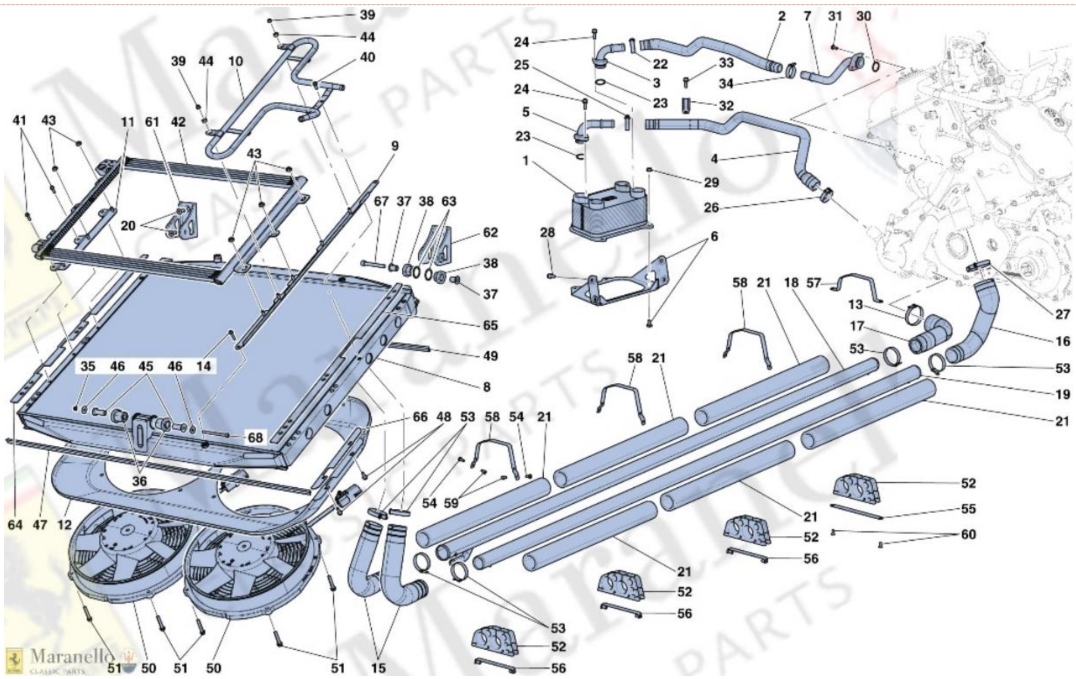
MECHANICAL VIBRATIONS

MATLAB TEAMWORK PROJECT

A.A. 2020/21

CONTINUOUS SYSTEMS

PIPES VIBRATION



Group D

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INTRODUCTION

In this project we will explore the vibrations of a pipe when a fluid is flowing through it.

To study this problem the theory of continuous systems will be used, aided by the preliminary knowledge of the complete PDE of this particular system.

The main goals of the project are to determine:

1. The natural frequencies of the system, the correlation between them and the fluid velocity, and the critical velocity of the fluid.
2. The modal shapes.
3. The response in the transversal direction of the pipe when it is excited by a point force applied on its midspan.

The pipe used as reference is a coolant return pipe taken from Ferrari LaFerrari, equipped with a mid-rear engine and front radiators.

THEORY

The straight pipe model

We define the coordinates x (axial) and z (vertical) to define the space (lagrangian coordinates). For every section we represent its transversal displacement with w and its axial displacement u respect to the equilibrium position.

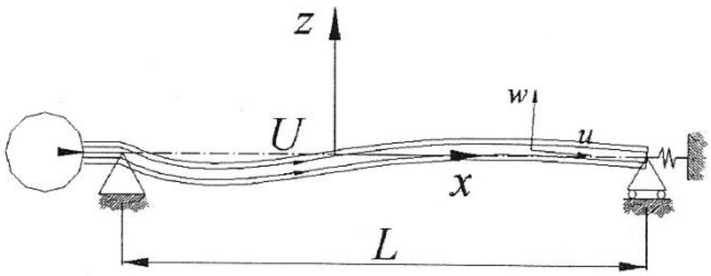

FIGURE 1: PIPE AND COORDINATES SYSTEM [1]

The model is based on the following hypothesis:

- Euler-Bernoulli beam equation
- Both supported ends
- Small section compared to length (slender pipe)
- Uncompressible fluid
- Coriolis and centrifugal forces
- Axial displacement negligible compared to the transversal one

The last hypothesis is very important because it makes possible a linear model that simplifies a lot the analysis.

The general equation of motion will be:

$$\left(E^*\frac{\partial}{\partial t}+E\right)I\frac{\partial^4 w}{\partial x^4}+\left\{MU^2\left(1+\frac{\lambda L}{4d^2}\right)-\bar{T}+\bar{p}A(1-2\nu\delta)-\left[(M+m)g-M\frac{dU}{dt}\right](L-x)\right\}\frac{\partial^2 w}{\partial x^2} + 2MU\frac{\partial^2 w}{\partial x\partial t}+(M+m)g\frac{\partial w}{\partial x}+c\frac{\partial w}{\partial t}+(M+m)\frac{\partial^2 w}{\partial t^2}=0 \quad (1)$$

Assuming that:

- Gravitational effects are neglected
- The fluid velocity U is constant
- The internal damping is neglected

We obtain the following partial differential equation:

$$EI\overbrace{\frac{\partial^4 w}{\partial x^4}}^{(1)} + \left[MU^2\left(1+\frac{\lambda L}{4d^2}\right) - \bar{T} + \bar{p}A(1-2\nu\delta)\right]\overbrace{\frac{\partial^2 w}{\partial x^2}}^{(2)} + 2MU\overbrace{\frac{\partial^2 w}{\partial x \partial t}}^{(3)} + (M+m)\overbrace{\frac{\partial^2 w}{\partial t^2}}^{(4)} = 0 \qquad (2)$$

1) Flexural restoring force: related to the elastic behavior of the pipe.
2) Centrifugal force: conservative force produced by the curved trajectory that the fluid must follow while the pipe is bending.
3) Coriolis force: non-conservative force produced by the fluid.
4) Inertia of the pipe and the fluid inside.
5) Friction between fluid and walls. It depends by the type of motion of the fluid (laminar or turbulent):

$$\lambda = \begin{cases} 75/Re & Re < 2300 \\ \dfrac{0{,}3164}{\sqrt[4]{Re}} & Re > 2300 \end{cases} \qquad (3)$$

E is the Young modulus, I the inertia moment of the pipe section related to the barycentric axis, M is the mass of fluid per unit length inside the pipe, T is the mean tensile force applied, p is the mean fluid pressure, A is the section area, ν the Poisson coefficient (δ is equal to 1 if the linear expansion of the pipe is constrained, 0 if not) and m is the pipe's mass per unit length.

This equation cannot be used directly for the analysis, but it must be transformed to an ordinary differential equation. The Galerkin method makes this possible.

The equation (2) can be written as dividing by M+m:

$$\ddot{w}(x,t) + \mathcal{L}\big(w(x,t)\big) = 0 \qquad (4)$$

$$where \quad \mathcal{L}(\cdot) = \frac{EI}{M+m}\frac{\partial^4}{\partial x^4} + \frac{MU^2\left(1+\frac{\lambda L}{4d^2}\right) - \bar{T} + \bar{p}A(1-2\nu\delta)}{M+m}\frac{\partial^2}{\partial x^2} + \frac{2MU}{M+m}\frac{\partial^2}{\partial x \partial t} \qquad (5)$$

The variable w can be written as:

$$w(x,t) = \sum_{j=1}^{\infty} q_j(t)\phi_j(x) \qquad (6)$$

Where $\phi_j(x)$ represents a complete set of functions. In particular, the set $\phi_j(x) = \sin\left(\frac{j\pi x}{L}\right)$ has been chosen as trial function.The result given by the (6) in the (4) is:

$$\mathcal{L}(w) = \sum_{j=1}^{\infty}\left[\left(\frac{EI}{M+m}\phi_j^{IV} + \frac{MU^2\left(1+\frac{\lambda L}{4d^2}\right) - \bar{T} + \bar{p}A(1-2\nu\delta)}{M+m}\phi_j^{II}\right)q_j(t) + \frac{2MU}{M+m}\phi_j^{I}\dot{q}_j(t) + \phi_j\ddot{q}_j(t)\right] \qquad (7)$$

And projecting it on the Hilbert space one obtains:

$$\sum_{j=1}^{\infty}\left[\left\langle\left(\frac{EI}{M+m}\phi_j^{IV}+\frac{MU^2\left(1+\frac{\lambda L}{4d^2}\right)-\bar{T}+\bar{p}A(1-2\nu\delta)}{M+m}\phi_j^{II}\right),\phi_n\right\rangle q_j(t)\right.$$
$$\left.+\left\langle\frac{2MU}{M+m}\phi_j^{I},\phi_n\right\rangle \dot{q}_j(t)+\langle\phi_j,\phi_n\rangle\ddot{q}_j(t)\right]=0 \tag{8}$$

Considering that $\langle f(x),g(x)\rangle=\int_a^b f(x)g(x)dx$ with $\mathcal{D}=[a,b]$ and solving the integrals the final result is:

$$\ddot{q}_n+\sum_{j=1}^{N}\frac{4MUj}{L(M+m)}\delta_{jn}\,\dot{q}_n+\left\{EI\frac{\pi^2n^2}{L^2(M+m)}-\frac{MU^2\left(1+\frac{\lambda L}{4d^2}\right)-\bar{T}+\bar{p}A(1-2\nu\delta)}{M+m}\right\}\frac{\pi^2n^2}{L^2}q_n$$
$$=0 \tag{9}$$

$$where\ \delta_{jn}=\begin{cases}0 & j=n\\ \dfrac{L}{j-n}+\dfrac{L}{j+n} & j\neq n\end{cases}$$

The result is an ordinary differential equation with N×N matrices:

$$\boldsymbol{M}\ddot{q}_n+\boldsymbol{C}\dot{q}_n+\boldsymbol{K}q_n=0\quad n=1,\ldots,N \tag{10}$$

By solving the eigenvalue problem is possible to obtain the natural frequencies and through the modal analysis the mode shapes.

Critical velocity

As shown in (2), the centrifugal force depends by the square of the flow speed U. This force acts producing a compressive stress on the pipe and if the flow speed is too high, the flexural restoring force is not sufficient to overcome it, leading to the instability of the system and to possible buckling phenomena.

First, let us define a dimensionless velocity:

$$u=\sqrt{\frac{M}{EI}}UL \tag{11}$$

When increasing U, the centrifugal force is more effective and this produces a loss of effective stiffness to the structure and subsequently, the natural frequencies will decrease. The critical velocity is the value in which the fundamental frequency is equal to 0 and the pipe loses its rigidity.

If $\bar{T},\bar{p}=0$, it is possible to prove that the dimensionless critical velocity is equal to π and therefore the critical velocity can be written as:

$$U_c=\frac{\pi}{L}\sqrt{\frac{EI}{M}} \tag{12}$$

Defining the point load

To evaluate the dynamic response of the pipe, a point force is applied initially. This force is not moving and can be described through the following equation:

$$F(x,t) = F_1(t)\delta(x - x_F) \quad where \quad x \in (0, L) \quad t > 0 \tag{13}$$

$F_1(t)$ describes the value of the force in time, while x_F represents the point of application.

NUMERICAL IMPLEMENTATION

At the beginning of the numerical implementation, nested `for` loops were put into the code for allowing the user to manage input data within specific ranges for each parameter. It is worth to remark that if no parameter is set, the script will automatically assign the default value, for each not given input.

For all the codes the number of trial functions Φ_j used to define the eigenfunctions is N=7. This number is a good trade-off between precision and computational load.

Point 1 – Natural frequencies and critical velocity

Every point has been developed first through numerical variables, and then with symbolic variables for validation purposes. After inserting all the data, the N trial functions Φ_j have been defined implicitly in the symbolic approach, while in the numerical approach are discretized in an N*10000 matrix.

Plotting the expression (8) and integrating numerically with the function `trapz` (`int` for the symbolic approach), one obtains the EOMs in a matrix form. The modal analysis is straightforward, calling the function `eig` it is possible to find the first N natural frequencies of the problem (it is to remember that a continuous system has infinite but numerable natural frequencies and the most important are the first ones).

This process has been applied to a range of velocities through a `for` cycle for the numerical approach, in order to plot the fundamental frequency in function of U. In the symbolic approach U is a variable, so it is possible to plot the frequency in function of the velocity. In this way it has been possible to find the critical velocity through a graphic approach.

Point 2 – Mode shapes

The definition of the problem in the code is similar to the point 1, where the EOMs are defined in a matrix form. After that, the equation can be written in the state-space:

$$\dot{\boldsymbol{p}} + \begin{bmatrix} -M^{-1}C & -M^{-1}K \\ I & 0 \end{bmatrix} \boldsymbol{p} = 0 \quad where \quad \boldsymbol{p} = \begin{bmatrix} \dot{\boldsymbol{q}} \\ \boldsymbol{q} \end{bmatrix} \tag{14}$$

With the command `eig` is possible to solve the eigenvalue problem and obtain the complex eigenvalues and eigenvectors. Finally, to start the oscillation the terms of the n-th eigenvector have been taken as initial conditions to obtain the n-th mode shape.

Point 3 – Response to a load point force

The problem has been defined exactly as in the previous codes, obtaining the ODEs in a matrix form. The force follows the expression (13) with $x_F = \frac{L}{2}$ and $F_1(t)$ defined as:

$$F_1(t) = A\left[0{,}5 + 0{,}5\cos\left(2\pi\frac{t-P}{W}\right)\right] \quad t \in \left(P - \frac{W}{2}, P + \frac{W}{2}\right) \tag{15}$$

After projecting $F(x,t)$ through the Galerkin method, the equation obtained will be like the (14) but with an excitation (see Eq. 16). Then, for both the approaches the equation has been defined with symbolic variables in order to use the command `ode45` and find the response. In this case, the initial conditions were the equilibrium position.

$$\dot{\boldsymbol{p}} + \begin{bmatrix} -M^{-1}C & -M^{-1}K \\ I & 0 \end{bmatrix} \boldsymbol{p} = \boldsymbol{F}_P \quad where \quad \boldsymbol{p} = \begin{bmatrix} \dot{\boldsymbol{q}} \\ \boldsymbol{q} \end{bmatrix} \quad and \quad \boldsymbol{F}_P = \begin{bmatrix} -F_1(t) \cdot \boldsymbol{\phi} \\ \boldsymbol{0} \end{bmatrix} \tag{16}$$

RESULTS

TABLE 1 –PIPE CHARACTERISTICS

EPDM rubber			
Length	2 m	**Young modulus**	100 MPa
External diameter	19 mm	**Poisson's ratio**	0,49
Thickness	3,9 mm	**Tensile force**	100 N
Internal diameter	11,2 mm	**Pressure (relative)**	2 bar
Density	2300 kg/m³		

TABLE 2 – FLUID CHARACTERISTICS

VW G13 coolant + water (50:50) [7]	
Fluid density	1077 kg/m³
Fluid velocity	20,3 m/s
Kinematic viscosity	$3{,}98 \cdot 10^{-6}$ m²/s

First of all, using the data in the tables above, we found the natural frequencies, their correlation with fluid velocity, and the critical velocity:

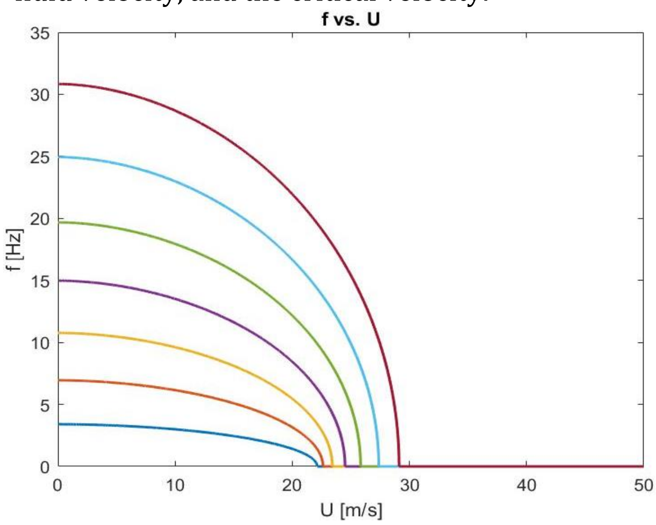

FIG. 2: NATURAL FREQ. VS. FLOW VELOCITY

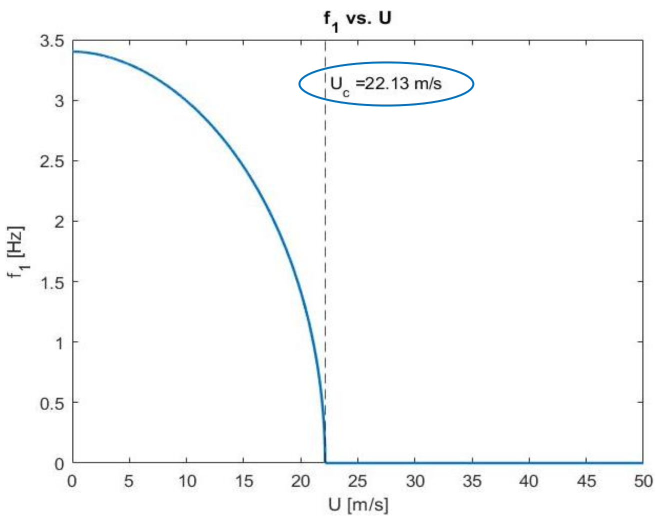

FIG. 3:FUNDAMENTAL FREQ. AND CRITICAL VELOCITY

i	1	2	3	4	5	6	7
f_i [Hz]	1.3170	2.9825	5.2304	8.1798	11.8851	16.3713	21.6502

TABLE 3: NATURAL FREQUENCIES

Then we found the modal shapes (complex) . In Fig. 4 we have for example the first and second modes:

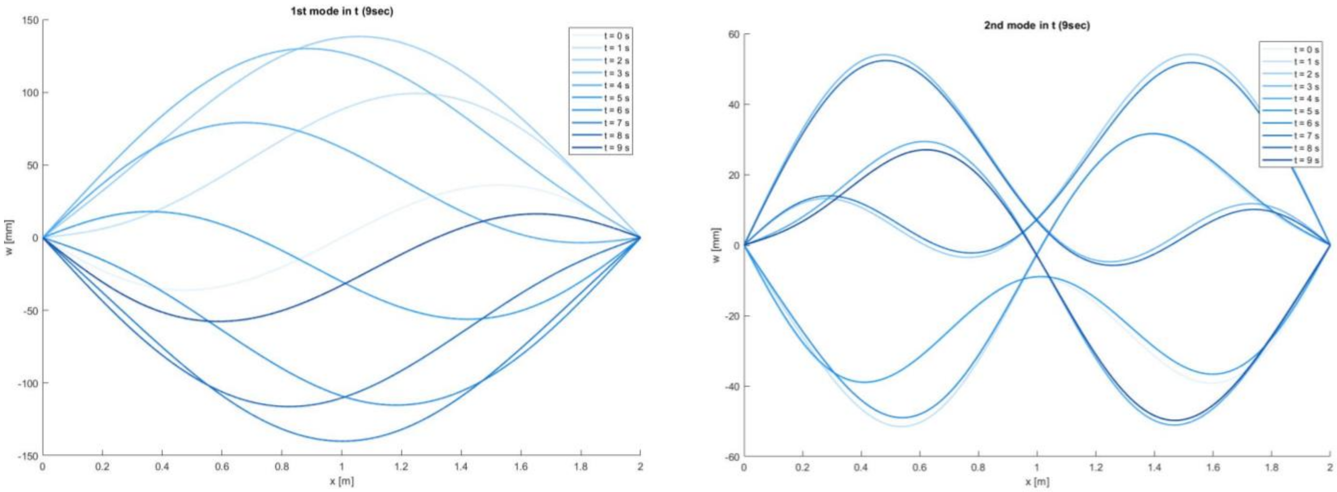

FIGURE 4: FIRST AND SECOND MODE SHAPES

Finally, we have calculated the transient response when an impulsive point force is applied at the midspan of the pipe, for the full animation please refer to the code (and/or to our presentation).

FURTHER DEVELOPMENTS

The model just studied contains many simplifications that can be removed to have a more precise simulation. In particular, the flow velocity can be considered not constant [1], making the model more realistic. Such development modifies the equation of motion that becomes:

$$EI\frac{\partial^4 w}{\partial x^4}+\left[MU^2\left(1+\frac{\lambda L}{4d^2}\right)-\bar{T}+\bar{p}A(1-2\nu\delta)+M\frac{dU}{dt}(L-x)\right]\frac{\partial^2 w}{\partial x^2}+2MU\frac{\partial^2 w}{\partial x\partial t} + (M+m)\frac{\partial^2 w}{\partial t^2}=0 \tag{17}$$

Let’s suppose that the flow U is subjected to a fluctuation:

$$U(t)=U_0+u\sin(2\pi\,\nu\,t) \tag{18}$$

U_0 is equal to the previous U, supposing a ±15% variation of the flow ($u=0{,}15U_0$) and an oscillation ν=100 Hz.

The new linear operator is:

$$\mathcal{L}(w)=\sum_{j=1}^{\infty}\left\{\left[\frac{EI}{M+m}\phi_j^{IV}+\frac{MU^2\left(1+\frac{\lambda L}{4d^2}\right)-\bar{T}+\bar{p}A(1-2\nu\delta)+M\frac{dU}{dt}(L-x)}{M+m}\phi_j^{II}\right]q_j(t) + \frac{2MU}{M+m}\phi_j^{I}\dot{q}_j(t)+\phi_j\ddot{q}_j(t)\right\} \tag{19}$$

Moreover, the response is different if the geometrical constrains change because the boundary conditions of the equation will be modified. For example, considering a clamped pipe on both edges instead of pinned, the set of functions $\phi_j(x)=\sin\left(\frac{j\pi x}{L}\right)$ is no longer suitable.

The new set of functions will be:

$$\phi_j(x) = sinh(\beta_j x) - sin(\beta_j x) + \alpha_j\left[cosh(\beta_j x) - cos(\beta_j x)\right] \tag{20}$$

$$where \quad \alpha_j = \frac{sinh(\beta_j L) - sin(\beta_j L)}{cos(\beta_j L) - cosh(\beta_j L)}$$

$$and \quad \beta_1 L = 4{,}730041 \quad \beta_2 L = 7{,}853205 \quad \beta_3 L = 10{,}995608 \quad \beta_4 L = 14{,}137165$$

So the critical velocity and the first mode shape for this clamped case are:

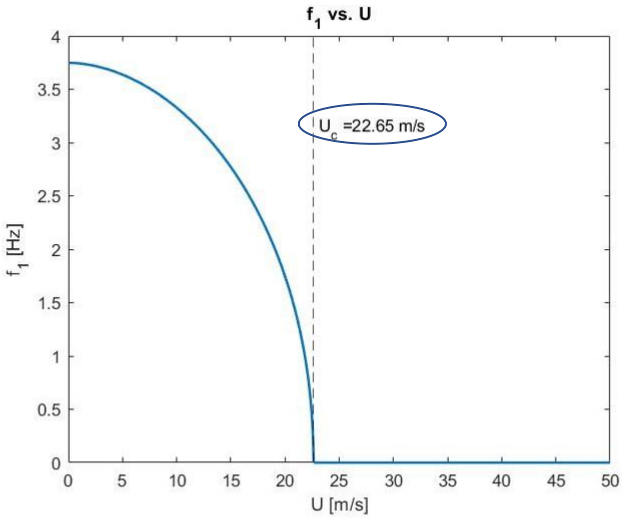

FIGURE 5: FUNDAMENTAL FREQ AND CRIT. VELOCITY CLAMPED PIPE

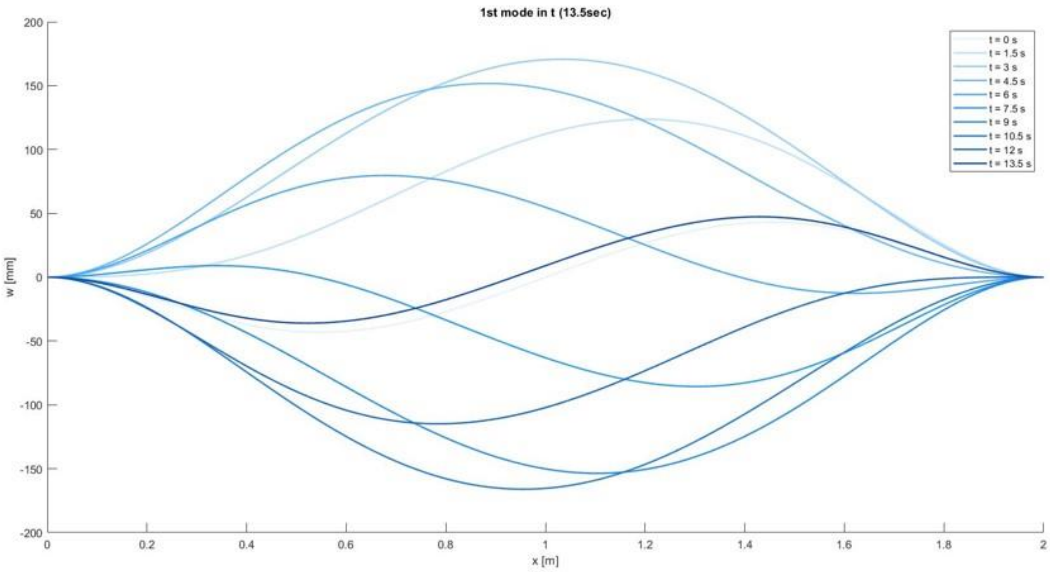

FIGURE 6: FIRST MODE SHAPE CLAMPED PIPE

Once again for the animation of the full transient response in the clamped case please refer to the code (and/or to our presentation)

CONCLUSIONS

In conclusion, we have achieved the goals of our projects with both a numerical and a symbolic approach in MATLAB for validation purposes. In certain points we have slight variations of the values between the two approaches (due to different computational precision) but we can consider them acceptable.

The behaviour of the natural frequencies is the one that we expected i.e. they decrease as the flow velocity increases. It is to note the important role of the tension which allows us to increase the critical velocity, making it suitable for this specific application.

The fluid motion substantially changes the mode shapes and the amplitude of the vibrations with respect to the case in which the fluid velocity is zero.

In the case of the application of the point force the amplitude of the vibration is not too high compared to the length of the pipe and so the linearity hypothesis holds.

About the further developments, in the application with *U≠const.* is difficult to appreciate the difference with the case of *U=const*. both in the mode shapes and the transient response to the point force. Probably this is due to the chosen U(t).

On the other hand, in the clamped case we have substantial differences, in particular the critical velocity results to be slightly higher, the mode shapes are different due to the different boundary conditions and finally the maximum amplitude of the vibrations is higher than in the hinged case.

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