hp-adaptive pitch

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Hello everyone, I'm Pietro Fumagalli and with my colleague Francesco Derme we will present our project: hp-adaptive strategies for multi-physics simulations. So let's dive into it. First of all, what's hp-adaptivity? Let's consider a simple case, namely the Laplacian on a polygonal mesh. Our goal is to approximate the solution of a pde through the use of discontinuous Galerkin FEM method: this gives us the right to modify h (the diameter of the element) and p (the polynomial degree of the approximant on the element), and we can do this to the different elements independently. In particular the idea is to construct the mesh as usual and the refine it, so take a really small h and a small polynomial degree p in the element where the solution is more problematic and a big h and an high polynomial degree where the solution is more constant. Now let's complicate it: we will consider a general time dependent heat equation with the addition of a non linear reaction term, with the goal of using the same technique. This of course comes with a lot of difficulties: in particular we have to understand where the equation is at a given time t, through the definition of some indicators, assign a particular value of this parameter to all the mesh elements and then through a cluster algorithm understand if the equation "is" in a given element or not, and this has to be done a priori for every time step. Once we understand where the solution is we need to refine the mesh in those particular elements: also this comes with a lot of challenges, how can we do it, how we can choose the perfect h and p? There is some optimal shape in which we can refine the element, in order to combine a light computational approach with good accuracy on the computation of the solution? And after we understand how to refine the mesh, how we can de-refine it once the solution is gone at the successive time steps? Of course keeping memory of how it was the mesh before refining it is too memory consuming, so how to deconstruct it? Now take all of this and consider it into a 3d mesh: here it's where our direct application take place. The idea is to construct a code that apply this techniques to waves equations that models the shock in the brain of an individual during an epileptic seizure. So this methods seems te best possible that can combine a reliable and computational friendly approach to a very difficult problem onto a complicated mesh.