MECHANICAL VIBRATIONS

MATLAB TEAMWORK PROJECT

A.A. 2020/21

I.C.E VIBRATIONS

Group D

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INTRODUCTION

In this project we will analyze, using a very simple 1 degree of freedom model, the vibrations produced by an internal combustion engine and how much these vibrations will transfer to the chassis of the car supposing that the engine is installed in the engine bay through a set of anti-vibrating mounts.

Two different tests will be performed.

One with constant angular velocity which will be the idle speed (Test 1)

The other with a speed ramp from stationary conditions to maximum rotational speed (Test 2)

The targets of the project are:

- Determine the inertial forcing due to an L4 2000cc engine. Considering only the first and second-order unbalanced forces caused by reciprocating components motion.
- Derive the equation of motion of the system, find the natural frequency, the critical damping, and plot the transmissibility $\tau(\omega)$.
- Plot the full transient response of the mass and the force transmitted to the chassis when the resonance is crossed. (only for Test 2)

The data that we used for the mass of the engine, the mass of the reciprocating components, the idle rpm and the redline rpm are taken based on Honda's F20c engine (inline-4 naturally aspirated).

THEORY

Dynamic analysis

The first step consists in defining the excitation force produced by the inertial forces inside the engine. From the theory of the engine balance [2] we have the following expressions for a single cylinder:

$$F_{rec}^{1st} = m_{rec} r \Omega^2 \cos(\Omega t) \tag{1}$$

$$F_{rec}^{2nd} = m_{rec} r \lambda \Omega^2 \cos(2\Omega t) \tag{2}$$

Where $m_{rec} = m_{Piston} + m_{Pin} + \frac{2}{3} m_{Conrod}$, r is the crank radius, l is the conrod length, $\lambda = \frac{r}{l}$ and Ω is the engine angular velocity.

Since we have an engine with *4 cylinders* and the phase displacement between them is 180°, the 1st order forces are intrinsically balanced and so we are left with only the 2nd order reciprocating forces.

So the excitation that our system will be subjected to is:

$$F(t) = F_0 \cos(2\Omega t) \quad with \quad F_0 = 4 m_{rec} r \lambda \Omega^2 \tag{3}$$

System analysis

Once that the excitation force is obtained, the next step consists in finding the equation of motion.

The system was simplified as a 1dof problem where the engine is the oscillating mass m, the chassis is the base and the engine mounts behave like a spring-damper system.

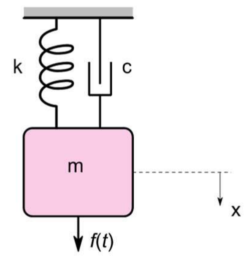

FIGURE 1 – REPRESENTATION OF THE SYSTEM

The excitation force is applied directly to the oscillating mass; the resulting equation of motion is:

$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{4}$$

Where c is the damping coefficient $[N \cdot s/m]$ and k is the elastic constant $[N/m]$.

The force applied to the chassis is transmitted through the damper and the spring:

$$T(t) = c\dot{x} + kx \tag{5}$$

The system can be now analyzed.

The natural frequency and the damped resonance frequency can now be determined:

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \quad with \quad \omega_n = \sqrt{\frac{k}{m}} \quad and \quad \zeta = \frac{c}{2\sqrt{k\,m}} = \frac{c}{2\,\omega_n\,m} \tag{6}$$

The critical damping is obtained by reverting the expression for ζ written in (5), setting its value to 1 and from that calculating $c_{critical}$:

$$c_{critical} = 2\sqrt{km} = 2\omega_n m \tag{7}$$

Another important parameter is the transmissibility, i.e. the ratio between the amplitude of the transmitted force and the amplitude of the excitation force:

$$\tau = \frac{|T|}{F_0} = \frac{|T|}{4m_{rec}r\lambda\Omega^2} \tag{8}$$

This expression can be applied only for the steady-state response with a fixed speed. It is possible to plot the transmissibility in function of ω.

To calculate the response in the first test we have the following differential equation:

$$m\ddot{x} + c\dot{x} + kx = F(t) = 4m_{rec}r\lambda\Omega^2\cos(2\Omega t) \tag{9}$$

To solve the problem in *Test 1* we can use the convolution integral:

$$x(t) = \int_0^t 4\,F_0 \cos(2\Omega\tau) \cdot h(t-\tau)\,d\tau \quad with \quad h(t) = \frac{1}{m\omega_d} e^{-\zeta\omega_n t} \sin(\omega_d t) \tag{10}$$

For *Test 2* Ω is no more fixed but it increases linearly with time:

$$\Omega(t) = \Omega_i + \frac{\Omega_f - \Omega_i}{L} t \tag{11}$$

Where Ω_i is the starting speed of the engine, that in this case is equal to 0, Ω_f is the maximum speed and L is the time needed to accelerate from Ω_i to Ω_f.

It is important to note that the speed is constantly changing: in this way the system has no time to enter the steady-state condition but it is always in a transient state.

As for *Test 1,* convolution integral can be used to calculate the response.

The resonance phenomenon is unavoidable, but it can be managed to be less harmful as possible. A typical car engine has an idle speed of more or less 800 rpm and under this speed it can be considered as not operative; the best solution is to have a resonance frequency under this value. In this way the resonance will be crossed only during the start, but in this condition the engine speed grows very quickly, so there is not enough time for the system to produce significant resonant effects. This will be achieved by selecting proper values of k and c with in mind also the target of having the lowest possible transmissibility value.

NUMERICAL IMPLEMENTATION

The numerical implementations were carried out with the help of MATLAB. Primarily, a definition of variables (for the system characterization) based on technical specifications for the Honda F20c engine, was made as follows:

TABLE 1: ENGINE CHARACTERISTICS

Honda F20 c			
Engine mass (M)	148 kg [5]	Idle speed (n_i)	800 rpm [3]
Piston mass (m_p)	0,355 kg [4]	Maximum speed (n_f)	8900 rpm [3]
Wrist pin mass (m_w)	0,109 kg [4]	Conrod length (l)	0,153 m [3]
Conrod mass (m_r)	0,636 kg [4]	Crank radius (r)	0,042 m [3]

The values of k and c have been considered as free variables to reach the following targets:

1) Resonance frequency below idle speed

2) Acceptable transmissibility peak

3) Adequate stiffness of engine mount to guarantee a good feeling to the driver

Supported by some references to understand the order of magnitude [6,7] and after various attempts we have chosen the most suitable values for our purpose:

TABLE 2 – FREE VARIABLES

Stiffness (k)	150000 $\left(\frac{N}{m}\right)$	Damping (c)	800 $\left(\frac{N*s}{m}\right)$

Test 1

As a matter of fact, for Test 1 in which the system is excited by a second order force with constant angular velocity, only the idle speed was considered (for the purpose of simplicity). Right after the definition of the engine, the system characteristics (such as ω_d, ω_n, ζ) are put into the code. These last ones were derived from the engine specifications according to equations (6) and the definition for m_{rec}.

Then, the discretization took place. Where a high sampling frequency was considered (to avoid aliasing), the vector of time was created, as well as the length of the signal to be analyzed (5 seconds). In order to have rotational speed in terms of $\left[\frac{rad}{s}\right]$, the idle speed was multiplied by a factor of $\frac{2*\pi}{60}$, which would be renamed as:

$$\Omega_i = n_i * 2 * \frac{\pi}{60} \; ; \qquad \Omega_f = n_f * 2 * \frac{\pi}{60}$$

From this point forward, with all constants and variables put into place, the solution for the differential equation [8] (the response of the system) falls upon the proper approach. Here we used two different ways of addressing the problem, one with symbolic variables and one with convolution.

Symbolic resolution

We decided to go for the use of a time symbolic variable in order to take advantage of the `dsolve` command provided by MATLAB. Even though, the solution is not precisely a numerical implementation, the symbolic approach allows to find a reliable result that can be compared with the one obtained by numerical methods. It is easily set up, requiring only initial conditions and the symbolic variables (`t_sym` which stands for the time, and `y_sym(t_sym)` referring to displacement variable) to do the math (refer to the code for setting a better understanding).

This implementation has its pros and drawbacks. The first impression is that the solution, because of its simplicity, was accurately calculated but at the cost of long computing time. Therefore, its main disadvantage is related to the poor trade-off between computing speed and resources used. For this reason, this method constrains are subjected to the complexity of the computational drills.

Convolution method

As for the convolution part, the correctness of the transfer function in time domain (response of the unitary impulse) and the convolution integral are more than enough for doing the calculations.

In this case we make use of the expressions (10) to solve the equation (9). This method is significantly faster, but it is directly joined with the transfer function and therefore affected by the correct identification of $h(t)$.

The rest of the code is meant for graph construction and creating the transmissibility function. It is worth to note that the graphs are plotted firstly in the range of five seconds and rescaled afterwards for one second in order to extensively appreciate the behavior of the response.

Test 2

The approach is practically the same as the one for Test 1, the main differences lie in the definition of the second order forces, that are no longer with constant angular velocity but with a ramp of velocities instead. This changes the equation (2) in the following way:

$$F_{rec}^{2nd} = m_{rec} r \lambda \frac{(\Omega_f t)}{L} \cos\left(2\lambda \frac{(\Omega_f\, t)}{L} t\right) \tag{12}$$

As for the methods put into work, we only have convolution, because in this case the symbolic approach has been dropped due to the high computational cost. Despite the fact that the force was dramatically changed, the convolution exhibits a much faster performance.

Finally, the transmissibility was calculated by the creation of a MATLAB function; using the transmissibility, the amplitude of the transmitted force to the chassis has been calculated.

RESULTS

The inertial forcing due to the engine and the complete EOM are presented in equations (3) and (4) respectively.

The natural frequency of the system is:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{150000\,\frac{N}{m}}{148\,kg}} = 31.84\,\frac{rad}{s} \tag{13}$$

The critical damping coefficient (7) is:

$$c_{critical} = 2\sqrt{km} = 2\sqrt{150000\frac{N}{m} * 148kg} = 9432\frac{Ns}{m} \tag{14}$$

Test 1

For Test 1 we have the response plot and the transmissibility plot:

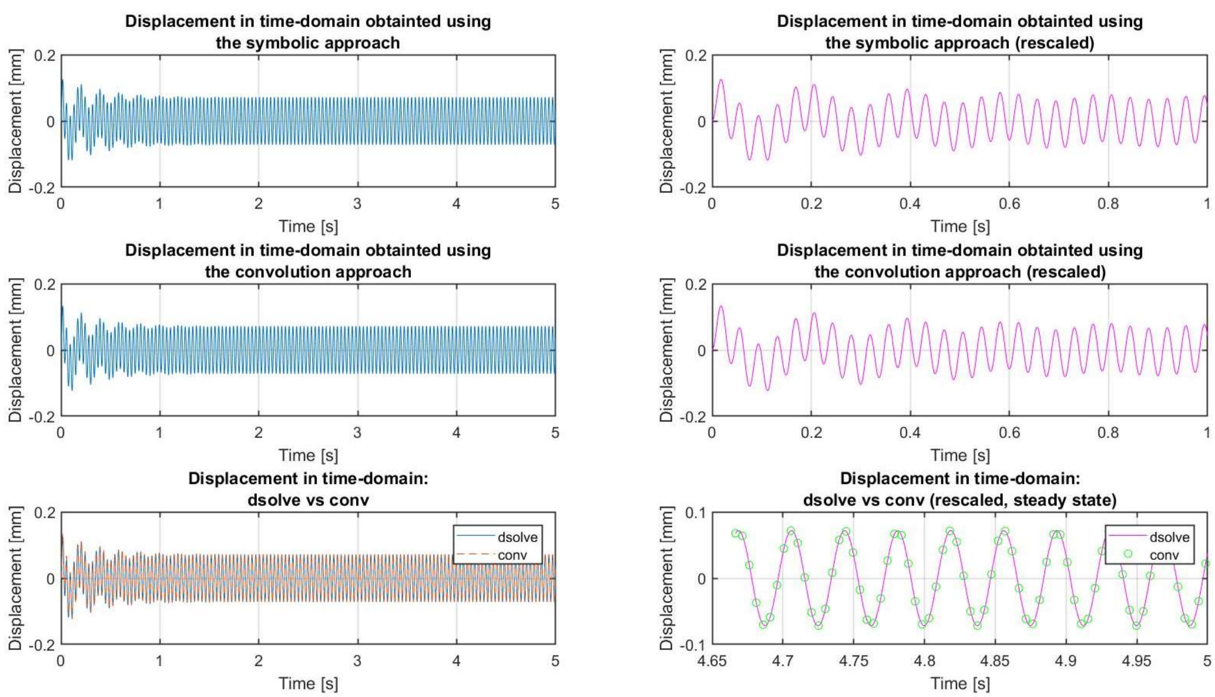

FIGURE 2 – PLOT OF THE RESPONSE OF TEST 1

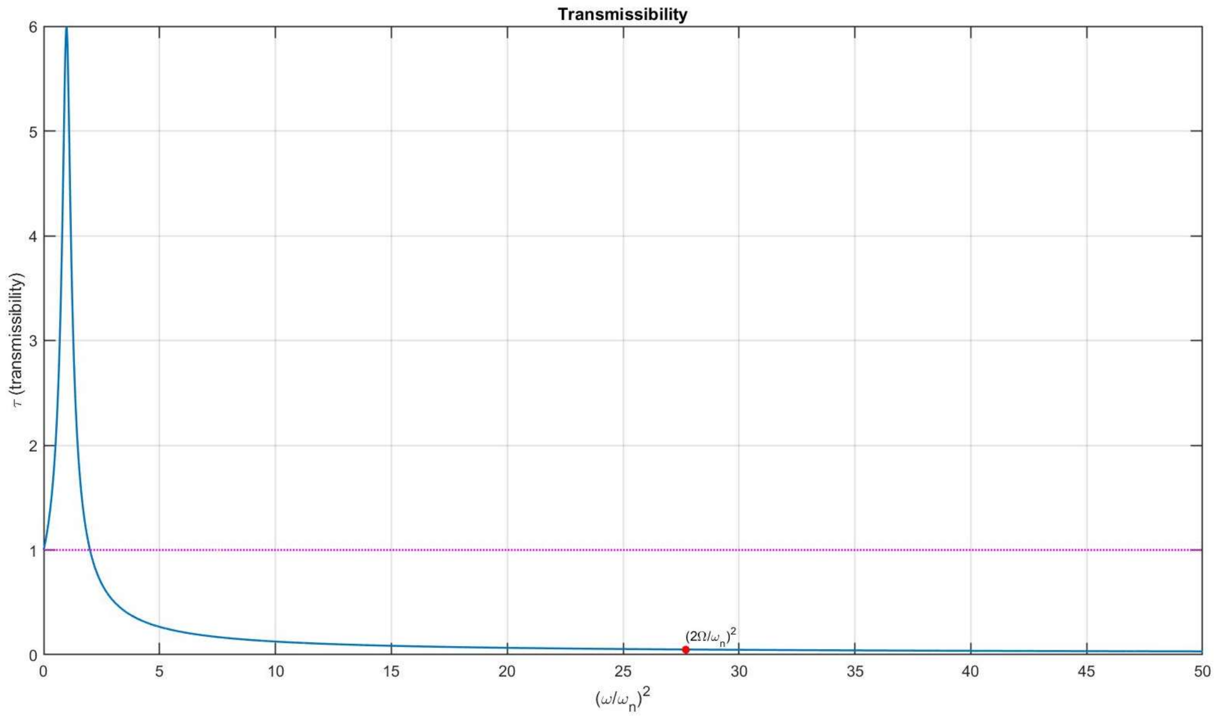

FIGURE 3 – PLOT OF THE TRANSMISSIBILITY IN TEST 1

In figure 3 the red dot, indicating the position of the idle speed of the engine, highlights the fact that in this test we are operating with a very low transmissibility value.

Test 2

For test two we have the full transient response plot and also a plot for the module of the force transmitted to the chassis:

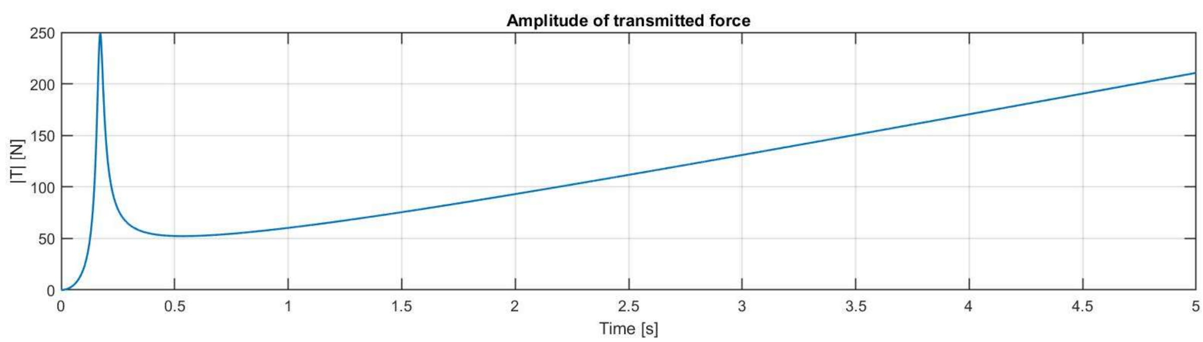

FIGURE 4 – PLOT OF THE FORCE TRANSMITTED IN TEST 2

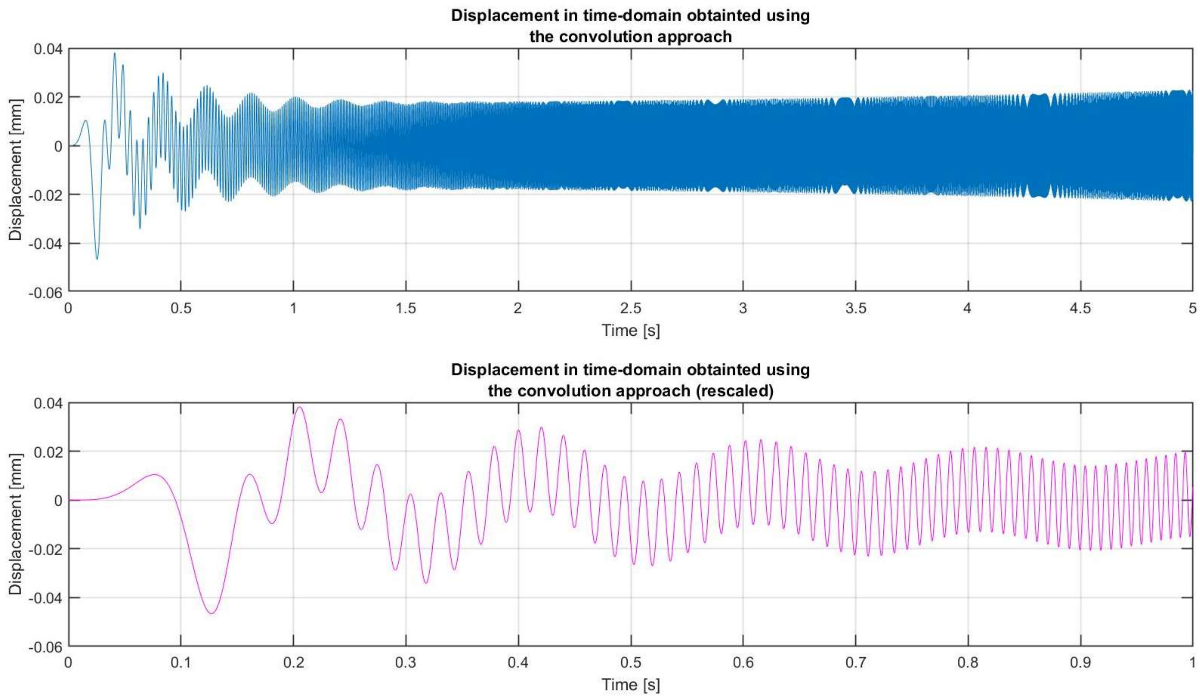

FIGURE 5 – PLOT OF THE FULL TRANSIENT RESPONSE OF TEST 2

As we can see even if the transmissibility peak is quite high, at the frequency where it is positioned the force produced by the engine is low and so also the transmitted force's module is not too high.

CONCLUSION

Using a 1DOF model of the Honda F20c engine EOM, $\omega_n, c_{critical}, \omega_{res}, \tau(\omega)$ have been calculated.

The $F(t)$ generated by pistons, pins and conrods movement and its response $x(t)$ have been studied in two different situations: $\Omega = \Omega_{idle}$ and Ω from 0 to Ω_f.

In the first case two different approaches have been used: symbolic and numerical (convolution integral) both implemented with MATLAB; their results are the same.
In the second case only numerical approach (convolution integral) has been used, furthermore amplitude of the transmitted force to the chassis has been calculated.

It has been seen that Ω_{idle} is higher than $\Omega_{resonance}$ so we can say that the problem of the amplification of the vibrations is only during the engine starting; the ζ allows to cross the resonance without a detrimental amplification of forces transmitted to the chassis, moreover it allows to have a good damping for Ω higher than the one that corresponds to $\left(\frac{\omega}{\omega_n}\right)^2 = 2$.

A possible improvement of our model is the one used in the document [7]; the authors studied the influence of varying the damping coefficient of the engine mount with a more complex model: 10 DOF, referred to the 4 wheels vertical movements, chassis rotations x2 and vertical movement, engine rotations x2 and vertical movement. Three different tests have been done, the first one in movement, the other two with the car still but with two different rpm. The results show that the engine vibrations affect a lot the vibrations of the chassis: circa 10 % for every DOF when the car is moving, moreover in all the 3 test is highlighted that an increase in the c of the engine mounts will lead to a reduced vibration of the chassis.

REFERENCES

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