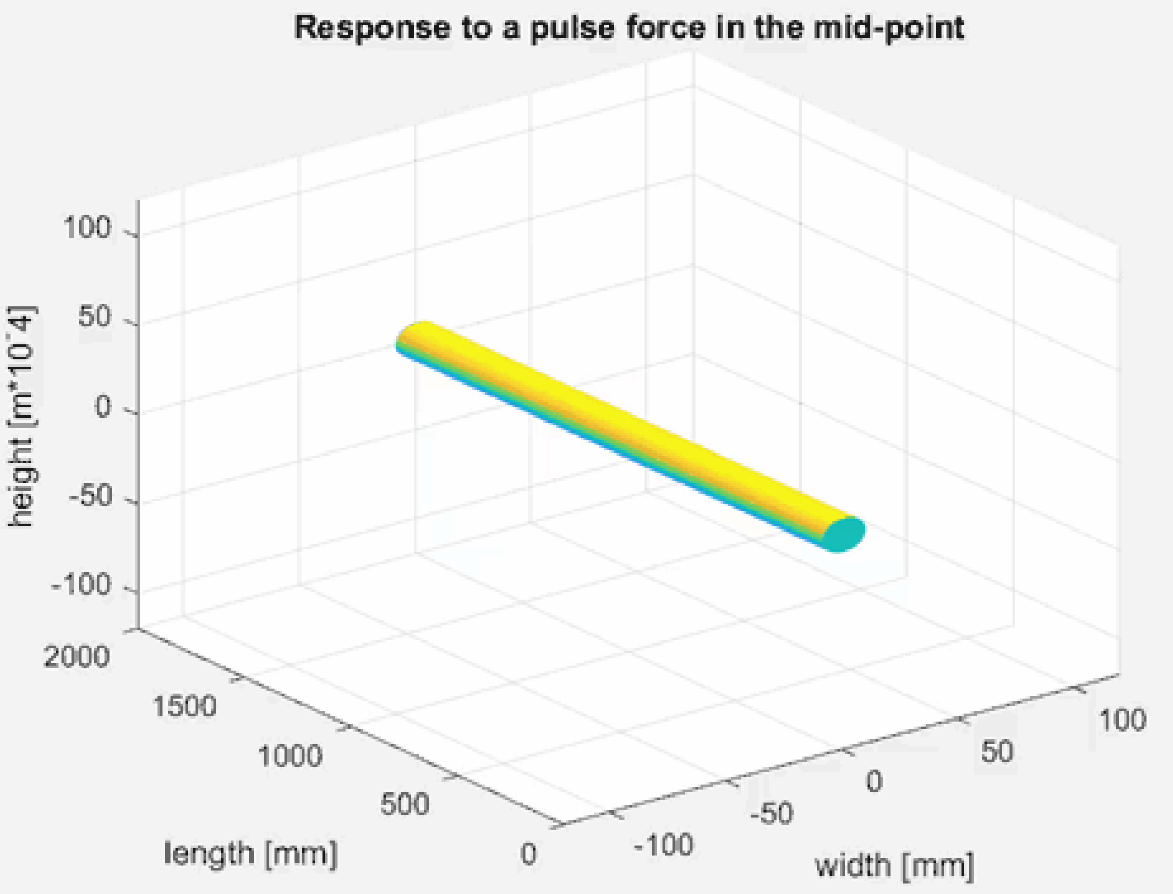


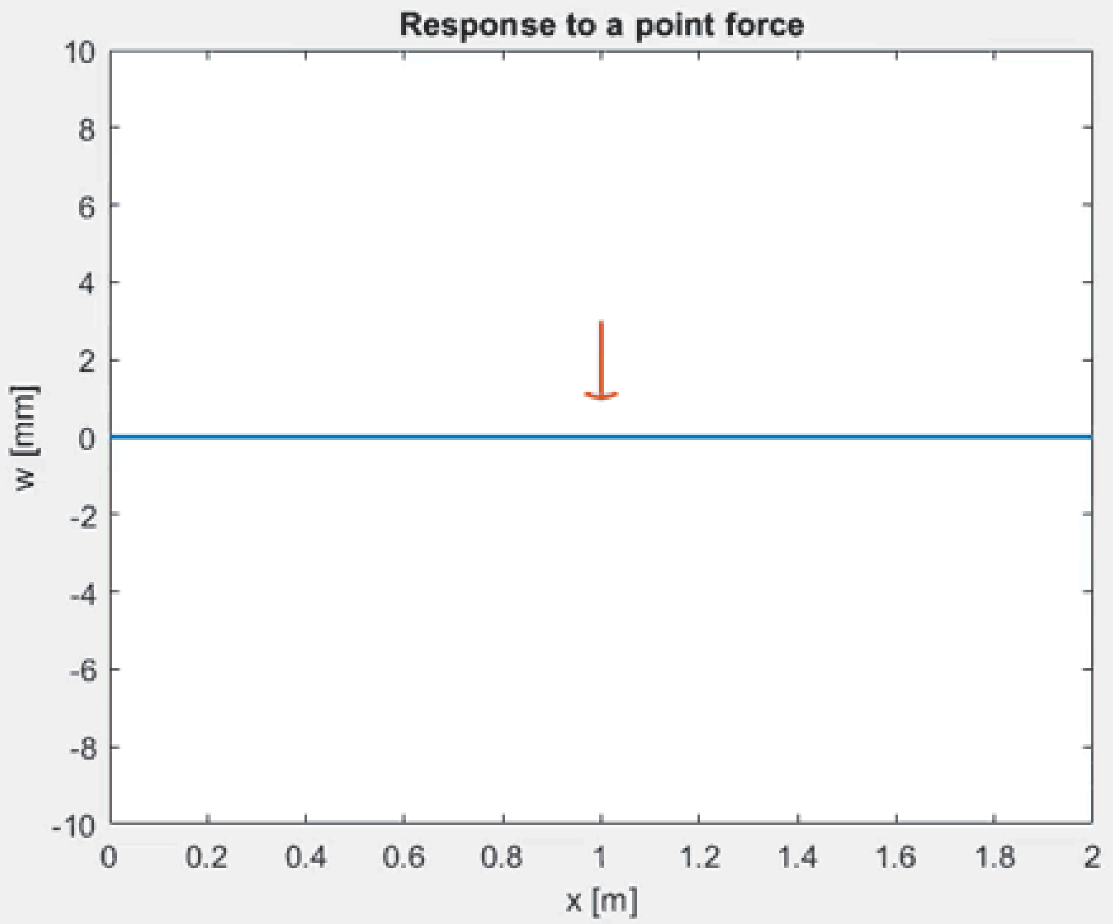
Symbolic approach



Numerical approach

RESULT: POINT 3.1, $U = const.$

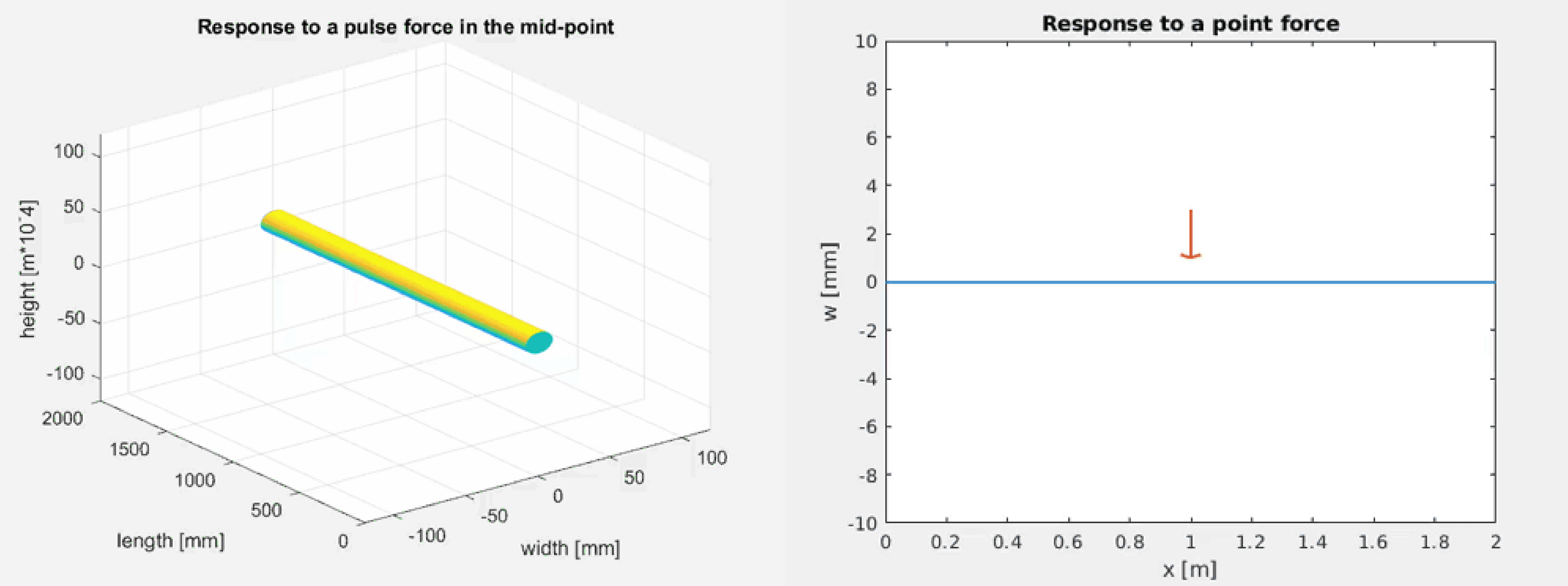




Numerical approach

MOTORVEHICLE UNIVERSITY OF EMILIA-ROMAGNA

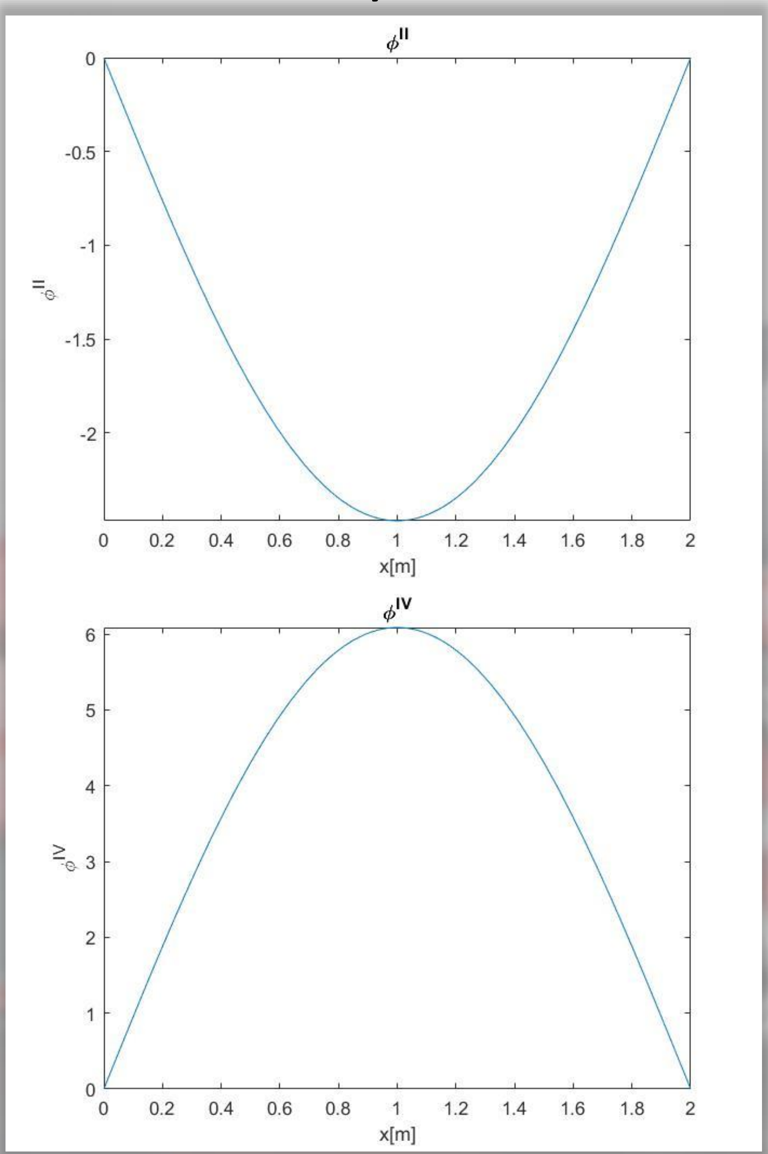
RESULT: POINT 3.1, $U = const.$

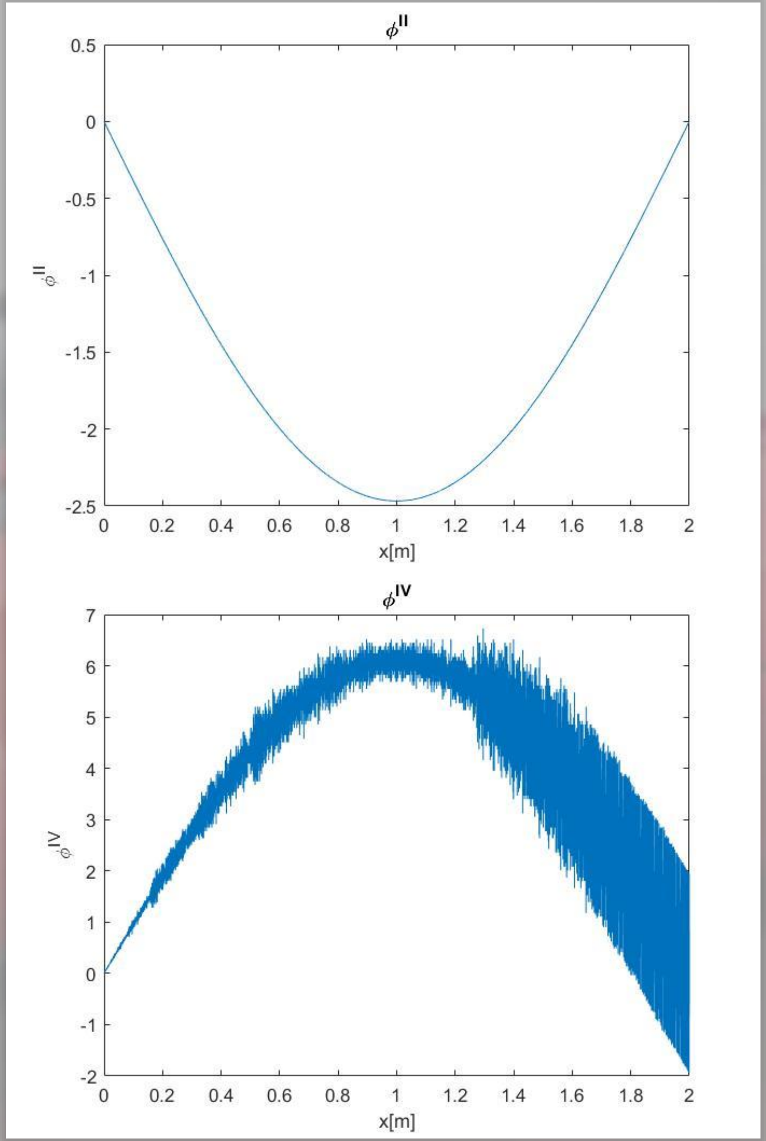


Symbolic approach

RESULT: POINT 1,2,3, COMPLETE MODEL

Symbolic vs. numerical derivatives of $\phi_1(x)$





FURTHER DEVELOPMENTS

- Simulation of a volumetric pump.

$$\frac{dU}{dt} \neq 0 \qquad U = 20{,}3 + 0{,}15\sin(100 \cdot 2\pi \cdot t)$$

EOM, PDE

$$EI\frac{\partial^4 w}{\partial x^4} + \left\{MU^2\left(\frac{\lambda L}{4D_i} + 1\right) - \bar{T} + \bar{p}A(1-2v) + M\frac{dU}{dt}(L-x)\right\}\frac{\partial^2 w}{\partial x^2} + 2MU\frac{\partial^2 w}{\partial x \partial t} + (M+m)\frac{\partial^2 w}{\partial t^2} = 0$$

Projecting in Hilbert space:

$$\int_0^L [(M+m)\phi_j\phi_n]dx\,\ddot{q}_n + \int_0^L [2MU\phi_j^I\phi_n]dx\,\dot{q}_n + \int_0^L \left[\left[EI\phi_j^{IV} + \left(MU^2\left(\frac{\lambda L}{4D_i} + 1\right) + \frac{dU}{dt}(L-x) - \bar{T} + \bar{p}A(1-2v)\right)\phi_j^{II}\right]\phi_n\right] dx q_n = 0$$

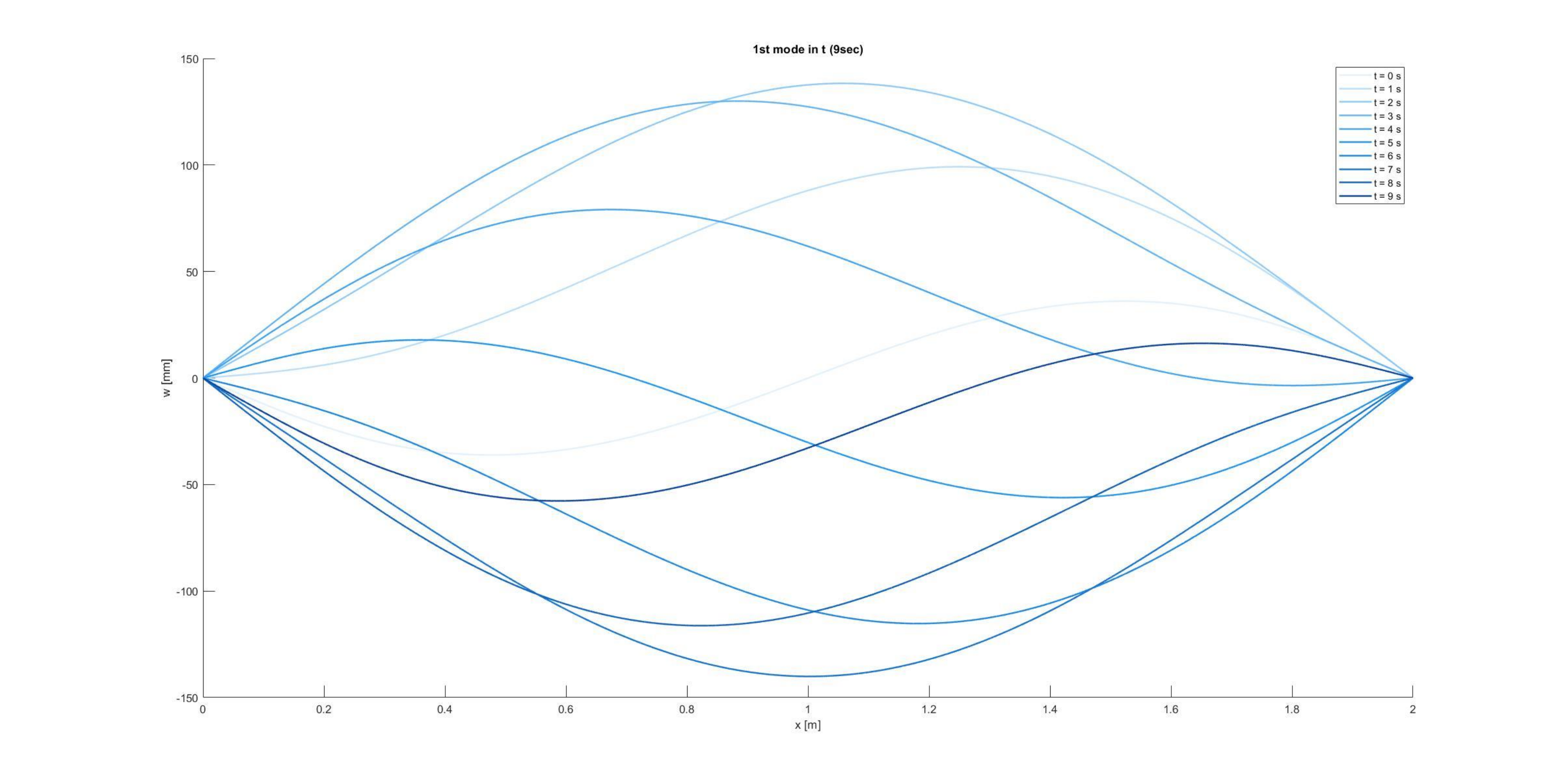
- Clamped pipe, trial function:

$$\phi_j(x) = \sinh(\beta_j x) - \sin(\beta_j x) + \alpha_j[cosh(\beta_j x) - \cos(\beta_j x)]\ with\ j = 1,..,4$$

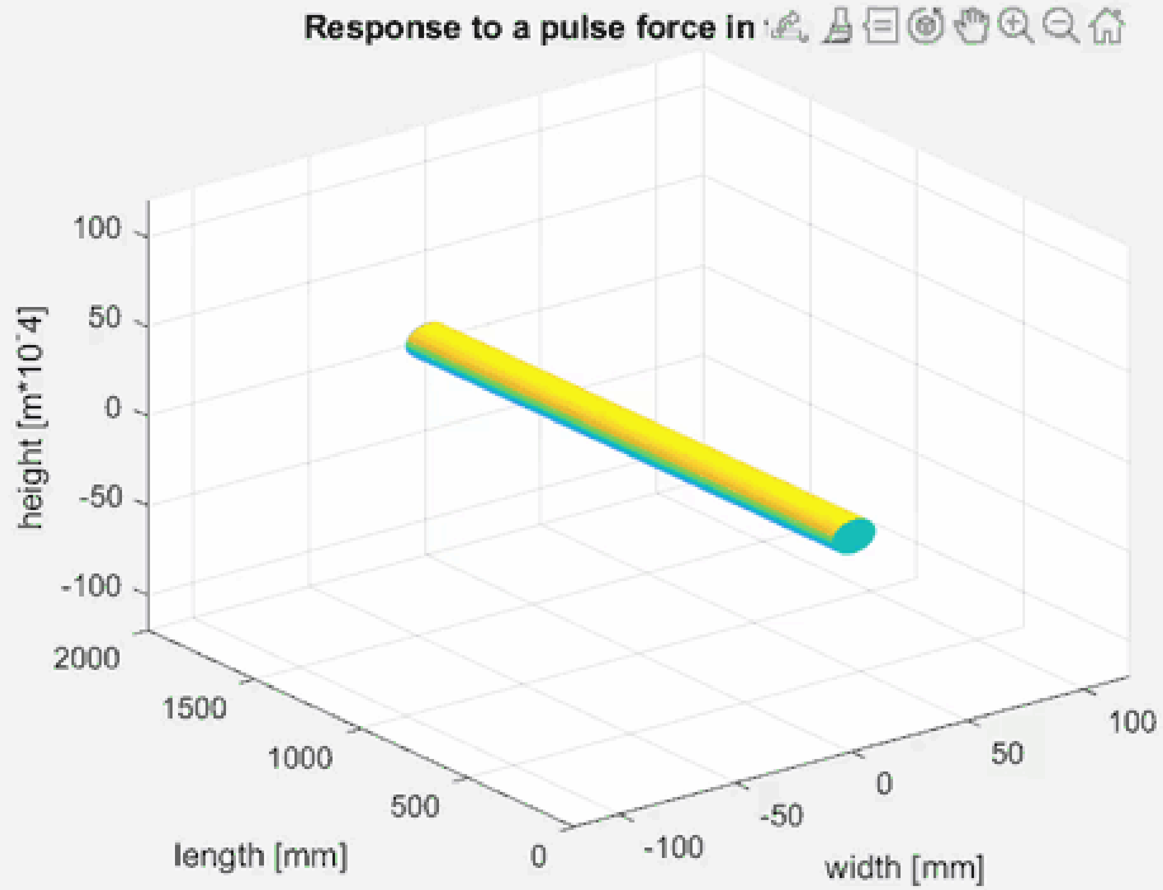
$$with\ \alpha_j = \frac{[\sinh(\beta_j L) - \sin(\beta_j \mathrm{L})]}{[\cos(\beta_j L) - \cosh(\beta_j L)]}$$

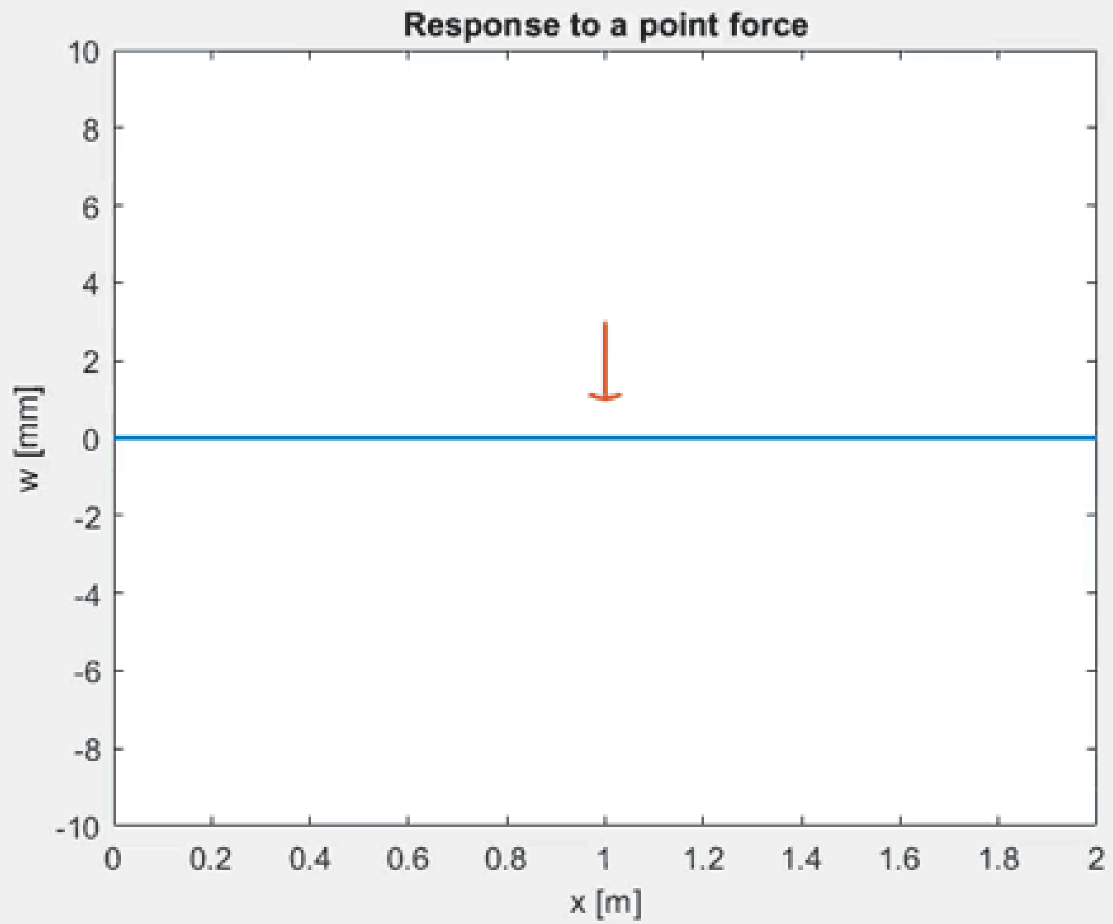
$$\beta_j\ from\ B.C.\,w(0) = w(L) = 0, w'(0) = w'(L) = 0$$

RESULT: FURTHER DEVELOPMENT POINT 2, $U \neq const.$



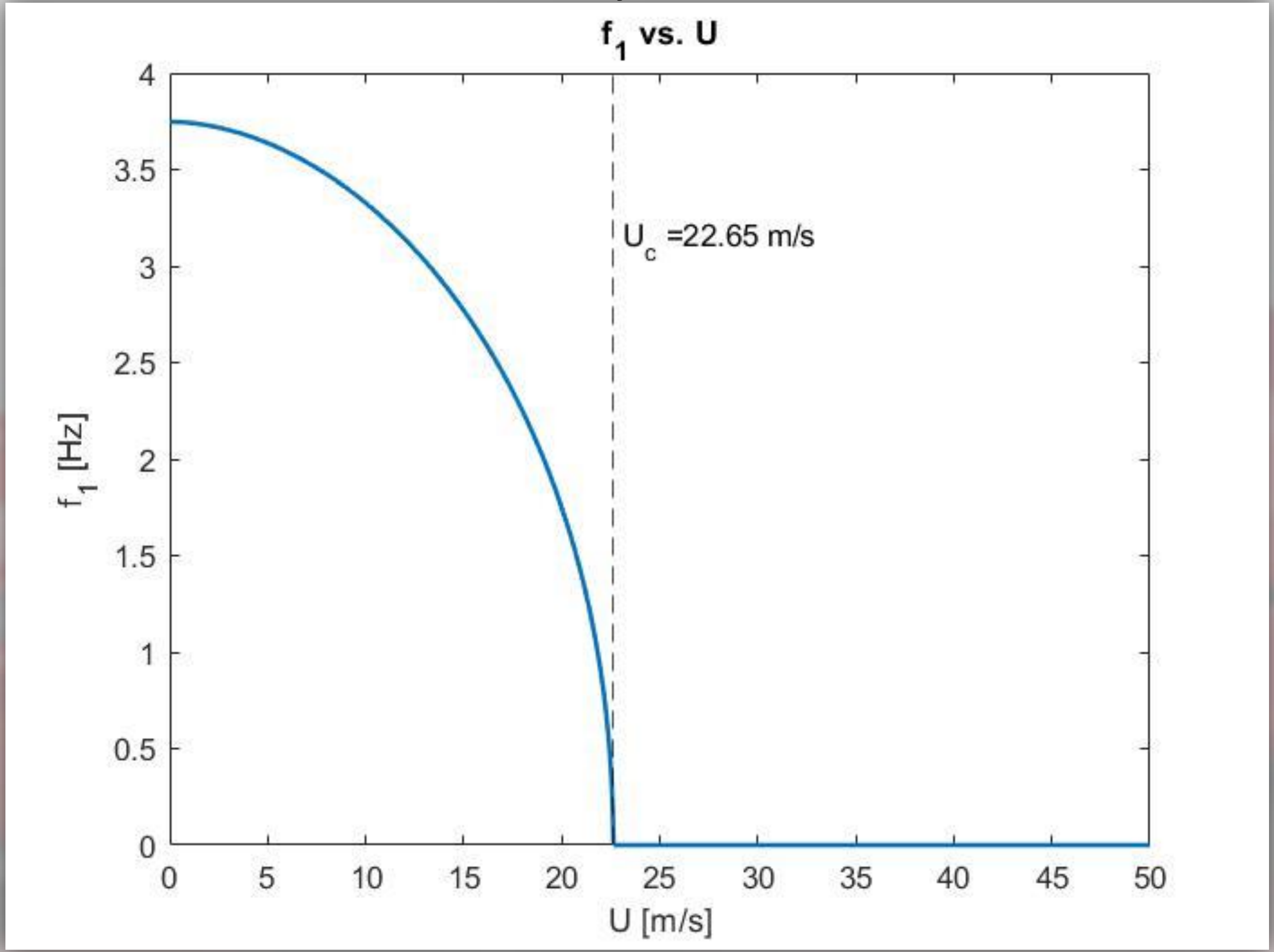
RESULT: FURTHER DEVELOPMENT
POINT 3, $U \neq const.$





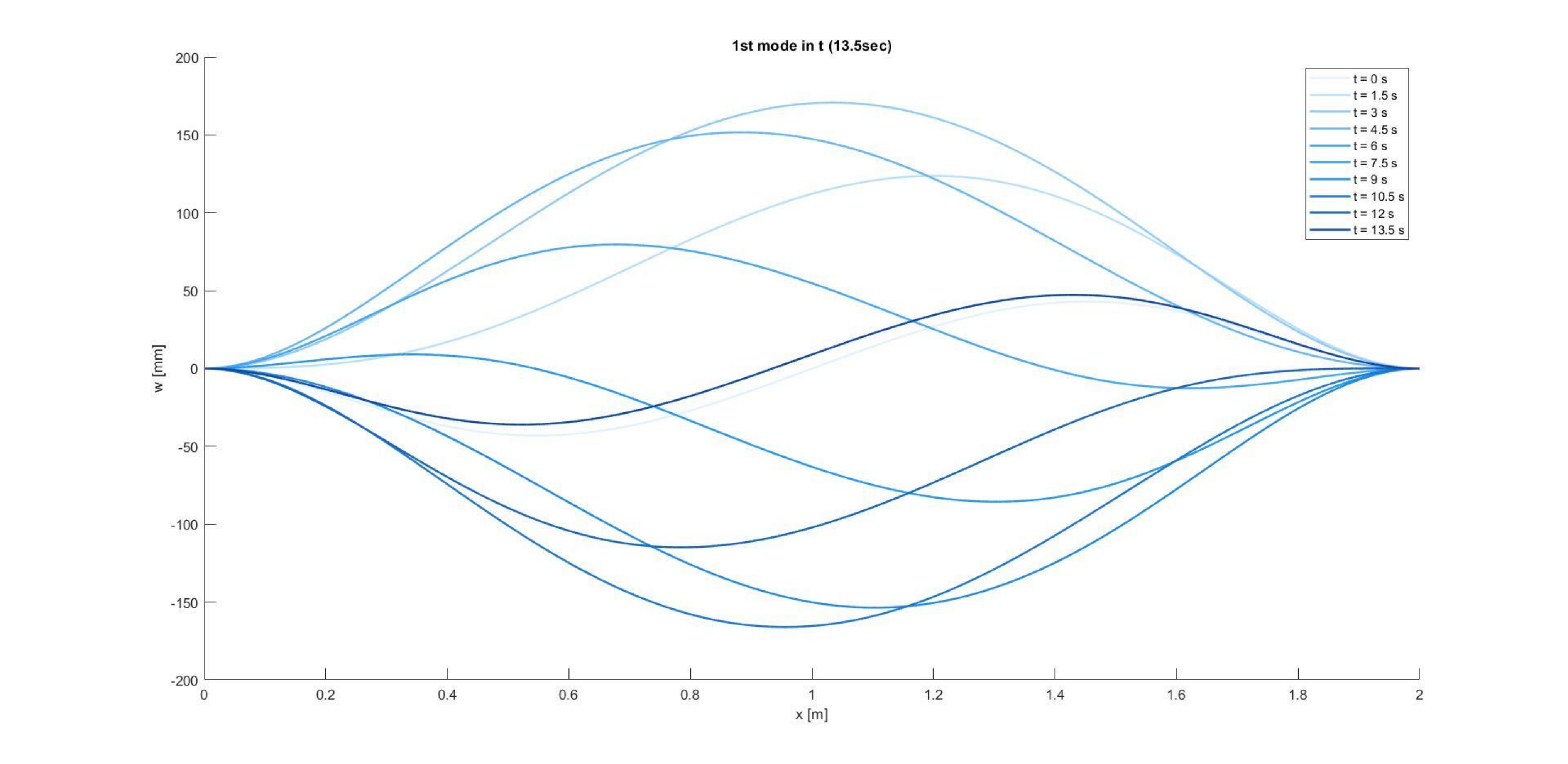
MOTORVEHICLE UNIVERSITY OF EMILIA-ROMAGNA

RESULT: FURTHER DEVELOPMENT POINT 1, CLAMPED



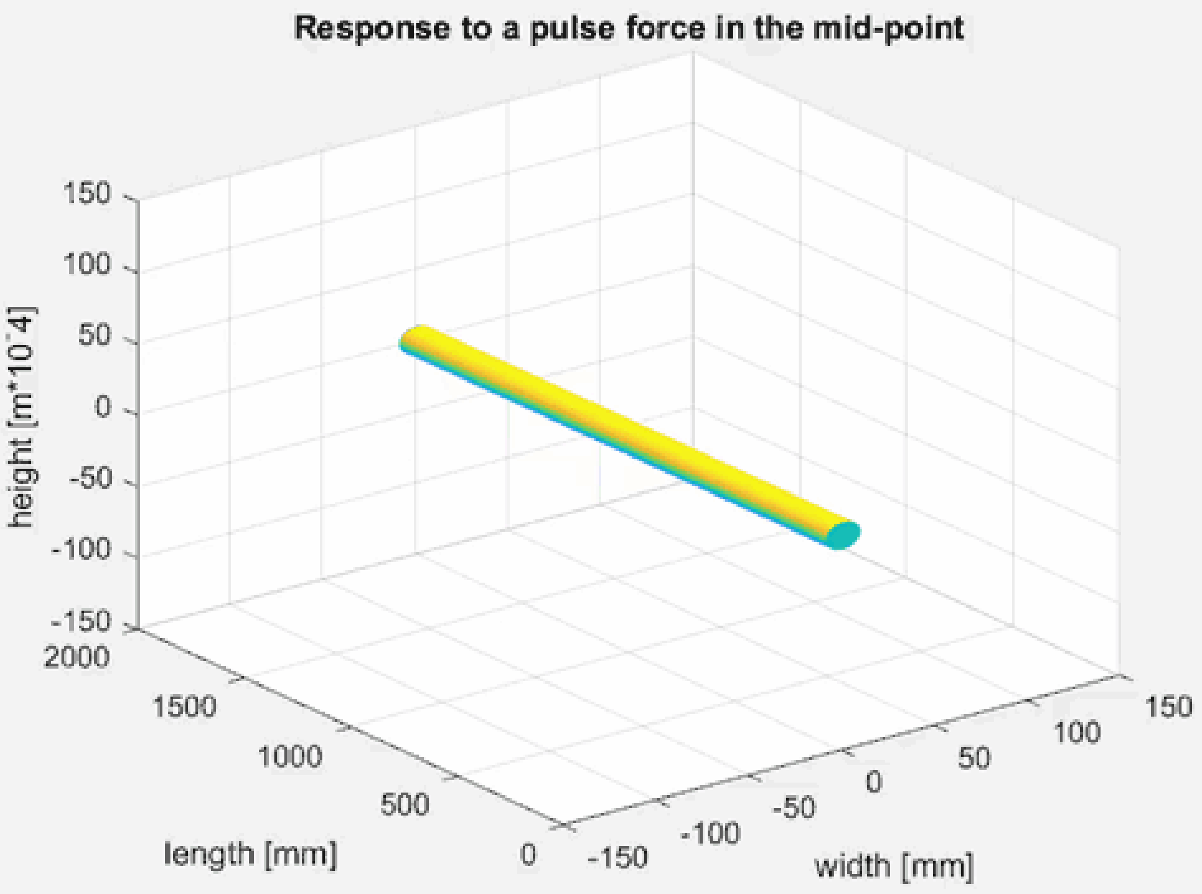
$$U_c > U$$

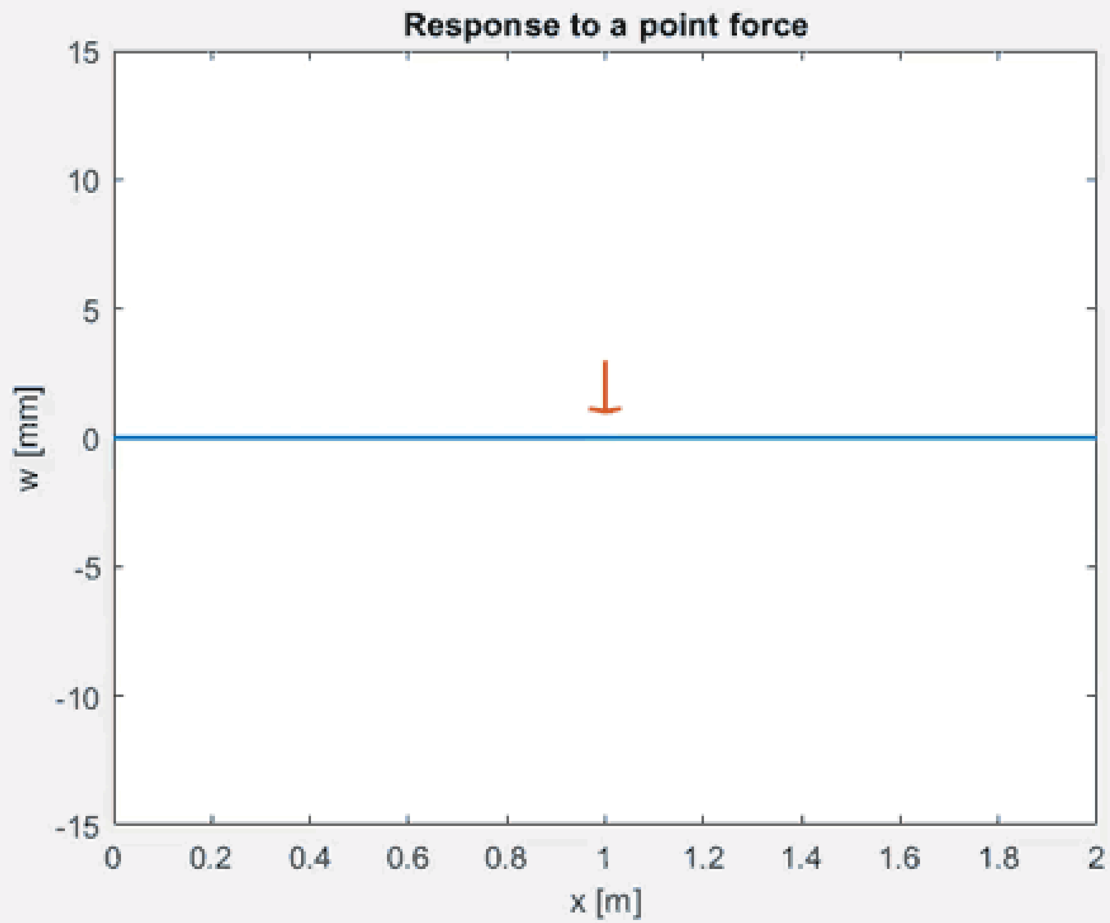
RESULT: FURTHER DEVELOPMENT POINT 2, CLAMPED, $U \neq const.$



$$U = 20{,}3 + 0{,}15\sin(100 \cdot 2\pi \cdot t)$$

RESULT: FURTHER DEVELOPMENT
POINT 3, CLAMPED, $U \neq const.$





$$U = 20{,}3 + 0{,}15\sin(100 \cdot 2\pi \cdot t)$$

CONCLUSIONS

Point 1

- Achieved f_i $v.s. U$, U_c for our complete model.
- Not complete models respect U_c condition only with $\bar{T}$.
- f_i have the expected trend with increasing U, both in symbolic and in numeric approaches.
- Symbolic vs. numerical approaches have similar results, they can be considered validated.

Point 2

- Mode shapes and representation of first mode have been calculated.
- Amplitude, more than 3 times bigger with $U \neq 0$ than $U = 0$.
- Practically no difference between numerical and symbolic.
- Numerical and symbolic approaches can be considered validated.

CONCLUSIONS

Point 3

- Animation of transient response from pulse excitation in mid point of pipe has been achieved.
- The value of vibrations' amplitude is consistent with the length of the pipe ($6mm\ vs.\ 2m$).
- Numerical and symbolic approach have similar amplitudes of vibration, but symbolic has lower prevalent modes; in general we can consider both approaches validated.

Symbolic vs. numeric

- For every point symbolic approach is slower than the numerical one beacuse has higher computational load.
- The slight differences between symbolic and numeric approaches are due to different approximations of derivatives (mainly the 4°) of $\phi_i(x)$ in point 1,2,3; in point 1 there is also the aforementioned approximations in λ (friction coefficient).

MOTORVEHICLE UNIVERSITY OF EMILIA-ROMAGNA

CONCLUSIONS

Further developments

- Point 2 with $U \neq const.$: is difficult to appreciate the difference with the case of $U = const.$, probably this is due to the chosen U timelaw, that has slight oscillation respect to mean value and it has also high frequency.
- Point 3 with $U \neq const.$: like Point 2 with $U \neq const.$
- Clamped, Point 1: U_C slightly bigger than hinged, f_1 for $U = 0$ slightly higher than hinged.
- Clamped, Point 2: different modal form respect hinged one because different B.C., higher max amplitude of vibrations respect hinged one.
- Clamped, Point 3: like Clamped, point 2.

Other possible improvements

- Consider gravity and dissipation effects.
- Consider nonlinear deformations.

MOTORVEHICLE UNIVERSITY OF EMILIA-ROMAGNA

Advanced Automotive Engineering

Any questions?

Thank you

REFERENCES

- [1] G. Catellani, M. Milani and F. Pellicano, “Dynamic Stability of a Pipe subjected to a pulsating Flow”, Dip. Scienze dell’Ingegneria, Univ. Modena e Reggio Emilia, 20020
- [2] F. Pellicano, F. Vestroni, “Nonlinear Dynamics and Bifurcations of an Axially Moving Beam”, Journal of Vibration and Acoustics, vol. 122 (21-30), 2000
- [3] F. S. Samani, F. Pellicano, “Vibration reduction on beams subjected to moving loads using linear and nonlinear dynamic absorbers”, Journal of Sound and Vibration, vol. 325 (742–754), 2009
- [4] A. Czerwiński, J. Łuczko, “Vibrations of Steel Pipes and Steel Hoses Induced by Periodically Variable Flow”, Mechanics and Control, vol. 31,2, 2012
- [5] Francesco Pellicano, Mechanical Vibrations lecture notes, 2020
- [6] M. P. Païdoussis, “Fluid-Structure Interations”, vol. 1, 1998
- [7] Viscosity of Automotive antifreeze – viscosity table and viscosity chart: Anton Paar Wiki (anton-paar.com)