POLITECNICO
MILANO 1863

Course of "Fluid Labs" A.A. 2024-2025

EXP Test Case 2
Analysis of the velocity field in the wake of a circular cylinder (experimental data from PSV)

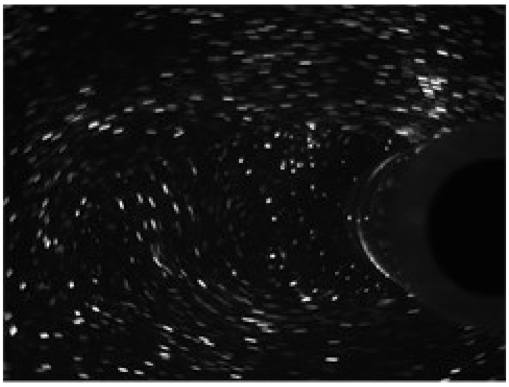

Figure 1. Exemplary image of PSV experiment

Case description and objectives

Goal of this laboratory is to analyze the velocity field around a cylinder in a water channel flow, focusing on the wake region. Reference is made to experimental data obtained through the Particle Streak Velocimetry (PSV) technique, which essentially resides on seeding the flow with small particles and shoot a video of the light-sheet illumination plane. The camera has a high exposure time, intended as the time duration of a frame, so that each particle leaves a streak on the image, as exemplified in Figure 1 in the previous page. The images then are processed to get the velocity field. In the case study proposed, the Reynolds number of the cylinder $Re_D=DU_\infty/\upsilon$ is within the range of the "sub-critical regime"*; therefore, it will be not surprising to notice that the flow separates at a certain distance from the front stagnation point, causing a recirculation zone behind of it, and that a turbulent oscillating wake is created by the shedding of two counter-rotating vortexes.

The scheme of the experiment is reported in the figure. The flow is from right to left in order to be consistent with the location of the PSV acquisition system in the water channel setup. The framing of the camera, which is the measurement field, is a vertical plane in the mid-section of the channel. The coordinate system of the PSV acquisition is (X,Y), as reported in the figure.

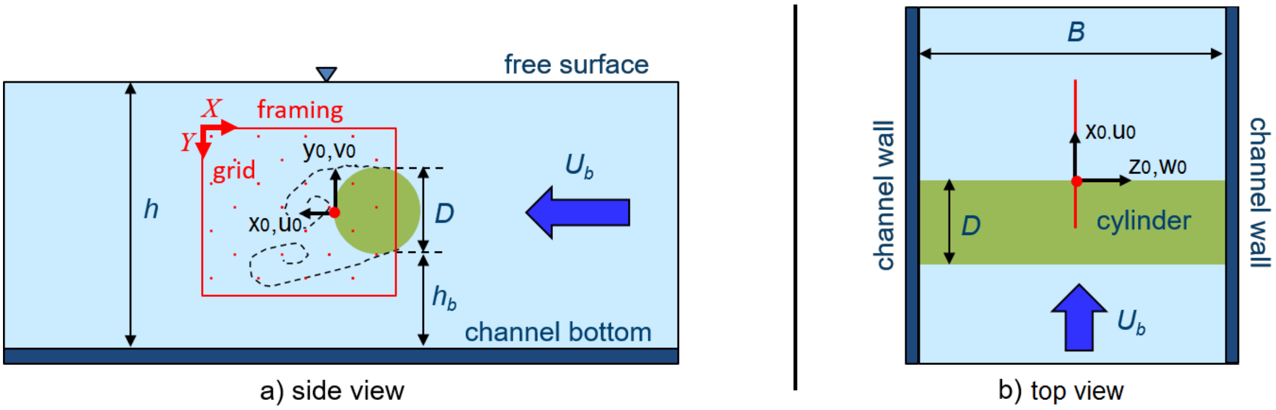

Figure 2. Sketch of the experiment with the PSV frame and the two coordinate systems

Input data

The input data of the problem are summarized in the table here below.

Symbol	Parameter	Value	Units
B	Width of the channel	0.5	m
D	Diameter of the cylinder	0.06	m
f_s	Sampling frequency	50	Hz
h	Water level in the channel upstream of the cylinder	0.42	m
h_b	Distance of the cilinder wall from the channel bottom	0.18	m
Q	Volumetric flow rate of water	35	l/s
res	Resolution of the images	3040	px/m
ρ	Density of water	998	kg/m^3
μ	Dynamic viscosity of water	0.001	Pa·s

* Note that the classification of flow regimes discussed in the class lecture refers to unconfined flows with uniform free stream flow. This is not the case of this experiment, in which the cylinder is located in a water channel with finite size cross section. However, it is reasonable to expect some similar behavior to the unconfined case.

For the post-processing of the PSV data, the frame was divided into a regular grid of 30 x 23 uniform cells along the directions X and Y, indicated in Figure 2a. The number of time steps is 1499, and the duration of each of them is equal to $1/f_s$, being f_s the sampling frequency. The complete set of acquisition data is provided in the MATLAB workspace "PSVdata.mat", which includes the following variables:

Variable	**Type**	**Value**
Grid_Xpx	23 x 30 matrix	X-coordinates of grid centers in the framing [in px]
Grid_Ypx	23 x 30 matrix	Y-coordinates of grid centers in the framing [in px]
SXpx	23 x 30 x 1499 matrix	X displacement [in px] for every time step
SYpx	23 x 30 x 1499 matrix	Y displacement [in px] for every time step

A static snapshot of the PSV frame with a ruler close to the cylinder has been taken before running the experiment, and it is provided as an image file ("ref_image.png"). A small preview of the image file is reported below in Figure 3. The scope of this preliminary step is two-fold. On the one hand, it has been used to estimate the resolution of the image as res = 3040 px/m, which is here already given as input for the sake of simplicity. On the other hand, it is used to turn the "pixel-based" matrixes Grid_Xpx, Grid_Ypx, SXpx, and Sypx into the corresponding dimensional matrixes in the new coordinate system (x0, y0), centered in the rear stagnation point and directed as shown in Figure 2a.

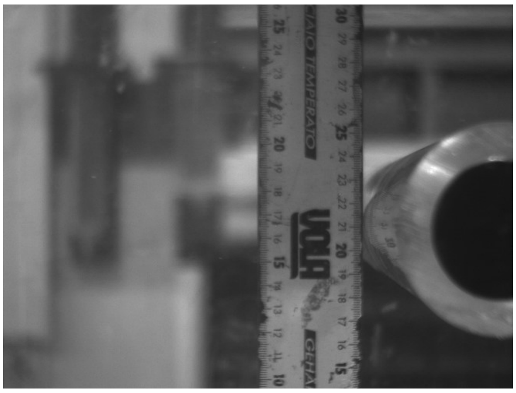

Figure 3. Small preview of the reference image.

Preliminary analyses

1. Calculate the channel bulk velocity U_b based on the measured quantities upstream of the cylinder Q, B, h. Assume that this velocity is the free stream one approaching the cylinder, U_∞, and then calculate the Reynolds number $Re_D=DU_\infty/\upsilon$. Note that $U_b \neq U_\infty$ since the experiments involve a water flow in a finite size channel. Nonetheless, this could be a reasonable first approximation.

2. Turn the "pixel based" matrixes Grid_Xpx, Grid_Ypx, SXpx, and SYpx into proper dimensionless matrixes. Reference is now made to the coordinate system (x0, y0), which is more physical than that of the PSV frame (X, Y). To this aim, make the following steps:

   2.1. Find the coordinates of the origin of the new coordinate system (x0, y0), both in pixels and in meters applying the suggested resolution res=3040 px/m. To accomplish this task, use must be made of the image file "ref_image.png" provided, and not to the preview shown in the previous page.

2.2. Start from Grix_Xpx and Grid_Ypx and build two new matrixes Grid_x0m, Grid_y0m, which contain the centre of the cells expressed in meters according to the new coordinate system (x0, y0). This requires converting the old “pixel-based” matrixes into dimensional matrixes (in meters), and then applying the transformation of coordinate system to switch from (X, Y) to (x0, y0) – not necessarily in this order. Finally, build the dimensionless matrixes Grid_x0_D and Grid_y0_D as obtained by normalizing the grid point coordinates by the diameter of the cylinder, D.

2.3. Start from SXpx, and SYpx and build new matrixes Sx0m, and Sy0m, containing the displacements in meters according to the new coordinate system (x0, y0). Obtain the matrixes of the “instantaneous”[†] velocity field matrixes u0, v0 by considering that the displacements were measured during the exposition time 1/fs. Finally, make the two matrixes dimensionless by dividing by the bulk velocity Ub; let us call the final matrixes u0_Ub, v0_Ub.

Analysis of the Reynolds-averaged flow field

3. Calculate the Reynolds-averaged velocity field through matrixes U0_Ub and V0_Ub. This requires performing the time average of the “instantaneous” velocity matrixes previously calculated in question 2.3. It is recommended to use the MATLAB command “nanmean[‡]”, so that the many “NaN” cells are ignored. Finally, visualize the Reynolds-averaged velocity field by showing the following quantities.

   3.1. The color plot of the Reynolds-averaged velocity magnitude. To this aim, you can use, for instance, the command using the commands “pcolor” with “shading interp”

   3.2. The vector map through the MATLAB command “quiver”. It is recommended to draw also the cylinder through the MATLAB command “rectangle”. Note that spurious vectors will be found also inside of the space occupied by the cylinder. Clearly, these vectors are unphysical and they are the consequence of perspective errors or issues in the PSV algorithm. They should be ignored.

   3.3. The streamlines with arrows through the MATLAB command “streamslice”. Once again, it is recommended to draw also the cylinder through the MATLAB command “rectangle”.

   3.4. The U0/Ub profiles along the horizontal line y0/D = 0 and along the two vertical lines at x0/D = 0.5 and x0/D = 1.5. This might require interpolating the grid values at selected space positions. You might use the MATLAB command “interp2”.

4. Calculate the Reynolds-averaged vorticity field applying the “curl” command to both normalized Reynolds-averaged velocities. Show the color plot of this variable (once again, you might use “pcolor” with “shading interp”). For better analysis, overlap the Reynolds-averaged streamlines (“streamslice” or “streamline” commands) and the cylinder.

Analysis of the dynamic evolution of the flow

5. Investigate the dynamic evolution of the flow, as follows.

   5.1. Plot the time history of the normalized vertical velocity v0/Ub in a point around the rear of the cylinder.

[†] Actually, this is not the instantaneous velocity field, but the average velocity field within the small time step 1/fs.
[‡] Some versions of MATLAB do not include the function “nanmean”. An alternative option is to use the usual function “mean” with option “omitnan”.

5.2. Use the MATLAB function “fft_of_v0_Ub_velocity” provided to compute the FFT (Fast Fourier Transfom) of the v0/Ub velocity signal and find out the frequency spectrum. Identify the vortex shedding frequency, f, as the main peak of the frequency spectrum.

5.3. Estimate the characteristic Strouhal number of the oscillating wake, as Sr=f· D/Ub, and compare with the reference value from the literature at the same Reynolds number for the unbounded case. As a reference solution, the plot provided in the paper by Fei et al. (1998) might be used, as reported in Figure 4.

5.4. Investigate the sensitivity of the estimated Strouhal number with respect to the initial choice of the monitoring point. Overlap the frequency spectra for all the points in the domain and discuss the resulting picture (note that for some points the “fft_of_v0_Ub_velocity” function will return an error due to the excessive number of NaN values – just exclude those points). Analyze the colour plot distribution of the vortex shedding frequency.

5.5. Apply a temporal moving average to the instantaneous velocity field (u0_Ub, v0_Ub), considering an averaging window of about 0.2-0.25T, being T=1/f the vortex shedding period. It is suggested to make use of the MATLAB command “movmean” with the option “omitnan” to exclude NaN values from the calculation. Analyze the time evolution of the velocity magnitude field, the velocity vector field, the streamslices, and the vorticity field at different time instants during a period.

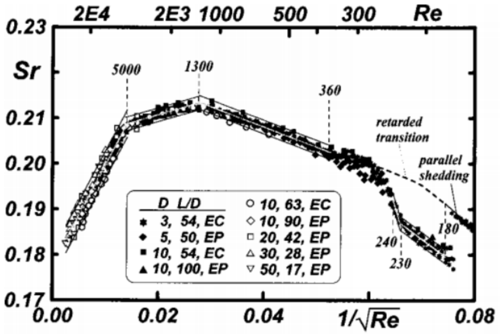

Figure 4. Strouhal number versus Reynolds number for unbounded flow over a circular cylinder, from Fey et al. [Physics of Fluids 10, 1547 (1998); doi.org/10.1063/1.869675].