Experimental laboratories, test case 3

Analysis of the hydrodynamic forces acting on a cylinder in steady free surface flow

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1 Introduction

We investigate the drag and lift forces acting on a cylinder submerged in steady-state free-surface water flow. The two force components, horizontal and vertical, have been measured through a balance. In this case, the Reynolds number of the cylinder $Re_D = D \cdot \frac{U_\infty}{\nu}$ is within the range of the *sub-critical regime*. Strictly speaking this classification of flow regimes refers to unconfined flows with uniform free-stream inlet which isn't the case here since the cylinder is located in a water channel with finite-size cross section. Still, it's reasonable to expect a behavior similar to that of the unconfined case, therefore it won't be surprising to notice that the flow separates at a certain distance from the front stagnation point causing a recirculation zone to form behind the cylinder and that a turbulent oscillating wake is created by the shedding of two counter-rotating vortexes. As a result, also drag and lift will show an oscillating behavior.

As for the experimental setup, the balance is fixed to the ground and connected to the cylinder through the *endplates*. The hydrodynamically shaped endplates allow to hold the cylinder in place and ensure some sort of "two-dimensional flow" around it by suppressing the three-dimensional effects at the cylinder ends. Two load cells are installed inside the balance and they provide an output voltage linearly proportional to the applied force. The calibration of the balance has been performed after the installation of the endplates and the cylinder on the balance, by applying weight standards to the cylinder without water in the channel. When developing the calibration function, it was taken into account that the real forces experienced by the cells are not only those produced by the applied load, but they also include the weight of the structure and other small contributions related with the deformability of the structure.

The calibration function has been determined in such a way that the condition $F_x = 0$, $F_y = 0$ corresponds to the absence of applied external forces, net of the weight and the small deformability-related contributions. When water flows in the channel, assuming that the dynamic forces on the endplates is negligible since they are hydrodynamically shaped, the horizontal external force F_x is equal to the drag force acting on the cylinder, F_D. Conversely, the vertical external force F_y is equal to the sum of the lift force acting on the cylinder F_L and the buoyancy force acting on the cylinder and the endplates, F_B. Thus, whereas F_D is simply taken as the calibration output F_x, F_L will be given as $F_y - F_B$. The buoyancy force, F_B, could be theoretically inferred by multiplying the volume of the immersed parts (cylinder and part of the endplates) by the specific weight of water. However, since knowing all the geometrical details with high accuracy is not trivial, directly measuring F_B with the balance is the preferred approach. This is achieved by making a test in still water with the same level of the flowing-water test. Since no drag and lift forces play a role in the static test, in this case the horizontal external force F_x will be zero and the vertical external force F_y will be equal to F_B.

Symbol	Parameter	Value	Unit
B	Width of the channel	0.5	m
D	Diameter of the cylinder	0.06	m
L	Width of the cylinder	0.185	m
t	Thickness of the endplates	0.002	m
b	Length of the endplates	0.18	m
h	Water level in the channel upstream of the cylinder	0.45	m
hb	Distance of the cylinder wall from the channel bottom	0.18	m
fs	Sampling frequency	200	Hz
Q	Volumetric flow rate of water	75	l/s
ρ	Density of water	998	kg/m^3
μ	Dynamic viscosity of water	0.001	Pa $\cdot$ s

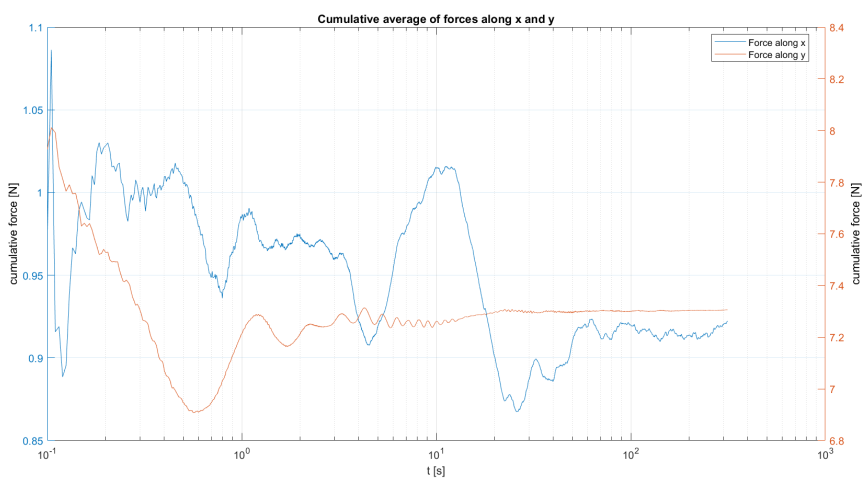
2 Analysis

1. We calculate the channel bulk velocity U_b based on quantities measured upstream of the cylinder, $U_b = \frac{Q}{B \cdot h} = 0.33333 \frac{m}{s}$. This velocity is assumed to be the free stream velocity approaching the cylinder, U_∞ so that the Reynolds number $Re_D = \frac{D \cdot U_\infty}{\nu}$ can be calculated. Note that $U_b \neq U_\infty$ since the experiment involves water flowing in a finite-size channel, nonetheless this is a reasonable approximation. The result indicates what, in an unconfined setting, would be the sub-critical regime.

$$5000 < Re_D = \frac{D \cdot U_\infty}{\nu} = 19960 < 3 \cdot 10^5 \tag{1}$$

2. We calculate the buoyancy force F_B acting on the immersed parts (cylinder and part of the endplates) as the average of F_y in the static case, this yields a value of $7.3236N$ which, coupled with a standard deviation of $0.24153N$ appears to be reasonable. We also take a look at the values of F_x in the static case to confirm that it is, on average, null. The mean value of F_x is $0.016718N$ with a standard deviation of 0.14359, again very reasonable.

3. We calculate the Reynolds-averaged drag and lift forces acting on the cylinder, $\langle F_D \rangle$ and $\langle F_L \rangle$, and the Reynolds averaged drag and lift coefficients, $\langle C_D \rangle$ and $\langle C_L \rangle$. Note that the drag and lift forces acting on the endplates are considered negligible. These Reynolds-averages are computed through a time average of the "instantaneous" quantities. Of course it's not guaranteed that the number of available frames is enough for the averaged variables to be representative of the Reynolds averaged flow. To test this hypothesis with a quantitative yet arbitrary metric, we define the *stable region* as the region where the cumulative average of a signal is **definitively** within a bound of $\pm 0.5\%$ of the final average. Then, for a signal to be *converged*, we require the length in seconds of this stable region to be bigger than or equal to 10 seconds. We use this test on the x- and y-components of the forces and, even if an oscillatory behavior is to be expected (especially in the y-component) due to the nature of the setup, still they both pass our test.



The results are $\langle F_D \rangle = 0.92234N$, $\langle F_L \rangle = -0.017207N$, $\langle C_D \rangle = 1.4987$ and $\langle C_L \rangle = -0.027959$. Recall that a reference value from the literature for C_D of a flow around a cylinder in the sub-critical regime is 1.22, this gives us some confidence in our results.

4. We're interested in investigating the uncertainties of C_D and C_L starting from the definition of these quantities and applying the error propagation law:

$$C_D = \frac{2 \cdot F_D}{\rho \cdot A_d \cdot U_\infty^2} \tag{2}$$

$$C_L = \frac{2 \cdot F_L}{\rho \cdot A_d \cdot U_\infty^2} \tag{3}$$

$$y = y(x1, x2, ..., xn) \rightarrow U(y) = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial y}{\partial x_i} \cdot U(x_i) \right)^2} \tag{4}$$

We make reference to the values of uncertainty in the following table.

Variable	Instrument	Uncertainty	Unit
B	Ruler	0.5	mm
D	Vernier caliber with scale value 0.05	0.05	mm
L	Ruler	0.5	mm
h	Piezometer	1	mm
Q	Proline Promag 50L installed in a 200 mm pipe (DN200)	0.51%	of reading
ρ	-	≈ 0	
F_x	From calibration of frame	0.045	N
F_y	From calibration of frame	0.025	N

$A_d = L \cdot D$ is the area of the projection of the cylinder in a plane perpendicular to the direction of the flow.

Note that the following formulas might be applied at the instantaneous level, that is, obtaining the uncertainty for every single value $C_D(t)$ and $C_L(t)$, in this case the uncertainties of the two coefficients will vary over time. However, it appears more practical to calculate directly the uncertainties of the Reynolds-averaged coefficient, $\langle C_D \rangle$ and $\langle C_L \rangle$, starting from $\langle F_D \rangle$ and $\langle F_L \rangle$. The uncertainties for F_x and F_y do reasonably hold also for their Reynolds-averaged values, $\langle F_x \rangle$ and $\langle F_y \rangle$.

$$U(U_\infty) = \left[\left(\frac{U(Q)}{B \cdot h} \right)^2 + \left(-\frac{h \cdot Q \cdot U(B)}{(B \cdot h)^2} \right)^2 + \left(-\frac{B \cdot Q \cdot U(h)}{(B \cdot h)^2} \right)^2 \right]^{\frac{1}{2}} \approx 1.889 \cdot 10^{-3}$$

$$U(U_\infty^2) = 2 \cdot U_\infty \cdot U(U_\infty) \approx 1.26 \cdot 10^{-3}$$

$$U(A_d) = \left[(D \cdot U(L))^2 + (L \cdot U(D))^2 \right]^{\frac{1}{2}} \approx 3.14 \cdot 10^{-5}$$

$$U(F_D) \approx U(F_x)$$

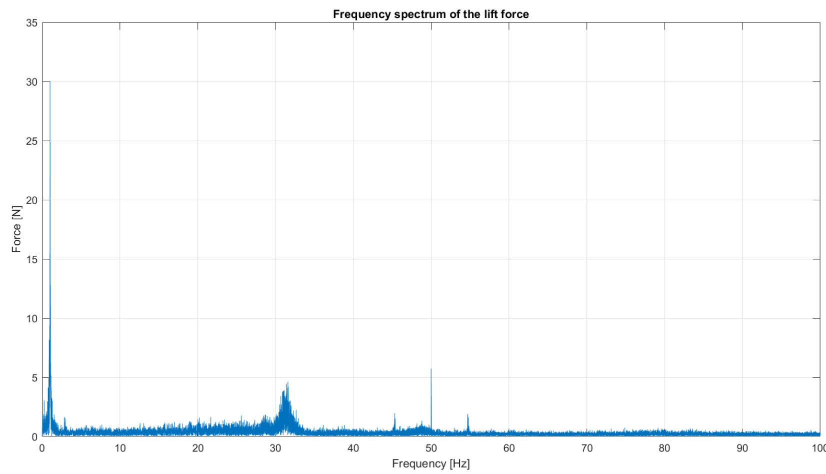
$$U(F_B) \approx U(F_y) \text{ in the still-water case}$$

$$U(F_L) = \left[(F_B \cdot U(F_y))^2 + (F_y \cdot U(F_B))^2 \right]^{\frac{1}{2}} \approx 0.2586$$

$$U(C_D) = \left[\left(\frac{2 \cdot U(F_D)}{\rho \cdot A_d \cdot U_\infty^2} \right)^2 + \left(\frac{2 \cdot \rho \cdot U_\infty^2 \cdot F_D \cdot U(A_d)}{(\rho \cdot A_d \cdot U_\infty^2)^2} \right)^2 + \left(\frac{2 \cdot \rho \cdot A_d \cdot F_D \cdot U(U_\infty^2)}{(\rho \cdot A_d \cdot U_\infty^2)^2} \right)^2 \right]^{\frac{1}{2}} \approx 0.07488$$

$$U(C_L) = \left[\left(\frac{2 \cdot U(F_L)}{\rho \cdot A_d \cdot U_\infty^2} \right)^2 + \left(\frac{2 \cdot \rho \cdot U_\infty^2 \cdot F_L \cdot U(A_d)}{(\rho \cdot A_d \cdot U_\infty^2)^2} \right)^2 + \left(\frac{2 \cdot \rho \cdot A_d \cdot F_L \cdot U(U_\infty^2)}{(\rho \cdot A_d \cdot U_\infty^2)^2} \right)^2 \right]^{\frac{1}{2}} \approx 0.42$$

5. We compute the *Fast Fourier Transfom* (FFT) of the F_L signal and find out the frequency spectrum. From this it's possible to identify the vortex shedding frequency, f, as the main peak of the frequency spectrum, this yields $f = 1.008Hz$. An estimate of the characteristic Strouhal number can be computed as $Sr = f \cdot \frac{D}{U_b} = 0.18144$ and compared with the reference value from the literature at the same Reynolds number for the unbounded case. As a reference solution, we use the plot provided in the paper by Fei et al. (1998) which yields $Sr_l = 0.195$. The quality of fit - good but not perfect - testifies to the similarities and the differences between the two experimental setups: on the one hand Reynolds numbers are similar, but on the other we are testing a confined flow whereas the cited paper studies the flow of air in a wind tunnel where the effect of the walls is negligible.



6. Finally it is necessary to ensure that the vortex shedding frequency is significantly different from the natural frequencies of the system. Otherwise, the structure could vibrate under the fluid loading, making the assumption of fixed body fail. In order to measure the natural frequencies of the structure in water, the cylinder is hit along the x-direction once using a stick, and left free to vibrate in still water. Based on the results of this test we calculate the natural frequencies of the balance structure in still water which are 7.5961 along y and 7.6586 along x, verifying that there's no risk of resonance-induced vibration.