Experimental laboratories, test case 1

Calibration of a load cell

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1 Introduction

The load cell is an instrument that can measure a force. It produces an electrical output (voltage) which is approximately proportional to the applied force. To estimate the value of a force with a load cell, it is necessary to determine the transfer function from *volt* to *newton*. The transfer function is obtained by using mass samples whose weight force F^* is accurately known. This operation is sometimes called *calibration* of the load cell. As a matter of fact, the values of V have a random variability, which makes the procedure of determining the transfer function analogous to the one of determining the calibration curve; this explains the reason why the term calibration is used in this context.

Each mass sample is applied to the load cell for a few seconds and the corresponding electrical output is recorded with a sampling frequency of 100 Hz, this produces a noisy signal.

The voltage output histories and the corresponding values of reference force F^* are available, with the convention that a positive value of F^* means the force is directed upward and a negative value means the force is directed downward. Our goal will be to determine the transfer function of the load cell and to estimate the uncertainty of the instrument.

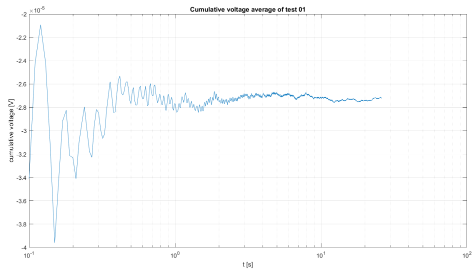
TestID	F^* $[N]$
01	0.000
02	-0.314
03	-0.411
04	-0.695
05	-0.891
06	-1.186
07	-1.676
08	-2.656

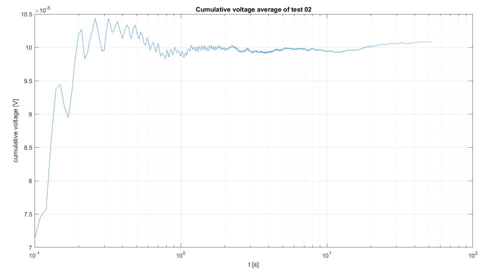
TestID	F^* $[N]$
09	-5.598
10	-10.501
11	0.126
12	1.097
13	2.078
14	4.039
15	8.942
16	13.845

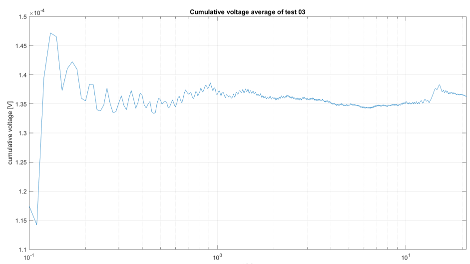
2 Analysis

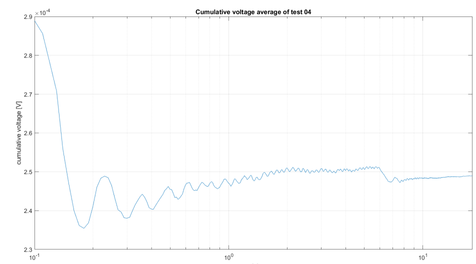
1. The noisy behavior of the voltage signal is due to the variability of the electrical output. The electric noise must be filtered out before finding the values of V_o which are to be correlated with F^* through the transfer function. This is achieved by calculating the cumulative average plots of the instantaneous voltage output. Such plots show, for each t_i, the average voltage output from 0 to t_i, with the time on the horizontal axis in log-scale.

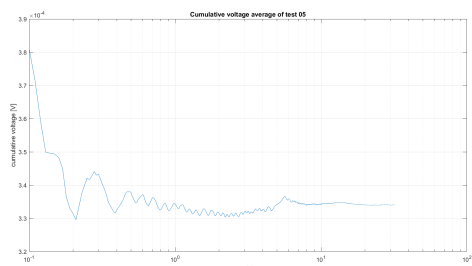
   By visual inspection of the curves, it can be assessed whether the time of acquisition of the signal is long enough to provide a reasonably stable cumulative average voltage. If this is the case, the value at the end of the curve, V, will be the voltage Vo associated with F^* in the next calculations.

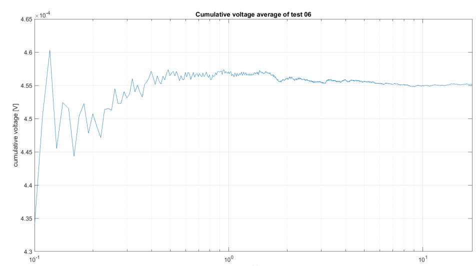


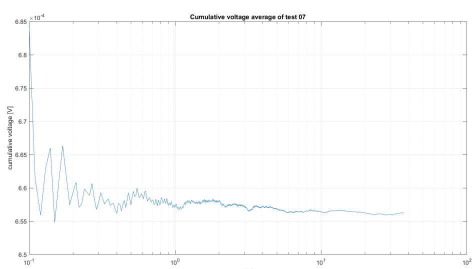


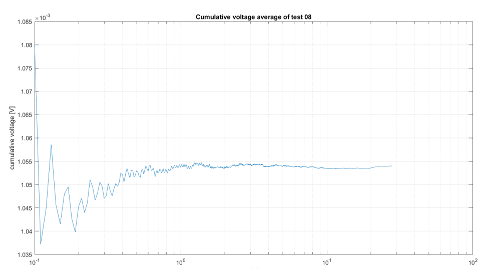


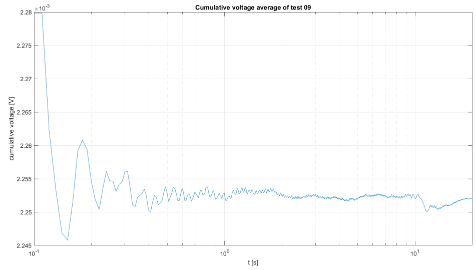


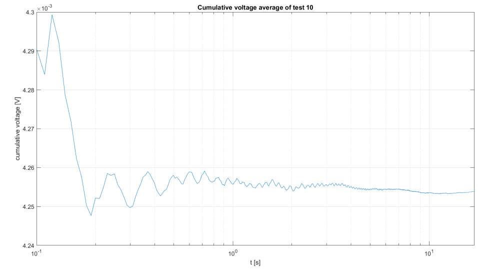


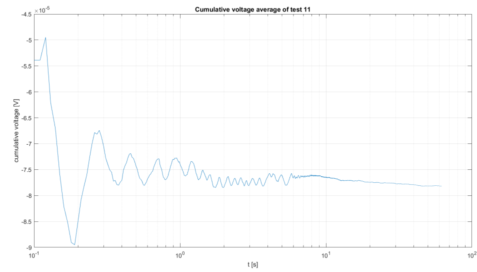


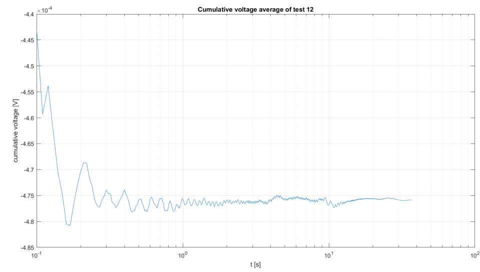


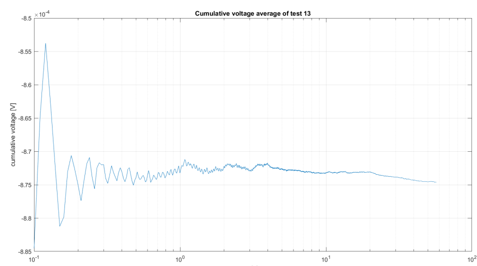


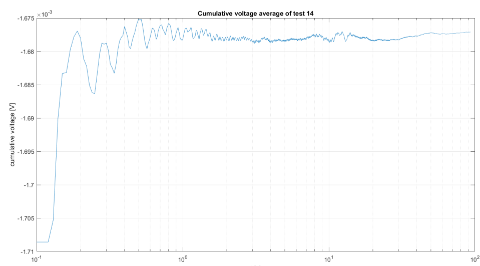


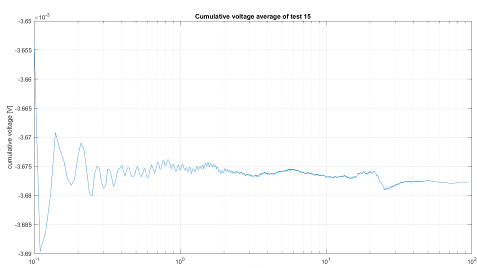


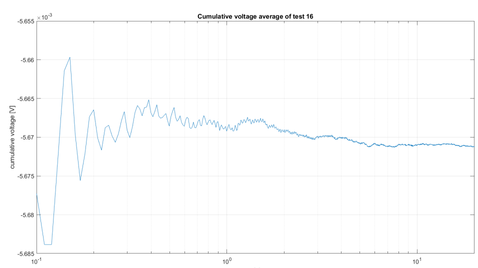












Every plot seems to stabilize enough for its data to be useful. To confirm this finding with a quantitative yet arbitrary metric, we define the *stable region* as the region where the cumulative average of the signal is **definitively** within a bound of $\pm 0.5\%$ of the final average. Then, for a signal to be *converged*, we require the length in seconds of this stable region to be bigger than or equal to 10 seconds. All the calibration experiments pass this test with flying colors, except for test 1 whose stable region only lasts 3.53 seconds and test 3 whose stable region only lasts 3.4 seconds. We examined these two tests more carefully by hand but in the end deemed them acceptable as well. Another proposed strategy was to take an average of the values in the stable region as V_o for tests 1 and 3, but since we're already talking about averages, it's not clear what statistical benefits this would have brought if any, hence this approach was discarded.

2. Note that the values of V_o are, in themselves, random. Indeed repeating the same test produces different values. A formula can be derived, which estimates the limiting mean of the Vo values, say μ_{V_o}, as a function of the reference force value, F^*. This can be obtained by linear regression of the (F^*, V_o) pairs. The formula will have the following form:

$$\mu_{V_o} = m \cdot F^* + b \tag{1}$$

The coefficients that minimize the sum of the squared differences between the actual values of V_o and the ones produced by our formula are $b = -2.89341664296959 \cdot 10^{-5}$ and $m = -0.000407713310134344$ and provide a wonderful fit to the data.

3. The practical interest is in finding a formula which, given a voltage reading V_o, provides an estimate of the corresponding "true" value F. This formula is called linear transfer function, and it can be obtained by rearranging the previous equation, as follows:

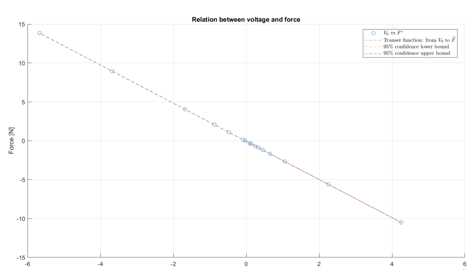
$$\widetilde{F} = \frac{V_o - b}{m} \tag{2}$$

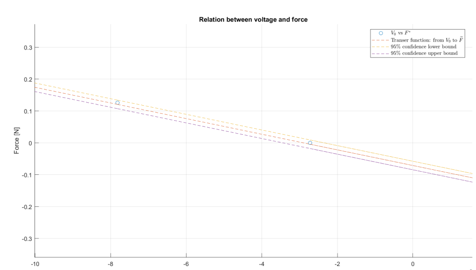
Here the tilde indicates that, for a given V_o, the formula above does not produce the real value of F, but just an estimate. This is due to the random nature of the measurement process (note that, in principle, the same V_o could correspond to every possible F). The sum of the squared differences between the actual values of F^* and the values of $\widetilde{F}$ produced by our formula is 0.00044196 which indicates, once again, a great fit.

We now would like to provide an estimate of the uncertainty associated with the estimate $\widetilde{F}$, that is, we would like to find U such that, for every V_o, the corresponding force lies within the range $\widetilde{F} \pm U$ with a certain confidence. A first approach is to set U as the maximum absolute difference between $\widetilde{F}$ and F^* for all the 16 calibration cases, this yields $U = 0.01342$. A second approach is to refer to the confidence intervals. For a perfect Gaussian PDF, 95% of the readings falls in the range $\mu \pm 2\sigma$. According to the theory of linear regression, we can estimate U at a 95% confidence as $2\bar{s}_{\widetilde{F}}$, where:

$$\bar{s}^2_{\widetilde{F}} = \frac{1}{N-2} \cdot \sum_{\text{calibration data}} \left(\frac{V_o - b}{m} - F^* \right)^2 \tag{3}$$

This yields $U = 0.011237$, so the first method has proven more stringent, but note that in principle, the formula above is reasonable only for very large populations, which isn't the case here.





4. The resolution of an instrument is the smallest change that can be measured. The resolution makes the possible voltage outputs of the load cell discrete, with a "step-wise" fashion. The resolution of the load cell in terms of voltage can be naively estimated as the minimum absolute difference among the averages of the voltages for each experiment, this yields $R_V = 3.5423 \cdot 10^{-5}$. Similarly, the resolution in terms of newtons was estimated as the minimum absolute difference among the values of $\widetilde{F}$ for each experiment, this yields $R_N = 0.086883$. Note that it wouldn't make sense to consider as resolution the minimum absolute difference among voltages within the same measurement, because those differences are due to electrical noise and can't be considered to be a *measured* quantity.

5. A gross estimate of the time of acquisition required to filter out the electric noise was obtained as the time time elapsed from the first measurement to the beginning of the stable region as it was defined earlier. Linear regression is much worse at estimating time of acquisition than it is at estimating F^*, this is because time of acquisition is a highly nonlinear variable: the load cell is fast in acquiring signals relatively close to 0 but it has huge difficulties when the signal is too close to 0. We tried higher-degree polynomials but quickly run into the Runge phenomenon.