LETTERS

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A new Strouhal–Reynolds-number relationship for the circular cylinder in the range $47<Re<2\times 10^5$

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Based on experiments a new law is proposed for the vortex shedding from a circular cylinder which describes in a consistent way the Strouhal–Reynolds-number dependency as $Sr=Sr^*+m/\sqrt{Re}$ from the beginning of the vortex shedding at $Re=47$ up to the laminar–turbulent transition of the cylinder boundary layer at $Re>2\times 10^5$. The various vortex shedding processes, occurring with increasing Reynolds number, are described by different coefficients Sr^* and m. [S1070-6631(98)01907-2]

New investigations of the circular-cylinder wake have shown that, for parallel shedding in the regular range ($47<Re<180$), the relation between Strouhal number $Sr=fD/U$ and Reynolds number $Re=UD/\nu$ (U: velocity of the oncoming flow, D: cylinder diameter, ν: kinematic viscosity, f: shedding frequency) can be described by a linear relationship if Sr is plotted as a function of $1/\sqrt{Re}$. Thereupon the investigations were extended to the irregular region ($Re>300$) during which it was found that also here $Sr(1/\sqrt{Re})$ can be represented piecewise by straight lines.

The measurements were carried out in three different facilities. In an Eiffel-type wind tunnel with a closed test section (this tunnel is the same as that used by Gerich and Eckelmann,[1] König *et al.*[2] and is described in Kastrinakis and Eckelmann[3]), in a recirculating water channel facility as described by Zhang *et al.*,[4] and in the $1\,m$ wind tunnel of the DLR in Göttingen which is a Prandtl-type wind tunnel with an open test section ($1\text{ m}\times 0.7\text{ m}$) and a closed loop return. In the Eiffel-type wind tunnel the speed was determined from the pressure measured with a Pitot tube and from the wall pressure measured at the same downstream location (Fey[5]). In the 1 m wind tunnel a Pitot-static tube was employed. In both cases MKS-Baratron pressure gauges were used. In the water channel the flow velocity was determined with the help of hydrogen-bubble time lines which were recorded by a CCD camera (Fey[6]).

To achieve parallel shedding in the regular range the cylinders were bounded by end cylinders (diameter $2D$, length $12D$) and end plates (diameter $10-20D$), as suggested by Eisenlohr and Eckelmann.[7] The aspect ratio L/D of the cylinders varied from 50 to 400. During the measurements Re was altered such that nearly equidistant steps in $1/\sqrt{Re}$ resulted. By this, a correct weighing at larger Reynolds numbers for the linear fit was achieved.

In the regular range from the beginning of the vortex shedding at $Re=47$ up to the transition of the wake at $Re\approx 180$ the wind tunnel measurements (open symbols in Fig. 1) can well be approximated by the straight line

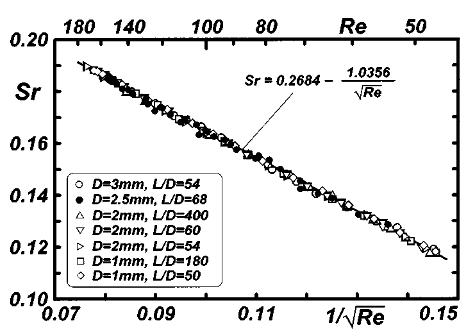

FIG. 1. Experimental data for parallel shedding of the vortices in the wake of a circular cylinder. The cylinders (diameter D, length L) were bounded by end cylinders and end plates. Open symbols: wind-tunnel measurements, solid symbols: water-channel measurements.

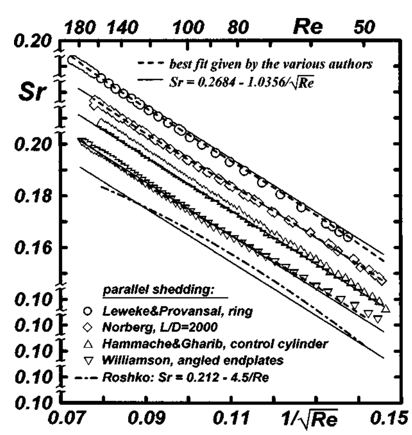

FIG. 2. The new relationship [Eq. (1)] in comparison to other authors who achieved parallel shedding by various means (see the text). The straight lines are shifted by $Sr=0.01$.

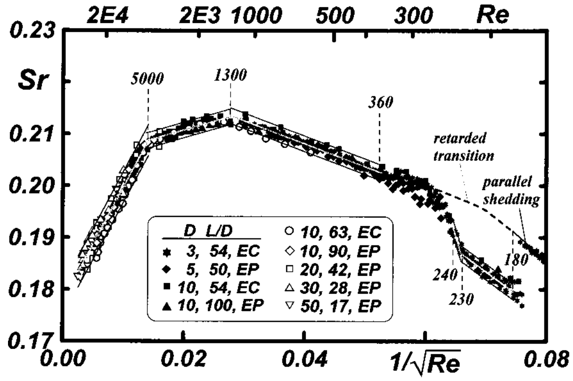

FIG. 3. Strouhal–Reynolds-number relationship for $Re>180$. Solid symbols: data measured in the Eiffel-type wind tunnel. Open symbols: data measured in the $1m$ wind tunnel up to $Re\approx 2\times 10^5$ (Schmiga[13]). Diameter D in mm, aspect ratio L/D and end conditions are given in the inset. Cylinders were bounded by end cylinders and end plates (EC) or by end plates only (EP). Black lines represent the graphically estimated error interval $\pm\delta Sr$, given in Table I. Retarded transition is obtained by suppressing the vortex adhesion- and A-mode instability with the help of control wires.

$$Sr(Re)=0.2684-\frac{1.0356}{\sqrt{Re}}. \tag{1}$$

Furthermore, this approximation is also in good agreement with the water channel measurements (solid symbols in Fig. 1).

In addition, the measurements of other authors are also well described by Eq. (1) (see Fig. 2). Here parallel shedding was achieved by different means. Leweke and Provansal[8] used cylindrical rings from which (in their mode $n=0$) parallel ring vortices were shed. Norberg[9] obtained parallel shedding by a large aspect ratio ($L/D=2000$), Hammache and Gharib[10] forced parallel shedding by two control cylinders mounted perpendicular to the cylinder and Williamson[11] applied angled end plates. Roshko,[12] who also used large aspect ratios, approximated his data by the linear relation $F=Sr\cdot Re=0.212Re-4.5$ which does not lead to a straight line in the $Sr(1/\sqrt{Re})$ representation. Leweke and Provansal, Norberg and Williamson used for their data a quadratic function of the form $F=A+B\cdot Re+C\cdot Re^2$ leading to a better agreement with Eq. (1).

Also for $Re>180$ the Strouhal–Reynolds-number dependency can be represented by piecewise linear relationships of the form

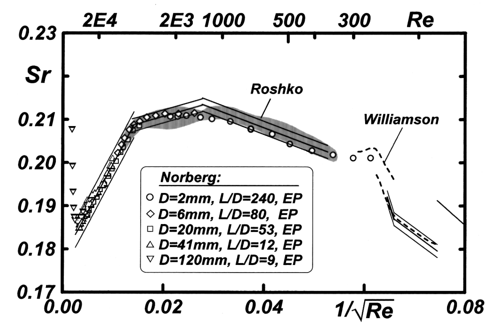

FIG. 4. Best fit lines for $Re>180$ in comparison to the measurements of other authors. Roshko's[12] data are represented by the shaded area, which show a bend at $Re\approx 5000$. Norberg's[9] data are given by the open symbols described in the inset.

$$Sr=Sr^*+m/\sqrt{Re} \tag{2}$$

with different constances Sr^* and m (Fig. 3). The black lines above and below the grey straight-line fit represent the graphically estimated error interval. The measurements were carried out both with end cylinders and end plates (EC) and with end plates alone (EP); L/D varied between 17 and 100.

In Fig. 3, the onset of the wake transition at $Re\approx 180$, caused by the vortex-adhesion mode and the A-mode instability (Williamson,[14] Zhang *et al.*[4]), produces a discontinuity and, compared to the parallel shedding, a small change in the slope. These instabilities can be suppressed by control wires placed in the vortex formation region (Fey and Eckelmann,[15] Fey,[5] Zhang *et al.*[4]). This leads in the $1/\sqrt{Re}$ representation to an extension of the straight line representing parallel shedding (dashed line in Fig. 3) and to a steady conversion into the shedding law for $Re>360$ (retarded transition). The steep increase in the small interval $230<Re<240$ is caused by the fading out of the A-mode instability. In the following interval $240<Re<360$ the measured Strouhal number is highly dependent on the boundary condition of the cylinder (i.e., end plates, end cylinders or tunnel walls) hence, here no generally valid relation can be given.

In the interval $360<Re<1300$ the data can again be approximated by a straight line which describes the vortex shedding under the influence of the B-mode instability (Fig. 3). Likewise in the intervals $1300<Re<5000$ and $5000<Re<2\times 10^5$ the measurements follow nearly a straight line in the $Sr(1/\sqrt{Re})$ representation. The bends at Re

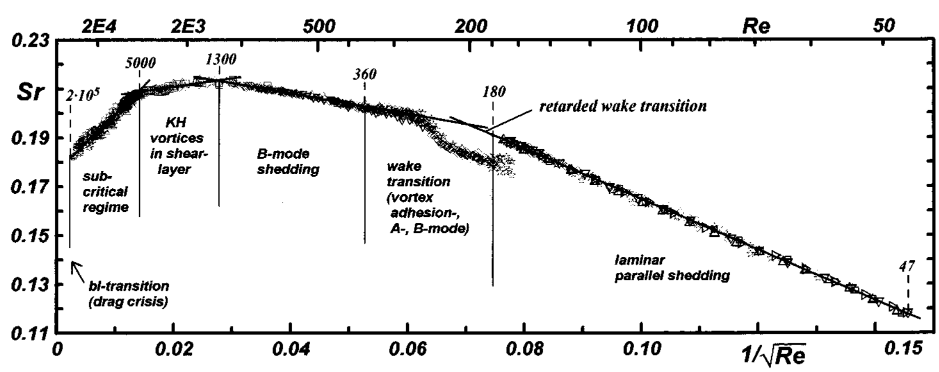

FIG. 5. Strouhal number–Reynolds number dependency in the range $47<Re<2\times 10^5$. The various changes in the slope are due to the different instabilities occurring in the near wake region.

TABLE I. Coefficients Sr^* and m in Eq. (2) for various Reynolds number intervals. δSr is the estimated error for the straight line approximation.

Re range	Sr^*	m	δSr	L/D	
$47<Re<180$	0.2684	-1.0356	0.0010	$\geqslant 50$	laminar parallel shedding
$180<Re<230$	0.2437	-0.8607	0.0015	$\geqslant 50$	wake transition:
$230<Re<240$	0.4291	-3.6735	0.0015	$\geqslant 50$	vortex adhesion-,
$240<Re<360$	depends on boundary condition				A- and B-mode instability
$360<Re<1300$	0.2257	-0.4402	0.0015	$\geqslant 50$	B-Mode shedding
$1300<Re<5000$	0.2040	$+0.3364$	0.0015	$\geqslant 50$	KH-instability in shear layer
$5000<Re<2\cdot 10^5$	0.1776	$+2.2023$	0.0030	$\geqslant 15$	subcritical regime

≈ 1300 and 5000 are correlated with the occurrence of further instabilities. It is known from Bloor[16] and from Wei and Smith[17] that at $Re\approx 1300$ vortices appear which are generated by a Kelvin Helmholtz instability in the separated shear layer. Under the influence of this instability the previously up to $Re\approx 1300$ growing length of the vortex formation region starts shrinking and reaches a minimum at $Re\approx 5000$ (Unal and Rockwell,[18] Norberg[9]). The reason, however, for the bending at $Re\approx 5000$ is not yet known.

A comparison of the here proposed Eq. (2) for $Re>180$ with measurements of Roshko,[12] Williamson[14] and Norberg[9] is given in Fig. 4. Here Roshko's data for $360<Re<8000$ are represented as a shaded area which also bends down at $Re\approx 5000$. For $180<Re<240$ Williamson's measurements are in good agreement with the straight lines obtained by the approximation to the present data. Norberg's measurements also follow the new law, only in the interval $360<Re<1300$ they fall systematically below our best fit line (Fig. 4). The steep increase at $Re\approx 2.5\times 10^5$ (∇ in Fig. 4) is caused by the laminar–turbulent transition of the cylinder boundary layer (drag crisis). With the facilities available the drag crisis could not be reached in the present investigation.

In Fig. 5 all measurements together with the different straight line approximations are plotted from the beginning of the vortex shedding at $Re=47$ up to the drag crisis at $Re>2\times 10^5$. The various coefficients Sr^* and m for the different Reynolds number intervals are put together in Table I. Roshko's formula $Sr=0.212-2.7/Re$ valid for $300<Re<2000$, which in practice is often also applied for higher Reynolds numbers, is still a good approximation to estimate the Strouhal number, however, it should not be used for $Re>5000$ since here Sr decreases stronger than predicted by the formula.

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